PROGRESS WITH VERIFICATION AND STABILIZATION OF REYNOLDS STRESS MODELS USING THE CFD SOFTWARE BY ONERA, DLR, AIRBUS (CODA)

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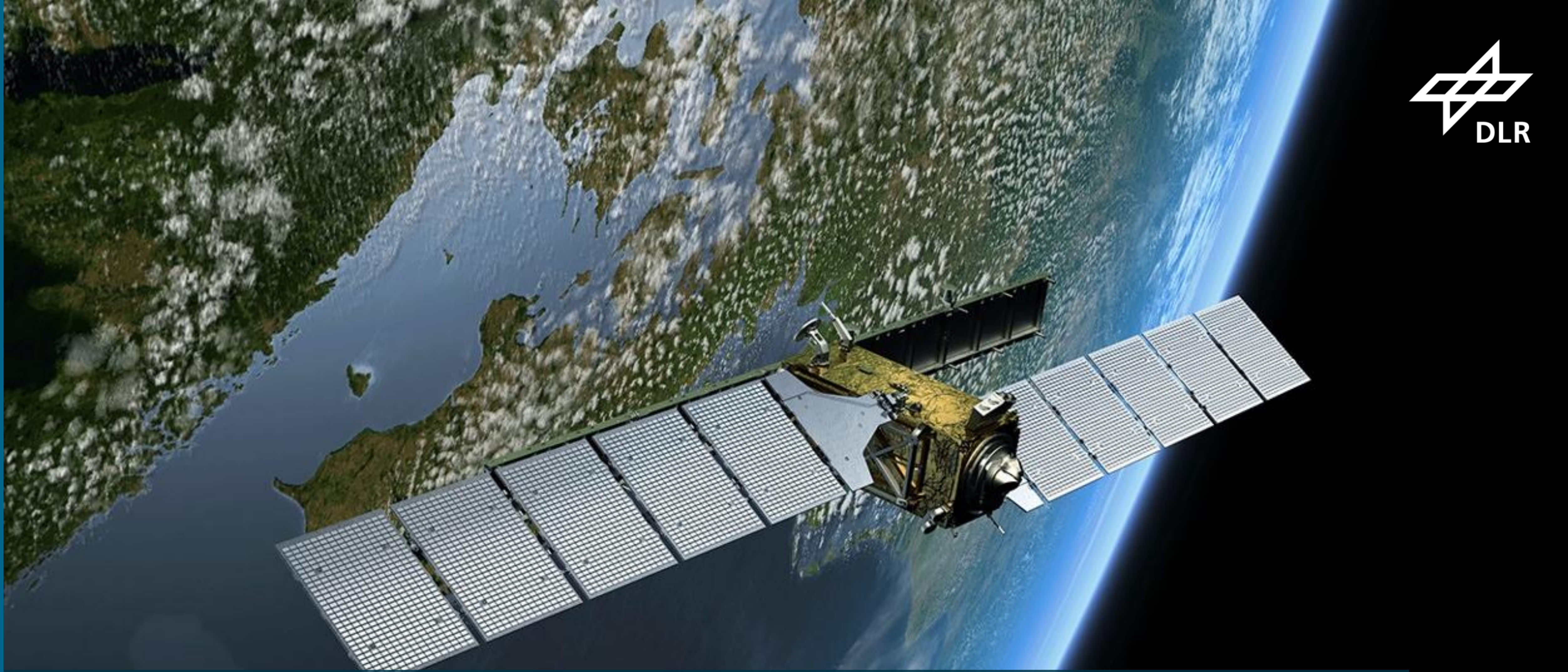
CLEAN AVIATION



INTRODUCTION

Reynolds stress models - RSM
SSG/LRR-ln(ω) model

- What are RSMs?
  - A family of RANS models providing transport equations for all the components of the symmetric Reynolds stress tensor.
  - **6 transport equations** for the Reynolds stresses (R_{ij}) and **1 additional equation** for the length scale (ω)
- When do we need RSMs?
  - **Accurate representation** of **anisotropic turbulence** and **complex flow phenomenon** like 3D separation, vortex dominated flows, etc.

Reynolds stresses equations

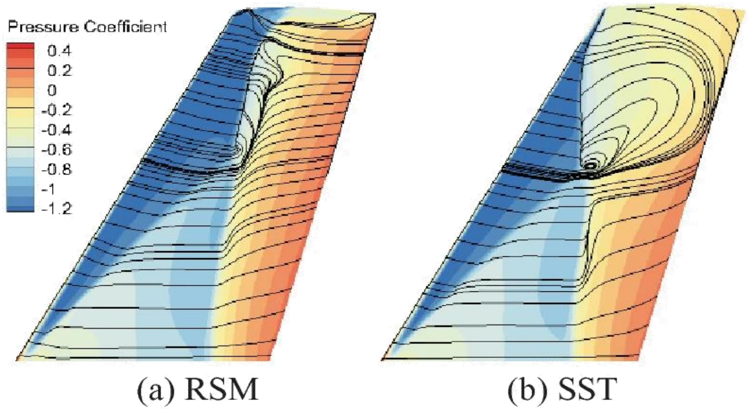
$$\frac{\partial(\bar{\rho}\widehat{R}_{ij})}{\partial t} + \frac{\partial(\bar{\rho}U_k\widehat{R}_{ij})}{\partial x_k} = \bar{\rho}P_{ij} + \bar{\rho}\Pi_{ij} - \bar{\rho}\varepsilon_{ij}$$

Where, P_{ij} → Production, Π_{ij} → Pressure-strain correlation, ε_{ij} → Dissipation tensor

Length scale equation

$$\frac{\partial(\bar{\rho}\hat{\omega})}{\partial t} + \frac{\partial(\bar{\rho}U_k\hat{\omega})}{\partial x_k} = \frac{\alpha_{\hat{\omega}}}{k}\frac{\bar{\rho}P_{kk}}{2} - \beta_{\hat{\omega}}\bar{\rho}e^{\hat{\omega}} + \frac{\partial}{\partial x_k}\left[\left(\bar{\mu} + \sigma_{\hat{\omega}}\frac{\bar{\rho}k}{e^{\hat{\omega}}}\right)\frac{\partial\hat{\omega}}{\partial x_k}\right] + \sigma_d\frac{\bar{\rho}}{e^{\hat{\omega}}}\max\left(\frac{\partial k}{\partial x_k}\frac{\partial\hat{\omega}}{\partial x_k}, 0\right) + \left(\bar{\mu} + \sigma_{\hat{\omega}}\frac{\bar{\rho}k}{e^{\hat{\omega}}}\right)\frac{\partial\hat{\omega}}{\partial x_k}\frac{\partial\hat{\omega}}{\partial x_k}$$

$\hat{\omega} = \ln\omega$, hence, $\omega = e^{\hat{\omega}}$



Upper-surface flow and C_p distribution comparison of different models for ONERA M6 wing at $Ma = 0.84$, $Re_{MAC} = 11.72 \times 10^6$, $\alpha = 6.06°$.

