Direct Sampled-Data Design of an Optimal Step-Tracking Control for a DC-DC Converter

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Abstract

This work considers the step-tracking problem on a DC-DC buck converter, whose objective is to both track an step voltage reference and reject step disturbances of the load current. A direct sampled-data approach is used for the synthesis of digital controllers for this task. The approach used is based on the concept of the Parametric Transfer Function, which is an analysis technique that allows to treat in the frequency domain systems with both continuous and discrete signals. Simulations are used to present the results of the synthesis and advantages and disadvantages are discussed.

1 Introduction

Switched power converters, composed of continuous electrical elements actuated by inherently discrete signals from a PWM generator, are a good example of a *sampled-data (SD) system*, a term usually applied to systems in which a discrete controller drives a continuous plant. This was already considered in [1], where the limitations of continuous state-space averaging and discrete modeling were compared and the idea of sampled-data modeling for these systems was introduced.

As discussed in [2], the strategies for analysis and control synthesis in SD systems can be classified into two types, known as *computer oriented* and *process oriented* approaches. Strategies belonging to the first type consider the system from the point of view of the digital computer and all signals are treated at the discrete sampling instants only. This can be done be either discretizing a controller previously designed in continuous time or by discretizing the model of the plant and performing the control synthesis using discrete methods. Most of the literature related to the control of DC-DC converters uses computer oriented approaches. Prime examples are [3], a work in which discrete models of the converter are considered from the beginning of the analysis, and [4], where an ideal continuous response is used to define the locations at which the closed loop poles will be placed.

Process oriented strategies, on the other hand, treat the whole SD system in continuous time. Since discrete elements become periodic-time-varying when treated in continuous time, process oriented methods require more elaborated tools to model the plant, the controller, and the signals present in the system. In [5] methods based on *lifting* the dimensionality of continuous signals are introduced, which transform continuous signals into discrete periodic signals of infinite dimension. Usual control synthesis problems, such as H_2 optimization, become then optimization problem for discrete systems. This is one of the most widely known process oriented approaches and was implemented in MATLAB as the *Sampled Data Control toolbox* [6, 7]. The lifting technique, however, is not applicable to some important practical cases, e.g., to systems with time delays.

In [8], the lifting technique was applied to the design of robust controllers for a DC-DC Buck converter following an H_∞ criterion. That work compares different approaches to the design of the robust controller, which they classify into an *indirect method*, in which a continuous controller is designed and then discretized, a *direct method*, in which the plant is discretized and then the controller is synthesized, and a method based on lifting. They show how the sampled-data design provides better results, particularly when switching frequencies are low.

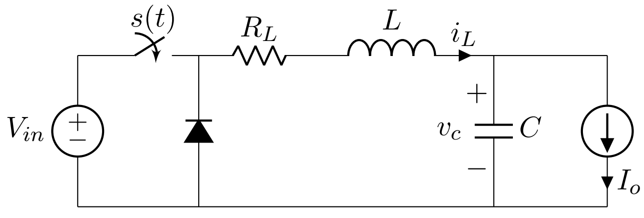


Figure 1: DC-DC Buck Converter

The frequency domain methods introduced by Rosenwasser and Lampe in [9, 10], based on the *Parametric Transfer Function* (PTF) concept, are a less-known alternative for the direct synthesis of controllers for SD systems. The PTF is a mathematical tool which can be used to represent both continuous and discrete systems, even simultaneously, making it specially well-suited to represent sampled-data systems. These methods can be used even when the continuous plant has time delays. Polynomial methods used for the synthesis of controllers under this approach were implemented in the he DirectSD toolbox for Matlab. In its initial version [11], this package was designed to implement these algorithms in the case of siso controllers. Further developments generated a mimo version [12], which uses the theory presented in [10]. These toolboxes became obsolete when the structure of the object-oriented programming model in Matlab was altered. Only some of the capabilities of the toolbox are now available.

The present paper shows an initial exploration of the application of the PTF approach to the design of controllers for switched power converters. A relatively simple problem is considered and described in section 2: a DC-DC buck converter operating as a voltage regulator in continuous conducting mode. This system is assumed to react to two input signals: a reference voltage to be tracked and a current signal demanded by the load. A controller structure is proposed based on feeding back the output voltage, and optimal controllers are found using the PTF approach. This is detailed in 3. Simulation results are presented and discussed in section 4 whereas some concluding remarks are presented in section 5.

