CENTRIFUGAL PUMP MODEL OF THE DLR THERMOFLUID STREAM LIBRARY

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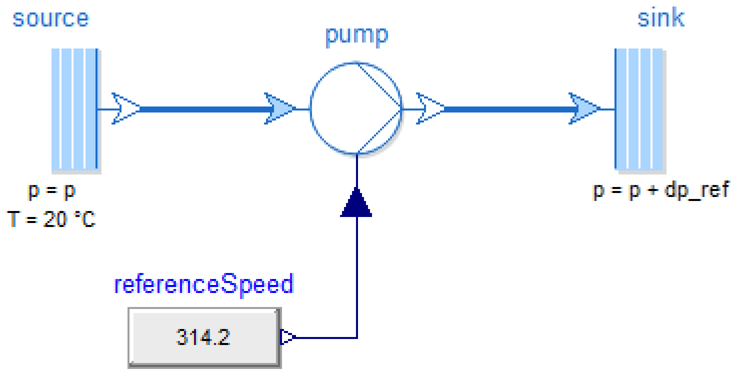
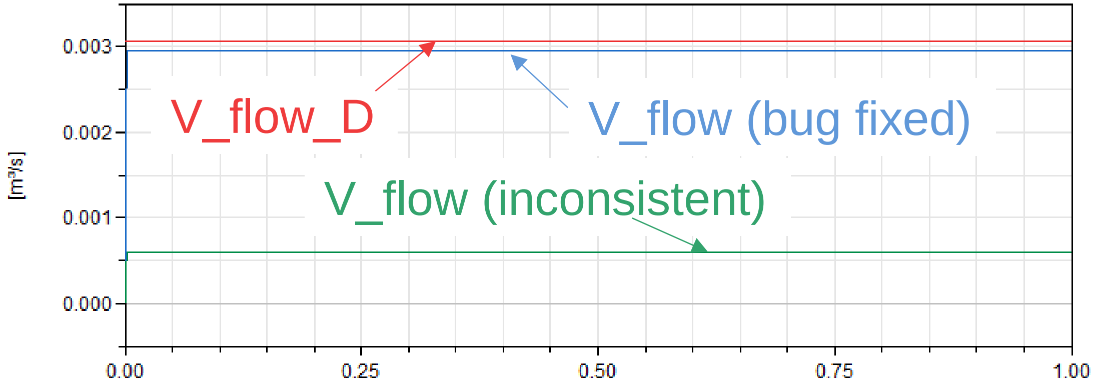
Inconsistent pump model

Scaling

TDH_D	3.6610	m	Design pressure head (max efficiency)
V_flow_D	3.06e-3	m³/s	Design volume flow (max efficiency)
omega_D	314.2	rad/s	Design angular velocity





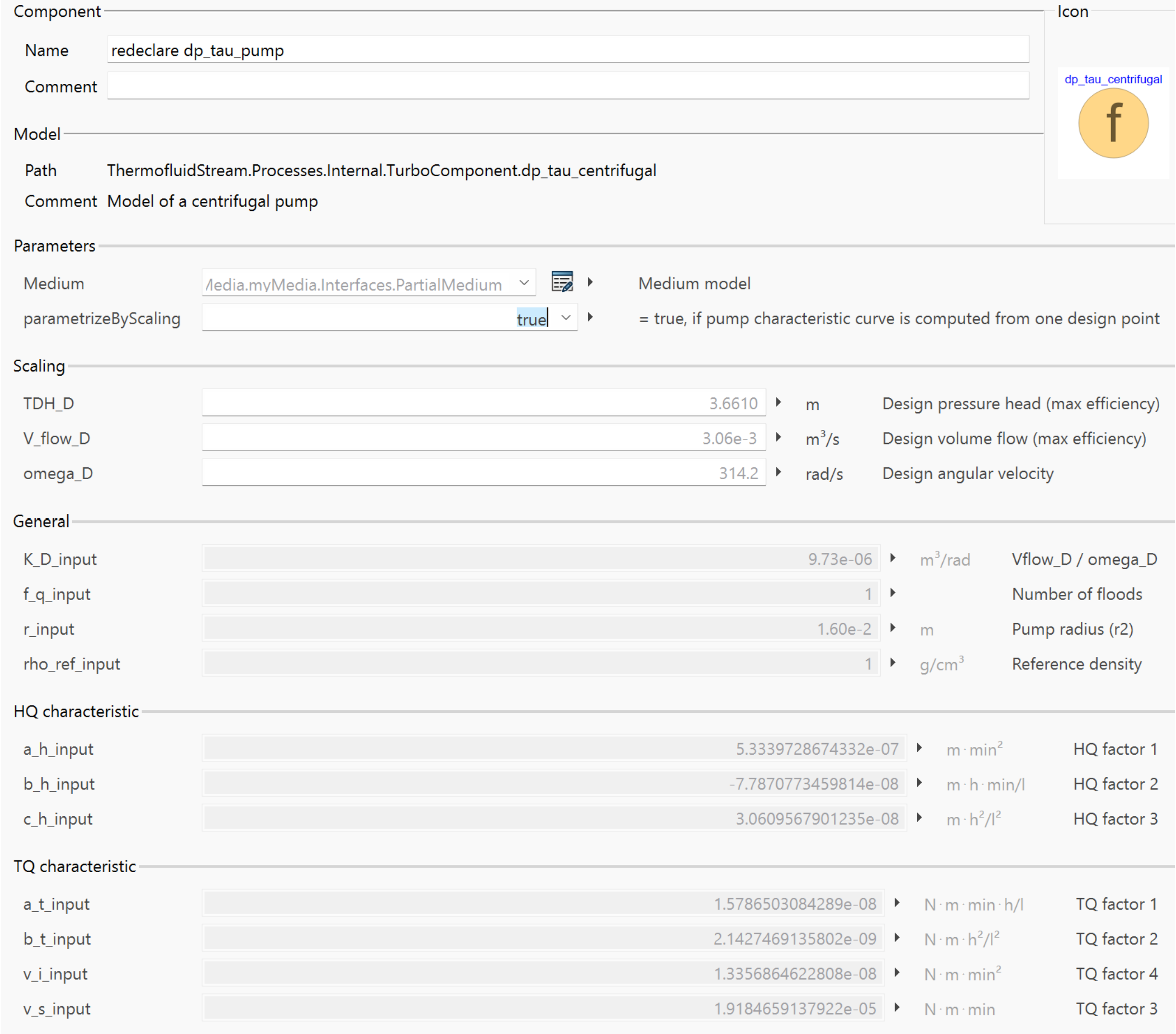

```
Re_mod     := (omega_hat*r^2)/(mu_in)*(omega_s^1.5*f_q^0.75);
```

```
SI.SpecificVolume mu_in = Medium.dynamicViscosity(state_in) "Specific volume at inlet";
```

- At *design angular velocity* and *design head* the volume flow rate is supposed to be the *design volume flow rate*… **but it isn‘t!**

- Implementation as a **function** → hard to debug
- Incorrect **Reynolds number** implementation → Fix removed ~95% of error
- Model approach (suitable for **extrapolation water → oil**) invalid for water itself (scaling factors ≠ 1) → **~5% error remain**

Inconsistent pump model



- At *design angular velocity* and *design head* the volume flow rate is supposed to be the *design volume flow rate…* **but it isn‘t!**

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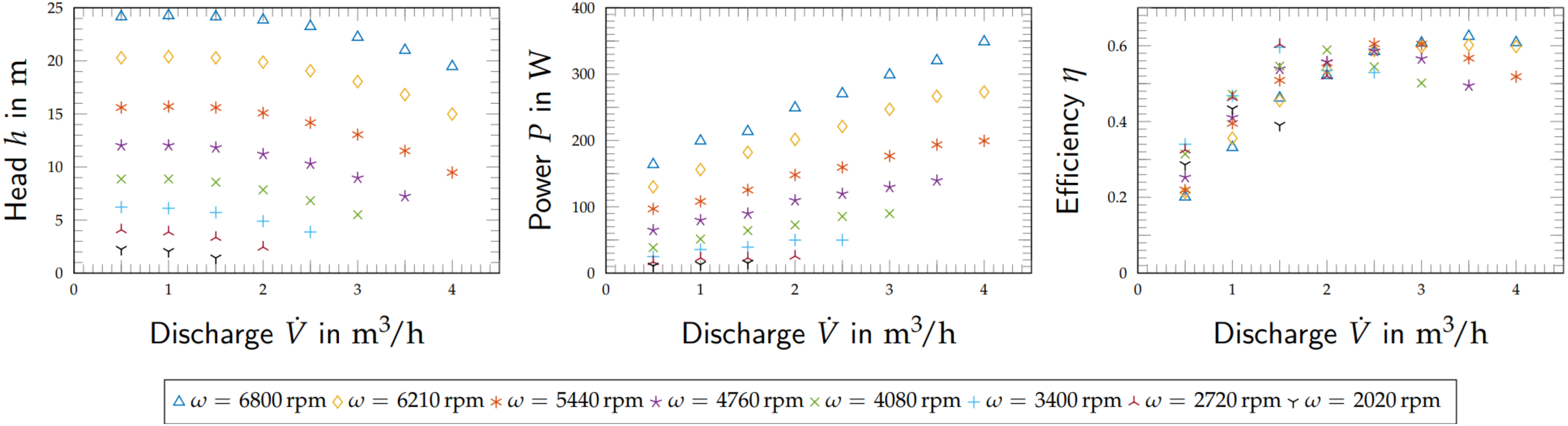
- Model approach (suitable for **extrapolation water → oil**) invalid for water itself (scaling factors ≠ 1) → **~5% error remain**
- Up to 14 Parameters → hard to use

→ Decision to design a **new pump model**

Measurement data
Sample development pump – automotive supplier HELLA GmbH & Co. KGaA



Head (or pressure difference) and mechanical power (or torque) for different discharges and different rotational speeds.

