EURO Journal on Transportation and Logistics 14 (2025) 100155

Contents lists available at ScienceDirect

EURO Journal on Transportation and Logistics

journal homepage: www.elsevier.com/locate/ejtl

50 years of OR in railway timetabling and rolling stock planning

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ARTICLE INFO

Keywords:
Operations Research
Railway planning
Timetabling
Rolling stock planning
Europe

ABSTRACT

This survey paper discusses the literature on Operations Research (OR) models and algorithms for railway planning in the last decades. Since infrastructure and rolling stock are two resources that are both very capital-intensive and characterize the railway system, we focus on timetabling and rolling stock planning problems. For timetabling, we also classify the literature among two dimensions, namely the decision level (strategic, tactical, operational) and the type of network infrastructure (dependent routes, independent routes). We also discuss robustness aspects in both planning problems.

We focus the discussion of the literature on the applicability of the models in the European context, where different types of trains (high-speed passenger trains, high-frequent suburban trains and freight trains) often share the same tracks and the organization of the railways is usually split between an infrastructure manager and one or more railway undertakings operating the trains.

We conclude the paper with some challenges and future research directions.

1. Introduction

Although the title of this article might suggest that Operations Research (OR) models and algorithms for the railway domain were only developed in the last 50 years, the first applications appeared much earlier. Schrijver (2002) was able to discover that the first paper discussing a transportation problem for a railway application dates back to 1939 (Tolstoı, 1939). Interestingly, a few years later, another railway application appeared in the paper that first described the minimum cut problem. Harris and Ross (1955) study the railway network in the former Soviet Union and affiliated Eastern European countries in a secret report written for the US Air Force in 1955 (downgraded as unclassified in 1999). The authors calculate the maximum flow through the network from origins in the Soviet Union to destinations in Eastern Europe, and then use that to find "the bottleneck" (nowadays known as the minimum cut). With this work, they laid the foundation for the famous paper of Ford and Fulkerson (1956).

In this survey paper, we focus on several (more recent) OR problems arising in railway planning. The railway domain is rich of challenging optimization problems that served as inspiration to many OR researchers. Good examples are the Edelman finalists Canadian Pacific Railway (CP) (Ireland et al., 2004), Netherlands Railways (NS) (Kroon et al., 2009) and Deutsche Bahn (DB) (Borndörfer et al., 2021), who all developed and applied sophisticated OR methods to solve real-world railway applications. With this paper, we want to review the basic models and the most important literature in the last decades of timetabling and rolling stock planning. We limit ourselves to these problems, because both topics use the two most capital-intensive resources to operate a railway system, namely infrastructure and rolling stock, and are both characteristic for railway operations. In addition, the building of infrastructure has a lead time of several decades, while the acquisition of rolling stock usually takes 5–10 years. Therefore, it is important to use these resources in an efficient way. Timetabling and rolling stock planning are both studied in the EU-Rail flagship project,

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https://doi.org/10.1016/j.ejtl.2025.100155
Received 27 February 2024; Received in revised form 9 February 2025; Accepted 20 February 2025
Available online 3 March 2025

MOTIONAL, funded by the European Union and the European railway sector, since there are also still many challenges for the future.

In this paper, we focus on the application of OR models to European railways. In Europe, long distance (often high-speed) passenger trains, regional/suburban passenger trains and freight trains often use the same railway infrastructure. Moreover, in most countries infrastructure management and railway operations are split in different organizations. As a result, there are different challenges in Europe than in North-America or Asia. Therefore, we focus on Operations Research models for European railway planning. For global surveys on railway planning, we refer to Harrod (2012). Other relevant topics in the railway domain are railway crew scheduling and disruption management. For these topics, we refer to recent overview papers by Heil et al. (2020) and Cacchiani et al. (2014), respectively.

1.1. Discussion on classifying the timetabling literature

Given a line plan and an infrastructure network, the *Train Timetabling Problem (TTP)* consists of finding a feasible schedule for the arrival and departure of trains at stations. The line plan is defined as the set of lines that all railway undertakings want to operate. For a passenger operator, a line is defined as a list of stations where the train is supposed to stop. For a freight operator, a line is defined simply by the origin and the destination terminal. Each line also has a certain frequency (e.g., twice per hour).

Railway timetabling can be classified in several ways. We distinguish between timetabling at the three classical decision levels (strategic, tactical and operational) and by the type of network infrastructure (independent routes versus dependent routes).

Regarding the decisions levels, **strategic** timetable planning is about developing timetables that can support decision-making in rail transport design, including adjusting the railway infrastructure, both w.r.t. network topology (e.g., a new track in a station) as well as technical aspects (e.g., modifications to the signaling system). This determines the capacity of a railway network. Usually, several timetables based on different input in terms of rail infrastructure and/or line plan are constructed in the strategic decision phase. Given the railway network and given the line plan, **tactical** timetable planning consists of constructing the yearly timetable. This is done once a year and constitutes the basis for the next level of planning. By **operational** planning we define the process of adjusting the current yearly timetable to create a new (possibly temporary) timetable that takes into account additional needs and constraints. This process can last from 1 day to 12 months ahead of the day of operations for the respective train. For instance, one may want to temporarily adjust the current timetable to account for maintenance activities which will block some tracks for a week. Or, one may want to add a special train to perform some extra service. We assume that the additional needs and constraints that are planned for *do not derive* from some unforeseen disruptive event (like a track failure) or from a primary train daily. These cases are handled by other railway processes, like *disruption management* or standard *dispatching*.

Regarding the way the railway network is modeled in the timetabling literature, either a single **independent** route is considered or multiple **dependent** routes are considered simultaneously. In the first case, the timetable of the route is assumed to be (almost) independent of the timetable of the other routes. This is for instance the case of dedicated high-speed railway lines. However, complex railway networks like the ones in Germany, the Netherlands or Switzerland have a strong inter-dependency between different routes, for instance, to ensure connections between each pair of trains at a major station. For example, at Zurich main station all trains arrive a few minutes before minute .00 and .30 and all trains depart a few minutes after minute .00 and .30. In this way, passengers between many origin stations and departure stations have a service every half an hour. For these types of networks, the timetable has to be constructed for the full network at once. This leads to different optimization models and other computational challenges. As a consequence, this resulted in a different stream of literature and we decided to devote a separate section to each stream.

1.2. Outline of the paper

The remainder of this paper is structured as follows. In Section 2, we discuss two basic models and the literature on railway timetabling for independent routes. Section 3 presents a basic model for network timetabling and station planning in the context of dependent routes. Robustness considerations in timetabling are discussed in Section 4. The rolling stock planning literature is reviewed in Section 5. We conclude this paper in Section 6 with propositions of new research directions and challenges.

2. Timetabling for independent routes

This section deals with timetabling problems that arise from railway networks that can be decomposed into single independent routes. Trains running within such routes have little to no interaction with trains from different routes. For example, high-speed trains running within a dedicated high-speed route usually interact with regional trains only in a few large stations, and this interaction is so limited that it can typically be ignored. Another example is the railway infrastructure in Norway, in which all major routes stem from Oslo forming a star-shaped network.

When dealing with a single independent route, passenger connections with other routes become less relevant and they can usually be ignored. This simplification allows the model to consider a rich set of realistic constraints, and to solve both network and station timetabling simultaneously. As we will see in the next section, this is typically not possible when dealing with dependent routes.

The literature on timetabling for independent routes (see Table 1) is overwhelmingly dominated by two formulations, one that makes use of the (infamous) big-M method and a time-indexed formulation. We will briefly introduce these two formulations before discussing the main recent contributions in strategic, tactical, and operational timetabling. In operational timetabling problems that require fast solution times, authors also consider more “exotic” formulations, which are then solved with custom heuristics.

2.1. Basic big-M formulation

A classical and most exploited way to model train scheduling problems as a mixed integer linear program (MILP) is by means of the so called big-M formulation. The first such formulation was proposed by Carey and Lockwood (1995) and focuses on determining the order in which trains should be scheduled. In this type of model, the order of trains is defined using binary variables and the event times are defined using continuous variables. A more recent and comprehensive basic model corresponds to the one introduced in Mascis and Pacciarelli (2002) for generic job-shop scheduling problems with no-wait and blocking constraints. The railway network is discretized into a set S of block sections. Each block section is preceded by a signal and can accommodate at most one train at a time. Let I be the set of trains. Assuming the route of each train $i \in I$ through the network is given, this can be seen as a sequence $S^i \subseteq S$ of block sections. Then, a real variable t^i_s is associated with each train $i \in I$ and each block section $s \in S^i$ on the route of i, representing the time in which i enters s. In the basic timetabling problem, we are given in input a wanted timetable $\mathbf{T}$. The timetable specifies, for each train, the departure times T^i_s from some special timing points $s \in D^i \subseteq S^i$, for instance at stations. We then have

$$t^i_s \geq T^i_s \qquad i \in I, s \in D^i \tag{1}$$

Next, let τ^i_s be the minimum time for train $i \in I$ to run through track section $s \in S^i$, and let denote by $d^i \in S^i$ the destination of i and by t^i_{s+1} the time i enters the block section following s in S^i, we have

$$t^i_{s+1} - t^i_s \geq \tau^i_s \qquad i \in I, s \in D^i \setminus \{d^i\} \tag{2}$$

Table 1
Classification of timetabling papers for independent routes in 6 categories.

	Big-M	Time-indexed	Others
Strategic	Sartor et al. (2023) Coviello et al. (2023)	–	–
Tactical	Carey and Lockwood (1995) Higgins et al. (1997) Kloster et al. (2023)	Brännlund et al. (1998) Caprara et al. (2002) Caprara et al. (2006) Fischer et al. (2008) Cacchiani et al. (2008) Cacchiani et al. (2010a) Fischer (2015) Caprara (2015)	Gestrelius et al. (2017)
Operational	Forsgren et al. (2013) Bababeik et al. (2019) D'Ariano et al. (2019) Lamorgese et al. (2017) Pellegrini et al. (2017)	Higgins et al. (1999) Luan et al. (2017) Lidén and Joborn (2017) Tan et al. (2020)	Jovanović and Harker (1991) Ingolotti et al. (2004) Burdett and Kozan (2009) Quiroga and Schneider (2010) Albrecht et al. (2013) Ljunggren et al. (2021) Erlandson et al. (2023)

Next, let i and $j \in I$ be distinct trains, and assume $r \in S^i$ and $s \in S^j$ are actually the same block section. Because the block section can accommodate at most one train at a time, then either train i exits r (and enters the next block section in its route) before train j enters s, or j exits s before i enters r. This is immediately translated into the following *disjunctive constraint*:

$$t^i_{r+1} \leq t^j_s \quad \textbf{OR} \quad t^j_{s+1} \leq t^i_r.$$

The above disjunctive constraint can be linearized by applying the so-called *big-M trick*, namely by introducing a suitable large constant M and a binary variable x^{ij}_{rs} which is equal to 1 if train i exits r before train j enters s and is equal to 0 if j exits s before i enters r. Then the disjunctive constraint can be replaced by the following pair of linear constraints:

$$\begin{aligned} t^i_{r+1} &\leq t^j_s - M(1 - x^{ij}_{rs}) \\ t^j_{s+1} &\leq t^i_r - M x^{ij}_{rs} \end{aligned} \tag{3}$$

for all distinct pair of trains (i, j) on $I \times I$ and all incompatible pairs or sections (r, s), with $r \in S^i$ and $s \in S^j$.

