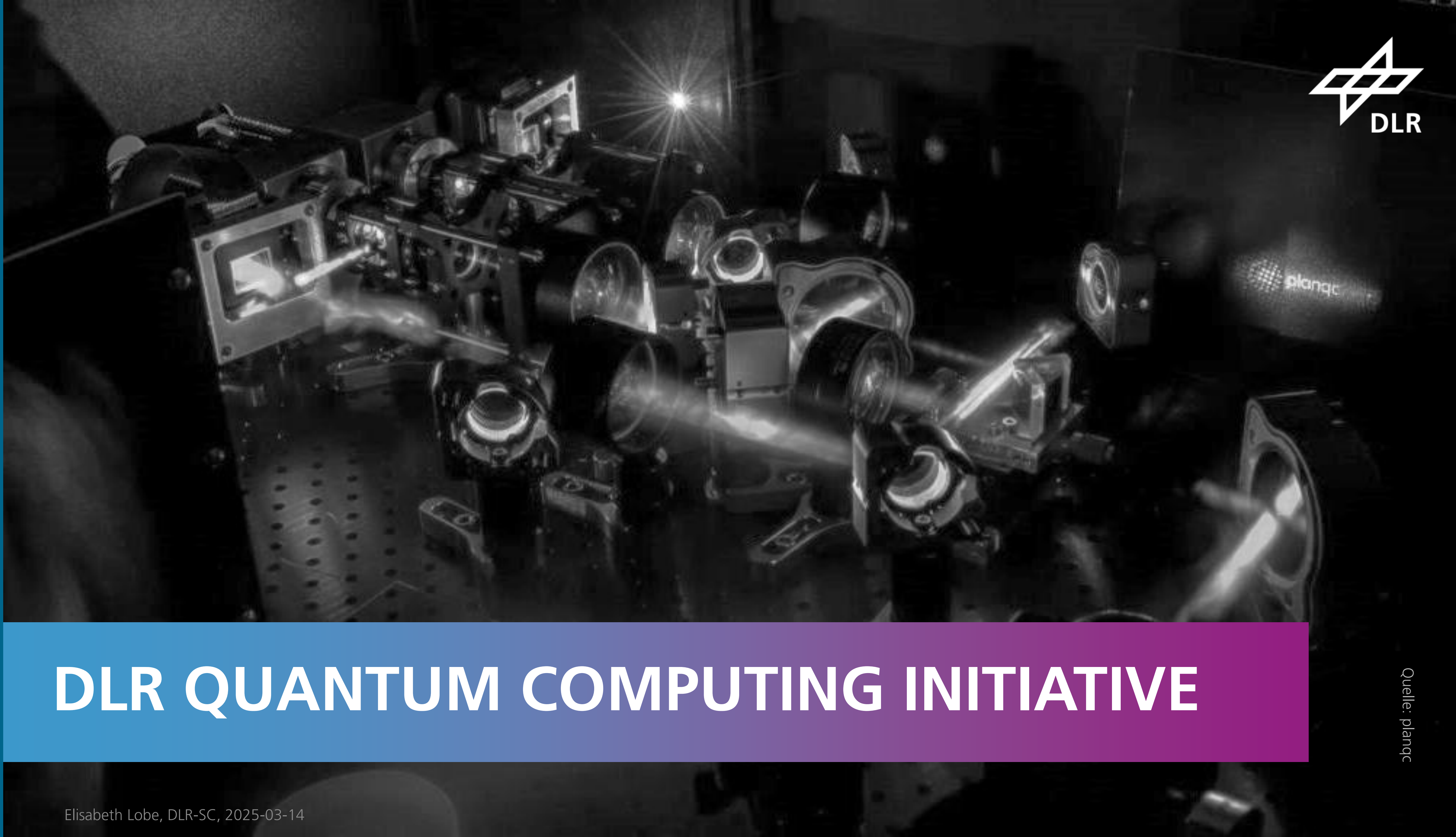


DLR QUANTUM COMPUTING INITIATIVE

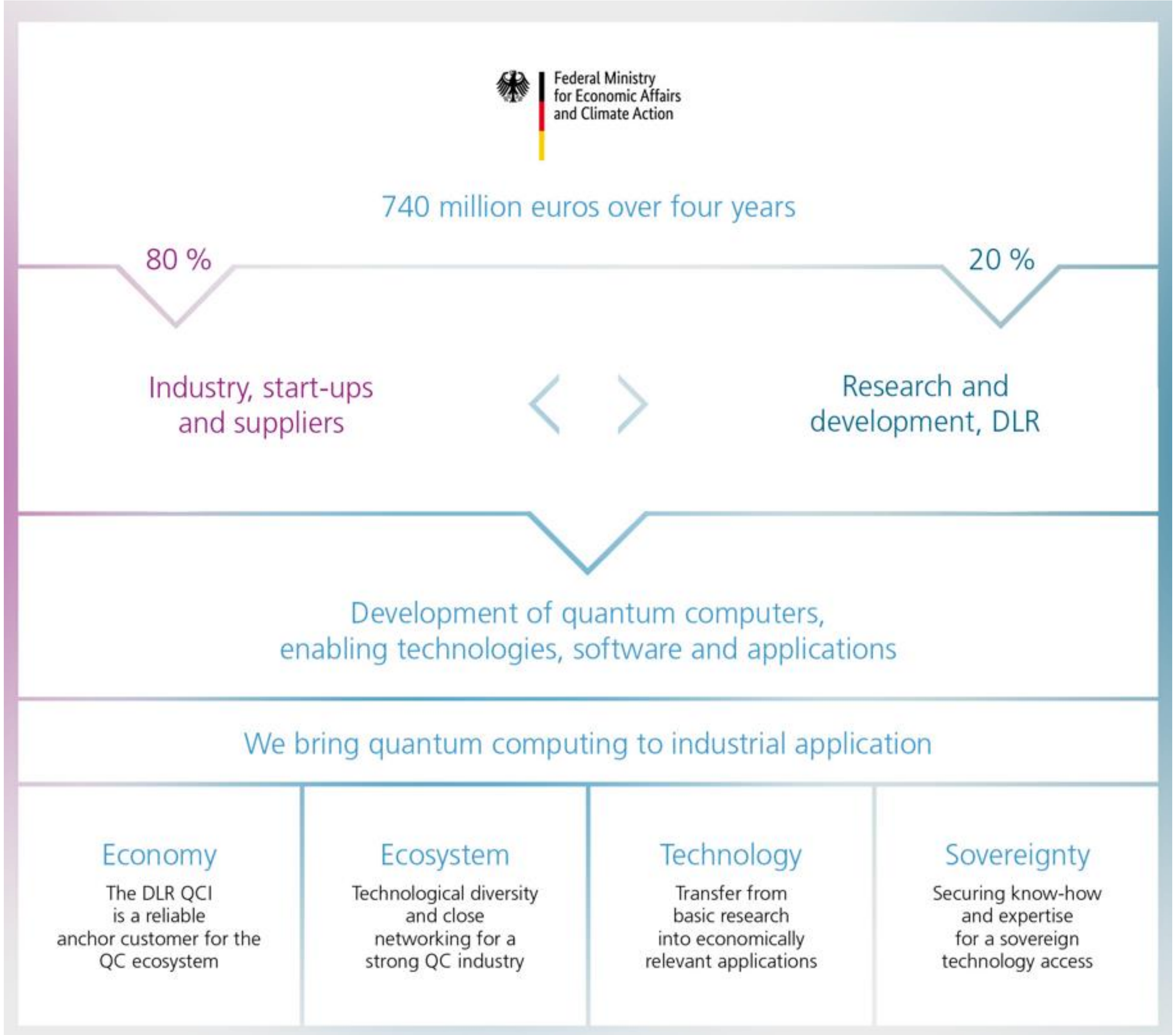
Quelle: planqc

Elisabeth Lobe, DLR-SC, 2025-03-14

DLR
Quantum Computing
Initiative

We shape the
quantum computing ecosystem

- Bundled technology, expertise & infrastructure
- Research and development on the basis of DLR areas of competence
- Procurement of complete QC systems and components through open-technology competitive tenders
- Upgrading by commissioning the necessary basic technologies and applications



qci.dlr.de

DLR
Quantum Computing
Initiative

We shape the
quantum computing ecosystem

1
Unique ecosystem

2
Innovation centers

19
DLR Institutes

22
Industry partners

6
Qubit technologies

17
Hardware projects

22
SW / App projects

DLR

QCI – Industry Partners

Technologies

There are many ways to realize qubits and thus quantum computers in hardware.
That is why we commission the development of different hardware platforms.

