Mitteilung

Fachgruppe: Turbulenz und Transition

Kinetic Energy Budget of Secondary Motions in Sinusoidally-tempered Vertical Turbulent Pipe Flow

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Solar power towers represent one of the most promising concentrated solar power technologies for future large-scale renewable energy generation. In such plants, a heliostat field focuses solar radiation onto one side of the central vertical receiver, imposing a highly non-uniform heat flux on the fluid as it ascends and descends within the receiver tubes [1]. To get a better understanding of the secondary motions occurring in such flows, the present study investigates the kinetic energy budget of secondary motions in a sinusoidally tempered vertical turbulent pipe flow using direct numerical simulations. Thus, the incompressible Navier–Stokes equations with the Boussinesq approximation and the energy equation in their dimensionless form,

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} + \nabla p = \left(\frac{1}{Re_b}\right) \nabla^2 \vec{u} + \left(\frac{Gr}{Re_b^2}\right) \theta \, \delta_{ij}, \tag{1}$$

$$\nabla \cdot \vec{u} = 0, \tag{2}$$

$$\frac{\partial \theta}{\partial t} + \vec{u} \cdot \nabla \theta = \left(\frac{1}{Pr \, Re_b}\right) \nabla^2 \theta, \tag{3}$$

are discretized using a finite-volume method with fourth-order spatial accuracy [2], and are integrated in time employing a semi-implicit second-order Euler–Leapfrog scheme [3]. The grid resolutions are adopted from previous works [4, 5, 6]. Normalizing equations (1) to (3) with the bulk velocity u_b and the pipe diameter D results in the bulk Reynolds number $Re_b = u_b D/\nu$, the Prandtl number $Pr = \nu/\kappa$, and the Grashof number $Gr = g\beta\Delta T u_b^3/\nu^2$. Here, ν is the kinematic viscosity, κ is the thermal conductivity, g is the gravitational acceleration, β is the thermal expansion coefficient, and ΔT is the temperature difference between the maximum and minimum imposed wall temperature. In the present case the characteristic parameters are set to $Re_b = 5300$, $Pr = 0.71$, and $Gr = 9.5 \times 10^6$. The flow geometry is a smooth-walled pipe of length $L = 21D$, with no-slip and impermeability boundary conditions at the wall and the wall temperature varied according to $\theta_w = 0.5 \, sin(\varphi)$, where φ is the azimuthal direction. The temperature is normalized with ΔT and the arithmetic mean of the wall temperatures T_0, leading to $\theta = (T - T_0)/\Delta T$.

The buoyancy force induces acceleration (deceleration) of the warm (cold) flow regions, which in turn gives rise to secondary motions, as shown in **Figure *1***(a), where the mean temperature profile is superimposed with the velocity field of the mean secondary flow, $\langle u_\varphi \rangle_{zt} \, \vec{e}_\varphi + \langle u_r \rangle_{zt} \, \vec{e}_r$. As expected, the fluid moves upward from the warmest to the coldest flow region, traveling through the pipe center where the the maximum temperature gradient occurs, and returns downward along the wall. **Figure *1***(b,c,d) show some examples of different terms of the kinetic energy budget of these secondary motions. **Figure *1***(b) depicts the production term, which is responsible for the energy transfer between secondary motions and the turbulent fluctuations. The production mechanism causes the strongest energy transfer into the turbulent velocity field in the region of maximum velocity gradients, i.e. near the lower part of the pipe wall, whereas in the remaining near-wall regions of the pipe, a reverse transfer occurs, feeding energy back from the fluctuating velocity field into the mean secondary flow. The dissipation from the secondary motions presented in **Figure *1***(c) is strictly negative and restricted to regions close to the pipe wall, attaining its minimum where the secondary motions are most intense.