To reduce the number of required measurements → use **affinity laws**

Affinity laws

List of physical variables influencing a centrifugal pump:

- diameter D, rotational speed ω, fluid density ρ, head gh (pressure difference Δp), volumetric flow rate $\dot{V}$ and mechanical power P (torque τ)
- Viscosity η is often neglected

Variables

Exponents

Units

$$\begin{array}{c|ccc|ccc} & D & \omega & \varrho & gh & \dot{V} & P \\ \hline \mathrm{m} & 1 & 0 & -3 & 2 & 3 & 2 \\ \mathrm{s} & 0 & -1 & 0 & -2 & -1 & -3 \\ \mathrm{kg} & 0 & 0 & 1 & 0 & 0 & 1 \end{array}$$

Row-wise Gaussian elimination

$$\begin{array}{ccc|ccc} D & \omega & \varrho & gh & \dot{V} & P \\ \hline 1 & 0 & 0 & 2 & 3 & 5 \\ 0 & 1 & 0 & 2 & 1 & 3 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}$$

Column-wise Gaussian elimination

$$\begin{array}{ccc|ccc} D & \omega^{-1} & \varrho D^3 & gh & \dot{V} & P \\ \hline 1 & 0 & 0 & 2 & 3 & 2 \\ 0 & 1 & 0 & -2 & -1 & -3 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}$$

Dimensionless numbers

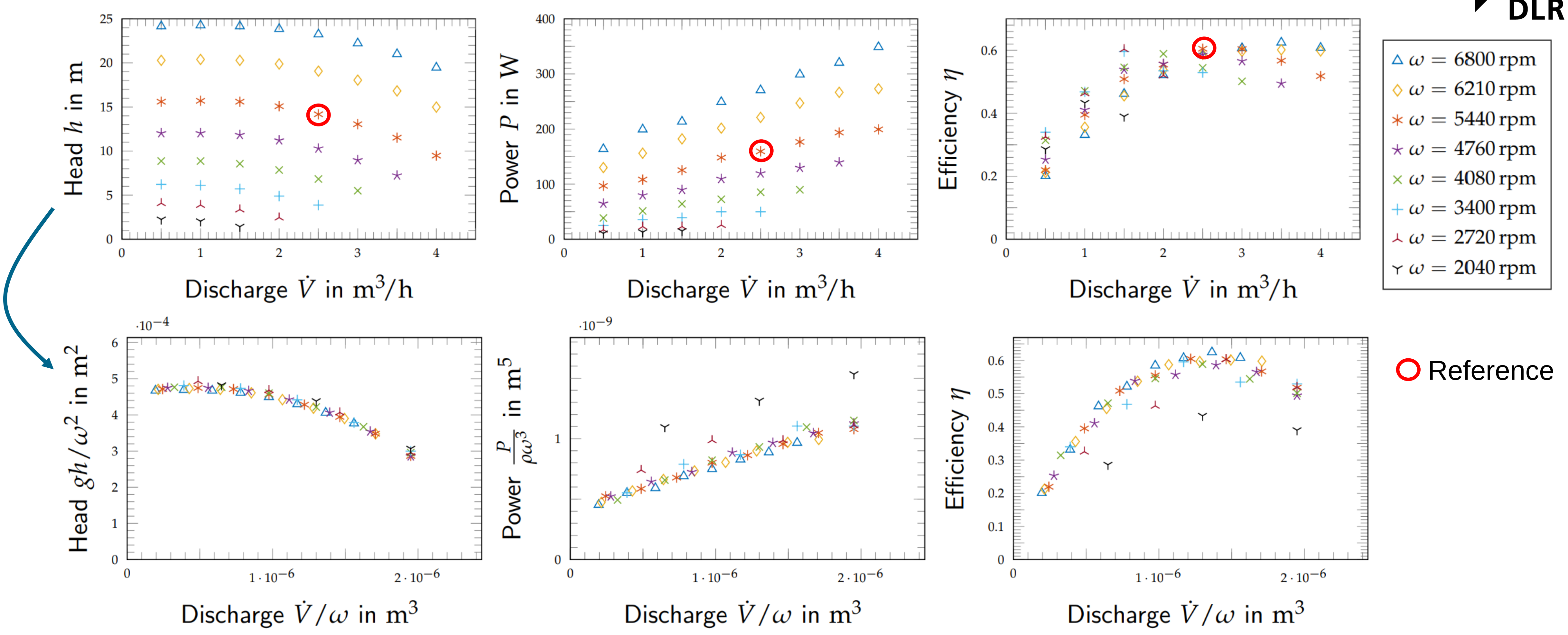
$$\Pi_{gh} = \frac{gh}{D^2\omega^2}$$

$$\Pi_{\dot{V}} = \frac{\dot{V}}{D^3\omega}$$

$$\Pi_{P} = \frac{P}{D^5\omega^3\rho}$$

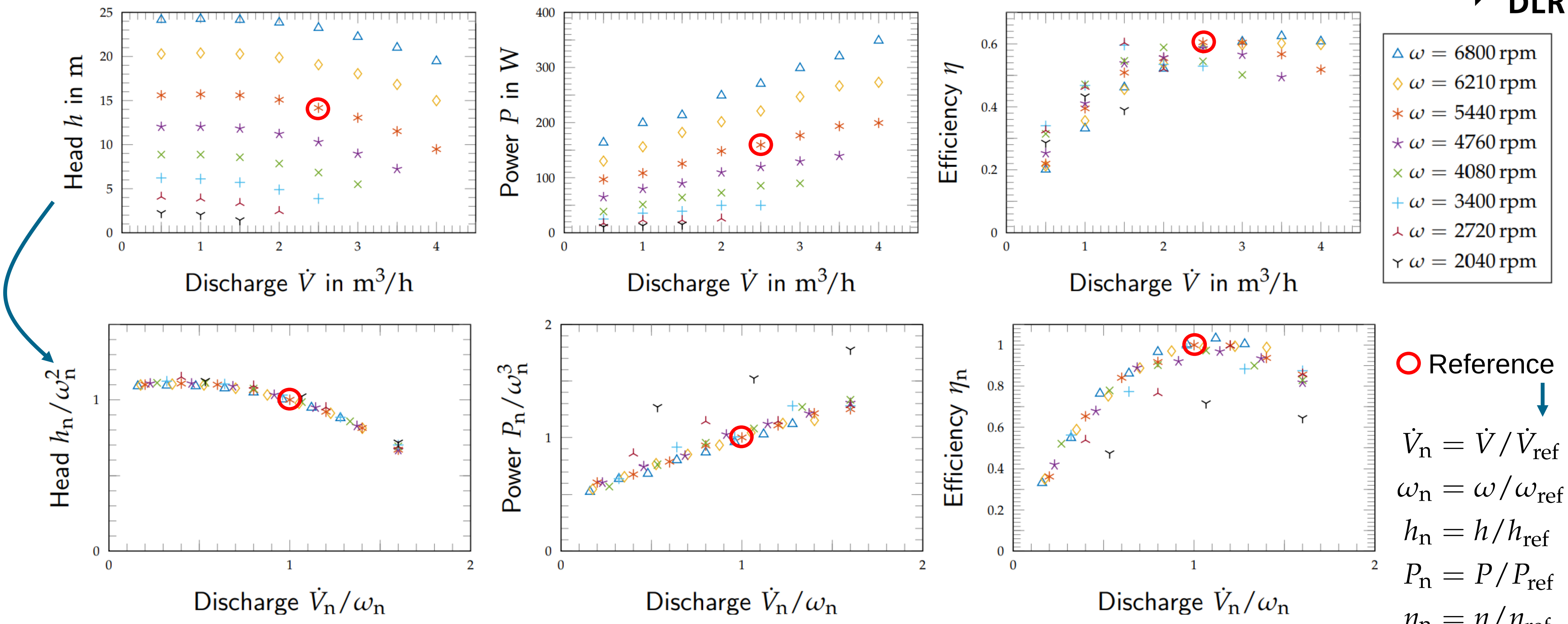
With fixed diameter D the well known similarity laws apply: $h \sim \omega^2$, $\dot{V} \sim \omega$, $P \sim \rho\omega^3$