As for the objective function, this consists usually in minimizing a cost associated with the deviation of the schedule $\mathbf{t}$ from a timetable $\mathbf{T}$. For instance, for each train $i \in I$, the delay in some timing points $A^i \subseteq S^i$ (as the incoming track in major stations or the final destination): in this case, the objective function can simply be:

$$\sum_{i \in I, s \in A^i} \max(0, t^i_s - T^i_s) \tag{4}$$

where T^i_s is the wanted arrival time in $s \in A^i$.

The objective function (4) can be easily linearized, and so (1)–(2), together with the binary stipulation on the x-variables (3) provide a big-M MILP formulation for the train timetabling problem.

Finally observe that it is not difficult to extend the model to cope with generic incompatible pairs of block sections, more complex section release mechanisms and to include the possibility of alternative routes for each train.

2.2. Basic time-indexed formulation

Big-M formulations like the one described above are compact but have weak relaxations, meaning that they usually return low quality bounds. A different approach for scheduling problems is introduced in Sousa and Wolsey (1992), where binary variables are used to represent events happening in a certain time window. It was first applied to train scheduling problems in Brännlund et al. (1998) and later improved upon by Caprara et al. (2002) and Caprara (2015).

First, the planning time horizon is partitioned into typically, but not necessarily, equally sized time intervals $H = \{h_1, \ldots, h_q\}$. For the sake of simplicity, for $l = 1, \ldots, q$ we let h_l be the starting time of the lth interval.

Then, we introduce, for each train $i \in I$, each block section $s \in S^i$ and each $t \in H$, a binary variable x^t_{is} which 1 if i enters s at time $t \in H$, and 0, otherwise.

Clearly, since every train must enter every block section in its route S^i at some time, we have that

$$\sum_{t \in H} x^t_{is} = 1 \qquad i \in I, s \in S^i \tag{5}$$

Next, we can rewrite constraints (1)–(3) as linear constraints in the new variables. In particular, a train $i \in I$ cannot leave $s \in D^i$ before T^i_s:

$$x^t_{is} = 0 \qquad i \in I, s \in D^i, t < T^i_s \tag{6}$$

Then, a train i cannot travel faster than its minimum running time between two successive block sections:

$$\sum_{h < t + \tau^i_s} x^h_{is+1} \leq 1 - x^t_{is} \qquad i \in I, s \in S^i, t \in H \tag{7}$$

where, for $s \in S^i$, $s+1$ denotes the block section on the route of i next to s.

The fact that not two trains can be at the same time on the same block section can be expressed by the following incompatibility constraints

$$x^t_{ir} + x^h_{js} \leq 1 \tag{8}$$

whenever i, j are distinct trains, $r \in S^i$ and $s \in S^j$ are the same block section (or incompatible block sections), and $t + \tau^i_r > h$ AND $h + \tau^j_s > t$. Indeed, suppose that constraint (8) is violated for a pair $(\bar{i}, \bar{r}, \bar{t}), (\bar{j}, \bar{s}, \bar{h})$, that is $x^{\bar{t}}_{\bar{i}\bar{r}} = x^{\bar{h}}_{\bar{j}\bar{s}} = 1$. This implies that $\bar{r}$ and $\bar{s}$ are incompatible block sections and train $\bar{i}$ enters $\bar{r}$ at time $\bar{t}$ and a distinct train $\bar{j}$ enters $\bar{s}$ at time $\bar{h}$. Now, since $\bar{t}, \bar{h}$ satisfy $\bar{t} + \tau^i_r > \bar{h}$ AND $\bar{h} + \tau^j_s > \bar{t}$, then train $\bar{i}$ exits $\bar{r}$ after $\bar{j}$ enters $\bar{s}$ AND $\bar{j}$ exits $\bar{s}$ after $\bar{i}$ enters $\bar{r}$, and the two trains will be on the same (or incompatible) block section at the same time.

Finally, the objective function can be expressed as

$$\min \sum_{i \in I} \sum_{s \in S^i} \sum_{t \in H} c^t_{is} x^t_{is} \tag{9}$$

where c^t_{is} is the cost of train i entering block section s at time t. Note that any objective function, even non-linear, which only depends on the entry or exit times of the trains in some block sections, and it is separable in such times, can be expressed in the form (9). This is indeed the case of the most common objective function, both in the literature and in the practice.

Note that the constraint of the time-indexed formulation can be easily strengthened in order to obtain stronger formulations. Also, the discretization allows to model rather complex physical and logical requirements by linear constraints. One major problem is that the number of variables and constraints can grow very large, especially if the time-discretization is fine, making the solution process very slow. On the other hand, a too gross time discretization may lead to unrealistic solutions which would turn to be infeasible in practice. For an interesting discussion on these and other issues related to the application of time-indexed formulations to train scheduling, we refer the reader to Harrod (2011).

Finally, it is customary to associate a graph with the variables and constraints of the time-indexed formulation, regarded to as a *time-space network* $G = (V, A)$ (see, e.g., Caprara et al. (2002)). You have a node $v = (i, t) \in V$ for every x_{it} variable of the formulation, which in turn represents entering a specific block section at a given time. An arc $((i, t), (j, t'))$ represents the fact that a train can enter block section j at t' if the train has entered block section i at time t. This graph is also the basis for another family of formulations, which are built by exploiting the oriented paths of the time-space network (see (12)–(14)).

2.3. Strategic timetabling

Being usually more difficult than its tactical and operational siblings, strategic timetabling for independent routes has not received much attention from the academic community. Only recently, two attempts were made into this direction.

Sartor et al. (2023) consider the problem of creating a feasible timetable starting from a given set of required periodic train services. While periodic trains are almost always preferred to non-periodic ones (at least by customers), they create a constraint that can substantially reduce the capacity of a route. In this work, the authors introduce the concept of quasi-periodic strategic timetables, which allows some deviation from strict periodicity in order to expand the set of feasible solutions, while guaranteeing a periodic timetable for the customers. The resulting MILP model is based on a big-M formulation solved by delayed row generation. Based on the outcome of the model, Norwegian Railway Directorate selected 3 out of the 4 scenarios for infrastructure expansion.

Coviello et al. (2023) introduce a new method for the efficient creation sets of strategic (not necessarily periodic) timetables using minimum infrastructure information. The method is based on a multi objective ant colony optimization algorithm and a mixed integer linear programming (MILP) formulation, and was verified on the real-life instance of the Bergen-Oslo railway line in Norway.

2.4. Tactical timetabling

Single line timetabling using a big-M MILP formulation was first discussed by Carey and Lockwood (1995), proposing a mix of heuristic and branching procedures to speed up the computation. Higgins et al. (1997) later proposed a similar formulation and develop a set of methods based on local search, genetic algorithms, tabu search, and hybrid combinations of these to solve the TTP.

Around the same time, time-indexed formulation for this problem were proposed in Brännlund et al. (1998) and Caprara et al. (2002). A bit later, Caprara et al. (2006) developed a Lagrangian heuristic algorithm tailored to real-world conditions of the Italian railway system. Fischer et al. (2008) employ a Lagrangian relaxation combined with a cutting plane approach to manage the substantial number of variables and constraints, focusing on a test instance that represents ten percent of the German network. Cacchiani et al. (2008) modify the time-space formulation to consider train paths instead of stations, and develop a column generation algorithm and local search heuristics to solve the TTP. They verify their algorithms on real-life instances of the Italian railway network. Cacchiani et al. (2010a) extend the model to account for the scheduling of extra freight trains on the network and use a similar Lagrangian heuristic as Caprara et al. (2006). Later, Fischer (2015) introduce ordering constraints improving the relaxation of the formulation and tested on instances of the RAS Informs competition (INFORMS, 2023).

More recently, Kloster et al. (2023) proposed an incremental approach to tactical timetabling based on a big-M formulation solved with a custom row-and-column generation algorithm. The idea is to find a new feasible timetable that is as close as possible to a (possibly infeasible) reference timetable. The route planner can then incrementally modify the reference timetable to steer the result towards the desired direction. The authors report a very good feedback from route planners who tested a prototype of such method on the Oslo-Trondheim line in Norway.

The models presented in this section can sometimes be extended to timetabling for dependent routes in case of an acyclic timetable. Often, heuristics are needed to solve the mixed-integer programming formulations in this case. For example, Gestrelius et al. (2017) developed an incremental fix and release heuristic and an improvement heuristic to solve tactical timetabling problems in the Swedish region around Hallsberg.

2.5. Operational timetabling

In operational timetabling, as in dispatching and disruption management, one has at hand an existing timetable T, and needs to modify T to factor in unforeseen or planned deviations. The modification can involve both the schedule of trains at different locations and their routes and one wants to minimize the distance of the modified timetable from T.

The major difference between operational timetabling and *dispatching* is that for the latter the available computation time is very short, typically a few seconds, while in operational timetabling one may have more time at disposal. Indeed, in dispatching, the modifications to the official timetable T are computed in real-time in order to tame the effects of primary delays and minimize knock-on effects. However, dispatching and operational timetabling share constraints and objectives. As a consequence, all the (huge) body of scientific literature devoted to automatic train dispatching and rescheduling can be regarded as a basis also for operational timetabling. Because the literature on train rescheduling is vast, here we will only mention a few survey papers on this topic as Cacchiani et al. (2014), Corman and Meng (2015), Fang et al. (2015), Lamorgese et al. (2018). Instead, we will consider in more detail the papers specifically devoted to operational timetabling as defined in Section 1.

In this survey we will look only at operational timetabling for some planned activities, and we consider two main streams:

1. Maintenance activities, which involve the closure of some tracks and require rescheduling and rerouting of trains.
2. Addition of individual train service to the current service.

In both cases, the objective will be to minimize the impact on the current timetable. In both cases, the existing body of literature is quite limited.

Before delving into the literature, it is worth mentioning that in the current practice, the operational timetabling process is performed manually and usually alternates between (1) making rough modifications to an existing timetable (e.g., shifting the departure of a train by half an hour) and then (2) making small adjustments to regain feasibility (e.g., reduce or increase the dwell time of some trains in some stations or “bending” the running time). The most time-consuming element of this process is related to the second step, that is to manually eliminate all conflicts that may arise after a timetable has been modified. Currently available commercial tools support the route-planners in various ways, and in particular in identifying train conflicts generated at step

(1). However, to our knowledge, there is no available commercial tool yet for conflict resolution to regain global feasibility and possibly optimality with respect to defined target KPIs like e.g., delay, capacity gain or mobility impact on end-customers.