Analogue Computers

Ion traps

Neutral atoms

NV Centers

Photons

Spin enabling

➔ **What is the best approach to a universal, error-corrected quantum computer?**

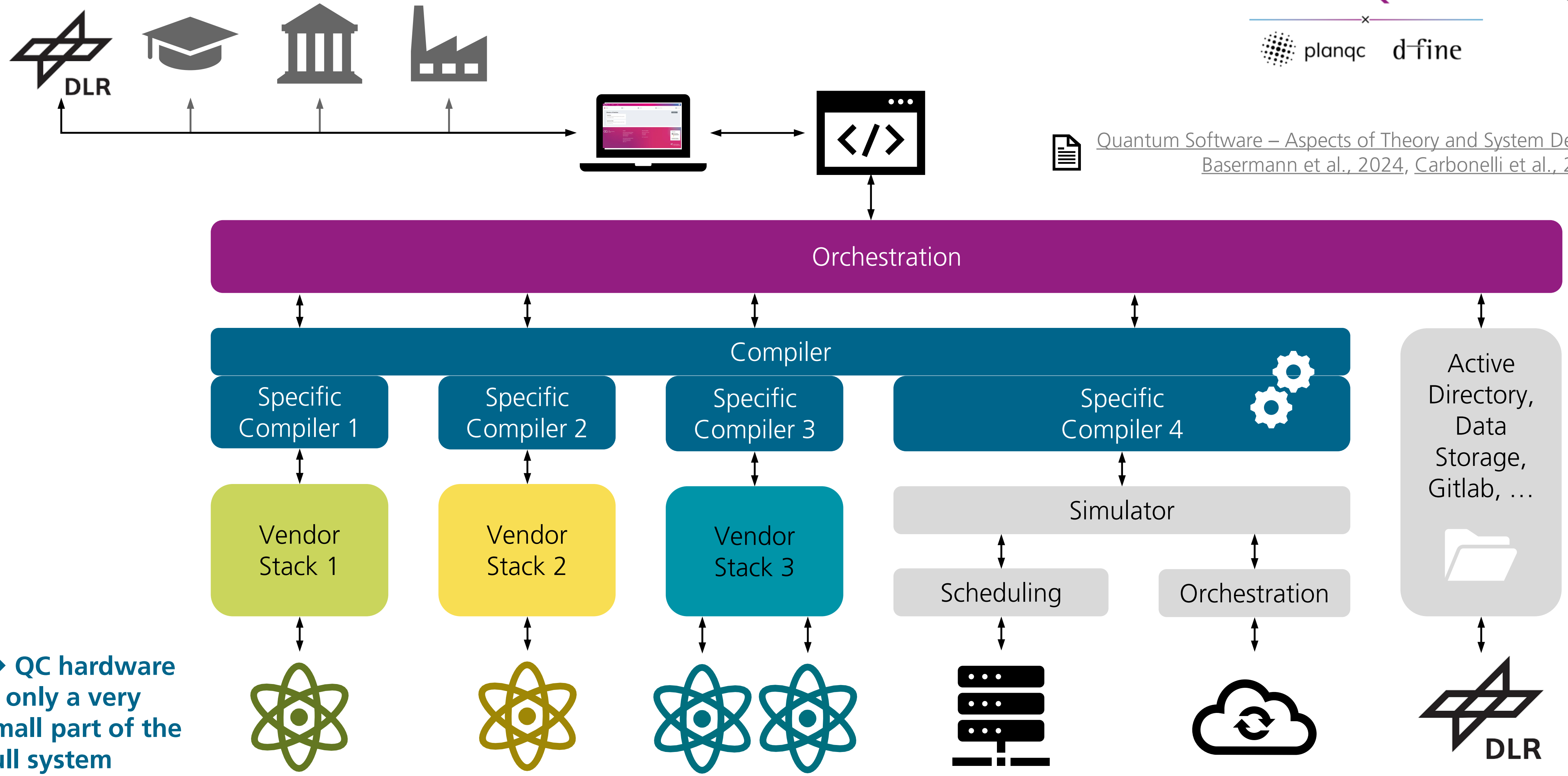
iStock.com

THE DLR QC ECOSYSTEM

Goal: Fully Integrative QC Hardware Software Stack

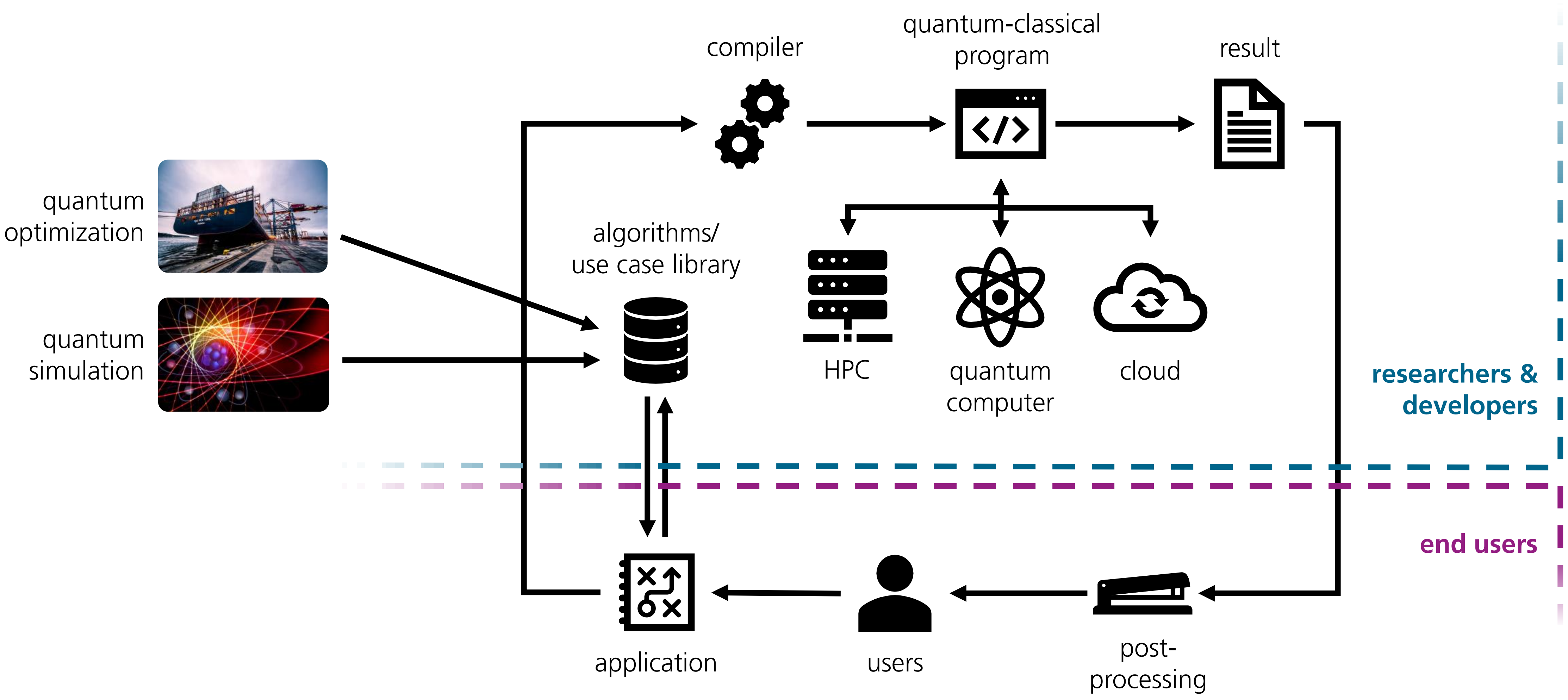
ALQU × planqc d-fine DLR

Quantum Software – Aspects of Theory and System Design
Basermann et al., 2024, Carbonelli et al., 2024



➔ QC hardware is only a very small part of the full system

Ideal Workflow of Using Quantum Computers



QC @ DLR-SC

Stock.adobe.com, Credit: Jackie Niam

Groups Working on QC at DLR-SC

DLR

Institute of Software Technology (SC)

High-Performance Computing (HPC)

Quantum Computing Application Software (QCA)

- Digital quantum simulation
- Development of algorithms for NISQ devices
- Combinatorial optimization with quantum computers / annealers
- Quantum machine learning

Quantum Computing Methods and Implementations (QMI)

