Mitteilung

Fachgruppe: Turbulenz und Transition

Study of the compressible mixing-layer correction for the SST k-ω model and for the SSG/LRR-ω model and application to the simulation of plumes.

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To calculate infrared (IR) signatures generated by plumes, CFD simulations using RANS turbulence models can be used to predict species and temperature distribution as input data for the subsequent tools. RANS models were mainly developed for subsonic flows, and in some flow situations, compressibility corrections need to be applied. One such situation is compressible free-shear-layer flow relevant for the simulation of plumes. The growth of the spreading rate of high-speed mixing layers is reduced significantly compared to the incompressible regime [1], leading to a much longer extent of the region effected by the plume.

The convective Mach number M_c appears as a parameter to describe compressibility effects in mixing-layer flow from theoretical studies (see [1]). It is based on the velocity difference U_1-U_2 and the sound speeds a_1, a_2 of the two streams and is given by $M_c=(U_1-U_2)/(a_1+a_2)$. The turbulence Mach number is defined by $M_t=2k/a^2$, with k being the turbulent kinetic energy and a being the local value of the speed of sound. As M_t is computed from local flow quantities, it is attractive for use in RANS turbulence modeling to parametrize compressibility effects [2].

Although the study of the underlying physical mechanisms is still under investigation, practical modifications of RANS models were devised which attempt to describe this effect. The present work describes the validation and application of the compressibility correction by Wilcox (2006) [2], which was developed for the Wilcox k-ω [2] model, for the SST k-ω model and for the SSG/LRR-ω RSM [3] in the DLR TAU code [4]. For the Wilcox k-ω model, the modification multiplies the coefficient β^* of the magnitude of dissipation ε of turbulent kinetic energy k, given by $\rho\varepsilon= \beta^*\rho k\omega$, and the coefficient β for the destruction of ω, denoted as Dest(ω)=$\beta\rho\omega^2$, of the specific rate of dissipation ω. The modification by Wilcox (2006) proposes $\beta^*=\beta^*_{inc}[1-2F(M_t)]$ and $\beta=\beta_{inc}-\beta^*_{inc}2F(M_t)$. Here the subscript 'inc' indicates the standard coefficient for incompressible flow, and effects of changing turbulent Mach number are modeled by $F(M_t)=\max(M_t^2-M_{t,0}^2,0)$, $M_{t,0}=0.25$. Regarding the application to the RSM, the corresponding coefficients of the SSG/LRR-ω model are changed. Following the implementation in the DLR TAU code for the SST model, the blending function of the SST model is used to activate the modification only outside boundary layers.

For the test of the compressibility modification, a mixing-layer test-case was developed based on the incompressible planar mixing layer case by Delville provided at the NASA turbulence modeling resource webpage [5]. A few modifications were found to be necessary for supersonic flow speeds. The distance of the farfield boundary from the splitter plate of length c is increased and the thickness of the plate is decreased. Moreover, the taper angle in the rear part of the splitter plate is reduced. The convective Mach number is varied by changing the velocity of the high-speed stream U_h and of the low-speed stream U_l. The factor of increase of $U_h/U_{h,inc}$ and $U_l/U_{l,inc}$ was the same as the decrease of the density ρ/ρ_{inc}. The subscript 'inc' denotes values for the incompressible case. Then there is no difference in pressure, density and temparature between the upper and lower stream. Self-similar solutions for the mean velocity and for the Reynolds shear stress were found for x/c>0.8. The layer thickness δ_{10} was evaluated using a post-processing tool. The 10% thickness is defined as the distance between the points where the velocity is equal to $U_h-0.1\Delta U$ and $U_l+0.1\Delta U$ where $\Delta U= U_h- U_l$. The results are shown in figure 1 (left) for the SST model and the SSG/LRR-ω model without and with compressible mixing-layer modification (ML). The figure shows the spreading rate $\delta'=d\delta/dx$ for the compressible flow conditions put in ratio to the incompressible case with spreading rate δ'_0, where x denotes the streamwise direction. Qualitatively, the ML modification is able to describe the reduction of δ'/δ'_0 for large M_t. However, the scatter in the reference data is obvious. Two correlations are included, i.e., the Langley curve and the curve obtained from linear stability theory by Day et al. (1998) for the amplification rate of the Kelvin-Helmholtz mode. The Langley curve was used to calibrate the function $F(M_t)$ in the ML modification by Wilcox. However, the correction appears to be a little too small if applied to the ε-equation rather than the ω-equation in the free shear layer for $0.5< M_t <1.3$, confirming the results by [6].