2.5.1. Planning maintenance activities

As observed in by Lidén (2015) and D'Ariano et al. (2019), the scheduling of preventive maintenance activities and the consequent rescheduling of trains are normally treated separately in two successive phases by infrastructure managers. D'Ariano et al. (2019) refer to the overall problem as the Tactical Traffic and Possession Scheduling Problem (TTPSP) where *decisions about retiming, re-sequencing and re-routing of trains have to be taken, before the operational day, with some knowledge of maintenance actions and traffic perturbations.*

There are already a number of surveys on planning maintenance activities in railroads, some as recent as 2021 (see Lidén (2015), Sedghi et al. (2021)). However, these surveys are centered on the maintenance schedule, and papers are classified according to this focus. In the initial works about maintenance, the planning of the activities was carried out without an exact re-schedule of train movements. We will only mention a couple of references, and focus instead on the papers which deals also with the re-computation of train timetables.

Only maintenance operations. One of the first works dealing with planning track maintenance activities is probably Higgins et al. (1999). They present a time-indexed formulation for scheduling maintenance operations in a corridor. The main limitation is that they do not produce a modified train timetable to take into account track blockages. The impact of the maintenance schedule on the train schedule is only considered by some approximating terms in the objective function. Also the more recent work such as Quiroga and Schnieder (2010) is affected by the limitation of only considering the scheduling of the maintenance (in this case, tamping) activities, without calculating a new feasible schedule for trains.

Integrating train timetabling and maintenance schedule. Albrecht et al. (2013) apply a probabilistic meta-heuristic to find a plan to schedule maintenance works and 50 trains in the 480 km long single-track North Coast Line in Queesland, Australia. The basic idea is to handle maintenance activities as fictitious train services, and then schedule real and fictitious train services simultaneously. The method produces several conflict-free timetables, sparing time of human planners which can instead spend it to choose the preferred timetable. Compared with the current practice, it led up to 34% delay reduction.

Fictitious (virtual) trains are also utilized in Luan et al. (2017) to represent maintenance activities. The model is a MILP time-indexed formulation (Section 2.2) equipped with multicommodity flow constraints to represent different routings. The model is single objective, namely the minimization of the deviation of trains schedule from the wanted one. The basic instance is derived from a portion of the Chinese railway network.

Modeling maintenance activities as fictitious, slow moving trains is smart but cannot be always applied. As observed in Lidén and Joborn (2017), this is not sufficient for situations where (a) a work activity can be interrupted (for letting real trains through), (b) the work closes off several tracks or line segments at the same time, or (c) the work inflicts speed restrictions or other operational restrictions on neighboring tracks. Moreover, there are additional features characterizing maintenance activities: (i) work tasks can be co-located, (ii) the costs have a non-linear dependency on the given shift and possession time and (iii) work force restrictions are differently handled. For this reasons, Lidén and Joborn (2017) introduce a comprehensive time-indexed (see Section 2.2) MILP model which considers maintenance activities and train movements as different type of events. The objective is the weighted sum of multiple terms to take into account the deviation of the train timetable from the wanted timetable, and the cost of executing a maintenance works on specific links (track sections) in specific time periods. The model considers an aggregated view of the train movements and therefore neglects the detailed resolution of conflicts and the meet/pass events precise specification. These are left for a hypothetical second phase, where the timetable problem is solved in detail. They finally consider a set of instances both with single and double tracks. Although it is unclear if the instances are real-life or realistic, the authors have made the valuable effort to make them available at https://github.com/TomasLiden/mwo-data.

Also Forsgren et al. (2013) schedule simultaneously train services and maintenance works (track possession), considering also train cancellations and rerouting. The model is an extended, multi-objective big-M formulation (Section 2.1), which is tested on two complex real-life scenario from the Swedish railway system. The objective cost takes into account train cancellations, potential conflicts in the use of tracks, and the deviation from the original timetable.

Bababeik et al. (2019) also considers the case of a single-track line, but factoring in with uncertain duration of maintenance works. The works will of course make some tracks unavailable to trains, and the schedule of the trains will be affected. A discrete probability distribution of duration is given for each maintenance operation, and this is used to constraint the minimum wanted duration of the same activity. A massive MILP model (based on the big-M model of Section 2.1) is built, and tested on a single instance with a 15 km line and 8 stations (probably real-life, but the authors do not say anything about that).

Also D'Ariano et al. (2019) address the problem of uncertain duration of future maintenance operation. These are handled by considering several alternative scenarios, weighted with their probability to occur. The basic model is the big-M MILP of Section 2.1 with multi-objective function, considering both the expected cost of delaying trains and the expected benefit of maximizing maintenance operations overlap. The two objectives are then combined in a single function by choosing suitable weights. In a second approach (the ϵ-constraint formulation), one of the objectives is instead modeled as a constraint while the other one is minimized. Another interesting feature of the model is to consider re-routing for the trains, which in turn is modeled as multi-commodity flow. This is particular relevant and realistic, because in many practical cases of maintenance track possessions, trains can and must be re-routed on alternative paths. The test instances were derived from the one used for the RAS 2012 railway challenge (INFORMS, 2023).

2.5.2. Adding extra train services

Adding extra services to a given timetable is a very common exercise carried out regularly by infrastructure managers. This is required to satisfy some extra demand, or to schedule work trains. Despite of this, the scientific literature on the topic seems to be not too large. When inserting new services into a given timetable, one is typically allowed, at some cost, to partially modify the schedule of the original trains. In particular, strict periodicity maybe relaxed, connections between trains may be lost, departure or arrival times at stations can be perturbed.

Jovanović and Harker (1991) presents one of the first MILP approaches to operational timetabling, called SCAN. The tool allowed to modify an existing schedule in order to add or delete trains, or meet new requirements. Although the authors claim the SCAN system was implemented and utilized by a class A American railroad company, Norfolk Southern, no details are given on the quality of the resulting plans or the actual quantitative benefits for the company.

In Tan et al. (2021) a MILP big-M model is developed for the problem of adding extra periodic train services to a timetable for high-speed trains. Numerical experiments are performed on 20 instances from Shanghai-Hangzhou High-Speed Railway in China. Besides considering several realistic constraints, such as acceleration and deceleration, the model also factors in periodicity, an important feature when handling passenger trains.

The problem of inserting non periodic extra-trains in a (possibly, partially) periodic timetable is instead addressed by several paper, such as Ingolotti et al. (2004), Burdett and Kozan (2009), Lamorgese et al.

(2017), Erlandson et al. (2023), Tan et al. (2020). These papers differ in the way some constraints are handled, such as losing connections or relaxing periodicity, and the solution methods, which vary from genetic algorithms (Tan et al., 2020) to sequential heuristics (Ingolotti et al., 2004), meta-heuristics (Erlandson et al., 2023; Burdett and Kozan, 2009), and Benders' decomposition (Lamorgese et al., 2017). In some cases, the original timetable cannot be modified at all, at any cost, typically because of contract clauses with the railway undertakings. This particular problem is addressed for instance in Ljunggren et al. (2021), where real-life instances from the Swedish railway network are solved by a variant of the Dijkstra's shortest path algorithm.

A slightly different perspective was investigated by Pellegrini et al. (2017). Here, the authors consider the railway saturation problem and propose the RECIFE-SAT, a MILP-based algorithm based on a hybrid time-indexed formulation with big-M constraints that estimates the capacity of a certain railway line by adding as many trains as possible while maintaining a feasible timetable. This method can then be used to evaluate infrastructure upgrades or other high-level decisions on the amount of trains that should serve a certain line.

3. Timetabling for dependent routes

In complex railway networks, routes cannot be scheduled independently. In addition, planning the stations is a complex tasks. Therefore, timetables are usually constructed in two steps (see Cacchiani et al. (2015)). First, a timetable on the **network** (or macroscopic) level is constructed. In other words, network timetabling aims at creating a feasible schedule with arrival and departure times of all trains at all stations. The considered infrastructure information is accounted for by headway times between the events (the arrival and departure of trains) to be scheduled. In a next step, **station planning** at the microscopic level is done for each (major) station. On microscopic level, individual switches and signals are taken into account.

Almost all European countries operating such a complex network also operate a **cyclic** timetable. This is a timetable that repeats itself after its cycle time. The cycle time is typically one hour (e.g. the Netherlands) or two hours (e.g. Germany). Cyclic timetables are generally preferred by passengers as they are easier to remember, leading to an increased demand. Therefore, many railway undertakings operate such timetables. However, cyclic timetables are also associated with high costs since the timetable must be repeated during the day, during and outside of peak hours (Cacchiani and Toth, 2012).

In the remainder of this section, we present the basic PESP formulation for constructing a cyclic timetable on the network level in Section 3.1. Section 3.2 discusses a basic mixed integer programming formulation for the station planning. Afterwards, we discuss the literature on strategic, tactical and operational level for network and station planning in Sections 3.3–3.8. An overview is presented in Table 2.

3.1. Basic PESP formulation

Most cyclic timetabling models are based on the Periodic Event Scheduling Problem (PESP) as defined by Serafini and Ukovich (1989). In PESP, we are given a cycle period T and event-activity network $G = (V, A)$ where V is the set of events to be scheduled and A is the set of activities linking those events. Each event $i \in V$ represent the arrival or departure of each train service at each station. An activity $(i, j) \in A$ is a directed arc that represents the time difference between two events i and j such that $(i, j) \in A$. The lower and upper bounds for the time duration that activities can take is defined by *activity constraints*. In its simplest form, the PESP is to find a feasible schedule of event times $\pi : V \to \{0, \dots, T-1\}$ satisfying the activity constraints

$$l_{(i,j)} \leq (\pi_j - \pi_i) \bmod (T) \leq u_{(i,j)} \quad \forall (i,j) \in A$$

where $l_{(i,j)}$ and $u_{(i,j)}$ are respectively the lower and upper bounds on the duration an activity $(i, j) \in A$ can take. Note that $l_{(i,j)}$ and $u_{(i,j)}$ must satisfy that $u_{(i,j)} - l_{(i,j)} \leq T - 1$, else the constraint becomes redundant as all time differences between π_i and π_j are allowed. Finally, it can be noted that the formulation above is non-linear due to the modulo function. Let $p_{(i,j)}$ be an integer variable used to model the modulo function MILP formulation of the PESP is

$$l_{(i,j)} \leq \pi_j - \pi_i + T p_{(i,j)} \leq u_{(i,j)} \qquad \forall (i,j) \in A \tag{10a}$$

$$p_{(i,j)} \in \mathbb{N}^0 \qquad \forall (i,j) \in A \tag{10b}$$

$$\pi_i \in \{0, \dots, T-1\} \qquad \forall i \in V \tag{10c}$$

Activities are used to model the movement of trains and passengers in the network, but also the infrastructure and safety requirements ensuring the safe operation of the timetable. As exemplification, we present here 5 common types of activities depicted on the event-activity network shown in Fig. 1 and how to model them using the PESP formulation:

Drive activities represent the time spent by a train traveling from one station to another. The lower bound of a drive activity constraint is defined by the minimum travel time given the distance and the maximum speed of the train between two stations. An upper bound is not necessary but can be defined by the maximum allowed deviation from the lower bound.