Applying similarity laws – $h \sim \omega^2$, $\dot{V} \sim \omega$, $P \sim \rho\omega^3$



- Head: excellent agreement
- Power: good agreement (expect outliers at low rotational speeds)
- The units are **impractical** → Normalize to **reference** point

Normalize with nominal/reference/rated conditions



Reference

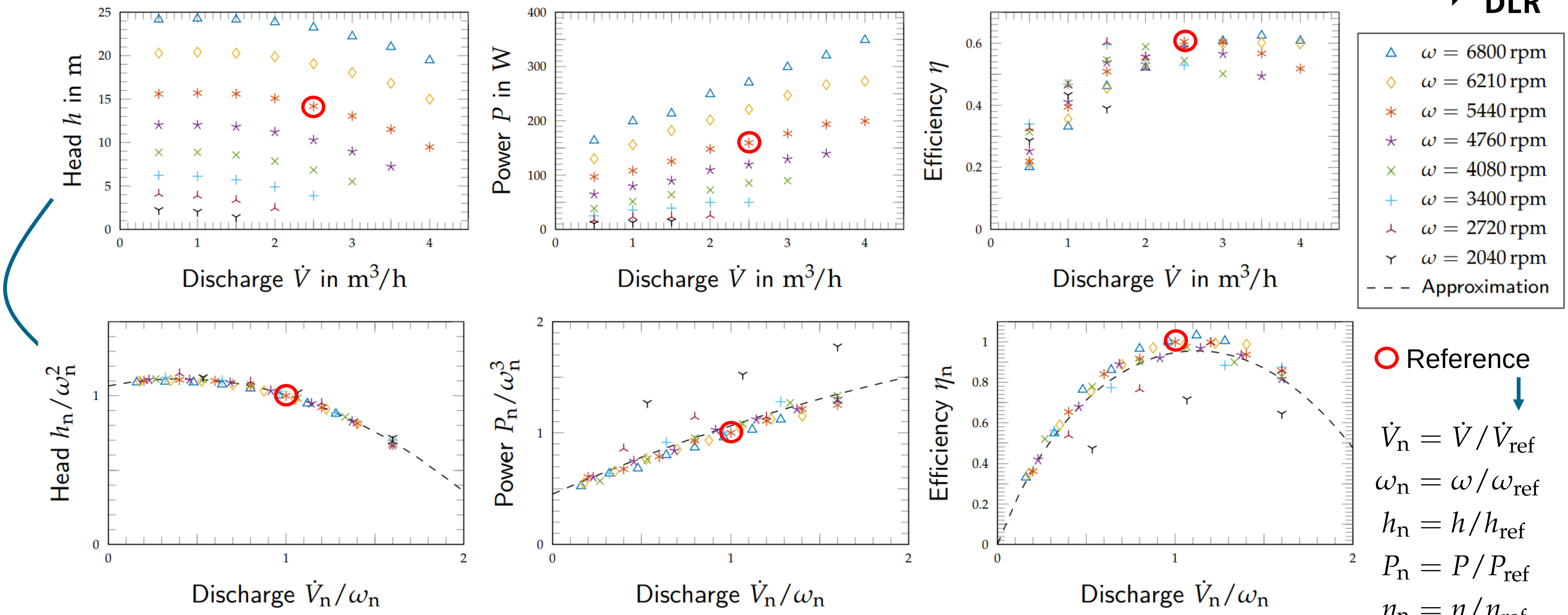
$$\dot{V}_\mathrm{n} = \dot{V} / \dot{V}_\mathrm{ref}$$
$$\omega_\mathrm{n} = \omega / \omega_\mathrm{ref}$$
$$h_\mathrm{n} = h / h_\mathrm{ref}$$
$$P_\mathrm{n} = P / P_\mathrm{ref}$$
$$\eta_\mathrm{n} = \eta / \eta_\mathrm{ref}$$

Normalized

- **Dimensionless** figures

→ **Approximate** head and power using **quadratic polynomials**

Approximation

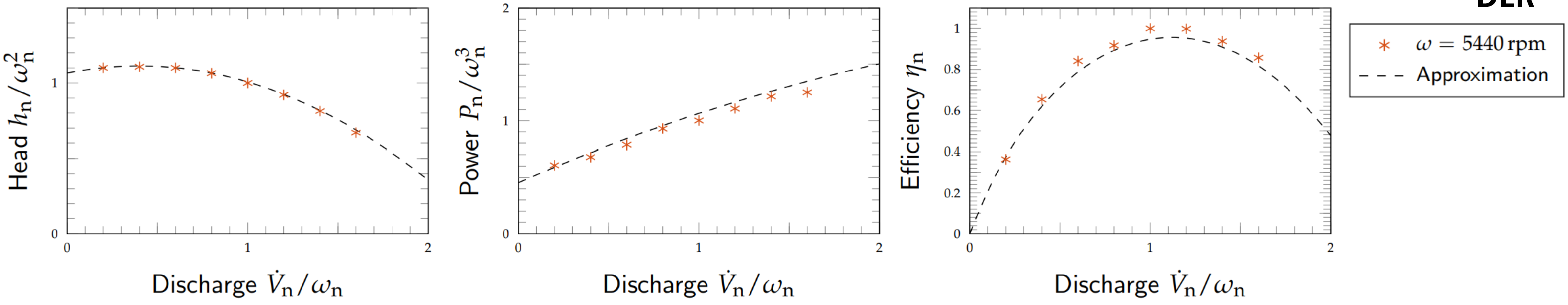


$$\dot{V}_n = \dot{V}/\dot{V}_{ref}$$
$$\omega_n = \omega/\omega_{ref}$$
$$h_n = h/h_{ref}$$
$$P_n = P/P_{ref}$$
$$\eta_n = \eta/\eta_{ref}$$