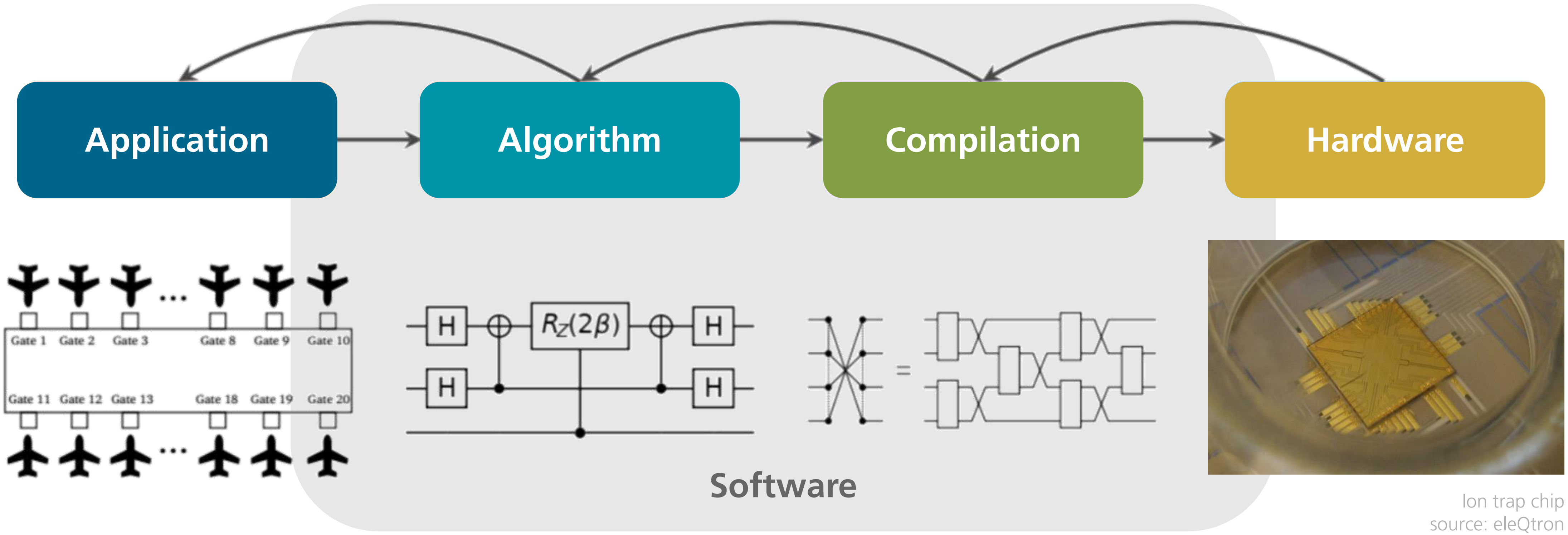
- Quantum error correction & mitigation
- (Hardware specific) compilation
- Circuit optimization
- Realistic noise models
- Hybrid computing

Intelligent and Distributed Systems (IVS)

Distributed Software Systems (VSS)

- Quantum software engineering
- User interfaces for quantum computing
- Software infrastructures for hybrid calculations
- Programming paradigms for quantum computing
- Verification and validation

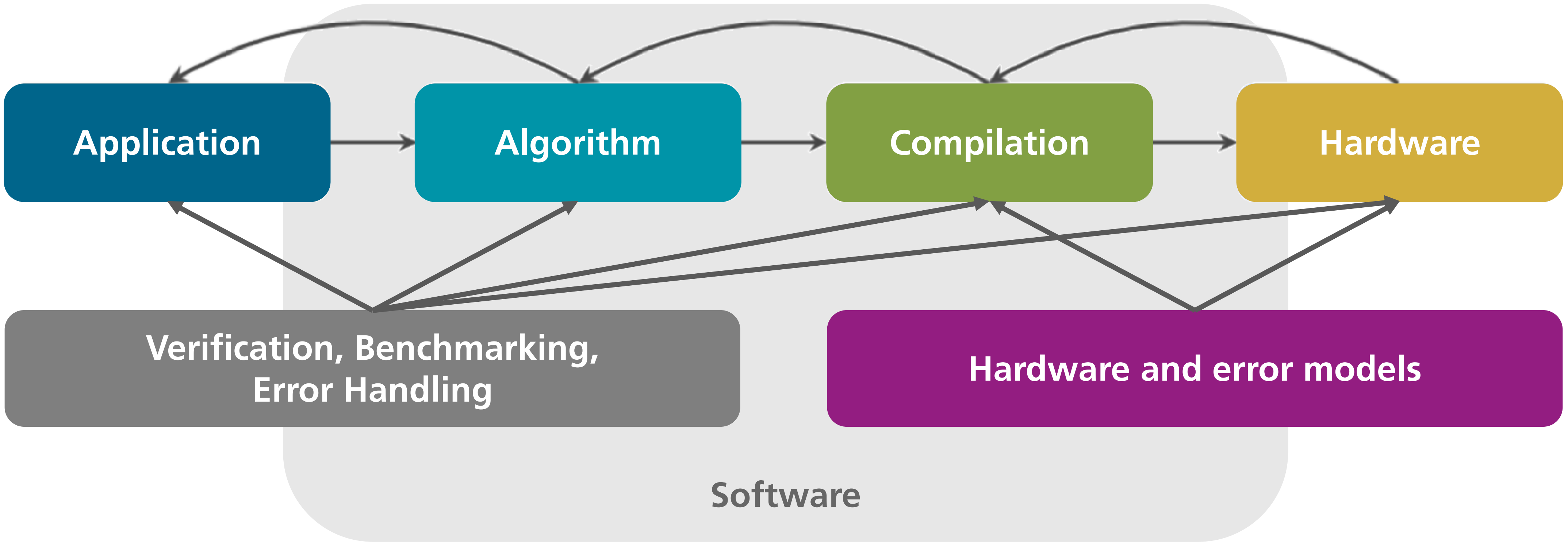
Hardware-Software Co-Design



Ion trap chip
source: eleQtron

- ➔ **Identification** of promising QC applications on the one hand as well as QC platforms (digital, analog, topological) on the other hand
- ➔ **Development** of new algorithms and application software for NISQ devices with the goal to find quantum advantage by optimally exploiting hardware specifics

Hardware-Software Co-Design

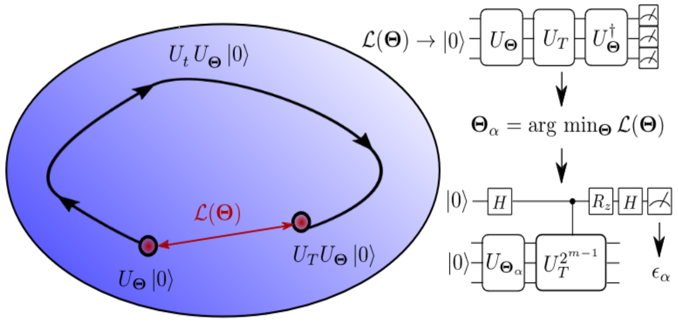


- Access and therefore adaptability at all levels of the QC stack
- Suitable abstraction layers and solution patterns, in particular enabling hybrid computing
- Integration of SW/HW manufacturers and users
- Comparison of DLR QC hardware with state-of-the-art classical methods

Quantum Applications

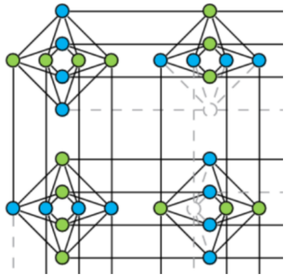
Digital Quantum Simulation

- New algorithms for quantum many body dynamics
- Development of quantum simulation software for NISQ devices



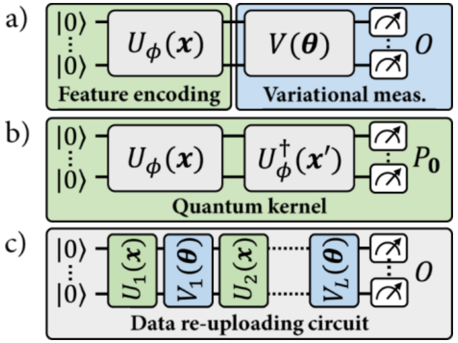
Combinatorial Optimization

- Combinatorial optimization problems around quantum annealers
- Development of open-source software library (quark and Co.)



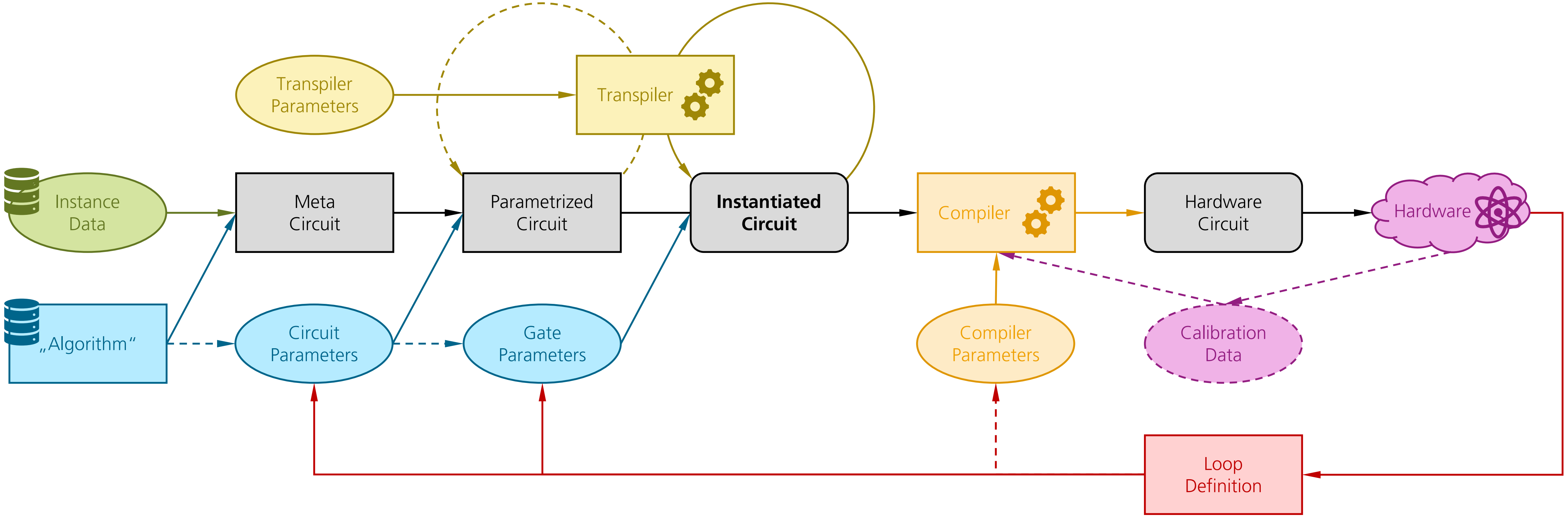
Other Quantum Applications and Quantum-inspired Algorithms