Dwell activities represent the time spent by a train at a station. The lower bound of a dwell activity represents the minimum time spent at a station by a train ensuring that the passengers have enough time to board or disembark the train. Similarly to drive activities, the upper bound can be defined by the maximum allowed deviation from the lower bound.

Transfer activities represent the time allocated for the transfer of a passenger from one train to another. Transfer activity constraints provide a lower and upper bound for transfer times.

Turnaround activities represent the reuse of rolling stock after the end of a train service. After the arrival of a train at its terminal station, the same train is often used to serve the line in the other direction. The minimum time restriction on this activity is at least the time that is needed for the driver to get to the other side of the train. An upper bound can be defined to avoid long turnaround times, as a measure aimed at reducing the number of vehicles needed to operate the timetable and thus reduce rolling stock costs.

Safety activities represent infrastructure constraints that guarantee a safe operation if all trains operate according to the timetable. Safety activity constraints define the minimum time difference between the arrival or departure of two trains using the same tracks. Given a headway time $h_{(i,j)}$ between two events i and j, the lower bound is equal to $h_{(i,j)}$ and the upper bound is defined as $u_{(i,j)} = T - h_{(i,j)}$ such that the headway time is accounted for no matter which event happens first during the cycle period.

Many more aspects of the timetable can be modeled using PESP activity constraints. We refer to Liebchen and Möhring (2007) for an extensive description of the modeling capabilities of the PESP for railway timetabling.

3.2. Basic routing formulation

When train timings are pre-defined (or restricted to a discrete set of possible values), routing decisions for trains can be modeled as a node packing problem, as first proposed by Zwaneveld et al. (1996). This is of particular interest in a station context, where arrival and departure times may be the output of a network-wide optimization in a previous

Table 2
Classification of timetabling papers for dependent routes in 6 categories.

	Network	Station
Strategic	Polinder et al. (2021)	Delorme et al. (2001)
	de Graaf (2021)	Sels et al. (2014)
		Jensen et al. (2017)
		Bešinović and Goverde (2019)
		Schmidt et al. (2019)
		Jovanović et al. (2020)
		Weik et al. (2020)
Tactical	Schrijver and Steenbeek (1993)	Zwaneveld et al. (1996)
	Nachtigall (1994)	Zwaneveld et al. (2001)
	Odijk (1996)	Lusby et al. (2011b)
	Peeters (2003)	Caimi et al. (2011)
	Kroon and Peeters (2003)	Caprara et al. (2011)
	Liebchen (2006)	Dewilde et al. (2013)
	Liebchen and Möhring (2007)	Dewilde et al. (2014)
	Liebchen (2008)	Caprara et al. (2014)
	Großmann et al. (2012)	Burggraeve and Vansteenwegen (2017)
	Gattermann et al. (2016)	
	Robenek et al. (2017)	
	Farina (2018)	
	Lindner and Liebchen (2019)	
	Matos et al. (2021)	
	Lindner and Liebchen (2022)	
	Martin-Iradi and Ropke (2022)	
	Polinder et al. (2022)	
Operational	Van Aken et al. (2017a)	–
	Van Aken et al. (2017b)	
	Bešinović et al. (2020)	

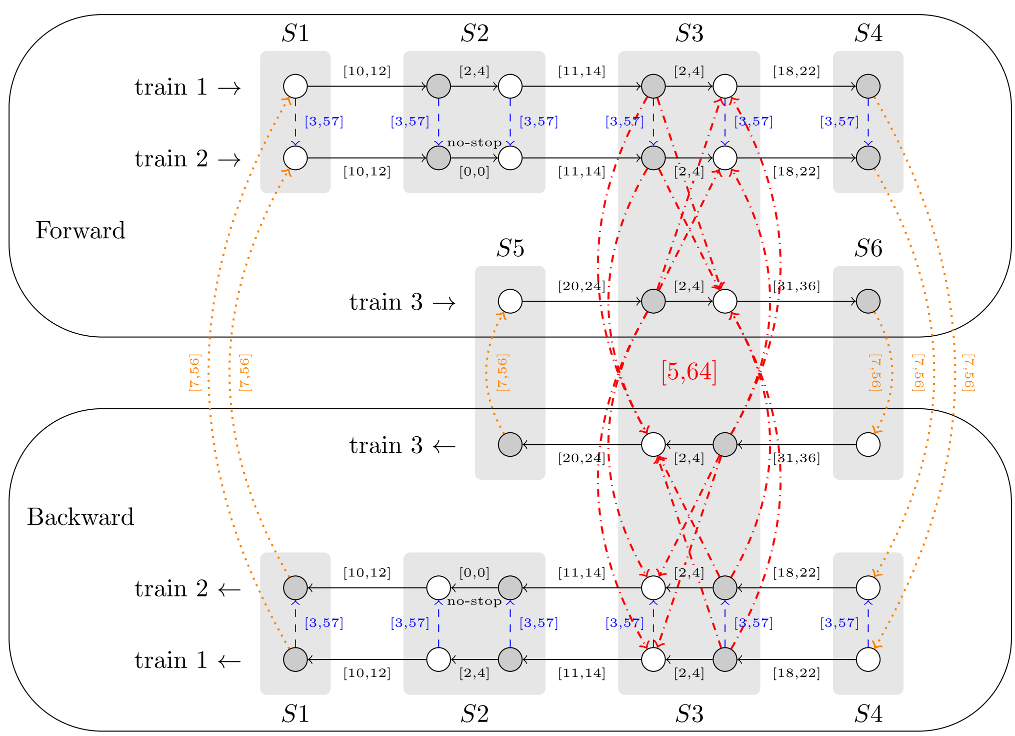

Fig. 1. Example of Event Activity Network for a cyclic timetable of cycle period $T = 60$; The black straight arrows represent the drive and dwell activities of a line, the dashed blue arrows represent the headway activities between trains, the red dashed-dotted arrows represent the transfer activities, and the orange dotted arrows represent the turnaround activities.

planning step and the proposed schedule must now be translated into a microscopically feasible train routing.

Train runs in stations are defined by their routes, as well as the traversal times along these routes. The combination of a physical route and the traversal time of a train along it, is called a *train path* (following Lusby et al. (2011a)). Thus, different paths may use the same route but differ in arrival or departure times. To judge whether two paths are in conflict, spatial and temporal constraints induced by the signaling system have to be respected. In particular, *block sections* referring to infrastructure segments which are exclusively usable by one train at a time, are fundamental for microscopically feasible routing concepts in station areas (for an introduction to infrastructure utilization and rail traffic control according to the blocking time see Pachl (2018)). Since different block sections need not be disjoint, e.g. in the case of merging or splitting routes, some approaches (Caimi et al., 2011; Lusby et al., 2011b; Pellegrini et al., 2014) divide them into multiple smaller, non-overlapping *track sections* to simplify conflict modeling. This has the added benefit of allowing a route-lock sectional-release traffic control system, where occupation for all track sections in a block section must start at the same time, but may end early for some track sections that the train has already freed. This extends the standard route-lock route-release traffic control system, where block sections are always occupied and released as a whole (see also Pellegrini et al. (2014) for a discussion of the two interlocking systems).

The train routing problem now consists of assigning a path to each train to be scheduled that connects its entry and exit points in the analyzed infrastructure. This problem is sometimes also referred to as the train platforming problem, particularly in cases where only one route is considered per platform. The set of all paths must be conflict-free according to the blocking time model. To this end, one has to find all possible physical routes available for a train and – through simulation or analytical evaluation – determine the occupation times of all block and track sections along these routes for the given arrival and departure times. Let us denote T as the set of all trains, then this yields for each train $t \in T$ a set of potential paths P_t, combined into the set of all potential paths $P = \bigcup_{t\in T} P_t$. Note that, while arrival and departure times are fixed, the occupation times themselves need not be discretized and can be expressed with arbitrary precision.

Not all paths are compatible with each other, which can be modeled through a set of conflicts

$$E = \{\{p, p'\} \mid p, p' \in P \text{ are incompatible}\}. \tag{11}$$

There are two types of important incompatibilities: First, all paths belonging to the same train are pairwise incompatible, as only one path can be chosen per train. Secondly, two train paths for different trains that occupy incompatible block sections for some overlapping time interval are also in conflict.

Now, the tuple (P, E) captures the routing problem in the form of a conflict graph. The objective is to select a maximal independent set, i.e. a maximal node set such that no two contained nodes are neighbors, by assigning binary selection variables $x_p \in \{0,1\}\ \forall p \in P$. A solution with objective value equal to the number of trains guarantees a feasible train routing. In its simplest form, this leads to the optimization problem

$$\max \quad \sum_{p\in P} x_p \tag{12}$$

$$\text{s.t.} \quad x_p + x_{p'} \leq 1, \qquad \forall \{p, p'\} \in E \tag{13}$$

$$\boldsymbol{x} \in \{0,1\}^{|P|} \tag{14}$$

Despite its intuitiveness, notice that this formulation suffers from a weak LP-relaxation and a high number of constraints. To remedy this somewhat, one can replace pairwise exclusions with valid clique inequalities, though these have to be carefully selected as their number is exponential. Several papers (Caprara et al., 2011; Dewilde et al., 2013, 2014; Zwaneveld et al., 2001) propose to use cliques of paths belonging to only one or two trains.

Finally, we note that the concept of generic train conflicts is a very flexible tool for setting up the optimization problem: Beyond modeling different traffic control systems as mentioned above, it can also encode desired relations between trains (e.g. routing transfer connections to the same platform) by forbidding those path combinations that violate the desired properties.

3.3. Strategic network timetabling

In complex railway networks, infrastructure can often be a limiting factor affecting the feasibility and quality of timetables. Hence, timetables are sometimes used to improve the quality of strategic decisions such as line planning and railway network design. For line planning, some attempts have been made at integrating (parts of) the line planning and timetabling problem to find better timetables (Michaelis and Schöbel, 2009; Burggraeve et al., 2017; Schöbel, 2017; Yan and Goverde, 2019; Correia Duarte et al., 2023). For railway network design, a strategic timetable is generated by abstracting part of the infrastructure requirements, and used as input for planning steps for network design. This method is employed in various Western European nations, such as Switzerland, Germany, and the Netherlands, but rarely covered in scientific literature (Friesen et al., 2024).

To the best of our knowledge, only a few recent papers explicitly focus on the creation of strategic timetables for railway network design. Polinder et al. (2021) study the creation of strategic cyclic timetables that dismiss headway constraints to find "ideal" passenger oriented timetables that can serve as blueprints for railway network design problems. The model, integrating passenger routing, aims at minimizing the passenger perceived travel time defined as a weighted sum of passenger waiting time and travel time and was validated on real life instances of the Intercity Network of NS. Furthermore, de Graaf (2021) investigates how timetables generated by the model of Polinder et al. (2021) can facilitate the strategic timetable design process. The research concludes that the emphasis of strategic timetables on waiting time and efficient transfers can provide valuable information that can be used to design tactical timetables, for instance in deciding which transfers to prioritize from a passenger perspective.

