

UNIVERSIDAD POLITÉCNICA DE MADRID
Escuela Técnica Superior de Ingeniería Aeronáutica y del Espacio



The Adaptive Harmonic Linearized Navier-Stokes methodology

DOCTORAL THESIS

Submitted for the degree of Doctor by:

Juan Alberto Franco Sumariva
MSc Aeronautical Engineer

Madrid, 2025



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Doctoral Degree in Fluid Mechanics

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Under the supervision of:
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*A mis padres, por todos los valores que,
con infinita paciencia, me habéis inculcado*

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Abstract

The constant seeking of drag reduction in the aircraft industry has lead to designers to extend the incoming laminar flow beyond the vicinity of the leading edge of aerodynamic surfaces, such as wings, nacelles, and vertical / horizontal stabilizers. However, the detrimental and unavoidable presence of surface imperfections (*e.g.* steps, gaps, etc.) usually plays a key role in the expected laminar-turbulent transition location. Although current methodologies such as Harmonic Linearized Navier-Stokes (HLNS) and Direct Numerical Simulations (DNS) can handle the presence of those irregularities, the high numerical cost inhibits their practical use for the parametric studies required in the design process.

The *Adaptive Harmonic Linearized Navier-Stokes* (AHLNS) methodology for compressible quasi three-dimensional boundary-layer instability analysis has been developed and implemented in a numerical code. The AHLNS equations represent a very efficient approach in cases where localized large streamwise gradients are present in the region of study (*e.g.* due to the presence of the above-mentioned surface imperfections). This efficiency is based on to take advantage of the *wave-like* character in streamwise direction of the convective flow instabilities along the *linear growth* stage of the laminar-turbulent transition process.

To illustrate the use of the AHLNS methodology, a relatively simple configuration has been chosen: a compressible flow on a flat plate without any external pressure gradient. Steps (both backward- and forward-facing) and humps of different heights, lengths and shapes (smooth and rectangular) have been studied. The reduced computational resources required by the AHLNS approach enables to increase the range of parametric variations. Therefore, a clearer perspective of the key parameters that have a more decisive influence on the expected laminar-turbulent transition has been achieved.

Finally, the applicability of the actual version of the AHLNS code for realistic large-aircraft geometries under typical transonic flight conditions has been successfully tested in several EC-funded projects: NACOR (for laminar nacelles), HLFC-WIN (for hybrid laminar flow control wings), and BLADE (for natural laminar wings). However, only results from the HLFC-WIN project are allowed to be included in this thesis. The results of the boundary-layer instability analysis computed with AHLNS suggest that the initial tolerances for the joint between different elements on the wings (in the form of superficial gaps) could be relaxed without compromising the expected transition location of the incoming flow.

To sum up, this thesis opens up the possibility of incorporating the AHLNS methodology in the design of future laminar and/or hybrid laminar wings including the influence of two-dimensional surface imperfections.

Resumen

La búsqueda constante de la disminución de la resistencia en la industria aeronáutica ha llevado a los diseñadores a extender el flujo laminar más allá del borde de ataque de las superficies aerodinámicas, tales como alas, góndolas de motores, y estabilizadores verticales / horizontales. Sin embargo, la perjudicial e inevitable presencia de imperfecciones superficiales (*e.g.* escalones, ranuras, etc.) generalmente juega un papel fundamental en la posición donde ocurre la transición del flujo laminar a turbulento. Aunque las metodologías actuales, tales como Harmonic Linearized Navier-Stokes (HLNS) y Direct Numerical Simulations (DNS) pueden incluir la presencia de dichas irregularidades, el alto coste computacional limita su uso práctico para los estudios paramétricos requeridos en el proceso de diseño.

La metodología *Adaptive Harmonic Linearized Navier-Stokes* (AHLNS) para el análisis de estabilidad de capas límite compresibles y cuasi-tridimensionales ha sido desarrollada e implementada en un código numérico. Las ecuaciones AHLNS representan un método muy eficiente en casos donde grandes gradientes en la dirección de la corriente están presentes en la región de estudio (debido a la presencia de las anteriormente mencionadas irregularidades superficiales). Esta eficiencia se basa en aprovechar el carácter ondulatorio en dirección de la corriente de las inestabilidades convectivas del flujo a lo largo de la etapa de *desarrollo lineal* en el proceso de transición de laminar a turbulento.

Para ilustrar el uso de la metodología AHLNS, se ha elegido una configuración relativamente simple: un flujo compresible en una placa plana sin ningún gradiente externo de presión. Escalones (tanto hacia arriba como hacia abajo) y crestas de diferentes alturas, longitudes y formas (rectangulares o redondeadas) han sido estudiados. Los reducidos requisitos computacionales requeridos por el método AHLNS permiten incrementar el rango de variaciones paramétricas. Por tanto, se ha alcanzado una perspectiva más clara de los parámetros fundamentales que tienen una mayor influencia en la posición de la transición de flujo laminar a turbulento.

Finalmente, la aplicación de la actual versión del código AHLNS para aviones de grandes dimensiones en condiciones típicas de vuelo transónico ha sido exitosamente comprobada en varios proyectos europeos: NACOR (para góndolas laminares), HLFC-WIN (para alas en flujo laminar con control híbrido), y BLADE (para alas en flujo laminar). Sin embargo, únicamente se ha permitido la publicación de resultados del proyecto HLFC-WIN. Los resultados del análisis de estabilidad de la capa límite calculados con AHLNS sugieren que las tolerancias iniciales para las uniones de los diferentes elementos de las alas (en forma de ranuras superficiales) podrían aumentarse sin afectar la posición donde el flujo cambie de laminar a turbulento.

En resumen, esta tesis abre la posibilidad de incorporar la metodología AHLNS en el diseño de futuras alas laminares y alas híbridas laminares incluyendo el efecto de imperfecciones superficiales bidimensionales.

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List of symbols

Latin

A	disturbance amplitude
A_u	disturbance amplitude based on velocity component u
c	chord length
c_f	skin-friction coefficient
c_p	specific heat at constant pressure
C	shape parameter for steps
f	frequency
F	reduced frequency
H	height of the step/hump
L	length of the step/hump
l_{ref}	reference length
m	shape parameter for humps
Ma	Mach number
Ma_∞	Mach number based on free-stream values
n	n -factor curve
N	N -factor envelope curve
n_x	number of grid points in x -direction
n_y	number of grid points in y -direction
p	pressure
Pr	Prandtl number
q	general quantity in the flow
Q	total free-stream velocity
$\Re$	real part
$\mathcal{R}$	specific gas constant
r	rectangular
Re	Reynolds number
$Re_{\bar{\delta}}$	Reynolds number based on Blasius length
Re_u	unit Reynolds number
s	arc-length along surface
t	time
T	Temperature
$\mathbf{u}$	velocity vector
u	x-velocity component
v	y-velocity component
w	z-velocity component
$\mathbf{x}$	position vector
x	streamwise direction
x_c	centre of the surface imperfection in x -direction
x_s	most upstream neutral position for n -factor computation
y	wall-normal direction

z spanwise direction

Greek

α wavenumber of the disturbance in x -direction
 α angle of attack
 β wavenumber of the disturbance in z -direction
 ξ streamwise direction in computational space
 η wall-normal direction in computational space
 η wall-normal direction in the body-attached coordinate system
 $\bar{\delta}$ Blasius length
 δ^* boundary-layer displacement thickness
 δ_c^* boundary-layer displacement thickness at the location of the surface imperfection
 γ ratio of specific heats
 γ sideslip angle
 θ_{max} maximum wall slope of a surface imperfection
 Θ phase function
 κ thermal conductivity coefficient
 λ second viscosity coefficient
 μ first viscosity coefficient
 ρ density
 σ growth rate
 ω circular frequency
 φ_∞ effective sweep angle
 ψ orientation of the wave vector w.r.t x_c -axis

Superscripts and marks

$\mathbf{T}$ transposed vector
 $*$ dimensional quantity
 $\dagger$ complex conjugate or conjugate transpose
 $-$ base flow quantity
 $\sim$ disturbance quantity
 $\wedge$ amplitude function

Subscripts

c body-fixed chordwise coordinate system
 $crit$ critical
 e value at boundary-layer edge
 i imaginary part
 r real part
 ref reference quantity

s	streamline-oriented coordinate system
S	spatial direction in which base flow and disturbance amplitude function are slowly varying
tr	transition
tr_0	transition for a clean / reference configuration
u	quantity based on disturbance velocity component in x -direction
∞	quantity in free stream

Abbreviations

AGARD	Advisory Group for Aerospace Research and Development
AHLNS	Adaptive Harmonic Linearized Navier-Stokes
BFS	Backward-Facing Step
BL	Boundary Layer
BLADE	Breakthrough Laminar Aircraft Demonstrator in Europe, an EC-funded project
CFI	CrossFlow Instability
DLR	Deutsches Zentrum für Luft- und Raumfahrt e. V.
DNS	Direct Numerical Simulation
EC	European Commission
FFS	Forward-Facing Step
FOI	Swedish Defence Research Agency
HLFC	Hybrid Laminar Flow Control
HLFC-WIN	HLFC WING components demonstrators, an EC-funded project
HLNS	Harmonic Linearized Navier-Stokes
LESP	Leading-Edge Suction Panel
KRG	Cryogenic Ludwig-Tube Göttingen
LNS	Linearized Navier-Stokes
LST	Local Stability Theory
LTT	Laminar-Turbulent Transition
MPI	Message Passing Interface
MUMPS	MULTifrontal Massively Parallel sparse direct Solver
NACOR	New innovative Aircraft COnfigurations and Related issues, an EC-funded project
NLF	Natural Laminar Flow
NOLOT	NOOnLocal Transition analysis, DLR/FOI-Code
NS	Navier-Stokes
PETSc	Portable Extensible Toolkit for Scientific computation
PSE	Parabolized Stability Equations
RANS	Reynolds-Averaged Navier-Stokes
TKE	Total Kinetic Energy
TS	Tollmien-Schlichting wave
UPM	Universidad Politécnica de Madrid
WB	Wing Box
2D	Two-Dimensional
2.5D	Infinite swept-wing condition
3D	Three-Dimensional

Chapter 1

Introduction

1.1 Motivation

The aeronautical industry has always sought for fuel-efficient aircraft. A straightforward way to achieve higher efficiency is to reduce the drag on wetted surfaces, *e.g.* wings, nacelles, vertical and horizontal stabilizers, etc. At typical flight conditions for commercial aircraft, turbulence levels are low, and the incoming flow is laminar. However, on current wing designs, this incoming flow remains laminar in the vicinity of the leading edge only. Laminar flow control (LFC) technology seeks a reduction in the total viscous drag of the aircraft. In particular, the design of *laminar* wings tries to achieve this viscous drag reduction by extending the region where the flow remains laminar. However, the presence of surface imperfections, such as joints between different aerodynamic elements, may play a key role in the process of laminar to turbulent transition. Such wall irregularities, in the form of two-dimensional steps, gaps, humps, etc. introduce very localized and relatively large changes in flow quantities in the streamwise direction of the boundary-layer region of the incoming laminar flow.

The physical mechanisms that trigger laminar to turbulent transition at those flight conditions are typically of convective nature: Tollmien-Schlichting (TS) waves and stationary crossflow (CF) vortices. From the perspective of research on the laminar-turbulent transition process, it is crucial to improve our understanding of the interaction between these physical mechanisms and the presence of such surface imperfections. Furthermore, from the perspective of designers, it is also critical to define certain maximum tolerances (in shape and/or dimensions) for the unavoidable presence of surface irregularities on the above-mentioned aerodynamic devices.

Hence, the increase in the aerodynamic efficiency of current and future designs in the aeronautical industry is closely related to the improvement on methodologies for prediction of laminar-turbulent transition.

1.2 Objectives and structure of the present work

Objectives

The e^N method is a very-well established methodology for laminar-turbulent transition prediction in the aeronautical industry. It requires computing the linear growth / decay of the flow instabilities that trigger transition. The spatial evolution of those flow instabilities is analysed by the linear stability theory, and the outcome of these computations is collected into the N -factor envelope curves. Within the framework of linear stability theory, several methodologies are available, according to the grade of assumptions included in each approach.

The influence of two-dimensional surface imperfections (or more in general, the presence of localized large streamwise gradients in flow quantities) is still an area where some space for improvement remains. Current methodologies which can take into account these localized large streamwise gradients within the context of linear stability theory (e.g. HLNS (Harmonic Linearized Navier-Stokes) and linear DNS (Direct Numerical Simulations)) require still a relatively large computational effort in order to correctly describe the spatial evolution of the typical convective instabilities that trigger the laminar-turbulent transition onset. One of the aims of this thesis is to develop the **AHLNS** (Adaptive Harmonic Linearized Navier-Stokes) equations, with the objective of to provide an alternative to those methodologies (HLNS and linear DNS), but requiring a significantly smaller amount of computational needs.

Once the AHLNS equations have been implemented in a numerical code, it is expected to test the applicability of the AHLNS method not only for relative simple configurations (e.g. rectangular steps on a flat plate in incompressible flow), but also for irregular surface imperfections on airfoil at transonic flow conditions. In this sense, Chapt. 9 summarizes the latest achievements done with AHLNS on recent EC-funded projects about laminar-turbulent transition on realistic wings. This task represents another important aim of this thesis: to introduce the AHLNS methodology as a very efficient tool for researches and designers in order to analyse the influence of two-dimensional surface imperfections in the linear stage of the laminar-turbulent transition process in practical scenarios, *e.g.* commercial laminar wings at typical flight regimes.

Structure

Chapters 2 and 3 introduce a brief review of the classical laminar-turbulent transition path, together with a small survey of the different models that have been historically developed in order to describe the various stages of the transition route. In particular, Chapt. 3 is devoted to describe the linear stability analysis up to certain detail. Starting from the Navier-Stokes equations, traditional methodologies such as LST (Local Stability Theory), PSE (Parabolized Stability Equations) and HLNS (Harmonic Linearized Navier-Stokes) are derived, focusing on the different assumptions made in each approach.

The core of this thesis, the **AHLNS** (Adaptive Harmonic Linearized Navier-Stokes) methodology, is fully described in Chapt. 4, 5, and 6. In particular, Chapt. 4 introduces the main ideas of the AHLNS approach. Chapt. 5 enumerates the key aspects of the numerical implementation of the AHLNS methodology in a computational code. This description includes the

discretization of the AHLNS equations and how the different boundary conditions are imposed. A special emphasis is put on the coupling of the AHLNS method with PSE, in order to enhance even more the efficiency of the approach. In order to be able to represent numerically a large variety of shapes of surface irregularities, a multi-block scheme together with a grid transformation have been implemented also. The last part of Chapt. 5 is devoted to explain briefly the solution procedure of the AHLNS equations. The verification and validation of the numerical implementation of the AHLNS methodology is conducted in Chapt. 6. The test cases presented here illustrate also the main differences of the AHLNS approach compared with similar methods, such as PSE, HLNS and linear DNS.

Chapters 7, 8, and 9 report practical applications of the AHLNS methodology in the study of the influence of two-dimensional surface imperfections in the linear development of Tollmien-Schlichting waves. Specifically, the influence of steps (Chapt. 7) and humps (Chapt. 8) on flat plates is analysed. Parametric studies about the height, shape (rounded/rectangular), type (back-/forward-facing steps), and length (humps) are conducted. A physical explanation of the instability analysis results, supported by the description of the steady laminar base flow characteristics for each surface irregularity, is given.

Chapter 9 summarizes the numerical investigations done within the framework of the EC-funded project HLFC-WIN. The aim of this chapter is to illustrate how the AHLNS methodology, in its current status, can be applied to transonic swept-wings at realistic flight conditions. Surface imperfections represent typically the joint of two aerodynamic devices (*e.g.* leading edge and wing box). Therefore, it is very plausible that the size of the surface imperfections changes slightly during flight conditions. This important aspect, and its effect on stability analysis, is discussed in this chapter also.

Finally, Chapt. 10 summarizes the key conclusions of previous chapters. Moreover, an outlook with possible research lines is presented. As an example, the studies conducted in this thesis have focused on the influence of two-dimensional surface imperfections on the linear development of TS waves. However, in three-dimensional flows, crossflow instabilities (CFI) can be the most important mechanisms that drive the laminar-turbulent transition. The extension of the research presented in this thesis to CFI is totally feasible without requiring any update for the current version of the AHLNS code.

Chapter 2

Background

2.1 The laminar-turbulent transition process in brief

In low-level disturbance environments such as free flight, the process in which an incoming laminar flow transitions into turbulent state is usually described by four different stages. The first or initial stage is often called receptivity. At this stage, environmental disturbances in the form of sound or vorticity fluctuations, superficial micro-roughness, vibrations or blowing/suction are converted into low-amplitude disturbances in the boundary layer. A survey about theory, modelisation and results can be found in the work of Saric *et al.* [1].

The second stage of the transition process consists of the initial growth of these low-amplitude disturbances. These disturbances, convected downstream after their generation, are usually named as *primary modes*. Since the amplitude of each mode is assumed to be small, they develop independently. This means that each mode grows or decays depending on the instability characteristics of the boundary layer (pressure gradients, surface effects, etc.). Typically, the linear growth/decay of the disturbances occurs over a relatively long length scale (compared with the region where non-linear interactions are no longer negligible). The evolution of the primary modes in this second stage is described by the linear stability theory.

The growth in amplitude of the *primary modes* can reach values where the assumption of *small* amplitude disturbances is no longer valid. The interaction between *primary modes* cannot be assumed to be negligible. Similarly, it cannot be neglected the distortion of the base flow caused by the *primary modes*. This third stage of the transition path is characterized by three-dimensional non-linear interactions and the rise of *secondary instabilities*. The non-linear growth of the disturbances is rapid (compared with the relatively slow spatial growth in the linear stage), and leads to the *breakdown* of the laminar flow (fourth stage) and turbulence occurs.

If the level of disturbance environment is increased, the exponential growth of flow disturbances described before in the *linear* stage might be bypassed, leading to a quick appearance of the turbulent state. This phenomenon is known as *bypass transition*. An example of bypass transition is the *transient growth* phenomenon (see Schmid [2], and Cantwell [3]).

Mathematically, it can be explained as it follows: The linearized Navier-Stokes operator is *non-normal*. This means that the eigenvectors of this operator are *non-orthogonal*. For the sake of clarification, a *normal* operator is such that, all eigenvectors are pairwise orthogonal. If these eigenvectors decay, even at very different rates, then a vector described by the linear superposition of those eigenvectors will decay also. However, for a *non-normal* operator such as the linear Navier-Stokes equations, it is feasible that, even if all (*non-orthogonal*) eigenvectors decay, a vector might initially grow (and finally decays) in response to the different decay rates of the respective eigenvectors.

Finally, for the completeness of this brief review about laminar-turbulent transition, the instability characteristics of the base flow and the disturbances is discussed. We will follow the ideas presented by Huerre and Monkewitz [4] for spatially evolving flows (*e.g.* boundary layers). Some ideas were originally established for parallel flows (i.e. flows which are independent of the streamwise x -position). However, those ideas can be applied still if the wavelength of the instability is much smaller compared with a length scale that characterizes the streamwise spatial changes in the flow. An example is the concept of *convectively/absolutely* unstable flow. In a convectively unstable flow, an impulse moves downstream away from the source and leaves the domain in its undisturbed state. However, a pulse-type disturbance contaminates completely an absolutely unstable flow. An instability is *convective* if its amplitude returns to zero in the laboratory frame while it grows infinitely in at least one reference frame that moves with non-zero velocity. The laboratory frame is simply the reference frame, defined by the boundary conditions of our particular problem (see Cossu and Loiseleux [5]). An instability is *absolute* if its amplitude grows unbounded with time at any fixed location in the reference laboratory frame.

2.2 Methods for analysis and prediction of transition

Receptivity is the process that establishes the initial conditions for disturbance amplitude, frequency, and phase. For ease, this review about the *Receptivity* stage is limited to subsonic flows only. Because boundary layers are convectively unstable flows, an unsteady disturbance is required to generate the instability waves. The birth of the disturbances is usually divided into *forced* and *natural* receptivity (Saric *et al.* [1]). *Forced* receptivity is referred to the unsteady and localized forced mechanisms that introduce energy into the flow with a combined frequency-wavelength similar to the disturbance sought. Superficial micro-roughness, surface vibrations or blowing/suction are examples of this type of mechanisms. On the contrary, *natural* receptivity mechanisms consist of acoustic disturbances that propagate at the speed of sound relative to the fluid and vortical disturbances that convect at the freestream speed. In contrast, instability waves have phase speeds that are a fraction of the freestream speed. Hence, these freestream (acoustic/vortical) disturbances have wavenumbers that are significantly different from the instability wavenumber sought. Therefore, natural receptivity mechanisms require a wavelength conversion process. The work of Bertolotti ([6], [7]) illustrates some examples of such *forced/natural* receptivity mechanisms. Another key reference is the work of Reshotko [8], where the receptivity to free-stream disturbances into the base flow is described as an initial value problem.

In two-dimensional flows the streamwise length where the linear development of the disturbances takes place is large compared with the extent of the non-linear region. This argument has justified that many tools for instability analysis and transition prediction have focused on the linear stage of the laminar-turbulent transition path only. However, since the initial conditions provided by the receptivity stage are not known, only correlations between two systems with similar environmental conditions are possible.

The instability characteristics of the convective disturbances in the linear stage of the laminar-turbulent transition path are usually computed by LST, PSE or HLNS approaches. These methods are described in Sect. 3.3. In cases where localized large streamwise gradients arise in the flow, *e.g.* due to the presence of surface imperfections, the most efficient approach is the AHLNS methodology (see Chapt. 4). The AHLNS equations are able to describe the linear spatial evolution of the flow disturbances in the presence of locally large flow gradients in streamwise direction. In contrast to more numerically demanding methods such as HLNS or linear DNS, the smaller streamwise grid resolution needed by AHLNS allows its use for parametric studies typically required in the industry.

Once the linear instability characteristics of the flow mechanisms that trigger laminar-turbulent transition are known, one of the most well-known approaches for prediction of the transition location is the e^N method. This method was developed independently by Smith and Gamberoni [9], and Van Ingen [10]. Authors integrated the growth rates of several disturbances, resulting in individual A/A_0 curves for each disturbance. An individual A/A_0 curve describes the relative amplitude A of the particular instability with respect to its initial amplitude A_0 . However, this initial amplitude is generally unknown, as mentioned before. Smith and Gamberoni [9], and Van Ingen [10] found that experimental transition location correlated reasonably well with values of the envelope curve of $A/A_0 = e^N$, with $N = 9$ for Smith and Gamberoni [9], and $N \sim 7 - 8$ for Van Ingen [10]*.

In three-dimensional boundary layers, the region of non-linear interactions in the laminar-to-turbulence path can extend much longer than for two-dimensional flows. The non-linear version of the PSE (see Hein *et al.* [11]) is able to reproduce features that indicate the onset of turbulence breakdown, for example the rise in skin-friction. However, adequate initial conditions (i.e. the initial disturbance spectrum) are required still.

Finally, all stages of the laminar-to-turbulence transition scenario could be covered by the use of DNS. However, the numerical resources required for cases of industrial interest (*e.g.* transonic swept wing at realistic flight conditions) are far from being available. Therefore, the use of DNS is limited to very particular cases or as validation of other approaches.

*Throughout this thesis, an individual A/A_0 curve associated with a particular disturbance will be mentioned as n -factor curve, while the envelope of these curves for an spectrum of disturbances will be denoted as N -factor envelope curve

Chapter 3

Linear stability theory

Linear Stability Theory is the branch of the fluid dynamics that studies the behaviour of a flow field upon the introduction of small-amplitude disturbances. One of the applications is the understanding of the processes of transition of a laminar flow to a turbulent regime.

Starting from the Navier-Stokes equations, and introducing certain assumptions, a number of instability analysis approaches will be described. The core of the thesis, the recently developed *Adaptive Harmonic Linearized Navier-Stokes* equations, will be described in detail separately in Chapt. 4. Finally, it is included a brief explanation about how the expected laminar-to-turbulent transition location can be related to the instability analysis results.

3.1 Navier-Stokes equations

The compressible Navier-Stokes equations describe the motion of a viscous fluid by imposing conservation of mass, momentum and energy. It is a system of non-linear partial differential equations which can be written as

$$\frac{\partial \rho^*}{\partial t^*} + \nabla \cdot (\rho^* \mathbf{u}^*) = 0, \quad (3.1)$$

$$\rho^* \left[\frac{\partial \mathbf{u}^*}{\partial t^*} + (\mathbf{u}^* \cdot \nabla) \mathbf{u}^* \right] = -\nabla p^* + \nabla [\lambda^* (\nabla \cdot \mathbf{u}^*)] + \nabla \cdot [\mu^* ((\nabla \mathbf{u}^*) + (\nabla \mathbf{u}^*)^T)], \quad (3.2)$$

$$\begin{aligned} \rho^* c_p^* \left[\frac{\partial T^*}{\partial t^*} + (\mathbf{u}^* \cdot \nabla) T^* \right] &= \nabla \cdot (\kappa^* \nabla T^*) + \frac{\partial p^*}{\partial t^*} + (\mathbf{u}^* \cdot \nabla) p^* + \lambda^* (\nabla \cdot \mathbf{u}^*)^2 \\ &\quad + \frac{\mu^*}{2} [(\nabla \mathbf{u}^*) + (\nabla \mathbf{u}^*)^T]^2, \end{aligned} \quad (3.3)$$

where ρ^* is the density, $\mathbf{u}^*$ is the velocity vector, t^* is time, κ^* is the thermal conductivity

coefficient, and μ^* and λ^* are the first and second coefficients of viscosity, respectively. c_p^* stands for the specific heat at constant pressure. It is assumed that the equation of state is given by the perfect gas relation

$$p^* = \rho^* \mathcal{R} T^*, \quad (3.4)$$

where $\mathcal{R}$ is the specific gas constant. The first and second coefficients of viscosity are related using the Stokes' hypothesis

$$\lambda^* = - (2/3) \mu^*. \quad (3.5)$$

The asterisk $*$ denotes dimensional quantities. The fluid variables are nondimensionalized as follows. Lengths are scaled with a fixed reference length l_{ref}^* . Typically, a characteristic length is chosen as reference length e.g. boundary-layer thickness, displacement thickness or Blasius length at a given reference position x_{ref}^* . Velocities are scaled with u_{ref}^* and temperature by T_{ref}^* (usually the boundary-layer edge velocity and temperature at x_{ref}^* , respectively). Pressure is made nondimensional by twice the corresponding dynamic pressure $p_{ref}^* = \rho_{ref}^* (u_{ref}^*)^2$. The reference time is defined by l_{ref}^* / u_{ref}^* . The subsequent nondimensional Navier-Stokes equations are

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \quad (3.6)$$

$$\rho \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right] = -\nabla p + \frac{1}{Re} \nabla [\lambda (\nabla \cdot \mathbf{u})] + \frac{1}{Re} \nabla \cdot [\mu ((\nabla \mathbf{u}) + (\nabla \mathbf{u})^T)], \quad (3.7)$$

$$\begin{aligned} \rho c_p \left[\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T \right] &= \frac{1}{Re Pr} \nabla \cdot (\kappa \nabla T) + (\gamma - 1) Ma^2 \left[\frac{\partial p}{\partial t} + (\mathbf{u} \cdot \nabla) p \right] \\ &+ \frac{(\gamma - 1) Ma^2 \mu}{Re} \left[-\frac{2}{3} (\nabla \cdot \mathbf{u})^2 + \frac{1}{2} [(\nabla \mathbf{u}) + (\nabla \mathbf{u})^T]^2 \right]. \end{aligned} \quad (3.8)$$

The resulting nondimensional parameters Re (Reynolds number), Pr (Prandtl number), and Ma (Mach number) are defined as follows

$$Re = \frac{\rho_{ref}^* u_{ref}^* l_{ref}^*}{\mu_{ref}^*}, \quad (3.9)$$

$$Pr = \frac{c_{p,ref}^* \mu_{ref}^*}{\kappa_{ref}^*}, \quad (3.10)$$

$$Ma = \frac{u_{ref}^*}{\sqrt{\gamma \mathcal{R} T_{ref}^*}}, \quad (3.11)$$

where γ is the ratio of specific heats. Throughout this thesis, the value of γ is fixed to 1.4 and the Prandtl number is set to 0.72.

In cases where the Mach number is relatively low (typically $Ma < 0.1$), a simplification of the system of equations (3.6)-(3.8) is possible by assuming that the nondimensional density ρ remains constant within a parcel of fluid of interest. The so-called "Incompressible Navier-Stokes equations" (in nondimensional form) can be written as

$$\nabla \cdot \mathbf{u} = 0, \quad (3.12)$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u}. \quad (3.13)$$

3.2 Linearized Navier-Stokes equations

The main idea of the linear stability theory is to superpose an infinitesimal small perturbation onto an undisturbed steady laminar basic state and to determine whether this perturbation grows or decays. Therefore, flow quantities $\mathbf{q}$ (velocity vector $\mathbf{u}$, temperature T , pressure p , and density ρ) are decomposed into a steady part $\bar{\mathbf{q}}$ and an unsteady perturbation $\tilde{\mathbf{q}}$

$$\mathbf{q}(\mathbf{x}, t) = \bar{\mathbf{q}}(\mathbf{x}) + \epsilon \tilde{\mathbf{q}}(\mathbf{x}, t), \quad \epsilon \ll 1 \quad (3.14)$$

where $\mathbf{x}$ is the position vector, and the parameter ϵ indicates that *small-amplitude* perturbations are considered only. It is very common that, in the context of *wall-bounded* flows, the three spatial coordinates (x, y, z) are called *streamwise*, *wall-normal* and *spanwise* directions, respectively. This flow decomposition (3.14) is introduced into the nondimensional Navier-Stokes equations (3.6)-(3.8), resulting in a system of equations where the set of equations satisfied by the steady basic state or *base flow* $\bar{\mathbf{q}}(\mathbf{x})$ can be subtracted. Moreover, non-linear terms with respect to the parameter ϵ are neglected. This procedure leads to the nondimensional *Linearized Navier-Stokes* (LNS) equations. Therefore, once a steady base flow is provided, the LNS equations describe the evolution of any small-amplitude perturbation and assess if the steady basic state $\bar{\mathbf{q}}(\mathbf{x})$ is linearly stable or not. If one would be interested in describing the *non-linear* behaviour of the perturbations, or how the perturbations interact with the base flow, without assuming any other hypothesis, then a *direct numerical simulation* (DNS) of the *perturbed* flow field would be required. It is important to remark that, throughout this thesis, DNS where the amplitude of the perturbation remains *small* (such that non-linear interactions or interactions with the base flow can be considered negligible) will be called *linear DNS*. Since we are interested only in the linear evolution of the flow disturbances, many results found in literature based on *linear* DNS will be simply presented here as *DNS* results.

Regarding the nature of the perturbations, and following the ideas presented by Huerre and Monkewitz [4], only convectively unstable flows will be discussed in this thesis. This means

that no self-sustained oscillations in the base flow might arise. The base flow acts purely as amplifier of external disturbances, and these disturbances grow and decay as convected instabilities.

3.3 Linear modal stability theory

The use of the complete set of LNS equations for realistic three-dimensional cases might be, numerically, very challenging. Alternatively, some assumptions can be introduced in the description of the perturbation $\tilde{\mathbf{q}}(\mathbf{x}, t)$ and the basic state $\bar{\mathbf{q}}(\mathbf{x})$, in order to reduce the complexity of the particular case. Within the context of *laminar-turbulent transition*, it is very common [12] to assume that the base flow is homogeneous in certain spatial directions, and perturbations are periodic in one or two spatial coordinates and/or in time. Via a *Fourier transformation*, a harmonic representation of the disturbances is possible in those spatial directions and/or in time. Throughout this thesis, only time-harmonic disturbances are considered. Therefore, discussions about the simplifications introduced in the base flow/perturbations will be limited to the spatial domain. Stability is determined separately for each Fourier mode, giving rise to the term *modal*. In this context, the linear perturbation $\tilde{\mathbf{q}}$ of each particular mode takes the form of the product of an amplitude or shape function $\hat{\mathbf{q}}$ and wave function $e^{i\Theta}$,

$$\tilde{\mathbf{q}}(\mathbf{x}, t) = \hat{\mathbf{q}}(\mathbf{x})e^{i\Theta}, \quad (3.15)$$

where Θ represents the phase function. Table 3.1 summarizes the linear modal stability theories that are described up to certain extent in the present thesis, in terms of the mathematical representation of the simplifications introduced into the steady base flow $\bar{\mathbf{q}}(\mathbf{x})$ and perturbation $\tilde{\mathbf{q}}(\mathbf{x}, t)$. For all these approaches, it is assumed that both the steady basic state $\bar{\mathbf{q}}(\mathbf{x})$ and amplitude function $\hat{\mathbf{q}}(\mathbf{x})$ do not change in spanwise z -direction.

LST denotes the *Local Stability Theory*. In the LST is assumed that base flow quantities and amplitude function of the disturbances are varying in wall-normal y -direction only*. Moreover, perturbations are assumed to be periodic in spanwise z -direction and in time t . PSE hold for *Parabolized Stability Equations*. Contrary to the LST, the PSE allow that base flow $\bar{\mathbf{q}}$ and amplitude $\hat{\mathbf{q}}$ of the disturbance quantities are slowly varying in streamwise x -direction (compared with the rapid wall-normal variations). The following approach, and the main object of discussion in this thesis, is the AHLNS (*Adaptive Harmonic Linearized Navier-Stokes*[†]) equations. Finally, HLNS stand for *Harmonic Linearized Navier-Stokes* equations. Identical to AHLNS, disturbances are assumed to be periodic in spanwise z -direction and in time t . However, no assumptions are made about the nature of the disturbances in the streamwise direction. Differences between AHLNS and HLNS will be described in Chapt. 4 in

*The assumption of a base flow homogeneous in two spatial directions is known as *parallel flow approximation*.

[†]The AHLNS approach, core of this thesis, is described in detail in the following chapter.

Name	Base flow $\bar{\mathbf{q}}$	Amplitude Function $\hat{\mathbf{q}}$	Phase Function Θ
LST	$\bar{\mathbf{q}}(y)$ $\partial_x \bar{\mathbf{q}} = \partial_z \bar{\mathbf{q}} = 0$	$\hat{\mathbf{q}}(y)$ $\partial_x \hat{\mathbf{q}} = \partial_z \hat{\mathbf{q}} = 0$	$\alpha x + \beta z - \omega t$
PSE	$\bar{\mathbf{q}}(x_S, y)$ $\partial_z \bar{\mathbf{q}} = 0$ $\partial_x \bar{\mathbf{q}} \ll \partial_y \bar{\mathbf{q}}$	$\hat{\mathbf{q}}(x_S, y)$ $\partial_z \hat{\mathbf{q}} = 0$ $\partial_x \hat{\mathbf{q}} \ll \partial_y \hat{\mathbf{q}}$	$\int \alpha(x_S) dx_S + \beta z - \omega t$
AHLNS	$\bar{\mathbf{q}}(x, y)$ $\partial_z \bar{\mathbf{q}} = 0$	$\hat{\mathbf{q}}(x, y)$ $\partial_z \hat{\mathbf{q}} = 0$	$\int \alpha(x) dx + \beta z - \omega t$
HLNS	$\bar{\mathbf{q}}(x, y)$ $\partial_z \bar{\mathbf{q}} = 0$	$\hat{\mathbf{q}}(x, y)$ $\partial_z \hat{\mathbf{q}} = 0$	$\beta z - \omega t$

Table 3.1: Classification of different linear modal stability approaches according to the basic state assumptions and the disturbance representation, with $\partial_i(\cdot) = \frac{\partial(\cdot)}{\partial i}$ (where $i = x, y, z$). The subscript S indicates the coordinate direction in which the corresponding base flow and disturbance amplitude function is assumed to be slowly varying. Wavenumbers α and β hold for the streamwise and spanwise components of the wave vector, respectively. ω represents the circular frequency of the disturbance. Adapted from [13].

detail. Each approach is described up to certain level in the following sections, pointing out its advantages and limitations as a consequence of the corresponding simplifications introduced.

3.3.1 Local stability theory

If the base flow is assumed to be homogeneous in two spatial directions, i.e. x and z (*parallel flow approximation*), then it is possible to express the disturbance $\tilde{\mathbf{q}}(\mathbf{x}, t)$ as

$$\tilde{\mathbf{q}}(\mathbf{x}, t) = \hat{\mathbf{q}}(y) e^{i(\alpha x + \beta z - \omega t)}. \quad (3.16)$$

This representation of the disturbances is called *normal-mode* form [14]. This approach is very often referred to in literature [15] as *local*, since the variations in streamwise x -direction of the base flow and shape function $\hat{\mathbf{q}}$ of the perturbations are neglected, and only variations along the wall-normal y -component are considered. Wavenumbers α and β hold for the streamwise and spanwise components of the wave vector, respectively. ω represents the circular frequency of the disturbance. In the most general case α , β and ω are complex, representing a *wave-like* disturbance whose amplitude varies both in time and space. If the wavenumbers α and β are real, and the frequency ω is complex, the individual Fourier mode will be periodic in streamwise and spanwise directions. This is known in literature (see Juniper *et al.* [12] or Mack [14]) as the *temporal-local* stability theory (*temporal-LST*). On the other hand,

if α and β are complex, and the frequency ω is real, the individual Fourier mode will be periodic in time. This framework, where the wavenumbers α and β are complex, and ω is a real-valued parameter, is known as *spatial*-local stability theory (*spatial*-LST). As mentioned before, throughout this thesis only *time*-periodic disturbances are considered. Therefore, the *spatial* approach will be our frame of study. An excellent review about *temporal*, *spatial*, and *temporal-spatial* approaches can be found in the work of Huerre and Monkewitz [4].

Regarding the spanwise wavenumber β , the homogeneity of the base flow in spanwise direction z , together with the *irrotationality* of the wave vector (see Mack [14] or Hein *et al.* [16]), support the assumption of

$$\frac{\partial \beta}{\partial x} = 0 \quad \Rightarrow \quad \beta = \text{const.} \quad (3.17)$$

Furthermore, it is assumed that the growth in amplitude of the disturbances is limited to the streamwise x -direction only. Therefore, β is a parameter with real part only. Introducing the normal-modal representation of the disturbance (3.16) into the LNS equations, together with the *parallel flow assumption* of the steady base flow, leads to a system of ordinary differential equations, whose coefficients are either constants or depend on the wall-normal component y only. The growth rate σ of a particular disturbance is given by the imaginary component of the wavenumber α :

$$\sigma = -\alpha_i. \quad (3.18)$$

3.3.2 Parabolized stability equations

In many wall-bounded flows, such as the *Blasius* boundary layer [17], the previous *parallel flow approximation* is not strictly valid, since the boundary layer varies in streamwise direction also. However, it is possible still to describe the linear perturbations $\tilde{\mathbf{q}}$ as the product of an amplitude function $\hat{\mathbf{q}}$ and a wave function $e^{i\Theta}$ (3.15).

For *non-parallel* flows, the amplitude function and the phase function takes the form (see Mack) [14] or Herbert [18])

$$\hat{\mathbf{q}}(\mathbf{x}) = \hat{\mathbf{q}}(x, y) \quad (3.19)$$

$$\Theta(\mathbf{x}, t) = \theta(x) + \beta z - \omega t \quad (3.20)$$

where $\frac{d\theta}{dx} = \alpha(x)$. Here, α is a complex-valued unknown, while β and ω are real-valued parameters. In contrast to the local stability theory, not only the phase function Θ , but the amplitude function $\hat{\mathbf{q}}$ are both allowed to vary in x -direction. In literature, this approach is also known as *nonlocal* [19]. Therefore, an extra condition or equation is needed to remove this ambiguity. This extra condition is called *Normalization condition* (very well described in the PSE context by Herbert *et al.* [20], Bertolotti *et al.* [21], and Hein *et al.* [16]). By means

of this extra equation, it is possible to specify how much of the streamwise changes of the disturbances are represented in the amplitude and phase functions, respectively. Ideally, the normalization condition must limit the rapid changes of the amplitude function in x , in such a way that most of the streamwise variations of the disturbance are covered by the phase function. Herbert [18], and Bertolotti *et al.* [21] discussed different choices of the normalization equation. Interestingly, Bertolotti *et al.* [21] showed that the growth rate of the instabilities is independent of the normalization condition chosen (between the different choices considered in their work). In fact, Chang *et al.* [22] used a slightly different normalization condition, and they arrived to the same conclusion. In particular, in the PSE code used throughout this thesis (*NOLOT*), and implemented by Hein *et al.* [16], the normalization condition is

$$\int_0^\infty \hat{\mathbf{q}}^\dagger \frac{\partial \hat{\mathbf{q}}}{\partial x} dy = 0, \quad (3.21)$$

where $\dagger$ means complex conjugate. A very detailed description about this particular normalization condition can be found in the seminal work of Bertolotti *et al.* [21]-Sect. 2.3.

It is crucial to notice that the *wave-like* representation of the disturbances introduced in (3.19)-(3.20), and the *normalization condition* represented by (3.21), does not change *per se* the *elliptic* nature of the LNS equations in streamwise direction. A second assumption is required to achieve the PSE. First, we introduce a scale separation between the small streamwise variations and the large wall-normal gradients of both base flow $\bar{\mathbf{q}}$ and disturbances amplitude quantities $\hat{\mathbf{q}}$. The scale separation is of order $1/Re$. Here, the Reynolds number Re , defined in Eqn. (3.9), is based on a length scale l_{ref}^* of the order of the boundary-layer thickness. Based on this scale separation, second derivatives of amplitude and phase functions in streamwise direction are negligibly small [16]. Base flows that hold this second assumption are known in literature as *weakly nonparallel flows*.

To sum up, the *wake-like* description of the disturbances, the *normalization condition* and the above-mentioned scaling, introduced into the LNS equations and keeping terms that scale with powers of Reynolds number up to $1/Re$, leads to the PSE (see Simen [19], and Hein *et al.* [16]). Since only disturbances that growth or decay as convected instabilities are considered, the upstream propagation of disturbances has a negligible influence, and the PSE can be solved by a marching procedure [21].

However, the use of a marching procedure to compute the spatial evolution of the disturbances by means of PSE is not straightforward. Strictly speaking, the PSE are not *parabolic*, but *nearly parabolic* (see [22]). Second-order derivatives in streamwise direction are not present in the formulation of PSE. However, a small ellipticity remains in the PSE which may lead to numerical instabilities. The influence of the remaining *ellipticity* in the PSE can be remedied by the use of a sufficiently large step size in the marching procedure. Many works have addressed this topic (the existence of a minimum step size), in particular in Li and Malik [23], and Chang *et al.* [22]. The work of Andersson *et al.* [24] comprises several alternative techniques that alleviate these numerical instabilities. Finally, it is very common to take a solution of the LST equations as initial condition of the marching procedure.

3.3.3 Harmonic linearized Navier-Stokes equations

As stated by Franco and Hein [25], three types of HLNS equations could be derived from the more general *Linearized Navier-Stokes* (*Standard* LNS) equations. We recall that the *harmonic* version of the LNS (HLNS) are obtained assuming that the base flow is homogeneous in z -direction, and the disturbances are periodic in spanwise z -direction and time t (see the description of modal analysis at the beginning of Sect. 3.3). No assumptions are made regarding the nature of the flow disturbances in the streamwise x -direction.

If the only assumption made is that disturbances are periodic in time, then it is possible to derive the *time-harmonic* version of the LNS equations. In contrast, there are the *spanwise harmonic*-LNS equations. These equations are derived from the LNS equations assuming that the base flow is spanwise invariant, and disturbances are periodic in spanwise z -direction. Finally, the third type of HLNS equations is the combination of the previous two approaches. The *harmonic*-LNS equations assume that the steady base flow is homogeneous in z -direction, and the disturbances are described by a Fourier-mode representation for both spanwise direction and time. A summary of the different approaches described here is presented in Table 3.2.

Note that, regarding the description of the disturbances in x -direction, there is no difference between the four LNS approaches described in this section. Therefore, the large computational resources required by *Standard* LNS/*linear* DNS and any of the three *Harmonic*-LNS (*time-harmonic*, *spanwise-harmonic* or simply *harmonic*), in terms of grid resolution in streamwise direction, are identical. Nevertheless, there may be order-of-magnitude differences concerning the computational resources required between the four types of LNS variants described in this section.