- Quantum machine learning
- Description of new quantum computing platforms (e.g. topological QCs)
- State-of-the-art numerical software for QC benchmarks



Support of Hybrid Quantum Algorithms in QC Ecosystem

- Problem: input output behaviour and definition of the interaction with classical systems
- Representation of loops with regard of scheduling and orchestration

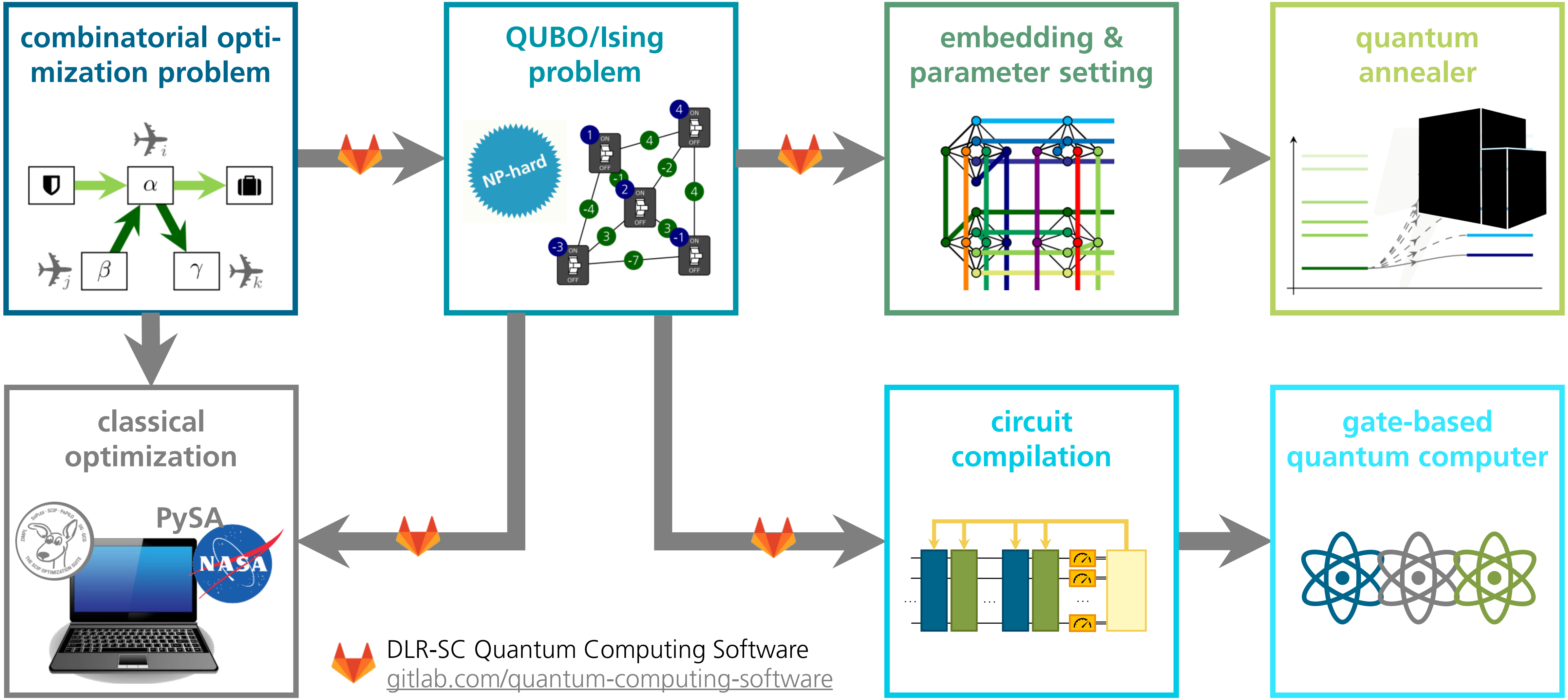


Elisabeth Lobe, DLR-SC, 2025-03-14

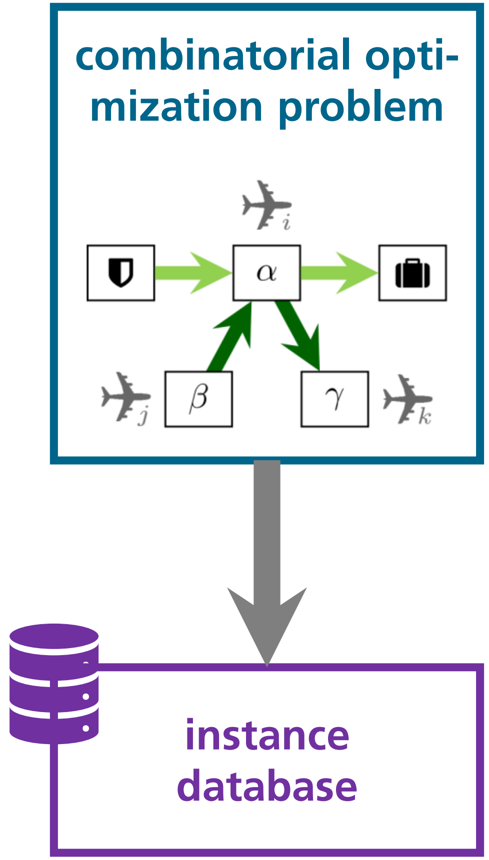
QUANTUM OPTIMIZATION SOFTWARE

Stock.adobe.com, Credit: Jackie Niam

Combinatorial Optimization QC Software Stack



Combinatorial Optimization Problems



10.5281/zenodo.13944133

Implemented Problems

- Minimum k-Union
- Graph Partitioning
- Subset Sum
- Knapsack

- Max Cut
- Prime Factorization
- Random/Arbitrary Ising Model
- Maximum Colorable Subgraph
- Flight-Gate Assignment
- Traveling Salesperson

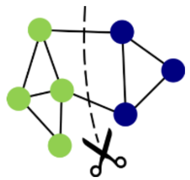
quapps

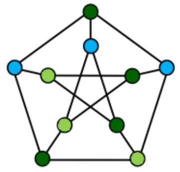
https://anaconda.org/dlr-sc/quapps

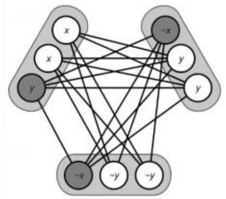
```
conda install dlr-sc::quapps
```

Planned

- Autoclave Loading
- Transmission Expansion Planning
- Antenna Alignment
- Satisfiability
- …

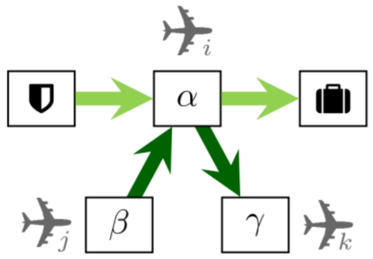




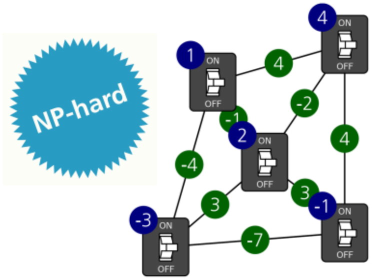


Transformation to Ising Problems

combinatorial optimization problem



QUBO/Ising problem



$$\begin{aligned} \text{minimize} \quad & p(x) \\ \text{subject to} \quad & \ell_1 \leq q_1(x) \leq u_1, \\ & \ldots, \\ & \ell_m \leq q_m(x) \leq u_m, \\ & x \in \mathbb{Z}^n \end{aligned}$$

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{N} W_i x_i + \sum_{i<j=1}^{N} S_{ij} x_i x_j \\ \text{subject to} \quad & x \in \{0,1\}^N / \{-1,1\}^N \end{aligned}$$

quark

https://anaconda.org/dlr-sc/quark

```
conda install dlr-sc::quark
```

10.18420/inf2023_123
10.5281/zenodo.13846762

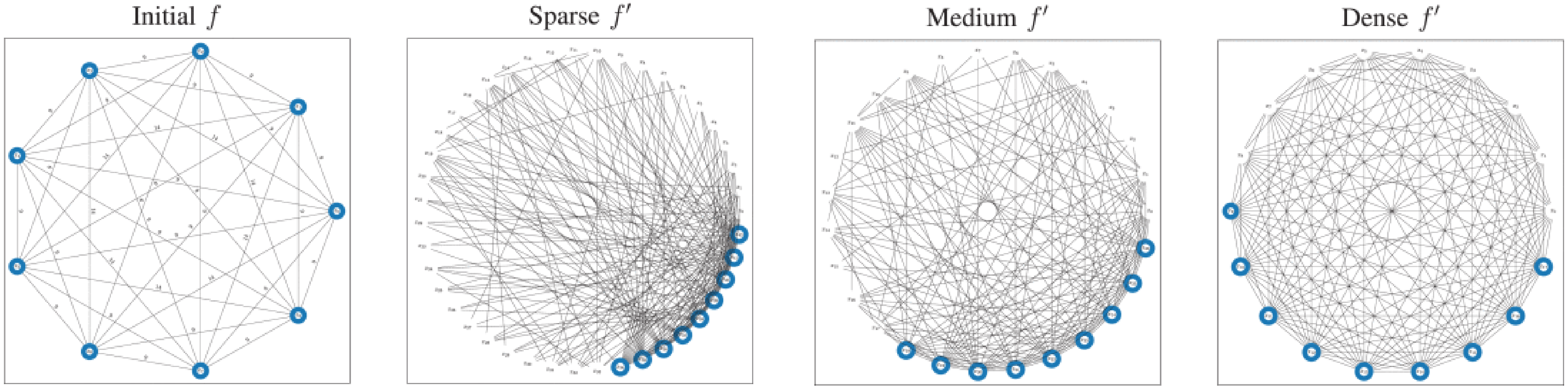
Main Transformation Steps:

- Encoding of the variables
- Reformulation of constraints to penalty terms added to the objective
- Choosing penalty weights
- Reduction of higher-degree polynomials

Reduction of Higher-degree Polynomials

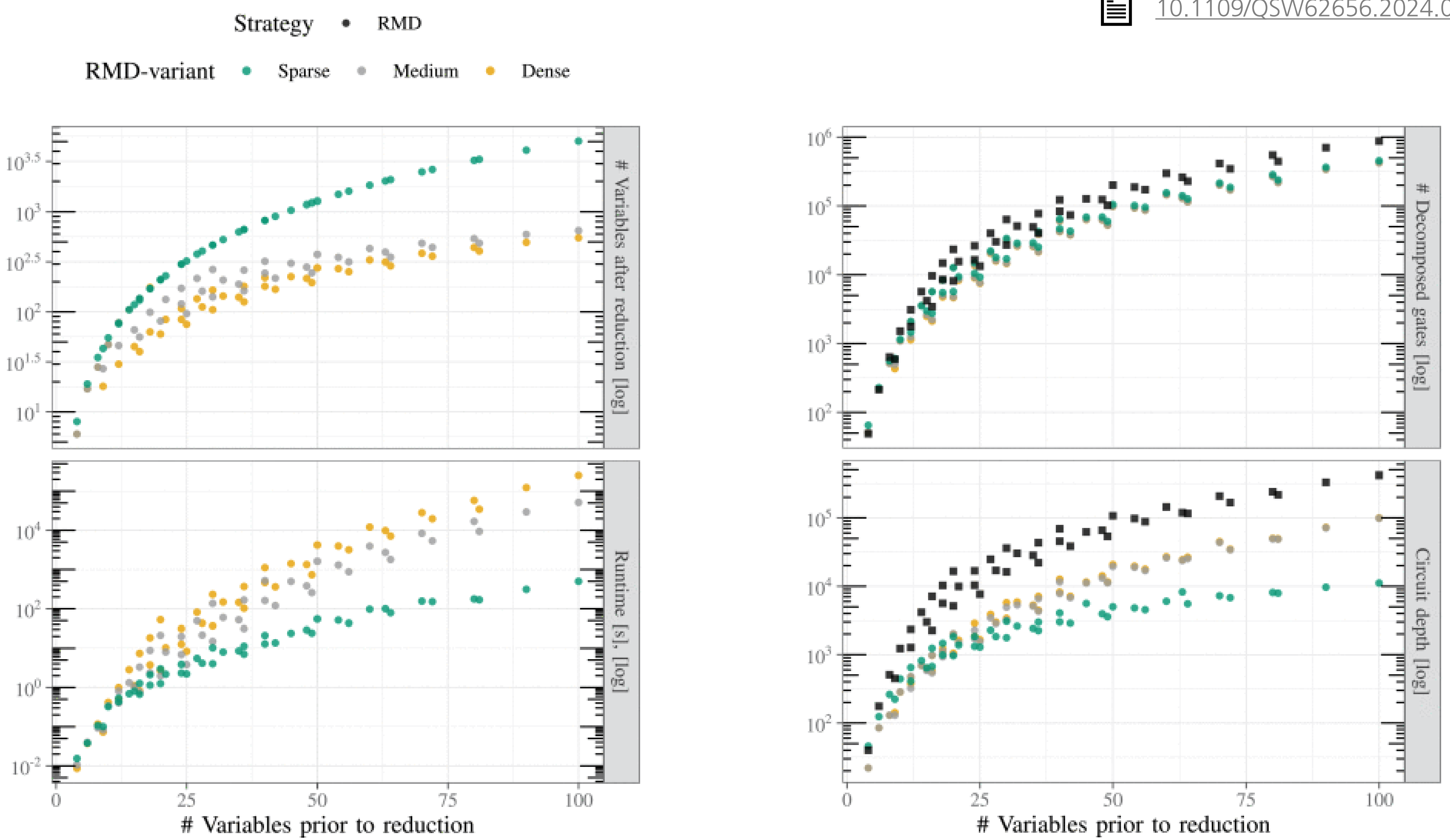
10.1109/QSW62656.2024.00018

- By Lukas Schmidbauer, Elisabeth Lobe, Karen Wintersperger, Wolfgang Mauerer
- Analyzed the influence of the reduction methods implemented in **`quark`** on the resulting Ising interaction graph
- And further on **their Impact on QAOA Circuits**



Reduction Impact on QAOA Circuits

10.1109/QSW62656.2024.00018

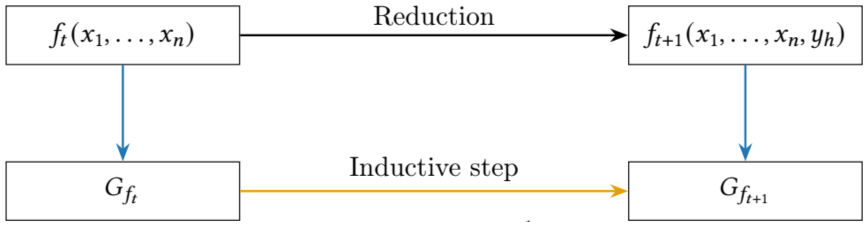


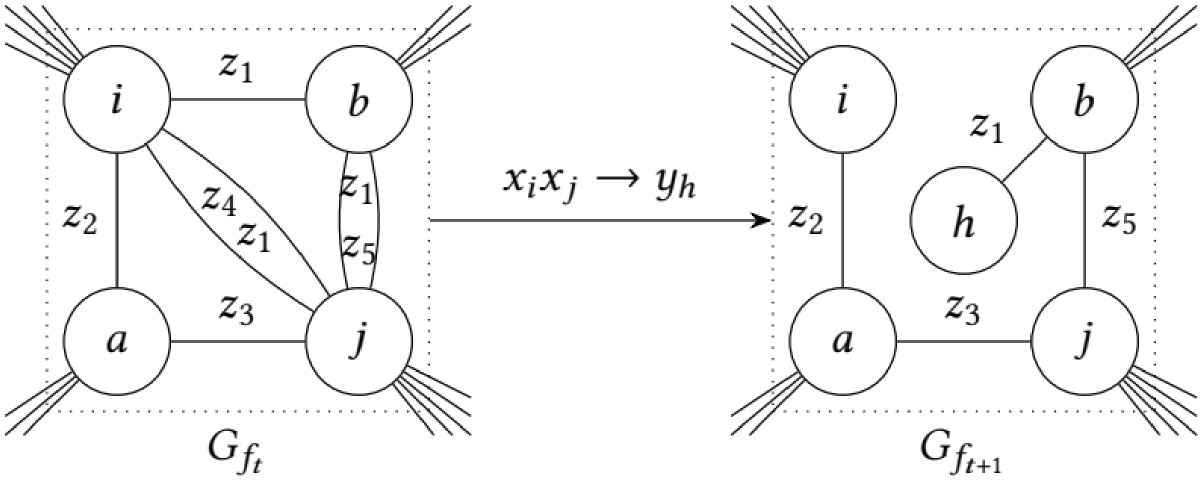
Faster Reduction of Higher-degree Polynomials

10.48550/arXiv.2411.19934

https://github.com/lfd/Fast_Quadratisation

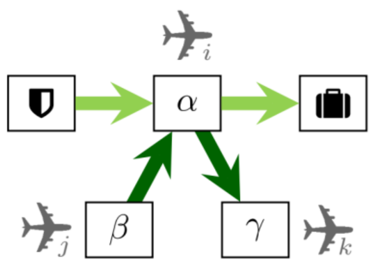
- By Lukas Schmidbauer, Elisabeth Lobe, Ina Schaefer, Wolfgang Mauerer
- Represented polynomials with a specific multigraph representation
- Based on that a new, **faster reduction** algorithm was developed
- Inductive step updates graph structure
- Rather than scanning the polynomial several times