2 Modeling

The schematic diagram of the DC-DC converter considered in this work is shown in Fig. 1. It is assumed that the inductor has a parasitic resistance R_L and that the semiconductors, i.e., the switch and the diode, both have a conducting resistance R_{on} (not shown in the diagram), so the total resistance of the LC circuit is $R_t = R_L + R_{on}$. The independent current source I_o represents the load. This circuit is described in state-space by

$$\begin{aligned} \begin{bmatrix} \dot{v}_c(t) \\ \dot{i}_L(t) \end{bmatrix} &= \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R_t}{L} \end{bmatrix} \begin{bmatrix} v_c(t) \\ i_L(t) \end{bmatrix} \\ &+ \begin{bmatrix} 0 & -\frac{1}{C} \\ \frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} v_u(t) \\ I_o(t) \end{bmatrix} \\ v_o(t) &= v_c(t), \end{aligned} \tag{1}$$

where $v_u(t)$ is a control voltage, which is applied as a PWM signal with switching frequency f_{sw} and period $T_{sw} = 1/f_{sw}$ by the gate signal represented by $s(t)$ in Fig. 1.

The objective of a voltage regulator is to ensure that $v_o(t)$, the voltage across the load, tracks a given reference value V_{ref}. For this, the control architecture shown in Fig. 2 is introduced. It consists on a feedforward path, which sets the proper operating point to account for large variations of the input values, and a discrete feedback controller $C(z)$. The total control command, i.e., the sum of the feedback and feedforward signals, is modulated by a PWM block and applied to the inductor, which in turn is affected by the voltage across the capacitor due to

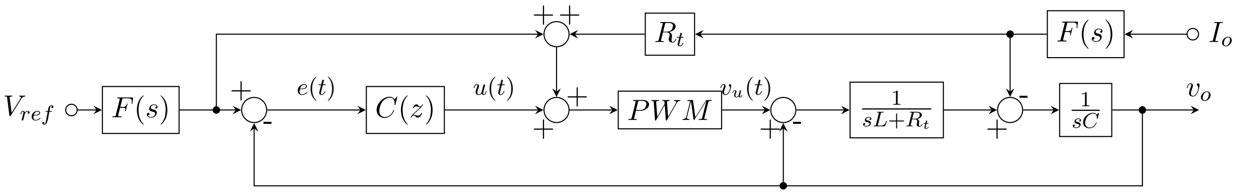


Figure 2: Block diagram of the voltage regulator control structure.

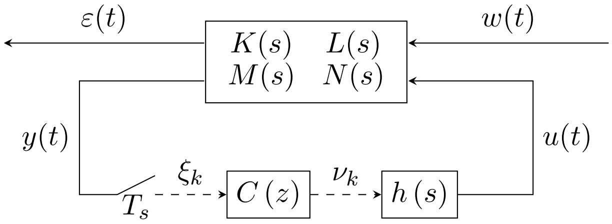


Figure 3: Standard sampled-data system.

the intrinsic feedback of the RC circuit. This voltage also represents the output of the system, and corresponds to a weighted integral of the difference between the currents through the inductor and the load.

The blocks with transfer functions $F(s)$ are forming filters required to define the optimization problem properly. Their usage will be explained in the following section.

It is assumed that the only signal that feedback controller measures is the output voltage, which is then compared to the reference to obtain an error signal. It is also assumed that the feedforward controller works in continuous time and can provide the value of both inputs instantaneously. Work is already ongoing to remove this very strong assumption made on the preliminary analysis.

The feedback controller $C(z)$ is assumed to be operating at a sampling frequency f_s considerably smaller than the PWM switching frequency f_{sw}, and to provide its output via a zero order hold. Following this, it can be assumed that the effective voltage applied to the circuit during a certain sampling period is approximately constant and equal to dV_{in}, where $d \in [0, 1]$ is the duty cycle of the PWM signal.

For the preliminary design of the feedback controller, the PWM actuation will be ignored. Its effects will be later discussed in section 4.