Name	Base flow $\bar{\mathbf{q}}$	Amplitude Function $\hat{\mathbf{q}}$	Phase Function Θ
Standard	$\bar{\mathbf{q}}(x, y, z)$	$\hat{\mathbf{q}}(x, y, z, t)$	—
Time-harmonic	$\bar{\mathbf{q}}(x, y, z)$	$\hat{\mathbf{q}}(x, y, z)$ $\partial_t \hat{\mathbf{q}} = 0$	$-\omega t$
Spanwise-harmonic	$\bar{\mathbf{q}}(x, y)$ $\partial_z \bar{\mathbf{q}} = 0$	$\hat{\mathbf{q}}(x, y, t)$ $\partial_z \hat{\mathbf{q}} = 0$	βz
Harmonic	$\bar{\mathbf{q}}(x, y)$ $\partial_z \bar{\mathbf{q}} = 0$	$\hat{\mathbf{q}}(x, y)$ $\partial_z \hat{\mathbf{q}} = 0$ $\partial_t \hat{\mathbf{q}} = 0$	$\beta z - \omega t$

Table 3.2: Classification of different LNS approaches according to the basic state assumptions and the disturbance representation, with $\partial_i(\cdot) = \frac{\partial(\cdot)}{\partial i}$ (where $i = x, y, z, t$). Adapted from [25].

3.4 Growth rate and n -factor

The classical *four-stages* path for the laminar-to-turbulent transition scenario in low turbulence levels (such as *e.g.* free-flight conditions), adapted from Morkovin [26], and described briefly in Sect. 2.1, can be summarized as:

$$\boxed{\text{receptivity} \rightarrow \text{linear growth} \rightarrow \text{non-linear interactions} \rightarrow \text{breakdown to turbulence}}$$

The main focus of this thesis is the description of the AHLNS methodology. In its current status, the AHLNS approach belongs completely to the *linear growth* stage of disturbances. For this reason, the three remaining processes (i.e. *receptivity*, *non-linear interactions*, and *breakdown to turbulence*) are not discussed throughout this thesis. The literature in these subjects is vast, and the fundamental ideas can be found in both AGARD reports 709 [14] and 793 [27].

The work of Arnal (*Predictions based on linear theory*, included in the AGARD report 793 [27]) summarizes the limitations of the linear stability theory, especially regarding the connexions with the remaining stages of the transition scenario (i.e. *receptivity*, *non-linear interactions*, and *breakdown to turbulence*). One of the major drawbacks of the linear theory is that the amplitude of the disturbances does not play any role. In other words, only the amplification ratio for a disturbance between two locations (x_1, x_2) can be computed. This amplification ratio is known in literature by the n -factor:

$$n = \ln \left(\frac{A(x_2)}{A(x_1)} \right) = \int_{x_1}^{x_2} \sigma dx, \quad (3.22)$$

where $A(x)$ denotes the disturbance amplitude, and $\sigma(x)$ the corresponding *growth rate* of such disturbance. It is very common to place x_1 as the most upstream location where the individual flow disturbance starts to grow [13]. In the LST approach, the growth rate is directly the imaginary component of the streamwise wavenumber (3.18). Fasel and Konzelmann [28] discussed the influence of each flow disturbance component and the wall-normal location in the choice of the *disturbance amplitude* $A(x)$ for their DNS studies.

The *nonlocal wave-like* representation of the flow disturbances used in both PSE and AHLNS (3.19)-(3.20) leads to the definition of the growth rate of a disturbance given by the expression:

$$\sigma = -\alpha_i + \Re \left\{ \frac{1}{\xi} \frac{\partial \xi}{\partial x} \right\}, \quad (3.23)$$

where ξ is usually a component of the amplitude function $\hat{\mathbf{q}}$, measured either at some fixed wall-normal y -position or where it reaches its maximum value. It is very common to use the streamwise velocity component of the amplitude function $\hat{u}$, measured at the wall normal y -position where it reaches its maximum value, to define the growth rate of a particular flow

disturbance. The growth rate and n -factors presented in this thesis will be computed using this definition based on $\hat{u}$, i.e. $\sigma_{\hat{u}}$.

Alternatively, in order to get rid of the influence of the choice of the wall-normal y -position, it is possible to define an integral quantity, as the total kinetic energy TKE, to measure of the amplitude of the disturbance:

$$TKE = \int_0^\infty \bar{\rho} \left(\|\hat{u}\|^2 + \|\hat{v}\|^2 + \|\hat{w}\|^2 \right) dy. \quad (3.24)$$

Then, the growth rate σ (3.23) is computed as

$$\sigma = -\alpha_i + \frac{\partial}{\partial x} \ln \left(\sqrt{TKE} \right). \quad (3.25)$$

Each individual n -factor is linked to a particular disturbance mode (defined for a particular frequency ω and spanwise wavenumber β). Within the context of modal spatial analysis, the n -factor must be computed for each mode. The envelope of all n -curves determines the maximum amplification of all possible disturbances at each x -location, the so-called N -factor envelope curve.

Chapter 4

The adaptive harmonic linearized Navier-Stokes methodology

4.1 Introduction

In many flows of practical interest there are localized regions where the flow variations in streamwise x -direction are similar (or even larger) to the gradients in wall-normal y -direction. A typical example is the presence of a surface irregularity such as steps, gaps and/or humps. The main interest lies on cases where these surface imperfections are located on aerodynamic devices such as wings or nacelles. The importance of such imperfections is due to, depending on their shape, size, position, etc., their impact on the expected laminar-turbulent transition location might be very large (compared with clean configurations where no irregularity is present). In those localized regions, the assumptions of *parallel flow* (for LST) or *slow streamwise variations* (for PSE) might be questionable and, eventually, lead to an incorrect description of the spatial evolution of the flow disturbances. In principle, a straight-forward alternative to LST/PSE approaches would be the HLNS method. The HLNS approach, described in Sect. 3.3.3, allows the presence of large streamwise gradients. However, since all changes in x -direction are collected in the amplitude function of the disturbance description (i.e. the wave function contains changes in spanwise z -direction and time t only), the grid requirements in x -axis might be quite demanding.

4.2 AHLNS vs HLNS

The *Adaptive Harmonic Linearized Navier-Stokes* (AHLNS) methodology share many of the assumptions made for the HLNS equations, i.e. the base flow is homogeneous in z -direction, and the disturbances are described by a Fourier-mode representation for both spanwise direction z and time t . However, the AHLNS approach takes the *wave-like* representation of the flow disturbances in streamwise x -direction (3.19)-(3.20) borrowed from the PSE approach

(Sect. 3.3.2). This means that streamwise variations are split between the amplitude function and the wave function. It is important to remark that *wave-like* representation does not imply any constraint regarding the streamwise gradients. Therefore, the *large* changes in x -direction mentioned before can be perfectly managed by the *wave-like* representation of the disturbances.

The introduction of the *wave-like* representation of the disturbances in x -direction leads to the requirement of the *normalization* condition, described already for PSE (Sect.3.3.2). The *normalization* or *auxiliary* condition defines how much of the streamwise gradients can be included in the wave function. This is described in more detail in the following Sect. 4.4.

Finally, there is a last point to mention when comparing HLNS and AHLNS methodologies. Some readers might already implicitly notice, but the HLNS approach could be described as a *particular* case of the more general AHLNS approach. This is a consequence of the *wave-like* representation of the flow disturbances. In the AHLNS approach, if *all* streamwise gradients are described by the amplitude function of the disturbance solely, then the HLNS methodology is recovered. Mathematically, this implies that the streamwise wavenumber α is fixed to $\alpha(x) \equiv 0$. An illustrative example about this idea is described in detail in the verification test case of Sect. 6.1.4.

4.3 AHLNS vs PSE

The AHLNS methodology is expected to be applied in regions where streamwise variations in base flow and disturbance amplitude quantities are large, i.e. where streamwise gradients are no longer significantly smaller than wall-normal variations. Therefore, the *scale separation* introduced in the PSE is no longer possible. Consequently, the AHLNS equations cannot be *parabolized* as PSE and must be solved as a fully elliptic problem, i.e. the AHLNS equations cannot be solved via a marching procedure. In contrast to PSE, the grid size in streamwise direction can be as small as required.

Bertolotti *et al.* [21] pointed that the PSE assumes that convective instabilities show a negligible influence on downstream conditions. This aspect is especially relevant in the presence of surface imperfections, where a *non-negligible* upstream influence of the irregularities on the spatial evolution of the flow instabilities must often be considered. While this is a limitation for the marching procedure, the fully-elliptic character of the AHLNS equations is able to manage such type of upstream influences in the spatial description of the flow instability mechanisms.

The marching procedure in x -direction in regions of *small* streamwise gradients is extremely efficient (from a numerical point of view). Therefore, if regions of *large* streamwise gradients are localized (i.e. do not extend along the whole domain of interest) then the join of PSE and AHLNS results in a very powerful and, to our knowledge, currently unbeatable instability analysis tool for describing the linear spatial evolution of convective instabilities, such as Tollmien-Schlichting waves and crossflow vortices. Moreover, sharing an identical ansatz for describing the flow disturbances offers a very useful property: the coupling between the

PSE and AHLNS methods is straightforward. This is particularly interesting for the study of surface imperfections. Typically, the large gradients associated with those irregularities extend few boundary-layer displacement thickness upstream and downstream of the location of the surface defect. Therefore, a significant numerical efficiency can be achieved if the AHLNS approach is applied in a limited region around the location of the surface imperfection, and PSE are used relatively far (upstream and downstream) from the irregularity position. This coupling is explained in more detail in Sect. 5.2.3.

4.4 The *adaptive* approach

The *adaptive* approach was originally introduced by Guo *et al.* [29]. Basically, the term *Adaptive* described in their work consisted in to incorporate the *wave-like* representation of the disturbances in streamwise direction in their HLNS solver. Surprisingly, Guo *et al.* [29] explicitly stated that "*So far the convergence property of the proposed iterative scheme is not fully understood, as is the case for the PSE approach*". This "*iterative scheme*", i.e. the normalization or auxiliary condition, is the key concept of the *adaptive* approach. In other words, the full potential of the adaptive approach was not exploited by Guo *et al.* [29] (they used an equally spaced point distribution in streamwise direction). However, it is worth to point out an important conclusion from their work: including the *wave-like* representation into their HLNS code allowed them to reproduce the spatial evolution of convective instabilities using 1-2 points per wavelength. Without the *wave-like* description of the flow instabilities in streamwise direction, they needed to increase the number of points per wavelength significantly (about 30 points per wavelength).

The key point of the *adaptive* approach was introduced by Franco and Hein [25], and it consists in the use of a variable number of grids points per wavelength of the flow disturbance in cases where localized large streamwise gradients in the base flow are present. This is explained as follows: the normalization condition is able to transfer the majority of the streamwise variations into the wave function of the flow disturbances. On the contrary, in areas where rapid changes in base flow profiles and growth rates of the disturbances occur (*e.g.* in the close vicinity of a surface imperfection), the sinusoidal representation of the disturbances given by the wave function is not able to reproduce those large gradients via the streamwise wavenumber α of the disturbances. Therefore, the amplitude function must collect these large variations by increasing the number of points of the grid in streamwise direction.

The normalization condition (3.21) was introduced in Sect. 3.3.2 for PSE. Details of this operator are given in [21]-Sect. 2.3. However, we describe briefly some ideas with the aim of clarifying how the normalization condition works. The integrand of (3.21) is the inner product $\langle \hat{\mathbf{q}}, \hat{\mathbf{q}}_x \rangle$. The projection p_q of $\hat{\mathbf{q}}_x$ onto $\hat{\mathbf{q}}$ is defined as

$$p_q = \frac{\langle \hat{\mathbf{q}}, \hat{\mathbf{q}}_x \rangle}{\langle \hat{\mathbf{q}}, \hat{\mathbf{q}} \rangle}. \quad (4.1)$$

The aim of the normalization condition is to find, iteratively, a value of the streamwise

wavenumber α that reduces the value of p_q . In other words, that the condition (3.21) is satisfied. As Herbert noted [18], this condition could be satisfied at some particular wall-normal location y_m (*e.g.* the location where the amplitude function reaches a maximum). However, if it expressed in a integral sense, undesirable effects due to possible jumps in the location of y_m are diminished.

Numerically, the finite-difference representation of p_q at one particular x_j location is

$$p_{q_j} = \frac{\langle \hat{\mathbf{q}}_j, \frac{\hat{\mathbf{q}}_{j+1} - \hat{\mathbf{q}}_j}{\Delta x} \rangle}{\langle \hat{\mathbf{q}}_j, \hat{\mathbf{q}}_j \rangle}. \quad (4.2)$$

If a suitable updated value of α that reduces the value of p_q is not found, then the streamwise variations $\hat{\mathbf{q}}_{j+1} - \hat{\mathbf{q}}_j$ must be reduced by decreasing the spatial resolution Δx of the grid in streamwise direction. Finally, the iterative procedure for finding the most suitable value of the streamwise wavenumber α is described in Sect. 5.7.

Once the normalization condition (3.21) is satisfied, the unique value of α transfers the streamwise variations collected in the amplitude function $\hat{\mathbf{q}}$ to the sinusoidal part of the disturbances (*i.e.* the wave function).

In summary, the AHLNS method is able to describe the spatial evolution of *wave-like* convective instabilities in regions with localized large streamwise gradients in the base flow by the use of an *adaptive* technique: a PSE-like grid resolution in regions far from those large streamwise gradients, and a HLNS/DNS-like grid resolution at the location of the large streamwise changes.

Chapter 5

Numerical implementation of the AHLNS equations

This chapter presents the numerical aspects that have to be considered in order to implement the AHLNS equations into a computational code.

5.1 The AHLNS equations

The *Adaptive Harmonic Linearized Navier-Stokes* (AHLNS) method, derived from the Navier-Stokes equations (3.6)-(3.8), and described in Chapt. 4, can be written as a system of equations in the form:

$$\mathbf{A}\hat{\mathbf{q}} + \mathbf{B}\frac{\partial\hat{\mathbf{q}}}{\partial y} + \mathbf{C}\frac{\partial^2\hat{\mathbf{q}}}{\partial y^2} + \mathbf{D}\frac{\partial\hat{\mathbf{q}}}{\partial x} + \mathbf{E}\frac{\partial^2\hat{\mathbf{q}}}{\partial x\partial y} + \mathbf{F}\frac{\partial^2\hat{\mathbf{q}}}{\partial x^2} = 0. \quad (5.1)$$

Coefficients $\mathbf{A} - \mathbf{F}$, which represent 5×5 matrices, contain base flow quantities, real-valued parameters spanwise wavenumber β and frequency $\omega^\dagger$, and the complex-valued streamwise wavenumber α . The vector of shape functions $\hat{\mathbf{q}}$ comprises the variables

$$\hat{\mathbf{q}} = (\hat{u}, \hat{v}, \hat{w}, \hat{p}, \hat{T})^\top. \quad (5.2)$$

Although vector $\hat{\mathbf{q}}$ could be defined by another selection of variables[‡], the choice of dependent variables in Eqn. (5.2) is useful because it makes numerically easier to study incompressible flows by simply changing the equation of state (3.4) from the perfect gas law to setting

[†]The frequency ω is also referred to as *circular* frequency.

[‡]The AHLNS equations have been implemented and tested also with the selection of variables $\hat{\mathbf{q}} = (\hat{u}, \hat{v}, \hat{w}, \hat{p}, \hat{T})^\top$ and $\hat{\mathbf{q}} = (\hat{u}, \hat{v}, \hat{w}, \hat{p}, \hat{\rho})^\top$.

the nondimensional base flow density to 1 (i.e. $\bar{\rho} \equiv 1$). This is the procedure followed by Herbert [20] in his compressible PSE code, called *PSH*.

5.2 Boundary conditions

5.2.1 Walls

At solid walls, the disturbance velocities are set to zero. It is assumed also that the temperature fluctuations vanish at the wall due to the thermal inertia of the surface. The non-zero pressure fluctuations at the wall can be retrieved from a suitable conservation equation, e.g. continuity, wall-normal momentum or even streamwise-momentum (in the particular case of a wall at which the normal vector is aligned with the x -axis, e.g. rectangular steps).

For porous walls, it is common to model the porosity of the wall by using a simplified version of Darcy's law that relates the wall-normal velocity component $\hat{v}$ and pressure $\hat{p}$ amplitude disturbances at the wall. An example of this type of modelling for porous walls can be found in the publication of Huang and Wu [30]. The work of Franco and Hein [25] reproduces some cases of [30] using the AHLNS equations.

5.2.2 Farfield

It is assumed that all disturbance quantities vanish outside the boundary layer far enough from the wall, i.e.

$$\hat{u} = \hat{v} = \hat{w} = \hat{p} = \hat{T} = 0 \quad \text{at} \quad y \rightarrow \infty. \quad (5.3)$$

The numerical implementation of the above physical boundary conditions has been done following the ideas included in the work of Bertolotti and Joslin [31], and Mack [14]. Two options have been implemented: Homogeneous Dirichlet boundary conditions and mixed[†] boundary conditions. In the first case, the disturbances are set to zero at some wall-normal distance far from the wall. In the second case, mixed conditions involve the shape function $\hat{\mathbf{q}}$ and its wall-normal derivative $\frac{\partial \hat{\mathbf{q}}}{\partial y}$. The mixed conditions are a particular case of the more general asymptotic boundary conditions (see Mack [14], and Bertolotti and Joslin [31]). The asymptotic boundary conditions assume that, outside the boundary layer, the base flow is uniform and the system of equations (5.1) reduces to a system of differential equations with constant coefficients. The 2^{nd} -order in y -direction can be rewritten as a system of ten 1^{st} -order differential equations. This system of equations admits exponential solutions in the form:

$$\hat{\mathbf{q}}(y) = \sum_{j=1}^{10} \Gamma_j \exp(i\lambda_j y), \quad (5.4)$$

[†]*Mixed* boundary conditions are sometimes called *Robin* conditions.

where Γ_j collects the corresponding eigenmodes, and each λ_j is the eigenvalue. It must be noted that, strictly speaking, the assumption of uniform base flow outside the boundary layer is quite restrictive. However, if the base flow is assumed to be slowly varying in streamwise direction outside the boundary layer (where the influence of the surface imperfections is negligible) then coefficients of the differential equations are slowly varying also, and Eqn. (5.4) is a good approximation of the original system.

We are interested only in exponentially decaying solutions, i.e. eigenvalues λ_j with positive imaginary part. Among those eigenvalues, the mixed conditions consider the eigenvalue with the smallest positive imaginary part (i.e. smallest decay rate). This is equivalent to use only the most slowly decaying exponential mode in wall-normal direction for describing the farfield evolution of the disturbances.

The mixed boundary conditions are more difficult to implement numerically than the Dirichlet conditions. However, the farfield boundary for mixed conditions can be placed slightly above of the boundary-layer edge, while the Dirichlet boundary conditions have to be located relatively far from the boundary-layer edge. Numerical tests indicated that no differences were found among the two types of farfield boundary conditions. As a result, a significant reduction in number of points in wall-normal direction is achieved when mixed boundary conditions are selected.

5.2.3 Inflow and outflow

The inflow and outflow locations are placed where the assumption of slow variation in streamwise direction of the laminar base flow and of the amplitude functions is still valid. At the inflow location, shape functions provided by an upstream LST or PSE computation are taken as inflow boundary condition for AHLNS, as well as the streamwise wavenumber α . At the outflow location, shape functions are linearly extrapolated from inner points.

In general, the region where a surface imperfection significantly impacts on the boundary-layer quantities and the flow instabilities is relatively small (compared with the whole extend to be studied, e.g. the chord length of an airfoil). Although the AHLNS approach could be employed for the complete domain of interest, it is more efficient to couple the PSE and AHLNS approaches as it is represented in Fig. 5.1. We recall that PSE are quasi-parabolic, and they can be solved by a marching procedure downstream (Chapt. 3.3.2), while the AHLNS equations are a fully-elliptic system (Chapt. 4), and the complete domain of study must be solved simultaneously. In this *multi-zonal* approach (Fig. 5.1), the instability analysis is computed using PSE upstream and downstream of the location of the surface imperfection, where the *slow* variation of the boundary-layer quantities (both base flow $\bar{\mathbf{q}}$ and amplitude function $\hat{\mathbf{q}}$ of the disturbances) still holds. Since the *wave-like* representation (3.19)-(3.20) of the flow disturbances is identical for both PSE and AHLNS equations, their coupling at the interfaces is straightforward.

The coupling of PSE and AHLNS at inflow/outflow locations assumes implicitly that only the convective disturbance mode provided by the upstream PSE computation is present at the

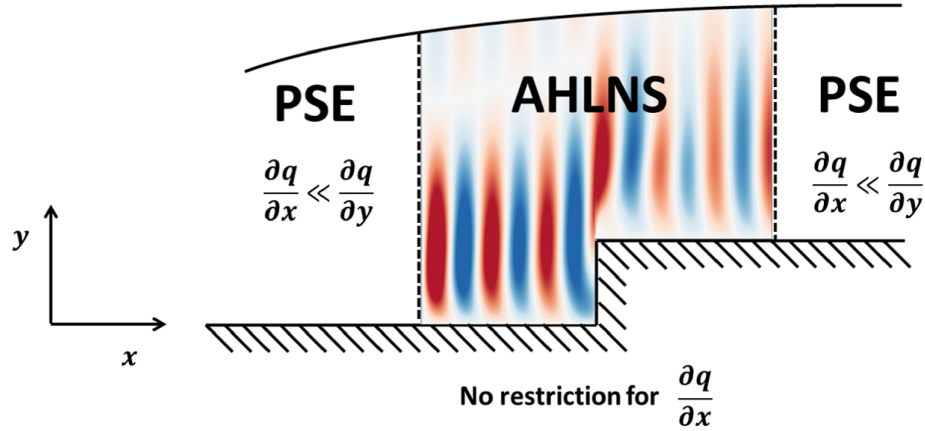


Figure 5.1: Sketch of the use of the multi-zonal technique for boundary-layer instability analysis in the presence of a forward-facing step. Coloured contours show a TS wave computed with AHLNS. The vertical dashed lines represent the inflow and outflow locations for AHLNS computations. Quantities $\mathbf{q}$ hold for both base flow $\bar{\mathbf{q}}$ and amplitude function $\hat{\mathbf{q}}$ of the disturbances. Flow direction is from left to right. Reproduced from [32].

boundaries. However, due to the fully-elliptic character of the AHLNS equations, acoustic modes moving upstream might be also captured (if the base flow is compressible) together with the disturbances[†]. In the same way, for incompressible flows, small variations in the pressure field caused by the interaction of the flow disturbance with the outflow boundary condition might produce a pressure pulse that it is transmitted everywhere. The work of Joslin *et al.* [34] explains this phenomenon for DNS. Those acoustic modes (for compressible flows) or pressure pulses (for incompressible flows) can be naturally damped at the inflow and outflow locations if a *relatively* large-spaced (i.e. *coarse*) point distribution is taken at those locations, acting similar to buffer regions. Here, *coarse* grid at the boundaries refers to a grid with a cell size at inflow/outflow boundaries similar to the step length used for the PSE computations.

The application of the AHLNS equations for all cases considered in this thesis did not require the explicit implementation of any numerical technique to damp out possible reflections at the boundaries. Actually, if the inflow interface with PSE is shifted upstream (and/or the outflow interface downstream) in order to remove undesired reflections, it does not imply a significant increase on the total number of streamwise points used. The large majority of grid points in streamwise direction are placed around the location of the surface imperfection. As mentioned before, the grid-elements length at inflow/outflow locations is of the order of the streamwise wavelength of the incoming convective instability. It was found that placing the upstream/downstream interfaces between AHLNS and PSE at locations where base flow quantities smoothly vary in streamwise direction results a good strategy (in order to define the domain where the AHLNS approach is used).

[†]There are not only acoustic waves propagating upstream, but viscosity-associated waves also. A complete review of the continuous and discrete spectrum of solutions for the LNS equations can be found in the work of Li & Malik [33].

The flexibility in the definition of the streamwise grid-point distribution for the AHLNS equations, and ease of the boundary condition implementation, do not hold for other approaches. Typically, DNS and HLNS codes need to incorporate sophisticated numerical techniques at inflow/outflow locations such as perfectly matched layer (see Appel *et al.* [35]), asymptotic expansions (Tourrette [36]), characteristic waves (Poinsot and Lele) [37] or buffer and sponge regions (Joslin *et al.* [34], and Guo *et al.* [38]), in order to numerically damp the acoustic waves or pressure pulses mentioned before. These techniques have the drawback of the effort required to be implemented in a computational code, and/or the significant increase in grid points needed in streamwise direction (typically one order of magnitude larger than AHLNS).

Finally, although throughout this thesis single surface imperfections are considered only, the use of the multi-zonal technique allows applying the AHLNS methodology for the study of several two-dimensional surface irregularities placed in downstream direction.

5.3 Spatial discretization

The spatial derivatives in streamwise and wall-normal coordinates are discretized using the high-order finite difference schemes given by Fornberg [39]. The order of the schemes implemented varies from 4th- to 10th-order, although in general the 4th-order scheme is sufficient for the computations performed using AHLNS.

The necessity of a high resolution in areas of strong streamwise gradients may lead to the appearance of spurious waves which cannot be damped by the low numerical dissipation of the high-order finite difference schemes. For those cases, a 10th-order compact filter is implemented. Filtering is applied in the amplitude function $\hat{\mathbf{q}}$ in both x and y directions independently. The filtering technique used here is explicitly described in detail in the work of Visbal and Gaitonde [40].

5.4 One-dimensional point distribution

One of the main features of the AHLNS approach is its intrinsic capacity to describe the spatial evolution of convective instabilities in regions with localized large streamwise gradients in a very efficient manner (Chapt. 4). In other words, the AHLNS method is able to combine a grid resolution similar to the streamwise wavelength of the disturbance (PSE-like step length) for those areas with small gradients, and a very fine grid resolution (DNS-like grid size) for those areas with large gradients. Typically, PSE-like step length and DNS-like grid spacing are length scales that may differ by one, two or even three orders of magnitude. Therefore, an adequate streamwise point distribution becomes a critical aspect in order to maximize the efficiency of the AHLNS method.

Point distributions in x - and y -directions are treated independently as one-dimensional point distribution. Throughout this work, two different one-dimensional point distributions are used for both the streamwise and wall-normal directions.

- A modified version (defined by Hein *et al.* [16]) of the original scheme used by Adams [41] in his DNS code. This scheme clusters the grid point distribution at one edge of the one-dimensional domain.
- The scheme used by Hein and Theofilis [42] in their BiGlobal code. With this scheme, grid points are accumulated in the middle of the one-dimensional discretization direction.

5.5 Multi-domain technique

The studies described in this thesis are focused on the presence of 2D surface irregularities such as steps, gaps, humps, etc. and their interaction with convective flow instabilities. If the shape of the imperfection is rectangular (i.e. the edges are sharp), one could try to replace the rectangular shape of the irregularity by a rounded shape that avoids the singularity at the corner. There are many examples in literature about this *smoothing* approach (see the works of Cooke *et al.* [43], Balakumar *et al.* [44], Dong and Zhang [45], and Hildebrand *et al.* [46]). In these examples, the singularity of the sharp corner is replaced by an analytical function that mimics up to certain detail the geometry of the rectangular imperfection. However, surface imperfections are regions where streamwise flow gradients might be very intense. This is especially true in forward-facing steps (see Chapt. 7). Therefore, small changes in the geometry of the step might lead to significant differences in base flow quantities $\bar{\mathbf{q}}$. These variations in base flow may result in important changes of n -factor curves. Examples of this effect can be found in the work of Zahn and Rist [47] or more recently in Franco *et al.* [48].

Alternatively, multi-domain decomposition methods are based on splitting the computational region into several domains, where each domain is discretized individually. The solution in each domain is calculated independently but taking into account information from the neighbouring domains [49]. This *multi-domain* approach has been implemented in the AHLNS code, and it is schematically represented in Fig. 5.2. At the interface between two adjacent domains, the amplitude functions ($\hat{\mathbf{q}}$) and their first-order derivatives ($\frac{\partial \hat{\mathbf{q}}}{\partial x}$ or $\frac{\partial \hat{\mathbf{q}}}{\partial y}$) are imposed to match at both sides of the interface. Mathematically, at one particular domain (i, j) which contains ($m(i), n(j)$)-points in (x, y)-directions, the *interface* boundary conditions are expressed as:

- Vertical interface

$$\begin{aligned} \hat{\mathbf{q}}(x_m, y_r) \Big|_{i,j} &= \hat{\mathbf{q}}(x_1, y_r) \Big|_{i+1,j}, \\ \frac{\partial \hat{\mathbf{q}}(x_m, y_r)}{\partial x} \Big|_{i,j} &= \frac{\partial \hat{\mathbf{q}}(x_1, y_r)}{\partial x} \Big|_{i+1,j}, \end{aligned} \tag{5.5}$$

with $r = 1, \dots, n(j)$.

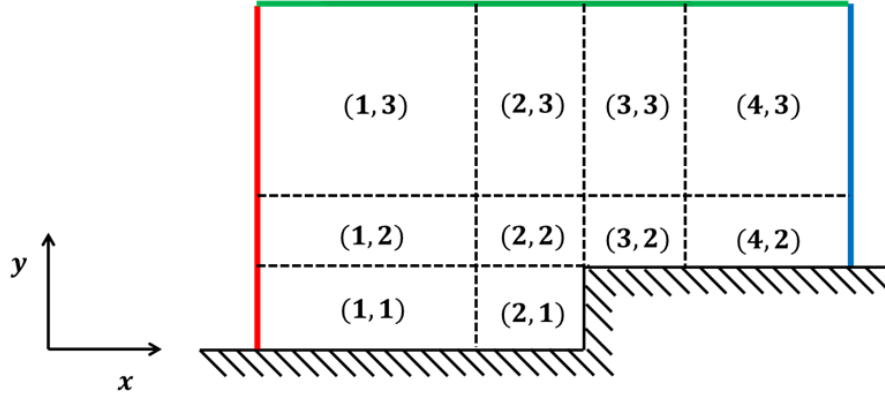


Figure 5.2: Sketch of the use of the multi-domain technique for boundary-layer instability analysis in the presence of a forward-facing step. The black dashed lines represent the interfaces between adjacent domains. Each domain is represented by an (i, j) pair. The red, green, and blue solid lines indicate inflow, farfield, and outflow boundary conditions, respectively. Solid black lines denote wall boundary condition. Flow direction is from left to right.

- Horizontal interface

$$\begin{aligned} \hat{\mathbf{q}}(x_s, y_n) \Big|_{i,j} &= \hat{\mathbf{q}}(x_s, y_1) \Big|_{i,j+1} \\ \frac{\partial \hat{\mathbf{q}}(x_s, y_n)}{\partial y} \Big|_{i,j} &= \frac{\partial \hat{\mathbf{q}}(x_s, y_1)}{\partial y} \Big|_{i,j+1}, \end{aligned} \tag{5.6}$$

with $s = 1, \dots, m(i)$.

At corner points that belong simultaneously to horizontal and vertical interfaces, only the vertical interface condition is imposed. Finally, it is worth to mention that the implementation of a multi-domain approach shows another advantage. As explained previously, the AHLNS equations can manage grid size variations in streamwise direction ranging from PSE-like step length to DNS-like grid size. These length scales may differ by several orders of magnitude. A point distribution (see Sect. 5.4) capable of adapting to such large variation of scales in one single domain is relatively difficult to achieve. However, if the same area of study is divided in few sub-domains (as it is depicted in Fig. 5.2), then the task of discretizing each sub-domain is simplified, and the total number of points in x - and y -directions can be easily optimized.

5.6 Coordinate transformation

In real applications, surface irregularities might not be perfectly rectangular, i.e. their shape may contain some rounded corners. This roundness might be due to manufacturing process, the result of environment effects (icing, dirt, erosion), or even by design requirements. Therefore, it is necessary to incorporate the capability of analysis of regions with large surface curvature. Examples of these types of surface imperfections can be found in Chapt. 6. Consequently, a coordinate transformation has been implemented to map the curvilinear physical domain (x, y) into the equally-spaced rectangular computational domain (ξ, η) . Figure 5.3 depicts a sketch of this coordinate transformation for a rounded indentation.

The numerical implementation of the coordinate transformation follows the ideas included in the work of Merle and Robinet [50]. By systematic application of the chain rule for partial derivatives, the derivatives w.r.t. x and y are expressed as derivatives w.r.t. ξ and η as:

$$\frac{\partial \hat{\mathbf{q}}}{\partial x} = \frac{1}{J_{2D}} \left[\frac{\partial \hat{\mathbf{q}}}{\partial \xi} \frac{\partial y}{\partial \eta} - \frac{\partial \hat{\mathbf{q}}}{\partial \eta} \frac{\partial y}{\partial \xi} \right], \quad (5.7)$$

$$\frac{\partial \hat{\mathbf{q}}}{\partial y} = \frac{1}{J_{2D}} \left[-\frac{\partial \hat{\mathbf{q}}}{\partial \xi} \frac{\partial x}{\partial \eta} + \frac{\partial \hat{\mathbf{q}}}{\partial \eta} \frac{\partial x}{\partial \xi} \right], \quad (5.8)$$

where the Jacobian J_{2D} is defined as:

$$J_{2D} = \frac{\partial x}{\partial \xi} \frac{\partial y}{\partial \eta} - \frac{\partial x}{\partial \eta} \frac{\partial y}{\partial \xi}. \quad (5.9)$$

Following the same procedure, the second derivatives are computed from the first derivatives [50]. Replacing the first and second derivatives w.r.t. x and y and reordering, the AHLNS equations (5.1) are expressed now as:

$$\mathcal{A}\hat{\mathbf{q}} + \mathcal{B}\frac{\partial \hat{\mathbf{q}}}{\partial \eta} + \mathcal{C}\frac{\partial^2 \hat{\mathbf{q}}}{\partial \eta^2} + \mathcal{D}\frac{\partial \hat{\mathbf{q}}}{\partial \xi} + \mathcal{E}\frac{\partial^2 \hat{\mathbf{q}}}{\partial \xi \partial \eta} + \mathcal{F}\frac{\partial^2 \hat{\mathbf{q}}}{\partial \xi^2} = 0. \quad (5.10)$$

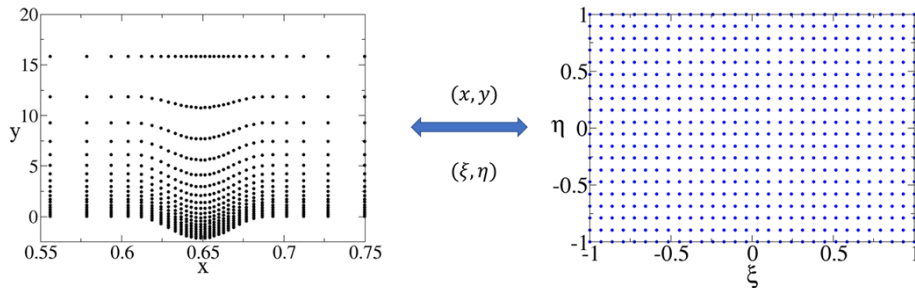


Figure 5.3: Sketch of the coordinate transformation for a rounded indentation between the physical space (x, y) [left], and the computational domain (ξ, η) [right].

As a reminder, matrices $\mathbf{A} - \mathbf{F}$, included in Eqn. (5.1), contain base flow quantities, real-valued parameters β and ω , and the complex-valued streamwise wavenumber α . Here, matrices $\mathcal{A} - \mathcal{F}$ include the same information plus the metric terms (e.g. $\frac{\partial \xi}{\partial x}$) that describe the geometry of the grid in physical space.

It is important to recall (see [50]) that we assume implicitly that functions $x(\xi, \eta)$ and $y(\xi, \eta)$ are $\mathcal{C}^{2\dagger}$. If a singularity is present in the real space (e.g. a sharp corner) or the physical grid is highly distorted (if the surface imperfection presents an area with very large curvature), then the use of multiple domains (see Sect. 5.5) must be considered.

5.7 Solution procedure

Once the boundary conditions are incorporated into the system of equations (5.10) and after discretization, a system of algebraic equations of the form $\mathbf{L}\hat{\mathbf{q}} = \mathbf{b}$ is reached, where vector $\mathbf{b}$ collects the information of the inflow boundary condition. This boundary-value problem is solved using the linear sparse direct solver MUMPS [51], provided by the package PETSc [52]. Matrices are implemented such that they can be easily solved in parallel (parallelized via Message Passing Interface (MPI)).

The streamwise wavenumber $\alpha(x)$ is computed iteratively using a procedure borrowed from the PSE method [16]:

$$\alpha_{new} = \alpha_{old} - i \frac{\int_0^\infty \hat{\mathbf{q}}^\dagger \frac{\partial \hat{\mathbf{q}}}{\partial x} dy}{\int_0^\infty (||\hat{\mathbf{q}}||^2) dy} \quad (5.11)$$

This iterative procedure corrects the value of $\alpha(x)$ at each x -position until the normalization condition (3.21) is satisfied. Typically, a first guess for α is given by the upstream PSE computation at the inflow location. Note that equation (5.11) is defined for a particular location x . Therefore, the iterative procedure continues until the adjustment of $\alpha(x)$ is considered converged for each streamwise location within the domain where the AHLNS equations are applied.

[†]A $\mathcal{C}^k$ function is a function with k continuous derivatives

Chapter 6

Verification and validation of the AHLNS method

6.1 Verification

The correct numerical implementation of the AHLNS equations (5.10), together with the boundary conditions and solution procedure (Sect. 5.2 and 5.7, respectively), is verified in the first part of the chapter. The test cases chosen for the verification task do not include any large gradients in streamwise direction. Therefore, the outcome of the AHLNS computations can be directly compared with PSE results. Moreover, the coupling of PSE and AHLNS mentioned in previous chapter (Sect. 5.2.3) is limited to the inflow only, i.e. the outflow of the AHLNS computations coincides with the end of the domain of study. For the first three test cases, the number of points in streamwise direction used for PSE and AHLNS computations is similar. Afterwards, a case where the number of points is increased significantly shows how the AHLNS equations can overcome some inherent limitations of the PSE. Finally, the role of the *normalization condition* and its influence on the AHLNS approach is explained in the last test case of the verification section.

6.1.1 Two-dimensional flows

Two test cases are analysed here. In both cases, the base flow corresponds to a two-dimensional flow bounded by a semi-infinite flat-plate. These cases were originally chosen by Hein *et al.* [16] in order to verify the correct implementation of the linear version of their PSE code *NOLOT*. Both cases showed an excellent agreement between PSE and published results using linear DNS. Therefore, the same cases will be studied here but using the AHLNS equations. These two cases have been chosen for two reasons: first, to check the correct implementation of the boundary conditions and the solution procedure. Second, to test that the AHLNS system of equations, derived for compressible flows, is able to describe the spatial evolution of convective instabilities in two different flow regimes: incompressible and supersonic flows.

6.1.1.1 Incompressible flow

This case, originally defined by Fasel and Konzelmann [28], is of particular interest since the original DNS [28] for a flat plate were done assuming an incompressible version of the Navier-Stokes equations. Both PSE [16] and AHLNS formulations are written for compressible flows. However, for the AHLNS computations, the base flow is explicitly treated as incompressible (see Sect. 5.1). Therefore, the main purpose of this verification test case is to assure the AHLNS equations can deal with incompressible flows. The base flow corresponds to the Blasius solution assuming that the Mach number is very small (in particular 10^{-6}). A particular two-dimensional ($\beta = 0$) TS wave of nondimensional frequency $F = 140$ is considered. Throughout this thesis, the nondimensional frequency $F^\dagger$ is defined as

$$F = 2\pi f^* \frac{\mu_{ref}^*}{\rho_{ref}^* (u_{ref}^*)^2} \times 10^6 \quad (6.1)$$

where f^* represents the dimensional physical frequency of the disturbance. The local Blasius length $\bar{\delta}$ [17] is defined as

$$\bar{\delta} = \sqrt{\frac{\mu_{ref}^* x^*}{\rho_{ref}^* u_{ref}^*}} \quad (6.2)$$

where x^* is the distance from the sharp leading edge of the flat plate. $Re_{\bar{\delta}}$ is the Reynolds number based on the local Blasius length $\bar{\delta}$ [17]

$$Re_{\bar{\delta}} = Re_u \bar{\delta} = \sqrt{\frac{\rho_{ref}^* u_{ref}^* x^*}{\mu_{ref}^*}} \quad (6.3)$$

The unit Reynolds number, defined as

$$Re_u = \frac{\rho_{ref}^* u_{ref}^*}{\mu_{ref}^*} \quad (6.4)$$

is set to $2 \times 10^6 \text{ m}^{-1}$. Figure 6.1 shows the comparison between PSE and AHLNS results in terms of the growth rate $\sigma_{\hat{u}}$ of the streamwise-velocity component of the individual TS wave. There is a perfect agreement between the results of both approaches.

The AHLNS computation takes the PSE solution at the inflow location ($Re_{\bar{\delta}} = 245$), and extends the region of study up to $Re_{\bar{\delta}} = 700$. The number of points in streamwise direction n_x required for each methodology is summarized in Table 6.1.

[†]The nondimensional frequency F is also referred to as *reduced* frequency.

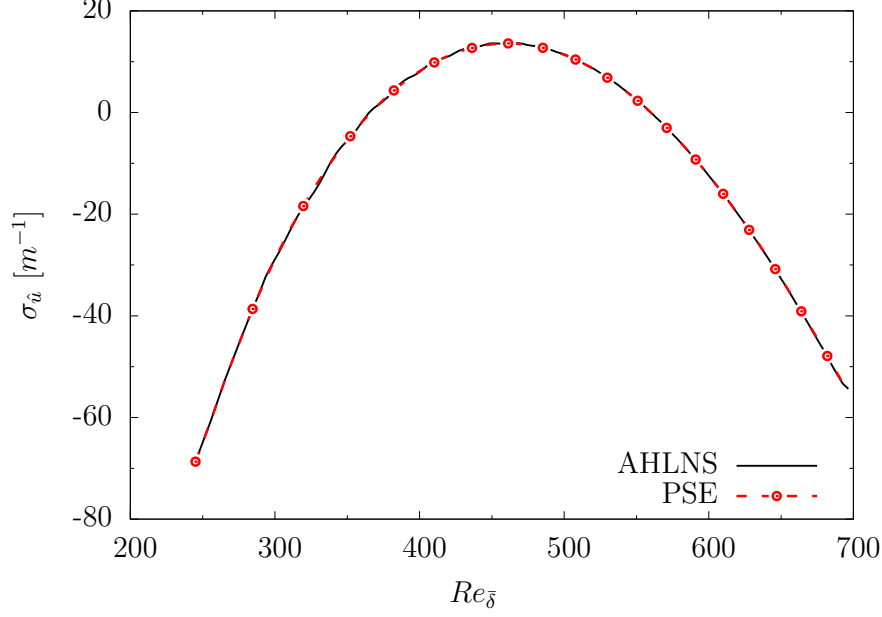


Figure 6.1: Growth rate $\sigma_{\hat{u}}$ of an incoming TS wave of reduced frequency $F = 140$. $Re_{\bar{\delta}}$ stands for Reynolds number based on local Blasius length $\bar{\delta}$.

6.1.1.2 Supersonic flow

The second test case for 2D flows corresponds to the spatial evolution of a two-dimensional ($\beta = 0$) second mode* of reduced frequency $F = 153$ in a supersonic flat-plate boundary-layer flow with an inflow Mach number $Ma_{\infty} = 4.5$. This case was defined and computed by Guo *et al.* [38] by means of DNS. The unit Reynolds number is $Re_u = 12000 \text{ m}^{-1}$, and the reference temperature $T_{ref}^* = 123 \text{ K}$.

Figure 6.2 depicts the spatial evolution of the growth rate $\sigma_{\hat{u}}$ computed with PSE and AHLNS equations. The growth rate of the streamwise-component of the disturbance $\sigma_{\hat{u}}$, defined in equation (3.22), is a quantity very sensitive to any numerical error (as Guo *et al.* [38]

*For supersonic flows, the second mode is also known as *Mack mode*.