3.4. Tactical network timetabling

In the tactical timetabling phase, the goal is to design a yearly timetable considering the network's capacity and service requirements. In complex railway networks, tactical network timetabling is often modeled as PESP (see Section 3.1) and solved using constraint programming or mixed integer programming methods. Notable early contributions include the development of the DONS decision support system for cyclic timetabling at NS by Schrijver and Steenbeek (1993), the development of the Cycle Periodicity Formulation for the PESP by Nachtigall (1994), and the work of Odijk (1996) on constraint generation algorithms for the PESP.

In particular, the Cycle Periodicity Formulation (CPF) developed by Nachtigall (1994) is the most studied formulation for the PESP. The CPF defines activity duration as variables and uses a new set of constraints to enforce the cyclic nature of the solution within the associated event-activity network. By doing so, fewer integer variables are created and tighter constraints are defined, leading to a stronger formulation for the linear programming formulation than that of Serafini and Ukovich (1989).

Peeters (2003) further studied applications of the CPF to the Dutch Railway Network using different objective functions, including the minimization of passenger travel times, the maximization of the timetable robustness, the minimization of required rolling stock, and the minimization of violation of soft constraints. Such timetables are generally created assuming that the travel time between stations is fixed. Kroon and Peeters (2003) extend the problem by considering variable travel times instead of fixed travel times, increasing the solution space and facilitating the search for feasible timetables using similar techniques as models using fixed travel times. Liebchen (2006) develop different methods to solve the PESP using Cut-and-Branch Algorithms for Integer Programs, Constraint Programming, and even Genetic Algorithms based on the CPF. Liebchen (2008) further describes how these optimization methods are used in the creation of a new Berlin subway timetable in December 2004. Few papers also aim at tackling cyclic timetabling for large instances by decomposing the problem into sub-problems. Lindner and Liebchen (2019) introduce a constraint programming based heuristic to generate T-partitions in the event activity graph to solve smaller problems and merge solutions. This approach is further explored by Lindner and Liebchen (2022) in which a timetable merging crossover heuristic is developed.

Polinder et al. (2022) introduce an iterative approach for constructing a tactical timetable with the objective of minimizing total perceived travel time of all passengers together. They iterate between a formulation which computes an ideal timetable without considering infrastructure constraints (Polinder et al., 2021) and a Lagrangian-based heuristic that satisfies the infrastructure constraints by modifying event times as little as possible.

Beyond common constraint programming approaches, some recent work takes advantage of developments in the performance of solvers for the Satisfiability (SAT) problem and develop new SAT-based formulations for network timetabling problems. Großmann et al. (2012) are the first to introduce a reformulation of the PESP as a SAT problem and provide results on the German network that outperform constraint programming solvers in finding feasible solutions. Gattermann et al. (2016) extend the SAT formulation of Großmann et al. (2012) by integrating passenger routing in the form of an objective aiming at minimizing passenger travel time, becoming a Maximum Satisfiability (MaxSAT) problem formulation. Matos et al. (2021) later develop a state-of-the-art reinforcement learning approach to solve the new MaxSAT formulation, outperforming existing methods over some of the publicly available instances of the PESPlib library.

Finally, some methods deviate from the PESP in the generation of tactical network timetables. For example, Robenek et al. (2017) consider a model between cyclic and acyclic timetabling and display the interest of keeping regularity for a fixed set of lines instead of all lines to solve the problem on a network level. The model is solved using a simulated annealing method and verified on the Israeli railway network. We further refer to Farina (2018) and Martin-Iradi and Ropke (2022) for recent state-of-the-art approaches to cyclic timetabling based on the acyclic TTP time-space formulation of Cacchiani et al. (2010a) using respectively local search methods and a column generation approach, both tested on parts of the Danish railway network.

3.5. Operational network timetabling

In the operational timetabling phase, the aim is to adjust the tactical timetable to account for planned changes in network infrastructure, such as maintenance or construction works that leave parts of the network unavailable. Since we do not want the whole network to be impacted by local changes, the alternative timetables should deviate as little as possible from the tactical timetable. In the process of adjusting a timetable, different changes from the tactical timetable can be expected, such as train retiming, i.e. changes in the scheduled event times, train reordering, i.e. changing the order of arrival of trains at stations, train cancellation, i.e. the complete removal of a train service or line from the timetable, or short-turning, i.e. changes in turnaround activities in stations close to the disruption location.

To the best of our knowledge, operational network timetabling has been the focus of only a small number of scholarly works. Van Aken et al. (2017a) extend the classical PESP model (Section 3.1) and introduce the Train Timetable Adjustment Problem (TTAP), in which the goal is to generate a timetable that minimizes the deviation from the original timetable to handle infrastructure maintenance. This model is tested on parts of the Dutch network. In this paper, short-turning is fixed in a preprocessing step. Due to the large complexity of the model, Van Aken et al. (2017b) develop solving methods and reduction techniques to tackle larger and more complex instances of the full Dutch network. This paper further extends the TTAP by introducing and evaluating different short-turning procedures. We also refer to Bešinović et al. (2020) for an extension of the model of Van Aken et al. (2017a) to consider freight trains in addition to the passenger trains in the timetable adjustment process.

3.6. Strategic station planning

In the following, we discuss the current state-of-technique for planning in the station context. In particular, we discuss methodological approaches for determining route plans and platform assignments for given macroscopic train routing requirements.

As local station planning is typically performed as a follow-up task after generating a (macroscopically) feasible timetable, strategic timetabling in the local timetable context is relatively rare. The topic has seen increased interest in the context of capacity planning and bottleneck resolution, as attractive (customer-friendly) network timetable concepts suffer from capacity restrictions in junctions or station areas. Given the change of paradigm in railway infrastructure planning towards timetable-driven network and infrastructure design, long-term station planning is hence also concerned with evaluating the characteristics of the available station infrastructure and developing it to meet growing travel demands.

To this end, station capacity, i.e. the number of trains that can be operated on the infrastructure, needs to be assessed. UIC Code 406 discusses the application of the well-known timetable compression method including quality-related threshold levels to assess infrastructure utilization in stations (International Union of Railways, 2013). An earlier method proposes to saturate the station by maximizing the number of trains operated in a given time frame through heuristic solutions for constraint programming and set packing models (Delorme et al., 2001). Successive insertion of additional trains in a timetable concept has also been used to assess the overall number of trains that can be operated in the station area (Sels et al., 2014). Drawing from scheduling approaches, Jovanović et al. (2020) proposes to optimize route sequence and to model utilization within junction areas using a graph-coloring method.

(Max,+)-approaches with fix or flexible timing of resource usage have also been applied in the station context, allowing the use of corresponding stability margins or recovery times as robustness metrics (Bešinović and Goverde, 2019). Another option is to use stochastic petri nets allowing to keep certain precedence constraints and timings (Schmidt et al., 2019; Burkolter, 2005; van der Aalst and Odijk, 1995). In order to abstract from a fix routing or timetable scheme, ensemble-averaging approaches have been described to assess infrastructure usage. Jensen et al. (2017) describe a statistical approach to assess capacity based on randomization of train sequences in the network. Weik et al. (2020) build on this method to describe an application-driven procedure to evaluate station capacity in Sweden.

3.7. Tactical station planning

On the tactical level, determining microscopically feasible platform assignments and station route plans for a given macroscopic network timetable concept is a common problem in railway timetabling. Microscopic feasibility is defined based on the blocking time concept which incorporates the design of the signaling system, as introduced in Section 3.2. There, we also described the node packing formulation (Zwaneveld et al., 1996), which is a common choice of model for this problem that forms the basis of multiple subsequent works (Zwaneveld et al., 2001; Caprara et al., 2011; Dewilde et al., 2013, 2014; Caprara et al., 2014).

Other approaches to find an efficiently solvable model formulation focus on the locality of conflicts: Lusby et al. (2011b) present a set packing model where *tints* (time interval track sections) signify the occupation of a certain track section in a certain time slot. The optimization problem then consists in choosing a set of train paths such that each tint is used by at most one path. Following an adjacent idea, Caimi et al. (2011) represent the problem as a multicommodity-flow model: The possible paths for a train are translated into a resource tree whose edges correspond to occupied track sections. With binary variables for all edges, choosing a path for a train is now equivalent to choosing a unit flow through the resource tree that respects standard flow constraints. Conflict constraints can then be formulated per track section, where the occupation times form an interval graph in which maximal cliques can be found efficiently.

Finally, it is also possible to model occupation and release times themselves as variables of a MILP model, with constraints requiring that for each pair of trains on the same track section, the release time of one precedes the occupation time of the other. This approach is used in Sels et al. (2014) and the timetabling model of Burggraeve and Vansteenwegen (2017). As an outlook, this modeling technique is also found in works on dispatching where it allows adding delays to different parts of the train path (Pellegrini et al., 2014).

3.8. Operational station planning

Operational station planning, i.e. the short-term re-planning in the station context for the purpose of maintenance or a time-limited increase in service, has received significantly less scientific attention. The reason for this presumably is that a local adaptation of the timetable or routing scheme in a single station is rarely sufficient in re-planning traffic. In principle, both manual adaptations of the timetable as well as a complete re-planning using methods of tactical station planning are possible depending on the size of the expected disruption. Robust routing approaches, such as the simultaneous scheduling of backup tracks allowing to cope with temporary inaccessibility of some tracks or platforms, to a certain extent, are applicable in this domain (cf. Caprara et al. (2010)).

4. Considerations of robustness in timetabling

Although the goal is often to construct efficient timetables, such timetables can easily become infeasible in the face of delays occurring at the operational level, leading to further delays and possibly train service cancellations. Therefore, many papers in the literature aim at creating timetables that are not only efficient but also robust to delays and delay propagation. A common approach to robust timetabling is the addition of buffer times in the planning phase, representing empty time slots that can be used to absorb delays. Decisions about the length and place of those buffer times in a timetable is a large part of the robust TTP (Cacchiani and Toth, 2012). See for research on the use of stochastic programming (Kroon et al., 2008b; Fischetti et al., 2009), light robustness (Fischetti et al., 2009; Goerigk et al., 2013; Cacchiani et al., 2020), recoverable robustness (Liebchen et al., 2009; Cicerone et al., 2009; Goerigk and Schöbel, 2014), and multi-objective methods (Yan et al., 2019) to tackle robustness in the TTP.

Improving robustness has received similar attention in the station context: While different metrics and concepts of robustness exist (cf. the survey by Lusby et al. (2018)), it is generally understood to mean the reduction or limitation of knock-on delays arising from small initial delays of arriving or departing trains.

One metric that has been proposed for measuring robustness is the maximization of buffers (i.e. time intervals) between trains occupying the same track sections, often weighted such that more importance is given to increasing particularly small buffers. The reasoning is straightforward: Any initial delay that is smaller than the buffer to the next train will not cause a knock-on delay. An early example of this approach is from Caimi et al. (2005), where a local search heuristic attempts to widen the buffers between trains starting from an initial feasible solution. Dewilde et al. (2013, 2014) define robustness as the minimization of weighted real travel times of passengers in case of small disturbances, but also improve it via a weighted buffer maximization heuristic: Through repeated iterative application of a route choice module, a timetabling module (which slightly shifts arrival or departure times) and (in Dewilde et al. (2013)) a platforming module, the algorithm reaches a routing with improved spread of the trains.