On the Correction of k-ω SST Turbulence Model to Three-Dimensional Shock Separated Flow

Challenges with RSMs

SSG/LRR-ln(ω) model

$$\frac{\partial(\bar{\rho}\widehat{R_{ij}})}{\partial t} + \frac{\partial(\bar{\rho}U_k\widehat{R_{ij}})}{\partial x_k} = \bar{\rho}P_{ij} + \bar{\rho}\Pi_{ij} - \bar{\rho}\varepsilon_{ij}$$

- **High non-linearity → dense Jacobian**
  - Mean flow (S_{ij}, W_{ij}) → Reynolds stresses
  - Reynolds stresses $(\partial R_{ij}/\partial x_j)$ → Mean flow
  - Reynolds stresses (P_{ij}) → Length scale equation
  - Length scale equation (ε) → Reynolds stresses
  - Reynolds stresses (All source terms, mainly Π_{ij}) ↔ Reynolds stresses
- **Different time scales → Ill conditioning** of Jacobian
  - $\frac{T_c}{T_S} \sim 10^3\ to\ 10^6$, i.e. $T_c \gg T_S$
- RSM vs Eddy Viscosity Models (SA or SST)
  - Less robust
  - More iterations
  - More memory

Production term

$$\bar{\rho}P_{ij} = -\bar{\rho}\widehat{R_{ik}}\frac{\partial\widehat{u_j}}{\partial x_k} - \bar{\rho}\widehat{R_{ik}}\frac{\partial\widehat{u_j}}{\partial x_k}$$

Dissipation term

$$\bar{\rho}\varepsilon_{ij} = \frac{2}{3}\bar{\rho}\varepsilon\delta_{ij}, \quad \varepsilon = C_\mu\hat{k}\omega, \quad \hat{k} = \frac{\widehat{R_{ii}}}{2}$$

Pressure-strain correlation term

$$\widehat{a_{ij}} = \frac{\widehat{R_{ij}}}{\hat{k}} - \frac{2}{3}\delta_{ij}$$

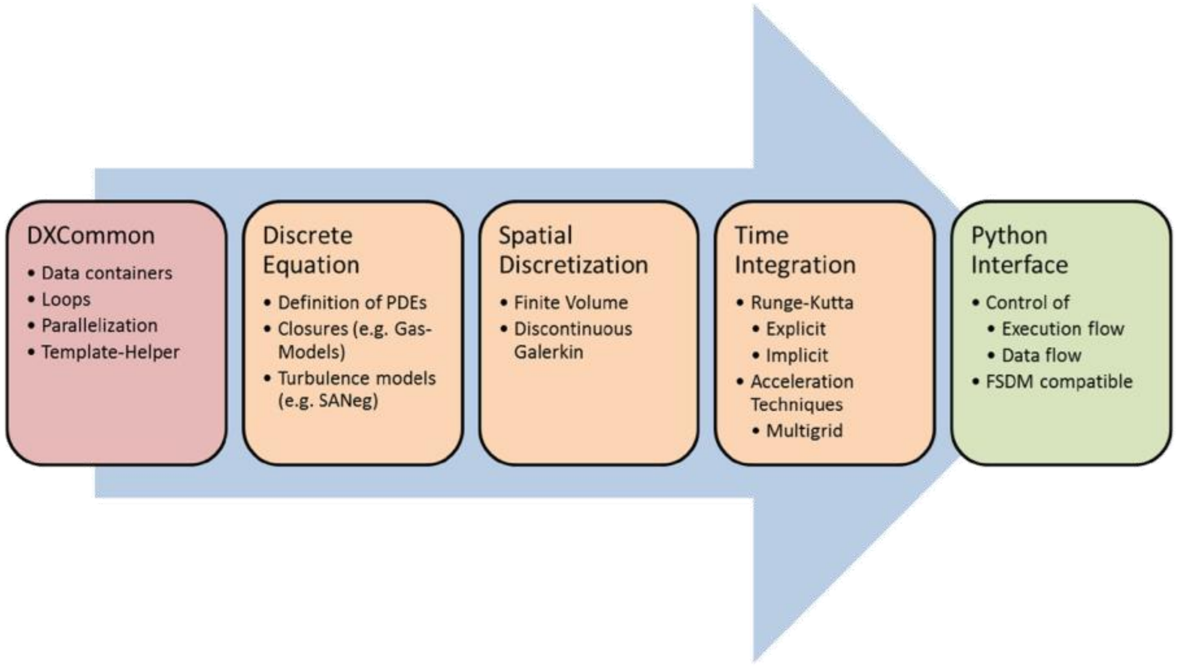
$$\begin{aligned}\bar{\rho}\Pi_{ij} = &-\left(C_1\bar{\rho}\varepsilon + \frac{1}{2}C_1^*\bar{\rho}P_{kk}\right)\widehat{a_{ij}} \\ &+ C_2\bar{\rho}\varepsilon\left(\hat{a}_{ik}\hat{a}_{kj} - \frac{1}{3}\widehat{a_{kl}}\widehat{a_{kl}}\delta_{ij}\right) \\ &+ \left(C_3 - C_3^*\sqrt{\widehat{a_{kl}}\widehat{a_{kl}}}\right)\bar{\rho}\hat{k}\widehat{S_{ij}^*} \\ &+ C_4\bar{\rho}\hat{k}\left(\widehat{a_{ik}}\widehat{S_{jk}} + \widehat{a_{jk}}\widehat{S_{ik}} - \frac{2}{3}\widehat{a_{kl}}\widehat{S_{kl}}\delta_{ij}\right) \\ &+ C_5\bar{\rho}\hat{k}\left(\widehat{a_{ik}}\widehat{W_{jk}} + \widehat{a_{jk}}\widehat{W_{ik}}\right)\end{aligned}$$

Generalized Gradient Diffusion

$$\bar{\rho}D_{ij} = \frac{\partial}{\partial x_k}\left[\left(\bar{\mu}\delta_{kl} + D\frac{\bar{\rho}\widehat{R_{kl}}}{C_\mu\omega}\right)\frac{\partial\widehat{R_{kl}}}{\partial x_l}\right]$$

CFD software by ONERA, Airbus and DLR (CODA)

- CODA is the CFD software being developed as part of a collaboration between the French Aerospace Lab (ONERA), the German Aerospace Center (DLR), Airbus, and their European research partners. CODA is jointly owned by ONERA, DLR and Airbus.
- Features of CODA:
  - ***Cell-centered unstructured fully coupled*** solver
  - Common framework for both second order ***Finite volume*** (FV) and ***Discrete Galerkin*** (DG) discretizations
  - ***Fully implicit*** time integration (using Newton method)
  - ***Algorthmic differentiation*** (AD) for calculating derivatives
  - Modular design to facilitate ***highly parallel*** simulations
  - Object oriented design using ***C++17*** with ***Python API*** to enable coupling to ***FlowSimulator*** framework
- The main aim of the work is to have a robust implementation of the SSG/LRR-ln(ω) RSM model in CODA.



Modular structure of the next generation flow solver (previously) Flucs (now CODA) software

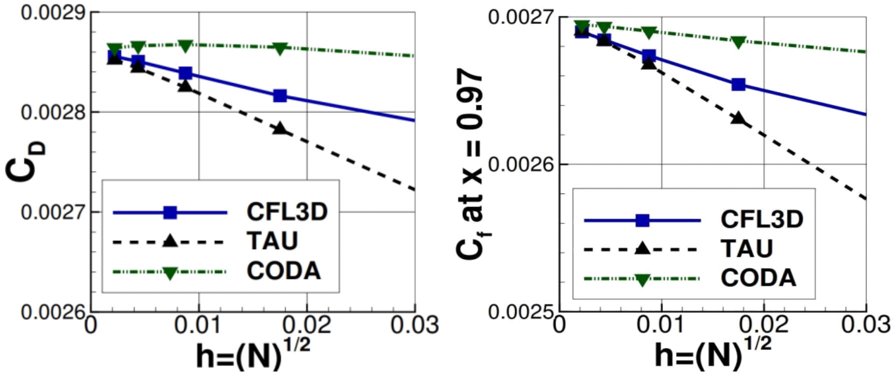
DLR project Digital-X: towards virtual aircraft design and flight testing based on high-fidelity methods

REYNOLDS STRESS MODELS – INITIAL IMPLEMENTATION & PRELIMINARY VERIFICATION

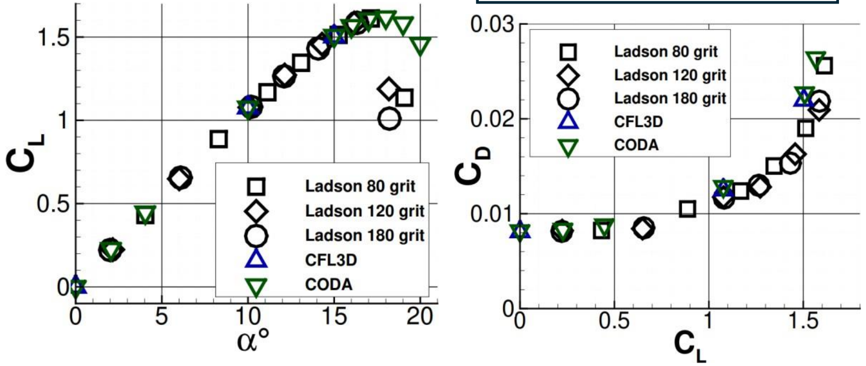
Preliminary verification & validation (V&V) of 2D cases