Methodology	n_x	Numerical scheme
PSE	75	1 st -order backwards
AHLNS	75	4 th -order centred
DNS [28]	600	4 th -order centred

Table 6.1: Summary of the number of points in streamwise direction n_x required for each methodology for the verification case described in Sect. 6.1.1.1 - Incompressible flow. DNS data were taken from the work of Fasel and Konzelmann [28].

pointed out). Therefore, the excellent agreement between PSE and AHLNS results certifies the correct implementation of the boundary conditions and the solution procedure for highly compressible flows. The number of points in streamwise direction used for PSE and AHLNS in the region $Re_{\bar{\delta}} \in [1400, 1725]$ is 100 for both approaches. This is equivalent to use two grid points per wavelength. As usual, PSE/AHLNS methods used 1st-order backward/4th-order centred schemes, respectively. On the other hand, Guo *et al.* [38] reported that they used about 15-20 points per wavelength for the DNS and a 6th-order Padé scheme, depending on the different inflow/outflow treatments considered in their work [38].

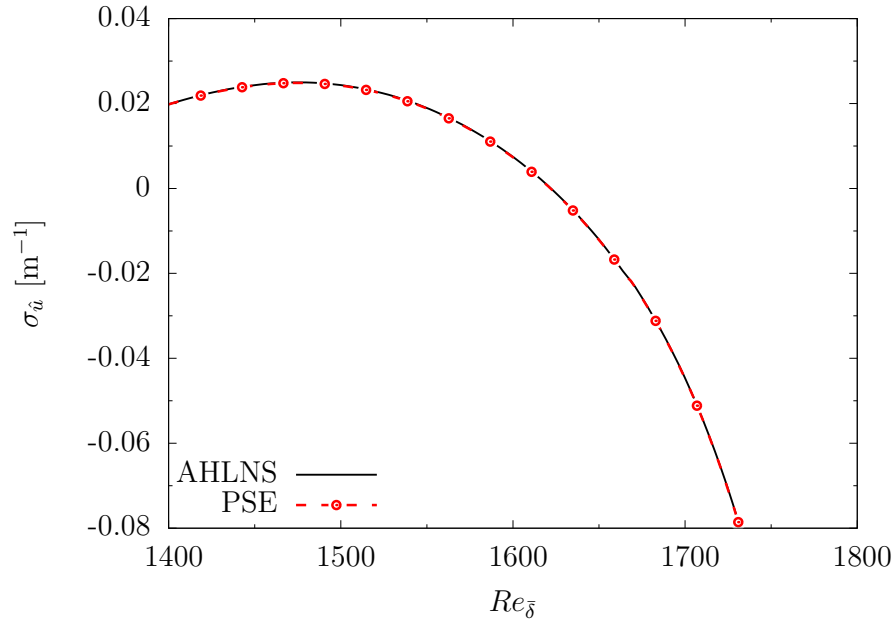


Figure 6.2: Growth rate evolution $\sigma_{\hat{u}}$ for the two-dimensional second mode wave of reduced frequency $F = 153$ in a supersonic flat plate boundary-layer flow ($Ma_{\infty} = 4.5$).

6.1.2 Three-dimensional flows

The AHLNS equations (5.10) were derived assuming that the base flow is *spanwise invariant* (Chapt. 4). Therefore, the verification of the correct implementation of the AHLNS approach should not be limited to two-dimensional instabilities in 2D flows, but including flows in which the spanwise velocity component of the base flow and the spanwise wavenumber of the instability are non-zero. In this section, we take a test case studied by Koch *et al.* [53]. The base flow used in [53] replicates numerically the experimental set-up originally defined by Bippes *et al.* [54] (see Fig. 6.3). The well-known experiment from Bippes *et al.* [54] consists of a swept flat plate, where the crossflow-generating pressure gradient is induced from outside (via a *displacement body*). The free-stream velocity is $Q_{\infty}^* = 19 \text{ ms}^{-1}$, and the effective sweep angle between the incoming flow and the leading edge of the flat plate is $\varphi_{\infty}^* = 42.5^\circ$. The kinematic viscosity is set to $\nu_{ref}^* = 0.151 \times 10^{-4} \text{ m}^2\text{s}^{-1}$.

Although the vast majority of cases studied and discussed along this thesis are related with the spatial evolution of TS waves, here it is shown that the AHLNS method is also perfectly

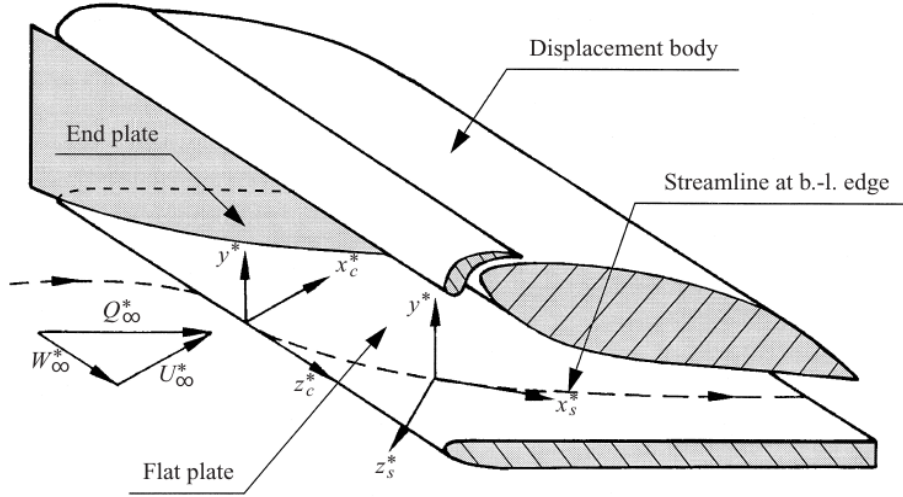


Figure 6.3: DLR infinite-swept-plate experiment. From [53].

valid for the linear stability analysis of crossflow vortices. The test case considered in this section consists of the spatial linear evolution of a stationary crossflow instability (CFI) of spanwise wavenumber $\beta^* = 523.6 \text{ m}^{-1}$ (wave angle $\psi^* \approx 86^\circ$). Figure 6.4 illustrates the very good agreement between the PSE and AHLNS results in terms of the growth rate of the streamwise-component of the stationary CFI.

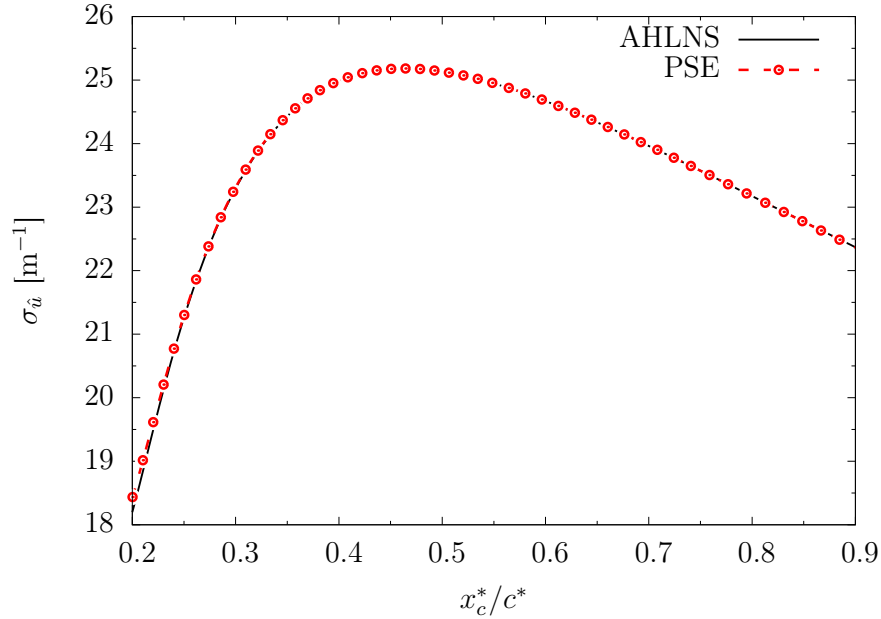


Figure 6.4: Growth rate evolution $\sigma_{\hat{u}}$ for a stationary crossflow instability of spanwise wavenumber $\beta^* = 523.6 \text{ m}^{-1}$ (wave angle $\psi^* \approx 86^\circ$) in incompressible flat plate boundary-layer flow. For this particular test case, x_c^* indicates the chordwise coordinate axis, and c^* is the chord length of the plate.

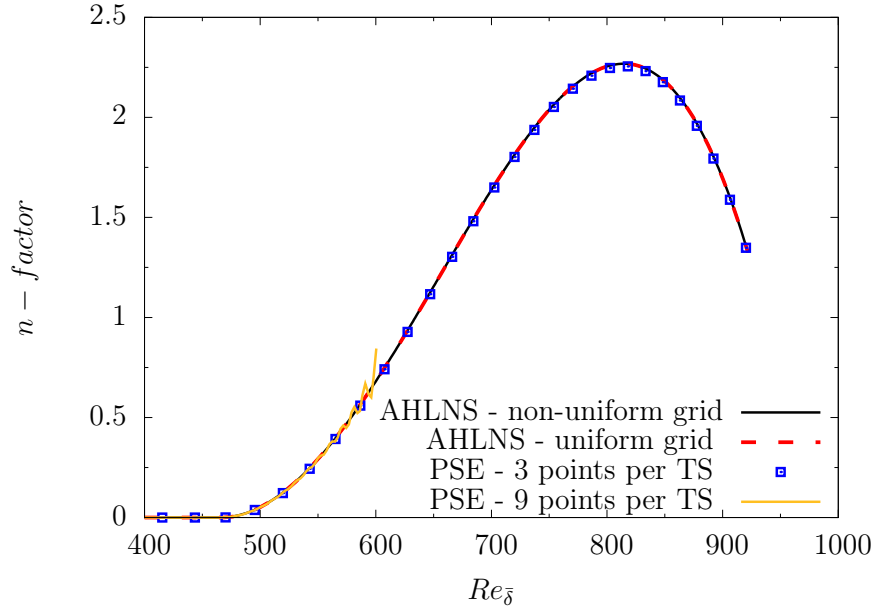


Figure 6.5: n -factor curves for an incoming 2D TS wave of frequency $F = 86$ in a flat-plate boundary-layer flow ($Ma_\infty = 0.1$), computed with PSE and AHLNS for different point distributions in streamwise direction. $Re_{\bar{\delta}}$ stands for Reynolds number based on local Blasius length.

6.1.3 Mesh size

The main characteristic of the AHLNS approach, in terms of streamwise grid resolution, is its inherent capacity of to handle large grid-spacing variations without compromising the quality of the results. This key idea was introduced already in Chapt. 4, where a numerical experiment was briefly described. Here, the same test is described with more detail.

The test case considered here was defined first by Joslin *et al.* [34] in order to validate their DNS code. Several comparisons between LST, PSE and DNS were shown there [34], and a good agreement was found. Here, the spatial evolution of a single 2D TS wave, with a reduced frequency $F = 86$ in a flat-plate boundary-layer flow (at $Ma_\infty = 0.1$), is studied by means of PSE and AHLNS. Figure 6.5 shows the outcome of this investigation. The blue symbols in Fig. 6.5 hold for a PSE computation where the TS wavelength is represented by three points only (a similar grid resolution was used in the work of Joslin *et al.* [34]). Only every third data point of the PSE result is depicted in Fig. 6.5. This means that the spacing between the blue symbols in Fig. 6.5 corresponds approximately to one TS wavelength. In other words, there are (roughly) 28 TS wavelengths within the region $Re_{\bar{\delta}} \in [400, 925]$. Increasing the streamwise resolution of the PSE computations up to nine points per TS wavelength (golden line, Fig. 6.5) leads to numerical instabilities, as explained in Sect. 3.3.2, or more in detail in the work of Li and Malik [23].

In the same Fig. 6.5, two AHLNS computations are included. Both simulations take the PSE solution at $Re_{\bar{\delta}} = 400$ as inflow boundary condition. In both cases, the AHLNS domain is

Methodology	n_x	Numerical scheme (x -direction)	Time (min)	Cores
PSE (3 points per TS)	84	1 st -order backward	~ 0.1	1
AHLNS (uniform grid)	90	4 th -order centred	~ 1	8
AHLNS (non-uniform grid)	600	4 th -order centred	~ 5	8

Table 6.2: Summary of the computational resources required for the three stability analysis completed for the verification test case described in Sect. 6.1.3. n_x holds for the number of points needed in x -direction for the stability analysis. The column *Cores* refers to the number of cores (the same local workstation was used for all cases).

extended up to $Re_{\bar{\delta}} = 925$ (i.e. no further coupling with PSE downstream). The red dashed line is the result of an AHLNS computation using a uniformly spaced grid with 90 points in streamwise direction, i.e. three points per wavelength, roughly. Finally, the black solid line depicts the result of a AHLNS simulation using a *non-uniform* mesh. This non-uniform grid was defined such that, at the edges of the domain (i.e. $Re_{\bar{\delta}} \approx 400$ and $Re_{\bar{\delta}} \approx 900$), the streamwise resolution is about three points per TS wavelength. However, in the middle of the domain (i.e. $Re_{\bar{\delta}} \approx 650$), the streamwise grid density is about 100 points per TS wavelength (almost twice the resolution used by Joslin *et al.* [34] in their DNS). In other words, there is about two orders of magnitude difference between the grid resolution at the edges and in the middle of the area of study for the *non-uniform* grid. A total number of 600 points are used in the streamwise direction for this grid. The good agreement of the AHLNS results using both a uniform and non-uniform grids indicates the expected flexibility of the AHLNS approach in terms of grid point distribution in streamwise direction. This test case highlights how the AHLNS approach can overcome one of the main limitations of the PSE approach. Table 6.2 summarizes the computational costs for the three stability analysis discussed in this section.

6.1.4 *Wave-like* disturbance representation and normalization condition

One of the key aspects of the AHLNS approach is the *wave-like* representation of the convective instabilities in the streamwise x -direction, similar to the PSE approach. In a nutshell, the streamwise variations are included in both amplitude function $\hat{\mathbf{q}}$ and the wave function $e^{i\Theta}$. Therefore, an extra *normalization* or *auxiliary* condition is needed to resolve this ambivalence.

Since the normalization condition has been *implicitly* checked in previous tests, the main purpose of the test case here is to show *explicitly* the benefits of the *wave-like* disturbance representation in x -direction in terms of grid requirements. This is easily tested by fixing the value of streamwise wavenumber $\alpha(x)$ to zero. This means that all streamwise variations of the disturbances are collected in the amplitude function $\hat{\mathbf{q}}$. In other words, this implementation ($\alpha(x) \equiv 0$) is equivalent to solve the HLNS equations.

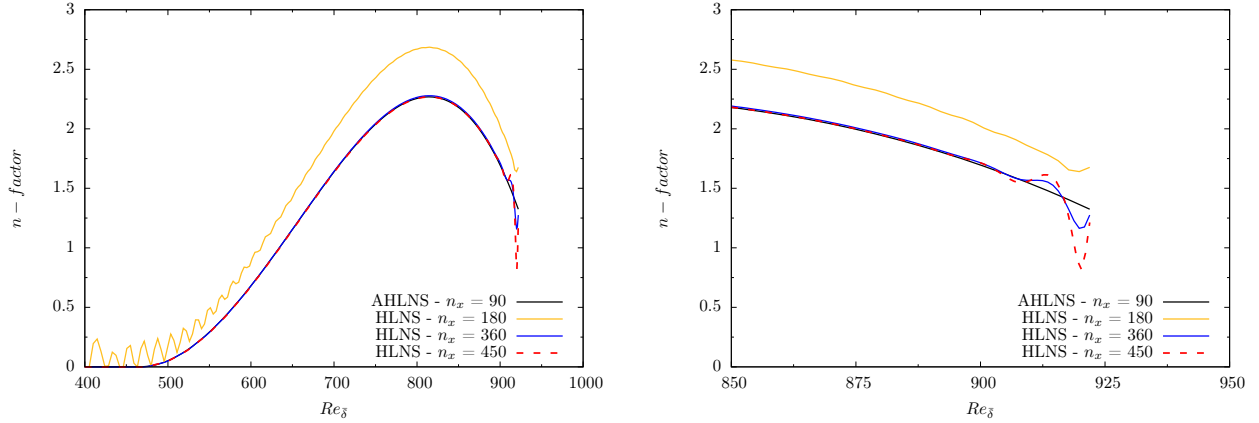


Figure 6.6: (*Left*) n -factor curves for an incoming TS wave of frequency $F = 86$ in a flat-plate boundary-layer flow ($Ma_{\infty} = 0.1$), computed with AHLNS for several equally-spaced grids in x -direction and switching on/off the *normalization* condition. Re_{δ} stands for Reynolds number based on local Blasius length. HLNS stands for AHLNS computations with $\alpha = 0$. (*Right*) Outflow detailed view.

The numerical experiment uses a set-up identical to the one already used in previous Sect. 6.1.3, i.e. a 2D TS wave ($F = 86$) evolving in a flat-plate boundary-layer flow ($Ma_{\infty} = 0.1$) [34]. Results of this numerical investigation are shown in Fig. 6.6. For all cases, the point distribution is equally distributed in x -direction. The black line is the outcome of an AHLNS computation taking a grid with 90 points in streamwise direction, and the normalization condition is active. This means that the streamwise wavenumber $\alpha(x)$ is updated iteratively, following the procedure described in Sect. 5.7. This simulation resembles the experiment done in Sect. 6.1.3 (Fig. 6.5, red dashed line).

The remaining curves in Fig. 6.6 (dark-yellow solid, blue solid, and red dashed lines) have been computed by fixing the value of the streamwise wavenumber $\alpha(x)$ to 0. This is the reason why these n -factor curves are labelled as *HLNS* in Fig. 6.6. Obviously, the value of α is not updated iteratively, i.e. the solution procedure described in Sect. 5.7 is not applied for these three computations. The dark-yellow solid line in Fig. 6.6 is a computation with a uniform grid that utilizes 180 points in streamwise direction. Clearly, the resolution is not enough and the spatial development of the TS wave is not well-resolved. If the number of points in streamwise direction is increased up to 360 (or 450) points (blue solid/red dashed lines, respectively) the corresponding n -factor curve is computed correctly (except in a small region at the outflow). We recall (see Sect. 6.1.3) that the DNS done by Joslin *et al.* [34] used 600 points. This comparison illustrates clearly the benefits, in terms of streamwise resolution, of the *wave-like* representation in x -direction complemented by the use of the *normalization* or *auxiliary* condition.

Regarding the n -factor curves at the outflow ($Re_{\delta} \approx 900$, Fig. 6.6-right), we recall that all grids used in this test are equally-distributed grids. Therefore, increasing the number of points leads to a higher grid resolution at the outflow. When the number of grid points is increased from 360 (blue solid line) to 450 points (red dashed line), the oscillations in the n -factor curve at the outflow are more pronounced. This effect was stated already in Sect. 5.2.3 or more in

detail by Joslin *et al.* [34]. Clearly, a simple extrapolation from the inner points is not a valid outflow boundary condition for *HLNS* computations, and another type of boundary condition treatment would be required for those grids. However, the AHLNS approach allows using a relatively coarse grid at inflow and outflow locations without compromising the quality of the results (check the numerical test case in Sect. 6.1.3). Therefore, this test case illustrates how the AHLNS approach allows the use of a relatively coarse grid at inflow/outflow locations, avoiding the need of implementing relatively complex boundary conditions.

6.2 Validation

The validation of the AHLNS approach is done by comparison with linear DNS results. We recall that the term *linear DNS* refers to non-linear DNS where the amplitude of the perturbations remains small enough (i.e. non-linear interactions can be considered negligible). Three different test cases are analysed in this section. The excellent agreement found between AHLNS and DNS (in terms of *n*-factor curves) confirms that the AHLNS equations are able to reproduce the spatial development of 2D and 3D TS waves along rounded and rectangular surface imperfections in incompressible and compressible boundary-layer flows. Moreover, it is shown that the numerical resources required by AHLNS (in terms of computational effort) are significantly smaller than those needed by linear DNS. Furthermore, it is demonstrated that, for the test cases described in this section, the PSE method by itself is not able to reproduce correctly the linear development of the boundary-layer instabilities in the presence of such surface imperfections.

6.2.1 Two-dimensional indentation in incompressible flow

This case was defined and studied by Xu *et al.* [55]. It is a flat plate with a two-dimensional single rounded indentation, where the center of the indentation is placed at $x_c^* = 0.649$ m, and the indentation depth h^* and width d^* are 1.620 mm and 81 mm, respectively. The nondimensional lengths of the indentation (in terms of the local boundary-layer displacement thickness δ_c^*) are $h = h^*/\delta_c^* = 1.28$ and $d = d^*/\delta_c^* = 64$, respectively. The base flow was replicated by Tocci [56] using the incompressible DNS code Nek5000 [57]. Figure 6.7 shows the nondimensional streamwise velocity contours $\bar{u}$ and streamlines of the steady laminar base flow.

In the original work of Xu *et al.* [55], only two-dimensional TS waves were considered. However, for this test case, a three-dimensional TS wave of frequency $f^* = 172$ Hz and spanwise wavenumber $\beta^* = 150 \text{ m}^{-1}$ (wave angle $\psi^* \approx 42^\circ$) is analysed [56]. The spatial evolution of this 3D TS wave is studied by means of three methods: DNS, combined AHLNS and PSE (labelled as *AHLNS*), and PSE only (see Fig. 6.8). The combined AHLNS and PSE approach means that AHLNS equations are used only in the vicinity of the indentation (i.e. in the region from $x^* = 0.55$ m to $x^* = 0.90$ m), where relatively *large variations* of base flow quantities and amplitude function disturbances in streamwise direction are expected to occur. In the remaining domain far from the indentation (i.e. from $x^* = 0.3$ m to $x^* = 0.55$ m, and

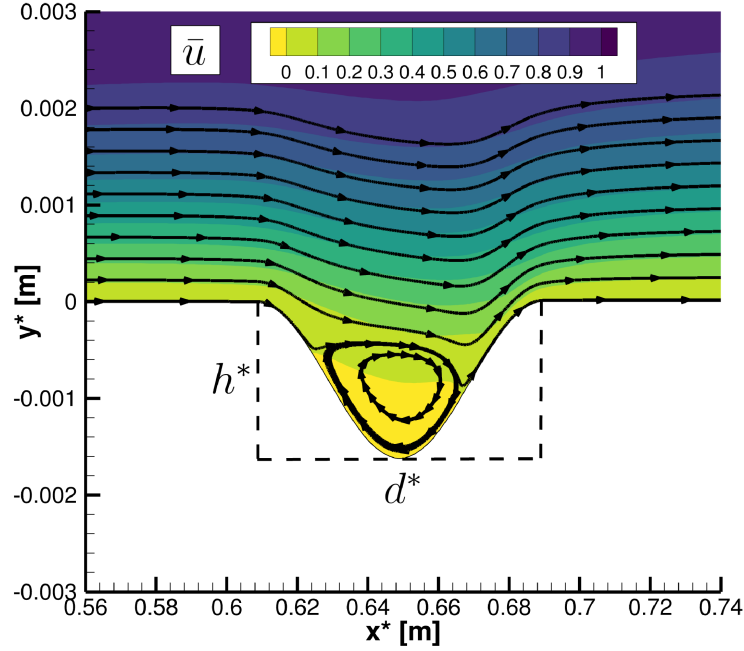


Figure 6.7: Streamlines and nondimensional streamwise velocity contours $\bar{u}$ for incompressible boundary-layer base flow along a rounded indentation, centred at $x_c^* = 0.649$ m. Its depth h^* and width d^* are 1.620 mm and 81 mm, respectively. Axes are not to scale. Flow direction is from left to right.

from $x^* = 0.9$ m to $x^* = 1.0$ m), PSE is used. The n -factor computed by the combination of PSE and AHLNS compared with the DNS result (black solid and red dashed lines, Fig. 6.8) shows a remarkable agreement.

On the other hand, the golden solid line in Fig. 6.8 is the result of the stability analysis using PSE only. The geometry of the indentation, with an aspect ratio $h^*/d^* = 0.02$, and a maximum wall angle $\theta_{max} = 3.6^\circ$, suggests that the streamwise gradients could be relatively small still, and the PSE approach solely might reproduce correctly the n -factor curve as well. However, some differences in n -factor appear, and therefore, the PSE approach only is not capable to compute correctly the linear spatial evolution of the TS wave along this type of surface imperfection. Although the maximum wall angle is relatively small, it is interesting to notice that the separation bubble fills a large space within the shallow indentation (Fig. 6.7). As a result, the n -factor rises rapidly along the indentation domain (from 1 to 5, Fig. 6.8).

Finally, the computational resources utilized for each methodology in streamwise direction are summarized in Table 6.3. The combination of PSE and AHLNS uses 110 points in x -direction (45 points for the PSE computations: 25 points upstream of the indentation and 20 points downstream, plus 65 points for AHLNS). The DNS code Nek5000 [57] does not include a Fourier mode representation in time t nor in z -direction. This explains why the computation of the linear evolution of a 3D TS wave using the incompressible DNS code Nek5000 needs three days in a HPC facility using 320 cores [56]. PSE and AHLNS computations were done on personal workstations.

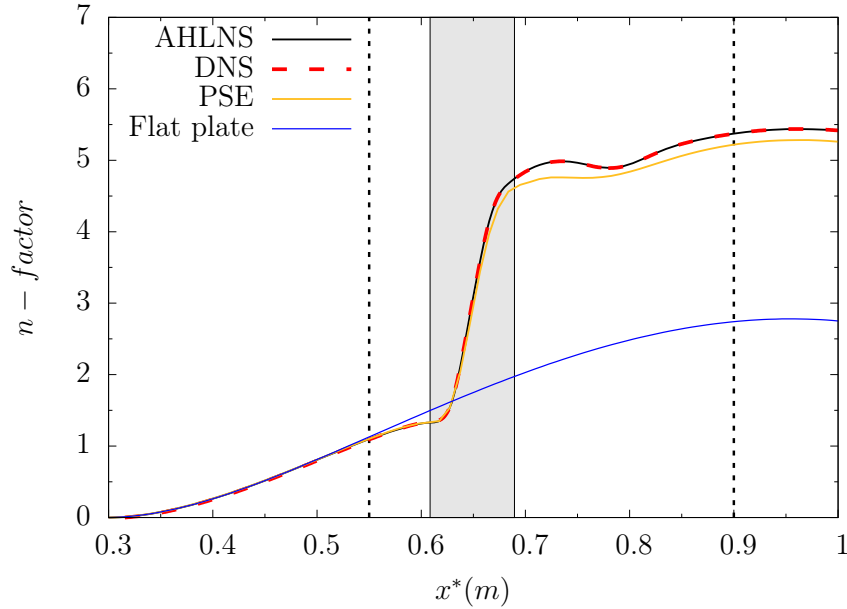


Figure 6.8: Spatial development of a 3D TS wave of frequency $f^* = 172$ Hz and spanwise wavenumber $\beta^* = 150 \text{ m}^{-1}$ in the presence of an indentation centred at $x_c^* = 0.649$ m. The blue line corresponds to the flat plate configuration (without indentation). The grey area represents the position and width of the indentation. Vertical dashed lines indicate the location of the interfaces between PSE and AHLNS methodologies. The golden solid line is a computation where only PSE are used. DNS data (red dashed line) has been extracted from [56].

6.2.2 Two-dimensional rod in incompressible flow

The test case studied here corresponds to a two-dimensional rod, placed on a flat plate, and originally defined in the work of Balakumar and Iyer [58]. The rod is centred at $x^* = 2.086$ m, and its height h^* and width d^* are 13.2 and 52.8 mm, respectively. The nondimensional lengths of the rod (in terms of the local boundary-layer displacement thickness δ_c^*) are $h = h^*/\delta_c^* = 1.38$ and $d = d^*/\delta_c^* = 5.53$, respectively. The geometry of the rod is such that the wall reaches a maximum deviation of $\theta_{max} = 63^\circ$ with respect to the horizontal. This case was selected in order to assure that the coordinate transformation (see Sect. 5.6) was able to handle such geometry by using a single domain. Tocci [56] reproduced the base flow and the geometry of the rod using the incompressible DNS code Nek5000 [57]. Figure 6.9 shows the nondimensional normal velocity contours $\bar{v}$ and streamlines of the steady laminar base flow. Two recirculation bubbles appear: a tiny one on the upstream side of the rod, and a very large separation bubble downstream of the rod.

The stability analysis for this test case is limited to an individual 2D TS wave of frequency $f^* = 200$ Hz. Again, three methods are considered here: DNS (from Tocci [56]), AHLNS and PSE. Results from Balakumar and Iyer [58] are not included in the present test case because they computed n -factor curves based on LST downstream of the rod location for several

Methodology	n_x	Numerical scheme (x -direction)	Time (min)	Cores
PSE	100	1 st -order backward	~ 0.1	1
PSE + AHLNS	45 + 65	4 th -order centred (AHLNS)	~ 1	8
DNS [56]	900	7 th -order spectral	4300	320

Table 6.3: Summary of the computational resources required for the three approaches used in the validation test case described in Sect. 6.2.1. n_x holds for the number of points needed in x -direction for the stability analysis. The column *Cores* refers to the number of cores used in each machine (local workstation for PSE and AHLNS, HPC for DNS).

Methodology	n_x	Numerical scheme (x -direction)
AHLNS	200	4 th -order centred
DNS [56]	1000	7 th -order spectral

Table 6.4: Summary of the computational resources required by AHLNS and DNS for the validation test case described in Sect. 6.2.2. n_x holds for the number of points needed in x -direction for the stability analysis.

frequencies. They performed also non-linear DNS [58] in order to study different secondary instability breakdown mechanisms, which is out of the scope of the thesis.

Due to the presence of a large recirculation area downstream of the rod, the AHLNS approach is extended to the end of the domain. Therefore, it is not coupled with a downstream computation using PSE. In other words, the AHLNS method is used in the region from $x^* = 1.4$ m to $x^* = 2.8$ m. Similar to previous validation test cases, the PSE result holds for a computation using PSE only. Findings from this stability analysis are summarized in Fig. 6.10. An excellent agreement is shown between DNS and AHLNS. However, PSE computation extends only up to slightly upstream of the rod. This is shown in detail in Fig. 6.10-right. The marching procedure used for PSE is not able to continue up to the location of the rod. It is very likely that the streamwise gradients upstream of the rod are very intense (as it could be concluded from Fig. 6.9) and, as a consequence, the PSE approach is not able to converge. The rapid variations in n -factor upstream of the rod (Fig. 6.10-right) are also a good indicator of the large variations of the base flow and amplitude function of the disturbances in streamwise direction.

Finally, Table 6.4 summarizes the computational effort (number of points and numerical scheme in streamwise direction) required by AHLNS and DNS [56].

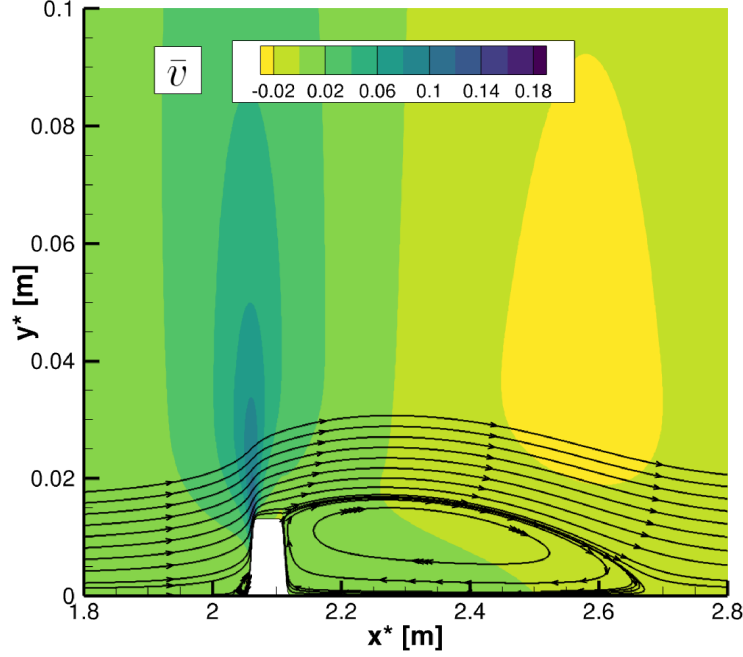


Figure 6.9: Streamlines and nondimensional normal velocity contours $\bar{v}$ for incompressible boundary-layer flow along a rounded rod, placed at $x^* = 2.086$ m. Its height h^* and width d^* are 13.2 mm and 52.8 mm, respectively. Axes are not to scale. Flow direction is from left to right.

6.2.3 Rectangular FFS in a compressible flow

This test case corresponds to a compressible flat-plate boundary-layer flow ($Ma_\infty = 0.8$), where a rectangular forward-facing step (FFS) is placed at $x^* = 1$ m. The unit Reynolds number is $Re_u = 2.45 \cdot 10^6 \text{ m}^{-1}$ and the Reynolds number based on the step height H^* is $Re_H = 1320$. Edelmann [59] studied the spatial evolution of a 2D TS wave of reduced

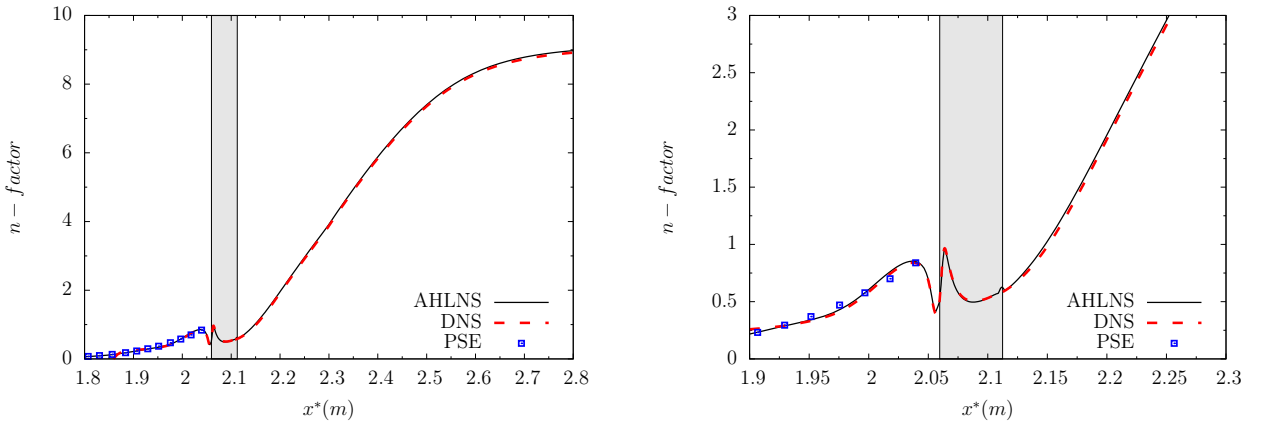


Figure 6.10: (Left) Spatial development of a 2D TS wave of frequency $f^* = 200$ Hz along a rounded rod, centred at $x_c^* = 2.086$ m. The grey area represents the position and width of the indentation. DNS data has been extracted from [56]. (Right) Detailed view.

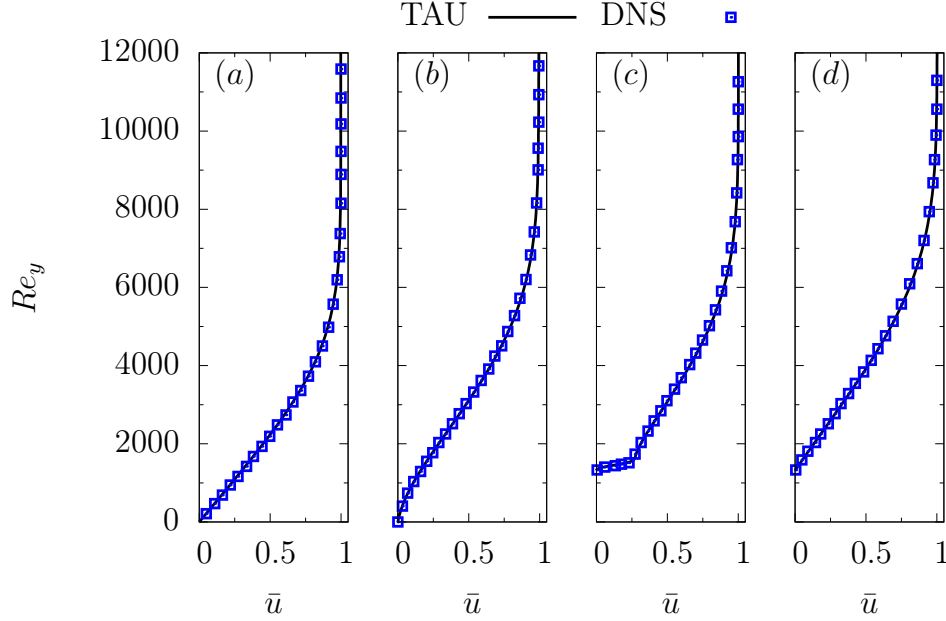


Figure 6.11: Nondimensional base flow streamwise velocity profiles $\bar{u}$ at four locations: a) $Re_x = 2000000$, b) $Re_x = 2445000$, c) $Re_x = 2450320$, d) $Re_x = 2800000$. The DNS data have been extracted from [59].

frequency $F = 21.875$ by means of DNS. This case is of particular interest because the geometry of the rectangular step is reproduced using several sub-domains (see Sect. 5.5).

In order to reproduce the stability analysis results given by Edelmann [59], the steady laminar base flow has to be computed first. This was done using the structured grid generator MEGACADS [60], together with the unstructured RANS solver TAU [61]. Both MEGACADS and TAU have been developed at DLR. Figure 6.11 depicts the nondimensional base flow streamwise velocity profiles $\bar{u}$ at four locations close to the step position, i.e. $Re_x = 2450000$. The streamwise location Re_x stands for $Re_x = Re_u \cdot x^*$. An excellent agreement was found between the published DNS results [59] and our steady laminar NS simulations using TAU.

Once the base flow was computed, the stability analysis was done by the combined use of PSE and AHLNS (labelled *AHLNS*). The AHLNS approach was used only in the vicinity of the step, i.e. in the region $x^* \in [0.75, 1.25] m$. In the remaining part of the domain, the instability analysis was done with the PSE method. Figure 6.12 shows the corresponding n -factor curve for the above-mentioned 2D TS wave. A very good agreement was found between the AHLNS computation and the published DNS result [59]. Besides this good agreement, it is also illustrative to compare the computational effort required by both methodologies in order to perform the stability analysis. The AHLNS computation needs 400 points in streamwise direction, taking about 5 minutes for reaching to a converged solution on a 8-cores workstation. On the other hand, Edelmann [59] stated that a single stability analysis with DNS took also about 5 minutes (depending on the finite difference scheme, 8th-order explicit or compact), but required 6144 processors on an HPC Cluster. The number of points in streamwise direction required by the linear DNS was not published [59].

Similar to previous validation cases, the stability analysis for this test case was done also using PSE solely. Figure 6.13 collects the results for the numerical investigation. It is interesting to notice that, unlike the 2D rod test case (see Sect. 6.2.2), the PSE approach is able to march through the rectangular step placed at $x^* = 1$ m (for relatively large-spaced point distributions). However, when the streamwise resolution is increased (golden dashed line, Fig. 6.13), the PSE method is not even able to continue the marching procedure over the FFS (see Fig. 6.14 for a detailed view). As a result, it was not possible to find a grid-independent result using PSE. Similar conclusions were reported in the work of Tocci *et al.* [62] for surface imperfections with rectangular corners present in an incompressible flow. As stated for the two-dimensional rod test case (Sect. 6.2.2), the rapid variations of n -factor computed by AHLNS and linear DNS [59] reflect the large streamwise variations in base flow quantities and amplitude function disturbances slightly upstream of the FFS.

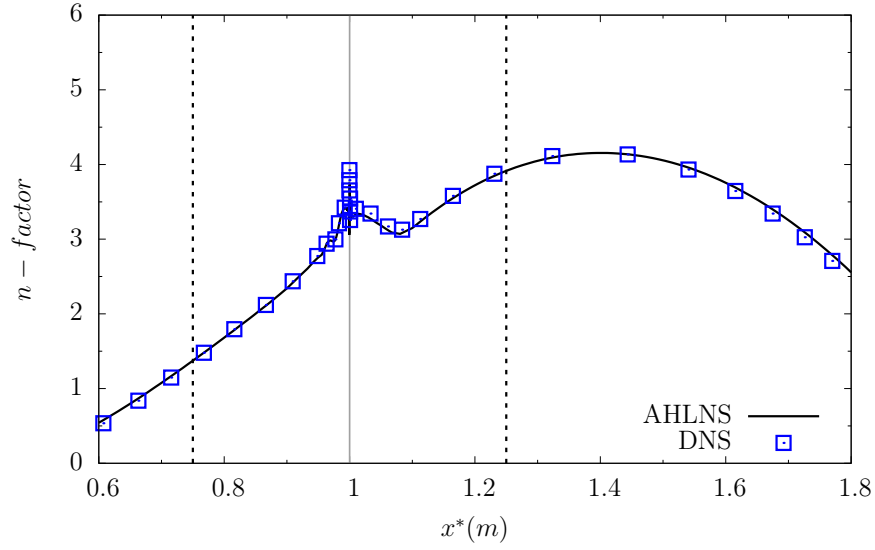


Figure 6.12: Spatial development of a single 2D TS wave ($F = 21.875$) in the presence of a rectangular forward-facing step. The vertical grey solid line represents the position of the step. The two vertical black dashed lines indicate the location of the interfaces between PSE and AHLNS methodologies. The DNS data (blue symbols) have been extracted from [59].

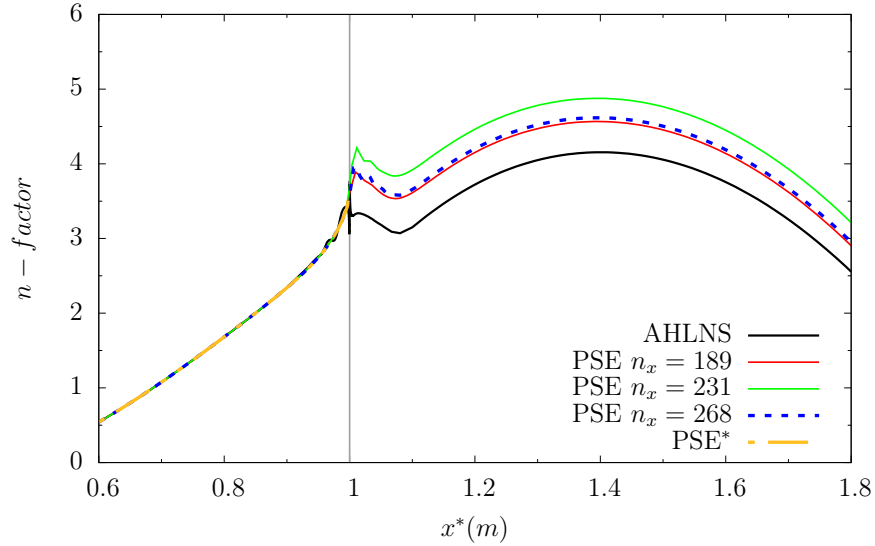


Figure 6.13: Spatial development of a single 2D TS wave ($F = 21.875$) in the presence of a rectangular forward-facing step. The vertical grey solid line represents the position of the step. PSE* holds for a PSE computation in which the marching procedure stopped converging at the step location $x^* = 1.0$ m.

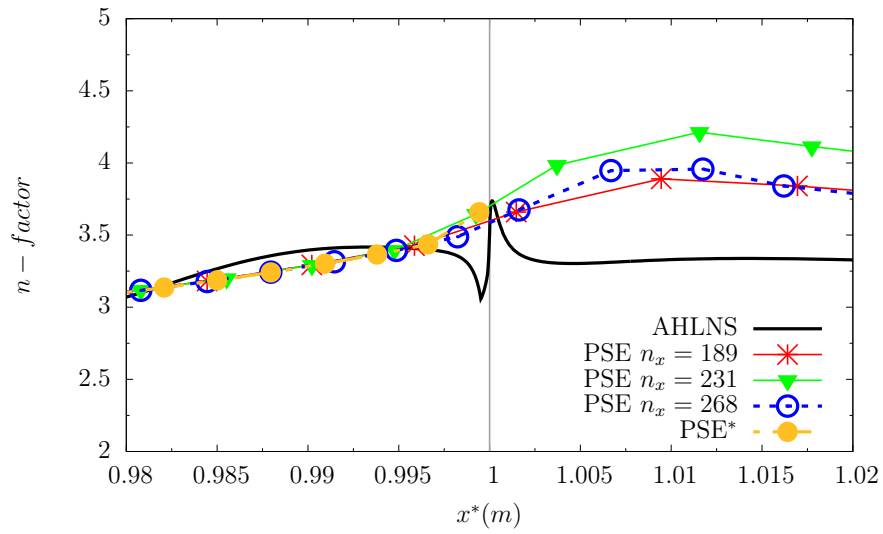


Figure 6.14: (*Detailed view*) Spatial development of a 2D TS wave ($F = 21.875$) in the presence of a rectangular forward-facing step. The vertical grey solid line represents the position of the step. PSE* holds for a PSE computation in which the marching procedure stopped converging at the step location $x^* = 1.0$ m. For the n -factor curves computed with PSE only, each symbol represents a streamwise location where PSE are computed.

