

# A Finite Element Approach to Contact Problems with Roughness

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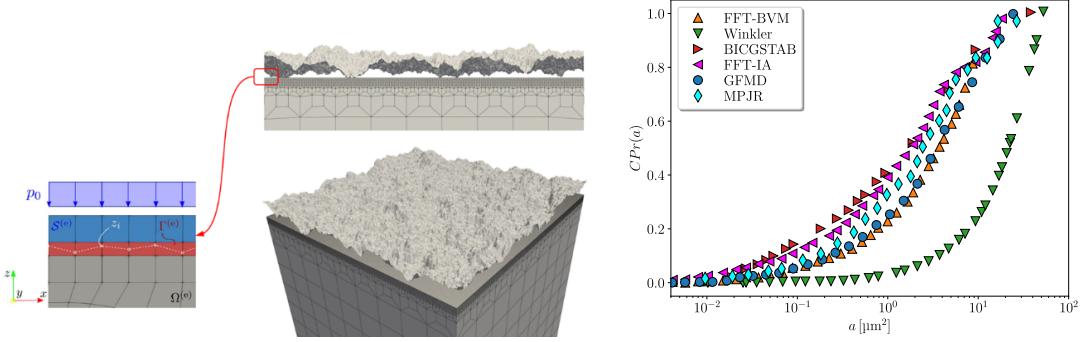
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The field of contact mechanics plays an important role in many different areas related to critical infrastructures protection (CIP) by providing a deep understanding of complex systems behavior under various loading conditions, also related to extreme scenarios far from the usual serviceability requirements, like extreme natural events or intentional attacks. Contact mechanics contributes to the development of more resilient design strategies thanks, e.g., a better comprehension of the mechanisms that lead to material failure, an essential asset for the development of predictive models to enforce CIP measures. More specifically, interactions occurring at the interface between contacting bodies play a major role in stress transfer, friction, wear, and heat and electric conduction, such that high-fidelity physics-based simulations accounting for surface texture and microscopic roughness are required [1]. Solutions based on the boundary element method (BEM) have been widely employed, since they can provide an accurate response at the interface level without the need to discretize the whole domain. However, they manifest limited flexibility in terms of material models and domain configurations. To overcome these limitations a solution approach based on the finite element method (FEM) has been proposed in [2], capable of circumventing the difficulties of standard FEM contact search algorithms in the presence of microscopic roughness. The proposed technique has been labelled eMbedded Profile for Joint Roughness (MPJR), and is based on directly embedding the deviation from planarity of a rough surface into interface finite elements, to be employed as a correction to the normal gap function computed if the surfaces were flat. The method can be applied to both rigid-deformable and deformable-deformable solids in contact and is efficient and adaptable since, regardless the actual geometry, (i) the interface can be treated as smooth, (ii) any desired height field can be incorporated, either coming from an analytical expression or from synthetic or experimental data, (iii) no smoothing nor regularization of the height field is necessary, since fixed pair of nodes are considered at the interface.

To enhance flexibility in utilizing the MPJR method within different analysis software, the surface data field is stored in a history variable containing elevation values associated with each spatial coordinate. This step is performed only once during the initial simulation setup. During the contact problem solution process, the normal gap between the interface elements is computed using standard kinematic relations and the calculated values are then adjusted by incorporating the actual embedded elevations at the nodal points as perturbations from the assumed planarity, Fig. 1a.

The method has been tested with both frictionless and frictional contact problems in partial slip [3] and for adhesive contacts [4]. Furthermore, it has been successfully employed in conjunction with a phase field approach to brittle fracture, to simulate the complex nonlinear coupled problem of contact-induced fracture determined by spherical rough indentation [5], with a very good agreement with available experimental data. Finally, the methodology has been also applied to the solution of large scale contact problems, leveraging the parallel computing implementation of the FEM analysis software FEAP,



(a) Sketch of the original contact problem together with the proposed solution approach. The information related to the original geometry is encoded in a single boundary layer of interface finite elements placed over the deformable bulk.

(b) Cumulative distribution of contact areas obtained using MPJR interface finite elements, compared with the reference solution of the CMC and results obtained employing different solution strategies.

Figure 1: Overview of the solution framework and capabilities of the proposed solution method.

addressing the problem posed by the well known contact mechanics challenge (CMC) [6]. The solution provided by the MPJR approach is shown in Fig. 1b in terms of cumulative distribution of contact areas, and is capable of reproducing with remarkable accuracy the results of the reference solution, obtained using a Green's function molecular dynamics (GFMD) approach. To the best of the authors knowledge, this is the first time that a FEM-based solution is proposed to tackle the challenge, paving the way for new opportunities in the field of high-fidelity modelling of large scale rough contact problems.

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