Quantum Optimization for Phase Unwrapping in SAR Interferometry

Kay Glatting, Jan Meyer, Sigurd Huber, Gerhard Krieger

German Aerospace Center (DLR), Microwaves and Radar Institute
Kay.glatting@dlr.de

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Phase Unwrapping

Phase Unwrapping (PU) is a postprocessing imaging technique used in interferometric synthetic aperture radar height and deformation estimation, tomography as well as medical applications such as magnetic resonance imaging. Given a wrapped phase interferogram, $\Psi \in [-\pi,\pi)^{n\times m}$ determine a per pixel ambiguity correction $k \in \mathbb{N}_0^{n\times m}$ to estimate the original phase $\Phi \in [-\pi,\infty)^{n\times m}$ through

$$\Phi = \Psi + 2\pi k$$

since

$$\mathrm{mod}_{2\pi}\Phi = \Psi.$$

This is a mathematically ill-posed inverse problem, unless the relative differences between two pixels are less than π in the original phase everywhere. In practice this is rarely the case, due to aspects such as low SNR, bad resolution, and complicated topographies.

PU is normally approached by minimizing

$$\mathbf{C}(k) = \left\| \sum_{(ij,\tilde{i}\tilde{j})\in E} k_{ij} - k_{\tilde{i}\tilde{j}} - \left\lceil \frac{\psi_{ij} - \psi_{\tilde{i}\tilde{j}}}{2\pi} - 0.5 \right\rceil \right\|_p$$

where E is the list of adjacent pixels and $p \geq 0$. The resulting ambiguity corrections will exhibit phase jumps (compared to the original phase), so called phase errors. The amount of phase errors decreases as p decreases, but the computational complexity conversely increases. Only $p = 2$ and $p = 1$ are used in practice, with the latter, sometimes called branch-cut/simplex approach, used almost exclusively due to the great tradeoff between computational speed and unwrapping accuracy. Fig 2. contains an example of such an approach. Note the unwrapping errors and the characteristic overall loss in height delta.

The best unwrapping results are expected by using the Hamming distance ($p = 0$), which turns the optimization into an NP problem.

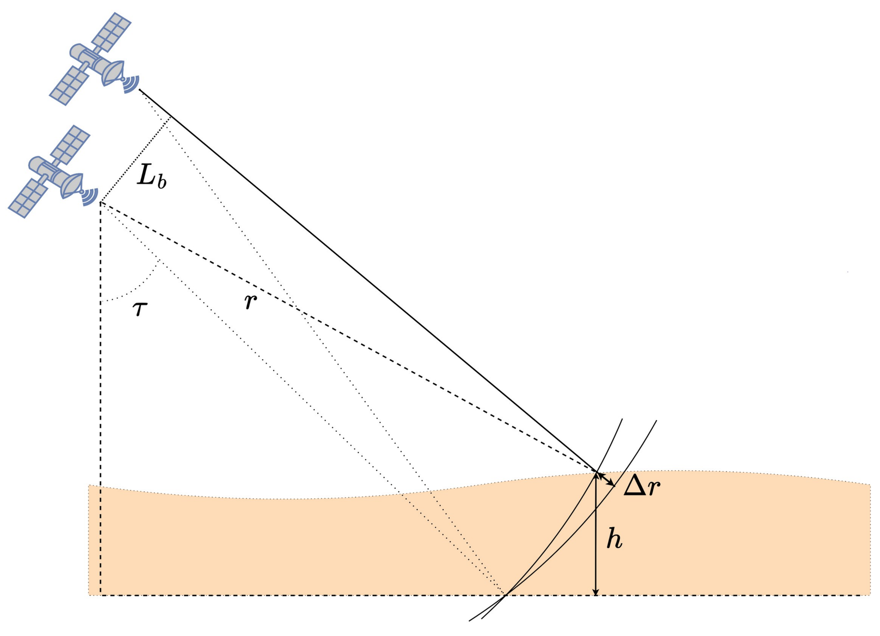


Fig 1: The principle InSAR geometry: Difference in the absolute phase between the two acquisition allows for height estimation