Chapter 7

AHLNS for step-like geometries

7.1 Introduction

The effect of steps on the onset of laminar-turbulent transition has been extensively studied, both numerically and experimentally. Therefore, only a short summary of the main works for two-dimensional flows in subsonic conditions is presented here. One of the first attempts to quantify the effect of steps on transition location was done by Nenni and Gluyas [63]. Based on a series of *in-flight* experiments on the X-21 wing, they set a critical Reynolds number based on the height of the step. According to this criterion, rectangular steps with a height smaller than the critical value hardly affect the transition location. Although the work of Nenni and Gluyas [63] was published almost 60 years ago, it is widely used in the aeronautical industry still. This criterion is often quite restrictive and very difficult to achieve for real aircraft in transonic flow conditions with very high Reynolds numbers.

More recently, Perraud and Séraudie [64] studied the effect of rectangular steps placed on a flat plate numerically by means of the PSE methodology. The authors also noticed that the main assumption in the PSE methodology, i.e. the streamwise variations of the base flow magnitudes are relatively small, '*is true almost everywhere, except in the vicinity of the step*' [64]. They proposed to include the effect of the steps in the N -factor envelope curve as a modification ΔN with respect to the N -factor envelope curve for the flat-plate configuration. Later, Wang and Gaster [65] made experimental investigations in a wind-tunnel for rectangular steps placed in a flat plate also, measuring the transition location and frequency spectrum of the disturbances for each step. In both studies [64, 65], the idea of the authors is to apply the well-known e^N methodology for cases where a surface imperfection is present.

For rectangular steps (both FFS/BFS), and due to the limitations of the LST/PSE approaches, recent efforts have been put on DNS in incompressible flow. Examples of these studies are the works of Sescu *et al.* [66], and Rizzeta and Visbal [67]. In both works, FFS and BFS were analysed. However, due to the numerical efforts required for these computations, parametric studies are very limited. Typically, only one frequency TS mode is studied, and the geometry of the step is changed with respect to its height only (i.e. no change in inflow conditions

neither the location of the step). Edelmann [59] studied the effect of sharp-edged FFS by means of DNS for compressible flows. Interestingly, he found that there was a good agreement between DNS and LST (Local Stability Theory) results. However, a small region around the step was omitted from the LST computations. Therefore, the question about the extension of the region excluded in the LST computation (where the large streamwise gradients are) remains open.

One of the earliest attempts of studying smooth steps was done by Ragab and Nayfeh [68]. They made use of LST to compute the spatial growth rate of incoming TS waves on BFS only. More recently, Xu *et al.* [69] studied the effect of rounded FFS by means of LST. For a very limited number of cases, they performed non-linear DNS simulations to extend the laminar-turbulent transition computations up to the fully turbulent state. They discovered [69] that a particular configuration with two smooth steps placed relatively close to each other showed a transition delay (compared with the flat-plate case). No rectangular steps were included in their investigation.

Another example of smooth steps is found in the recent work of Hildebrand *et al.* [46]. They aimed to reproduce numerically the findings of the experiments of Wang and Gaster [65] (for BFS only) by modelling the step as slightly rounded, instead of rectangular. Hildebrand *et al.* [46] combined the use of HLNS for the instability analysis with the e^N methodology. In spite of considering smooth steps instead of sharp-edge steps, they achieved a relatively good agreement between the predicted transition location with the experimental data [65]. Franco *et al.* [48] presented similar conclusions for BFS. However, they found notable differences between very slightly rounded and rectangular FFS [48].

For compressible subsonic conditions, most of the published work has been done experimentally in the Cryogenic Ludwig-Tube Göttingen (KRG). The work of Costantini *et al.* [70] represents an excellent summary of many experimental test campaigns done at KRG for several years, focusing mainly on rectangular FFS. Many parametric studies regarding the influence of step height, Reynolds number, Mach number, etc. are discussed there [70]. For incompressible conditions, the experimental work of Methel *et al.* [71] combines the influence of FFS and wall suction on the expected transition location. The wall suction is a key aspect to be considered in the development of the HLFC (Hybrid Laminar Flow Control) technology for future wing designs. The principal aim of these new designs is to extend the region on the wing surface where the flow remains in a laminar state. The implementation of such technologies seeks for a reduction in viscous drag, as Reneaux [72] remarked.

The investigation conducted in this chapter applies the AHLNS methodology to a flow configuration where localized large streamwise gradients are present (in this case, surface steps). The results present here extend the investigations done before by *et al.* [32]. Due to the much lower computational cost required by the AHLNS computations compared with linear DNS, the range of parameters can be significantly enhanced. For example, varying the frequency of the TS modes, the corresponding N -factor envelope curve for each step can be computed. The e^N methodology, supported with the information given by these N -factor curves, provides an estimation of the expected transition location. In other words, systematic parametric variations such as inflow conditions, step type (BFS/FFS), step shape, step location and step height (just for citing some of them) can be easily translated into

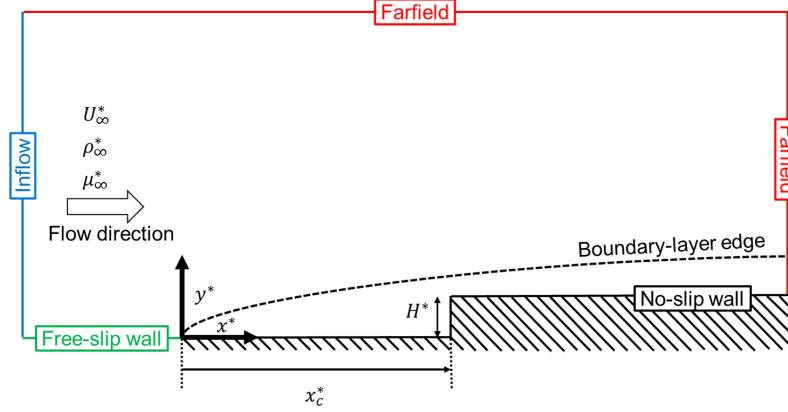


Figure 7.1: Schematic representation of the computational domain used for the present study. The labels at the edges of the domain indicate the type of boundary conditions selected for the base flow computations.

transition location. These results help to researchers and designers to quantify the influence of such parameters.

We remark that the AHLNS methodology belongs to the linear stability theory. Therefore, the initial stage (receptivity) and subsequent stages of the classical path of laminar-turbulent transition (non-linear interactions and breakdown to turbulence) are not present in our studies. Moreover, we limit ourselves by fixing inflow conditions and step location. However, systematic variations on step type, step shape and step height are presented. It is expected that the research conducted in this chapter will enhance the knowledge about the influence of surface steps on the laminar-turbulent transition process driven by TS waves.

7.2 Problem description

We study numerically the effect of a single step on boundary-layer transition of a two-dimensional laminar flow on a semi-infinite flat plate. Figure 7.1 shows schematically the computational domain used in the present study, including the type of boundary conditions imposed for computing the steady laminar base flow. All dimensional quantities are marked with an asterisk *. We place the origin of the Cartesian coordinate system at the leading edge of the plate, where x^* , y^* and z^* are the streamwise, normal, and spanwise coordinate components of the position vector $\mathbf{x}^*$. The free-stream Mach number Ma_∞ is set to 0.5 for all cases. The shape of the step is defined by the expression

$$y^* = \frac{H^*}{2} \left(1 \pm \operatorname{erf} \left(\frac{1}{C} \frac{x^* - x_c^*}{\delta_c^*} \right) \right), \quad (7.1)$$

where the parameter H^* indicates the height of the step. The *error function* erf is used for describing the shape of a rounded step. This idea was inspired by the work of Xu et al. [69].

Here, '+' refers to FFS and '-' to BFS. The centre of the step is placed at x_c^* . The geometrical parameter C controls the shape of the step. For rounded steps, some plots included in this chapter are described using the term *width*. Actually, it refers to the domain, in streamwise direction, where $x - x_c \in [-C, C]$. The rectangular shape can be considered the limiting case when $C \rightarrow 0$ (also examined in the present study). The rectangular step case is defined by the piecewise expression

$$y^* = \begin{cases} 0, & \text{if } x^* < x_c^*, \\ H^*, & \text{if } x^* \geq x_c^*, \end{cases} \quad (7.2)$$

in the case of rectangular FFS. For the rectangular BFS, the wall surface is defined by

$$y^* = \begin{cases} H^*, & \text{if } x^* \leq x_c^*, \\ 0, & \text{if } x^* > x_c^*. \end{cases} \quad (7.3)$$

Geometrical quantities are nondimensionalized with δ_c^* (the compressible boundary-layer displacement thickness at the x_c^* position for a flat plate at zero pressure gradient). The value of δ_c^* is given by the expression (see Schlichting [17])

$$\delta_c^* = \int_0^\infty \left(1 - \frac{\rho^* u^*}{\rho_e^* U_e^*} \right) dy^*, \quad (7.4)$$

where ρ^* and u^* stand for density and streamwise velocity component, while ρ_e^* and U_e^* refer to density and velocity at the boundary-layer edge at the step location x_c^* , respectively. The Reynolds number based on δ_c^* is defined as

$$Re_{\delta^*} = \frac{\rho_\infty^* U_\infty^* \delta_c^*}{\mu_\infty^*}, \quad (7.5)$$

where ρ_∞^* , U_∞^* , and μ_∞^* denote the density, streamwise velocity, and dynamic viscosity at the inflow, respectively. In the present study, the value of Re_{δ^*} is fixed to 1823. We will consider three values for the geometric parameter C : 10, 50, and the above-mentioned rectangular case r , i.e. $C \rightarrow 0$.

For the nondimensional height H , three values will be examined here: 0.4, 0.8, and 1.2. The Reynolds number based on the step height Re_H is 729, 1429, and 2187, respectively. In the present work, the height of the step will be also referred to as *small* ($H = 0.4$), *medium* ($H = 0.8$), and *large* ($H = 1.2$). The combination of parameters H and C for both FFS and BFS gives a total of 18 types of surface irregularities, which are summarized in Table 7.1. In this table, the maximum slope θ_{max} of the wall is included also. Although the cases of smooth steps have a very small θ_{max} (in other words, these rounded steps are very shallow), it will be shown that, in certain cases, their influence in the expected laminar-turbulent transition location is similar (and for specific cases, almost identical) to their corresponding rectangular steps. Finally, the link between the critical Reynolds number based on the height of the rectangular steps ($Re_{crit} = 1800$ for FFS and $Re_{crit} = 900$ for BFS) established by Nenni and Gluyas [63], and the step heights considered in this work ($Re_H = 729, 1429$, and 2187 , for both FFS and BFS) will be discussed.

Case name	H	C	θ_{max} ($^{\circ}$)
BFS_H04_C50	0.4	50	-0.26
BFS_H04_C10	0.4	10	-1.29
BFS_H04_r	0.4	r	-90
BFS_H08_C50	0.8	50	-0.52
BFS_H08_C10	0.8	10	-2.58
BFS_H08_r	0.8	r	-90
BFS_H12_C50	1.2	50	-0.78
BFS_H12_C10	1.2	10	-3.87
BFS_H12_r	1.2	r	-90
FFS_H04_C50	0.4	50	0.26
FFS_H04_C10	0.4	10	1.29
FFS_H04_r	0.4	r	90
FFS_H08_C50	0.8	50	0.52
FFS_H08_C10	0.8	10	2.58
FFS_H08_r	0.8	r	90
FFS_H12_C50	1.2	50	0.78
FFS_H12_C10	1.2	10	3.87
FFS_H12_r	1.2	r	90

Table 7.1: Values of the nondimensional parameters H and C considered in the present study. The case $C = r$ refers to the rectangular step. The last column indicates the maximum slope θ_{max} of the wall.

7.3 Base flow computations

7.3.1 Approach

The laminar steady two-dimensional flow on a flat plate in the presence of a step was computed numerically. We used the compressible Navier-Stokes solver TAU [61], developed at DLR. Grids were generated using the structured grid generator MEGACADS [60], also developed at DLR. A schematic representation of the computational domain and the boundary conditions imposed for computing the base flows is depicted in Fig. 7.1. Farfield boundary conditions were imposed sufficiently far from the surface irregularity (about $5000 \delta_c^*$ in both streamwise and normal directions). Simulations were run until the variations in lift and drag coefficients within the last 40 iterations were smaller than 10^{-7} and $3 \cdot 10^{-8}$, respectively.

7.3.2 Grid convergence

Two sets of grids were used to verify the grid-independence of the base flow results. In both sets of grids, points were clustered near the walls, where the largest variations of flow

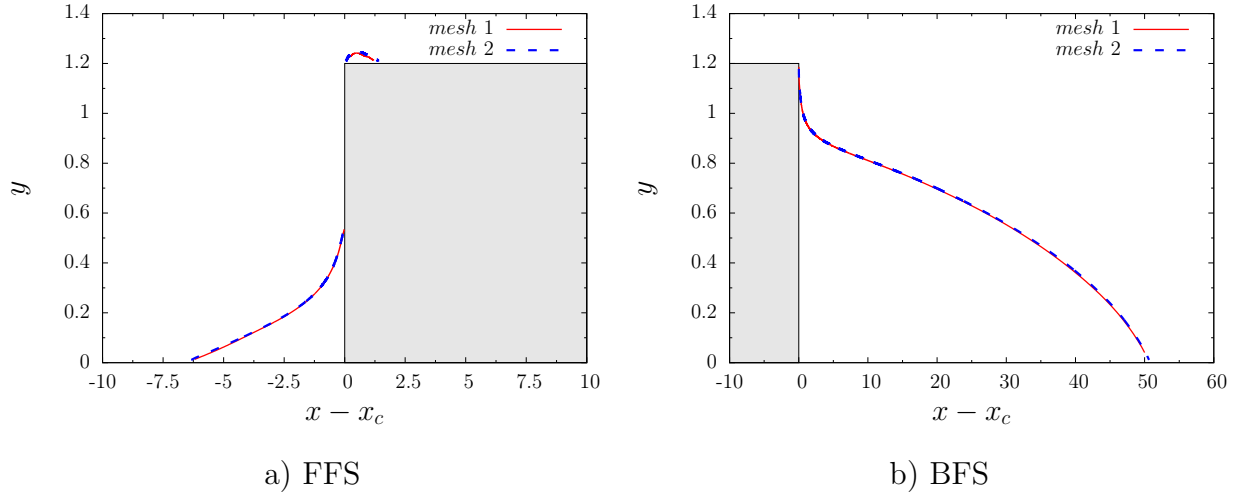


Figure 7.2: Base flow grid convergence study for cases *FFS_H12_r* (left), and *BFS_H12_r* (right). Both the red solid line (*mesh 1*) and the blue dashed line (*mesh 2*) depict the points in the base flow where $\bar{u} = 0$. The grey area represents the corresponding step. Flow direction is from left to right. Axes are not to scale.

quantities are expected. The first set of grids is referred to *mesh 1*. These grids have about 1200 and 300 points in streamwise and normal directions, respectively. The number of points is varied slightly in normal direction when the value of H changes (in order to keep the same grid spacing close to the wall). Another set of grids, named *mesh 2*, was also used to compute the base flows. In this second set of grids, the number of points was increased in both directions: about 2000 and 500 points in streamwise and normal directions, respectively.

Figure 7.2 shows the results of the grid convergence study for the case *H12_r* for both FFS (a) and BFS (b). The dividing line $\bar{u} = 0$ limits the region where there is reverse flow ($\bar{u} < 0$). For the FFS (Fig. 7.2-a)), there are two recirculation areas: one in front of the step and one on top of the step. For the BFS (Fig. 7.2-b)), a single large separation region appears downstream of the vertical wall. In both cases (FFS/BFS), the dividing lines $\bar{u} = 0$ are the same if the base flow is computed with a grid of set *mesh 1* or *mesh 2*. Therefore, the grid independence of the base flows for rectangular steps is demonstrated.

For rounded steps, Fig. 7.3 shows the results of the grid convergence study for the case *H12_C10* (FFS and BFS), in terms of the skin-friction coefficient c_f (left) and the nondimensional displacement thickness δ distribution (right).

The value of c_f is given by the expression (see Schlichting [17]):

$$c_f = \frac{(\mu \frac{\partial u}{\partial y})_{y_{wall}}}{\frac{1}{2} \rho_{\infty} (U_{\infty})^2}. \quad (7.6)$$

Results for the flat-plate case have been added to Fig. 7.3 (black solid line) for comparison. Similar to the above-mentioned rectangular case *H12_r*, the excellent agreement using the

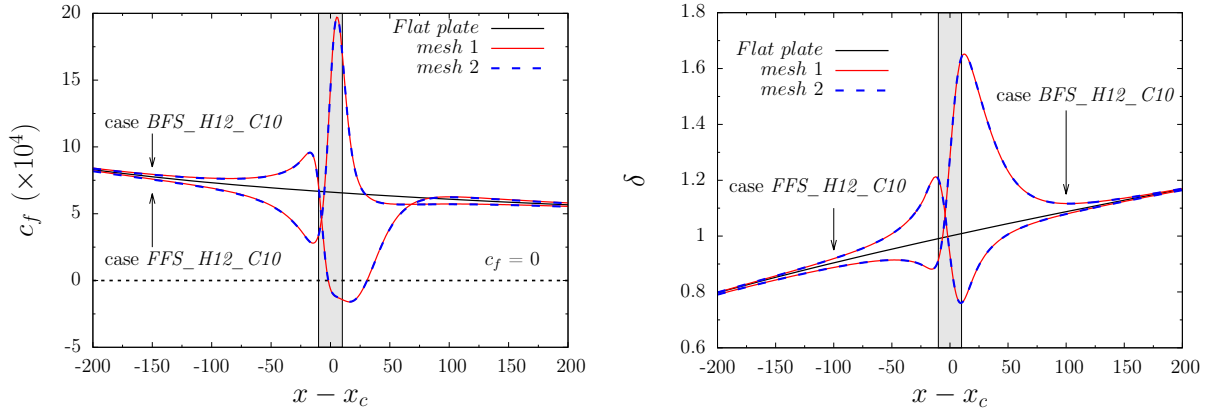


Figure 7.3: Base flow grid convergence study for cases *FFS_H12_C10* and *BFS_H12_C10*, based on skin-friction coefficient c_f (left) and nondimensional displacement thickness δ (right). The vertical grey bar indicates the location and width of the rounded step. Results for the flat-plate case (black solid line) have been added for comparison.

two grids *mesh 1* and *mesh 2* indicates the grid-independence of the base flow quantities. Therefore, the base flow computed with meshes of the set *mesh 1* are used for stability analysis. The number of points for *mesh 1* (1200×300) and *mesh 2* (2000×500) is similar, regardless of whether the step is rectangular or smooth.

Figure 7.3 illustrates another aspect that has to be pointed out: The presence of the step affects strongly the base flow characteristics, but this effect is limited to the vicinity of the step. In other words, at certain distance from the step (upstream and downstream), the streamwise gradients of the quantities $c_f(x)$ and $\delta(x)$ become very small. This circumstance (localized influence of the surface irregularity) permits to apply the multi-zonal approach described in Sect. 5.2.3 for a subsequent stability analysis. Hereby, the PSE method is used relatively far from where the step is located, and it is coupled with the AHLNS approach in the region where the surface imperfection is placed.

7.3.3 Base flow results

The steady laminar base flow pressure $\bar{p}^*$ is made nondimensional $\bar{p}$ by twice the incoming dynamic pressure, i.e. $\rho_\infty^*(U_\infty^*)^2$. Figure 7.4 shows the contours of $\bar{p}$ and streamlines for all *large* steps ($H = 1.2$) studied. For FFS, the flow field remains attached for rounded steps (Fig. 7.4-a) and -c)), while for the rectangular case (Fig. 7.4-e)), a tiny recirculation area appears in front of the step. For BFS (Fig. 7.4-b), -d), -f)), the parameter C has a crucial influence on the presence and size of the separation bubble. In particular, for very smooth steps ($C = 50$) the flow remains attached. If the shape parameter C is reduced (cases *BFS_H12_C10* and *BFS_H12_r*), a large bubble appears right after the step.

In terms of pressure contours, it is interesting to notice that for BFS (Fig. 7.4-b), -d), -f)) the pressure contours barely change with the shape of the step. However, for FFS (Fig. 7.4-a),

-c), -e)) the pressure gradients in the vicinity of the step become significantly more intense as the step shape approaches to the rectangular case.

7.4 Instability analysis

Once the steady laminar base flow has been computed, it is possible to start with the subsequent boundary-layer instability analysis. The numerical simulation of the spatial development of a broadband discrete spectrum of TS waves (each one computed individually and defined by its reduced frequency F and spanwise wavenumber β) is described in this section. The spatial growth/decay in amplitude of each TS wave is collected in the corresponding n -factor curve (defined in Sect. 3.4).

The instability analysis is limited to the domain $(x - x_c) \in [-500, 2000]$. An initial investigation performed on a flat plate using PSE revealed that two-dimensional ($\beta = 0$) TS waves of frequencies $F \in [20, 75]$ produce the highest n -factor curves in the domain mentioned above. Therefore, present parametric study is limited to 2D TS waves. However, the instability characteristics of 3D TS waves when a step is present will be briefly discussed in Sect. 7.4.4.

When there is a step, the multi-zonal approach described in Sect. 5.2.3 is applied. The interfaces between PSE and AHLNS are placed at $(x - x_c) = \pm 165$, sufficiently far away from the region with separated flow and large pressure gradients (see Fig. 7.4). Figure 7.5 illustrates how the N -factor envelope curve is built for each step (here the case *BFS_H08_r* is considered). A total of 34 TS waves, with reduced frequencies in the range between $F = 20$ and $F = 75$ are computed. At each streamwise location, the maximum n -factor value for all frequencies computed is taken as the corresponding N -factor envelope value at that particular location. This process is repeated for the 18 steps considered in the present work (see Table 7.1). Therefore, circa 600 individual n -factor curves have been computed for the parametric studies presented in this chapter. In terms of computational effort, each individual n -factor curve took about 5 min. on a personal workstation with eight cores. The use of HLNS or linear DNS would have required the order of few hours for a single n -factor curve. This example illustrates clearly the huge advantage of the AHLNS approach for parametric studies.

7.4.1 Grid convergence

Two sets of grids, called *reference* and *fine*, were used to verify the grid-independence of the stability analysis results. For simplicity, only results regarding the area around the steps $(x - x_c) \in [-165, 165]$ will be discussed. This verification was done computing the corresponding n -factor curve for a single TS wave of frequency $F = 48$. The number of grid points in each mesh depends on whether a rounded or rectangular step is considered, as it is shown in Table 7.2. For rounded steps, cases *BFS_H12_C10* and *FFS_H12_C10* are studied (see Fig. 7.6-a)). For rectangular steps, the grid independence of the results for cases *BFS_H12_r* and *FFS_H12_r* is shown in Fig. 7.6-b)). No differences in terms of n -factor

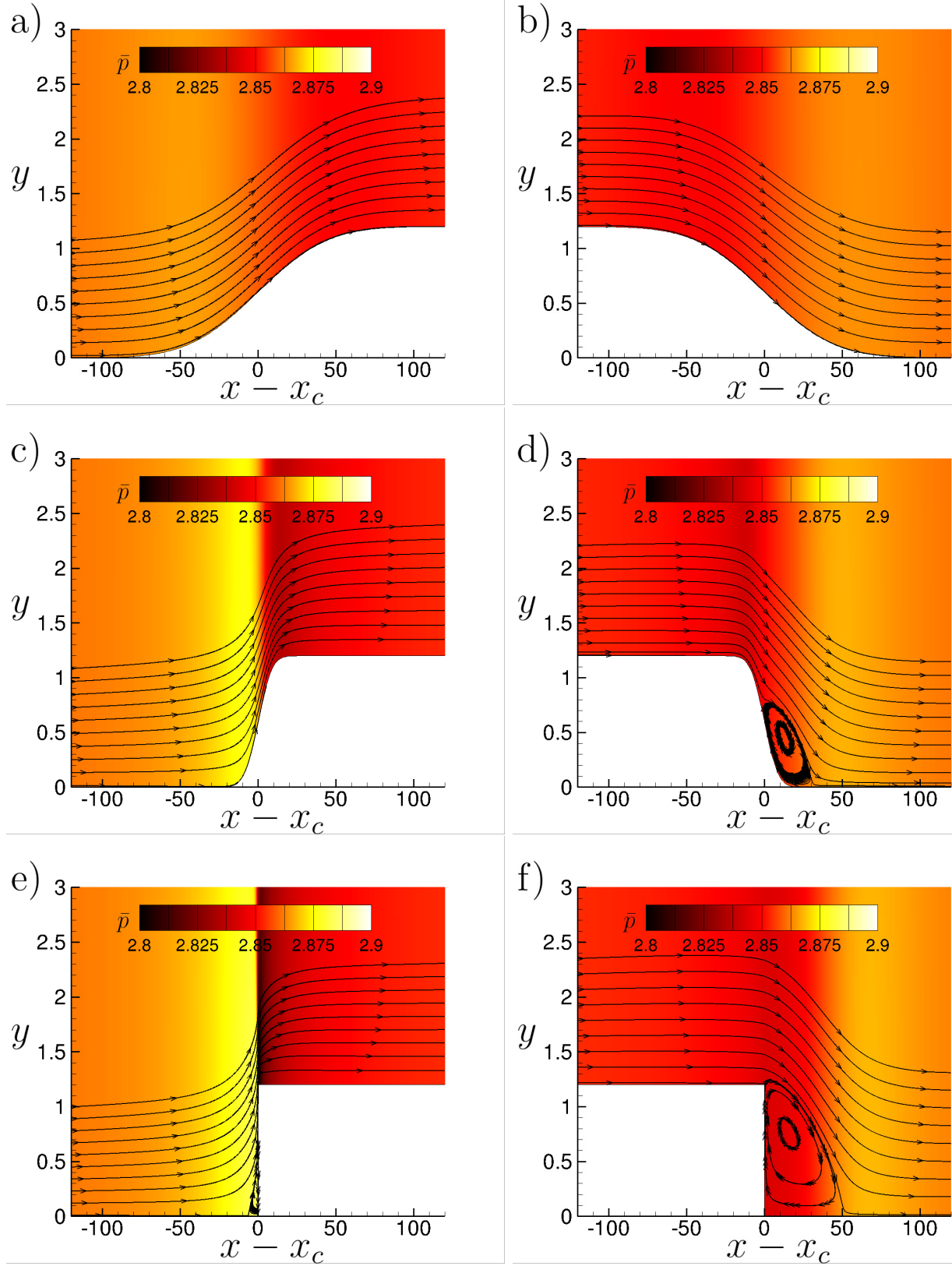


Figure 7.4: Steady laminar base flow pressure $\bar{p}$ contours and streamlines for cases (a) *FFS_H12_C50*; (b) *BFS_H12_C50*; (c) *FFS_H12_C10*; (d) *BFS_H12_C10*; (e) *FFS_H12_r*; (f) *BFS_H12_r*. Flow direction is from left to right. Axes are not to scale.

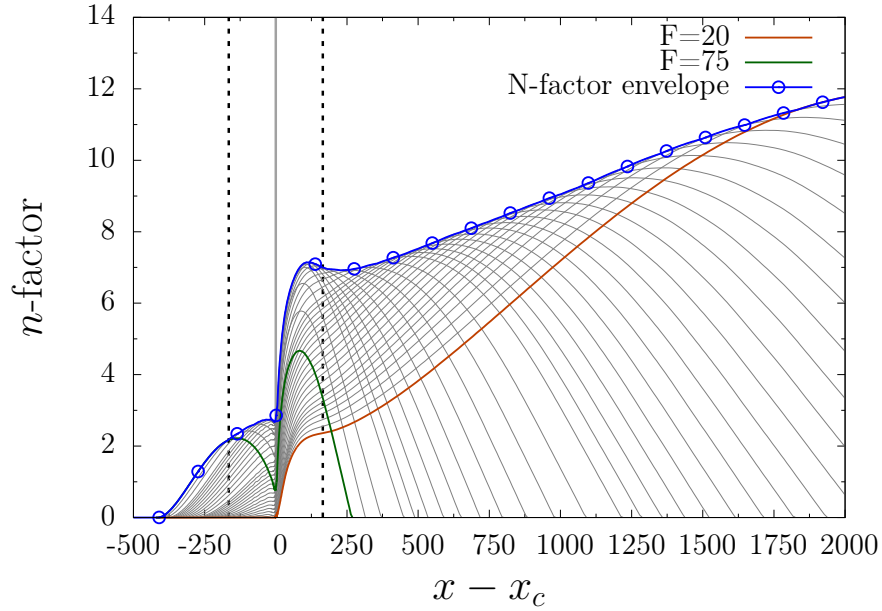


Figure 7.5: Definition of the N -factor envelope curve (blue line with circles) for the case *BFS_H08_r*. A total of 34 TS waves (grey solid lines) were computed. The smallest ($F = 20$) and largest ($F = 75$) frequencies considered have been highlighted for ease. Vertical dashed lines indicate the location of the interfaces between PSE and AHLNS methodologies. The vertical grey solid line represents the position of the rectangular step.

curves were found neither for rectangular steps nor rounded steps. Therefore, the *reference* grid specifications (either for rounded or rectangular steps) of Table 7.2 are used for stability analysis purposes.

7.4.2 Effect of step height and shape

The boundary-layer instability analysis results will be presented in following sections by grouping the different steps according to their height (*small*, *medium*, and *large*).

<i>shape</i>	n_x, n_y	
	<i>Reference grid</i>	<i>Fine grid</i>
Rounded steps	150, 100	250, 150
Rectangular steps	400, 200	600, 300

Table 7.2: Number of points n_x and n_y (streamwise and normal directions, respectively) for grid convergence verification using AHLNS.

7.4.2.1 Small steps

Figures 7.7-a) and 7.8-a), for FFS and BFS respectively, show the N -factor envelope curves for *small* steps ($H = 0.4$) as function of the shape parameter C . The Reynolds number based on the step height is $Re_H = 729$. This value is below the critical values of 1800 for FFS and 900 for BFS, established by Nenni and Gluyas [63]. The Nenni and Gluyas criteria, defined for rectangular steps and based on experimental data, sets the maximum step height H_{crit} such that, below this value the transition location is generally unaffected by the presence of the step. In the case of FFS (Fig. 7.7-a)), the presence of the step is barely noticeable in the N -factor curves. For BFS (Fig. 7.8-a)), the presence of the step increases slightly the values of the corresponding N -factor curve far downstream of the step location x_c . The largest increase in N -factor is produced by the rectangular step. In both cases, either FFS or BFS, and based on the N -factor curves, it is not expected that a *small* step changes considerably the predicted transition location compared with the flat-plate case. This result holds for all values of shape parameter C . Therefore, the outcome of the instability analysis for FFS and BFS agrees qualitatively with the Nenni and Gluyas criteria mentioned above.

As expected, the rapid (but small) changes in the N -factor envelope curves occur in the vicinity of the step. These variations are driven mainly by the pressure gradients introduced by the step into the flow field (see Fig. 7.7-b), and 7.8-b)). In the case of FFS (Fig. 7.7-b)), there is an adverse pressure gradient in front of the step. This adverse gradient is followed by a favourable pressure gradient along the step. It is well known (see Drazin and Reid [73]) that an acceleration in the flow usually has a stabilizing effect. And similarly, a deceleration leads to a destabilizing effect. Therefore, it is possible to relate the smooth adverse pressure gradient in front of the step with the slight increase in the N -factor curves. Similarly, the rapid drop in pressure at the step location is linked with the decrease in N -factor over the

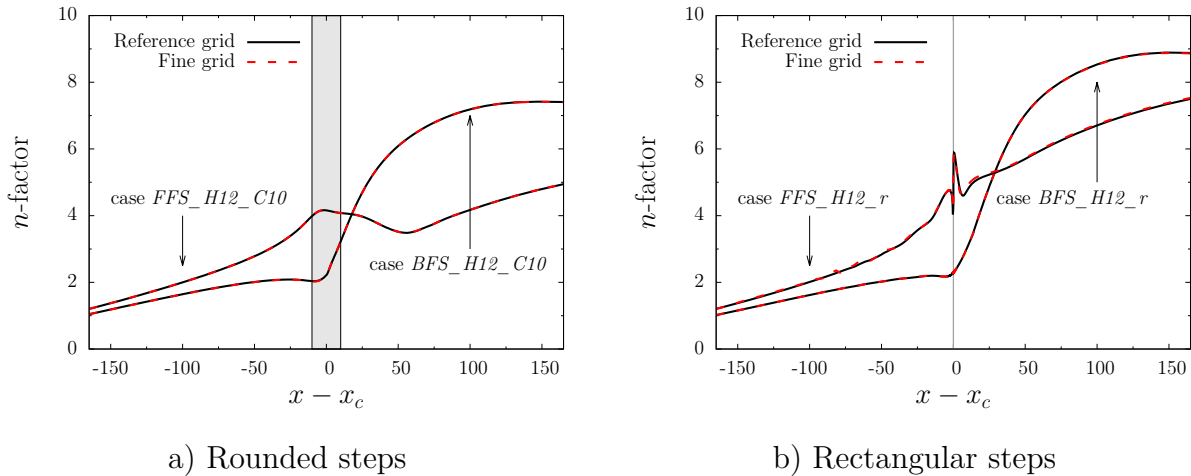


Figure 7.6: Stability analysis grid convergence study for cases $H12_C10$ (rounded steps) and $H12_r$ (rectangular steps), both FFS and BFS, based on the n -factor curve for a 2D TS wave of frequency $F = 48$. The grey area (rounded steps) indicates the position and width of the steps. The vertical grey line (rectangular steps) represents the location of the step.

step location. Finally, at certain distance downstream from the step, the pressure distribution recovers the constant value of the flat-plate case. Similarly, the growth rate of the TS waves returns to the values of the flat-plate case. For BFS (Fig. 7.8-b)), the pressure distribution develops in the opposite way. A smooth favourable pressure gradient upstream of the step is followed by a relatively rapid adverse pressure gradient over the step. We recall that the pressure gradients in the vicinity of the step are significantly smaller for BFS than their FFS counterpart, especially for rectangular steps. The slightly larger increase in the N -factor downstream of the step for BFS (compared with FFS) is due to the presence of the relatively large separation bubble.

7.4.2.2 Medium steps

Figures 7.7-c) (FFS) and 7.8-c) (BFS) show the N -factor envelope curves for *medium* steps ($H = 0.8$) as function of the shape parameter C . The Reynolds number based on the step height is $Re_H = 1458$. In the case of FFS (Fig. 7.7-c)), the presence of the step increases slightly the values of the N -factor curves. This small increase is larger when the shape parameter C is reduced. This result is in line with the Nenni and Gluyas criteria mentioned previously, which sets the critical Reynolds number for rectangular FFS as $Re_{H_{crit}} = 1800$. In other words, it is not expected that any of the *medium* FFS considered here alters significantly the predicted transition location for the clean flat plate. For BFS (Fig. 7.8-c)) the presence of the step increases noticeably the value of the N -factor curves. Clearly, the rectangular step produces the largest increase in N -factor. Interestingly, the influence of the shape parameter C suggests that the values of the N -factor for the rectangular step could be reproduced by a smooth step if the parameter C is reduced enough. Finally, the outcome of the instability analysis for the rectangular BFS agrees qualitatively with the Nenni and Gluyas criteria, which sets the critical Reynolds number for rectangular BFS as $Re_{H_{crit}} = 900$. This means that the *medium* BFS (at least the rectangular step) are expected to have a significant effect on the transition location. In terms of pressure distribution (Fig. 7.7-d) and 7.8-d)), the scenario described before for *small* steps holds also for *medium* steps.

7.4.2.3 Large steps

Finally, Fig. 7.7-e) (FFS) and 7.8-e) (BFS) illustrate how the N -factor envelope curves change for *large* steps ($H = 1.2$) as function of the shape parameter C . The Reynolds number based on the step height is $Re_H = 2187$. This value is above the critical Reynolds number for both FFS and BFS, according to the Nenni and Gluyas criteria [63] (it is important to remark that these criteria were defined for sharp-edged steps only). In our study, the presence of a sharp-edged step increases noticeably the values of the N -factor curves with respect to a shallow step. For the smoothest steps considered in this work ($C = 50$), the presence of the step barely increases the values of the N -factor envelope curves for both FFS (case *FFS_H12_C50*) and BFS (case *BFS_H12_C50*) compared to the flat plate N -factor envelope curve. The results for the rounded step with $C = 10$ show a different behaviour: the presence of the step *FFS_H12_C10* causes a small increase in the values of the N -factor curve (compared with the flat plate), while its counterpart *BFS_H12_C10* affects strongly

the boundary-layer instability characteristics (i.e. the values of the N -factor envelope curve are significantly higher).

The analysis of the N -factor envelope curves suggest that, if the shape parameter C is sufficiently reduced, the influence of a sharp-edged step might be reproduced by a rounded-edged step. This conclusion holds for both BFS and FFS. However, the parameter C should be significantly smaller for FFS than for BFS (in order to replicate the instability characteristics of a rectangular step by a rounded step). Our work [\[48\]](#) presents a detailed comparative between slightly rounded and rectangular steps using AHLNS.

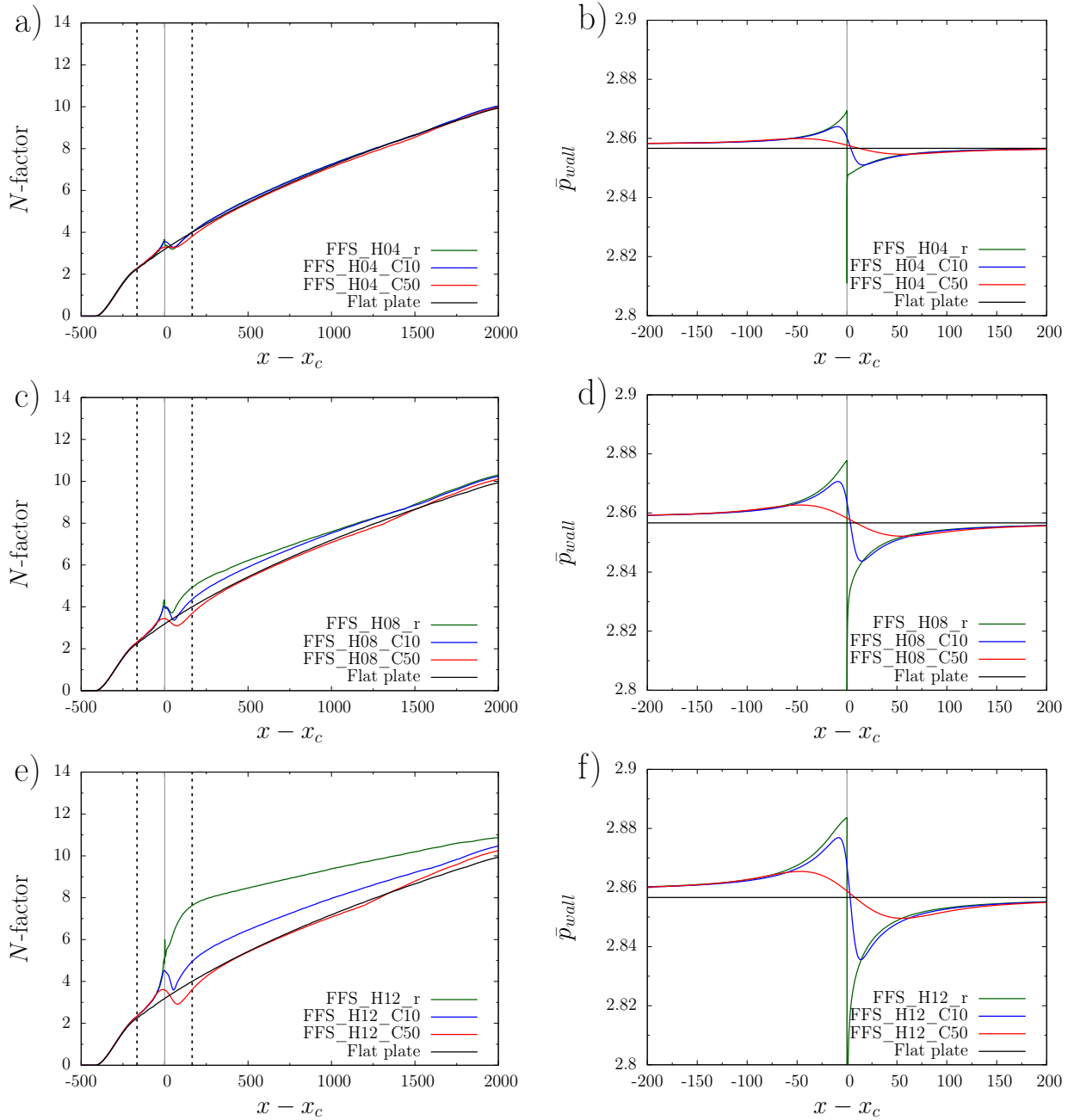


Figure 7.7: (Left) Effect of shape parameter C on N -factor envelope curves for FFS cases (a) *small*; (c) *medium*; (e) *large*. (Right) Effect of shape parameter C on pressure distribution at the wall $\bar{p}_{wall}$ for FFS cases (b) *small*; (d) *medium*; (f) *large*. Vertical dashed lines indicate the location of the interfaces between PSE and AHLNS methodologies. The vertical grey solid line represents the position of the step. Results for a flat plate (black solid lines) are included as reference.

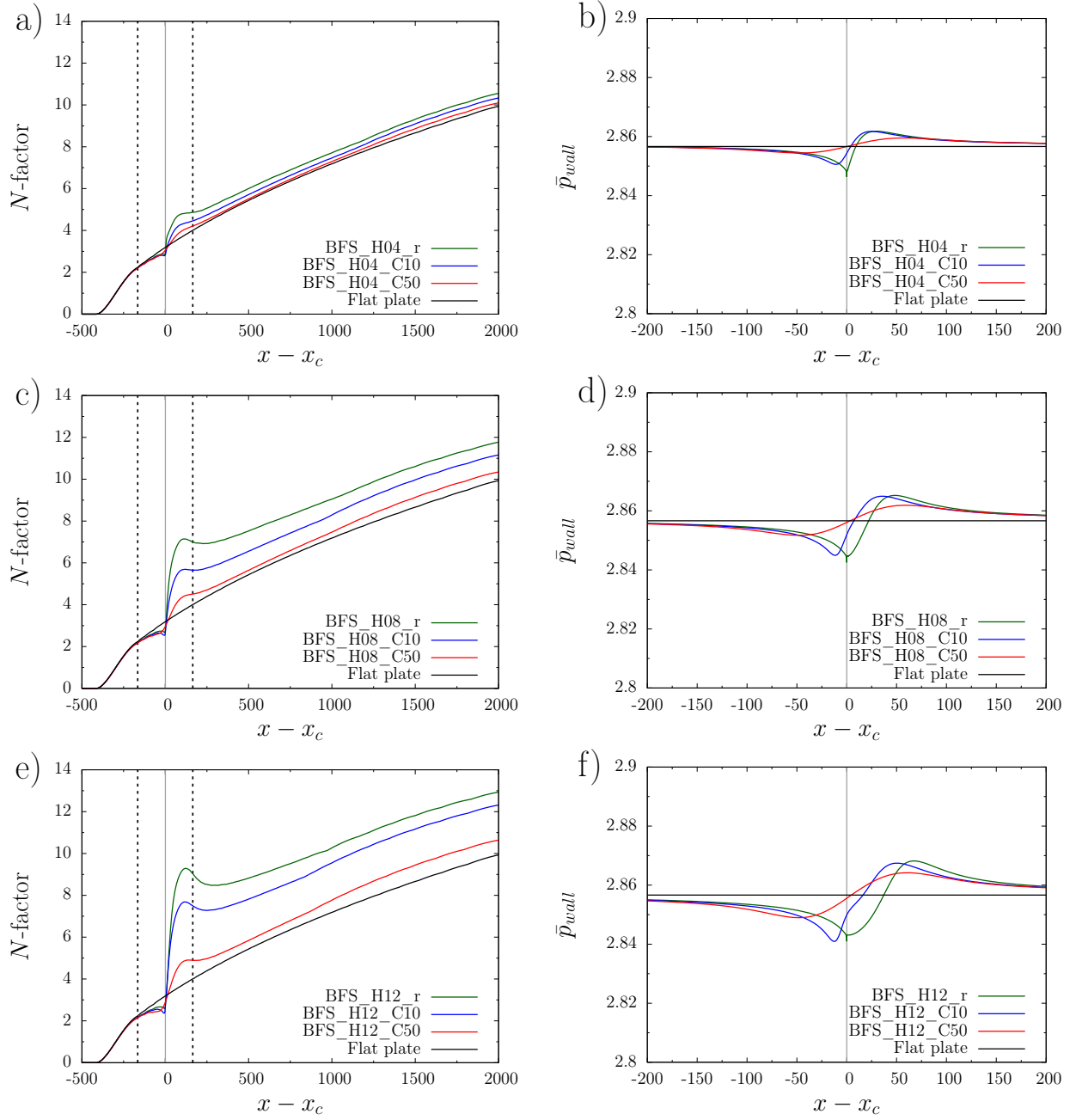


Figure 7.8: (Left) Effect of shape parameter C on N -factor envelope curves for BFS cases (a) *small*; (c) *medium*; (e) *large*. (Right) Effect of shape parameter C on pressure distribution at the wall $\bar{p}_{wall}$ for BFS cases (b) *small*; (d) *medium*; (f) *large*. Vertical dashed lines indicate the location of the interfaces between PSE and AHLNS methodologies. The vertical grey solid line represents the position of the step. Results for a flat plate (black solid lines) are included as reference.