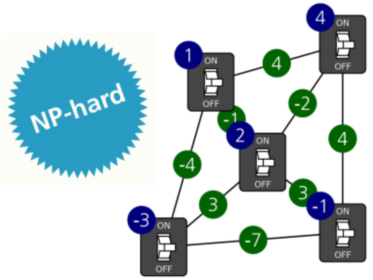


Use Cases Transformation for QAOA

combinatorial optimization problem



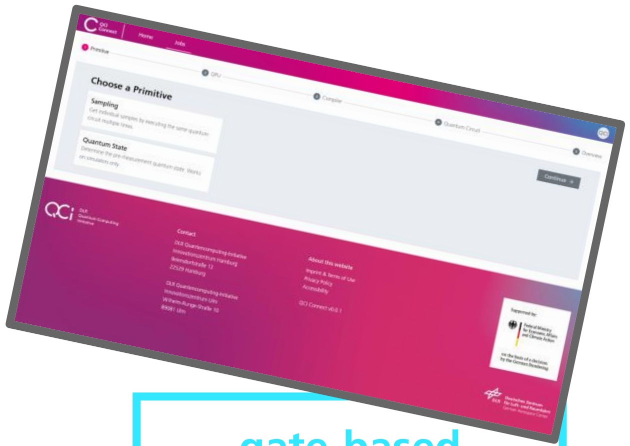
QUBO/Ising problem



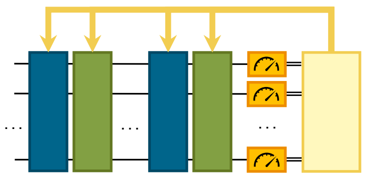
other optimization algorithms +

quantum simulation applications +

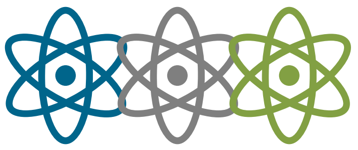
+



circuit compilation

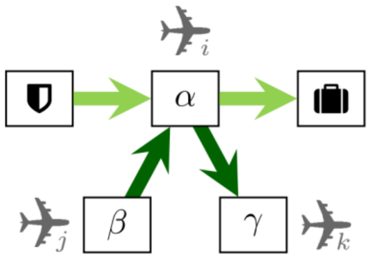


gate-based quantum computer



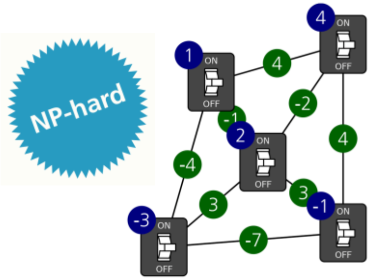
Classical Solving

combinatorial optimization problem



+ …

QUBO/Ising problem

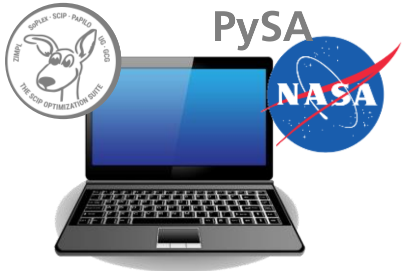


+ cvxpy solver (DLR-VE)