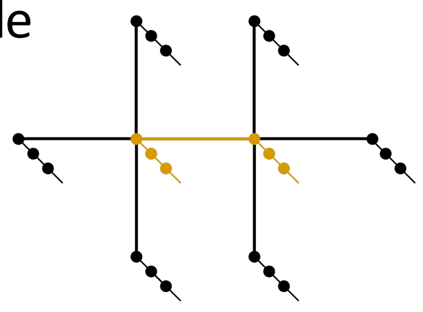
L0-PU via Quantum Approximate Optimization Algorithm

We therefore explore the potential usage of the quantum approximate optimization algorithm (QAOA) for L0 PU.

The QAOA algorithm consists of two principal components: Determining optimal classical parameters $\alpha \in [-\pi,\pi)^q, \beta \in [0,2\pi)^q$ and using quantum circuit Fig 3. to create the state

$$\frac{1}{\sqrt{2^N}} \prod_{l=0}^{q-1} \mathbf{U}(\beta_{q-l},\mathbf{X})\mathbf{U}(\alpha_{q-l},\mathbf{C}) \sum_{s=0}^{2^N-1} |s\rangle$$

We can significantly reduce the classical optimization complexity by treating phase differences between adjacent pixels as regular four-graphs (here example for $q = 1$). Resulting subgraphs contribute equally if they are the same. This allows us to count the occurring subgraphs, leading to a reduced classical function of the form



$$\langle \alpha,\beta|\,\mathbf{C}\,|\alpha,\beta\rangle = \sum_g \omega_g \sum_{\substack{s\in\{0,..,2^{n_{q,b}}-1\} \\ \mathbf{C}_{g_0}(s)=1}} \left\| \eta_{q,b}^{\alpha,\beta}(s) \right\|_2^2$$

Where g is the subgraph types and ω_g the number of occurrences. The complexity class now only scales with the circuit depth. We can further reduce this representation by collecting terms. The overall approach is summarized in Algorithm 1. on the right.

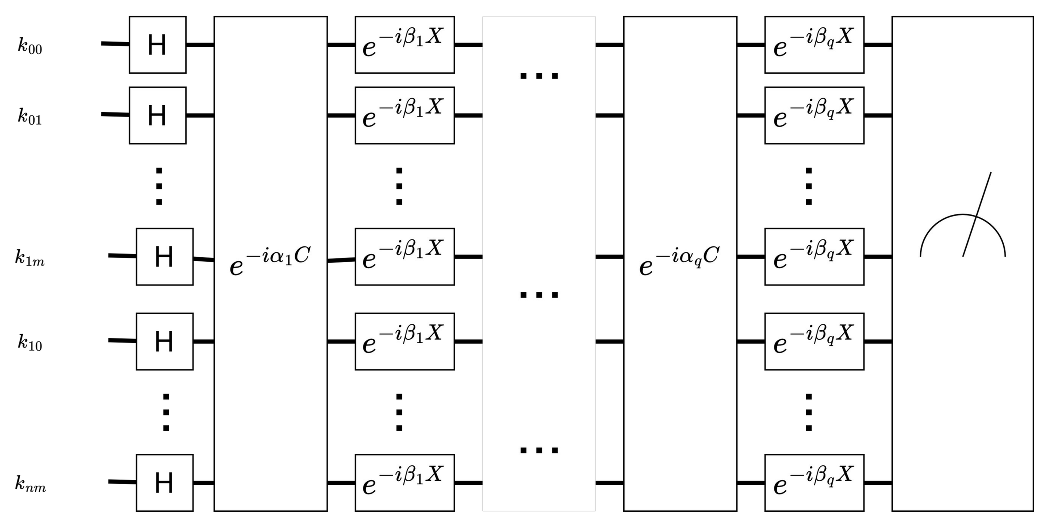


Fig 3: The QAOA circuit

Algorithm 1: Phase Unwrapping via L^0 QAOA.

