Estimation of Calving Law Parameters from Satellite Data

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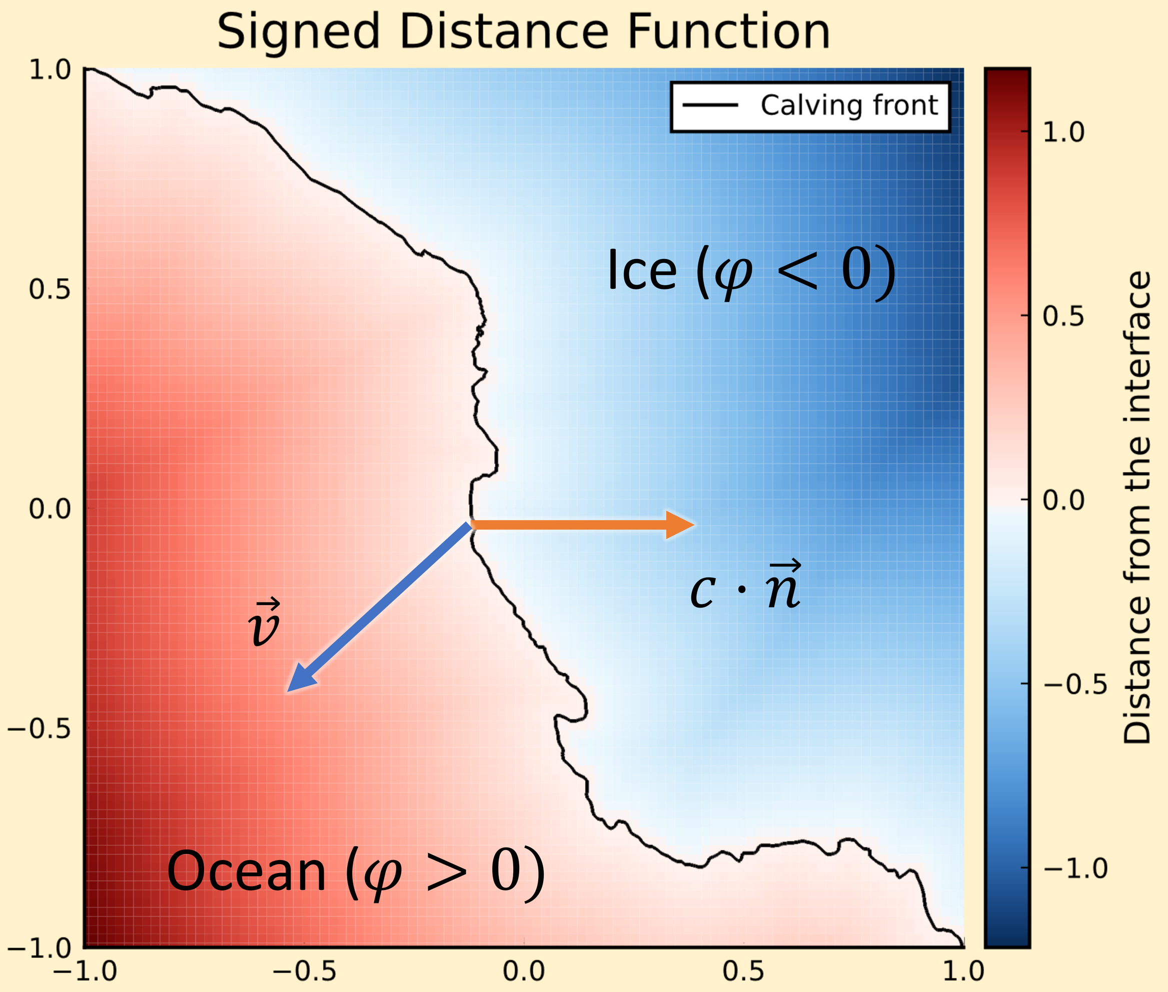
Summary

- Development of a new Level-set solver for tracking calving fronts of ice sheets
- Improve scaling over existing solver
- Coupled with ice sheet model
- Optimization to estimate parameters

Level-set Method for Calving Fronts



- Implicit representation of the calving front as Level-set field
- Level-set Equation:

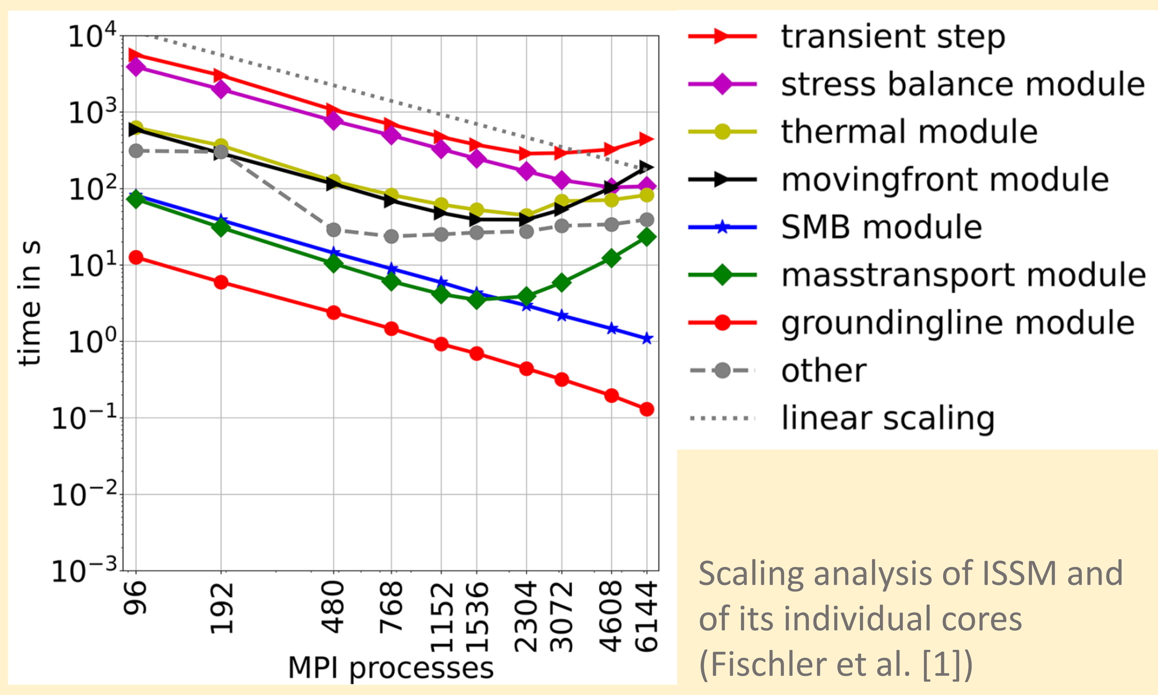
$$\frac{\partial \varphi}{\partial t} + \vec{v}\ \nabla\varphi = c\,\|\nabla\varphi\|$$

- Calving law, e.g. von Mises:

$$c = \|\vec{v}\|\frac{\sigma_{VM}}{\sigma_{max}}$$

- Ice velocity $\vec{v}$, von Mises stress σ_{VM} from ice sheet model
- Calibrated stress threshold σ_{max}

Scaling Analysis of ISSM



Scaling analysis of ISSM and of its individual cores (Fischler et al. [1])

- Existing Level-set core integrated into ice sheet model ISSM identified as scaling bottleneck
- Motivation for replacement with new DG solver

Discontinuous Galerkin for Hamilton Jacobi Equations

- Nonlinear Hamilton-Jacobi equations:

$$\varphi_t + H(\varphi_x) = 0$$

- Traditionally solved with (W)ENO Finite Differences
- Direct DG method due to Yan and Osher [2] (for 1D, analog. for 2D)

$$\int_E \varphi_t u\,dx + \int_E \widehat{H}(p_1,p_2)u\,dx = 0 \quad \text{with} \quad p_1 = p_2 = \varphi_x$$

- Compute $p_{1/2}$ with left/right upwind flux:

$$\int_E p_{1/2}u\,dx + \int_E \varphi u_x\,dx - \int_{\partial E}\varphi^{+/-}n_x u\,dx = 0$$