3 Controller Synthesis

Any sampled-data system, like the one under consideration, can be represented by the standard form of Fig. 3, for which

$$\begin{aligned} \varepsilon(s) &= K(s)w(s) + L(s)u(s) \\ y(s) &= M(s)w(s) + N(s)u(s) \end{aligned}. \tag{2}$$

The process of designing a controller using the PTF approach begins with the determination of the transfer functions in (2). From the block diagram in Fig. 2 it is clear that the system under consideration has two inputs: the voltage reference $V_{ref}(t)$ and the load current $I_o(t)$, so $w(t) = [V_{ref}(t)\ I_o(t)]^T$. The feedback controller $C(z)$ uses as input the error signal $y(t) = V_{ref}(t) - v_o(t)$ and produces a control output $u(t)$. In a computer implementation of the controller, the error $y(t)$ is sampled with a period T_s to generate a discrete sequence $\xi_k = y(kT_s)$, $k = 0, 1, 2, \ldots$ and the discrete output of the digital controller, ν_k, passes through a zero-order hold device $h(s)$ to generate the continuous control action $u(t) = (t - kT_s)\nu_k$, for $kT_s < t < (k+1)T_s$.

The cost function for the optimization problem is the $\mathcal{L}_2$-norm of the output signal $\varepsilon(t)$, so selecting the components of the output signal influences the optimization target and the controller synthesis. Since the main interest in this work is step tracking, the goal should be to keep the error signal $e(t) = V_{ref}(t) - v_o(t)$ as small as possible. This signal is therefore to be considered at the output. However, it is always a good practice to consider also the control effort in the optimization routine, so $v_u(t)$ will be also included, weighted by a factor $\lambda > 0$. The output of the system is then taken as $\varepsilon(t) = [e(t)\ \lambda v_u(t)]^T$, and the optimal cost to be minimized is the total variance of this signal

$$J = \int_0^\infty e^2(t) + \lambda^2 v_u^2(t) dt. \tag{3}$$

The quadratic optimization based on the PTF provides a controller that minimizes (3) assuming impulse signals at the inputs. Since the objective of the work is to provide optimal step tracking, the forming filters $F(s) = \frac{1}{s}$ are introduced in Fig. 2 to transform them into step signals.

With the definitions of the inputs and outputs of the standard system, it is straightforward to find that, for the system under consideration, the continuous transfer functions in (2) are

$$\begin{aligned}
K(s) &= \begin{bmatrix} \frac{s^2+\frac{R_t}{L}s}{s^3+\frac{R_t}{L}s^2+\frac{1}{LC}s} & \frac{\frac{1}{C}}{s^2+\frac{R_t}{L}s+\frac{1}{LC}} \\ \frac{\lambda}{s} & \frac{\lambda R_t}{s} \end{bmatrix}, \\
L(s) &= \begin{bmatrix} \frac{-\frac{1}{LC}}{s^2+\frac{R_t}{L}s+\frac{1}{LC}} \\ \lambda \end{bmatrix}, \\
M(s) &= \begin{bmatrix} \frac{s^2+\frac{R_t}{L}s}{s^3+\frac{R_t}{L}s^2+\frac{1}{LC}s} & \frac{\frac{1}{C}}{s^2+\frac{R_t}{L}s+\frac{1}{LC}} \end{bmatrix}, \\
N(s) &= \frac{-\frac{1}{LC}}{s^2+\frac{R_t}{L}s+\frac{1}{LC}}.
\end{aligned} \tag{4}$$

The quadratic error (3) can be expressed in the frequency domain, using Parseval's formula, as

$$J = \frac{1}{j2\pi} \int_{-j\infty}^{j\infty} e(s)e(-s) + \lambda^2 u(s)u(-s) ds. \tag{5}$$

The integral (5) requires the frequency domain representation of signals that have passed through both continuous and discrete processes. The usual transformations into the frequency domain apply only for signals in a single time domain: the Laplace transformation for continuous time signals and the Z-transformation for those in discrete time. The transfer functions of systems in one domain do not mix with those of systems in the other domain, so a different approach is needed here, one that allows the representation of both type of signals and systems simultaneously. This can be achieved by using the *Parametric Transfer Function* (PTF). Introduced in [9], the PTF can be used to represent both continuous and discrete systems, which makes it specially suited for the analysis of sampled-data systems.