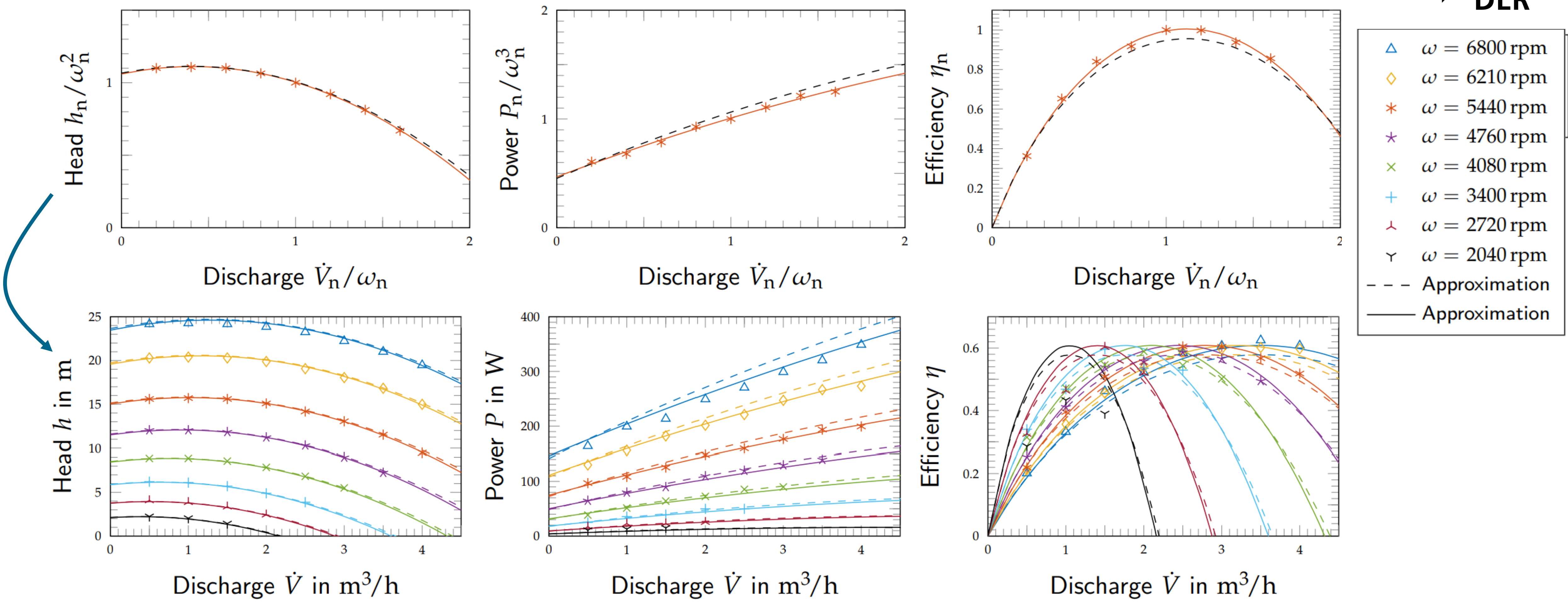
Normalized

→ Check reference rotational speed

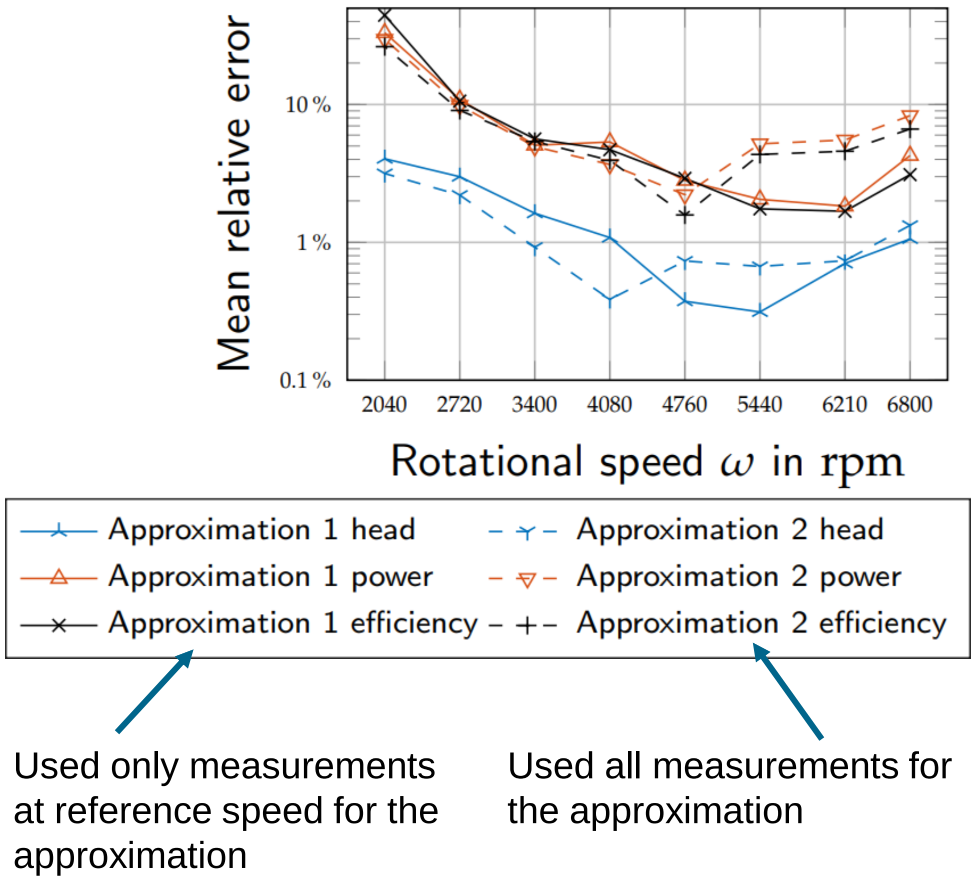
Approximation of data at reference rotational speed



Final result



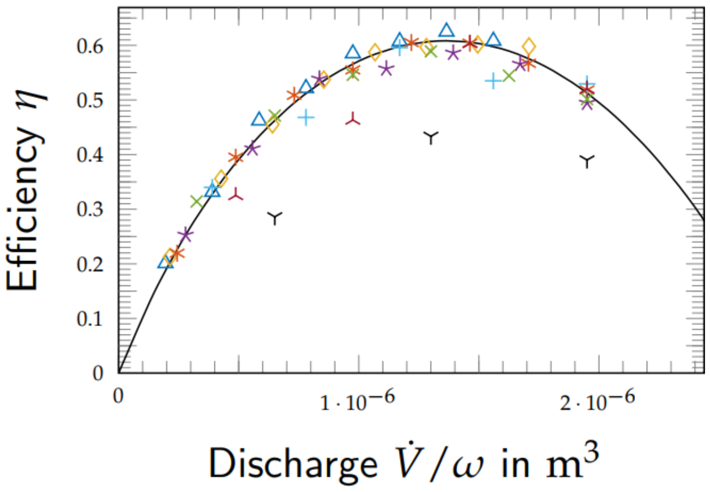
Mean relative error



Mean relative error:

- Head $<\approx 1\ \%$
- Power, Efficiency $<\approx 5\ \%$ except low rotational speeds with up to $40\ \%$
- Adding data at different rotational speeds does not lead to overall improvement

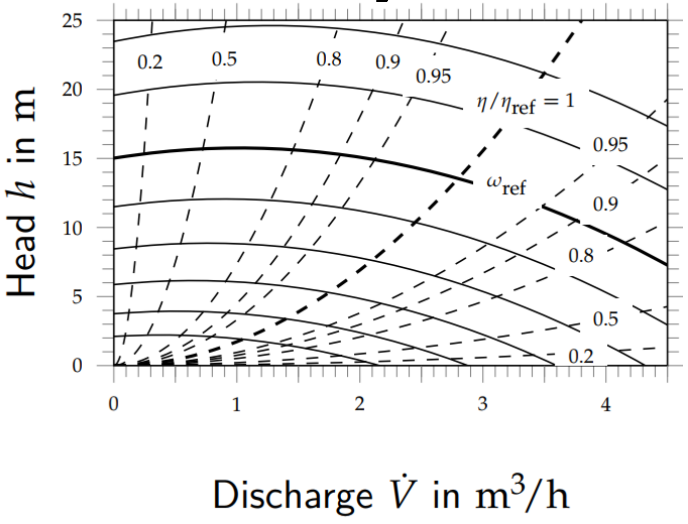
Efficiency contour



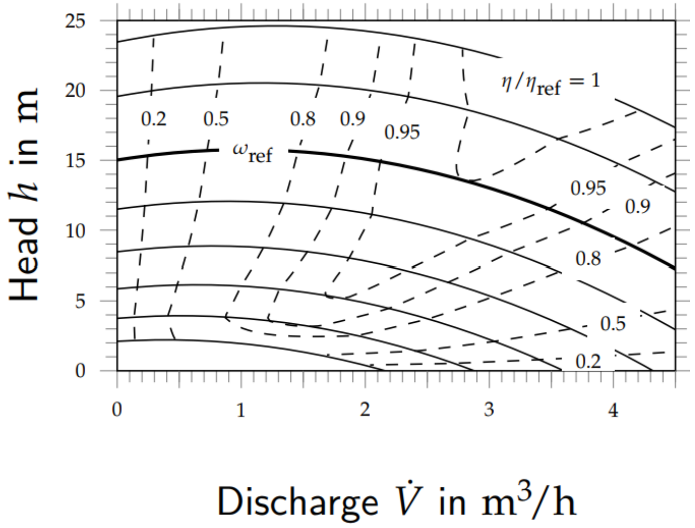
$$\eta = \text{const.} \implies \frac{\dot{V}}{\omega} = \text{const.} \implies \frac{\dot{V}}{\sqrt{h}} = \text{const.} \implies h(\eta = \text{const.}) \sim \dot{V}^2$$