+ Benders' Decomposition

classical optimization

PySA

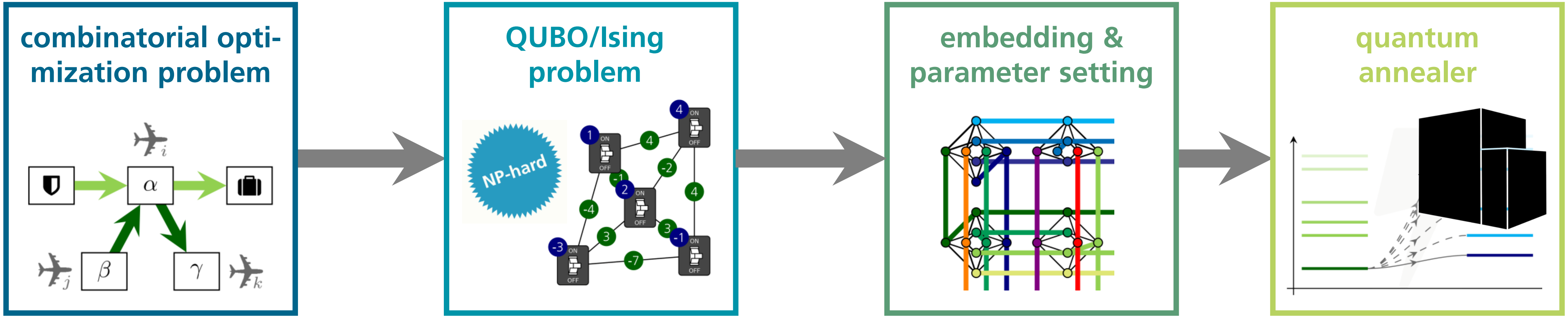


+ McSparse/SpinGlass solver

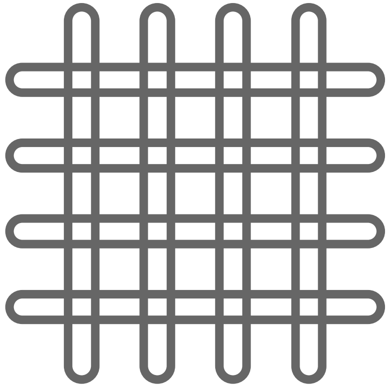
+ Selby algorithm

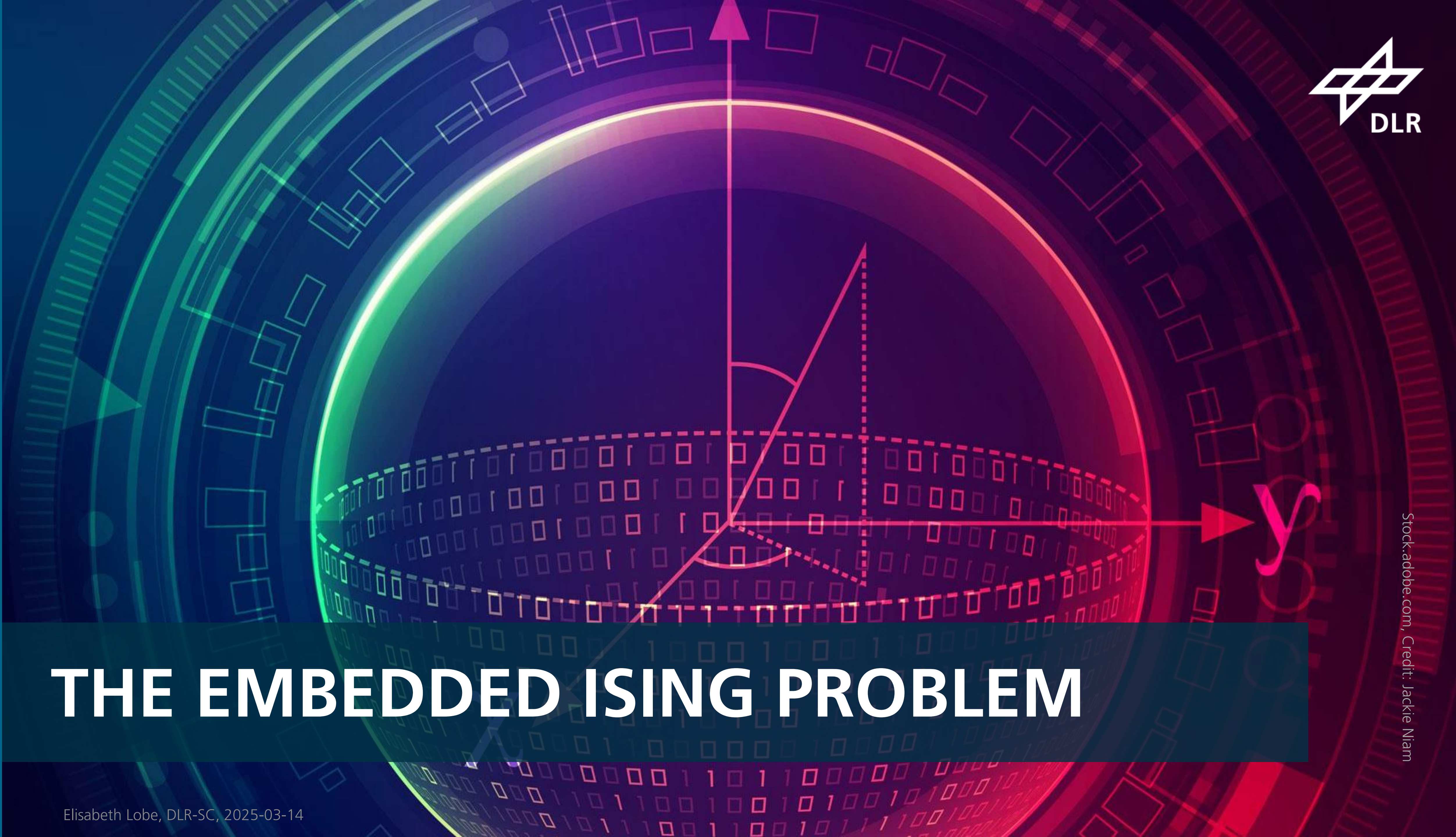
UNIVERSITÄT BONN

Use Cases Transformation for Quantum Annealing



- Restricted hardware graph
- “Broken” qubits
- Limited parameter precision

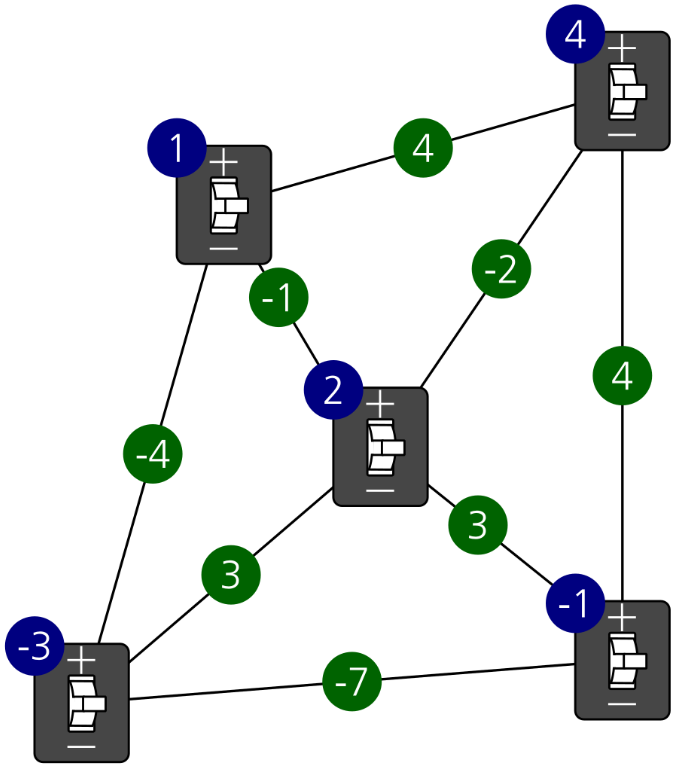




THE EMBEDDED ISING PROBLEM

Stock.adobe.com, Credit: Jackie Niam

Ising Problem



Definition

An **Ising model** over graph G with **weights** $\boldsymbol{W} \in \mathbb{R}^{V(G)}$ and **strengths** $\boldsymbol{S} \in \mathbb{R}^{E(G)}_{\neq 0}$ is a function $\boldsymbol{I_{W,S}} : \{-1,1\}^{V(G)} \to \mathbb{R}$ with

$$I_{W,S}(s) := \sum_{v \in V(G)} W_v s_v + \sum_{vw \in E(G)} S_{vw} s_v s_w .$$

We call G the **interaction graph** of the Ising model.

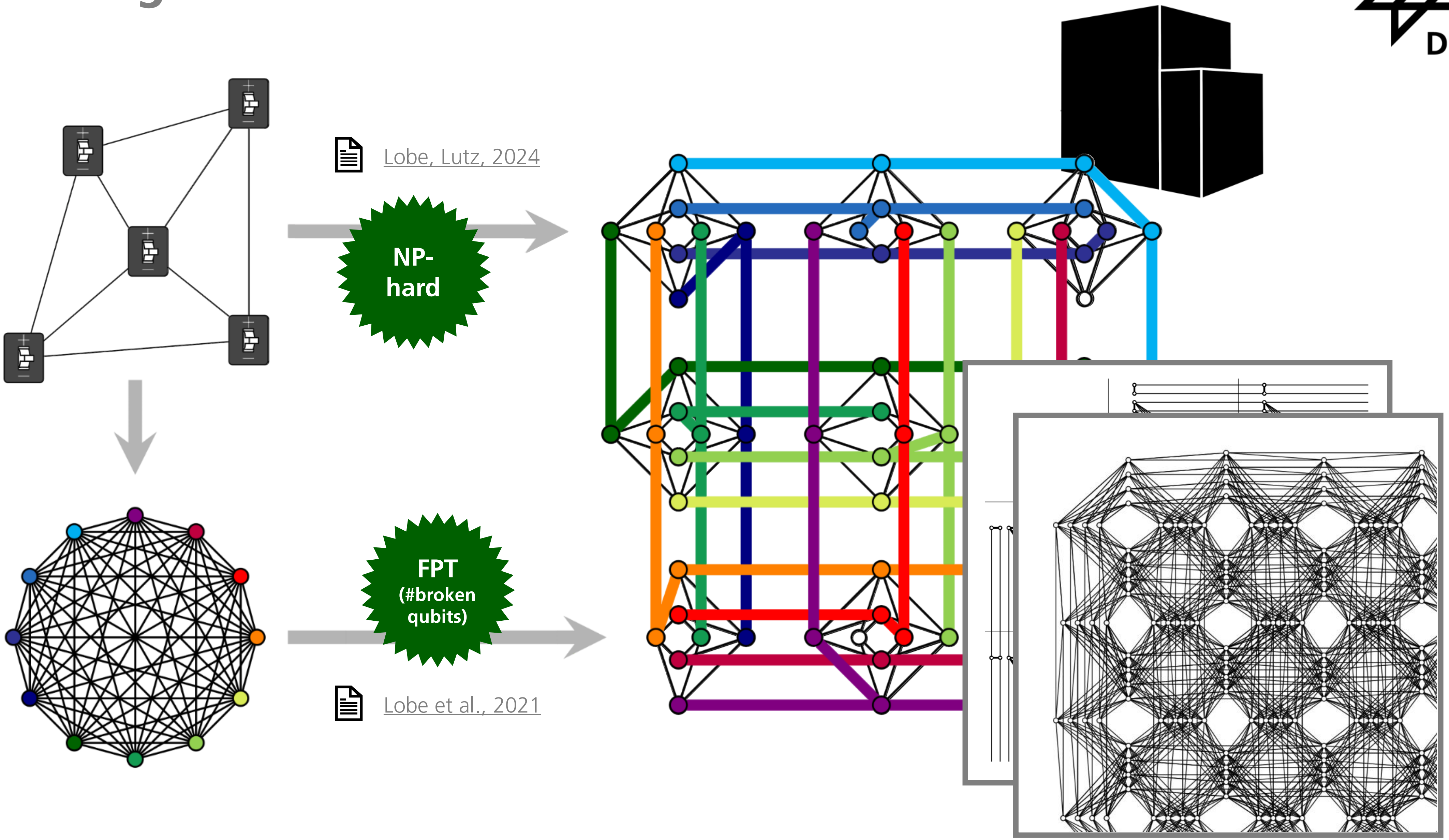
Ising Problem

Given a graph G, $W \in \mathbb{R}^{V(G)}$ and $S \in \mathbb{R}^{E(G)}$, find s that solves

$$\min_{s \in \{-1,1\}^{V(G)}} I_{W,S}(s).$$

NP-hard

Embedding Problem for D-Wave Hardware



Embedded Ising Problem

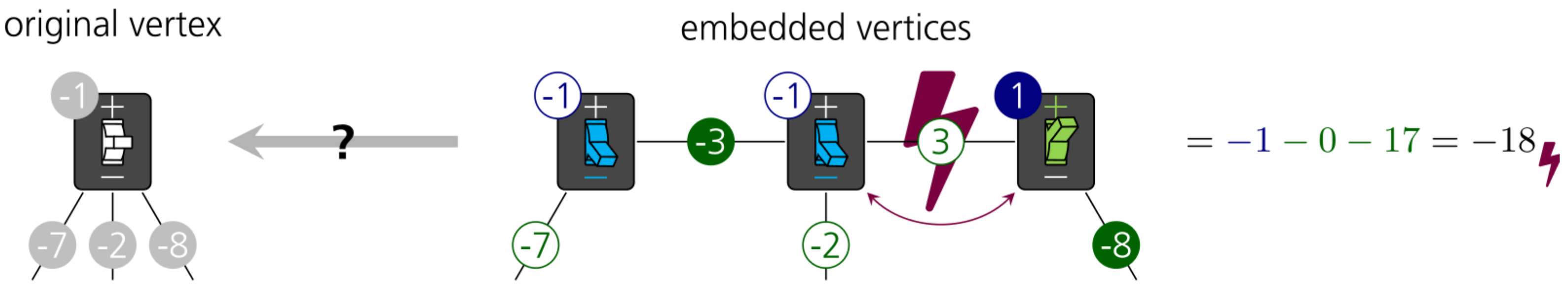
Lobe, Kaibel, 2023

$$\bar{I}_{\overline{W},\bar{S}}(s) := \sum_{q \in V(H_\varphi)} \overline{W}_q s_q + \sum_{pq \in E_\varphi \cup E_\delta} \bar{S}_{pq} s_p s_q$$

$$= \sum_{v \in V(G)} \left(\sum_{q \in \varphi_v} \overline{W}_q s_q + \sum_{pq \in E(H[\varphi_v])} \bar{S}_{pq} s_p s_q \right) + \sum_{vw \in E(G)} \left(\sum_{pq \in \delta_{vw}} \bar{S}_{pq} s_p s_q \right)$$

$\overset{?}{\swarrow} W_v t_v$ $\overset{?}{\swarrow} S_{vw} t_v t_w$

- with **proven equivalence** to original problem
- such that (at least optimal) solutions correspond to each other
- based on **synchronization** of the embedded variables