First of all, a new transformation, the *Discrete Laplace Transformation* (DLT) for signals in time domain, is defined. For a continuous signal $f(t)$ the DLT is defined as

$$\mathcal{D}_f(T, s, t) = \sum_{k=-\infty}^{k=\infty} f(t + kT)e^{-ksT} \tag{6}$$

where $T > 0$ is a real parameter and $\omega = 2\pi/T$. The DLT can also be defined in terms of the Laplace image of the signal. Assuming $F(s) = \mathscr{L}\{f(t)\}$,

$$\mathcal{D}_F(T, s, t) = \sum_{k=-\infty}^{k=\infty} F(s + kj\omega)e^{(s+kj\omega)t}. \tag{7}$$

Notice that the sum of the series in (6) and (7) are identical.

The DLT is also defined for a discrete sequence $\{x_k\}$. In this case it takes the form

$$X^*(s) = \sum_{k=-\infty}^{k=\infty} x_k e^{-skT}, \tag{8}$$

which not entirely by coincidence looks like the z-transform when $z = e^{sT}$. In fact, for continuous signals with rational Laplace images, the DLT, for a given t, becomes a function of $\zeta = z^{-1} = e^{-sT}$.

The convergence criteria of the series in (6), (7), and several other properties, including closed formulas for the sums in most common cases, are presented in detail within [9].

Just like the traditional transfer functions for continuous and discrete systems are based on the Laplace and Z- transformations of the corresponding input and output signals, the PTF can be obtained by using the DLT of the input and output signals. Formally, however, for a system with input $x(t)$ and output $y(t)$, represented by an operator in the form

$$y(t) = \mathsf{U}\left[x(t)\right], \tag{9}$$

the Parametric Transfer Function $W(s,t)$ is defined assuming that the input has the form $x(t) = e^{st}$, with s being a complex constant, such that

$$\mathsf{U}\left[e^{st}\right] = W(s,t)e^{st}. \tag{10}$$

Notice that the PTF depends on two parameters: $t \in \mathbb{R}$ and $s \in \mathbb{C}$. It is important to take into account that s is a complex parameter and is not exactly the same Laplace variable [9].

For LTI systems, the PTF loses its dependency on t and is identical to the standard transfer function in the Laplace or Z-domain. For sampled data systems, in which continuous and discrete system are connected together, the PTF becomes periodic with respect to t. All these definitions and properties are used to formulate a single transfer function for a closed-loop system that includes discrete and continuous blocks simultaneously. Several different optimization problems can be then formulated on this basis.

By employing the DLT and PTF, the integral (5) can be discretized and represented as

$$\begin{aligned} J = \frac{1}{j2\pi} \int_{-j\infty}^{j\infty} & A(\zeta)W_d(\zeta)W_d(\zeta^{-1}) - B(\zeta)W_d(\zeta) \\ & -B(\zeta^{-1})W_d(\zeta^{-1}) + E(\zeta)\frac{d\zeta}{\zeta}, \end{aligned} \tag{11}$$

where $\zeta = z^{-1} = e^{-sT}$. How the coefficients $A(\zeta)$, $B(\zeta)$, $E(\zeta)$ are related to the transfer functions of the individual elements in Fig. 3 is presented with all details in [9, 10, 13]. The most important point is that, in (11), $W_d(\zeta)$ is the only element that depends on the transfer function of the controller $C(z)$.

The process of obtaining the optimal transfer function $C_{opt}(z)$ which gives the minimum value of a functional of the form (11) was first presented in [14]. In [9] it was shown how control synthesis problems within the framework of the PTF also lead to this type of functional, and polynomial methods to solve for the optimization problem are introduced in [13]. These polynomial methods were implemented in the MATLAB toolbox DIRECTSD [11, 12, 15]. Due to some obsolescence issues, the full toolbox runs only in relatively old versions of the software. A minimal set of function has nevertheless been adapted to the latest versions and was used to run the simulations described in the following section.