7.4.3 Effect of the step on the TS wave structure

The presence of a step, either FFS or BFS, affects the spatial structure of an incoming TS wave. As a representative example of a FFS, the case *FFS_H04_r* is chosen. Figure 7.9 shows the isocontours of the real part of the streamwise velocity disturbance $\Re(\tilde{u}/\tilde{u}_{max})$ for a particular instant of time when an incoming TS wave of frequency $F = 25$ encounters the rectangular step. These particular step case and TS wave have been selected due to the moderate overall variation of the TS wave amplitude in the plotted domain, which facilitates a visualization by isocontours in Fig. 7.9. The figure shows that the shape of the TS wave varies significantly in the very vicinity of the step only. This phenomenon, which holds also for *medium* and *large* FFS, is reflected in the corresponding n -factor and growth rate $\sigma_{\hat{u}}$ of the TS wave (Fig. 7.10). Upstream of the step, the growth rate $\sigma_{\hat{u}}$ of the TS wave (Fig. 7.10-right) is similar to the well-known development of a TS wave in a flat-plate boundary layer. At the step location, extremely large variations in growth rate happen in a very short distance (the order of few δ_c^*). Beyond a relatively short distance downstream of the step location, the growth rate $\sigma_{\hat{u}}$ of the TS wave recovers the slow streamwise variations typical of the flat-plate case. It is interesting to notice that, for this particular step case and TS wave, the corresponding individual n -factor (Fig. 7.10-left) shows slightly smaller values than the equivalent flat-plate case downstream of the step. However, we remark that the study presented in this section is focused on how the presence of a step modifies locally the structure of a particular TS wave. The N -factor envelope (Fig. 7.7-a)) indicates that, considering a large range of TS modes on a sufficiently large domain, no transition delay (with respect to the flat plate) is expected.

For BFS, the counterpart case has been chosen (i.e. step case *BFS_H04_r* and individual TS mode of frequency $F = 25$). The most remarkable effect of the BFS on the TS structure is the significant enhancement of the TS amplitude slightly downstream of the step position (Fig. 7.11). This large increase in amplitude is reflected in the evolution of the n -factor curve right downstream of the step location (Fig. 7.12-left). It is worth to mention that the very intense changes arising over a short distance described for the FFS case are not present here. The variations in the spatial evolution for the BFS, measured in terms of the growth rate $\sigma_{\hat{u}}$ (Fig. 7.12-right), are considerable smaller at the step location.

7.4.4 2D vs. 3D TS waves

At the beginning of Sect. 7.4, it was stated that the boundary-layer instability analysis for configurations with surface imperfections will be limited to the study of 2D TS waves (i.e. with spanwise wavenumber $\beta = 0$). It is well-known that, for 2D compressible subsonic wall-bounded flows without any surface irregularity present and no external pressure gradient, the 2D TS waves exhibit the largest individual n -factor curves. In the case of external pressure gradients, 3D TS waves might be more relevant for transition (see the PhD thesis of Stemmer [74]). In general, although Drazin and Reid [73] cited the Squire's theorem [75] as '*to obtain the minimum critical Reynolds number it is sufficient to consider only two-dimensional disturbances*', Mack [14] pointed out that this theorem only applies to **incompressible self-similar** boundary layers in the **temporal** approach, and it refers '*to the minimum*

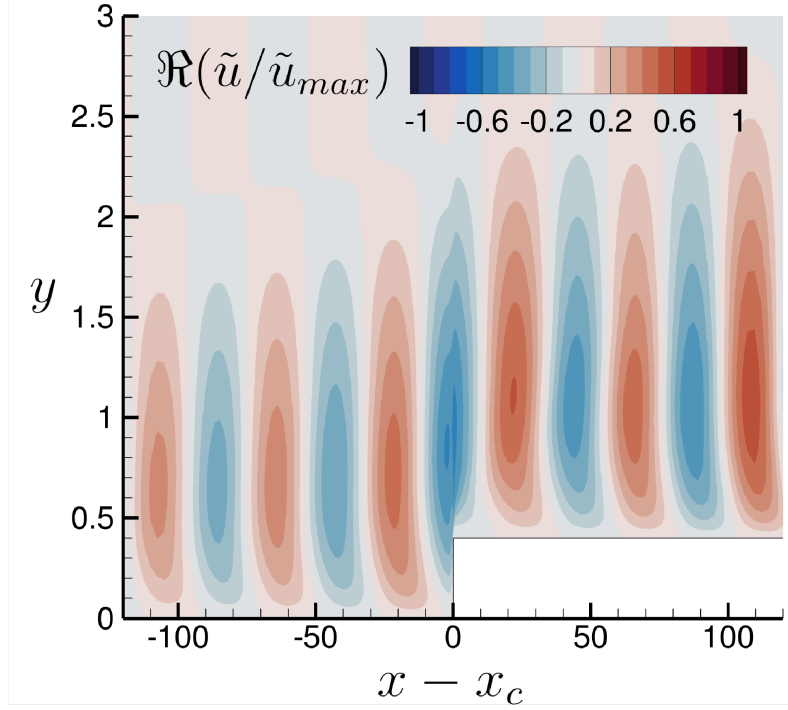


Figure 7.9: Influence of a FFS (case *FFS_H04_r*) on the real part of the streamwise velocity disturbance $\Re(\tilde{u}/\tilde{u}_{max})$ for an incoming TS wave of frequency $F = 25$ at an arbitrary instant of time. Flow direction is from left to right. Axes are not to scale.

critical Reynolds number and not to the critical Reynolds number of a particular frequency'. In the present work, the approach is focused on the **spatial** development of TS disturbances along **compressible** boundary-layer flows without external pressure gradient where **locally** there is a strong streamwise gradient caused by a surface irregularity. Therefore, it cannot be

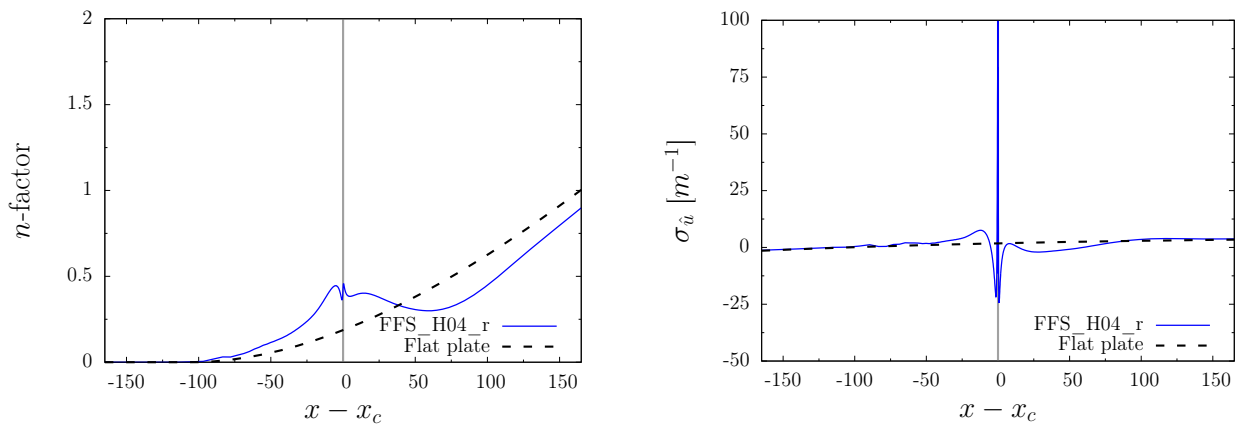


Figure 7.10: n -factor (*left*) and growth rate $\sigma_{\tilde{u}}$ (*right*) evolution for an incoming TS wave of frequency $F = 25$ in the presence of a rectangular FFS (case *FFS_H04_r*). The grey vertical line represents the position of the step. Results for a flat plate (black dashed lines) are included as reference.

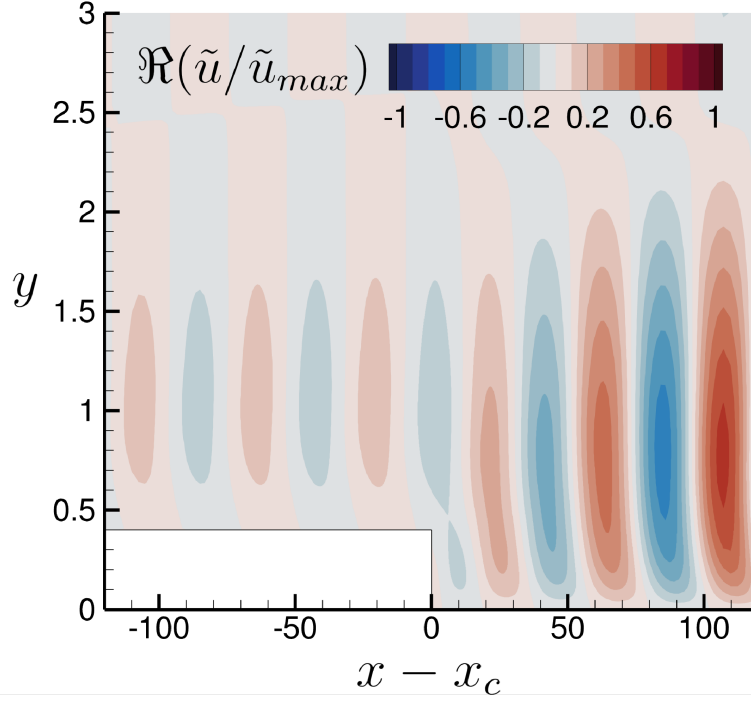


Figure 7.11: Influence of a BFS (case *BFS_H04_r*) on the real part of the streamwise velocity disturbance $\Re(\tilde{u}/\tilde{u}_{max})$ for an incoming TS wave of frequency $F = 25$ at an arbitrary instant of time. Flow direction is from left to right. Axes are not to scale.

assured *a priori* that the most unstable disturbances are two-dimensional. For this reason, we study TS waves of frequency $F = 48$ and different spanwise wavenumbers β . In terms of wave angle ψ , the maximum spanwise wavenumber considered here ($\beta^* = 200 \text{ m}^{-1}$) corresponds to $\psi^* \approx 62^\circ$. Both FFS (case *FFS_H12_r*) and BFS (case *BFS_H12_r*) have been considered

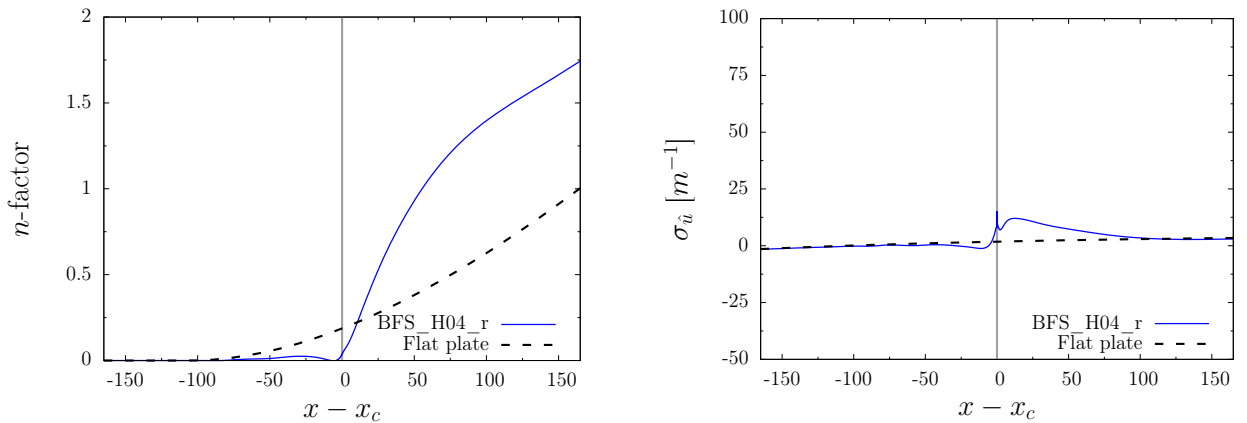


Figure 7.12: n -factor (*left*) and growth rate $\sigma_{\hat{u}}$ (*right*) evolution for an incoming TS wave of frequency $F = 25$ in the presence of a rectangular BFS (case *BFS_H04_r*). The grey vertical line represents the position of the step. Results for a flat plate (black dashed lines) are included as reference.

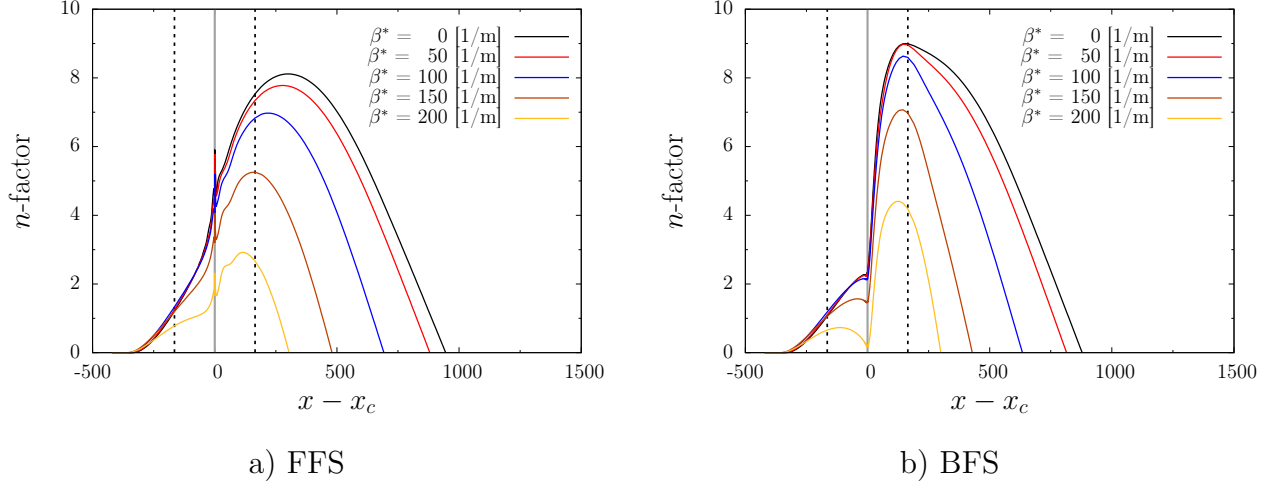


Figure 7.13: n -factor curves for TS waves of frequency $F = 48$ and several spanwise wavenumbers β^* . Steps *FFS_H12_r* (left) and *BFS_H12_r* (right) are considered. Vertical dashed lines indicate the location of the interfaces between PSE and AHLNS methodologies. The vertical grey solid line represents the position of the rectangular step.

for this study. The results of this investigation are shown in Fig. 7.13. For both steps the highest amplification is produced by the two-dimensional wave (black solid line). Based on these results, and for the purpose of the investigations (2D base flow with no external pressure gradient), it is assumed that the 2D TS disturbances are the most relevant in terms of n -factor curves.

Therefore, the working hypothesis is confirmed and, for the purpose of the investigations, the 2D TS disturbances are the most relevant in terms of n -factor curves.

7.5 Prediction of transition location

7.5.1 e^N methodology

The results of the linear instability analysis presented in previous section can be related with a predicted transition location x_{tr} through the e^N methodology (see Sect. 3.4). The semi-empirical e^N methodology assumes that the amplification ratio A/A_0 remains constant at the transition point x_{tr} . In other words, the expected transition location x_{tr} occurs at the closest point to the leading edge where the amplification curves reach a certain threshold value N_{tr} . Starting upstream of the step location, we follow the N -factor envelope curve for each step downstream until it reaches this threshold value. Without having any further information, it is very common to set the transition N -factor to $N_{tr} = 9$.

In spite of the shortcomings of the e^N methodology (see Arnal [27]) for predicting laminar-turbulent transition, it still can illustrate some interesting features of the different cases considered in the present work. The resulting expected transition location x_{tr} for each case

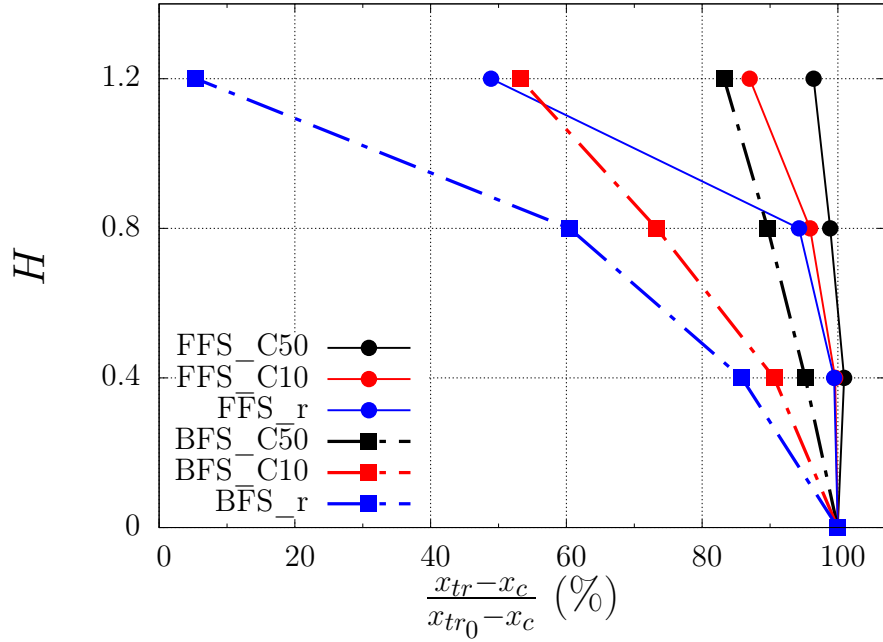


Figure 7.14: Expected transition location x_{tr} , referred to the predicted transition position for a flat plate x_{tr0} , for all steps considered in the present chapter. The e^N methodology, with a transition N -factor of $N_{tr} = 9$, is applied.

is summarized in Fig. 7.14. The abscissa $(x_{tr}-x_c)/(x_{tr0}-x_c)$ expresses the *relative* change in expected transition location x_{tr} with respect to the step position x_c as a percentage of the transition location without any surface irregularity x_{tr0} . In this manner, an abscissa of 0 % means that the transition from laminar to turbulent is expected to occur at the step location. On the other hand, an abscissa of 100 % indicates that the respective step has no effect on transition location, and transition is likely to take place at the same x -location as the flat plate.

7.5.2 Analysis of the expected transition location x_{tr}

A first conclusion that it can be extracted from Fig. 7.14 is that, for a given step with height H and shape C , the corresponding BFS (dashed lines) promotes the expected transition location more upstream than its equivalent FFS (solid lines). On the other hand, for most of the FFS studied here, the transition location x_{tr} is barely influenced by the presence of the step. Only the rectangular step with the highest height studied (case FFS_{H12_r}) leads to a significant upstream movement of the expected laminar-turbulent transition.

For BFS (Fig. 7.14 - dashed lines), it is interesting to notice that, given a particular shape of the step (i.e. a fixed value of the parameter C), the variation of transition location x_{tr} with step height H is nearly linear. However, this outcome does not hold for the rectangular case with the highest height: BFS_{H12_r} . For this particular case, the e^N methodology predicts a transition location very close to the step position. The reason must be sought in the

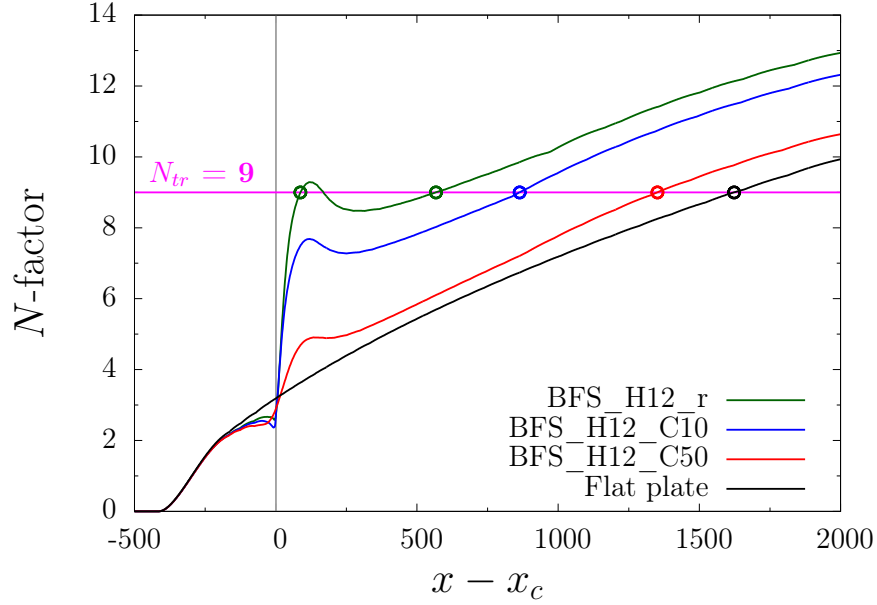


Figure 7.15: N -factor envelope curves for BFS with height $H = 1.2$. Circles indicate the streamwise location where the N -factor curves reach the threshold value $N_{tr} = 9$. The vertical grey line represents the position of the step. The N -factor envelope curve for a flat plate (black line) is included as reference.

N -factor envelope curves. Figure 7.15, which is based on Fig. 7.8-e) but adding the threshold transition N -factor $N_{tr} = 9$, indicates the expected transition location for each step x_{tr} based on the e^N methodology. This transition location is represented in Fig. 7.15 by circles. For the case *BFS_H12_r* (Fig. 7.15, green line), the N -factor curve shows a peak in the vicinity downstream of the step that reaches the threshold value $N_{tr} = 9$. The more upstream green circle, placed at $x - x_c = 90$, is given by the n -factor of a TS mode of frequency $F = 58$. Downstream of this peak, the N -factor curve diminishes and grows again, reaching the value of $N_{tr} = 9$ once again at a different downstream location. This second green circle, placed at $x - x_c = 567$, is defined by a TS wave of frequency $F = 35$. For clarification, Fig. 7.16 depicts the N -factor envelope curve for this particular case (*BFS_H12_r*) and the two individual n -factor curves for both TS waves.

This is an example of the limitations of the e^N methodology. Only the amplification ratio is considered (via N_{tr}). The information about the frequency range of the disturbances that triggers the transition process is, in principle, omitted. This information cannot be provided by the linear stability analysis itself but, *e.g.* by comparison with experimental data.

Moreover, it is important to recall that the whole transition prediction discussed in this section has been based on a transition N -factor $N_{tr} = 9$. Although the main idea in this section was to illustrate a practical example of the e^N methodology, a sensitivity analysis varying the value of N_{tr} would provide a better perspective of the stability analysis done in this chapter.

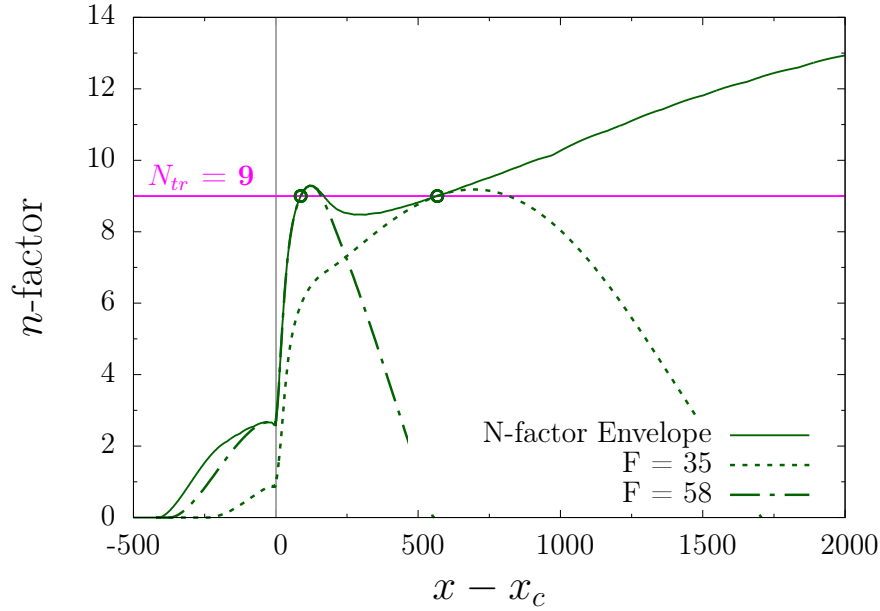


Figure 7.16: N -factor envelope curve for the rectangular BFS with height $H = 1.2$ (case *BFS_H12_r*). Circles indicate the streamwise location where the N -factor curve reaches the threshold value $N_{tr} = 9$. These circles correspond to the individual n -factor curves for frequencies $F = 35$ (dashed green line) and $F = 58$ (point-dashed green line). The solid grey vertical line represents the position of the step.

7.5.3 Sensitivity analysis for N_{tr}

The expected laminar-turbulent transition location for all steps considered in this chapter has been determined assuming that the threshold value of the N -factor is $N_{tr} = 9$. However, no empirical data supports this assumption for the particular configuration studied here. Therefore, a plausible question might arise: how the results presented previously might change if the value of N_{tr} is (slightly) different? In order to elucidate about this question, the e^N methodology has been applied again to the stability analysis results but assuming that N_{tr} is 8 and 10. The findings of this exercise are depicted in Fig. 7.17, which resembles previously commented Fig. 7.14, but adding the results from $N_{tr} = 8$ (dashed lines) and $N_{tr} = 10$ (point-dashed lines). The outcomes have been split into FFS (Fig. 7.17-a)) and BFS (Fig. 7.17-b)).

Results for FFS are relatively unaffected by the choice of the N_{tr} (8, 9, or 10). The *relative* expected transition location $(x_{tr}-x_c)/(x_{tr0}-x_c)$ barely changes for all FFS considered (rounded and rectangular). Only the rectangular step with the highest height (case *FFS_H12_r*) shows clearly a large scattering in the expected transition location depending on the value of N_{tr} selected. On the other hand, for the smoothest FFS ($C = 50$) and $N_{tr} = 8$ (Fig. 7.17-a), black dashed line), the e^N methodology predicts a small transition delay compared with the flat-plate case. It is important to notice that changing the value of N_{tr} implies, implicitly, that the range of frequencies of TS modes that triggers the transition process is different. This *predicted transition delay* is an indication of how of the pressure distribution around the step

can alter in such a way the spatial evolution of the TS waves, leading to even a small delay in the growth of the disturbances (compared with the case with no surface imperfections).

For BFS (Fig. 7.17-b)), the scenario is very similar as already described for FFS. The *relative* expected transition location is relatively insensible to the value of the N_{tr} (8, 9, or 10) for all steps, either rounded or rectangular. Again, only the rectangular step with the highest height (case *BFS_H12_r*) presents a large variation in the expected transition location x_{tr} if N_{tr} is 8 or 9 (x_{tr} is very close to the step location x_c) or N_{tr} is 10 (x_{tr} is located almost in the middle between x_c and the expected transition location for the flat plate x_{tr0}).

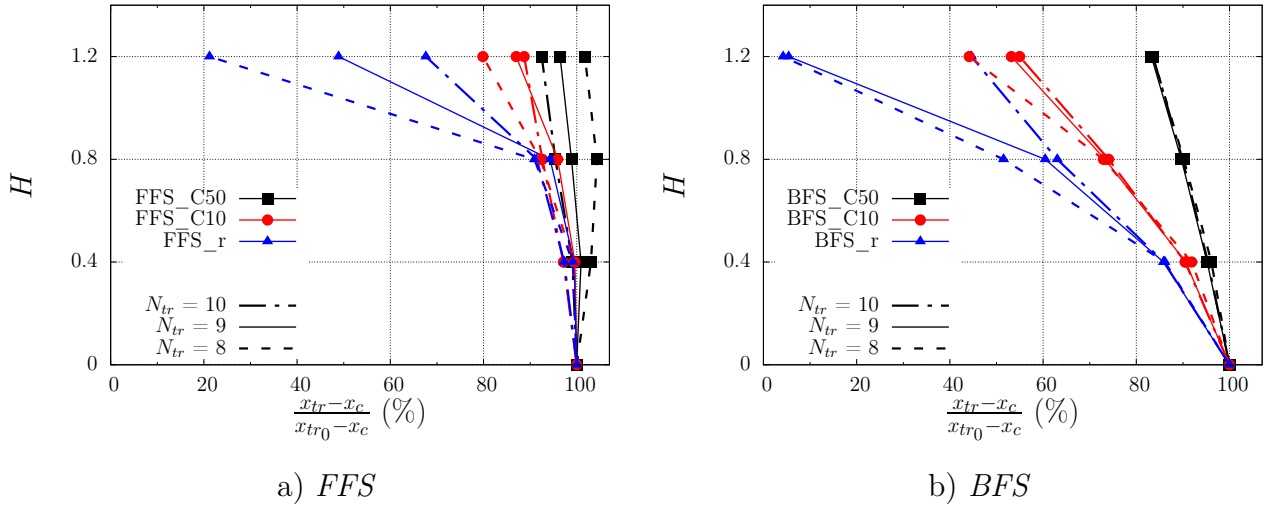


Figure 7.17: Expected transition location x_{tr} , referred to the expected transition location for a flat plate x_{tr0} , for all steps considered in the present chapter. The e^N methodology is applied. Dashed, solid, and point-dashed lines correspond to transition N -factors $N_{tr} = 8$, 9, and 10, respectively.

Chapter 8

AHLNS for hump-like geometries

8.1 Introduction

The influence of surface imperfections on the laminar-turbulence transition phenomena has been studied in depth for different types of irregularities, not only for steps. In the particular case of humps, a detailed description of how an incoming TS wave is affected by the presence of a two-dimensional hump on a flat plate was done experimentally by Boiko *et al.* [76].

Later, Dovgal and Kozlov [77] extended those studies by changing the shape of the hump and the frequency of the incoming waves. They were able to reduce by a factor of four the maximum amplitude reached by a single TS wave by placing a ramp downstream next to the original rectangular hump. Some years later, Danabasoglu *et al.* [78] reproduced the initial findings of Boiko *et al.* [76] by using Direct Numerical Simulations (DNS). They considered also rectangular humps of different heights. They suggested that disturbances growing in the separation zone have an inviscid character and are controlled by the instability of the shear layer at the edge of the separation zone.

Similarly, Liu and Liu [79] used also DNS in order to reproduce the findings of Dovgal and Kozlov [77]. However, in the work of Liu and Liu [79] the shape of the roughness was defined by a hyperbolic cosine function rather than a rectangular hump. They stated that the differences in the shape of the hump were the reason of the different growth of the instabilities when comparing their DNS results with the experimental results from Dovgal and Kozlov [77]. Instead, Masad and Iyer [80] used Linear Stability Theory (LST) for reproducing the results of Dovgal and Kozlov [77]. Masad and Iyer [80] extended their studies by considering also compressibility effects. They investigated the effects of height, length, location, and shape of the hump. However, they examined humps with rounded corners only. The transition location was defined by applying the e^N transition criterion: the predicted transition location takes places at the position where the N -factor reaches the threshold value of $N_{tr} = 9$.

Some years later, Wörner *et al.* [81] applied DNS to study the effect of height and length on rectangular humps on a single TS wave in incompressible flow. They included results for a smooth hump in their paper. Among the geometrical parameters considered (height,

length and shape), they identified the height of the hump as the geometrical parameter that has more influence on the amplification of the incoming TS wave. However, because of the computational requirements of the DNS simulations, their studies were limited to very few geometries (six in total) and one particular TS mode.

Later, Perraud *et al.* [82] developed a mathematical model for predicting the increment in N -factor due to the presence of a small bump based on numerical and experimental results. Their mathematical model considered smooth bumps of different shapes. However, they noticed that the model is restricted to cases where the height of the bumps is small enough that laminar separation does not occur. Similarly, Costantini *et al.* [83] have performed experimental investigations for smooth bumps, but considering also cases in which the height of the bump promotes laminar separation.

Alternatively to LST and DNS, Parabolized Stability Equations (PSE) have been successfully applied to the study of growth of amplitude for convective instabilities in boundary-layer flows and, based on the e^N methodology, to provide a reasonable estimate of the transition location. In the work of Thomas *et al.* [84], they successfully applied PSE for studying the effect of small indentations, where the depth of the indentations was limited to 0.2 times the boundary-layer thickness. Similarly, Park and Park [85] studied the effect of humps on incompressible flows using PSE. They were able to reproduce the DNS results of Wörner *et al.* [81], but only for the smooth hump case. More recently, Franco *et al.* [86] studied the influence of rounded humps and indentations using the AHLNS methodology. The work of Tocci *et al.* [62] describes the spatial evolution of a particular TS wave in the presence of surface irregularities when the height of the irregularity is similar to the local boundary-layer displacement thickness and the shape of the surface irregularity contains sharp corners. In this work [62], linear DNS, AHLNS, PSE and LST were used. The streamwise gradients introduced by the surface irregularities violates the weakly-nonparallel assumption of the PSE approach, and therefore the PSE method is not suited for cases with sharp-edged surface imperfections. LST over-predicts the growth of the TS wave for all cases, including cases with rounded edges. On the other hand, an excellent agreement between linear DNS and AHLNS, in terms of n -factor curve, was found.

In this chapter, we are taking the same approach as described in previous Chapt. 7. In fact, the results present here extend the investigations done before by Franco *et al.* [87]. Again, due to the relatively low computational cost of AHLNS (each n -factor curve is computed in about 5 min in a personal workstation), several parametric studies (*e.g.* shape, height, length, etc.) can be easily performed. Once the N -factor envelope curve for each hump is available, the e^N methodology provides an expected location for the laminar-turbulent transition x_{tr} . The variation of the expected transition location is used to quantify the influence of each parameter mentioned before. The limits of this approach were already mentioned in previous Chapt. 7 for steps. On the other hand, our overall goal is not to focus on the study of particular parameter, but to describe the AHLNS methodology applied for a specific case study. We expect that designers and researches can benefit from our methodology in order to enrich the knowledge of the influence of surface imperfections on the laminar-transition process.

8.2 Problem description

The numerical set-up and inflow parameters considered here for humps resemble the configuration described already in Sect. 7.2 for steps. Therefore, readers will be directed to Chapt. 7 for ease in several occasions in order to avoid repeated (and therefore, unnecessary) explanations.

We study numerically the effect of a single surface roughness on boundary-layer transition of a two-dimensional laminar flow on a flat plate. The shape of the surface roughness is defined by the expression

$$y^* = H^* \exp \left(- \left(\frac{x^* - x_c^*}{L^*/2} \right)^{2m} \right), \quad \text{with } m = 1, 2, 3, 4, \quad (8.1)$$

where the parameter H^* indicates the height of the hump and parameter L^* controls its length. For ease, parameter L^* will be referred also as *length* of the hump, even for smooth humps. The centre of the hump is placed at x_c^* . Figure 8.1 shows schematically the computational domain used in the present study, including the type of boundary conditions imposed for computing the laminar base flow. The geometrical parameter m is varied from 1 to 4, where the case $m = 1$ corresponds to a *Gaussian-shaped* roughness. The rectangular shape can be considered the limiting case when $m \rightarrow \infty$ (also examined in the present study). The rectangular hump case is defined by the piecewise expression

$$y^* = \begin{cases} 0, & \text{if } x^* < x_c^* - \frac{L^*}{2}, \\ H^*, & \text{if } x_c^* - \frac{L^*}{2} \leq x^* \leq x_c^* + \frac{L^*}{2}, \\ 0, & \text{if } x^* > x_c^* + \frac{L^*}{2}. \end{cases} \quad (8.2)$$

Geometrical quantities are nondimensionalized with δ_c^* (see Sect. 7.2). The Reynolds number based on δ_c^* is $Re_{\delta^*} = 1823$. Figure 8.2 depicts a sketch of the wall shape for several values of the geometrical parameter m in the case of humps with values of H and L fixed to 1.2 and 100, respectively. For the nondimensional height H , three values are examined here: 0.4, 0.8, and 1.2. These values were already used in Chapt. 7. Therefore, names *small*, *medium*, and *large* refer to values of the height $H = 0.4$, 0.8, and 1.2, respectively. The nondimensional parameter L is fixed to 100 for *small* and *medium* roughnesses. For *large* roughnesses, three different values of L are considered: 50, 100 and 150. The combination of parameters m , H , and L gives a total of 25 different humps, which are summarized in Table 8.1. In this table, the maximum slope θ_{max} of the wall is also included. Although the cases of smooth humps ($m = 1, 2, 3, 4$) present a very small value of θ_{max} , it will be shown that their influence on the expected laminar-turbulent transition location is similar to their corresponding rectangular humps for certain cases.

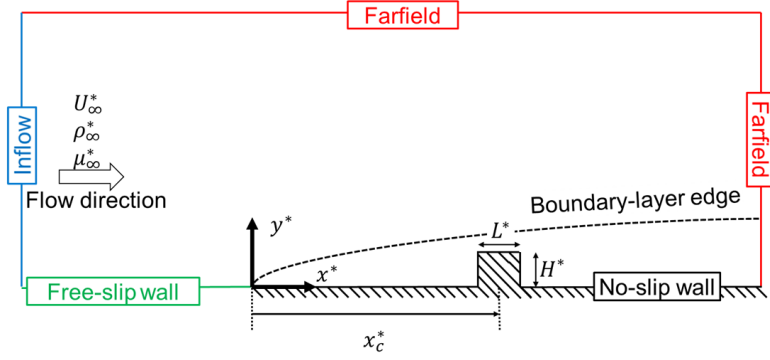


Figure 8.1: Schematic representation of the computational domain defined for the present study. The labels at the edges of the domain indicate the type of boundary conditions used for the base flow computations.

8.3 Base flow computations

8.3.1 Approach

The laminar steady two-dimensional flow on a flat plate in the presence of a hump is computed numerically. Grid generator (MEGACADS [60]), Navier-Stokes solver (TAU [61]) and boundary conditions (Fig. 8.1) were described already in Sect. 7.3.1. Therefore, details about how to compute the laminar steady base flows for each test case are omitted here.

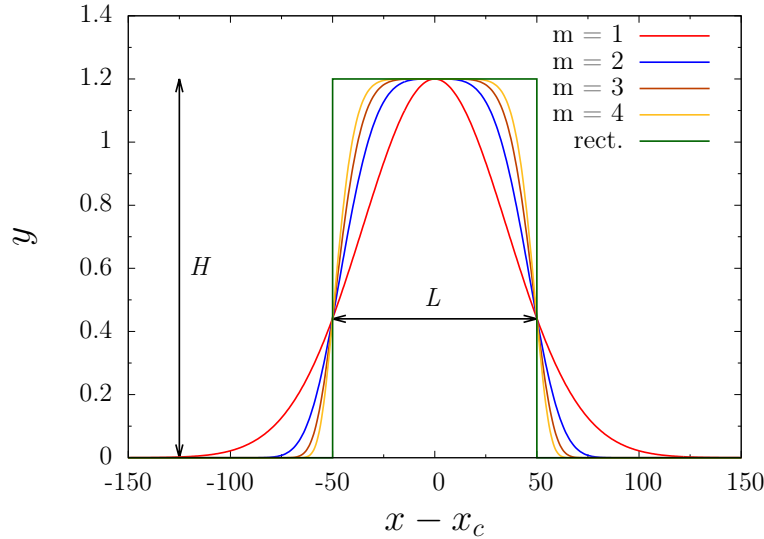


Figure 8.2: Sketch of the wall shape for several values of the geometrical parameter m in the case of humps with values of H and L fixed to 1.2 and 100, respectively. Axes are not to scale. The rectangular case can be considered the limiting case when $m \rightarrow \infty$.

Case name	H	L	m	θ_{max} ($^{\circ}$)
H04_m1	0.4	100	1	0.39
H04_m2	0.4	100	2	0.70
H04_m3	0.4	100	3	1.02
H04_m4	0.4	100	4	1.36
H04_r	0.4	100	r	90
H08_m1	0.8	100	1	0.79
H08_m2	0.8	100	2	1.40
H08_m3	0.8	100	3	2.04
H08_m4	0.8	100	4	2.71
H08_r	0.8	100	r	90
H12_m1	1.2	100	1	1.18
H12_m2	1.2	100	2	2.08
H12_m3	1.2	100	3	3.07
H12_m4	1.2	100	4	4.07
H12_r	1.2	100	r	90
H12_L50_m1	1.2	50	1	2.36
H12_L50_m2	1.2	50	2	4.15
H12_L50_m3	1.2	50	3	6.12
H12_L50_m4	1.2	50	4	8.06
H12_L50_r	1.2	50	r	90
H12_L150_m1	1.2	150	1	0.79
H12_L150_m2	1.2	150	2	1.40
H12_L150_m3	1.2	150	3	2.05
H12_L150_m4	1.2	150	4	2.72
H12_L150_r	1.2	150	r	90

Table 8.1: Values of nondimensional parameters H , L , and m considered in the present study. The case $m = r$ refers to the rectangular humps. The last column indicates the maximum slope θ_{max} of the wall.

8.3.2 Grid convergence

The grid independence of the base flow results for humps is proved in a very similar procedure as it was done for steps (see Sect. 7.3.2). The grid-independence analysis helps us to briefly introduce some base flow characteristics of this particular type of surface imperfections. Two sets of grids (*mesh 1* and *mesh 2*) were defined. *Mesh 2* is similar to *mesh 1*, but increasing significantly the number of points around the hump. The total number of points in streamwise and normal directions (n_x and n_y , respectively) are detailed in Table 8.2.

Number of grid points	<i>mesh 1</i>	<i>mesh 2</i>
n_x	1100	1500
n_y	225	300

Table 8.2: Number of points in streamwise and normal directions (n_x and n_y , respectively) for grid convergence verification of the steady laminar base flow results.