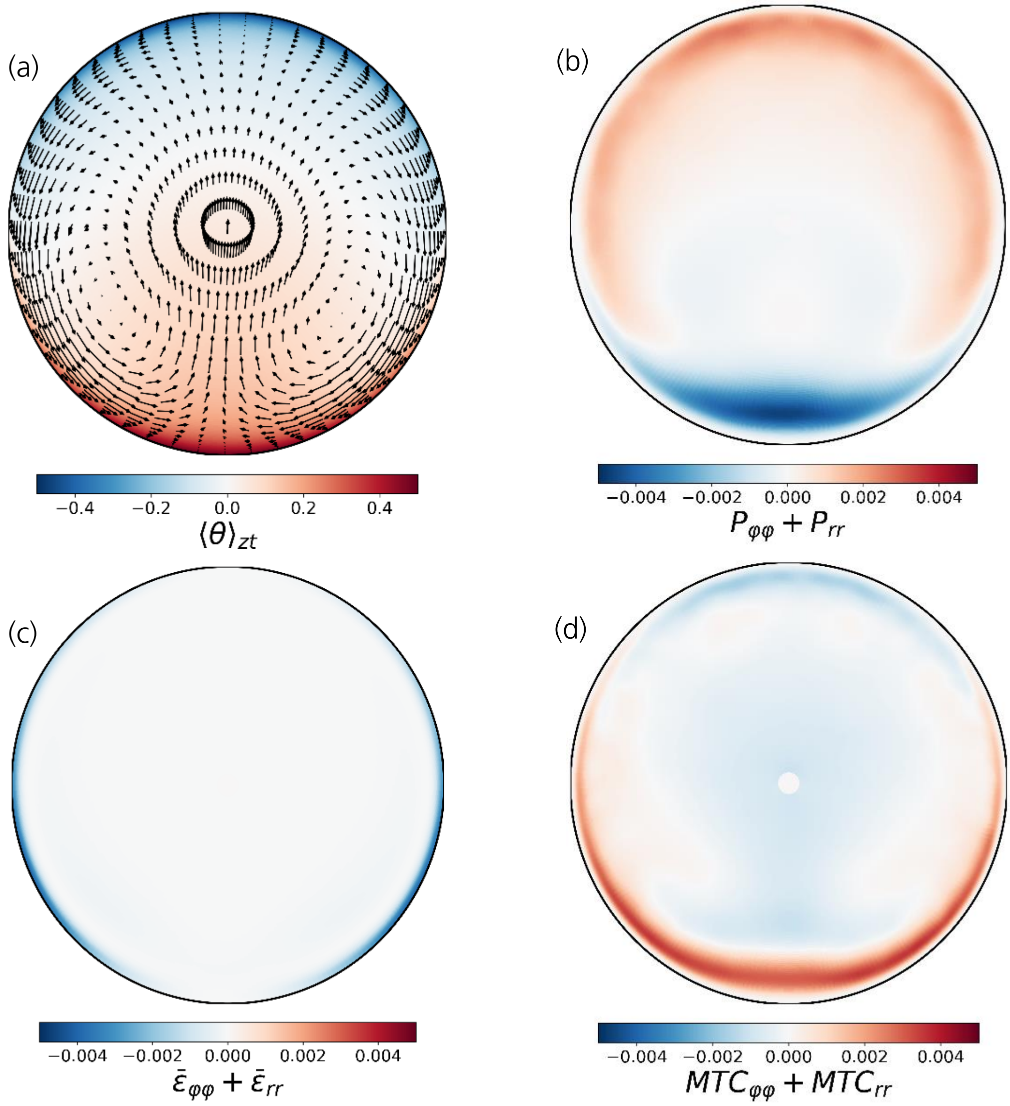

Figure 1: *(a) Mean temperature* $\boldsymbol{\theta}$ *and velocity profile of the mean secondary flow . (b) Production of turbulent kinetic energy due to the mean secondary flow. (c) Dissipation due to the mean secondary flow. (d) Convection of the mean secondary flow due to turbulent fluctuations.*

At the bottom center, the wall-parallel velocity component vanishes due to flow symmetry, which minimizes shear between the fluid and the wall and thus reduces the dissipation to nearly zero in this region. **Figure 1**(d) shows the convection of secondary flow kinetic energy due to turbulent fluctuations. This term reaches its largest positive values in the warmest flow regions along the lower part of the pipe wall. On the opposite wall, a less pronounced negative minimum appears. The kinetic energy budget of the secondary motions will be discussed in more detail at the workshop.

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