Buffer maximization also forms part of the two-step approach by Burggraeve and Vansteenwegen (2017): First, the route choice of trains in the station is optimized with the objective of using the infrastructure evenly. Only in a second step, a timetabling module is used to find an optimal schedule that maximizes the buffers between train pairs on these fixed routes. Note that this approach establishes its own timetable and is thus not easily compatible with a given network timetable optimizer, but could instead be used as a decomposition step for the timetabling of independent routes.

Other robustness criteria have also been studied in the literature: A heuristic method by Bešinović and Goverde (2019) combines multiple metrics, aiming for small capacity occupation, small average train delay and balanced use of infrastructure resources. These objectives are evaluated in subroutines and a greedy multi-start local search heuristic is used to replace routes based on problem-specific route permutation rules. Finally, Caprara et al. (2014) propose a scenario-based approach to model and minimize worst-case delay propagation directly, following concepts of recoverable robustness.

5. Rolling stock planning

Rolling stock is the most expensive and critical resource for a railway undertaking. Rolling stock refers to the powered and unpowered vehicles required to move passengers or freight. In the early literature on rolling stock scheduling, the rolling stock units often correspond to locomotives. In the last decades, passenger railway operator mainly use Electric Multiple Units (EMUs). An EMU is a powered train unit that can drive by itself and can be combined with other EMUs to form a longer train.

Based on the required train paths, decided in train time timetabling, that must be covered, the goal in rolling stock scheduling is to minimize the total cost to operate the rolling stock. This typically means that the rolling stock units should move empty (typically for repositioning) as little as possible. Another related criterion is the minimization of the number of rolling stock units required to cover the transportation demands. Various technical and functional constraints need to be satisfied, in particular the available number of rolling stock units of each type, the eligibility of a rolling stock unit to operate on its assigned train paths and specific requirements on maintenance of rolling stock units.

The literature has initially focused on the so-called locomotive assignment problem, which is the most simple version of the deterministic rolling stock scheduling problem. Additional constraints or criteria have later been considered in the problem.

5.1. Freight rolling stock

Ziarati et al. (1997) model the locomotive assignment problem as a multi-commodity flow problem with supplementary constraints, and propose a Dantzig–Wolfe decomposition method. To solve a similar problem, Ziarati et al. (1999) present a branch-and-cut approach. The approaches presented in these two papers are validated using real-life data from the Canadian National railway company. Rouillon et al. (2006) improve the approaches of Ziarati et al. (1997) and Ziarati et al. (1999) by introducing a new backtracking mechanism, associated branching methods and new ways to compute upper bounds.

Fuegenschuh et al. (2008) present a multi-commodity minimum cost flow model, written as an integer linear program, that considers cyclic departures of trains, time windows on starting and arrival times, network load-dependent travel times, and that cars can be transferred between trains. Several improvements of the integer linear program are proposed, together with a solution approach that uses a randomized greedy heuristic combined with a standard mathematical programming solver. The approach is validated on real-world instances of the German railway company Deutsche Bahn.

Following Vaidyanathan et al. (2008a), Vaidyanathan et al. (2008b) solve a rather general version of the rolling stock scheduling problem. In particular, the authors consider locomotive fueling constraints and locomotive servicing constraints. Both types of constraints require that every locomotive visits a fueling station or a servicing station every time it has reached a given number of miles. An integer programming model is proposed and, as the model has a huge number of variables for relevant instances, an aggregation–disaggregation based algorithm is developed to solve the problem in a few minutes. Computational experiments validate the proposed algorithm on the real-life data of a U.S. railway company.

Recently, Ortiz-Astorquiza et al. (2021) study the locomotive assignment problem for the Canadian national railway company. Two integer linear programming models are proposed, whose originalities lie in the way in which the decisions on the operating mode of each train is modeled. Various improvements are presented for one of the models, as well as a Benders decomposition–based algorithm to determine feasible solutions. These contributions are validated by convincing computational experiments.

5.2. Passenger rolling stock

Cordeau et al. (2000) and Cordeau et al. (2001a) solve the problem of assigning locomotives and cars in passenger transportation, Cordeau et al. (2001b) propose a multi-commodity network flow-based model. A branch-and-bound method that relies on a Benders decomposition approach is introduced, and validated on real-life instances, in particular from VIA Rail Canada.

Rolling stock scheduling problems in passenger transportation are also studied in Abbink et al. (2004), Fioole et al. (2006) and Alfieri et al. (2006), with computational experiments relying on industrial instances of NS (the main Dutch Railway operator). Abbink et al. (2004) present a model that optimizes the allocation of train units to the railway lines, and whose application shows better results compared to manual planning. This model is used in the tactical planning phase. Alfieri et al. (2006) focus on optimizing the numbers of rolling stock units of different types based on an integer multicommodity flow model. The coupling and uncoupling of rolling stock units to meet the demand in number of passengers are taken into account, together with the shunting constraints in stations. Focusing on optimizing the rolling stock circulation in the operational phase, Fioole et al. (2006) consider multiple objectives, in particular the maximization of service quality and reliability instead of only the total operational cost. Moreover, the proposed model allows trains to be combined and split. The method developed in this paper resulted in an application that is used on a regular basis at NS (Kroon et al., 2009). This application was part of the work NS won the Franz Edelman Award in 2008.

About a decade later, DB reached the Edelman final with their application of optimizing algorithms for rolling stock scheduling. Their approach, based on a hypergraph model (see Borndörfer et al. (2016)), allowed to not only take the order of the train units into account but also their orientation. The latter is especially relevant for scheduling ICE train units, where first class carriages and second class carriages are on the opposite sides of the train. In addition, maintenance and regularity constraints are considered.

Cacchiani et al. (2010b), Cacchiani et al. (2013) and Cacchiani et al. (2019) propose a heuristic to solve a real-world rolling stock scheduling problem where a set of train units, each with a cost and a capacity in terms of number of available seats, must be assigned to a set of trips.

5.3. Robust rolling stock scheduling

The robustness of rolling stock plans for passenger trains is investigated in Nielsen et al. (2012), Cacchiani et al. (2012) and Kroon et al. (2015). In these three papers, the proposed approaches are validated on real-life instances of NS. Nielsen et al. (2012) introduce a generic framework for real-time disruption management of rolling stock schedules. A rolling horizon approach is proposed, where schedules are regularly adjusted, and computational experiments are conducted on a set of disruptions. Also to deal with major disruptions, Cacchiani et al. (2012) present a two-stage optimization model, formalized as a mixed integer linear program, to determine robust rolling stock schedules. Benders decomposition is used to solve the linear relaxation of the mixed integer linear program, and to derive a heuristic to determine robust schedules. The computational experiments show that it is much easier to recover from robust rolling stock schedules than from non-robust rolling stock schedules. Kroon et al. (2015) also focus on real-time rolling stock rescheduling in case of large-scale disruptions, and present a simulation–optimization framework. Note that simulation and optimization are often combined to tackle railway rescheduling problems. The proposed framework takes dynamic passenger flows into account, as passengers search for alternative routes in case of disruptions. The numerical results show that the average delay of the passengers can significantly be reduced.

Tréfond et al. (2017) also study the problem of robust rolling stock scheduling for passenger trains. After characterizing robustness indicators, the paper focuses on the determination of the turning times of rolling stock units at stations in a schedule to absorb potential delays. An approach with multiple steps is proposed to determine robust rolling stock schedules. Using a simulation model and real-life instances from the French national railway company, the robustness indicators are significantly improved while maintaining low operating costs and satisfying the maintenance requirements.

More details can be found in the review on robustness in railway planning of Lusby et al. (2018), where a section is dedicated to robustness in rolling stock planning.

5.4. Rolling stock stabling

Passenger train units have to be parked and serviced during the night. This problem is called rolling stock stabling, which is a relevant and hard problem to solve when infrastructure is limited. This is especially the case in dense urban areas. Freling et al. (2005) were the first to study the so-called train units shunting problem, where arrival and departing units need to be matched, parked, cleaned, and routed from the station to a parking track and back. They solved all these subproblems in a sequential way. Kroon et al. (2008a) considered a partial integration of the matching and parking step and solved this problem with a mixed-integer programming model. van den Broek et al. (2022) considered a further integration of all steps and solved it with a local search heuristic.

5.5. Integrated planning

Train timetabling remains currently most often not concerned by rolling stock planning, typically because infrastructure managers do not own or are not in charge of the rolling stock. However, a better integration of these two types of decisions could help to improve the profitability of train operators but also, and maybe more importantly, to better use infrastructure and rolling stock by offering both a cheaper and more effective service. The integration of train timetabling and locomotive assignment is studied in Xu et al. (2018). A three-dimensional state-space–time network is introduced, and the problem is modeled as a minimum cost multi-commodity network flow problem with incompatible arcs and integer flow restrictions. A Lagrangian heuristic is proposed, and computational results are reported to illustrate the benefits of integrating timetabling and locomotive planning.

An interesting remark in Lusby et al. (2018) is that rolling stock planning "is one area of railway planning in which the robustness introduced in the earlier planning problems, in particular line planning and timetabling, can be adversely affected". This motivates the research on the integration of timetabling and rolling stock scheduling decisions, to better balance the buffers required to handle disruptions.

To our knowledge, Dauzère-Pérès et al. (2015) are the only ones to have tackled the integration of rolling stock scheduling and crew scheduling decisions. A Lagrangian relaxation heuristic is proposed, and promising results are obtained on real-life instances of a French region.

6. Conclusions and directions for future research

In this paper, we looked back at 50 years of applying Operations Research methods in railway planning. The two most important and domain specific planning problems in railways are timetabling and rolling stock planning. We classified the literature in railway timetabling in decision levels (strategic, tactical and operational) and by the type of network infrastructure. For the latter, we consider two extreme cases, namely independent routes where the timetable of one route is completely independent of the other routes like in a star network and dependent routes, where complex railway networks with many inter-dependencies are considered. In these networks, a distinction between network level and station level is necessary to be able to solve real-world instances. Moreover, such networks often consider cyclic timetables. We think that this distinction is not only useful for classifying the literature as we did, but also from other perspectives. For instance, when introducing new European legislation the topology of the network in different countries could be considered. Moreover, we considered the state-of-the-art in rolling stock planning including its link with infrastructure planning in the rolling stock stabling problem.

Both timetabling and rolling stock planning received much attention since the 1990s, when the first optimization models and solution methods for these problems were developed. These initial approaches covered the tactical level, i.e. construction of the annual timetable or rolling stock plan. At that time, only small-scale instances could be solved.

In the first decade of the 21st century some major achievements have been accomplished, for instance the mathematical model used to construct the timetable of the Berlin subway in December 2004 and the use of OR methods for both timetabling and rolling stock planning to develop and introduce the completely new timetable in the Netherlands in December 2006.

In the last 10 years, new models and solution methods have been introduced in the literature to tackle the strategic and operational planning level. However, to the best of our knowledge, these methods have hardly been applied in practice, in particular, in the context of complex networks with dependent routes. Therefore, we consider this as a major challenge for the European railway sector.