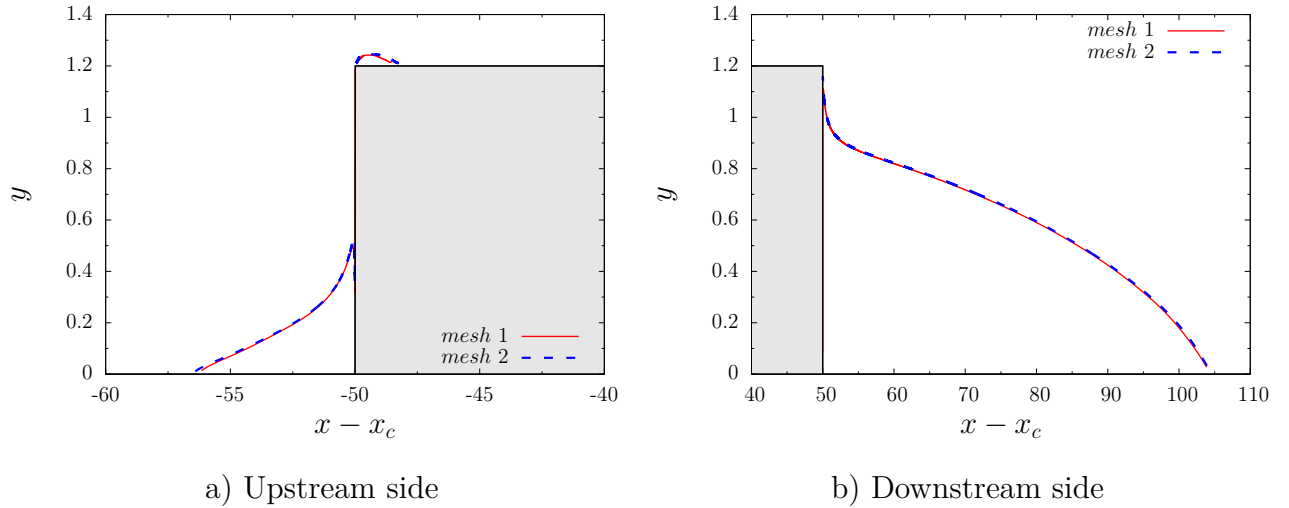


Figure 8.3: Base flow grid convergence study for case *H12_r*. Both the continuous red line (*mesh 1*) and the dashed blue line (*mesh 2*) represent the points in the base flow where $\bar{u} = 0$. The grey area represents the rectangular hump. Flow direction is from left to right. Axes are not to scale.

Figure 8.3 shows the results of the grid convergence study for the case *H12_r*. The dividing line $\bar{u} = 0$ limits the region where the reverse flow ($\bar{u} < 0$) is present. For this case, three recirculation areas appear: one in front of the hump, one on top of the hump (both represented in Fig. 8.3-a)), and a large recirculation area downstream of the hump (shown in Fig. 8.3-b)). A perfect agreement between the base flow obtained using both grids is achieved. Interestingly, these two plots resemble the results presented for both FFS and BFS in previous Chapt. 7 (Fig. 7.2). This fact will be discussed in more detail later in Sect. 8.4.6.

For rounded humps, the case *H12_m4* is chosen for the grid convergence verification. Figure 8.4 shows the skin-friction coefficient c_f and nondimensional displacement thickness δ distribution. No difference is found for these quantities when the laminar steady base flow is computed with *mesh 1* or *mesh 2*. Therefore, as a result of the grid convergence study, the base flow computed with meshes of the set *mesh 1* will be used for stability analysis.

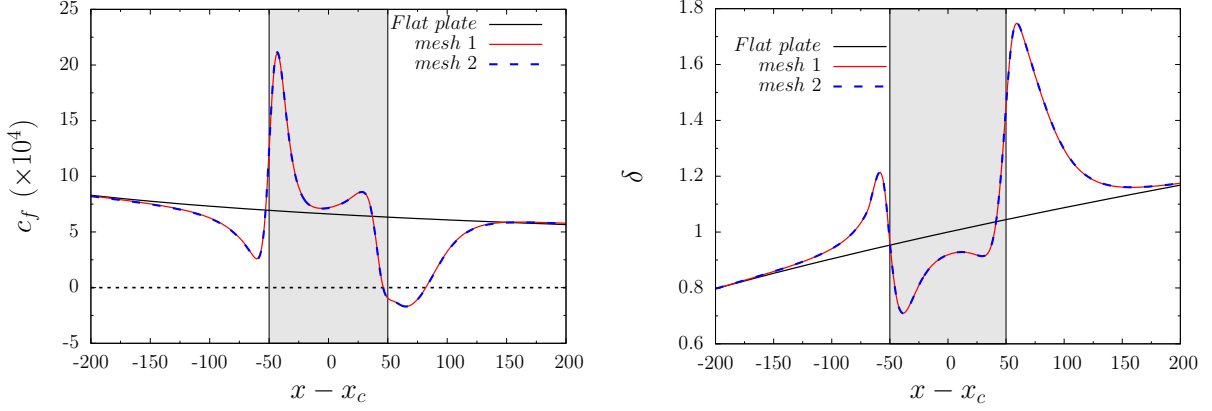


Figure 8.4: Base flow grid convergence study for case $H12_m4$, based on skin-friction coefficient c_f (left) and nondimensional displacement thickness δ (right). The grey area indicates the position of the hump. Results for the flat-plate case (black solid line) have been added for comparison.

8.3.3 Base flow results

The base flow pressure $\bar{p}^*$ is made nondimensional $\bar{p}$ by twice the incoming dynamic pressure, i.e. $\rho_\infty^*(U_\infty^*)^2$. Figure 8.5 compares the contours of $\bar{p}$ and streamlines for several cases. For *small* ($H = 0.4$) roughnesses (Fig. 8.5-a), -c), -e)) the flow field exhibits little variations in the three cases considered. Only in the rectangular case $H04_r$ (Fig. 8.5-e) there is a tiny separation bubble at the downstream side of the hump (plus an almost imperceptible recirculation area in front of the hump). The streamwise pressure gradients, especially at the leading edge of the humps, are more intense as the humps tend to the rectangular shape (i.e. the value of m increases). For *large* ($H = 1.2$) roughnesses (Fig. 8.5-b), -d), -f)), variations in the flow field are more relevant. The flow in case $H12_m1$ (Fig. 8.5-b)) remains fully attached, although the pressure gradients are more pronounced than in case $H04_m1$. For case $H12_m4$ (Fig. 8.5-d)) a separation bubble appears at the trailing edge of the hump. The length of this bubble (in x -direction) is approximately one quarter of the length L of the hump. The last case examined here, namely $H12_r$ (Fig. 8.5-f)) exhibits a large separation bubble downstream of the hump (the length is approximately one half of the parameter length L in streamwise direction). Similar to the steps studied in previous Chapt. 7 (FFS and BFS), changes in the base flow provoked by the presence of the roughness are limited to the vicinity of the hump. At a certain distance upstream and downstream from the hump, the flow field recovers the key characteristics of the undisturbed flow field (i.e. small variations of flow quantities in streamwise direction).

8.4 Instability analysis

The methodology used for the instability analysis here was described already in Sect. 7.4. The individual n -factor curve for each TS mode (defined for a particular frequency F and spanwise

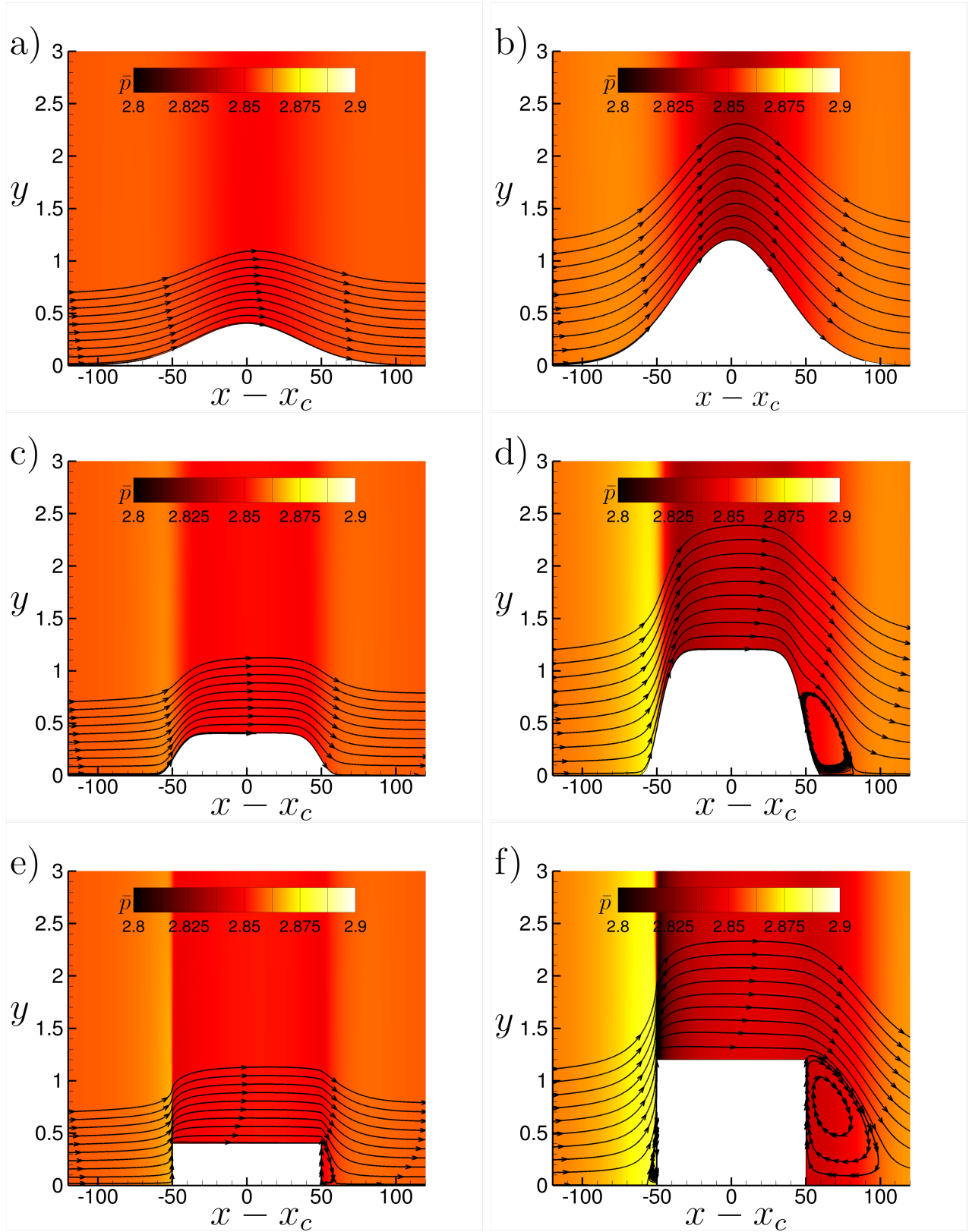


Figure 8.5: Steady laminar base flow pressure $\bar{p}$ contours and streamlines for cases (a) $H04_m1$; (b) $H12_m1$; (c) $H04_m4$; (d) $H12_m4$; (e) $H04_r$; (f) $H12_r$. Flow direction is from left to right. Axes are not to scale.

wavenumber β) is computed using the multizonal approach (PSE+AHLNS+PSE) explained in Sect. 5.2.3. The interfaces between PSE and AHLNS are placed at $(x - x_c) = \pm 165$ also, sufficiently far away from the recirculation regions and large pressure gradients (see Fig. 8.4 and 8.5). Initially, 2D TS waves ($\beta = 0$) will be computed only. The influence of 3D TS waves ($\beta \neq 0$) is discussed in Sect. 8.4.5.

8.4.1 Grid convergence

The grid independence of the instability analysis is done computing the corresponding n -factor curve for a single TS wave of frequency $F = 48$. The number of grid points depends on whether the hump considered is rounded or rectangular, as it is shown in Table 8.3.

The presence of sharp corners increases significantly the number of points required for a correct representation of the TS waves in the vicinity of the hump. This is the reason for choosing two humps, one rectangular and one rounded (*H08_r* and *H08_m4*, respectively) to prove that the instability results presented here are grid-independent. For rounded humps, the *Reference grid* is made by 100 points in streamwise direction and 100 points in normal direction. However, for cases where sharp corners are present, the *Reference grid* is made of 500×300 points in streamwise and normal direction, respectively. Moreover, we will refer here only to the number of points used for the AHLNS computations (i.e. in the domain $(x - x_c) \in [-165, 165]$). For each hump, the number of grid points for the stability analysis is increased by 40% in each direction independently with respect to the corresponding *Reference grid*. Figure 8.6 reports the results of the grid independence investigation. There is an excellent agreement between the results for *Reference grid* and the different refinements considered. Therefore, *Reference grids* for both rectangular and rounded humps (see Table 8.3) are used in this work.

8.4.2 Effect of hump height and shape

The results of the stability analysis will be presented here by grouping the different roughnesses according to their height (*small*, *medium* and *large*). For ease, humps with the same value for the parameter length $L = 100$ are discussed here.

<i>shape</i>	n_x	n_y
Rounded humps	100	100
Rectangular humps	500	300

Table 8.3: Number of points n_x and n_y (streamwise and normal directions, respectively) of the *Reference grid* for mesh-convergence verification using AHLNS.

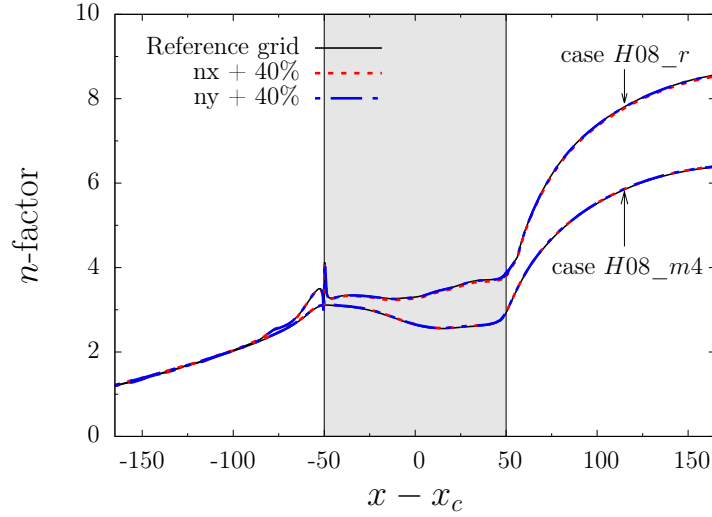


Figure 8.6: Stability analysis grid convergence study for cases *H08_r* and *H08_m4*, based on the n -factor curve for a 2D TS wave of frequency $F = 48$. Solid black curves represent AHLNS computations using the *Reference grid*. The results marked by red dashed and blue dash-dotted lines were obtained with AHLNS also, increasing the number of points in streamwise and normal direction by 40%, respectively. The grey area indicates the position and length of the hump.

8.4.2.1 Small humps

Figure 8.7-a) compares the N -factor envelope curves when *small* humps ($H = 0.4$) are considered. The N -factor curves are similar for all five humps ($m = 1, 2, 3, 4, r$) analysed here. In the vicinity of the humps, the development of the TS waves is driven by the pressure gradients introduced into the flow field by the presence of the hump. Figure 8.7-b) shows the nondimensional pressure distribution at the wall $\bar{p}_{wall}$ for the five corresponding humps. The flat-plate case is also included for comparison. In all cases, in the leading edge region of the hump ($x - x_c \approx -50$), there is an initial smooth adverse pressure gradient of the flow followed by a rapid favourable pressure gradient. This phenomenon resembles the description made for FFS in terms of pressure distribution (see previous Chapt. 7. This is the reason that the N -factor curves in the upstream side of the humps compare quite well with the N -factor evolution of the FFS (i.e. small increase in the N -factor curves followed by a rapid reduction).

On the downstream side of the hump ($x - x_c \approx 50$), the situation is the opposite: a rapid adverse pressure gradient followed by a smooth favourable pressure gradient. This pressure distribution is very similar to the base flow description made for BFS (see previous Chapt. 7. As expected, variations in the N -factor curves here ($x - x_c \approx 50$) resemble the evolution of the amplitude curves for BFS: a significant local increase in N -factor due to the separation bubble.

The effect of reducing the smoothness of the hump (increasing m) provokes that the pressure variations concentrate at the borders of the hump. Downstream of the hump, where the zero pressure gradient is recovered, the N -factor curves grow almost parallel to the flat-plate case.

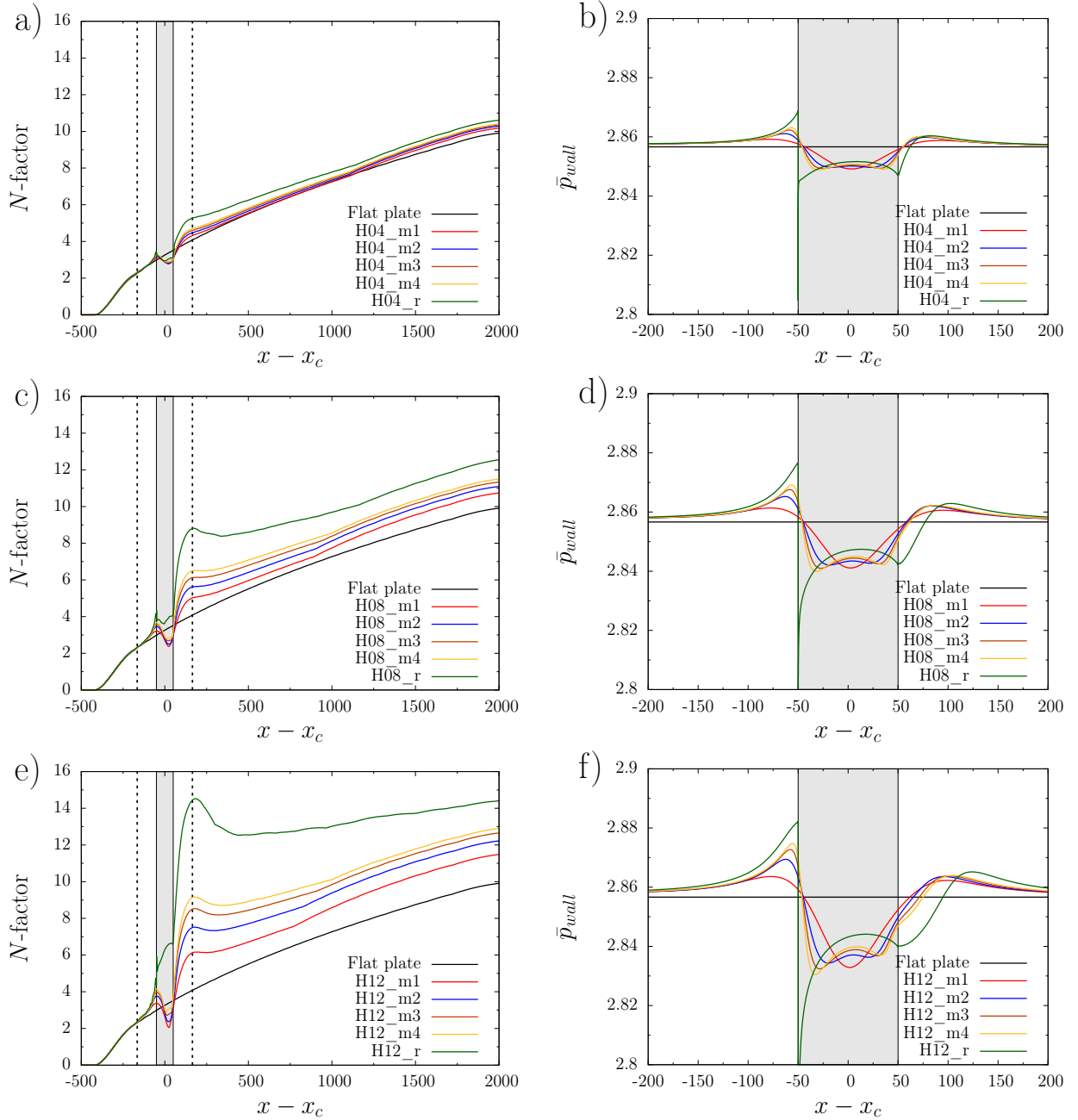


Figure 8.7: (Left) Effect of shape parameter m on N -factor envelope curves for hump cases (a) *small*; (c) *medium*; (e) *large*. (Right) Effect of shape parameter m on pressure distribution at the wall $\bar{p}_{wall}$ curves for hump cases (b) *small*; (d) *medium*; (f) *large*. Vertical dashed lines indicate the location of the interfaces between PSE and AHLNS methodologies. The grey area indicates the position and length ($L = 100$) of the hump. The black curves for a flat plate case are included as reference.

8.4.2.2 Medium humps

Figure 8.7-c) shows the N -factor envelope curves for *medium* humps ($H = 0.8$). Similar to the *small* humps, beyond a certain distance downstream of the hump the N -factor curves grow parallel to the N -factor of the flat-plate case. The pressure gradients around the hump are notably higher (see Fig. 8.7-d)) compared with *small* humps. Once the pressure distribution reaches again the corresponding constant value of the flat-plate case ($x - x_c \geq 200$), the N -factor curves grow parallel to the flat-plate case. It is interesting to notice that, while for *small* humps there were no significant differences between the rounded humps and the rectangular one, here the situation is different. The rectangular *medium* hump produces an N -factor envelope curve significantly higher than the corresponding N -factor curves from rounded humps.

8.4.2.3 Large humps

As a reminder, only humps with length $L = 100$ are discussed here. Variations of the parameter L are presented in the following section. Figure 8.7-e) plots the amplification curves for the five humps considered here. The pressure gradients (Fig. 8.7-f)) follow a similar behaviour as it was already described for previous cases (*small* / *medium* humps). Readers can observe (Fig. 8.7-f)) that differences between the rectangular hump (case $H12_r$) and rounded humps are especially noticeable in the hump region. The smooth humps mitigate the intensity of the pressure gradients around the hump and therefore, they reduce the intensity of the rapid variations of the N -factor curve present in the rectangular-shaped case. The overall effect of the rounded humps is a significant reduction in terms of N -factor compared with the rectangular case. In principle, the N -factor curve for the rectangular hump could be retrieved by a rounded hump, but the corresponding value of the shape factor m should be much larger than the values considered in this work.

8.4.3 Effect of hump length

In this section, only *large* humps ($H = 1.2$) are considered. Figure 8.8-a) shows the N -factor curves when the shape parameter is fixed to $m = 1$. Clearly, the length parameter L has a strong influence on the N -factor curves and the pressure distribution $\bar{p}_{wall}$ (see Fig. 8.8-b)). Reducing the length L of the hump provokes that the pressure gradients around the rounded humps are significantly increased, resulting in an overall increase in the N -factor curves for the rounded humps.

If the shape parameter is fixed to $m = 4$, the influence of the length parameter L in terms of N -factor curves (Fig. 8.8-c)) and pressure distribution $\bar{p}_{wall}$ (Fig. 8.8-d)) is smaller than previous cases with $m = 1$. Finally, for rectangular humps ($m = r$), the length parameter L has no effect on the N -factor curves (Fig. 8.8-e)) for the three lengths $L = 50, 100, 150$ considered here. It is interesting to notice that the pressure gradients at the hump corners (Fig. 8.8-f)) are very similar for the three rectangular cases considered here. This result suggests that the local pressure gradients are one of the drivers (if not the main one) of the evolution

of the n -factor curves around this type of surface imperfections. However, the size of the recirculation bubble might play an important role also in other cases.

8.4.4 Effect of the hump on the TS wave structure

Similar to the discussion on Sect. 7.4.3, the spatial changes in the structure of a particular TS wave passing along a hump are discussed here. The rectangular hump $H04_r$ and the TS mode with frequency $F = 25$ have been chosen due to its moderate overall variation of the mode amplitude in the plotted domain, which facilitates a visualization by isocontours in Fig. 8.9. This figure confirms that the main structural changes in the TS wave occur in two particular areas:

- In a very small region close to the upstream corner of the hump
- At the location of the large separation bubble downstream of the hump

These changes are quantified in terms of the particular n -factor and growth rate $\sigma_{\hat{u}}$ in Fig. 8.10. At the upstream corner of the hump, i.e. at $x - x_c \approx -50$, very large variations in the growth rate take place but in a very limited extension (the order of few δ_c^*). The n -factor reflects these large variations. However, since these changes extend in a very small region, the overall n -factor value at $x - x_c \approx -50$ remains about $n \approx 0.5$.

On the other hand, downstream of the hump, i.e. at $x - x_c \approx 50$, the large separation bubble smooths out the streamwise changes in growth rate at the corner. This relatively large separation bubble increases significantly the n -factor values in that region.

8.4.5 2D vs. 3D TS waves

In Sect. 7.4.4, a detailed discussion about the amplification of a singular 2D TS wave compared with several 3D TS waves of the same frequency was presented. It was concluded there that, for both FFS and BFS, the most amplified TS wave is the 2D mode ($\beta = 0$). Here, a similar study is done but considering a rectangular hump instead. In particular, a rectangular hump with the same height (case $H08_r$) and the same individual TS mode (frequency $F = 48$) is chosen. The corresponding n -factor of the 2D TS wave ($\beta = 0$) is depicted in Fig. 8.11, together with the n -factor of several 3D TS waves of different spanwise wavenumbers. Clearly, the 2D TS mode produces the highest n -factors in the region of interest. Therefore, this result assures that, for the purpose of our investigations, the 2D TS disturbances are the most relevant in terms of n -factor curves.

8.4.6 Steps vs. Humps

We mentioned in Sect. 8.3.2 that the structure of the laminar steady base flow around the hump (case $H12_r$), in terms of the size of the separation bubbles (upstream and downstream of the hump, Fig. 8.3), resembles the flow structure found in the equivalent FFS and BFS

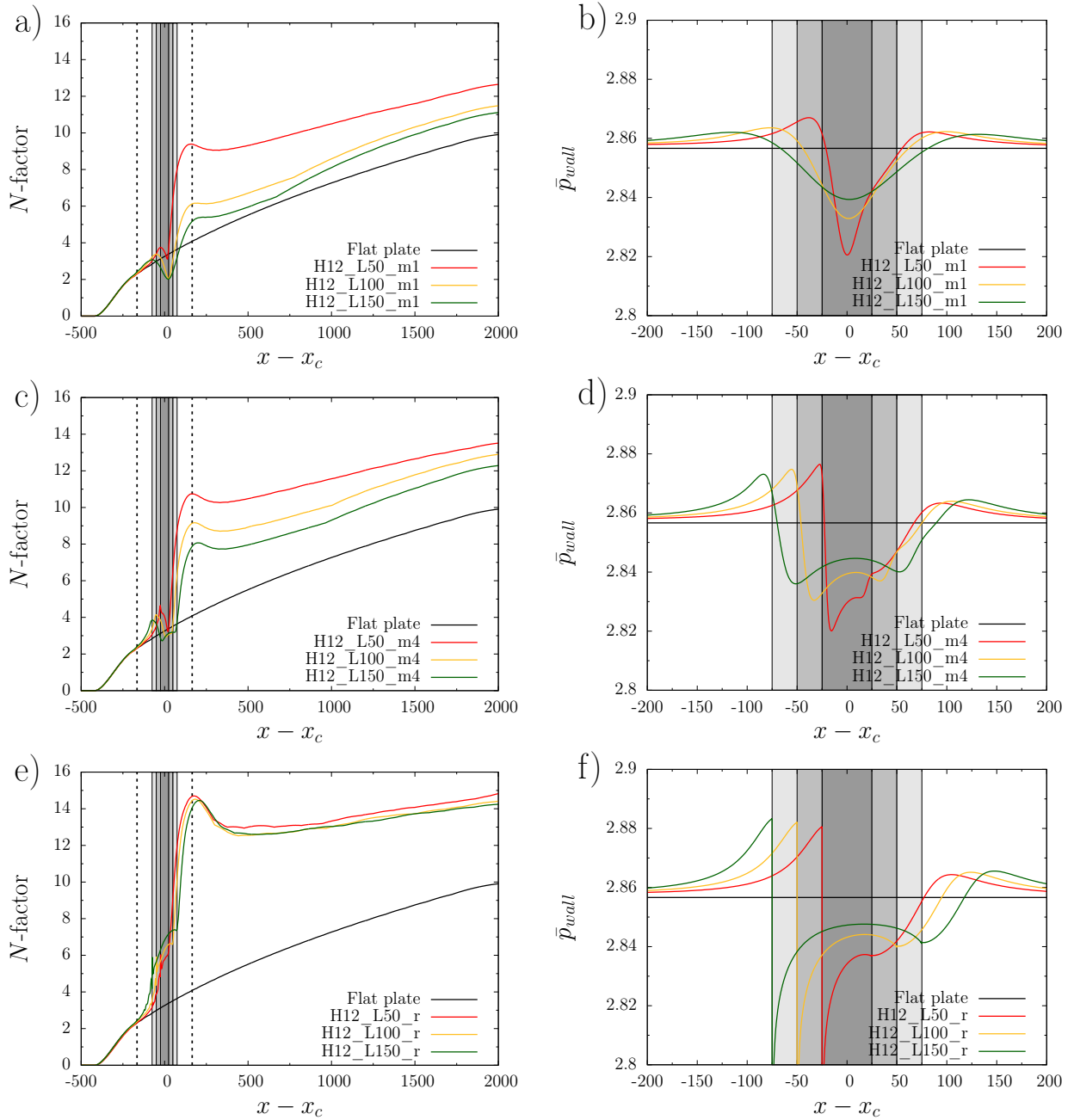


Figure 8.8: (Left) Effect of length L on N -factor envelope curves for cases (a) $H12_m1$; (c) $H12_m4$; (e) $H12_r$. (Right) Effect of length L on pressure distribution at the wall $\bar{p}_{wall}$ curves for cases (b) $H12_m1$; (d) $H12_m4$; (f) $H12_r$. Vertical dashed lines indicate the location of the interfaces between PSE and AHLNS methodologies. The grey areas indicate the position of the humps and their length ($L = 50, 100, 150$). The black curves for a flat plate case are included as reference.

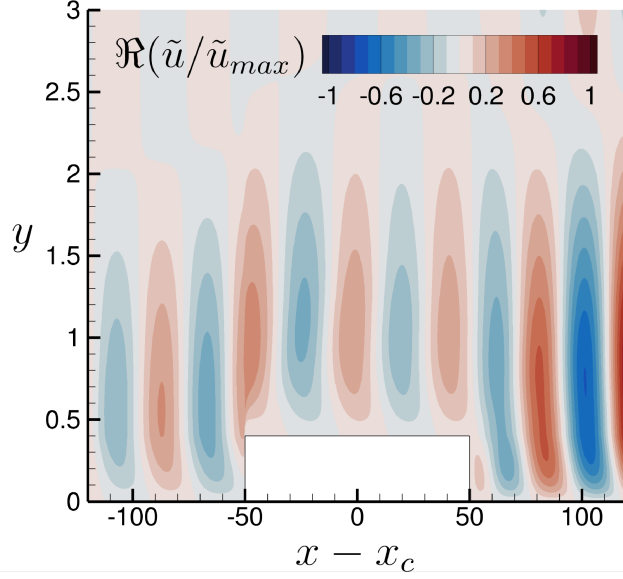


Figure 8.9: Influence of the presence of a hump (case $H04_r$) on the real part of the streamwise velocity disturbance $\Re(\tilde{u}/\tilde{u}_{max})$ for an incoming TS wave of frequency $F = 25$ at an arbitrary instant of time. Flow direction is from left to right. Axes are not to scale.

(cases FFS_H12_r and BFS_H12_r , respectively. See Fig. 7.2). One question that might arise is: would it be possible to combine the results of the stability analysis (N -factor envelope curves) of the FFS and BFS in order to reproduce the instability characteristics of their equivalent hump? Figure 8.12 shows the N -factor envelope curves for three cases: FFS_H12_r , BFS_H12_r and the rectangular hump $H12_r^\dagger$. The N -factor curve $BFS + FFS_H12_r$ (green solid line) combines the individual effect of the steps FFS_H12_r and BFS_H12_r .

[†]The three *large* rectangular humps ($L = 50, 100, 150$) have a very similar N -factor envelope curve (see Fig. 8.8-e)). Therefore, the case $L = 100$ has been chosen for ease.

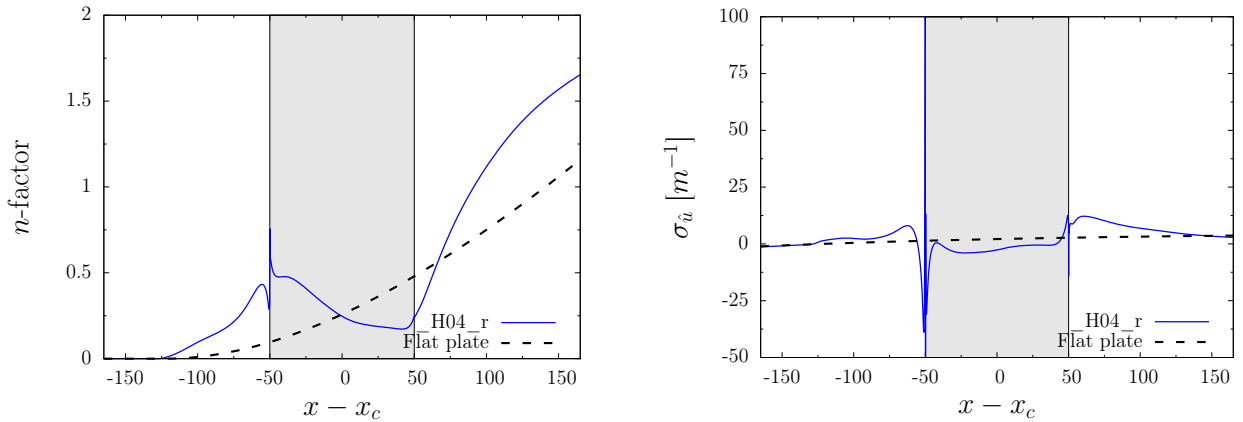


Figure 8.10: n -factor (left) and growth rate $\sigma_{\tilde{u}}$ (right) evolution for an incoming TS wave of frequency $F = 25$ in the presence of a rectangular hump (case $H04_r$). The grey area represents the position and length of the hump.

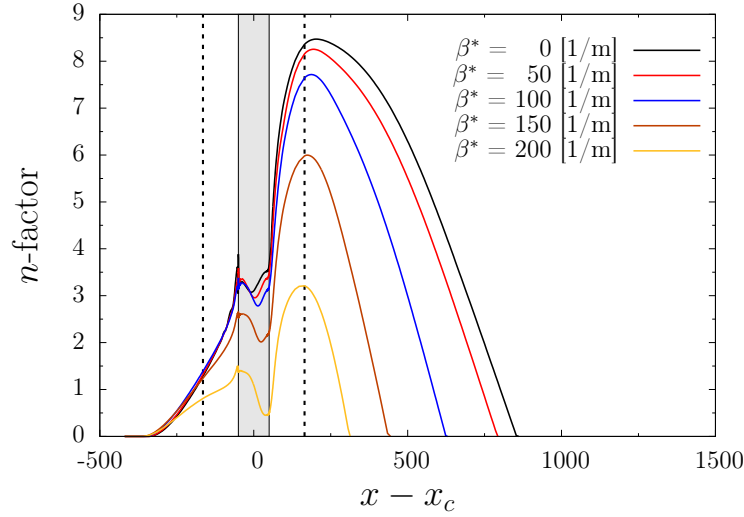


Figure 8.11: n -factor curves for TS waves of frequency $F = 48$ and several spanwise wavenumbers β^* . Case *H08_r* is considered. Vertical dashed lines indicate the location of the interfaces between PSE and AHLNS methodologies. The grey area represents the position and length of the hump.

This combination consists simply in to compute the increase in N -factor with respect to the flat-plate case of the FFS and the BFS individually (i.e. ΔN_{FFS} and ΔN_{BFS} , respectively), and add both increments to the N -factor of the flat plate. Mathematically,

$$N_{FFS+BFS} = N_{FP} + \Delta N_{FFS} + \Delta N_{BFS} \quad (8.3)$$

It is interesting to notice that, beyond certain distance downstream of the hump location, the N -factor curves for the rectangular hump and the combined BFS + FFS grow parallel, with only a small constant offset of $\Delta N \approx 0.4$ (Fig. 8.12, red and green solid lines, respectively).

A similar analysis has been done for *medium* and *small* rectangular imperfections (FFS, BFS and hump, see Fig. 8.13). Similar to *large* imperfections, there is a small constant offset in N -factor between the rectangular hump and the combination of BFS + FFS downstream of the hump location. Interestingly, this constant N -factor offset for *medium* imperfections ($\Delta N \approx 0.3$) downstream of the surface imperfection is very similar to the offset for *large* irregularities ($\Delta N \approx 0.4$). For *small* rectangular imperfections (Fig. 8.13-b)), the offset is almost zero.

As a summary, the combination of the N -factor curves of rectangular BFS and FFS can reproduce (assuming a small constant offset) the N -factor curve for the rectangular humps (with the same height) relatively far downstream of the irregularity location. This result holds for rectangular humps whose length L is significantly larger than their height H . This constant offset is practically zero for *small* imperfections, and approximately 0.3 and 0.4 for *medium* and *large* imperfections, respectively. However, right downstream of the hump

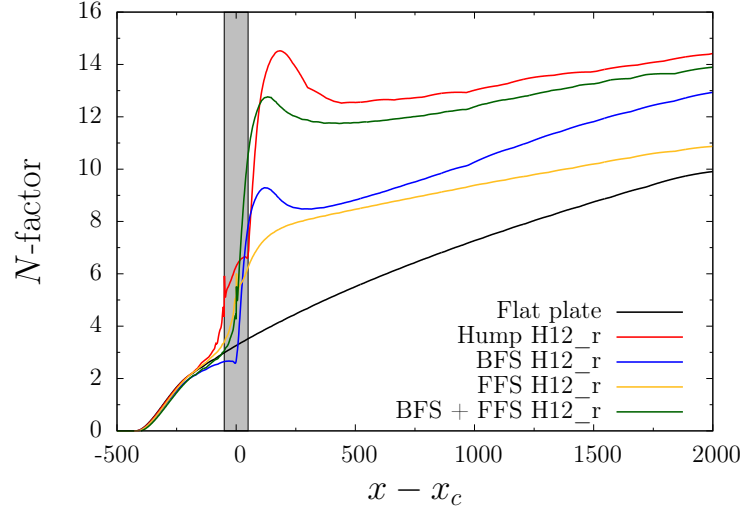
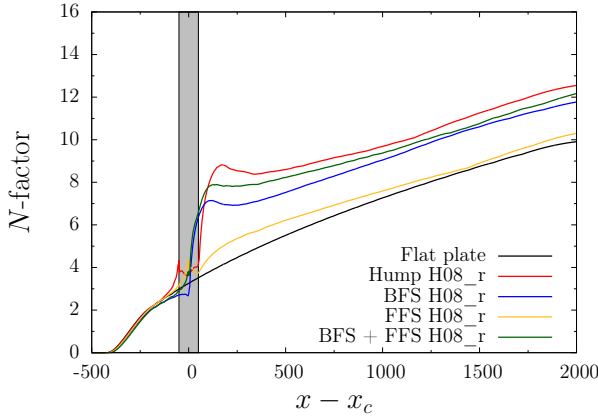
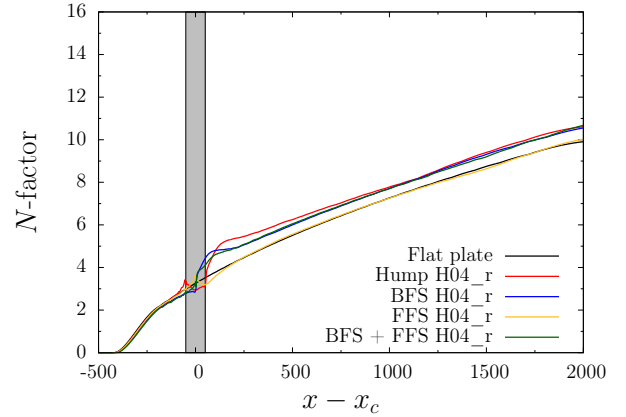


Figure 8.12: N -factor envelope curves for rectangular steps and hump of height $H = 1.2$ considered in Chapt. 7 and 8. The N -factor $BFS + FFS$ (green curve) depicts the combined effect of both individual BFS and FFS N -factors. The grey area represent the position and length of the hump ($L = 100$). The N -factor envelope curve for a flat plate (black curve) is included as reference.



a) *Medium* imperfections



b) *Small* imperfections

Figure 8.13: N -factor envelope curves for rectangular steps and hump of height $H = 0.8$ (left) and $H = 0.4$ (right) considered in Chapt. 7 and 8. The N -factor $BFS + FFS$ (green curve) depicts the combined effect of both individual BFS and FFS N -factors. The grey area represent the position and length of the hump ($L = 100$). The N -factor envelope curve for a flat plate (black curve) is included as reference.

location, the rapid variations of the N -factor curve of rectangular humps are not so well reproduced by the combination of $BFS + FFS$.

8.5 Prediction of transition location

8.5.1 e^N methodology

Once the stability analysis results (in terms of N -factor envelope curves) are available, the expected location of the laminar-turbulent transition x_{tr} is calculated based on the e^N methodology. The procedure used here has been described in detail in Sect. 7.5. Therefore, some aspects will be omitted here for simplicity. Once again, a threshold transition N -factor value of $N_{tr} = 9$ has been chosen. The laminar-turbulent transition is expected to take place at the most upstream location where the corresponding N -factor curve reaches the N_{tr} value. The resulting expected transition location x_{tr} for each case is summarized in Fig. 8.14. The abscissa $(x_{tr}-x_c)/(x_{tr0}-x_c)$ expresses the relative change in the expected transition location with respect to the hump centre position x_c as a percentage of the transition location without any surface irregularity, x_{tr0} . In this manner, an abscissa of 0 % means that the transition from laminar to turbulent is expected to occur at the hump location. On the other hand, an abscissa of 100 % indicates that the respective hump has a negligible effect on transition location, and transition is likely to take place at the same x -location as the flat plate.

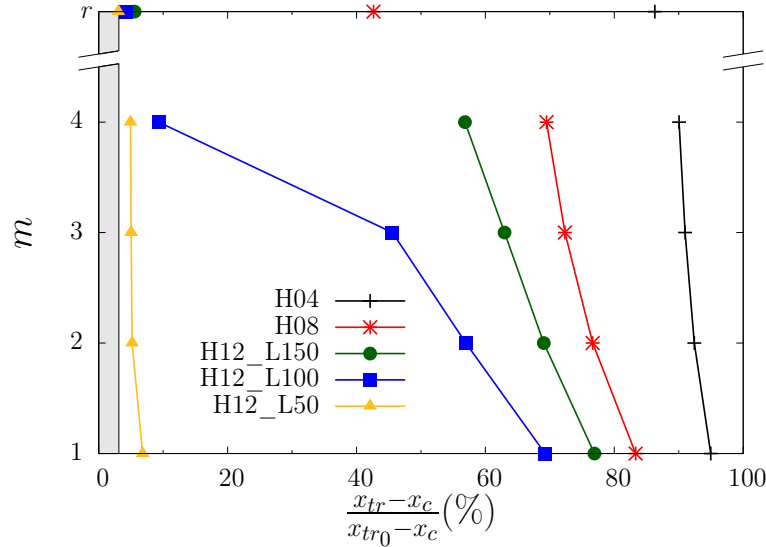


Figure 8.14: Expected transition location x_{tr} , referred to the expected transition location for a flat plate x_{tr0} , for all humps considered in the present chapter. The e^N methodology, with a transition N -factor of $N_{tr} = 9$, is applied. The grey area represents the position and length of the hump (for $L = 100$). Rectangular cases ($m = r$) can be considered as limiting cases when $m \rightarrow \infty$.

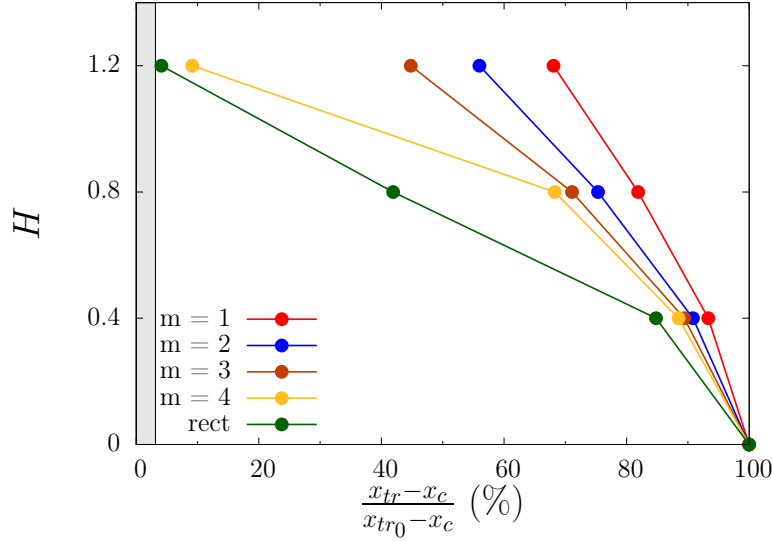


Figure 8.15: Expected transition location x_{tr} , referred to the expected transition position for a flat plate x_{tr0} , for all humps with length $L = 100$. The e^N methodology, with a transition N -factor of $N_{tr} = 9$, is applied. The grey area represents the position and length of the hump.