Data: Wrapped phase $\psi \in [-\pi,\pi)^{n\times m}$
Result: Optimal correction term k that maximises $\mathbf{C}(k)$

1. g, ω_g: Count the occurring subgraphs per isomorphism class in ψ;
2. Λ_g: Determine the weights per isomorphism class;
3. Λ: Calculate the overall weights for the reduced representation
4. α, β: Use a non-convex optimisation algorithm for the reduced optimisation problem to determine the optimal set of control parameters;
5. k: Run circuit in Fig. 3 using the set of control parameters α, β;
6. k: Repeat step 5 to achieve statistical significance, keeping the result with the highest weighting function value $\mathbf{C}(k)$;

Small-Scale Demonstration

Following is a small scale PU demonstration of a 5 by 3 pixels scene using the L0-PU simulated in qiskit. Fig 6. top left is a terrain model of a typical scene that L1 unwrapping struggles with: A steep and a gradual decent close to each other. L1 unwrapping produces a flattened scene while the L0 QAOA properly reconstructs the scene. 10^5 measurements were used (compare $2^{30} \approx 10^9$ tries for brute force) to find the optimal solution (outlined in Fig. 4). The classical optimization function is represented in Fig. 5

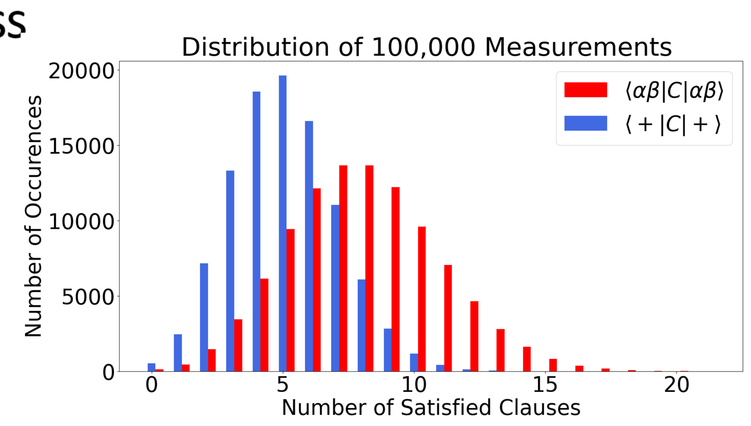


Fig 4: Statistical evaluation of the measurement ofc quantum circuit

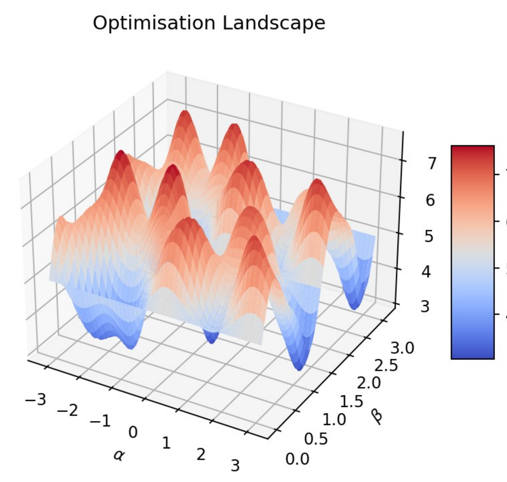


Fig 5: Energy landscape of the classical optimization function

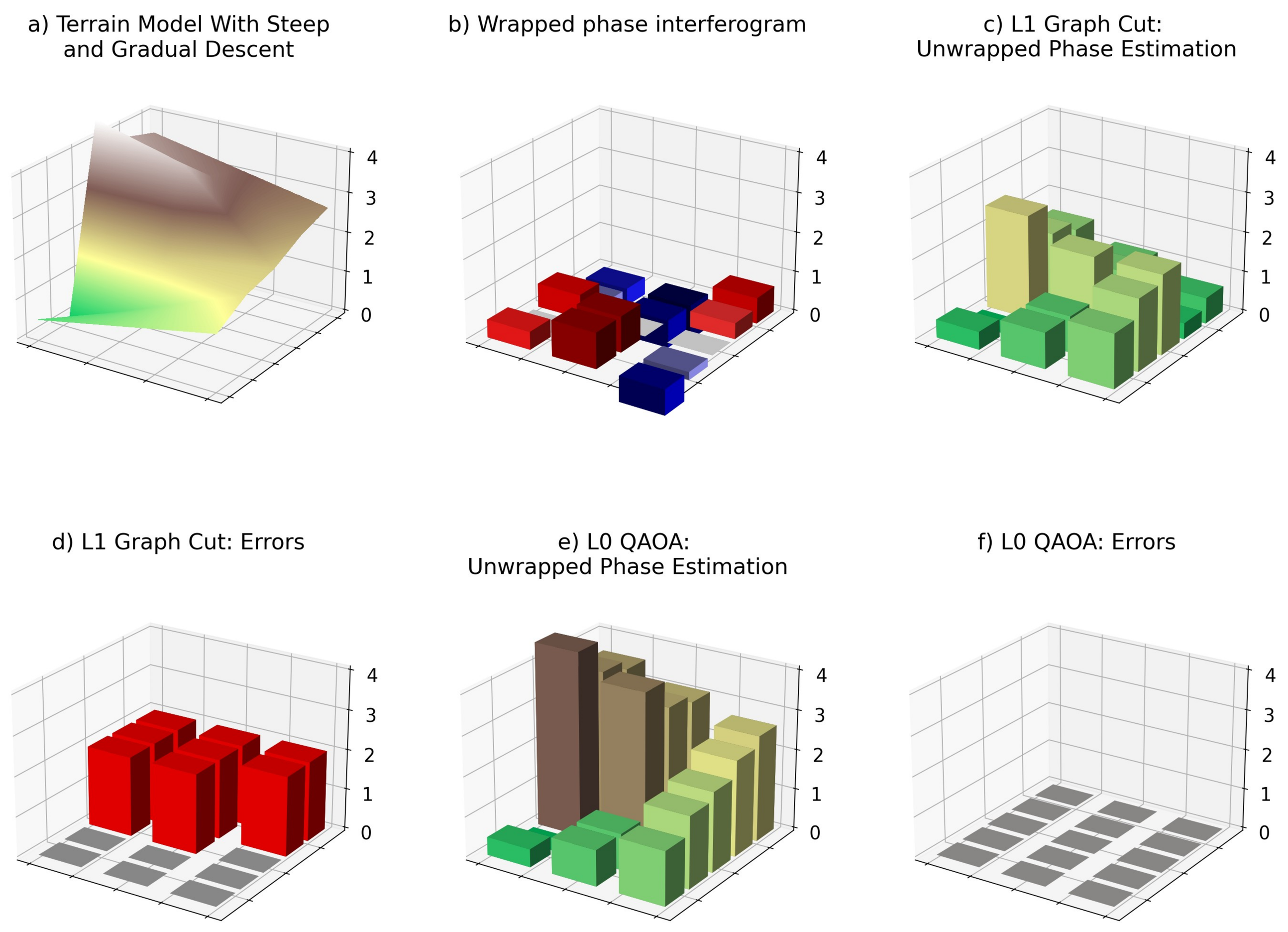


Fig 6: Small scale example. The L0 QAOA (bottom center and right) manages to reconstruct the original scene (top left and center), while the L1 algorithm produces errors