- **Zero pressure gradient flat plate (2DZP):** Very good convergence for all meshes with dependence on CFL ramping
- **Bump in channel (2DB):** Convergence was possible only with simple diffusion (SD) and GGD failed due to realizability violation.
- **RAE2822 – Case 9:** A two stage convergence with first order in the first stage and second order in mean flow during the second stage.
- **NACA0012 – Full Polar:** The simulations converged for the full polar until maximum lift with a two stage approach as well.
- **NACA4412 – Trailing edge separation:** A two stage approach worked well for this case as well.
- The meshes for all the below cases were taken from NASA TMR (except for NACA4412 & RAE2822)

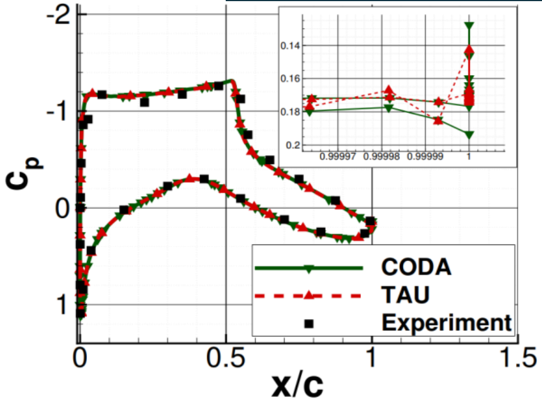
2DZP mesh convergence



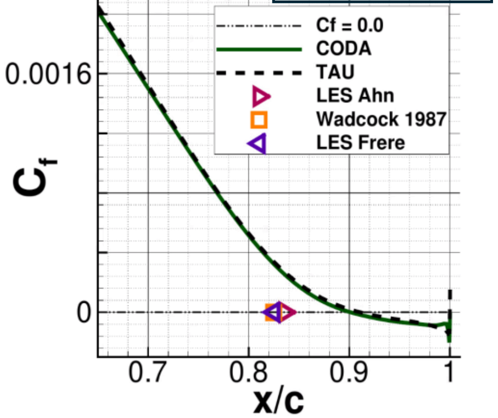
Polar for NACA0012



RAE2822 – Case 9



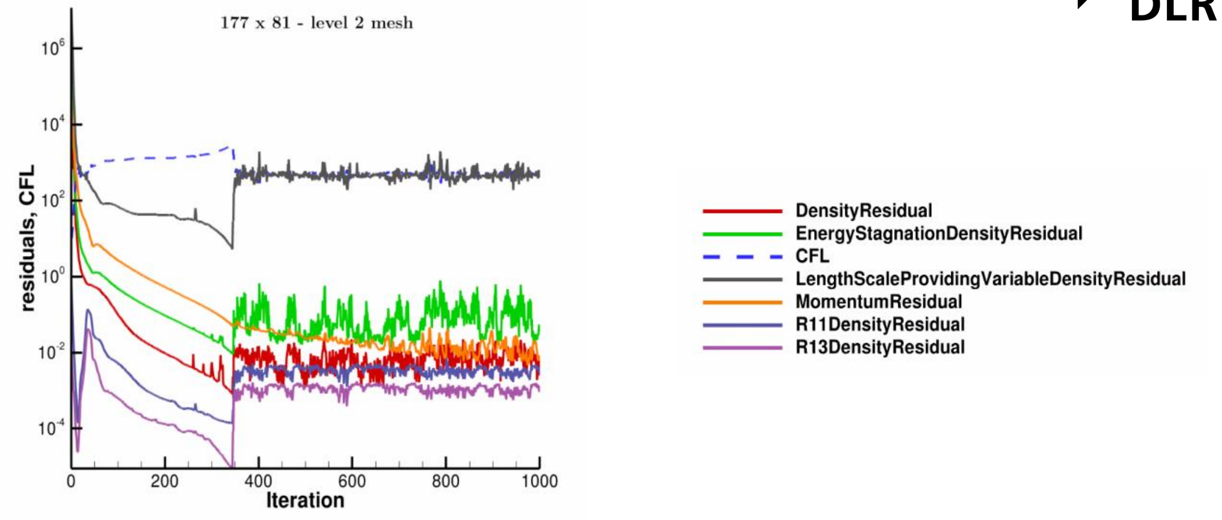
NACA4412



Challenges from initial implementation

- Robustness issues leading to realizability constraints (2DB case)
- Failure of traditional clipping due to AD (differentiability of Reynolds stresses in Jacobian)
- Mesh & parameter studies (NACA4412) – factors affecting convergence:
  - Mesh & Case/Configuration
  - Realizability constraints
  - Gradient reconstruction
  - CFL setting
  - Entropy fix
  - Convection scheme
- Moving to 3D cases like ONERA M6 lead to even more challenges – failure of convergence without realizability constraints.

Spurious residual behaviour in 2DB after implementing realizability constraints



Mesh & Parameter study for NACA4412

Ma = 0.085, Re = 1.64e6, AoA = 10°, Realizability OFF in Last Stage	Mesh_yp1_blunt_TE	NASA_HexMesh_897x257	Re_1-6Mio_wake_panel_yp1_v1b	Re_1-6Mio_yp1_blunt_TE_v1c_fine
run_1c_composite-scheme_grad-recon-stdGG_Efix-0.2-0.0			Divergence in 2nd Stage (very high residuals, 10^10 to 10^40)	Residual stall
run_1c1_composite-scheme_grad-recon-stdGG_Efix-0.2-0.1-0.0		Changing second stage Efix to 0.2 and increasing number of iterations lead to	Inf in 2nd Stage	Residual stall
run_1c2_composite-scheme_grad-recon-stdGG_Efix-0.2-0.1-0.05		Changing second stage Efix to 0.2 and increasing number of iterations lead to	Inf in 2nd Stage	Residual stall
run_2c_composite-scheme_grad-recon-extGG_Efix-0.2-0.0			Divergence in 2nd Stage(very high residuals, 10^10 to 10^40)	Residual stall
run_2c1_composite-scheme_grad-recon-extGG_Efix-0.2-0.1-0.0		Residual stall - changing Stage 2 Efix to 0.2 and Stage 3 SER exp to 0.4, integral		Residual stall
run_2c2_composite-scheme_grad-recon-extGG_Efix-0.2-0.1-0.05		Residual stall - changing Stage 2 Efix to 0.2 and Stage 3 SER exp to 0.4, integral	Divergence in 3rd Stage(very high residuals, 10^10 to 10^40)	Residual stall
run_7c_composite-scheme_grad-recon-LSQ_Efix-0.2-0.0			Divergence in 2nd Stage (very high residuals, 10^10 to 10^40)	Residual stall
run_7c1_composite-scheme_grad-recon-LSQ_Efix-0.2-0.1-0.0				Residual stall
run_7c2_composite-scheme_grad-recon-LSQ_Efix-0.2-0.1-0.05				Residual stall