8.5.2 Analysis of the expected transition location x_{tr}

Figure 8.14 groups all humps according to its height H (*small*, *medium*, *large*), and its length L (50, 100, 150). For *small* humps ($H = 0.4$), the transition location is expected to move slightly upstream. The change of x_{tr} with respect to the flat-plate case x_{tr0} is larger when the value of the geometrical parameter m is increased, although this change in transition location is relatively small even for the rectangular hump. On the other hand, for *large* humps ($H = 1.2$) and length L fixed to 50, the onset of transition is expected to occur in the vicinity downstream of the hump, independent of its shape. For the remaining cases, the findings of the investigation suggest that transition location depends on not only height H and length L of the hump, but on the shape parameter m also.

Figure 8.15 groups all cases with length $L = 100$ in order to clarify the influence of the shape parameter m and height H . For *medium* and *large* humps, the results indicate that the influence of a rectangular hump (Fig. 8.15, green line) can be significantly reduced by smoothing its shape. On the other hand, for a given value of the shape parameter m , the expected transition location moves upstream quasi-linearly with the hump height H , except for the case $H12_m4$. This particular case will be studied in detail later.

In order to illustrate the influence of the parameter L , Fig. 8.16 shows the expected transition location for all cases with height $H = 1.2$. All *large* rectangular humps (Fig. 8.16, green line) are expected to promote transition at the hump location. However, for smooth humps there is a clear dependence with respect to the shape parameter m . Again, for the hump $H12_m4$ (with length $L = 100$), the e^N methodology predicts a transition location very close to the hump position. This result is not aligned with the trends shown by the rest of cases studied here. The reason must be sought in the N -factor envelope curves.

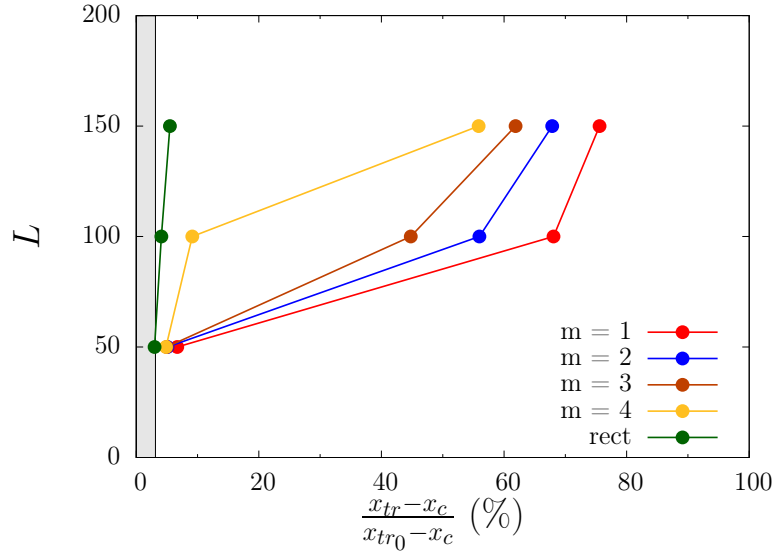


Figure 8.16: Expected transition location x_{tr} , referred to the expected transition position for a flat plate x_{tr0} , for all humps with height $H = 1.2$ (*large* humps). The e^N methodology, with a transition N -factor of $N_{tr} = 9$, is applied. The grey area represents the position and length of the hump (for $L = 100$).

Figure 8.17 indicates the expected transition location x_{tr} for each hump (with height $H = 1.2$ and length $L = 100$) based on the e^N methodology. This transition location is represented in Fig. 8.17 by circles. For the case *H12_m4* (Fig. 8.17, golden line), the N -factor curve shows a local maximum in the vicinity downstream of the hump that just exceeds the threshold value $N_{tr} = 9$. Downstream of this local maximum, the N -factor curve diminishes and grows again, reaching the value of $N_{tr} = 9$ once again at a different downstream location. The more upstream golden circle is placed at $x - x_c = 148$, while the second golden circle is located at $x - x_c = 530$.

Figure 8.18 depicts this particular case (*H12_m4*) and the two TS modes that define the two x -locations where the N -factor reaches a value of $N_{tr} = 9$: a TS mode of frequency $F = 56$ (which corresponds to the location $x - x_c = 148$), and a TS mode of frequency $F = 36$ (which corresponds to the location $x - x_c = 530$). Clearly, this case resembles the particular BFS case studied in Sect. 7.5 (case *BFS_H12_r*). In principle, based on the e^N methodology, one should take the position $x - x_c = 148$, given by the *higher-frequency** TS mode ($F = 56$) as the location x_{tr} where transition is expected to take place. However, these *higher-frequency* TS modes are quickly damped, and it is very plausible that they cannot transfer the energy required to higher harmonics to promote the laminar-turbulent transition. In any case, in the context of *linear stability analysis* and without extra information from experiments and / or non-linear computations, it is not possible to distinguish if the onset of laminar-turbulent transition is promoted by *lower-* or *higher-frequency* modes.

*Terms *lower-* and *higher-frequency* modes are referred to the frequency spectrum $F \in [20, 75]$ of relevant TS for this particular flow configuration.

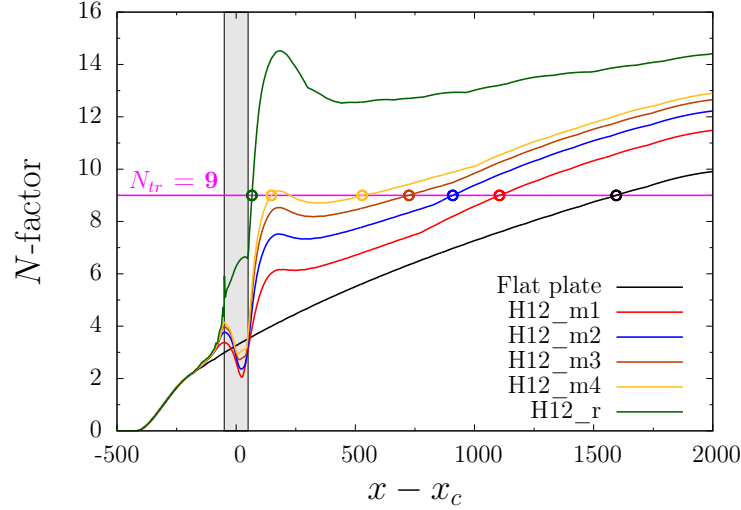


Figure 8.17: N -factor envelope curves for humps with height $H = 1.2$ and length $L = 100$. Circles indicate the streamwise location where the N -factor curves reach the threshold value $N_{tr} = 9$ (horizontal magenta line). The grey area represents the position and length of the hump. The N -factor envelope curve for a flat plate (black curve) is included as reference.

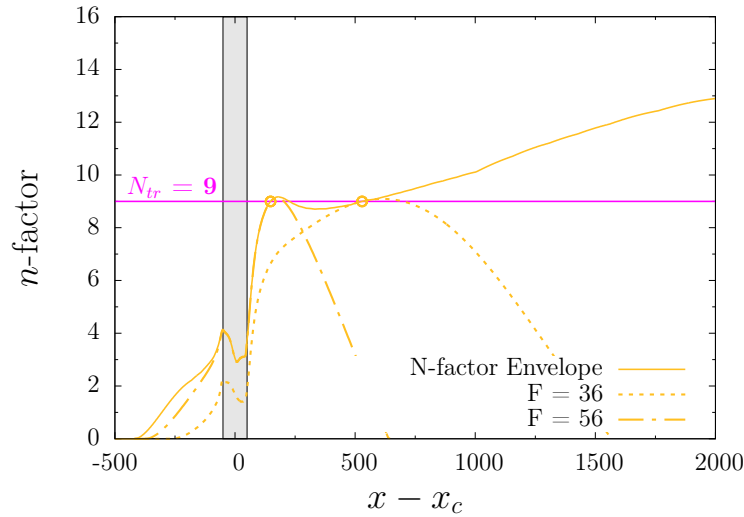


Figure 8.18: N -factor envelope curve for the rounded hump with height $H = 1.2$ and shape parameter $m = 4$. Circles indicate the streamwise location where the N -factor curve reaches the threshold value $N_{tr} = 9$. These circles correspond to the individual n -factor curves for frequencies $F = 36$ (dashed line) and $F = 56$ (point-dashed line). The grey area represents the position and length of the hump.

8.5.3 Sensitivity analysis for N_{tr}

The sensitivity analysis for the value of N_{tr} for steps conducted in Chapt. 7 is done now for humps. The expected transition location x_{tr} for each hump computed with the e^N methodology for a value of $N_{tr} = 9$ is recomputed by setting the value of N_{tr} to 8 and 10.

This sensitivity analysis is split in two parts. Initially, only humps with parameter length $L = 100$ are considered (see Fig. 8.19). The idea is to compare humps with the same height (*small*, *medium*, and *large*) and same length L , while the shape factor m varies. Later, only *large* humps are studied (Fig. 8.20). For these cases, the influence of the parameter L is discussed.

Humps are initially classified according to their height (*small*, *medium*, and *large*, Fig. 8.19-a), -b), and -c), respectively). For *small* humps, the *relative* expected transition location x_{tr} with respect to the expected transition location for the flat-plate case x_{tr0} (i.e. $(x_{tr}-x_c)/(x_{tr0}-x_c)$) is barely influenced by the choice of N_{tr} (8, 9, or 10). No significant differences have been found for all *small* humps (neither rounded nor rectangular). For *medium* humps, the smooth cases are also relatively indifferent to the value of N_{tr} . However, for rectangular humps ($m = r$), the *relative* expected transition location is strongly influenced by the choice of the N_{tr} value. Finally, for *large* humps, there is a large scattering of results depending on the choice of N_{tr} . Only for very smooth ($m = 1$, *Gaussian* hump) or rectangular hump, the *relative* location $(x_{tr}-x_c)/(x_{tr0}-x_c)$ does not change with the value of N_{tr} .

Hereafter, we focus on *large* humps only (Fig. 8.20). They are grouped according to their length L (150, 100, 50, Fig. 8.20-a),-b),-c), respectively). For $L = 150$, the *relative* transition location $(x_{tr}-x_c)/(x_{tr0}-x_c)$ is barely affected by the choice of N_{tr} , except for a particular case: when the shape parameter is $m = 4$. Considering a value of $N_{tr} = 8$ leads to a transition location x_{tr} very close to the hump. This particular case resembles the discussion on Sect. 8.5 for the case *H12_m4* with $N_{tr} = 9$.

Large humps with length $L = 100$ were discussed above (Fig. 8.19-c)). Finally, if $L = 50$ (Fig. 8.20-c)), transition is expected to occur right downstream of the hump for all cases except for a particular case: a *Gaussian* hump ($m = 1$), and $N_{tr} = 10$.

In summary, for many of the humps studied in this chapter, the *relative* expected transition location $(x_{tr}-x_c)/(x_{tr0}-x_c)$ based on the e^N methodology is barely affected by the choice of the threshold value N_{tr} (8, 9, or 10). However, for certain combinations of parameters height H , length L , and shape m , the *relative* transition location depends significantly on the choice of N_{tr} . This conclusion reinforces the need of supporting the value of N_{tr} based on experiments and / or numerical simulations (fully non-linear DNS) for very similar configurations as studied here (*e.g.* for the flat-plate case with identical inflow conditions).

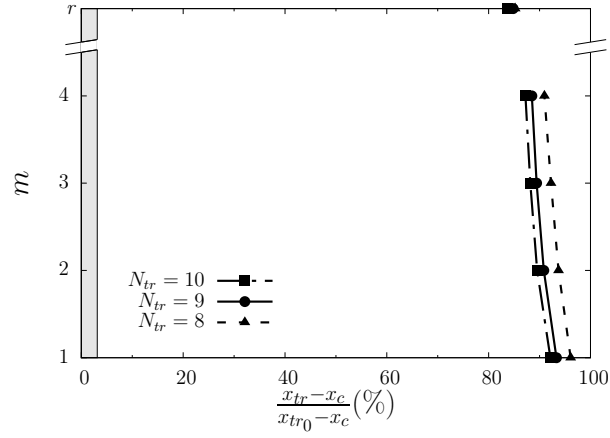
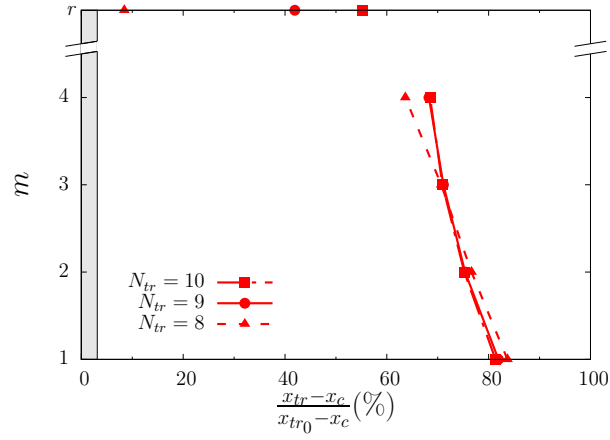
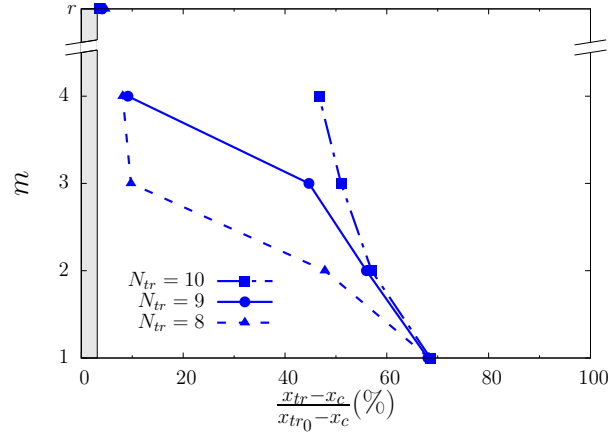
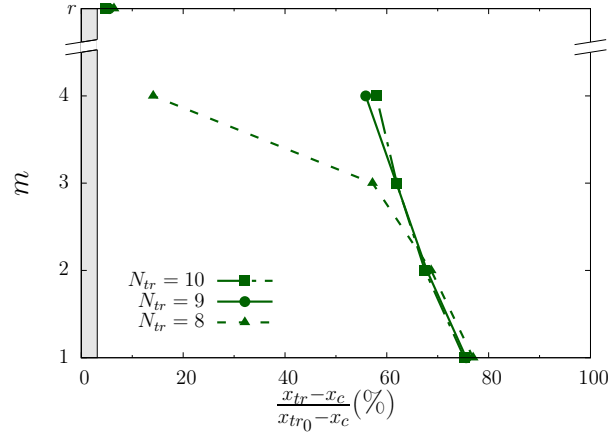
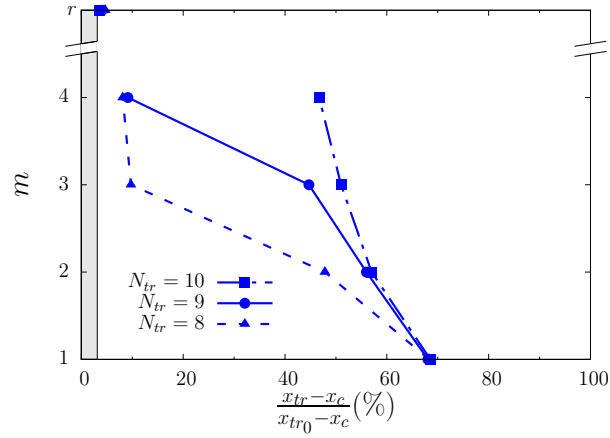

 a) *Small humps*

 b) *Medium humps*

 c) *Large humps*

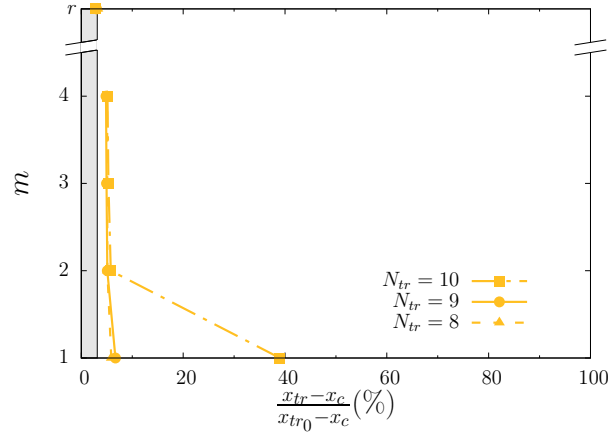
Figure 8.19: Expected transition location x_{tr} , referred to the expected transition location for a flat plate x_{tr0} , for all humps (with length $L = 100$) considered in the present chapter. The e^N methodology is applied. Dashed, solid, and point-dashed lines correspond to transition N -factors $N_{tr} = 8, 9$, and 10 , respectively.



a) $L = 150$



b) $L = 100$



c) $L = 50$

Figure 8.20: Expected transition location x_{tr} , referred to the expected transition location for a flat plate x_{tr0} , for all *large* humps considered in the present work. The e^N methodology is applied. Dashed, solid, and point-dashed lines correspond to transition N -factor $N_{tr} = 8, 9$, and 10 , respectively.

Chapter 9

Surface imperfections on HLFC wings

9.1 Introduction

Hybrid Laminar Flow Control (HLFC) is one of the most promising techniques for savings in CO_2 and NO_x emissions because of the potential of boundary-layer suction for reducing viscous drag and, therefore, fuel consumption, on modern commercial aircraft. An excellent review about this technology can be found in the work of Krishnan *et al.* [88]. This reduction in drag is mainly due to the impact on laminar-turbulent transition (LTT), extending the region along the wings where the flow remains laminar [72]. At typical flight conditions (low turbulence level, free-flight Reynolds number and transonic Mach number), the growth of flow instabilities (e.g. Tollmien-Schlichting (TS) waves or crossflow (CF) vortices), triggers the LTT process. Typically, on HLFC wings, suction is applied only in the leading-edge region (before the front spar). The flow suction modifies the boundary-layer (BL) profiles in such a way that the growth of the above-mentioned flow instabilities is delayed.

However, the joint between the Leading-Edge Suction Panel (LESP) and the Wing Box (WB) may introduce surface imperfections (e.g. backward/forward-facing steps (BFS/FFS), gaps, etc.) that can enhance the spatial development of those flow instabilities and, therefore, reduce the potential benefit of the suction system. For this reason, it is crucial during the design process to quantify the influence of such surface irregularities in terms of BL instability. This was one of the tasks of the EC-funded project HLFC-WIN [89], part of the Clean Sky 2 Joint Undertaking. In this chapter, a short description about the methodology used in the project, and the main stability analysis results are presented. In total, eight surface irregularities (seven gaps and one BFS) were selected in order to describe the influence, in terms of N -factor envelope curves, on the expected LTT location for HLFC wings at transonic flow conditions. Based on a reference regular gap (i.e. a gap with constant depth), variations in length and depth were studied. Furthermore, irregular gaps (i.e. gaps with streamwise-varying depth) were analysed also. These irregular gaps represent more realistic joints between different aerodynamic devices, since the manufacturing process or loads distribution in flight conditions can lead to small but relevant changes in the geometry of the joint. To the author's knowledge, no parametric studies for realistic three-dimensional wing geometries, at

transonic flight conditions, including suction and irregular surface imperfections have been published before the development of the AHLNS approach. The use of AHLNS applied for such complex configurations can be found in our recent works (see Franco *et al.* [90], and Franco and Hein [91, 92]). This is an important step forward in the study and understanding of the influence of surface imperfections in realistic flight configurations, and it shows the high level of maturity that the AHLNS methodology has achieved. It is expected that the results presented in this chapter will contribute to the selection of a suitable joint design and the specification of the corresponding manufacturing tolerances.

On the other hand, it is very common in the industry to use the criteria defined in the work of Nenni & Gluyas [63], who gave a critical Reynolds number for certain types of surface irregularities, based on incoming flow conditions and geometrical dimensions of the surface irregularity only. The criteria were established for a representative flow condition among all flight conditions tested on the X21, a LFC (Laminar-flow control) aircraft which includes suction. Therefore, although the use of the Nenni & Gluyas criteria provides a rapid estimation of the maximum allowable dimensions for a surface irregularity, the range of validity of such estimation is very limited.

The literature about the combined effect of suction and surface irregularities is limited. The more recent contributions about this topic are the experimental work from Methel *et al.* [93] & [71], Dimond *et al.* [94], and the numerical work by Zahn and Rist [95] using DNS. In these cases, the study was focused on flat-plate configurations. Small external pressure gradients were included in the studies of Dimond *et al.* [94] only.

The level of maturity of the AHLNS methodology allows its use for a wide range of aeronautical applications. In particular, the AHLNS approach have been used for EC-funded projects BLADE [96] (for laminar wings) and NACOR [97] (for laminar nacelles). In both cases, realistic geometries at transonic flight conditions were numerically evaluated. The technology considered in these two projects is the NLF (Natural Laminar Flow), which extends the laminar region on an airfoil by shaping the airfoil's geometry only (see [88]). Although the NLF technology do not include a suction system, surface imperfections in the form of joints are still present (*e.g.* access doors in laminar nacelles). The use of the AHLNS methodology has allowed designers to relax the initial tolerances for those joints given by the Nenni & Gluyas [63] criteria.

9.2 Infinite swept-wing approximation

The original fully three-dimensional (3D) wing geometry corresponds to a design with a LESP that reaches up to 20% of the chord on the upper side of the wing. The computational demands for the entire 3D flow field simulation, including the joint between the Leading-Edge Suction Panel and the Wing Box, are excessively high. In order to ease the computational effort and thus study various joint geometries and assess their impact on the boundary-layer instability characteristics, the flow field of the original 3D wing at a selected line-in-flight cut is approximated by an infinite swept-wing (2.5D) flow with similar BL properties.

A preliminary study [90] selected the line-in-flight cut located at the wingspan position $y = 19.34$ m. This particular spanwise section was chosen because it has the most conical flow condition compared to other spanwise cuts. This means that the spanwise variations for the 3D configuration at that particular spanwise position were the smallest ones we could find.

To ensure that the boundary layer and instability characteristics in the selected cut are comparable to the original 3D flow field, a parametric study was done in which the freestream Mach number Ma_∞ , the angle of attack α , and the sideslip angle γ were varied systematically [90]. In contrast, the airfoil geometry, its chord length c , and the suction distribution remained unchanged. Moreover, the transition location predicted for the 3D reference configuration in the selected slice ($x_{tr}/c \approx 0.54$) is prescribed as the transition location in the 2.5D (i.e. infinite swept-wing) simulations done with the DLR Navier-Stokes solver TAU [61]. The parameter combination that best approximates both the pressure distribution and N -factor curve is $Ma_\infty = 0.78$, $\alpha = 2.6^\circ$ and $\gamma = 14^\circ$.

9.3 Base flow computations

9.3.1 Approach

Once the spanwise invariant 2.5D base flow for a particular line-in-flight cut of the complete 3D wing is defined (see Sect. 9.2), then an intermediate step is introduced in order to ease the steady Navier-Stokes (NS) computations. This intermediate seeks to reduce the high grid requirements for computing the flow quantities around the joint (see Sect. 9.3.3).

The infinite swept-wing base flow (without any joint included, i.e. a clean configuration), described in Sect. 9.2, was computed using the DLR TAU code [61]. A hybrid CENTAUR [98] grid was used for this task (Fig. 9.1 - red hybrid mesh). This hybrid grid meshes the whole flow field around the 2D airfoil up to a distance of 100 chord lengths.

Then, a new reduced structured mesh is built with the DLR grid generator MEGACADS [60] (Fig. 9.1 - black structured mesh). The boundary conditions for the NS computations at the *Inflow* and *Upper side* edges of the MEGACADS grids are interpolated from the 2.5D TAU solution of the clean configuration, while boundary conditions at the *Outflow* are simply extrapolated from inside points. The *Inflow* and *Upper side* boundaries of the reduced domain are placed relatively far from the joint location, in order to assure that the presence of the joint does not affect the flow field variables at these borders. Finally, the surface irregularities are incorporated into this new structured mesh. The prescribed transition location for the 3D and 2.5D base flows ($x_{tr}/c \approx 0.54$) is kept for the steady NS computations in the reduced structured mesh.

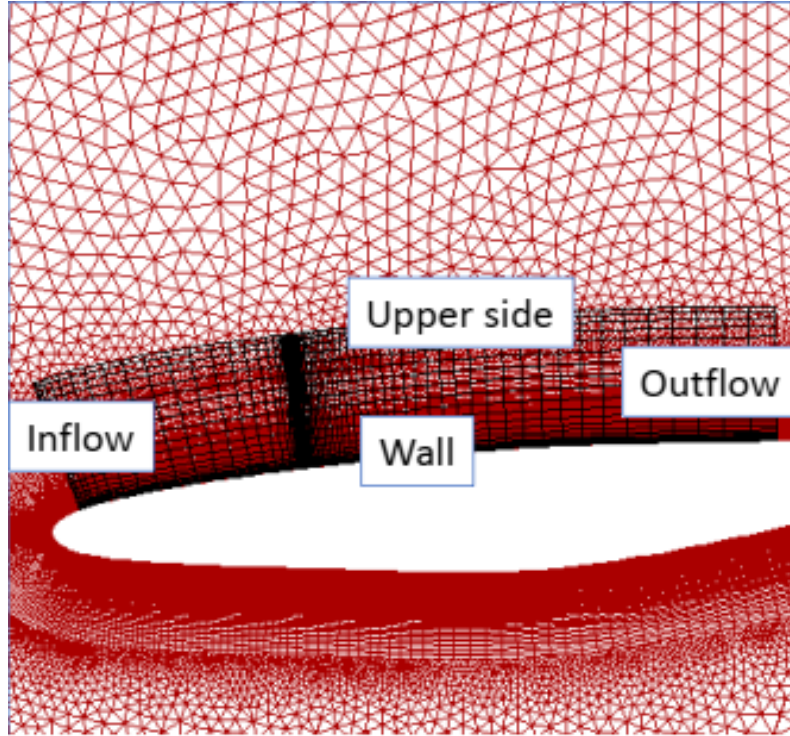


Figure 9.1: Example of the hybrid CENTAUR mesh (red) used for computing the 2.5D base flow (without any surface irregularity) and the structured MEGACADS grid (black) where the 2D surface irregularities are incorporated. For the fully-structured grid, only one out of ten points (in streamwise / wall-normal directions) is shown. Labels indicate the boundary conditions imposed in the computation of the base flow with the surface imperfection incorporated.

9.3.2 Joint geometry

The different cases described and studied in the present chapter are grouped based on the geometrical quantities that define their geometry. In particular, three parameters will be considered:

- H_1 : Height of the surface irregularity on the side of the Leading-Edge Suction Panel (LESP)
- H_2 : Height of the surface irregularity on the side of the Wing Box (WB)
- L : Length of the surface irregularity

A schematic representation of the three parameters is shown in Fig. 9.2. The nomenclature used to refer to a particular joint will be

$$TYPE_H_1_H_2_L,$$

where *TYPE* denotes the surface irregularity (BFS, Gap). Throughout this thesis, gap cases for which $H_1 = H_2$ are named as *regular* gaps. From a geometrical point of view, a gap can be seen as the combination of a backward-facing step (BFS) of height H_1 placed at the LESP side, and a forward-facing step (FFS) of height H_2 placed at the WB side, both separated by a distance L .

Based on the nomenclature and parameters defined before, a list with eight different types of joints is shown in Table 9.1. These surface irregularities represent the 2D geometry of the joint between the LESP and the WB. The choice of their geometrical dimensions was driven by the classical Nenni & Gluyas criteria [63]. However, we are also interested in checking the influence of small changes on a given irregularity geometry, i.e. to investigate the influence of manufacturing and/or geometrical tolerances due to different loads and deformations at both sides of the joint. Therefore, from a *reference* case, small variations in parameters H_1 , H_2 , and L are introduced. The leading-edge position of the surface irregularities (i.e. the end of the LESP) is fixed to 20% of the chord airfoil length for all cases.

The *reference* case is *Gap_0p8_0p8_3p5*. Cases *Gap_0p8_0p8_3p0* and *Gap_0p8_0p8_1p0* keep the same depth of the reference case (0.8 mm) but the length of the gap varies. The influence of the depth is studied by analysing *Gap_1p0_1p0_3p5*. These four gaps belong to the class of *regular* gaps (the depth H remains constant, i.e. $H_1 = H_2$).

Irregular gaps (*Gap_0p8_1p0_3p5* and *Gap_1p0_0p8_3p5*) reflect possible changes that might occur at both sides of the joint (the LESP and the WB parts) with respect to the *reference* case *Gap_0p8_0p8_3p5*. In particular, the depth is increased by 0.2 mm at each side independently.

Finally, two particular cases are also studied: the irregular *Gap_1p0_0p8_1p5* and the *BFS_0p2*. Although the two cases are geometrically different and do not have a direct link to the *reference* gap case, the flow topology shows similar features in both cases. Later, it will be discussed how these similarities in base flow topology relate with the boundary-layer instability analysis done with AHLNS.

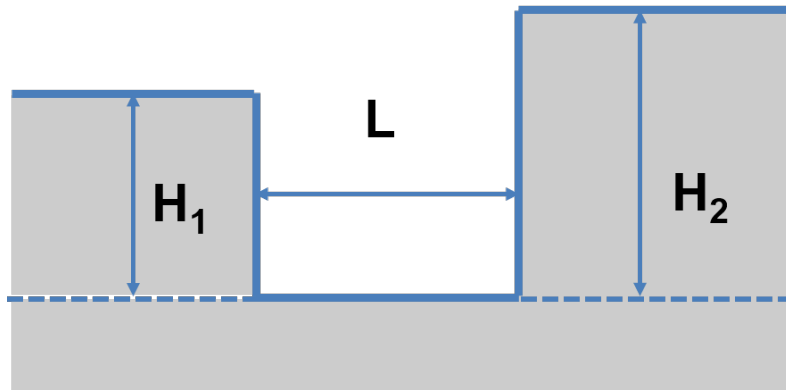


Figure 9.2: Parameters that define the joint geometry.

Name	Type	H_1	H_2	L
Gap_0p8_0p8_3p5	Gap	0.8	0.8	3.5
Gap_0p8_0p8_3p0	Gap	0.8	0.8	3.0
Gap_0p8_0p8_1p0	Gap	0.8	0.8	1.0
Gap_1p0_1p0_3p5	Gap	1.0	1.0	3.5
Gap_0p8_1p0_3p5	Gap	0.8	1.0	3.5
Gap_1p0_0p8_3p5	Gap	1.0	0.8	3.5
Gap_1p0_0p8_1p5	Gap	1.0	0.8	1.5
BFS_0p2	BFS	0.2	-	-

Table 9.1: Geometric parameters of the different surface irregularities considered in the present study. Parameters H_1 , H_2 , and L are expressed in mm.

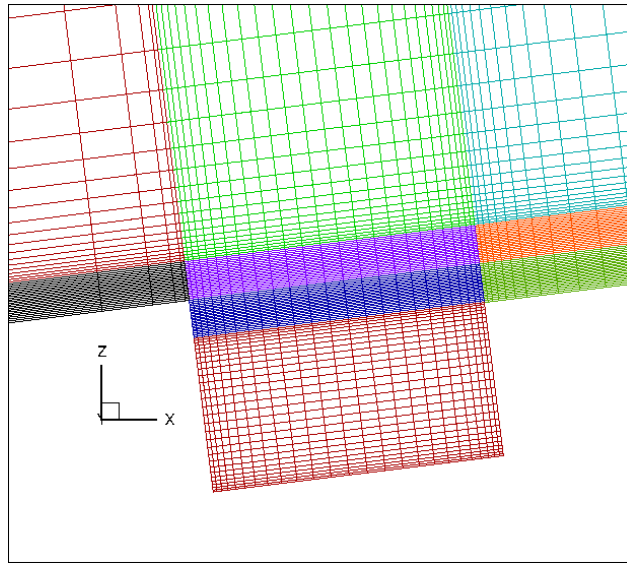


Figure 9.3: Example of the multi-block mesh built with MEGACADS for 2D surface irregularities. Only one out of ten points is shown in both streamwise/wall-normal directions.

9.3.3 Meshing of the surface imperfection

Figure 9.3 shows an example of a structured mesh built with the DLR grid generator MEGACADS [60], where a 2D surface irregularity is incorporated into the airfoil. This particular surface imperfection represents an *irregular* gap with $H_1 > H_2$. The geometry of a particular surface irregularity is reproduced by activating/deactivating the corresponding mesh block. About 800000 grid points are used for generating the complete structured mesh (circa 2000 points in chord direction, and 400 points in wall-normal direction). Inside the gap, about 300 points are placed along its length L , and a similar amount of points (≈ 300) are used in wall-normal direction (only for the depth H , depending on the particular joint considered).

9.3.4 Base flow results

Mach number contours and sectional streamlines are presented for the eight cases. For ease, different cases are grouped according to a particular criterion, taking the case *Gap_0p8_0p8_3p5* as reference. The aim in this section is to describe the main characteristics of the base flow topology. Later, the description presented here will support the boundary-layer instability analysis computed with AHLNS.

9.3.4.1 Influence of length

Figure 9.4 shows the Mach number contours and sectional streamlines for the following three gaps:

- *Gap_0p8_0p8_3p5* (*reference*)
- *Gap_0p8_0p8_3p0*
- *Gap_0p8_0p8_1p0*

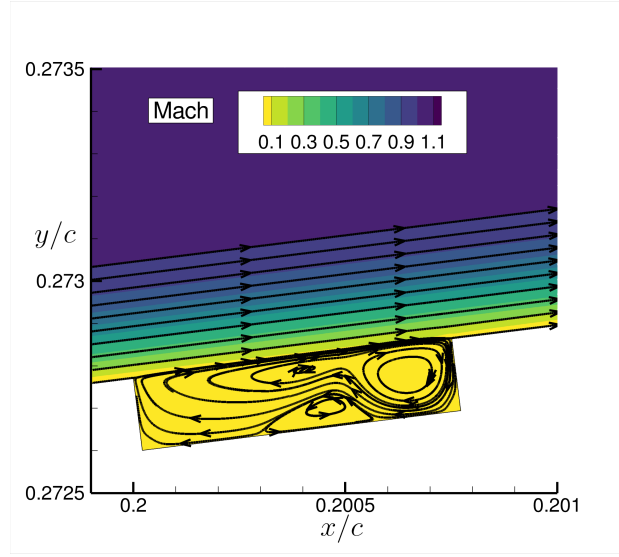
The reference case and cases *Gap_0p8_0p8_3p0* and *Gap_0p8_0p8_1p0* are all *regular* gaps with the same depth (0.8 mm). The reference case (Fig. 9.4-a)) presents a small separation zone at the bottom of the gap, surrounded by a large recirculation bubble. The streamlines indicate that the flow rotates in opposite direction in each bubble. Interestingly, the incoming upstream streamlines are barely deflected once they encounter the gap. A similar portrait describes the flow topology of the *Gap_0p8_0p8_3p0* (Fig. 9.4-b)). Finally, case *Gap_0p8_0p8_1p0* shows a single large recirculation area that fills the entire gap. For the three cases, there is a tiny recirculation bubble at each inner corners of the gap.

9.3.4.2 Influence of depth

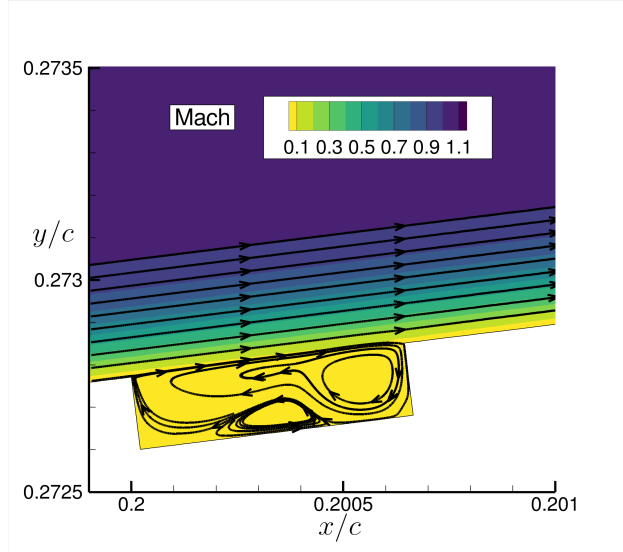
Figure 9.5 compares the Mach number contours and sectional streamlines for the two *regular* gaps:

- *Gap_0p8_0p8_3p5* (*reference*)
- *Gap_1p0_1p0_3p5*

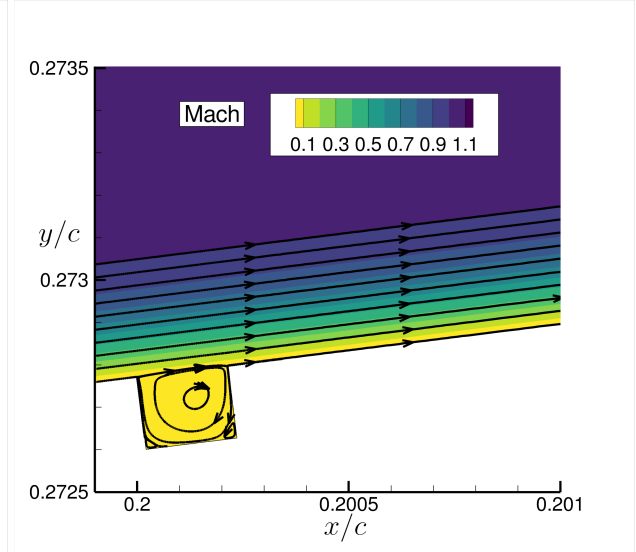
These two gaps have the same length (3.5 mm), but the depth of the *Gap_1p0_1p0_3p5* is 0.2 mm larger than the *reference* case. The most noticeable effect of increasing the depth of the *reference* case is the growth of the separation bubble at the bottom of the gap (Fig. 9.5-b)). The incoming upstream sectional streamlines remain barely deflected by the increase in depth of case *Gap_1p0_1p0_3p5*.



a) *Gap_0p8_0p8_3p5*



b) *Gap_0p8_0p8_3p0*

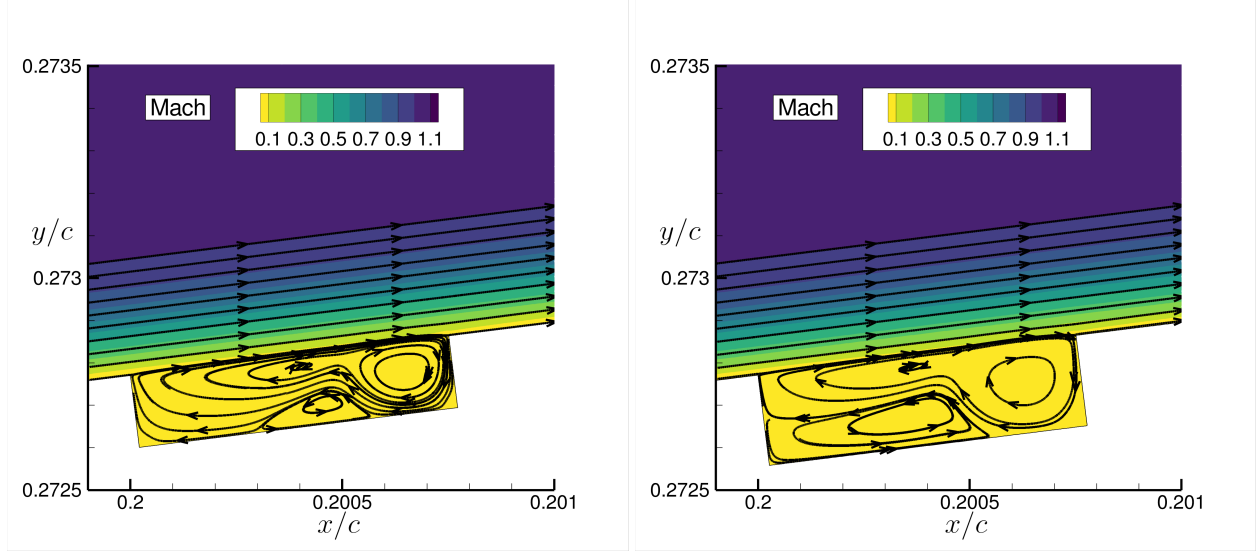


c) *Gap_0p8_0p8_1p0*

Figure 9.4: Mach number contours and sectional streamlines for cases (a) *Gap_0p8_0p8_3p5*; (b) *Gap_0p8_0p8_3p0*; (c) *Gap_0p8_0p8_1p0*. Flow direction is from left to right.

9.3.4.3 Irregular gaps

An *irregular* gap represents a small modification of the *regular reference* gap, where the depth H of the gap is no longer constant. Since the gap is a representation of the joint of two aerodynamics elements (here the LESP and the WB), an *irregular* gap can be seen as the joint of the two devices when one of them has a relative displacement with respect to the other (due to manufacturing, different wing loads, etc.). Here, Fig. 9.6 depicts the Mach number contours and two-sectional streamlines for the following three joints:


 a) *Gap_0p8_0p8_3p5*

 b) *Gap_1p0_1p0_3p5*

Figure 9.5: Mach number contours and sectional streamlines for cases (a) *Gap_0p8_0p8_3p5*; (b) *Gap_1p0_1p0_3p5*. Flow direction is from left to right.

- *Gap_0p8_0p8_3p5* (*reference*)
- *Gap_0p8_1p0_3p5*
- *Gap_1p0_0p8_3p5*

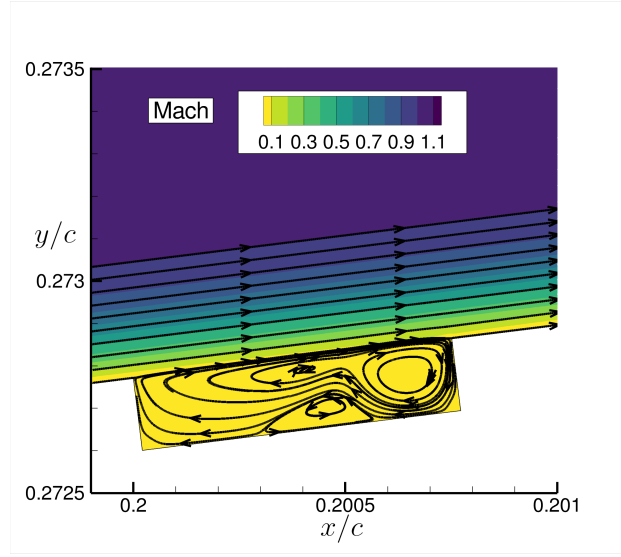
Case *Gap_0p8_1p0_3p5* (Fig. 9.6-b)) introduces a small increase in depth of 0.2 mm on the WB side (H_2) only with respect to the *reference* gap. The flow structure inside the gap is very similar to the *reference* *Gap_0p8_0p8_3p5* (Fig. 9.6-a)). However, the flow field at the WB side is different: the streamlines have to rapidly deflect in order to accommodate for these extra 0.2 mm. Interestingly, the trajectories of the incoming upstream streamlines resemble the flow topology of a forward-facing step of 0.2 mm height.

A steady laminar flow for case *Gap_1p0_0p8_3p5* could not be achieved. Figure 9.6-c) depicts the Navier-Stokes computation at a particular iteration. This phenomenon might indicate that the separated flow exceeds the gap space and an *absolute* instability develops there. For the sake of clarification, a similar *irregular* gap with a shorter length is studied in the following section.