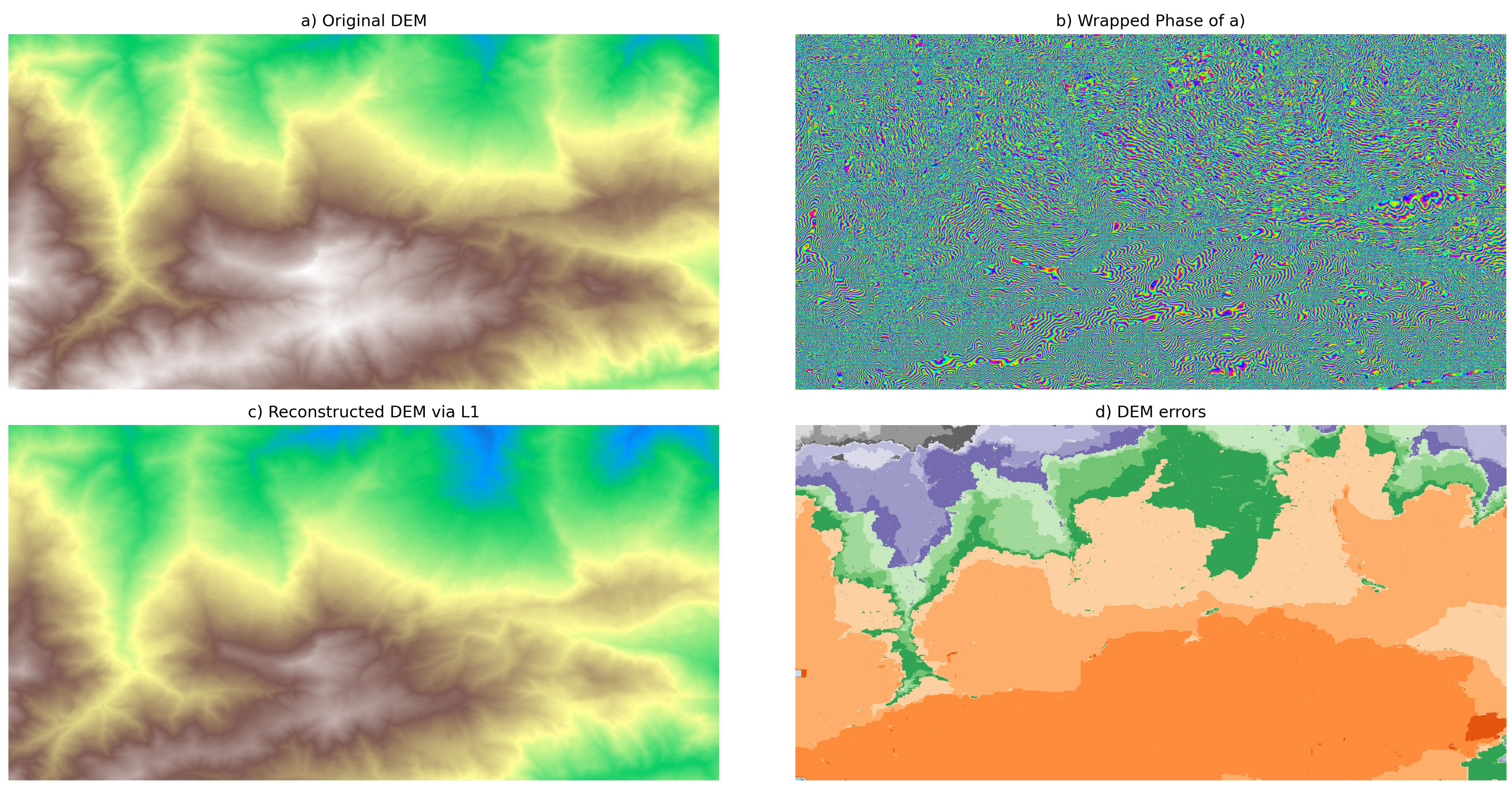


Fig 2: The L1 Phase Unwrapping Process. Acquisition of the top left scene generates the wrapped phase top right.. Bottom left is the reconstructed DEM via L1 phase unwrapping. Bottom right is the differences to the original. Bottom right denotes the differences to the original. Each color shift represents a phase error of 2π, with bright orange denoting the highest loss.

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