Table 1: Parametric Values used in the Paper

Parameter	Value
C	94 µF
L	1.34 mH
R_L	1.34 Ω
R_{on}	0.8 Ω
f_{sw}	20 kHz
f_s	5 kHz
V_{in}	20 V
$V_{ref}(0)$	3.3 V
$I_o(0)$	0.825 A

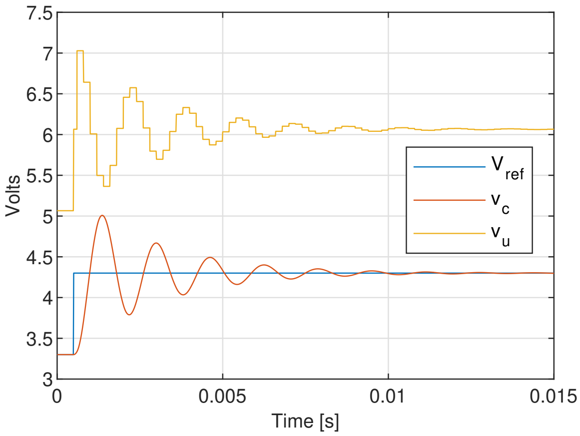


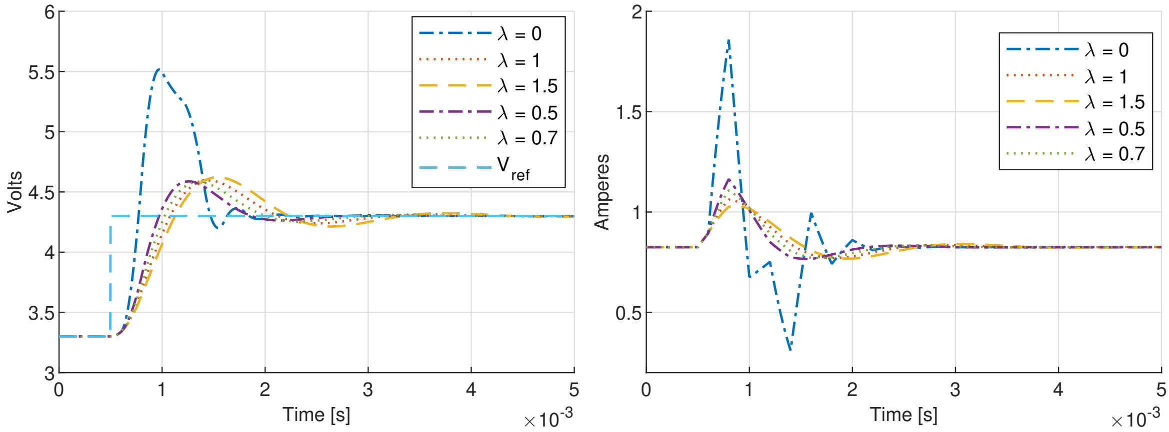
Figure 4: Response of the uncompensated system to a step in the voltage reference.

4 Simulations

Dynamic simulations of the DC-DC buck converter under study were executed to observe and compare the performance of different compensators. The Step response is the main focus of the observations. The system is started from equilibrium, with the voltage reference set to $V_{ref}(0) = 3.3$ V and the load current to $I_o(0) = 0.825$ A. Values of other parameters are as described in Table 1.

To have a starting point to compare the results obtained with different controllers, the system was first simulated without a compensator. The controller $C(z)$ is selected a a transfer function with unit gain, so the control action is simply a first order hold. The response of the system to a step of 1V in V_{ref} is presented in Fig. 4.

Compensators were synthesized for five different values of λ. The first value, $\lambda = 0$, was selected to observe the behavior of the system when only the tracking error is considered in the objective function, ignoring the control action. The other values consider both signals as equally important ($\lambda = 1$), the control action more important than the tracking error ($\lambda = 1.5$), the error more important than the control action ($\lambda = 0.5$), and a value which provides a qualitatively balanced result ($\lambda = 0.7$). The responses of the system to a step of 1 V in V_{ref} and 1 A in I_o are respectively shown in Fig. 5 and Fig. 6. One of the main drawbacks of this approach is the lack of direct relation between the parameter λ in cost function and the characteristics of the system response, like settling time or percentage of overshoot. This makes trial and error always necessary to obtain a desired response. This is an issue common to almost any optimal control problem in which the cost function is



(a) Voltage across the capacitor

(b) Current through the inductor

Figure 5: Responses of the system to a step of 1 V in V_{ref} for different values of λ.

not directly related to certain characteristics of the system.