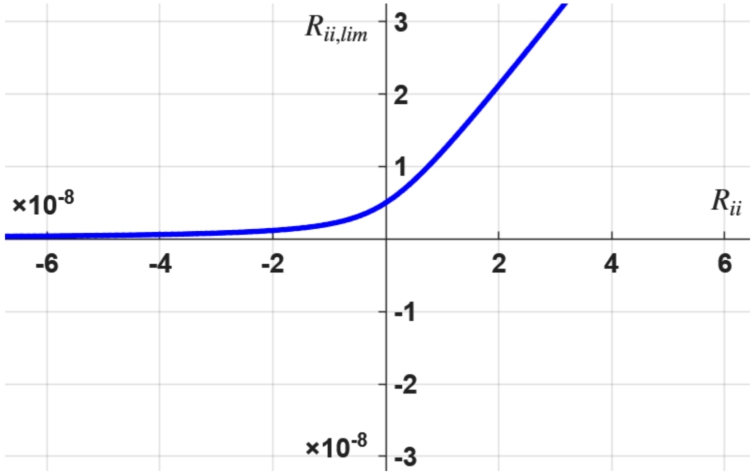
Realizability constraints

- Non-physical turbulence can arise due to turbulence modelling and discretization errors affecting convergence - Schumann
- **Realizability constraints**: Reynolds stress tensor should be Symmetric Positive Semi Definite (SPSD) tensor throughout the simulation.
  - $R_{ij} \geq 0, \ \forall \ i = j$
  - $R_{ij} \ \leq \ \sqrt{R_{ii} R_{jj}}, \ \forall \ i \ \neq j$
- Several attempts were made to implement the realizability constraints
  - Pure **hard clipping** directly at the source terms: Failed as it is not AD differentiable.
  - Use of **non-linear positivity filter** with eigen value decomposition of Reynolds stress tensor: Eigen decomposition becomes expensive as more cells violate differentiability.
  - **Hard clipping** in the python layer (for testing): Simulations stalled even the ones which may converge otherwise.
  - **Realizability preserving time stepping** (through implicit/explicit linearization of source terms): Worked only for basic test cases, further investigation needed.
  - **Smoothed clipping** of Reynolds stress variables in closure: The **most possible way** which worked for several 2D/3D cases like NACA4412, coarse meshes of ONERA M6 and could be integrated easily to the CODA architecture.

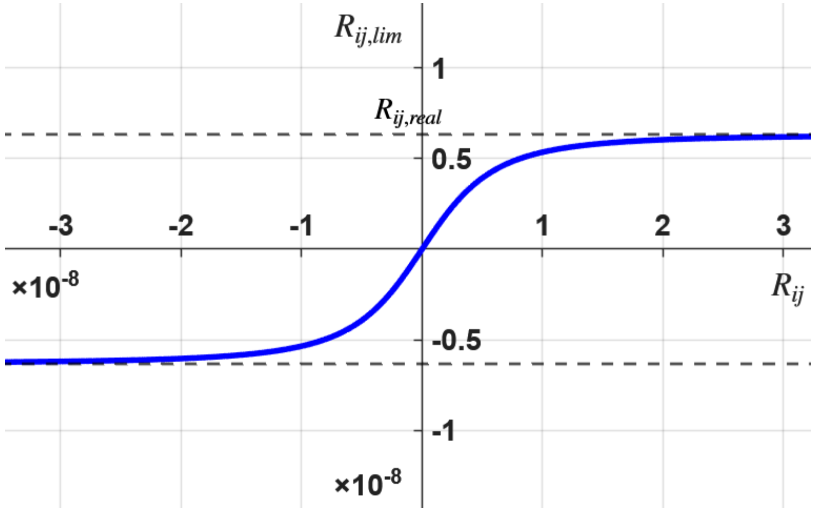
Smooth clipping for realizability constraints

For normal stress, R_{ii} :

- $R_{ii,min} \rightarrow$ **Minimum normal stress**
- $R_{ii,lim} = 0.5\,(R_{ii} + \sqrt{R_{ii}^2 + R_{ii,min}^2})$



A plot for $R_{ii,lim}$ when $R_{ii,min} = 1 \times 10^{-8}$



A plot for $R_{ij,lim}$ when $R_{ii,lim} = 1 \times 10^{-8}, R_{jj,lim} = 4 \times 10^{-9}$

For shear stress, R_{ij}:

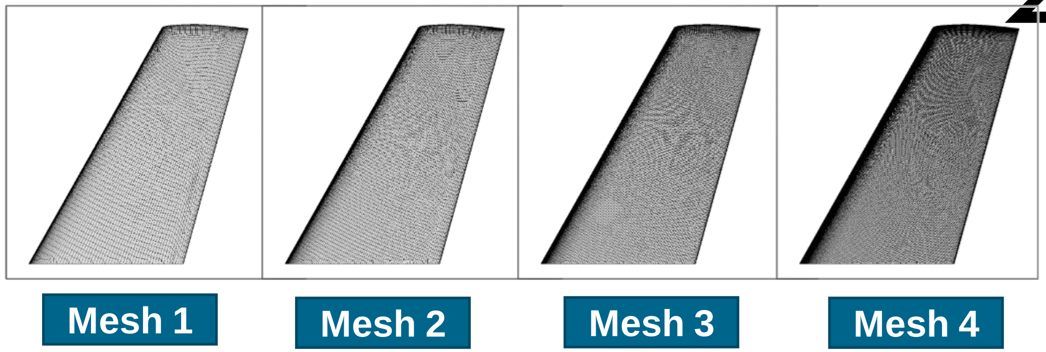
$$R_{ij,real} = \sqrt{R_{ii,lim}\, R_{jj,lim}}$$

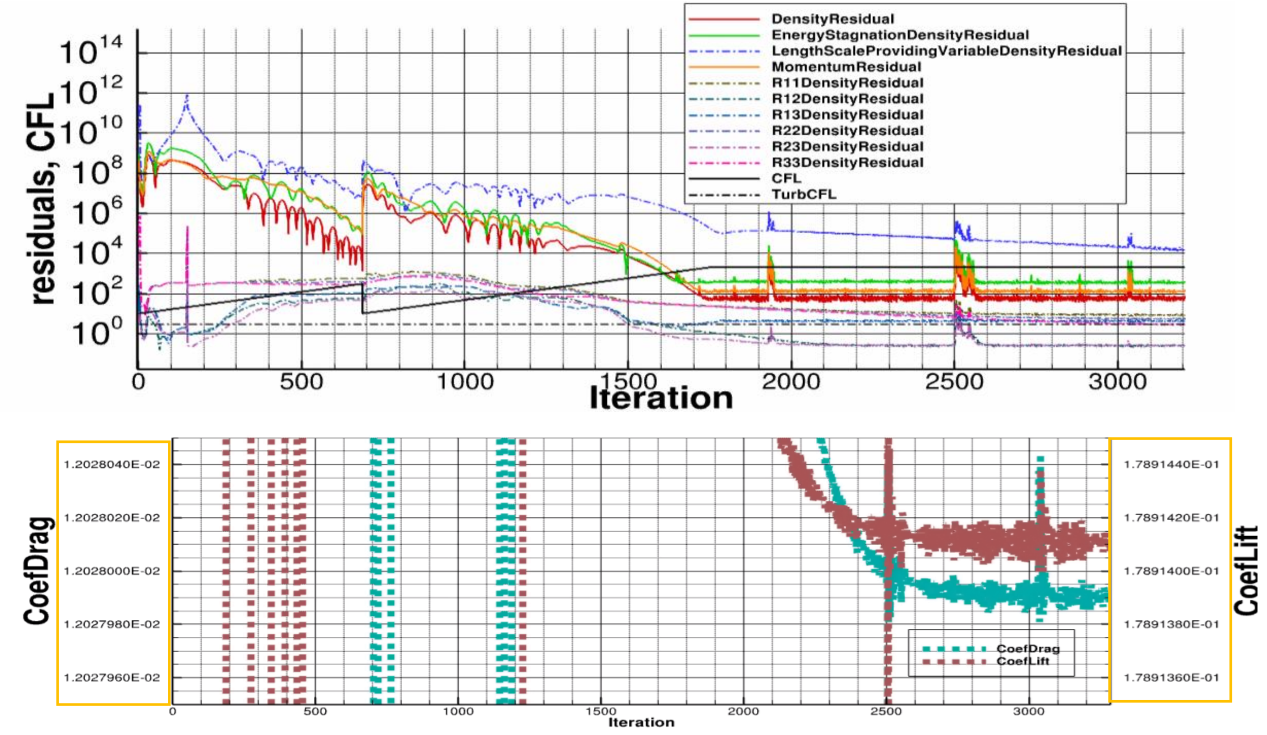
$$R_{ij,lim} = \frac{R_{ij}}{\sqrt{1 + \left(\frac{R_{ij}}{R_{ij,real}}\right)^2}}$$