Regarding the calculation of IR-signatures, figure 1 (right) shows one of the driving species creating IR-signature combined with the relevant temperature. The upper part of the figure is based on the SST turbulence model using the mixing-layer correction. At the moment this CFD setting shows best comparison to experimental data in IR signature predictions. In the middle of the figure SST without mixing-layer correction is plotted which shows a far too short plume propagation in comparison to experimental data. This also is true for the RSM turbulence model given at the lower part of the figure. For RSM the mixing layer correction was not yet implemented and the CFD calculations for RSM compared to SST have been much more stable compared to SST versions. So, RSM with available mixing-layer correction has a great potential to improve CFD calculations for IR signature predictions.

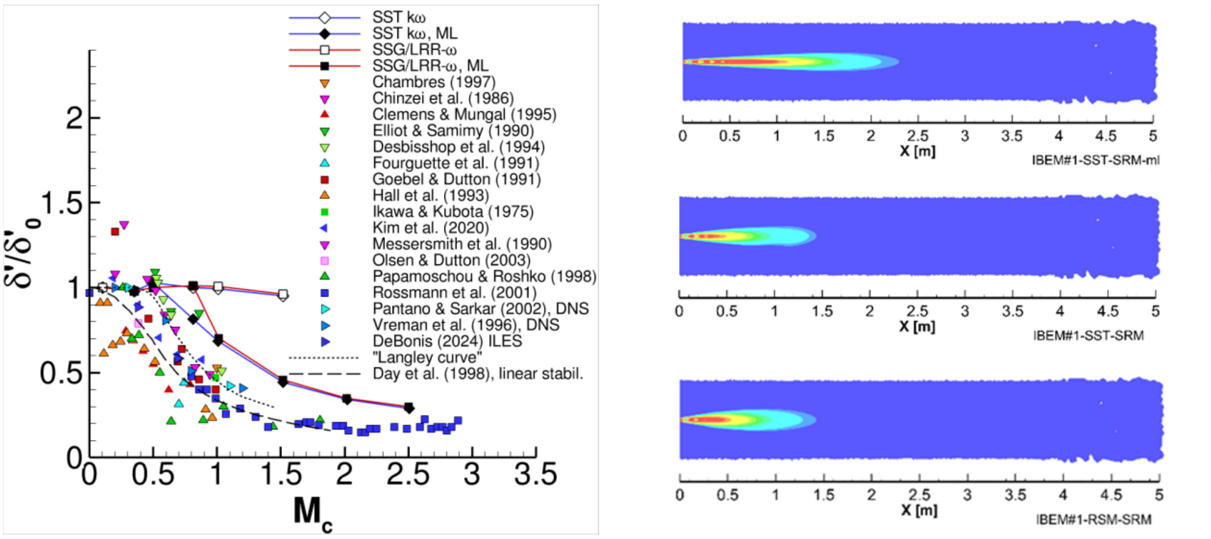

Figure 1. Left: Computed and measured spreading rate $\delta'=d\delta/dx$ for the compressible mixing layer in relation to the incompressible case with spreading rate δ'_0. Computations are for the SST model and for the SSG/LRR-ω model without and with compressible mixing layer modification denoted by ML. Right: Driving species creating IR-signature combined with the relevant temperature distribution of plumes using the SST model with compressible mixing-layer modification (ml) (top), and without modification for both the SST model (middle) and the SSG/LRR-ω (bottom).

As a conclusion, the compressible mixing-layer modification by Wilcox (2006) shows to be useful to capture compressibility effects in free-shear layer flows and in plumes. A more detailed study of the transitional regime between very small and very large values of the convective Mach number seems to be of interest for future research.

References :

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