→ Curves of constant efficiency are parabolas in the head – discharge plot

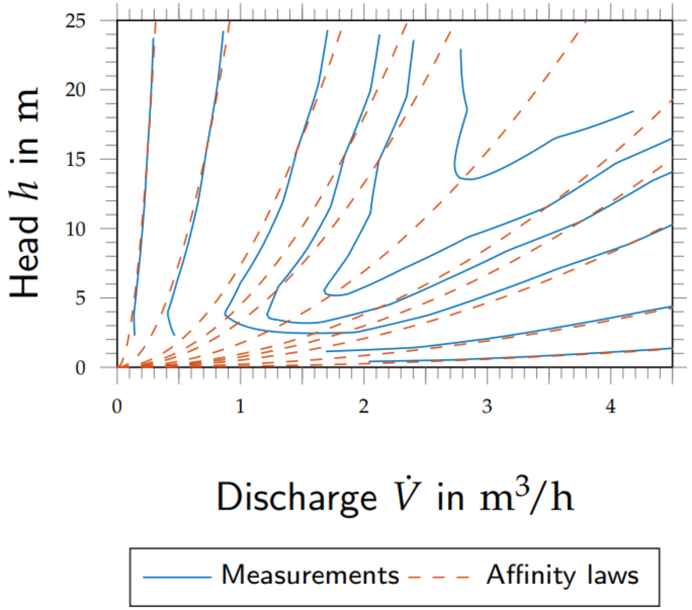
Affinity laws



Measurements



Comparison



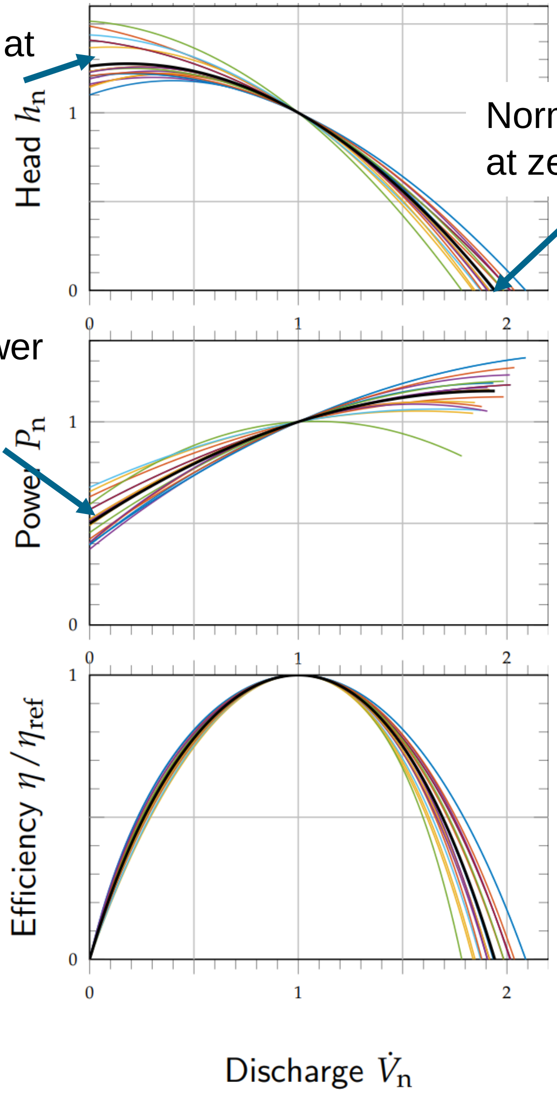
Comparison of different (radial flow) pumps

18 Wilo pumps available at the Buildings Library

Normalized head at zero flow $h_{0,\mathrm{n}}$

Normalized discharge at zero head $\dot{V}_{0,\mathrm{n}}$

Normalized power at zero flow $P_{0,\mathrm{n}}$



$$h_\mathrm{n} = c_{h,0}\omega_\mathrm{n}^2 + c_{h,1}\omega_\mathrm{n}\dot{V}_\mathrm{n} + c_{h,2}\dot{V}_\mathrm{n}^2$$

$$P_\mathrm{n} = \omega_\mathrm{n}\tau_\mathrm{n} = \omega_\mathrm{n}\rho_\mathrm{n}\left(c_{P,0}\omega_\mathrm{n}^2 + c_{P,1}\omega_\mathrm{n}\dot{V}_\mathrm{n} + c_{P,2}\dot{V}_\mathrm{n}^2\right)$$

To determine the six coefficients:

- 3 reference values: $h_\mathrm{ref}, \dot{V}_\mathrm{ref}, \eta_\mathrm{ref}$
- 3 „shape“ parameters: $h_{\mathrm{n},0}, \dot{V}_{n,0}, P_{n,0}$

	h_ref in m	$\dot{V}_\mathrm{ref}$ in m^3/h	P_ref in W	η_ref in %	ω_specific
min	1.4	2.9	27	24	0.42
max	22	73	4200	77	1.21

$$\omega_\mathrm{specific} = \omega\sqrt{\frac{\dot{V}}{\sqrt{gh}^3}}$$

	mean	std	min	max
$h_{0,\mathrm{n}}$	1.273	±0.128	1.101	1.516
$\dot{V}_{0,\mathrm{n}}$	1.946	±0.087	1.784	2.090
$P_{0,\mathrm{n}}$	0.499	±0.099	0.372	0.677

Advantages

$$\frac{h}{h_{\mathrm{ref}}} = \underbrace{\tilde{c}_{h,0}\frac{\omega_{\mathrm{ref}}^2}{h_{\mathrm{ref}}}}_{c_{h,0}}\left(\frac{\omega}{\omega_{\mathrm{ref}}}\right)^2 + \underbrace{\tilde{c}_{h,1}\frac{\omega_{\mathrm{ref}}\dot{V}_{\mathrm{ref}}}{h_{\mathrm{ref}}}}_{c_{h,1}}\frac{\omega}{\omega_{\mathrm{ref}}}\frac{\dot{V}}{\dot{V}_{\mathrm{ref}}} + \underbrace{\tilde{c}_{h,2}\frac{\dot{V}_{\mathrm{ref}}^2}{h_{\mathrm{ref}}}}_{c_{h,2}}\left(\frac{\dot{V}}{\dot{V}_{\mathrm{ref}}}\right)^2$$

$$\frac{P}{P_{\mathrm{ref}}} = \frac{\tau}{\tau_{\mathrm{ref}}}\frac{\omega}{\omega_{\mathrm{ref}}} = \frac{\rho}{\rho_{\mathrm{ref}}}\frac{\omega}{\omega_{\mathrm{ref}}}\left(\underbrace{\tilde{c}_{P,0}\frac{\rho_{\mathrm{ref}}\omega_{\mathrm{ref}}^3}{P_{\mathrm{ref}}}}_{c_{P,0}}\left(\frac{\omega}{\omega_{\mathrm{ref}}}\right)^2 + \underbrace{\tilde{c}_{P,1}\frac{\rho_{\mathrm{ref}}\omega_{\mathrm{ref}}^2\dot{V}_{\mathrm{ref}}}{P_{\mathrm{ref}}}}_{c_{P,1}}\frac{\dot{V}}{\dot{V}_{\mathrm{ref}}}\frac{\omega}{\omega_{\mathrm{ref}}} + \underbrace{\tilde{c}_{P,2}\frac{\rho_{\mathrm{ref}}\omega_{\mathrm{ref}}\dot{V}_{\mathrm{ref}}^2}{P_{\mathrm{ref}}}}_{c_{P,2}}\left(\frac{\dot{V}}{\dot{V}_{\mathrm{ref}}}\right)^2\right)$$

Rewrite quadratic polynomials

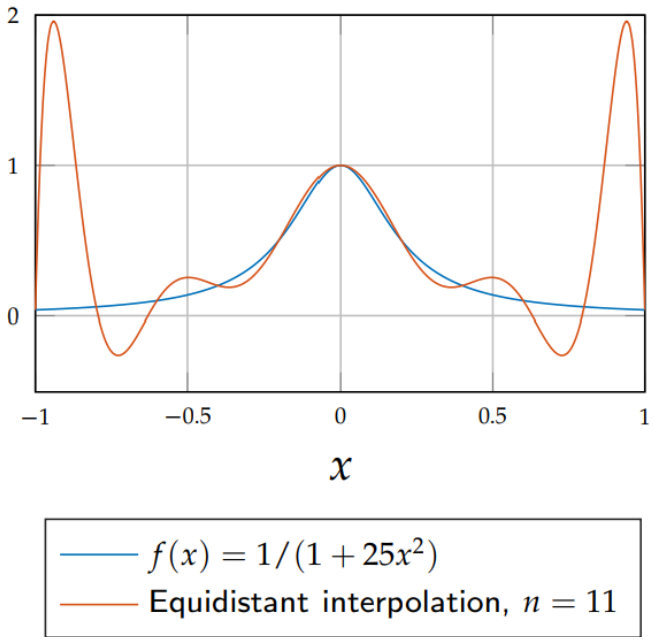
$$\frac{h}{\omega^2} = \tilde{c}_{h,0} + \tilde{c}_{h,1}\frac{\dot{V}}{\omega} + \tilde{c}_{h,2}\left(\frac{\dot{V}}{\omega}\right)^2$$