VERIFICATION OF 3D CASES AND WEAK COUPLING OF EQUATIONS

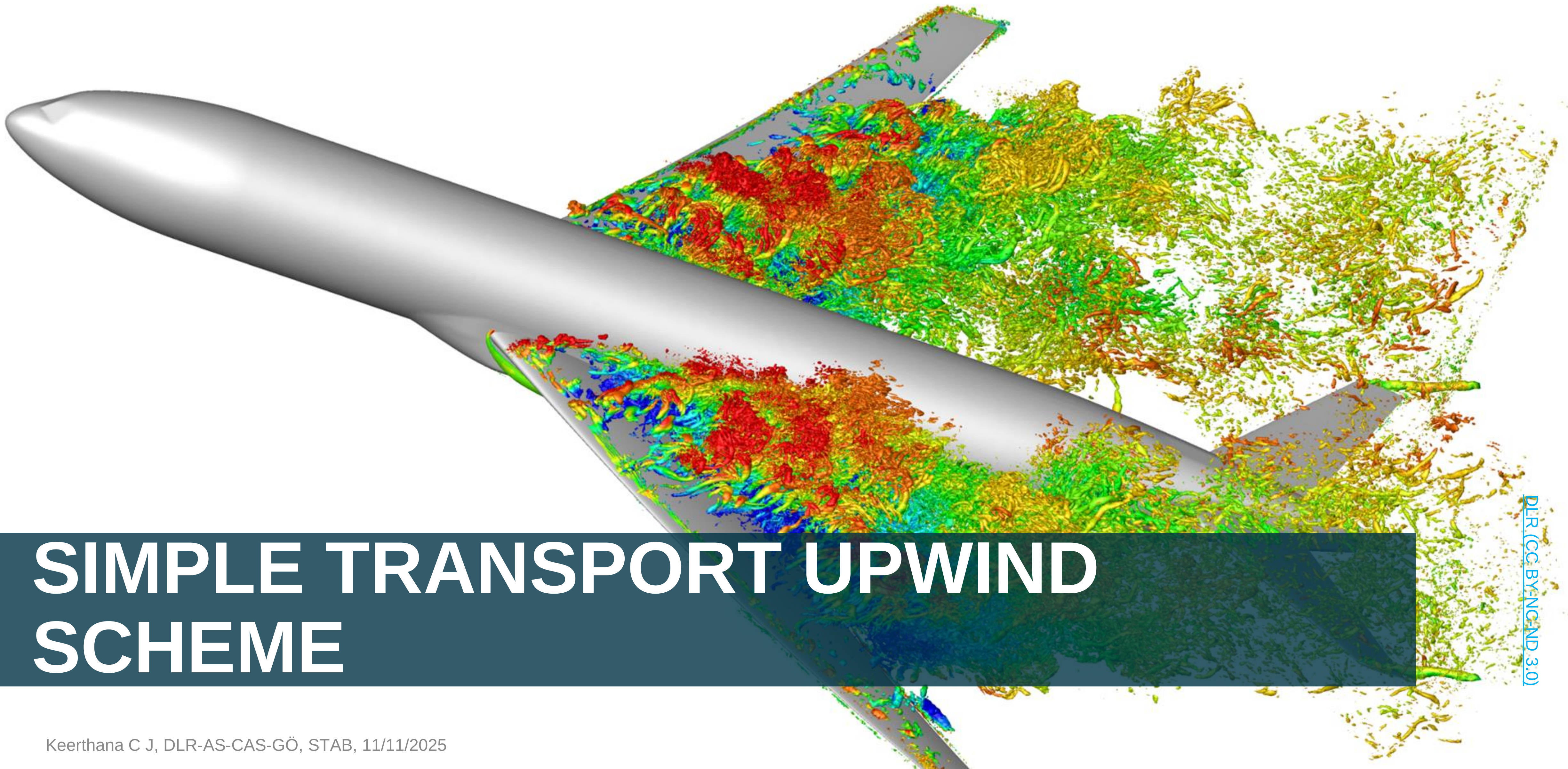
Verification of 3D cases
ONERA M6

- Centaur meshes
- **Flow conditions:** $Ma = 0.84,\ \alpha = 3.06°, Re = 14.6 \times 10^6, L_{Re} = 1.$
- Challenging to converge even with realizability constraints in place.
- Converged only with **LLF Upwinding** scheme for turbulence or with a **weak coupling** of the mean flow and turbulent equations.
- However, weak coupling approach for the equations did not always improve convergence.





Convergence plots for Mesh 1 with weak coupling



SIMPLE TRANSPORT UPWIND SCHEME

Simple upwinding scheme for turbulence equations

Previous Roe scheme for transport eqns. in CODA

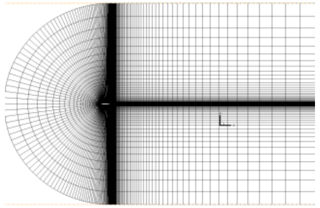
$$flux = \frac{1}{2}\left[\left(v_L(\rho\varphi)_L + v_R(\rho\varphi)_R\right) - \left|\left(\frac{(\rho v)_L + (\rho v)_R}{\rho_L + \rho_R}\right)\right|_{max}\left((\rho\varphi)_R - (\rho\varphi)_L\right)\right]$$

Simple upwinding (from DLR-TAU)

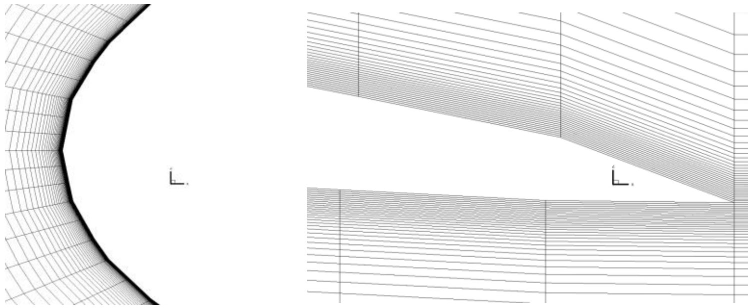
$$flux = \frac{1}{2}\left[(v_L + v_R)(\rho_L\varphi_L + \rho_R\varphi_R) - |(v_L + v_R)|(\rho_R\varphi_R - \rho_L\varphi_L)\right]$$