Optimization methods on the strategic level are becoming more important, because if we look at the coming decades where on one hand rail plays a major role in the European Green Deal, but on the other hand financial sources for large investments in the railway infrastructure are limited, an efficient use of the railway infrastructure is required.

At the operational level, many challenges are resulting from the ageing infrastructure that needs a lot of maintenance in the coming years.

Other important topics for future research are the further evaluation and development of cross-links between network timetabling and station planning in complex networks, and the integrating of different planning problems. Especially, the integration of timetabling and rolling stock planning has the potential to reduce the capital and operational costs of European railways. The split in many European countries between infrastructure manager and railway undertaking makes this also an organizational challenge. Fairness considerations and game-theoretic approaches could therefore be interesting research directions as well.

CRediT authorship contribution statement

Pedro José Correia Duarte: Writing – original draft. **Stéphane Dauzère-Pérès:** Writing – original draft. **Dennis Huisman:** Writing – review & editing, Writing – original draft. **Carlo Mannino:** Writing – review & editing, Writing – original draft. **Giorgio Sartor:** Writing – review & editing, Writing – original draft. **Norman Weik:** Writing – original draft. **Philipp Widmann:** Writing – original draft.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Dennis Huisman reports financial support was provided by Horizon Europe. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

Most research leading to this paper has been part of EU-Rail flagship project MOTIONAL, funded by the European Union under Grant Agreement 101101973 (call HORIZON-ER-JU-2022-01).

References

Abbink, E., Van den Berg, B., Kroon, L., Salomon, M., 2004. Allocation of railway rolling stock for passenger trains. Transp. Sci. 38 (1), 33–41.

Albrecht, A.R., Panton, D.M., Lee, D.H., 2013. Rescheduling rail networks with maintenance disruptions using problem space search. Comput. Oper. Res. 40 (3), 703–712.

Alfieri, A., Groot, R., Kroon, L., Schrijver, A., 2006. Efficient circulation of railway rolling stock. Transp. Sci. 40 (3), 378–391.

Bababeik, M., Zerguini, S., Farjad-Amin, M., Khademi, N., Bagheri, M., 2019. Developing a train timetable according to track maintenance plans: A stochastic optimization of buffer time schedules. Transp. Res. Procedia 37, 27–34.

Bešinović, N., Goverde, R.M., 2019. Stable and robust train routing in station areas with balanced infrastructure capacity occupation. Public Transp. 11, 211–236.

Bešinović, N., Widarno, B., Goverde, R.M., 2020. Adjusting freight train paths to infrastructure possessions. In: 2020 IEEE 23rd International Conference on Intelligent Transportation Systems. ITSC, IEEE, pp. 1–6.

Borndörfer, R., Eßer, T., Frankenberger, P., Huck, A., Jobmann, C., Krostitz, B., Kuchenbecker, K., Mohrhagen, K., Nagl, P., Peterson, M., et al., 2021. Deutsche bahn schedules train rotations using hypergraph optimization. INFORMS J. Appl. Anal. 51 (1), 42–62.

Borndörfer, R., Reuther, M., Schlechte, T., Waas, K., Weider, S., 2016. Integrated optimization of rolling stock rotations for intercity railways. Transp. Sci. 50 (3), 863–877.

Brännlund, U., Lindberg, P.O., Nou, A., Nilsson, J.-E., 1998. Railway timetabling using Lagrangian relaxation. Transp. Sci. 32 (4), 358–369.

van den Broek, R., Hoogeveen, H., van den Akker, M., Huisman, B., 2022. A local search algorithm for train unit shunting with service scheduling. Transp. Sci. 56 (1), 141–161.

Burdett, R.L., Kozan, E., 2009. Techniques for inserting additional trains into existing timetables. Transp. Res. Part B: Methodol. 43 (8–9), 821–836.

Burggraeve, S., Bull, S.H., Vansteenwegen, P., Lusby, R.M., 2017. Integrating robust timetabling in line plan optimization for railway systems. Transp. Res. Part C: Emerg. Technol. 77, 134–160.

Burggraeve, S., Vansteenwegen, P., 2017. Robust routing and timetabling in complex railway stations. Transp. Res. Part B: Methodol. 101, 228–244.

Burkolter, D., 2005. Capacity of Railways in Station Areas using Petri Nets (Ph.D. thesis). ETH Zurich.

Cacchiani, V., Caprara, A., Galli, L., Kroon, L., Maróti, G., Toth, P., 2012. Railway rolling stock planning: Robustness against large disruptions. Transp. Sci. 46 (2), 217–232.

Cacchiani, V., Caprara, A., Toth, P., 2008. A column generation approach to train timetabling on a corridor. 4OR 6, 125–142.

Cacchiani, V., Caprara, A., Toth, P., 2010a. Scheduling extra freight trains on railway networks. Transp. Res. Part B: Methodol. 44 (2), 215–231.

Cacchiani, V., Caprara, A., Toth, P., 2010b. Solving a real-world train-unit assignment problem. Math. Program. 124, 207–231.

Cacchiani, V., Caprara, A., Toth, P., 2013. A Lagrangian heuristic for a train-unit assignment problem. Discrete Appl. Math. 161 (12), 1707–1718.

Cacchiani, V., Caprara, A., Toth, P., 2019. An effective peak period heuristic for railway rolling stock planning. Transp. Sci. 53 (3), 746–762.

Cacchiani, V., Galli, L., Toth, P., 2015. A tutorial on non-periodic train timetabling and platforming problems. Euro J. Transp. Logist. 4 (3), 285–320.

Cacchiani, V., Huisman, D., Kidd, M., Kroon, L., Toth, P., Veelenturf, L., Wagenaar, J., 2014. An overview of recovery models and algorithms for real-time railway rescheduling. Transp. Res. Part B: Methodol. 63, 15–37.

Cacchiani, V., Qi, J., Yang, L., 2020. Robust optimization models for integrated train stop planning and timetabling with passenger demand uncertainty. Transp. Res. Part B: Methodol. 136, 1–29.

Cacchiani, V., Toth, P., 2012. Nominal and robust train timetabling problems. European J. Oper. Res. 219 (3), 727–737.

Caimi, G., Burkolter, D., Herrmann, T., 2005. Finding delay-tolerant train routings through stations. In: Operations Research Proceedings 2004: Selected Papers of the Annual International Conference of the German Operations Research Society (GOR). Jointly Organized with the Netherlands Society for Operations Research (NGB) Tilburg, September 1–3, 2004. Springer, pp. 136–143.

Caimi, G., Chudak, F., Fuchsberger, M., Laumanns, M., Zenklusen, R., 2011. A new resource-constrained multicommodity flow model for conflict-free train routing and scheduling. Transp. Sci. 45 (2), 212–227.

Caprara, A., 2015. Timetabling and assignment problems in railway planning and integer multicommodity flow. Networks 66 (1), 1–10.

Caprara, A., Fischetti, M., Toth, P., 2002. Modeling and solving the train timetabling problem. Oper. Res. 50 (5), 851–861.

Caprara, A., Galli, L., Kroon, L., Maróti, G., Toth, P., 2010. Robust train routing and online re-scheduling. In: 10th Workshop on Algorithmic Approaches for Transportation Modelling, Optimization, and Systems. ATMOS'10, Schloss-Dagstuhl-Leibniz Zentrum für Informatik.

Caprara, A., Galli, L., Stiller, S., Toth, P., 2014. Delay-robust event scheduling. Oper. Res. 62 (2), 274–283.

Caprara, A., Galli, L., Toth, P., 2011. Solution of the train platforming problem. Transp. Sci. 45 (2), 246–257.

Caprara, A., Monaci, M., Toth, P., Guida, P.L., 2006. A Lagrangian heuristic algorithm for a real-world train timetabling problem. Discrete Appl. Math. 154 (5), 738–753.

Carey, M., Lockwood, D., 1995. A model, algorithms and strategy for train pathing. J. Oper. Res. Soc. 46 (8), 988–1005.

Cicerone, S., D'Angelo, G., Di Stefano, G., Frigioni, D., Navarra, A., Schachtebeck, M., Schöbel, A., 2009. Recoverable robustness in shunting and timetabling. In: Robust and Online Large-Scale Optimization: Models and Techniques for Transportation Systems. Springer, pp. 28–60.

Cordeau, J.-F., Desaulniers, G., Lingaya, N., Soumis, F., Desrosiers, J., 2001a. Simultaneous locomotive and car assignment at VIA rail Canada. Transp. Res. Part B: Methodol. 35 (8), 767–787.

Cordeau, J.-F., Soumis, F., Desrosiers, J., 2000. A benders decomposition approach for the locomotive and car assignment problem. Transp. Sci. 34 (2), 133–149.

Cordeau, J.-F., Soumis, F., Desrosiers, J., 2001b. Simultaneous assignment of locomotives and cars to passenger trains. Oper. Res. 49 (4), 531–548.

Corman, F., Meng, L., 2015. A review of online dynamic models and algorithms for railway traffic management. IEEE Trans. Intell. Transp. Syst. 16 (3), 1274–1284.

Correia Duarte, P.J., Schmidt, M., Huisman, D., Veelenturf, L.P., 2023. Fewer trains for better timetables: The price of fixed line frequencies in the passenger-oriented timetabling problem. In: 23rd Symposium on Algorithmic Approaches for Transportation Modelling, Optimization, and Systems (ATMOS 2023). Schloss-Dagstuhl-Leibniz Zentrum für Informatik.

Coviello, N., Medeossi, G., Nash, A., Nygreen, T., Pellegrini, P., Rodriguez, J., 2023. Multiobjective timetable development tool for railway strategic planning in Norway. Transp. Res. Rec. 2677 (1), 720–729.

D'Ariano, A., Meng, L., Centulio, G., Corman, F., 2019. Integrated stochastic optimization approaches for tactical scheduling of trains and railway infrastructure maintenance. Comput. Ind. Eng. 127, 1315–1335.

Dauzère-Pérès, S., De Almeida, D., Guyon, O., Benhizia, F., 2015. A Lagrangian heuristic framework for a real-life integrated planning problem of railway transportation resources. Transp. Res. Part B: Methodol. 74, 138–150.

de Graaf, Y., 2021. Strategic Passenger-Oriented Timetable Design: Long-Term Timetable Designs with Minimised Passenger Inconvenience (Master's thesis). KTH Royal Institute of Technology.

Delorme, X., Rodriguez, J., Gandibleux, X., 2001. Heuristics for railway infrastructure saturation. Electron. Notes Theor. Comput. Sci. 50 (1), 39–53.

Dewilde, T., Sels, P., Cattrysse, D., Vansteenwegen, P., 2013. Robust railway station planning: An interaction between routing, timetabling and platforming. J. Rail Transp. Plan. Manag. 3 (3), 68–77.

Dewilde, T., Sels, P., Cattrysse, D., Vansteenwegen, P., 2014. Improving the robustness in railway station areas. European J. Oper. Res. 235 (1), 276–286.