From the plots it is possible to see how aggressive the control action results when it is not considered in the cost function. The system reaches the steady state very fast: the settling time for $\lambda = 0$ is bout 10% of what it was for the uncontrolled system. This, however, leads to very large overshoots in v_c and i_L, which is an undesirable behavior. This highlights again the importance of considering the control action in the cost function.

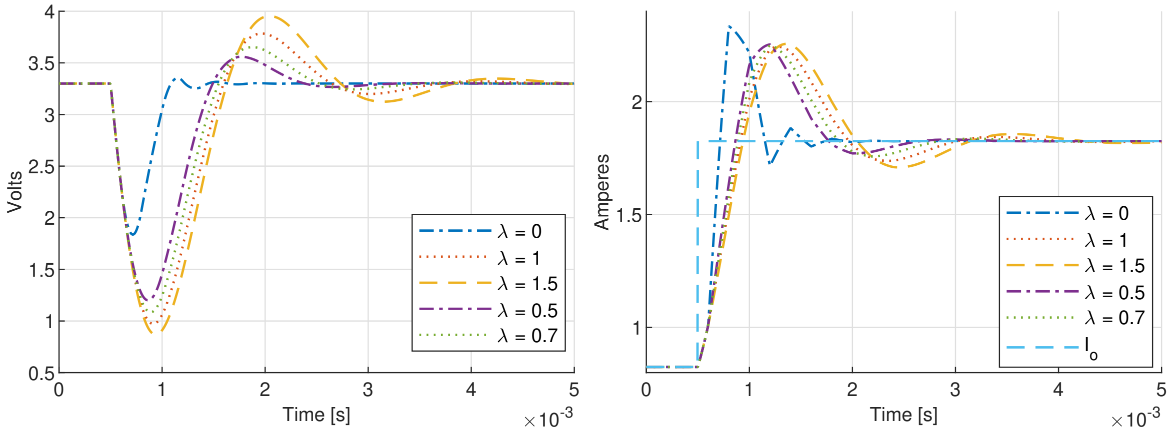
When the control action is included, the system response is smoother, but slower. For example, $\lambda = 1.5$ results in the slower system response, with a settling time of about 4 ms. Furthermore, including the control action in the cost function also leads to a worsening of the system response to changes in the load current. As seen in the top plot of Fig. 6, the drop on $v_c(t)$ after a step change in I_o increases with λ, whereas the peak value of the current through the inductor $i_L(t)$ is not affected that much by this parameter.

Figures 7 and 8 show the simulation results when the PWM actuator is included, for the case $\lambda = 0.7$. The modulation is run with a switching frequency of $f_{sw} = 20$ kHz, such that there are always four PWM switching cycles for each period of the discrete controller. The general performance of the system does not seem affected by the usage of the PWM.

5 Conclusions

The problem of designing a discrete controller for a DC-DC buck converter operating as a voltage regulator in continuous conduction mode is treated in this paper. The objective is to track the reference voltage and to maintain the regulation even when the current load is increased in a step. An optimal controller was synthesized to minimize a certain quadratic cost. A parametric study was performed to showcase the effect of different weighting values on the dynamic behavior of the closed loop system.

The main intention of this work is to showcase the advantages of the optimal approach for the synthesis of controllers for sampled-data systems based on the PTF. This is a tool has not yet been applied in the realm of switched power converters. The simulation results show that this is an approach that could work, but further explorations are needed to cover several issues. Some ways in which the performance of the controller could be improved are considering also the current on the inductor as part of the cost function, and the usage of MIMO controllers that feed back not only the output voltage but also the currents. Furthermore, any actual implementation of the feedforward structure for the load current will necessary incur in a time delay and its



(a) Voltage across the capacitor

(b) Current through the inductor

Figure 6: Responses of the system to a step of 1 A in I_o for different values of λ.

effect on the control performance should also be considered.