L, R → left, right fluxes, v → convective velocity, φ → turbulent variable

RAE2822

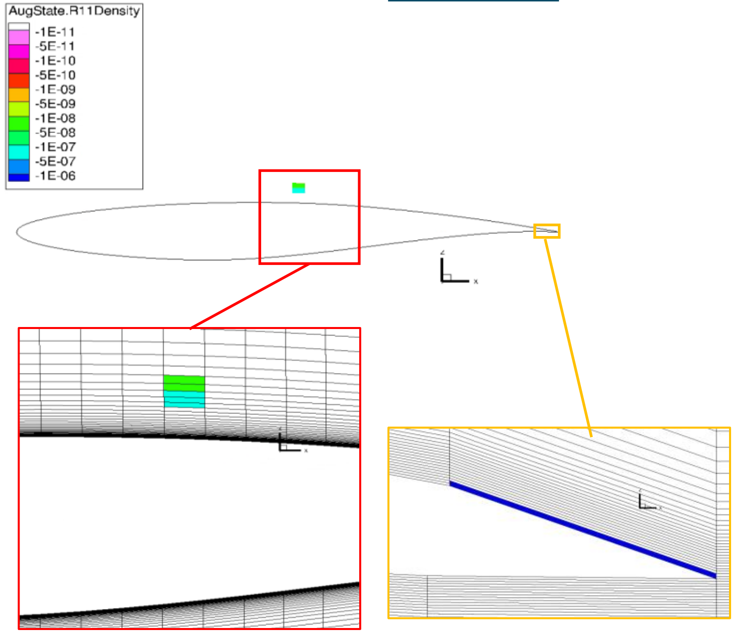


Thanks to Axel Probst for the idea! ☺

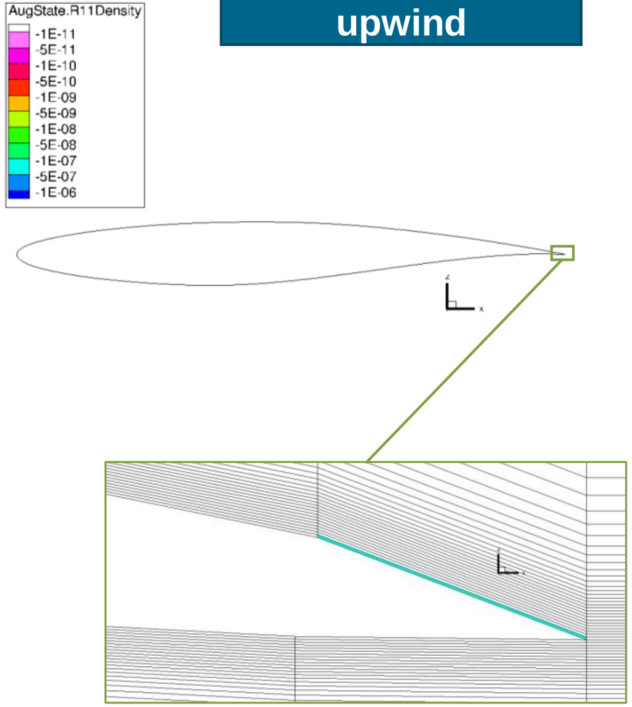


Not a perfect RAE2822 mesh!

Roe



Simple transport upwind



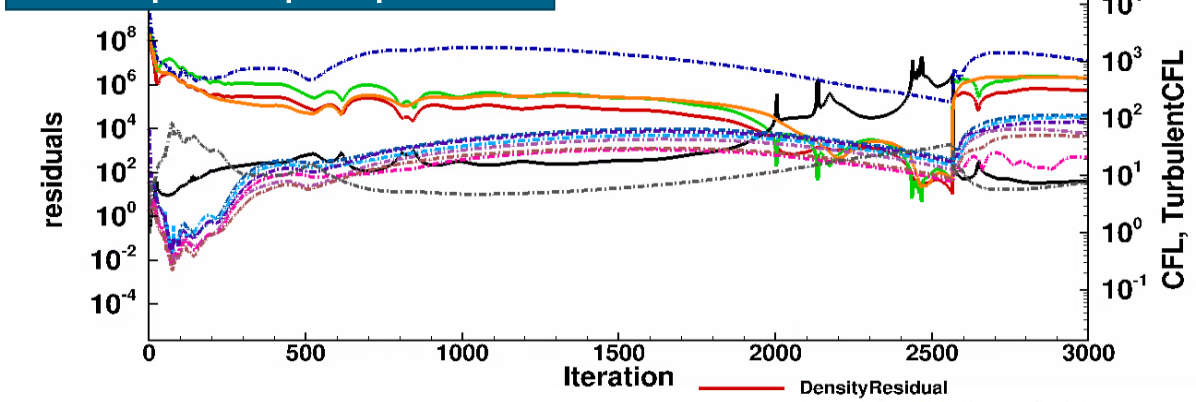
Realizability violations in the shock region

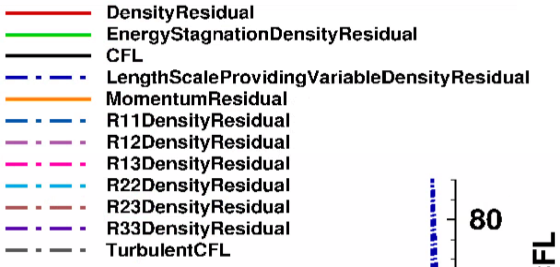
Simple transport upwind scheme

- 2D Bump in Channel converged for the first time with coupled solver approach.
- All the meshes of ONERA M6 converged for the coupled solver and also for second order (except finest mesh)!
- Turbulent CFL numbers can be higher (x2 or x3) compared to Roe scheme.