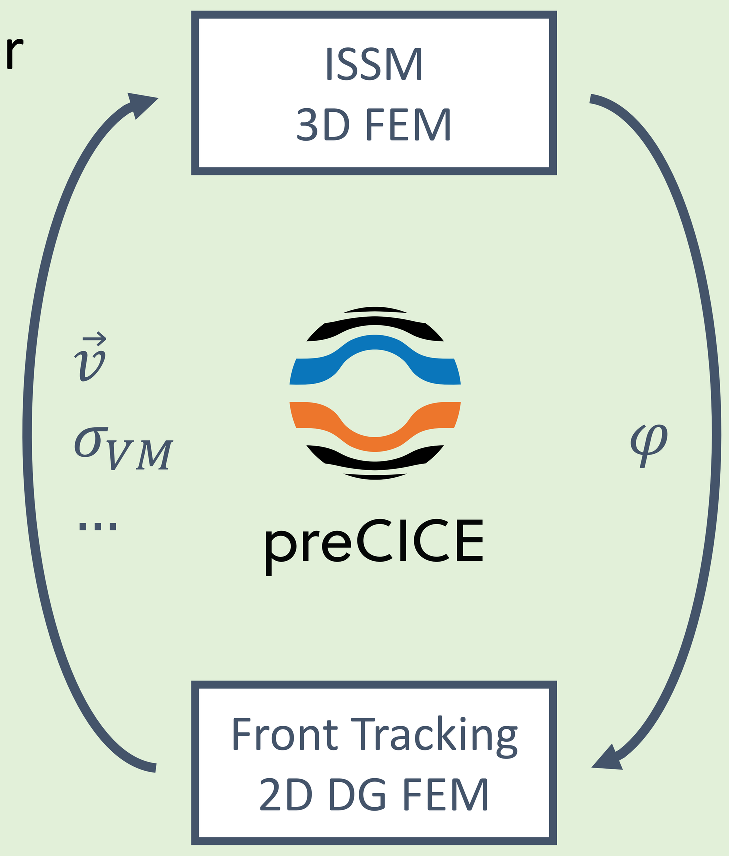
- Lax-Friedrichs Numerical Hamiltonian:

$$\widehat{H}(p_1,p_2) = H\left(\frac{p_1+p_2}{2}\right) - \frac{1}{2}\theta(p_1-p_2) \quad \text{with} \quad \theta = \max_p\left(\frac{\partial H(p)}{\partial p}\right)$$

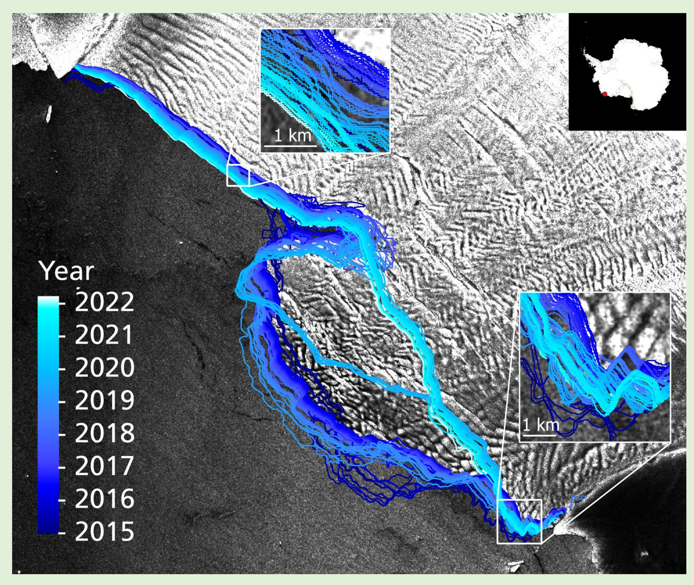
Coupling with Ice Sheet Model ISSM

- Development of a generic coupling adapter for ISSM [3] with preCICE coupling library [4]
- Useable for calving fronts, subglacial hydrology, oceans, etc.
- (Almost) any of 100+ ice sheet variables
- Good scaling and negligible overhead
- Independent development of solvers

https://git.rwth-aachen.de/terrabyte-dnn2sim/issm-precice



Parameter Estimation



Time series of Calving front at Devicq Clacier, Getz Ice Shelf, Antarctica generated with automated neural network processing (Baumhoer et al. [5])

Optimization using satellite observations

Challenges:

- Only area around front **-> Weighting**
- Coupling to ice sheet model
- Reinitialization of SDF

} **-> Limit Δt**

- Solve with adjoint of Level-set Equation:

$$\min_{\sigma_{max}} \frac{1}{2}\sum_i \|(\varphi(t_i) - \varphi_i^*)w_i\|^2$$

$$\text{s.t.} \quad \frac{\partial\varphi}{\partial t} + \vec{v}_i\ \nabla\varphi = \|\vec{v}_i\|\frac{\sigma_{VM,i}}{\sigma_{max}}$$

$$\varphi(t_{i-1}) = \varphi_{i-1}^*$$

- Observations φ_i^* and state $\vec{v}_i$, $\sigma_{VM,i}$ at time t_i
- Gaussian weights $w_i = \exp(-{(\varphi_i^*)^2}/{\alpha^2})$

Test with synthetic data (grid resolution Δx)

$\alpha/\Delta x$	Rel. Err.
1	$1.7 \cdot 10^{-3}$
1/2	$1.3 \cdot 10^{-3}$
1/4	$1.3 \cdot 10^{-3}$
1/8	$0.9 \cdot 10^{-3}$
1/16	$1.0 \cdot 10^{-3}$

Ongoing Work

- Optimization and performance analysis of Level-set solver
- Verification of parameter estimation method with real life data

References

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