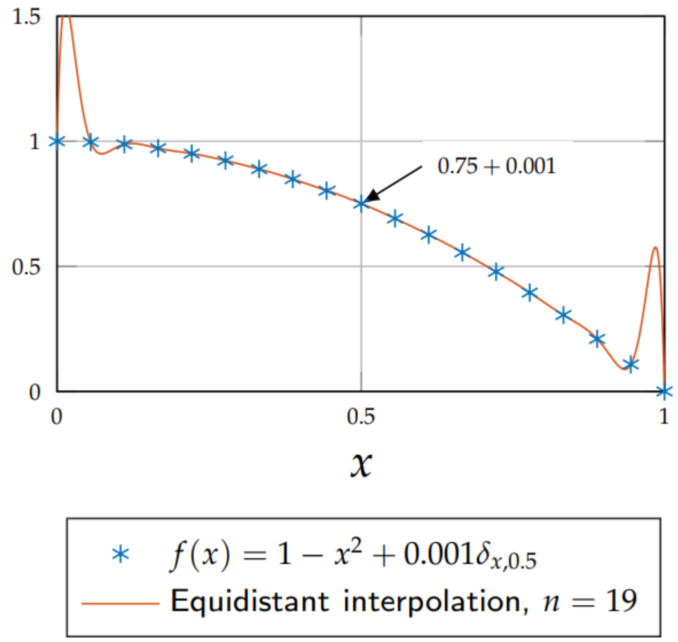
$$\frac{P}{\rho\omega^3} = \frac{\tau\omega}{\rho\omega^3} = \tilde{c}_{P,0} + \tilde{c}_{P,1}\frac{\dot{V}}{\omega} + \tilde{c}_{P,2}\left(\frac{\dot{V}}{\omega}\right)^2$$

$$h_{\mathrm{n}} = c_{h,0}\omega_{\mathrm{n}}^2 + c_{h,1}\omega_{\mathrm{n}}\dot{V}_{\mathrm{n}} + c_{h,2}\dot{V}_{\mathrm{n}}^2$$

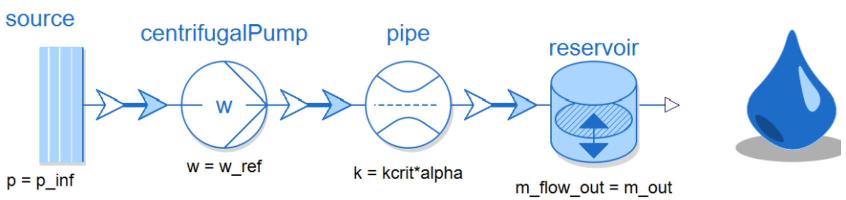
$$P_{\mathrm{n}} = \omega_{\mathrm{n}}\tau_{\mathrm{n}} = \omega_{\mathrm{n}}\rho_{\mathrm{n}}\left(c_{P,0}\omega_{\mathrm{n}}^2 + c_{P,1}\omega_{\mathrm{n}}\dot{V}_{\mathrm{n}} + c_{P,2}\dot{V}_{\mathrm{n}}^2\right)$$

- No division by zero (rotational speed)
- No oscillations (*Runge‘s phenomenon*)
- No numerical workarounds (due to the TFS)





Waterhammer limit cycle



$$A\frac{\mathrm{d}h_\mathrm{R}}{\mathrm{d}t} = \dot{V} - \dot{V}_\mathrm{out}$$

$$\rho L \frac{\mathrm{d}\dot{V}}{\mathrm{d}t} = \Delta p_\mathrm{P} - (\Delta p_\mathrm{sys} + \rho g h_\mathrm{R})$$

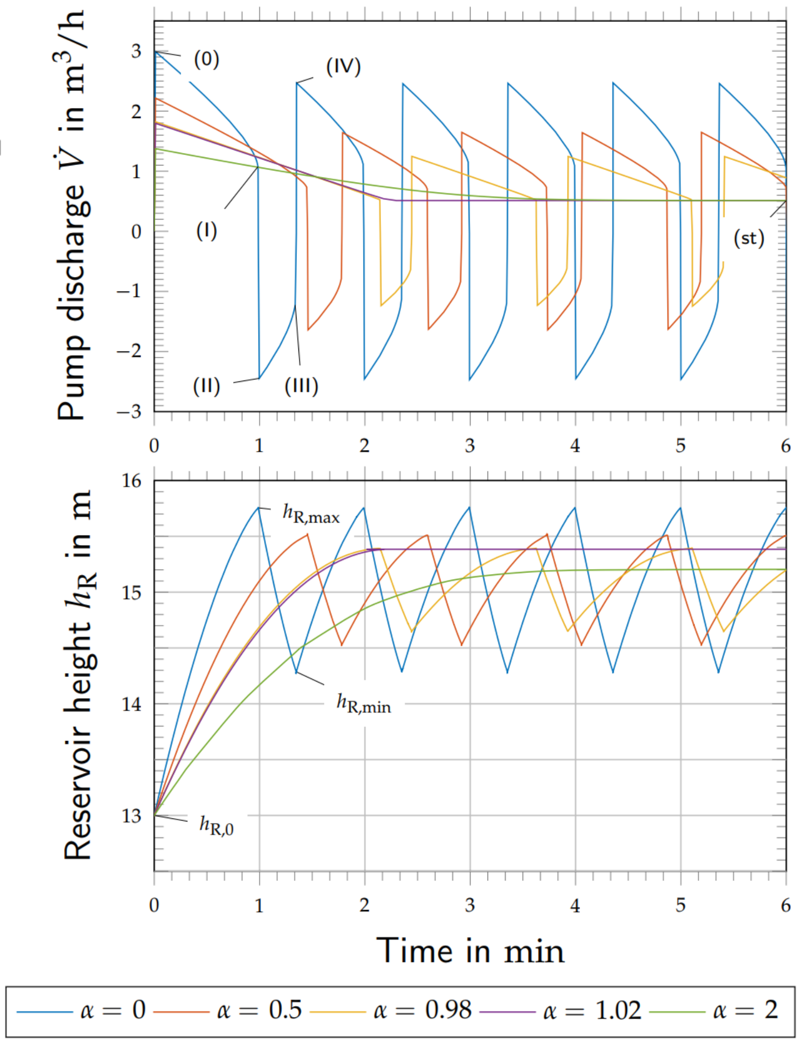
Inertia Pump Pipe Reservoir

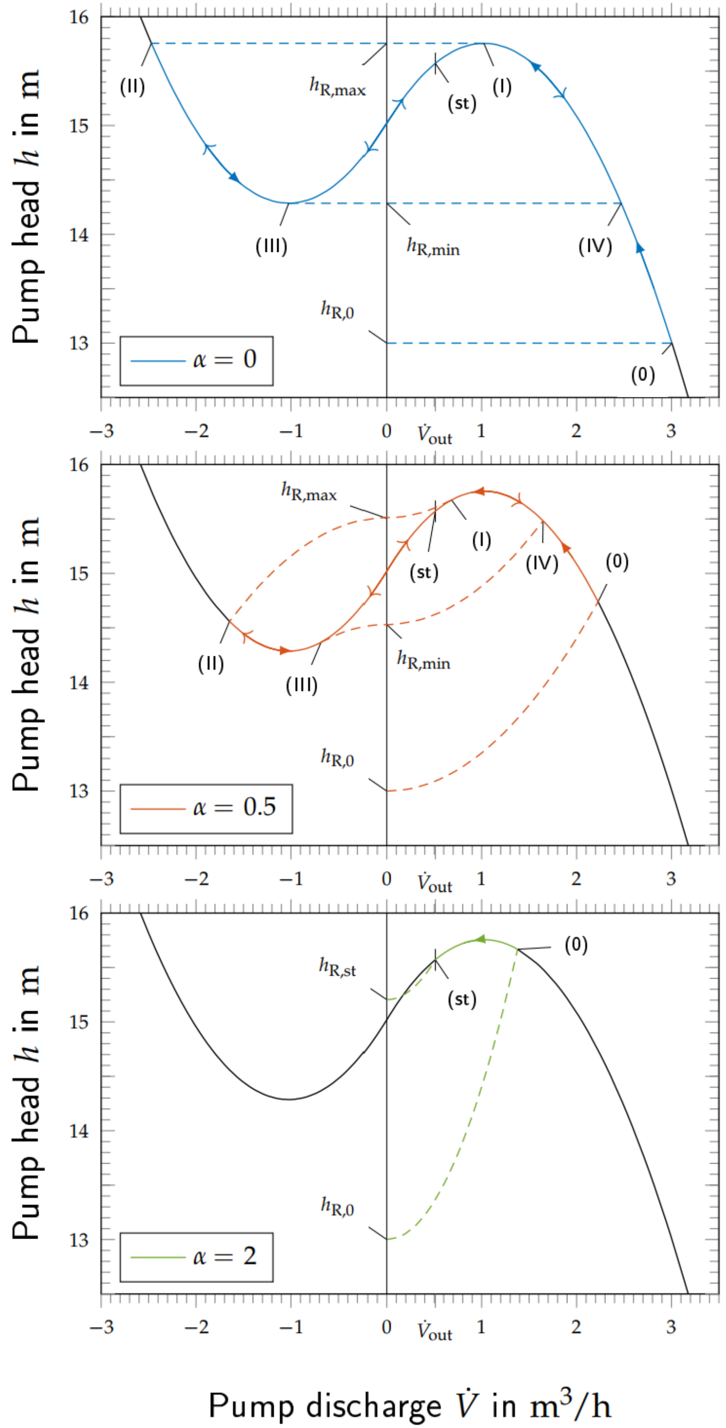
$$ma = \sum F$$

$$h_\mathrm{n} = c_{h,0}\omega_\mathrm{n}^2 + c_{h,1}\omega_\mathrm{n}\dot{V}_\mathrm{n} + c_{h,2}\dot{V}_\mathrm{n}^2$$