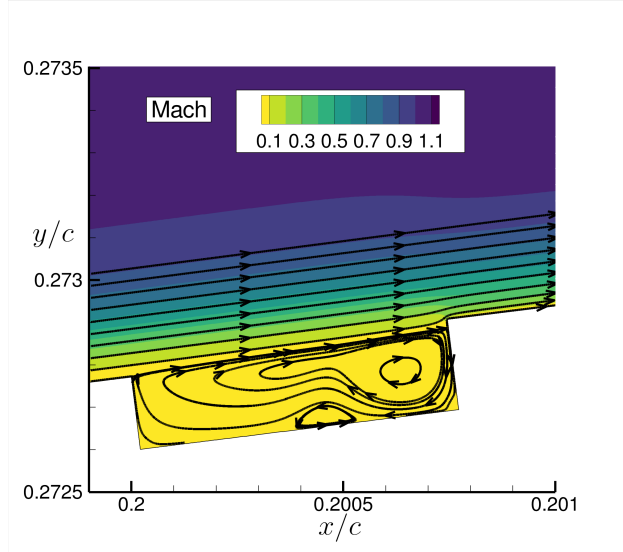
9.3.4.4 BFS

Finally, the last two joints to analyse are:

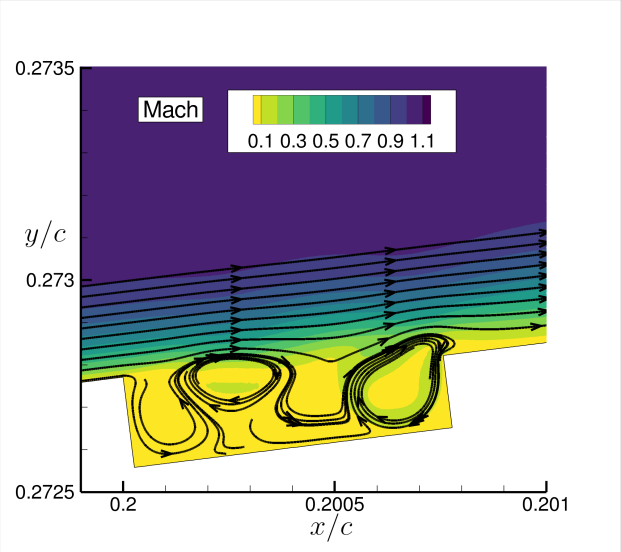
- *Gap_1p0_0p8_1p5*
- *BFS_0p2*



a) *Gap_0p8_0p8_3p5*



b) *Gap_0p8_1p0_3p5*



c) *Gap_1p0_0p8_3p5*

Figure 9.6: Mach number contours and sectional streamlines for cases (a) *Gap_0p8_0p8_3p5*; (b) *Gap_0p8_1p0_3p5*; (c) *Gap_1p0_0p8_3p5* (for this particular case, the figure represents the not-converged NS computation at a particular iteration). Flow direction is from left to right.

These two cases are not directly related with the *reference* case *Gap_0p8_0p8_3p5*. However, the comparison of both irregularities is a simple proof of the variety of cases (in terms of flow topologies) that may exist in the study of surface imperfections. In previous sections, it has been shown that incoming upstream streamlines are barely deflected by the presence of a *regular* gap. Therefore, we asked ourselves if the effect of an *irregular* gap (where $H_1 > H_2$) would be similar to a backward-facing step with a height $H_1 - H_2$.

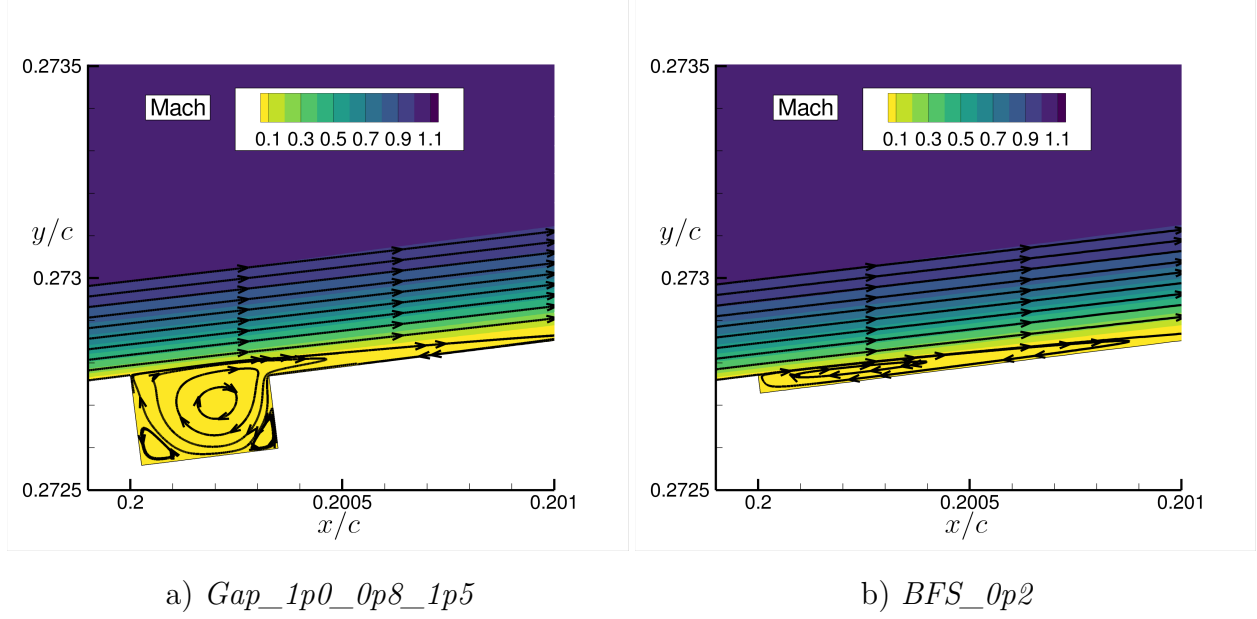


Figure 9.7: Mach number contours and sectional streamlines for cases (a) *Gap_1p0_0p8_1p5*; (b) *BFS_0p2*. Flow direction is from left to right.

Figure 9.7 shows the Mach number contours and sectional streamlines of the two imperfections: *Gap_1p0_0p8_1p5* and *BFS_0p2*. The flow topology of *Gap_1p0_0p8_1p5* (Fig. 9.7-a)) shows that the recirculating flow exceeds the space inside the gap, resulting in a large separation area downstream of the gap. Interestingly, the length in streamwise direction of the separation region is similar to the extent of the recirculation bubble produced by the *BFS_0p2* (Fig. 9.7-b)). Later, in Sect. 9.4.1.4, the boundary-layer instability analysis of the two irregularities is compared.

9.4 Instability analysis

A preliminary instability analysis study for the infinite swept wing clean configuration was done using the PSE approach. For this preliminary study, suction is acting in the LESP up to 20% of the chord length, but no superficial joint incorporated into the shape of the airfoil. Results of this initial analysis can be found in Franco *et al.* [90]. To sum up, this study revealed that the most relevant n-factor curves for TS instabilities (each one obtained for a particular spanwise wavenumber β^* and frequency f^*) were found in the parameter space $\beta^* \in [200, 400] \text{ m}^{-1}$ and $f^* \in [1000, 6000] \text{ Hz}$. The corresponding wave angle $\psi^{*\dagger}$ shows a large variety of values (in the range of 10° to 60°), which might indicate that not only TS modes but travelling CF modes are relevant in the boundary layer instability analysis. A clear distinction between both types of convective instabilities has not been done yet. For ease, all travelling modes ($f^* \neq 0 \text{ Hz}$) are labelled as TS modes.

[†]relative to the boundary-layer edge streamline direction

Regarding the influence of stationary ($f^* = 0$ Hz) CF vortices, it was shown [90] that, for the complete three-dimensional wing including suction but without any surface irregularity, the N -factor envelope curve of the stationary CF vortices is significantly smaller than the corresponding N -factor envelope curve for travelling modes. Therefore, only *non-zero* frequency modes are considered in the present boundary-layer instability analysis.

9.4.1 N -factor envelope curves

The outcomes of the instability analysis (in terms of N -factor envelope curves) for all cases are grouped with the same criteria that were used for the base flow results (Sect. 9.3.4). Moreover, the examination of the pressure distribution $\bar{p}_{wall}^\dagger$ helps for a better physical understanding of the influence of each particular surface imperfection in the boundary-layer instability analysis.

9.4.1.1 Influence of length

Figure 9.8 collects the boundary-layer instability results (in terms of N -factor envelope curves) of the three *regular* gaps with same depth (*reference* case *Gap_0p8_0p8_3p5* plus cases *Gap_0p8_0p8_3p0* and *Gap_0p8_0p8_1p0*).

Only minor differences are found in a small region downstream of the gap between the N -factor curve for the clean configuration and the three gaps. As expected, the suction applied in the LESP (up to 20% of the chord) for all cases inhibits the growth of any TS wave up to the streamwise location where the WB part starts. The pressure distribution $\bar{p}_{wall}$ (Fig. 9.9) shows that none of the three gaps discussed here leads to a noticeable change in the $\bar{p}_{wall}$ distribution compared with the *clean* configuration (label '*No irregularity*'). Only small peaks are barely visible at the edges of the gaps, which are responsible for the minor differences in the N -factor curves commented before. Overall, it is important to notice that the highest N -factor values reached by the *reference* case and the *no irregularity* case are very similar. Therefore, it is expected that the presence of the *reference* gap has a negligible effect on the laminar-turbulent transition location.

9.4.1.2 Influence of depth

The instability analysis results for the *reference* case (*Gap_0p8_0p8_3p5*) are compared with the N -factor curve from case *Gap_1p0_1p0_3p5*, i.e. a gap with identical length (3.5 mm) but 0.2 mm deeper. Both N -factor curves (Fig. 9.10) and pressure distribution $\bar{p}_{wall}$ (Fig. 9.11) are practically identical along the chordwise direction for both gaps. Therefore, we can state that a small increase in depth of the *reference* gap has no influence on the stability analysis results.

[†]the base flow pressure distribution $\bar{p}_{wall}^*$ is made nondimensional $\bar{p}_{wall}$ with the static inflow pressure $\bar{p}_\infty^*$

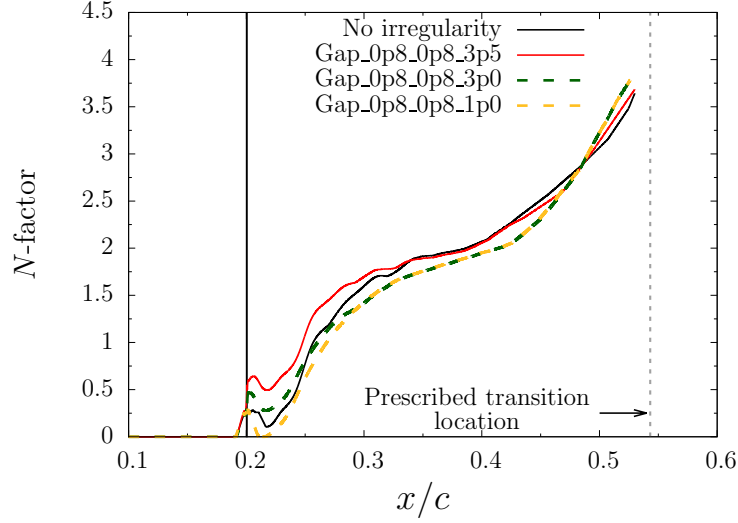


Figure 9.8: N -factor envelope curves for cases *Gap_0p8_0p8_3p5* (red solid line); *Gap_0p8_0p8_3p0* (green dashed line); and *Gap_0p8_0p8_1p0* (golden dashed line). The clean configuration (black solid line) has been included as reference. The vertical solid grey line represents the location of the surface imperfection. The vertical grey dashed line indicates the prescribed LTT location in the 2.5D base flow TAU computations of the clean configuration.

9.4.1.3 Irregular gaps

Since the steady base flow for case *Gap_1p0_0p8_3p5* could not be computed, only the stability analysis results for the *irregular Gap_0p8_1p0_3p5* are compared with the results of the *reference Gap_0p8_0p8_3p5* (Fig. 9.12). The amplification curve of case *Gap_0p8_1p0_3p5* shows a very pronounced peak at the joint location, followed by rapid growth up to $x/c \approx 0.3$, from where it remains almost constant (with a value about 2.5), and finally it grows again (with a growth rate similar to the *No irregularity* case) up to the prescribed transition location.

Overall, the N -factor curve for *Gap_1p0_0p8_1p5* shows a small increment ΔN with respect to the amplification curve for the *No irregularity* case in the proximity of the prescribed LTT location. Therefore, it is expected that the presence of the joint does not significantly move the transition location upstream.

In terms of pressure distribution $\bar{p}_{wall}$ (Fig. 9.13), the flow is decelerated upstream of the joint and rapidly accelerated at the side of the WB. The base flow analysis (Sect. 9.3.4.3) suggested that the flow topology for this *irregular* gap resembles an FFS of 0.2 mm height ($H_2 - H_1$). In fact, the pressure distribution that arose in the vicinity of the gap is very similar to that of the FFS cases studied in Chapt. 7.

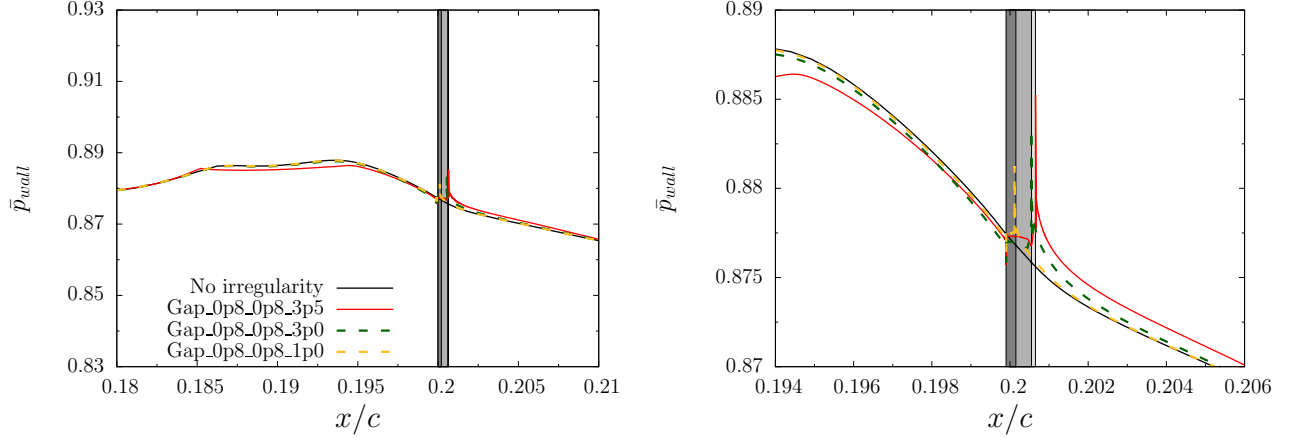


Figure 9.9: (Left) Non-dimensional pressure distribution at the wall $\bar{p}_{wall}$ for cases *Gap_0p8_0p8_3p5* (red solid line); *Gap_0p8_0p8_3p0* (green dashed line); and *Gap_0p8_0p8_1p0* (golden dashed line). The clean configuration (black solid line) has been included as reference. The vertical grey area represents the location and width (3.5/3.0/1.0 mm) of the gap. (Right) Detailed view.

9.4.1.4 BFS

As stated previously (Sect. 9.3.4.4), the aim of this investigation is to compare the influence of an *irregular* gap (with $H_1 > H_2$) and a BFS of height $H_1 - H_2$. The comparison here is by means of the stability analysis results (Fig. 9.14).

The amplification curve of case *Gap_1p0_0p8_1p5*, (Fig. 9.14 - red solid line) grows similar to case *BFS_0p2* (blue dashed line), i.e. there is a significant increase in N -factor downstream of the gap up to $x/c \approx 0.25$, from where the two curves remains nearly constant (with a value about 2.5-3). Finally, both N -factor curves (*Gap_1p0_0p8_1p5* and *BFS_0p2*) grow again up to the prescribed transition location with a growth rate similar to the *No irregularity* case. Close to the prescribed laminar-turbulent transition location, the N -factor values of both *Gap_1p0_0p8_1p5* and *BFS_0p2* are very similar and only slightly higher than the *No irregularity* case. Therefore, it is not expected that the presence of the *Gap_1p0_0p8_1p5* or the *BFS_0p2* provokes a noticeable upstream displacement of the transition location (compared with the *No irregularity* case). The pressure distribution $\bar{p}_{wall}$ (Fig. 9.15) for both cases is almost identical. Very small differences at $x/c \approx 0.2$ are noticeable only.

9.4.2 Influence of the joint on the TS wave structure

The boundary-layer instability analysis computed with AHLNS for *regular* gaps showed that the presence of the gap modifies the instability scenario predicted for the clean configuration (*No irregularity*). However, these changes are localized in the vicinity of the joint and imply relatively small increases in the N -factor values, but only in the vicinity of the gap. Indeed, it is not expected that any of the *regular* gaps modify the predicted transition location. Therefore, it might be of interest to check in more detail the structure of a particular TS mode

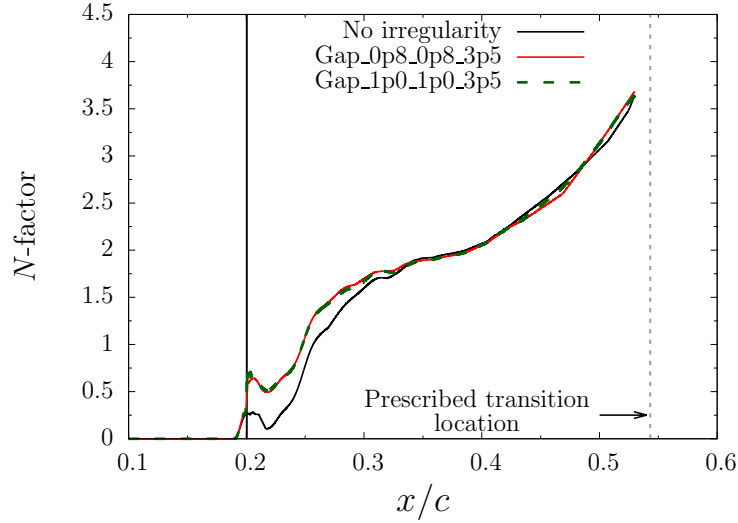


Figure 9.10: N -factor envelope curves for cases *Gap_0p8_0p8_3p5* (red solid line) and *Gap_1p0_1p0_3p5* (green dashed line). The clean configuration (black solid line) has been included as reference. The vertical solid grey line represents the location of the surface imperfection. The vertical grey dashed line indicates the prescribed LTT location in the 2.5D base flow TAU computations of the clean configuration.

when passing along one of these *regular* gaps. Figure 9.16 shows the spatial evolution of an incoming TS wave of frequency $f^* = 2000$ Hz and spanwise wavenumber $\beta^* = 200 \text{ m}^{-1}$ (wave angle $\psi^* \approx 55^\circ$) passing along the case *Gap_0p8_0p8_3p0*. This particular instability mode has been chosen because it produces the individual n -factor curve with the highest value at the end of the domain ($x/c \approx 0.53$). The body-attached coordinates (s, η) in Fig. 9.16 are defined as: s represents the arc-length along the airfoil surface (with origin at the leading edge), and η holds for the surface-normal coordinate. For this particular TS mode, the streamwise wavelength is about 6 cm, while the gap length L is 3 mm. Clearly, the large difference in streamwise length scales between surface imperfection and instability wavelength mode explains the small influence of the gap on the structure of the instability mode. Figure 9.17 depicts the growth rate of the streamwise velocity disturbance $\sigma_{\hat{u}}$ for this particular TS mode. The presence of the gap introduces very large gradients in the growth rate of the instability mode. These very rapid changes justify the use of the AHLNS methodology. The use of PSE would not be possible due the lower step-size limit (see Sect. 3.3.2) that inhibits the correct description of such rapid changes. However, the strong gradients are limited to the vicinity of the gap. In other words, the wavelength is still one order-of-magnitude larger than the extent of this region of large streamwise gradients. In addition, for this particular example of HLFC wing, the presence of the *regular* gaps barely changes the pressure distribution $\bar{p}_{wall}$ compared with the clean configuration (see Fig. 9.9 and 9.11). As a result, the spatial development of the instability mode is almost identical in both configurations: with or without surface irregularity.

For *irregular* gaps or BFS the scenario is different. These imperfections introduce changes that modify significantly the pressure distribution of the reference clean case (see Fig. 9.13 and 9.15).

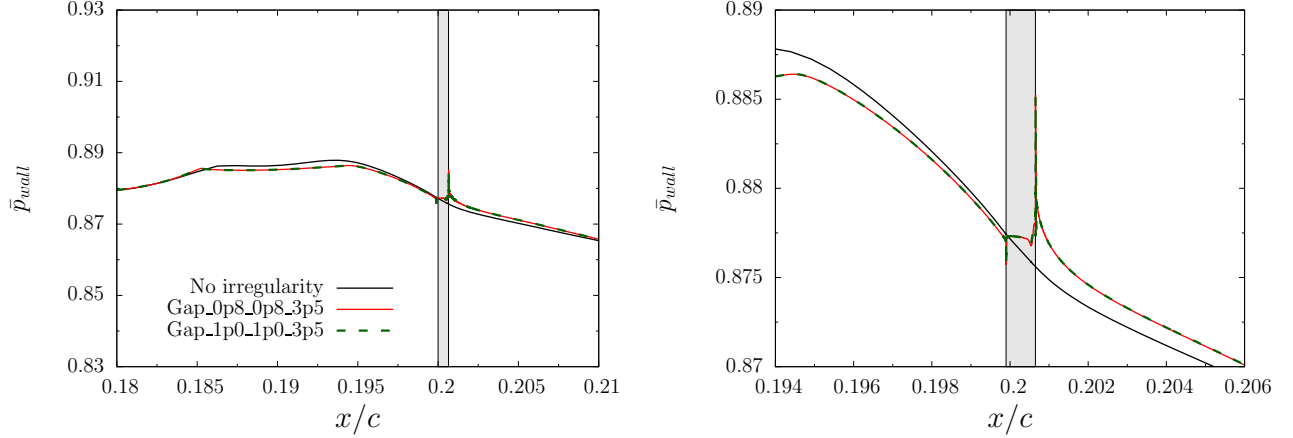


Figure 9.11: (Left) Non-dimensional pressure distribution at the wall $\bar{p}_{wall}$ for cases *Gap_0p8_0p8_3p5* (red solid line) and *Gap_1p0_1p0_3p5* (green dashed line). The clean configuration (black solid line) has been included as reference. The vertical grey area represents the location and width (3.5 mm) of the gap. (Right) Detailed view.

Moreover, these pressure variations extend in chordwise direction over distances similar of the wavelength of the TS waves. Although the *irregular* gaps or BFS (cases *Gap_0p8_1p0_3p5*, *Gap_1p0_0p8_1p5*, and *BFS_0p2*) are not expected to alter the predicted transition location compared with the clean configuration, the corresponding N -factor curves arise noticeable differences. These variations highlight the sensitivity to small changes in H_2 relative to H_1 (due to manufacturing tolerances, wing loads, etc.)

9.4.3 Comparison with classical criteria

It is very common in the aeronautical industry to use the criteria for allowable surface tolerances established by Nenny & Gluyas [63]. The authors stated that *'these criteria do not necessarily represent the maximum permissible instability[†] to avoid transition, however, but represent what are considered safe levels for a wide variety of flight conditions. The permissible degree of instability is most conveniently expressed in terms of a Reynolds number based upon characteristic thickness to which the boundary layer is allowed to grow'*. In particular, the criteria for rectangular BFS and regular gaps set

$$H_{crit} \approx \frac{900}{Re_u}, \quad \text{and} \quad L_{crit} \approx \frac{15000}{Re_u}, \quad (9.1)$$

where H_{crit} is the '*allowable step height*', and L_{crit} is the '*allowable gap width*'. In both cases, the criterion was defined for a unit Reynolds number $Re_u = 2 \times 10^6$ per ft, and a chord length of 45 ft. As a result, a representative value of H_{crit} for BFS is 0.005 in (0.127 mm). For gaps, an indicative value of L_{crit} is 0.09 in (2.286 mm).

[†]authors explicitly referred here to *attachment-line*, *Tollmien-Schlichting*, and *crossflow* instabilities

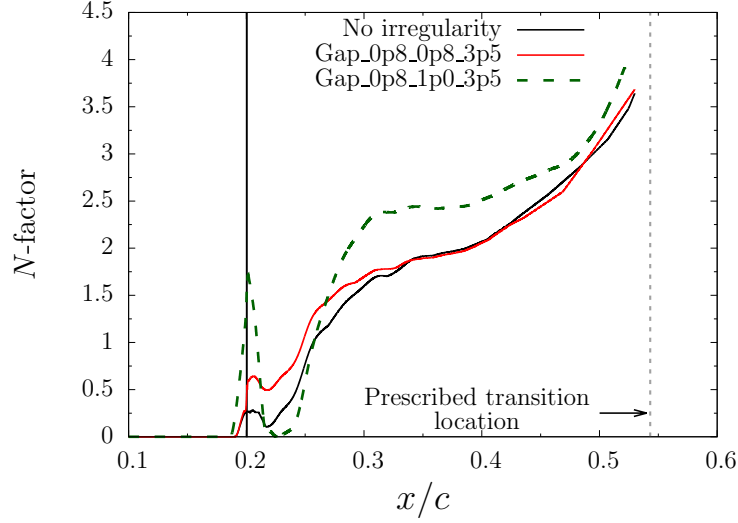


Figure 9.12: N -factor envelope curves for cases *Gap_0p8_0p8_3p5* (red solid line) and *Gap_0p8_1p0_3p5* (green dashed line). The clean configuration (black solid line) has been included as reference. The vertical solid grey line represents the location of the surface imperfection. The vertical grey dashed line indicates the prescribed LTT location in the 2.5D base flow TAU computations of the clean configuration.

Although the applicability of those criteria is straightforward, it is worth to recall their limitations. In particular, a major drawback is that those expressions are defined based on incoming flow magnitudes (through the Re_u) solely. Therefore, the BL characteristics at the joint location are not considered. Moreover, these criteria apply for relatively simple surface irregularities. Shapes like ramps, rounded steps, gaps of variable height (i.e. *irregular* gaps), or a combination of different irregularities are out of the scope of these criteria, as Holmes *et al.* pointed out [99]. In particular, for the particular case of *regular* gaps, the Nenni and Gluyas criterion neglects the influence of the depth H of the gap.

Taking the freestream conditions that define the infinite swept-wing approximation (see Sect. 9.2), the maximum allowable step height H_{crit} for BFS would be 0.144 mm. For gaps, the maximum allowable gap width L_{crit} would be 2.4 mm. Clearly, these dimensions are much more restrictive (in terms of tolerances for manufacturing) than the values considered in the current study. Since the stability analysis results obtained from the AHLNS approach indicate that none of the joints discussed here promotes a significant upstream movement of the predicted transition location (compared with the clean configuration), the potential for less restrictive joint tolerances could be exploited still.

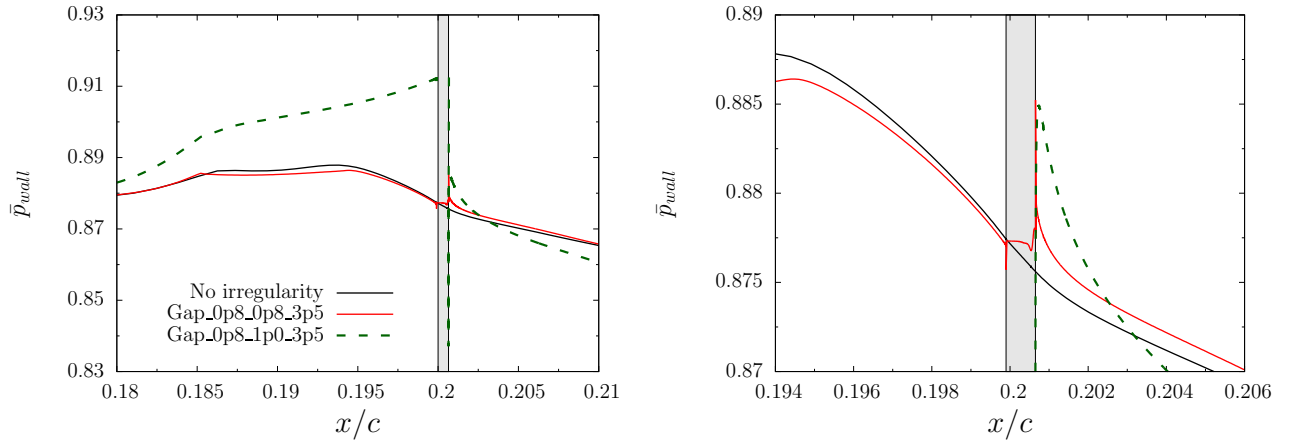


Figure 9.13: (Left) Non-dimensional pressure distribution at the wall $\bar{p}_{wall}$ for cases *Gap_0p8_0p8_3p5* (red solid line) and *Gap_0p8_1p0_3p5* (green dashed line). The clean configuration (black solid line) has been included as reference. The vertical grey area represents the location and width (3.5 mm) of the gap. (Right) Detailed view.

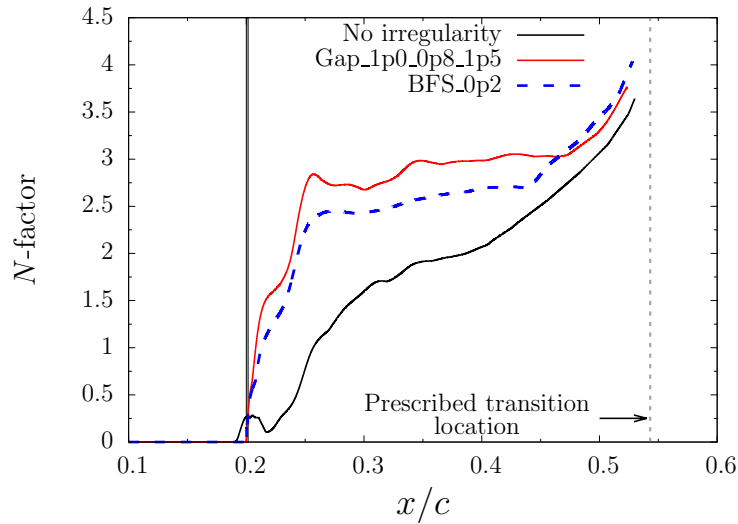


Figure 9.14: N -factor envelope curves for cases *Gap_1p0_0p8_1p5* (red solid line) and *BFS_0p2* (blue dashed line). The clean configuration (black solid line) has been included as reference. The vertical solid grey line represents the location of the surface imperfection. The vertical grey dashed line indicates the prescribed LTT location in the 2.5D base flow TAU computations of the clean configuration.

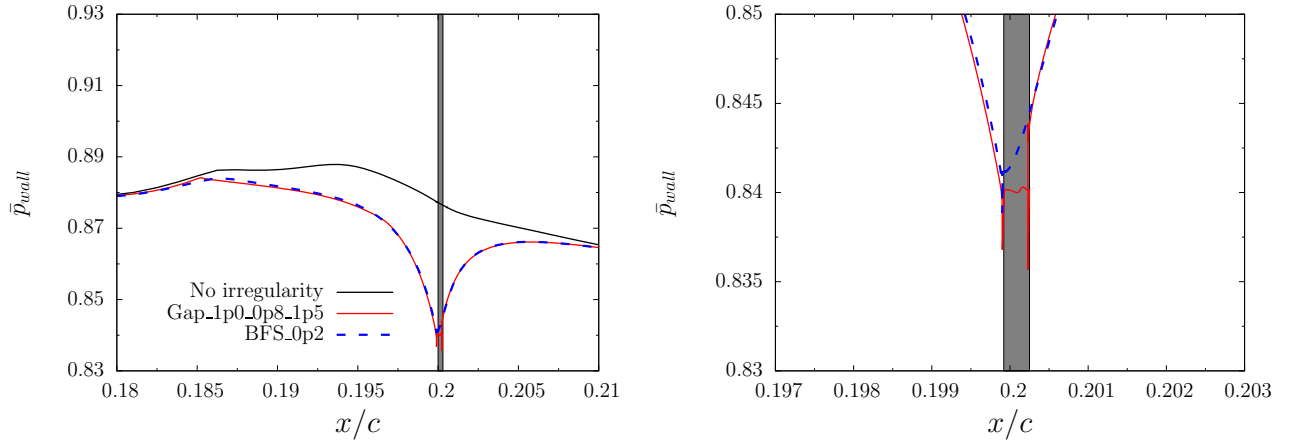


Figure 9.15: (Left) Non-dimensional pressure distribution at the wall $\bar{p}_{wall}$ for cases *Gap_1p0_0p8_1p5* (red solid line) and *BFS_0p2* (blue dashed line). The clean configuration (black solid line) has been included as reference. The vertical grey area represents the location and width (1.5 mm) of the gap. (Right) Detailed view.

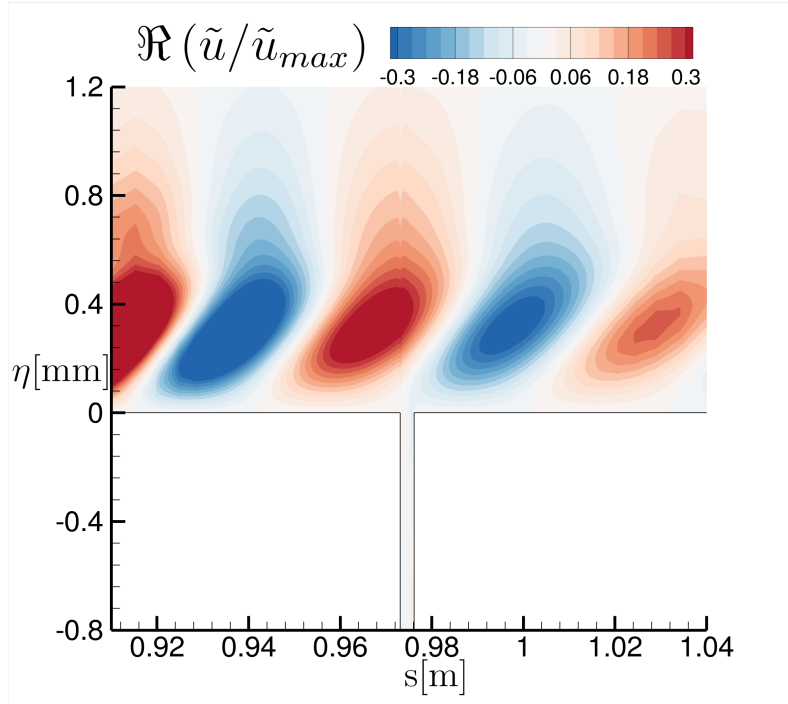


Figure 9.16: Influence of the presence of a regular gap (case *Gap_0p8_0p8_3p0*) on the real part of the streamwise velocity disturbance $\Re(\tilde{u}/\tilde{u}_{max})$ for an incoming TS wave of frequency $f^* = 2000$ Hz and spanwise wavenumber $\beta^* = 200 \text{ m}^{-1}$ (wave angle $\psi^* \approx 55^\circ$) at an arbitrary instant of time. Flow direction is from left to right. Body-attached coordinates (s, η) [axes are not to scale].

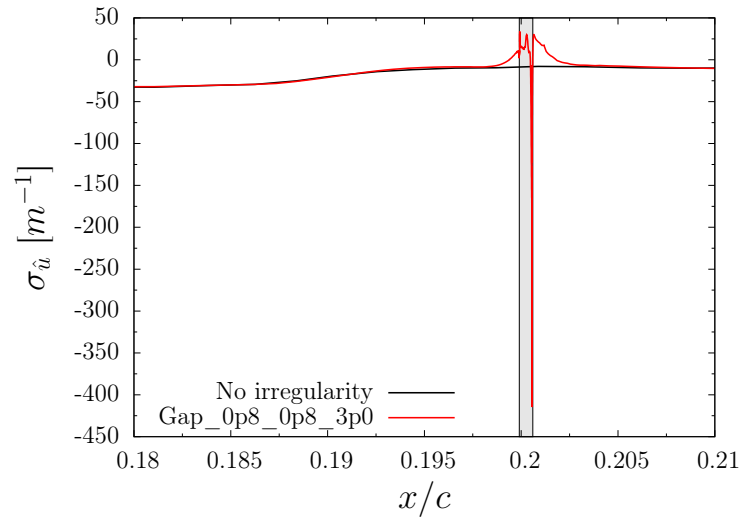


Figure 9.17: Influence of the presence of a regular gap (case *Gap_0p8_0p8_3p0*) on the growth rate of the streamwise velocity disturbance $\sigma_{\hat{u}}$ for an incoming TS wave of frequency $f^* = 2000$ Hz and spanwise wavenumber $\beta^* = 200$ m⁻¹ (wave angle $\psi^* \approx 55^\circ$). The grey vertical area represents the position and width of the gap.

Chapter 10

Concluding remarks

10.1 Summary

The AHLNS methodology

One of the key areas for present and future aircraft design is to achieve a significant drag reduction by delaying the transition of the incoming laminar free-stream air to turbulent state. However, the unavoidable existence of surface imperfections (*e.g.* steps, gaps, etc.) usually plays a key (and detrimental) role in the expected laminar-turbulent transition location.

The compressible *Adaptive Harmonic Linearized Navier-Stokes* (AHLNS) methodology for quasi three-dimensional boundary-layer flows represent a very efficient approach in cases where there are localized large streamwise gradients in the region of study (*e.g.* due to the presence of the above-mentioned surface imperfections). This efficiency is based on to take advantage of the *wave-like* character in streamwise direction of the convective flow instabilities along the *linear growth* stage of the laminar-turbulent transition process.

The implementation of the AHLNS approach into a numerical code has been successfully tested and validated. The test cases highlight key features of the current version of the AHLNS code. The multi-block technique enables the study of surface imperfections with sharps edges. Similarly, the coordinate transformation permits to analyse rounded wall irregularities. The straightforward coupling with the Parabolized Stability Equations (PSE) at the inflow and outflow locations greatly simplifies the boundary conditions at those positions and increases (even more) the numerical efficiency of the AHLNS methodology. Moreover, boundary-layer instability analysis for compressible and quasi three-dimensional flows are feasible also. Of particular interest are the numerical experiments where the streamwise grid-point distribution shows very large variations. Those experiments have confirmed the flexibility of the AHLNS approach for highly non-uniform meshes.

The comparison between standard Harmonic Linearized Navier-Stokes (HLNS)/linear Direct Numerical Simulations (DNS) and AHLNS, in terms of n -factor curves, have shown an

excellent agreement for cases where a two-dimensional surface imperfection interacts with incoming Tollmien-Schlichting (TS) waves. For these cases, it has been proved also that the use of the PSE approach leads to an incorrect description of the spatial evolution of the flow instabilities. Moreover, it has been demonstrated that the grid requirements in streamwise direction for AHLNS are significantly smaller than those needed by HLNS/linear DNS.

To sum up, the initial aim of building a boundary-layer instability analysis tool suitable for cases with (localized) large streamwise gradients have been achieved. This new approach is able to reproduce results from HLNS computations but requiring significantly less numerical effort (as expected).

Influence of steps and humps on the predicted transition location

As an example of one possible application of the AHLNS methodology, a large variety of steps (both forward- and backward-facing steps (FFS/BFS)) and humps placed on a flat plate in a compressible flow has been studied. In order to reduce the whole parametric space, incoming flow properties have been kept fixed. The geometrical parameters which define the surface irregularity (height, shape and length (in the case of humps)) have been systematically varied.

A physical interpretation of the instability results (N -factor curves), based on the local changes on pressure distribution introduced by the presence of the step, is also described with detail. The results of the stability analysis have been related with an expected transition location via the e^N methodology, assuming that a threshold value of $N_{tr} = 9$ triggers the onset of transition to turbulence. The sensibility of the transition location to the value of N_{tr} has been discussed also (setting the value of N_{tr} to 8 and 10). Limits of the e^N methodology and the need of extra information (in the form of frequency spectrum of the disturbances that trigger transition) have been highlighted also.

The Nenni-Gluyas criteria (defined for rectangular steps, but not for humps) is widely used in aeronautical design. It defines a critical Reynolds number based on incoming flow quantities and the rectangular step height. However, the Nenni-Gluyas criteria provides binary information: it is not possible to measure how much the transition location changes if the height of the rectangular step is larger than the critical value. The results of the boundary-layer instability analysis computed with AHLNS agree quite well with the Nenni-Gluyas criteria, i.e. for rectangular steps (both FFS and BFS) with height smaller than the critical value, the presence of the step barely affects the expected transition location. In contrast, if the step height exceeds the critical value, then there is an upstream movement of the transition location towards the position of the step (compared with the reference flat-plate case). The influence of each step and hump has been quantified in terms of this upstream movement of the expected transition location.

The comparison between rectangular (sharp-edged) and shallow (smooth-shaped) surface imperfections reveals that smoothing the shape of sharp-edged wall irregularities is a possible alternative to reduce their detrimental effect in terms of expected transition location. Although this is a well-known effect, the use of the current version of AHLNS allows extending the

number of parametric studies in order to enhance our knowledge and quantify the influence of the shape of the surface imperfection in the expected transition location.

Surface imperfections on HLFC wings

The effect of two-dimensional surface irregularities, in particular gaps, on laminar-turbulent transition on a realistic Hybrid Laminar Flow Control (HLFC) wing at transonic flow conditions has been analysed by means of the AHLNS methodology. These surface imperfections represent the joint between the Leading-Edge Suction Panel (LESP) and the Wing Box (WB). The combined effect of infinite swept-wing configuration, airfoil geometry, transonic flow conditions, and suction represents a qualitative step forward with respect to previous applications of the AHLNS approach, where only zero-pressure gradient and flat-plate configurations were considered. The current status of the AHLNS stability analysis tool allows considering irregular gaps. In these type of gaps, the depth at the upstream and downstream side of the gap is different. To the author's knowledge, instability analysis results using HLNS/linear DNS for such configurations are not available in literature. Thus, the AHLNS code can be used to study more realistic joint configurations not covered by classical criteria for single rectangular FFS/BFS and simple regular gaps.

A regular gap of 0.8 mm depth and 3.5 mm length was chosen as *reference* case in the HLFC-WIN project. In terms of N -factor envelope curve, this *reference* case barely modifies the N -factor curve of the *clean* case (i.e. no surface imperfection). Modifications in depth and length were introduced. However, none of these modifications result in a relevant change of the instability analysis results compared with the *clean* case. Two physical aspects explain this result: first, the length scale difference between the wavelength of the TS waves relevant for the instability analysis (about 6 cm), and the length of the gaps (about 3 mm). Second, the pressure distribution at the wall $\bar{p}_{wall}$ is barely affected by the presence of the *regular* gaps studied in this thesis.

However, for irregular gaps, the corresponding N -factor envelope curves show a significant deviation with respect to the *clean* case. The flow topology of these irregular gaps resembles the typical flow structures found in rectangular steps (BFS/FFS) with height equal to the difference in depth between both sides of the gap (upstream/downstream). Despite this, none of the irregular gaps studied leads to a significant upstream movement of the expected transition location compared with the clean case. This result suggests that there might be some potential to relax the surface tolerances initially specified in the HLFC-WIN project using classical criteria.

10.2 Outlook

The studies presented in this thesis represent a very narrow spectrum of the multiple scenarios where the use of the AHLNS methodology can offer a significant benefit over other approaches. In particular, for spanwise-invariant three-dimensional flows, the effect of two-dimensional

surface irregularities on the spatial evolution of crossflow instabilities (CFI) (stationary and travelling) could be studied by means of the current version of the AHLNS methodology without requiring any upgrade. As an example, a common publication with TU Delft (Westerbeek *et al.* [100]) describes the influence of smooth humps on stationary CFI studied by means of AHLNS and standard HLNS. There is an excellent agreement between HLNS and AHLNS results. Differences (in terms of numerical efficiency) between both approaches are discussed also.

Other examples of flow instabilities where the influence of surface imperfections might alter significantly the spatial linear development are Görtler vortices and *streaks** [18]. Another common publication with TU Delft (Casacuberta *et al.* [101]) shows the influence of FFS in the spatial development of stationary CFI and streaks. This study uses AHLNS, standard HLNS and DNS. Results show a very good agreement between the three methodologies.

Regarding the type of flow, an important area of research are supersonic and hypersonic flows. Clearly, surface imperfections might play a key role in the transition scenario. In principle, the linear stage of the flow instabilities could be analysed by the AHLNS methodology also. However, the linearized Rankine-Hugonist conditions should be implemented at the perturbed shock boundary [102].

Finally, the study of the *receptivity stage* of the laminar-turbulent transition process may be another area where the use of AHLNS could be useful. Hein *et al.* [11] mentioned the strong influence of the initial disturbance spectrum on the laminar-turbulent transition, and the need of modelling the boundary-layer receptivity to external disturbances such as wall-roughness or wall suction and blowing. Traditionally, the characteristics of the inflow disturbance (amplitude shape and phase) for PSE computations is provided by a LST search. In cases where this procedure shows difficulties (e.g. generation of crossflow vortices), the use of AHLNS may represent a useful alternative. A practical example can be found in the work of Guo *et al.* [29].

**streaks* or longitudinal vortices are known in literature as *Klebanoff* modes.

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