Further work is already ongoing along different paths. For example, in the simulations run here the load current was independent of the output voltage, a behavior that is not realistic. Simulations not based on the ideal dynamic behavior of the circuit elements but on more detailed physical models need to be used to evaluate the algorithms more appropriately. A physical implementation to conduct experiments on actual hardware is the next logical step, which is already planned.

Under the PTF approach, problems like robust optimization under H_∞ criteria can also be treated using. These methods and techniques will also be transferred to other topologies for switched power converters, including boost and buck-boost DC-DC converters and also DC-AC inverters.

Dedication

The author would like to dedicate this work to the memory of Professor Efim Natanovich Rosenwasser, one of the main propossers of the theory on which this paper is based. Prof. Rosenwasser passed away on the 14th of August 2025 at the age of 93 in his house in Selenogorsk, while the final version of this paper was being prepared.

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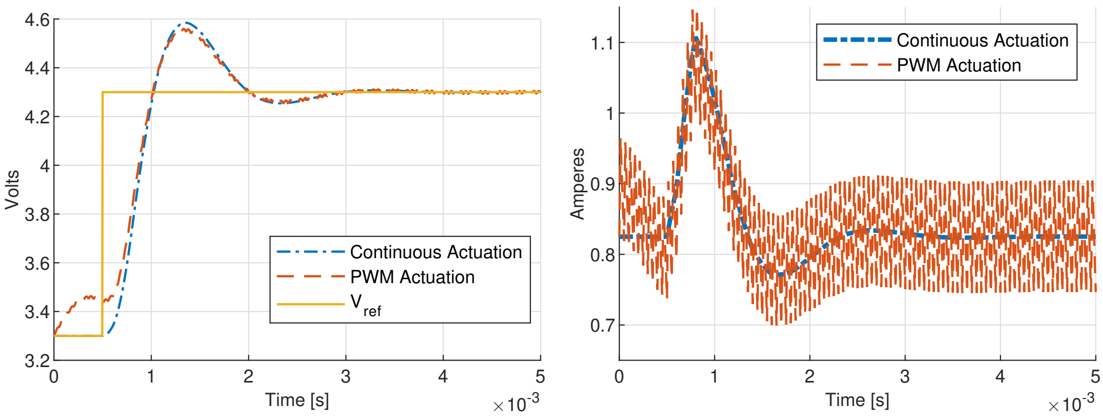
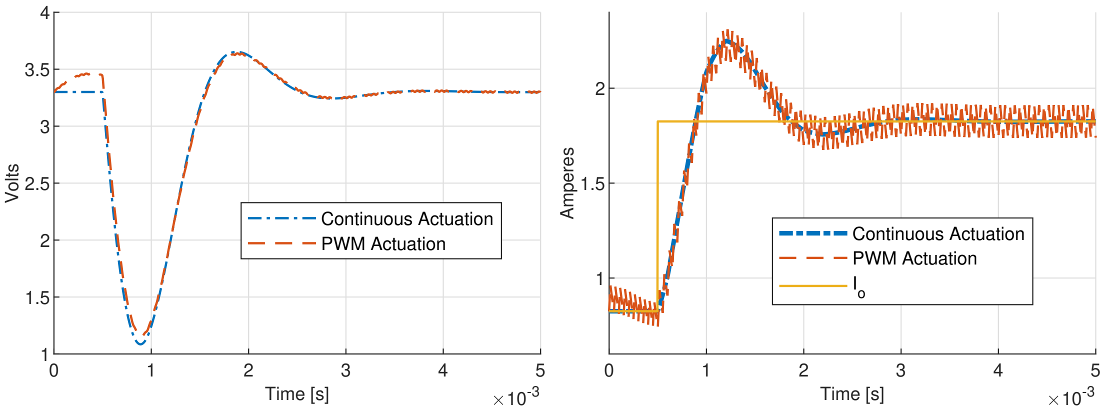


(a) Voltage across the capacitor (b) Current through the inductor

Figure 7: Responses of the system to a step of 1 V in V_{ref} for $\lambda = 0.7$.



(a) Voltage across the capacitor (b) Current through the inductor

Figure 8: Responses of the system to a step of 1 A in I_o for $\lambda = 0.7$.

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