Qualitative extension
for negative flow

$$h_\mathrm{n} = c_{h,0}\omega_\mathrm{n}^2 + c_{h,1}\omega_\mathrm{n}\dot{V}_\mathrm{n} + c_{h,2}\dot{V}_\mathrm{n}|\dot{V}_\mathrm{n}|$$





→ Without static equilibrium, inertia matters

Analytic solution exists
read the paper if you are interested

Limit points

$$\hat{\dot{V}}_{\mathrm{I,n}} = \frac{\hat{c}}{2(1+\hat{c}+\hat{k})}\,,$$

$$\hat{h}_{\mathrm{R,max,n}} = 1 + \hat{c}\hat{\dot{V}}_{\mathrm{I,n}} - (1+\hat{c}+\hat{k})\hat{\dot{V}}_{\mathrm{I,n}}^2\,,$$

$$\hat{\dot{V}}_{\mathrm{II,n}} = \frac{\hat{c} + \sqrt{\hat{c}^2 - 4(1+\hat{c}+\hat{k})\left(1-\hat{h}_{\mathrm{R,max,n}}\right)}}{-2(1+\hat{c}+\hat{k})}$$

$$\hat{\dot{V}}_{\mathrm{III,n}} = -\hat{\dot{V}}_{\mathrm{I,n}}\,,$$

$$\hat{\dot{V}}_{\mathrm{IV,n}} = -\hat{\dot{V}}_{\mathrm{II,n}}\,,$$

$$\hat{h}_{\mathrm{R,min,n}} = 1 - (\hat{h}_{\mathrm{R,max,n}} - 1)$$

Differential equation

$$\frac{\mathrm{d}\hat{h}_{\mathrm{R,n}}}{\mathrm{d}\tau} = -\hat{\dot{V}}_{\mathrm{out,n}} \pm \hat{\dot{V}}_{\mathrm{I,n}}\left(1 + \sqrt{1 \pm \gamma(1-\hat{h}_{\mathrm{R,n}})}\right)$$

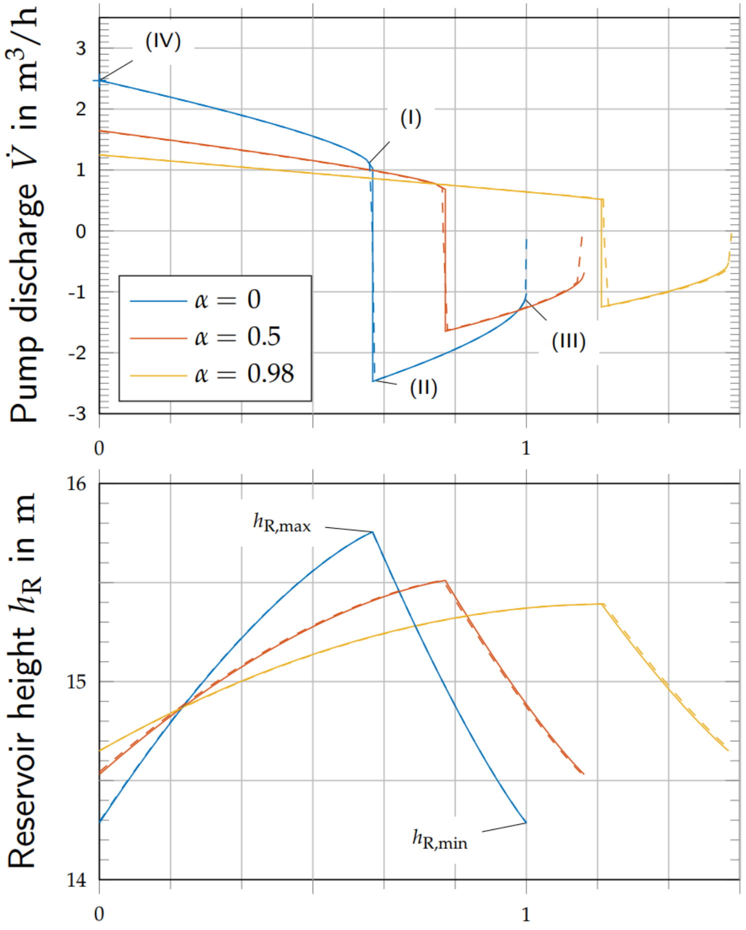
Implicit analytical solution

$$\tau = f_{\pm}(\hat{h}_{\mathrm{R,n}}) - f_{\pm}(\hat{h}_{\mathrm{R,0,n}})\,,$$

$$f_{\pm} = -\frac{2}{\hat{\dot{V}}_{\mathrm{I,n}}^2\gamma}\Big(\hat{\dot{V}}_{\mathrm{I,n}} g_{\pm} +$$

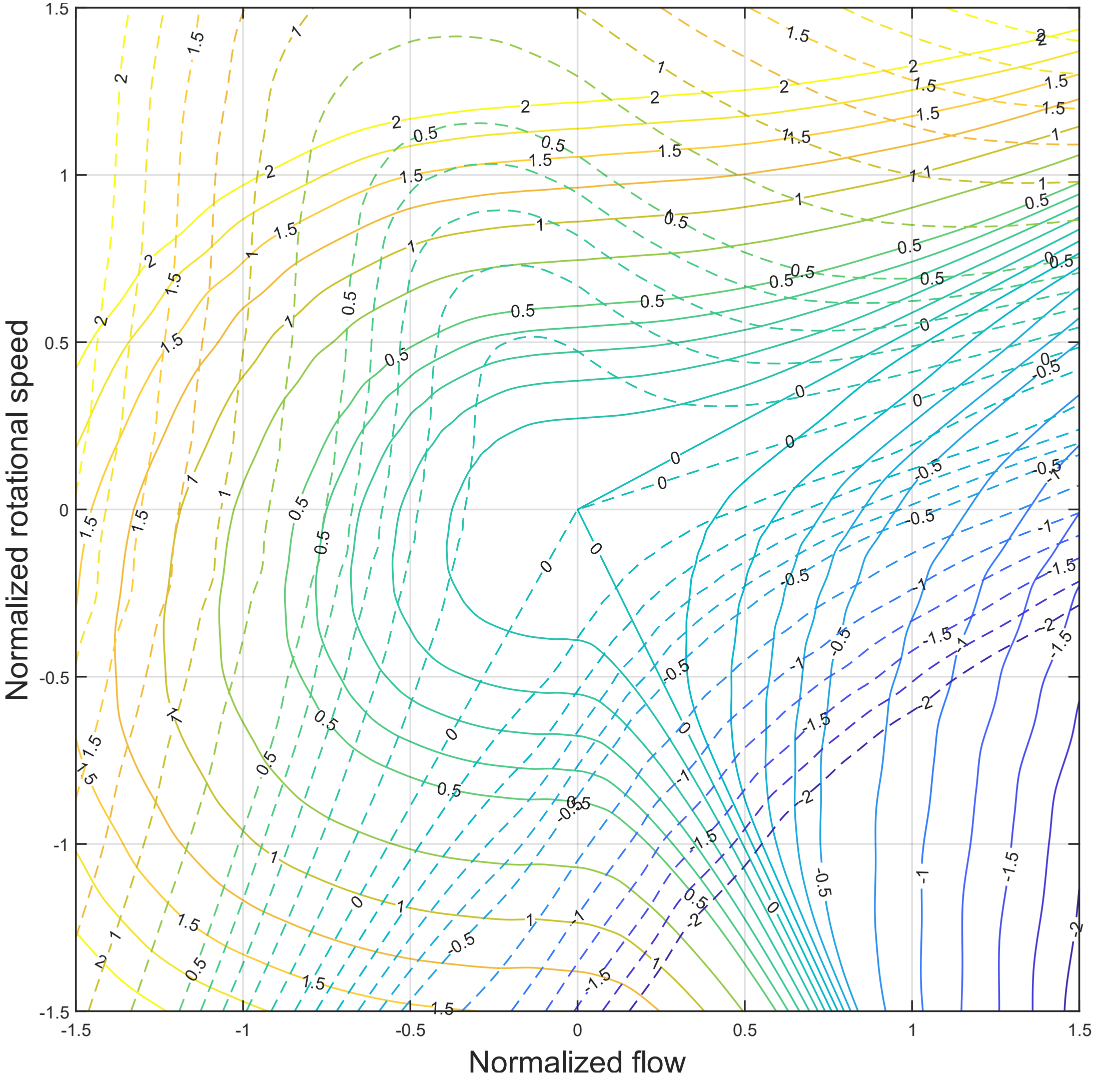
$$+ (\pm\hat{\dot{V}}_{\mathrm{out,n}} - \hat{\dot{V}}_{\mathrm{I,n}}) \ln\left|\mp\hat{\dot{V}}_{\mathrm{out,n}} + \hat{\dot{V}}_{\mathrm{I,n}}(1+g_{\pm})\right|\Big)\,,$$