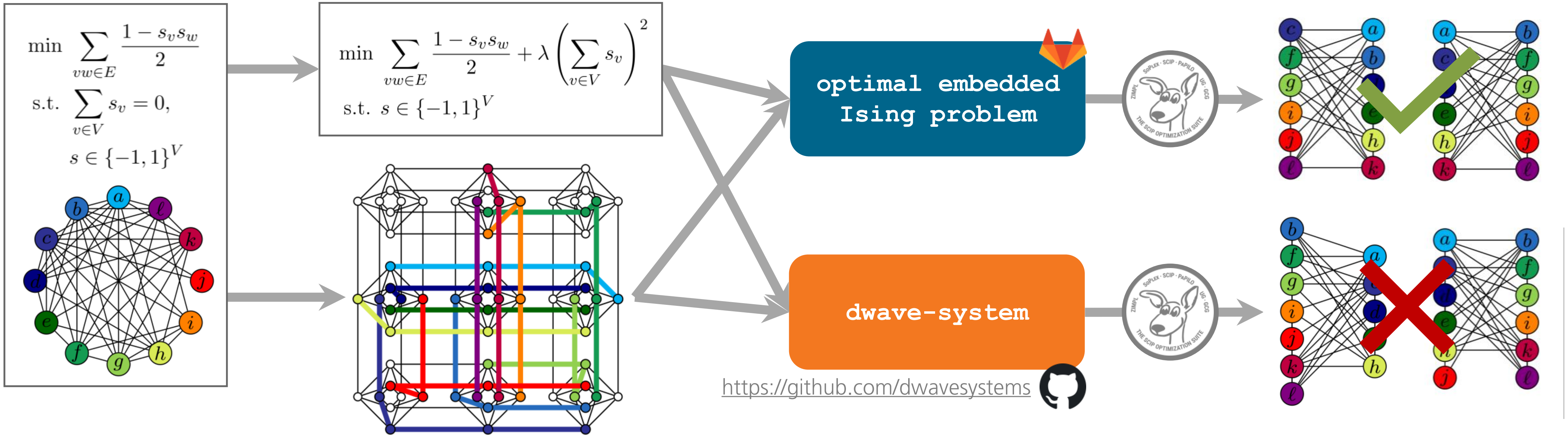


How large does the coupling strength need to be, but how small can it be?

Get Embedded Ising via "Uniform Torque Compensation"

- Current implementation in D-Wave `dwave-ocean-sdk` with method `embed_ising`
  - calls `dwave-system.dwave.embedding.chain_strength.uniform_torque_compensation`

- Found **counterexample**
  - where the method does **not** provide an equivalent embedded Ising problem
  - based on the Graph Partitioning Problem with 12 nodes
  - embedded by D-Wave using `minorminer.find_embedding`



Get the Optimal Embedded Ising Problem

Lobe, Kaibel, 2023

Requires to solve for each individual original vertex:

Gapped Weight Distribution Problem

Given graph $G = (V, E)$, $\sigma \in \mathbb{R}^V_{\geq 0}$, $W \in \mathbb{R}_{\geq 0}$ with $W < \sigma(V)$ and $\gamma \in \mathbb{R}_{>0}$:

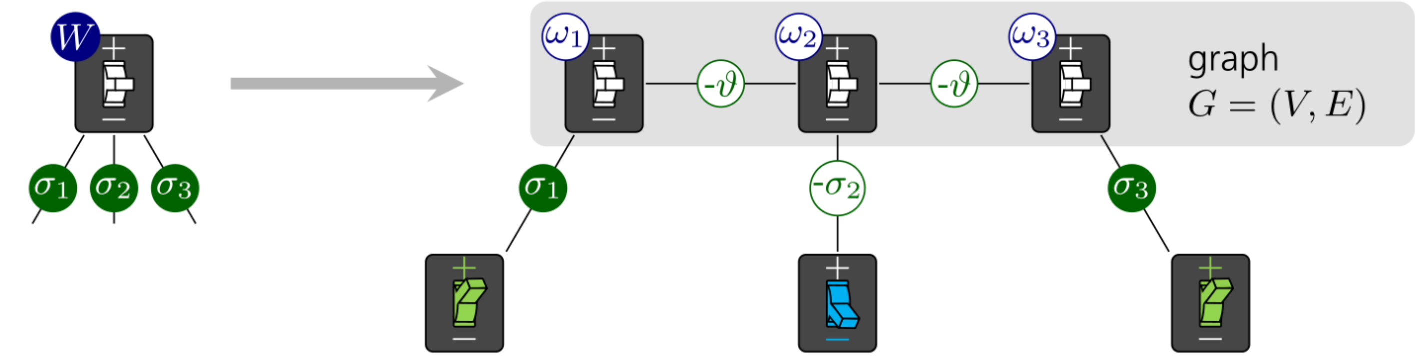
$$\min \vartheta$$

$$\text{s.t. } \vartheta \in \mathbb{R},\ \omega \in \mathbb{R}^V$$

$$\vartheta \geq \frac{\min\{\sigma(S) + \omega(S), \sigma(V \setminus S) - \omega(V \setminus S)\} + \gamma}{|\delta(S)|} \quad \forall \emptyset \neq S \subsetneq V,$$

$$\omega(V) = W$$

polynomial for trees



- cut constraints are redundant for S or $V \setminus S$ being not connected
- **efficiently solvable** in practical embedding setup

How large does the coupling strength ϑ need to be, but how small can it be?

Get the Optimal Embedded Ising Problem

Lobe, Kaibel, 2023

Lobe, PhD Thesis, 2022

DLR

Requires to solve for each individual original vertex:

GAPPED INTEGER WEIGHT DISTRIBUTION PROBLEM

Given graph $G = (V, E)$, $\sigma \in \mathbb{N}^V$, $W \in \mathbb{N}$ with $W < \sigma(V)$ and $\gamma \in \mathbb{N}_+$:

$$\min \vartheta$$

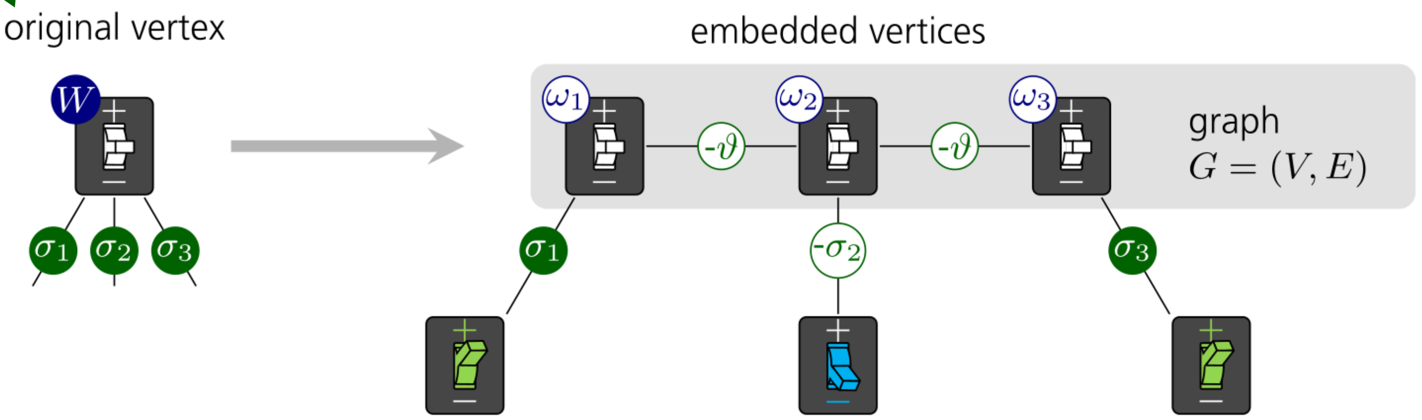
$$\text{s.t. } \vartheta \in \mathbb{Z},\ \omega \in \mathbb{Z}^V,$$

$$\vartheta \geq \frac{\min\{\sigma(S) + \omega(S), \sigma(V \setminus S) - \omega(V \setminus S)\} + \gamma}{|\delta(S)|} \quad \forall \emptyset \neq S \subsetneq V,$$

$$\omega(V) = W$$

polynomial for trees

- cut constraints are redundant for S or $V \setminus S$ being not connected
- **efficiently solvable** in practical embedding setup
- even for integer problem



optimal embedded Ising problem

DLR-SC Quantum Computing Software
gitlab.com/quantum-computing-software

First Insights after Annealing Runs

➔ Comparison to the current implementation by D-Wave

Random Ising Problems	Graph Partitioning Problems

- Integer version of OEIP produces significantly better coefficient ratios
- Best raw solution or solution with best chain break fraction does in general not correspond to best de-embedded solution
- Certain embeddings seem to be produce higher chain breaks than others for diverse instances, in particular the clique embedding

Random Ising Problems

- Very slightly reduced chain break fraction
- Significantly worse best objective values

Graph Partitioning Problems

- Significantly improved chain break fractions
- Slightly improved best objective values

Imprint

Author:	Elisabeth Lobe
Institute:	German Aerospace Center (DLR) – Institute of Software Technology (SC)
Date:	2025-03-14
Image sources:	All images “DLR (CC BY-NC-ND 3.0)” unless otherwise stated