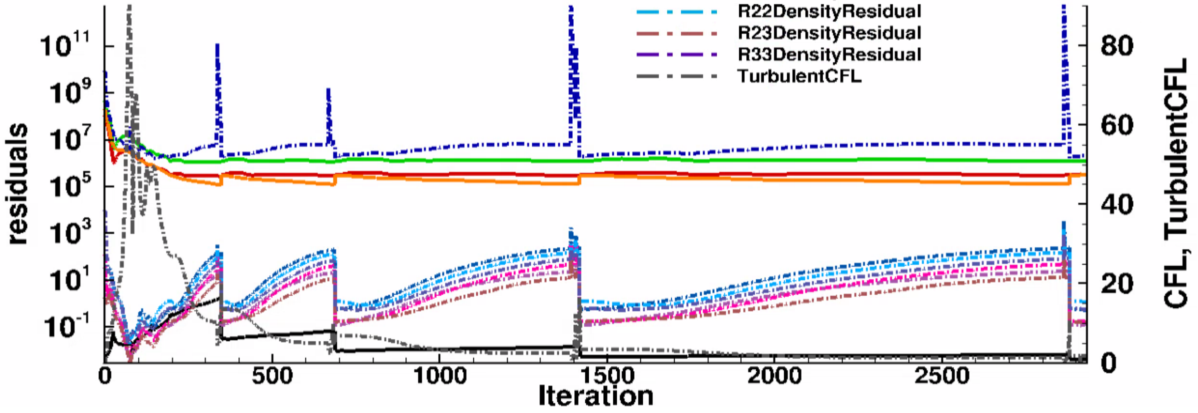
ONERA M6 – Mesh 3

Simple transport upwind





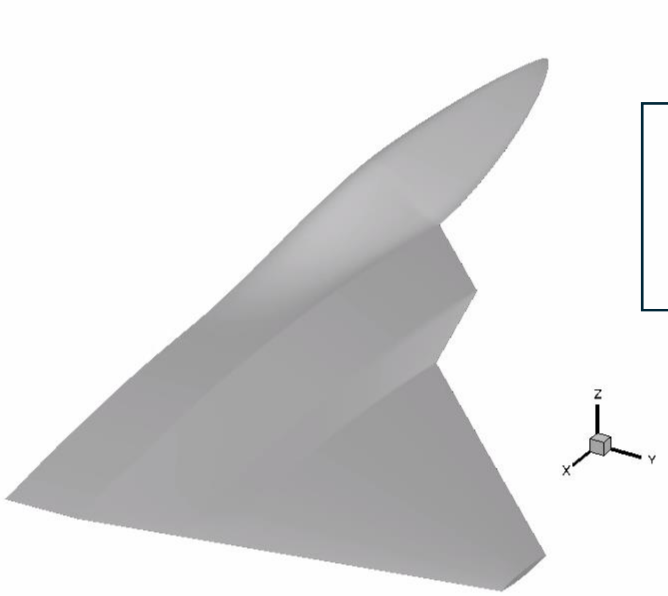
Roe



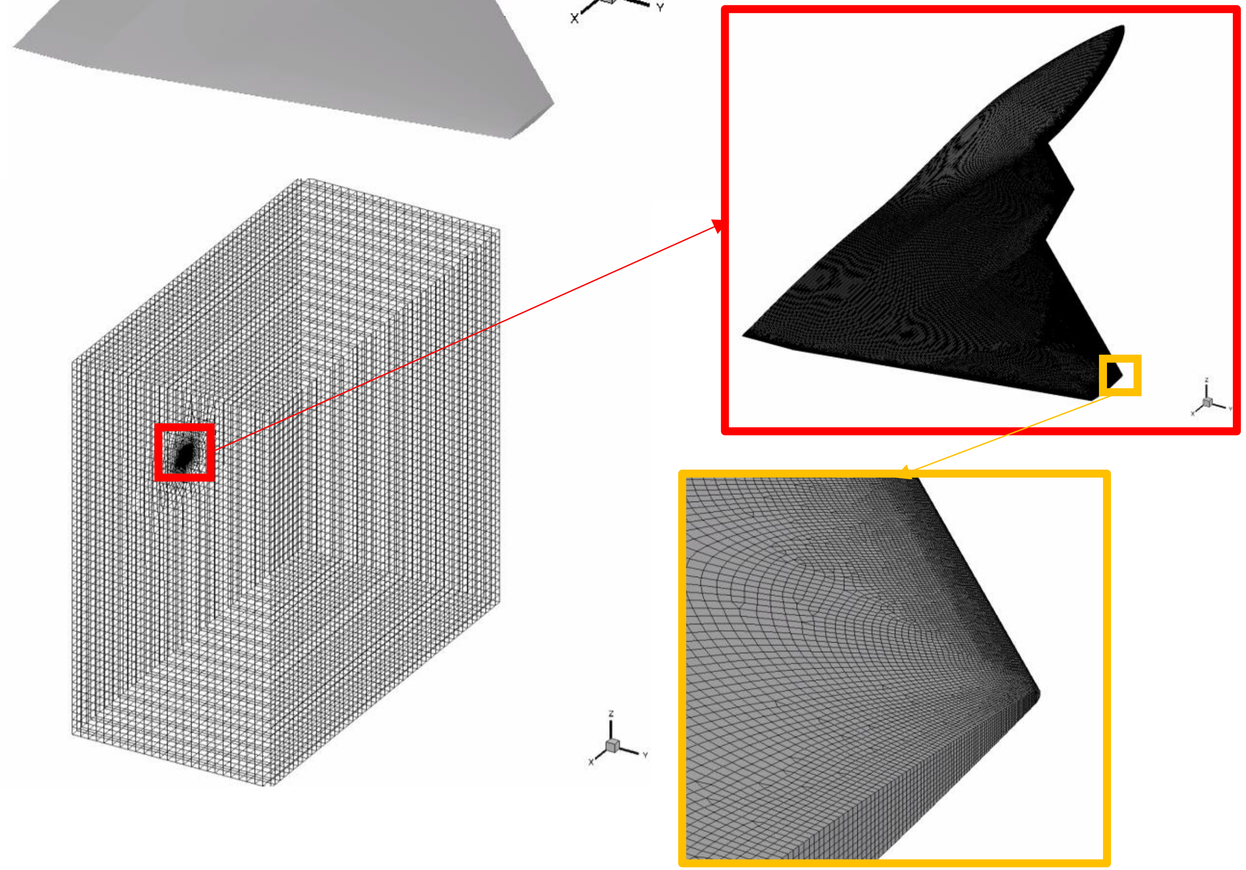
Thanks to Deepak Kunhappan for the combined framework of Simple Upwind scheme ☺

Verification of 3D cases
DLR – F23

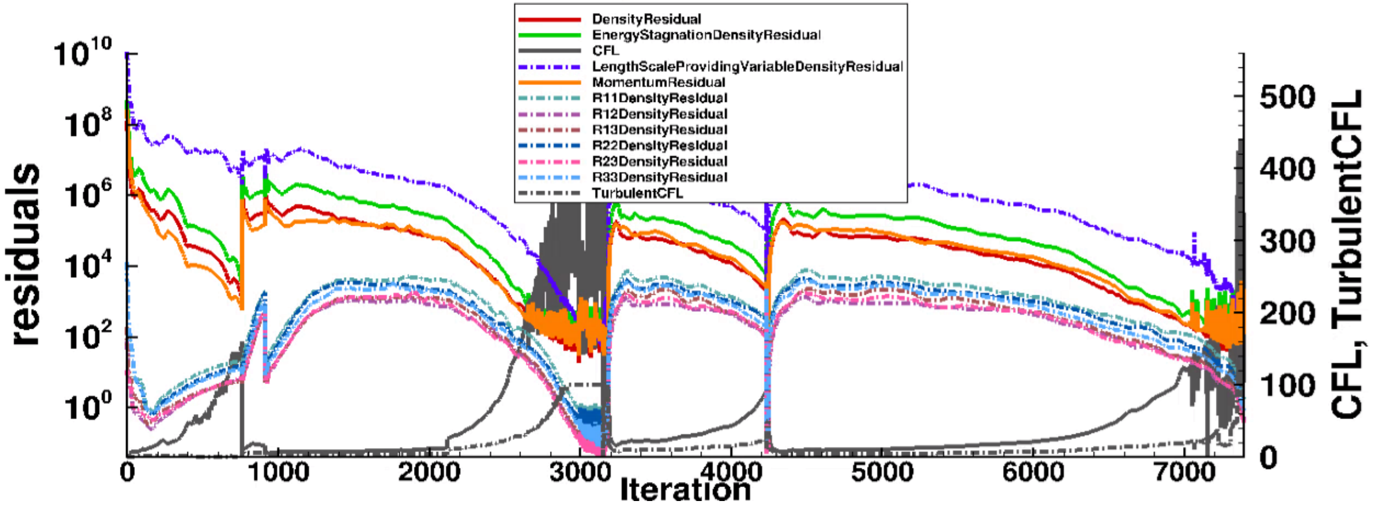
- Case & meshes from Michael Werner (DLR–AS-HGK): **Delta wing** with 32725358 hexas or 36267940 nodes – **ANSA hex-dominant mesh**
- **Flow conditions**: $Ma = 0.85, \alpha = 12°, \ \ Re = 3022728.49491777, L_{Re} = 0.382.$
- **Dimensional specs:**
  $p = \ 37410.7109 \ kg/ms^2 \ , T = \ 270.8606 \ K,$
  $R = 287 \ m^2/Ks^2 \ , grid \ unit \ length = 0.382 \ m$
- Initially tried to start with α =12° which failed for any run – weak coupling/coupled, LLF/Roe/Upwind. Restart from α =9° worked well for upwind scheme.
- For α =12°, stalling of residuals with unsteadiness in the solution – vortex breakdown in the flow.

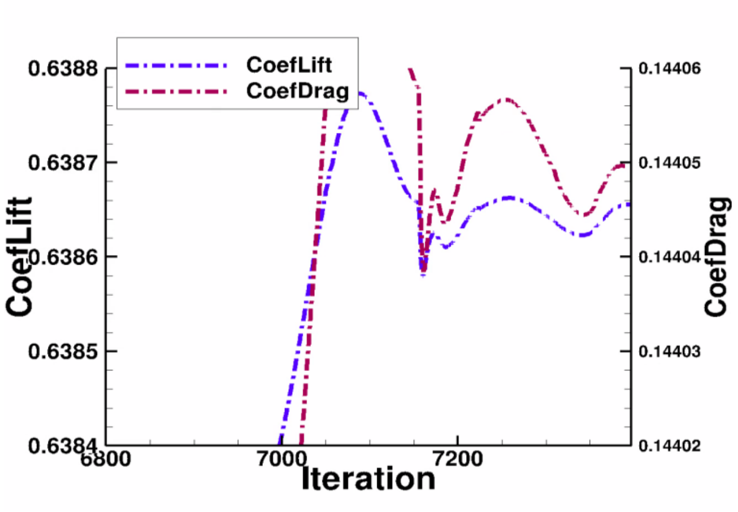


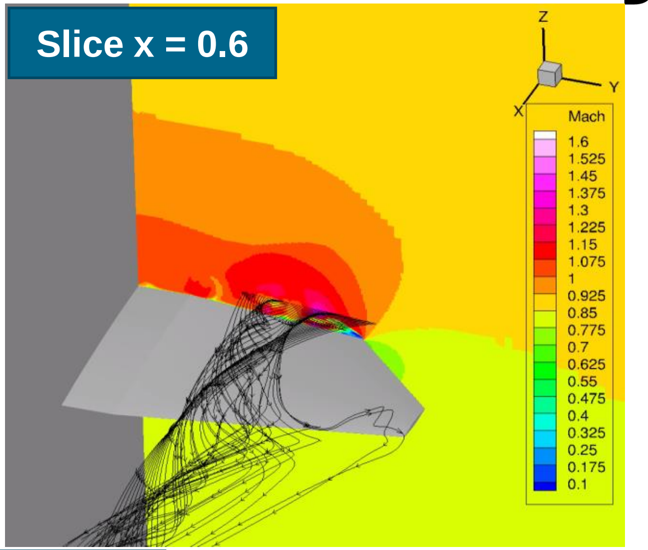
Thanks to Michael Werner for the case, mesh and his inputs for the analysis! ☺