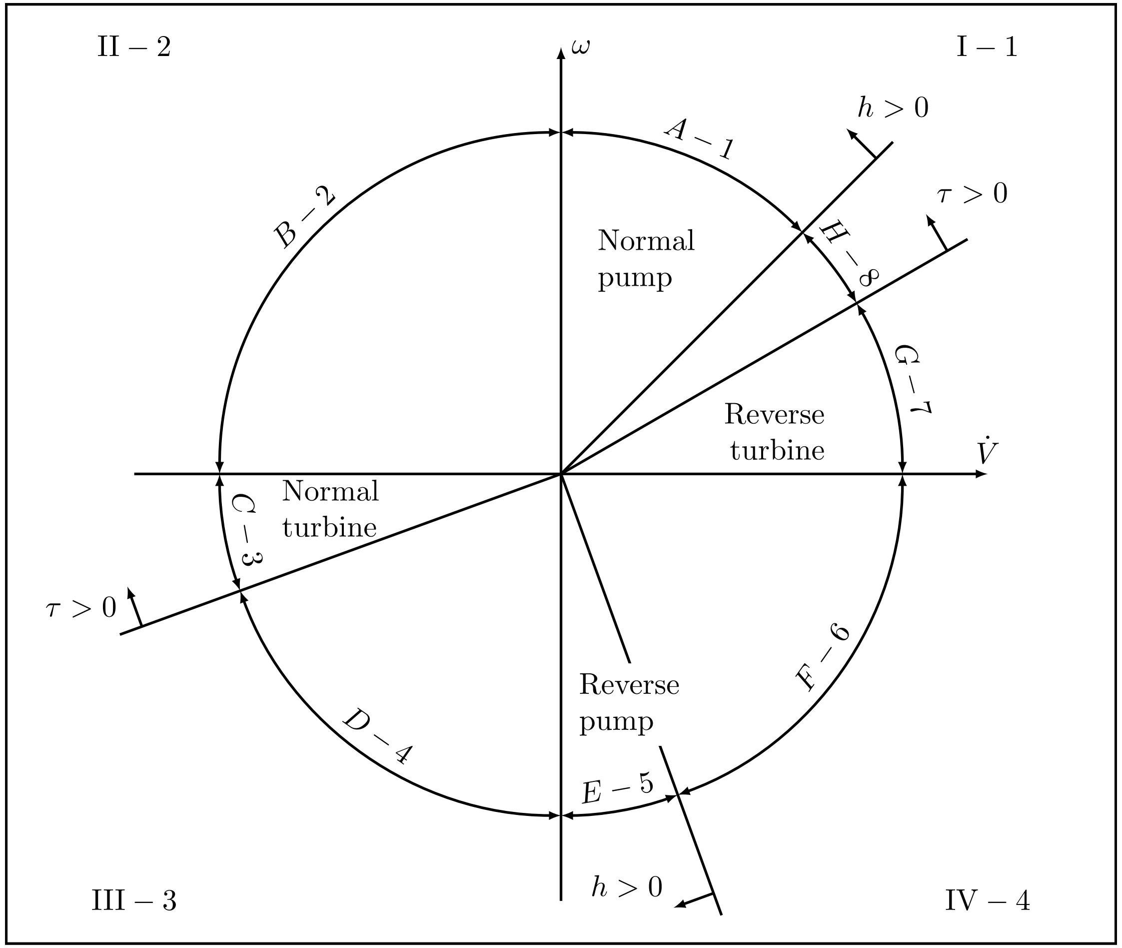
$$g_{\pm} = \sqrt{1 \pm \gamma(1-\hat{h}_{\mathrm{R,n}})}\,,$$



→ **Simulation agrees with analytical solution**

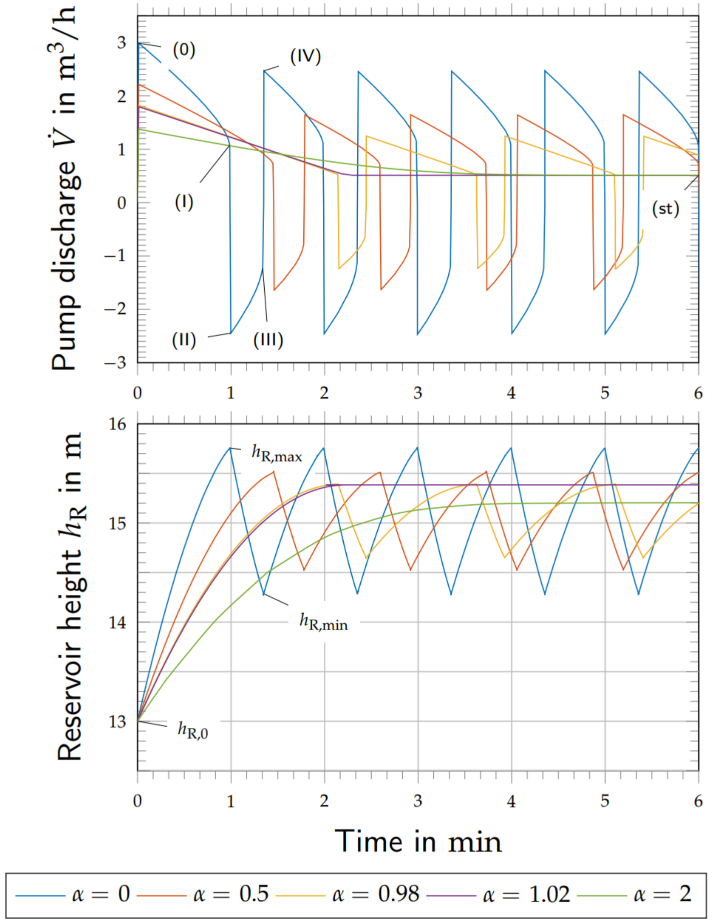
Outlook – 4 quadrant pump model – head/torque - countour

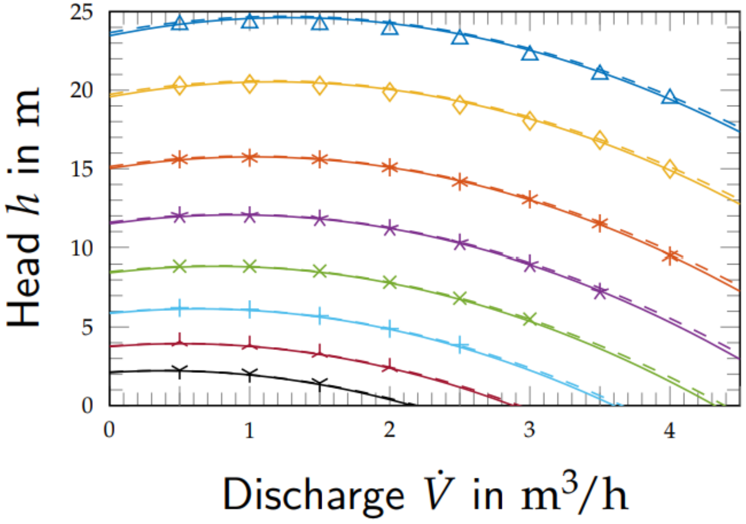


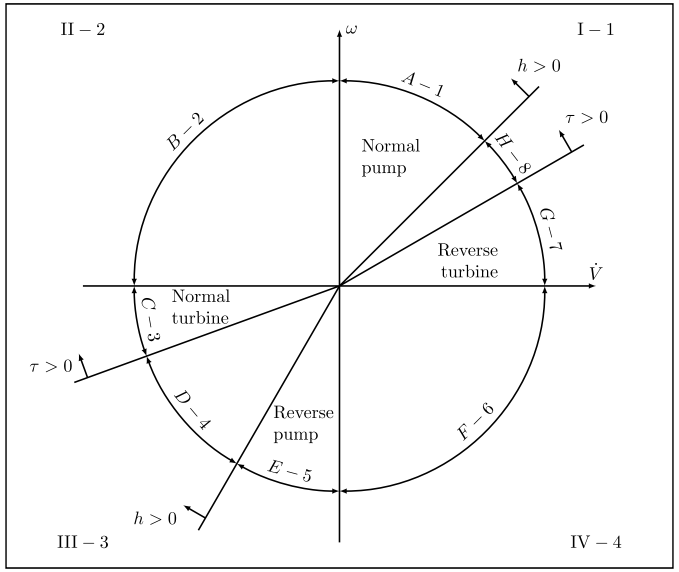


Conclusion

- Robust pump model
- Easy to parametrize
- Further work ongoing







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Imprint

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