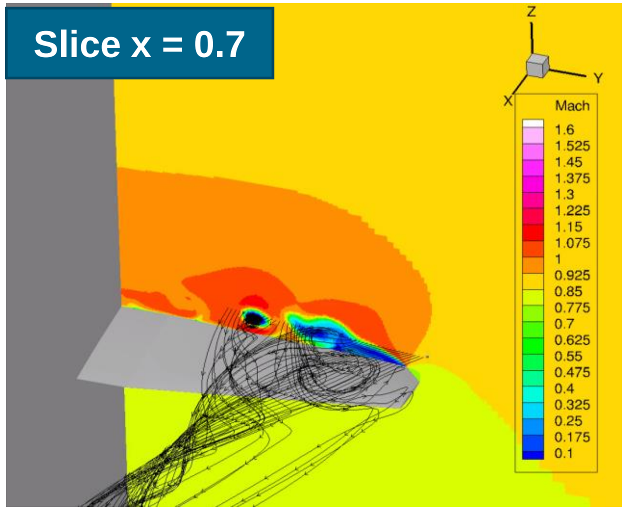
DLR – F23 Results, $\alpha = 12°$

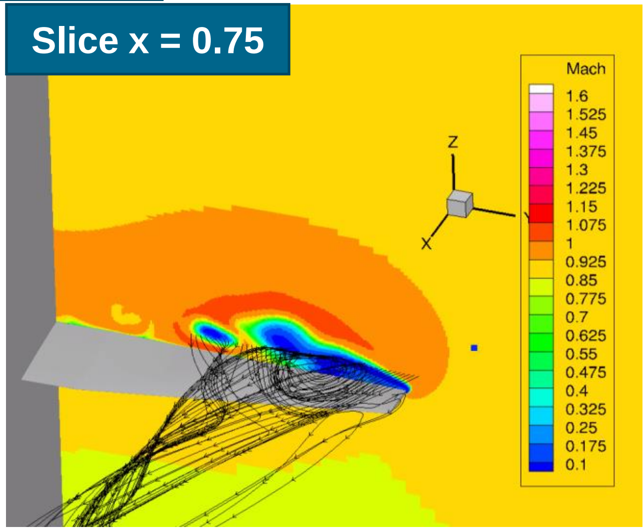






Vortex break down

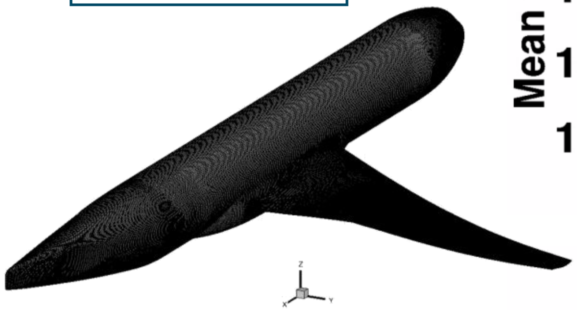


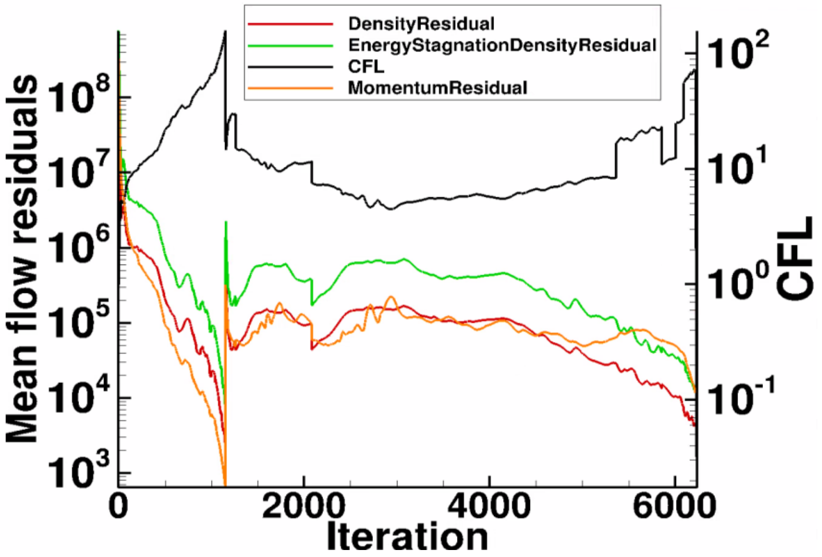


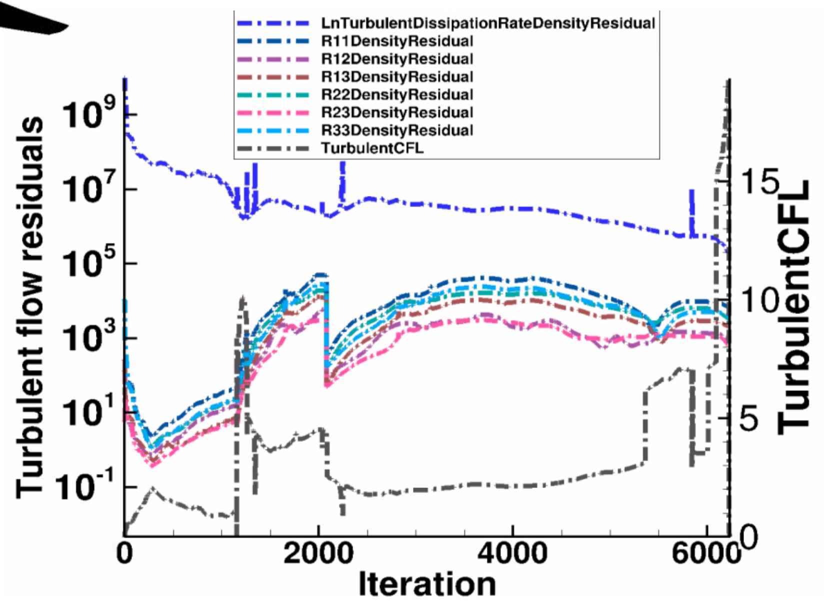
Cases with convergence issues & challenges yet to be solved

- We may still have meshes/cases which don't converge
- **DPW CRM:** Mesh obtained from Tobias Knopp, results available for Tau for SST and RSM - wing body configuration, solar mesh
- **Flow conditions:** $Ma = 0.85,\ \alpha = 2.5°, Re = 5 \times 10^6,\ \ L_{Re} = 0.18914364.$
- Appearance of **NaN – Still under investigation!**

DPW CRM







Conclusion & further work

- With every CODA version, we obtain improvements. A better convergence of RSM has been possible due to several additional improvements like CFL splitting, equations scaling etc. from other developers. Thanks to all CODA developers and verification engineers!
- A textbook implementation can lead to some struggles (Simple upwinding for turbulence).
- Further knowledge of implicit solvers for turbulence models is still needed – stability analysis for RSM in implicit solvers.
- **Appearance of NaN** is of primary concern. Weak coupling may help to improve but further tests are needed.
- **Linearization of source terms**: Previous attempts to linearize source terms were not that successful, a careful linearization may help.
- **Effect of linear solvers**: Algebraic multigrid may help to have better linear system convergence. With integration of PetSc interface, this can be tested.
- **Tackling the issue at discretization level:** Efforts are being undertaken currently for Eddy viscosity models (SST) in CODA, outcomes from these efforts can be extrapolated to RSM.

Special thanks to

- Tobias Knopp, Johannes Löwe for their ideas & support.
- Andreas Krumbein, Axel Probst for their guidance.
- Matthias Lühmann (Airbus) for meshes & verification activities.
- Roberto Sanchez, Deepak Kunhappan for their support with debugging & settings.

THANKS!

Impressum

Thema: Progress with verification and stabilization of Reynolds stress models using the CFD Software by ONERA, DLR, Airbus (CODA)

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