Erlandson, W., Häll, C.H., Peterson, A., Schmidt, C., 2023. Meta-heuristic for inserting a robust train path in a non-cyclic timetable. Transp. Plan. Technol. 46 (7), 842–863.

Fang, W., Yang, S., Yao, X., 2015. A survey on problem models and solution approaches to rescheduling in railway networks. IEEE Trans. Intell. Transp. Syst. 16 (6), 2997–3016.

Farina, F., 2018. Optimization of Operations in Public Transportation (Ph.D. thesis). Technical University of Denmark.

Fioole, P.-J., Kroon, L., Maróti, G., Schrijver, A., 2006. A rolling stock circulation model for combining and splitting of passenger trains. European J. Oper. Res. 174 (2), 1281–1297.

Fischer, F., 2015. Ordering constraints in time expanded networks for train timetabling problems. In: 15th Workshop on Algorithmic Approaches for Transportation Modelling, Optimization, and Systems (ATMOS 2015). Schloss Dagstuhl-Leibniz-Zentrum fuer Informatik.

Fischer, F., Helmberg, C., Janßen, J., Krostitz, B., 2008. Towards solving very large scale train timetabling problems by lagrangian relaxation. In: 8th Workshop on Algorithmic Approaches for Transportation Modeling, Optimization, and Systems. ATMOS'08, Schloss Dagstuhl-Leibniz-Zentrum für Informatik.

Fischetti, M., Salvagnin, D., Zanette, A., 2009. Fast approaches to improve the robustness of a railway timetable. Transp. Sci. 43 (3), 321–335.

Ford, L.R., Fulkerson, D.R., 1956. Maximal flow through a network. Canad. J. Math. 8, 399–404.

Forsgren, M., Aronsson, M., Gestrelius, S., 2013. Maintaining tracks and traffic flow at the same time. J. Rail Transp. Plan. Manag. 3 (3), 111–123.

Freling, R., Lentink, R.M., Kroon, L., Huisman, D., 2005. Shunting of passenger train units in a railway station. Transp. Sci. 39 (2), 261–272.

Friesen, N., Sander, T., Büsing, C., Nachtigall, K., Nießen, N., 2024. The complexity of the timetable-based railway network design problem. Networks 83 (2), 289–299.

Fuegenschuh, A., Homfeld, H., Huck, A., Martin, A., Yuan, Z., 2008. Scheduling locomotives and car transfers in freight transport. Transp. Sci. 42 (4), 478–491.

Gattermann, P., Großmann, P., Nachtigall, K., Schöbel, A., 2016. Integrating passengers' routes in periodic timetabling: a sat approach. In: 16th Workshop on Algorithmic Approaches for Transportation Modelling, Optimization, and Systems (ATMOS 2016). Schloss Dagstuhl-Leibniz-Zentrum fuer Informatik.

Gestrelius, S., Aronsson, M., Peterson, A., 2017. A MILP-based heuristic for a commercial train timetabling problem. Transp. Res. Procedia 27, 569–576.

Goerigk, M., Schachtebeck, M., Schöbel, A., 2013. Evaluating line concepts using travel times and robustness: Simulations with the lintim toolbox. Public Transp. 5, 267–284.

Goerigk, M., Schöbel, A., 2014. Recovery-to-optimality: A new two-stage approach to robustness with an application to aperiodic timetabling. Comput. Oper. Res. 52, 1–15.

Großmann, P., Hölldobler, S., Manthey, N., Nachtigall, K., Opitz, J., Steinke, P., 2012. Solving periodic event scheduling problems with SAT. In: Advanced Research in Applied Artificial Intelligence: 25th International Conference on Industrial Engineering and Other Applications of Applied Intelligent Systems, IEA/AIE 2012, Dalian, China, June 9–12, 2012. Proceedings 25. Springer, pp. 166–175.

Harris, T., Ross, F., 1955. Fundamentals of a Method for Evaluating Rail Net Capacities. Tech. Rep., Rand Corporation.

Harrod, S., 2011. Modeling network transition constraints with hypergraphs. Transp. Sci. 45 (1), 81–97.

Harrod, S., 2012. A tutorial on fundamental model structures for railway timetable optimization. Surv. Oper. Res. Manag. Sci. 17, 85–96.

Heil, J., Hoffmann, K., Buscher, U., 2020. Railway crew scheduling: Models, methods and applications. European J. Oper. Res. 283 (2), 405–425.

Higgins, A., Ferreira, L., Lake, M., 1999. Scheduling rail track maintenance to minimise overall delays. In: Transportation and Traffic Theory: Proceedings of the 14th International Symposium on Transportation and Traffic Theory. Elsevier Science Ltd, pp. 779–796.

Higgins, A., Kozan, E., Ferreira, L., 1997. Heuristic techniques for single line train scheduling. J. Heuristics 3, 43–62.

INFORMS, 2023. RAS problem solving competition 2012 archive. https://connect.informs.org/railway-applications/new-item3/problem-solving-competition681/new-item6.

Ingolotti, L., Barber, F., Tormos, P., Lova, A., Salido, M.A., Abril, M., 2004. An efficient method to schedule new trains on a heavily loaded railway network. In: Advances in Artificial Intelligence–IBERAMIA 2004: 9th Ibero-American Conference on AI, Puebla, Mexico, November 22–26, 2004. Proceedings 9. Springer, pp. 164–173.

International Union of Railways, 2013. UIC code 406.

Ireland, P., Case, R., Fallis, J., Van Dyke, C., Kuehn, J., Meketon, M., 2004. The Canadian Pacific railway transforms operations by using models to develop its operating plans. Interfaces 34 (1), 5–14.

Jensen, L.W., Landex, A., Nielsen, O.A., Kroon, L., Schmidt, M., 2017. Strategic assessment of capacity consumption in railway networks: Framework and model. Transp. Res. Part C: Emerg. Technol. 74, 126–149.

Jovanović, D., Harker, P.T., 1991. Tactical scheduling of rail operations: the SCAN I system. Transp. Sci. 25 (1), 46–64.

Jovanović, P., Pavlović, N., Belošević, I., Milinković, S., 2020. Graph coloring-based approach for railway station design analysis and capacity determination. European J. Oper. Res. 287 (1), 348–360.

Kloster, O., Luteberget, B., Mannino, C., Sartor, G., 2023. An optimization-based decision support tool for incremental train timetabling. Oper. Res. Forum 4.

Kroon, L., Huisman, D., Abbink, E., Fioole, P.-J., Fischetti, M., Maróti, G., Schrijver, A., Steenbeek, A., Ybema, R., 2009. The new Dutch timetable: The OR revolution. Interfaces 39 (1), 6–17.

Kroon, L., Lentink, R.M., Schrijver, A., 2008a. Shunting of passenger train units: an integrated approach. Transp. Sci. 42 (4), 436–449.

Kroon, L., Maróti, G., Helmrich, M.R., Vromans, M., Dekker, R., 2008b. Stochastic improvement of cyclic railway timetables. Transp. Res. Part B: Methodol. 42 (6), 553–570.

Kroon, L., Maróti, G., Nielsen, L., 2015. Rescheduling of railway rolling stock with dynamic passenger flows. Transp. Sci. 49 (2), 165–184.

Kroon, L., Peeters, L.W., 2003. A variable trip time model for cyclic railway timetabling. Transp. Sci. 37 (2), 198–212.

Lamorgese, L., Mannino, C., Natvig, E., 2017. An exact micro–macro approach to cyclic and non-cyclic train timetabling. Omega 72, 59–70.

Lamorgese, L., Mannino, C., Pacciarelli, D., Krasemann, J.T., 2018. Train dispatching. Handb. Optim. Railw. Ind. 265–283.

Lidén, T., 2015. Railway infrastructure maintenance-a survey of planning problems and conducted research. Transp. Res. Procedia 10, 574–583.

Lidén, T., Joborn, M., 2017. An optimization model for integrated planning of railway traffic and network maintenance. Transp. Res. Part C: Emerg. Technol. 74, 327–347.

Liebchen, C., 2006. Periodic Timetable Optimization in Public Transport (Ph.D. thesis). Technical University Berlin.

Liebchen, C., 2008. The first optimized railway timetable in practice. Transp. Sci. 42 (4), 420–435.

Liebchen, C., Lübbecke, M., Möhring, R., Stiller, S., 2009. The concept of recoverable robustness, linear programming recovery, and railway applications. In: Robust and Online Large-Scale Optimization: Models and Techniques for Transportation Systems. Springer, pp. 1–27.

Liebchen, C., Möhring, R.H., 2007. The modeling power of the periodic event scheduling problem: railway timetables—and beyond. In: Algorithmic Methods for Railway Optimization: International Dagstuhl Workshop, Dagstuhl Castle, Germany, June 20–25, 2004, 4th International Workshop, ATMOS 2004, Bergen, Norway, September 16–17, 2004, Revised Selected Papers. Springer, pp. 3–40.

Lindner, N., Liebchen, C., 2019. New perspectives on PESP: T-partitions and separators. In: 19th Symposium on Algorithmic Approaches for Transportation Modelling, Optimization, and Systems (ATMOS 2019). Schloss Dagstuhl-Leibniz-Zentrum fuer Informatik.

Lindner, N., Liebchen, C., 2022. Timetable merging for the periodic event scheduling problem. EURO J. Transp. Logist. 11, 100081.

Ljunggren, F., Persson, K., Peterson, A., Schmidt, C., 2021. Railway timetabling: a maximum bottleneck path algorithm for finding an additional train path. Public Transp. 13, 597–623.

Luan, X., Miao, J., Meng, L., Corman, F., Lodewijks, G., 2017. Integrated optimization on train scheduling and preventive maintenance time slots planning. Transp. Res. Part C: Emerg. Technol. 80, 329–359.

Lusby, R.M., Larsen, J., Bull, S., 2018. A survey on robustness in railway planning. European J. Oper. Res. 266 (1), 1–15.

Lusby, R.M., Larsen, J., Ehrgott, M., Ryan, D., 2011a. Railway track allocation: models and methods. OR Spectrum 33, 843–883.

Lusby, R.M., Larsen, J., Ryan, D., Ehrgott, M., 2011b. Routing trains through railway junctions: a new set-packing approach. Transp. Sci. 45 (2), 228–245.

Martin-Iradi, B., Ropke, S., 2022. A column-generation-based matheuristic for periodic and symmetric train timetabling with integrated passenger routing. European J. Oper. Res. 297 (2), 511–531.

Mascis, A., Pacciarelli, D., 2002. Job-shop scheduling with blocking and no-wait constraints. European J. Oper. Res. 143 (3), 498–517.

Matos, G.P., Albino, L.M., Saldanha, R.L., Morgado, E.M., 2021. Solving periodic timetabling problems with SAT and machine learning. Public Transp. 13 (3), 625–648.

Michaelis, M., Schöbel, A., 2009. Integrating line planning, timetabling, and vehicle scheduling: a customer-oriented heuristic. Public Transp. 1, 211–232.

Nachtigall, K., 1994. A Branch and Cut Approach for Periodic Network Programming. Univ., Inst. für Mathematik.

Nielsen, L.K., Kroon, L., Maróti, G., 2012. A rolling horizon approach for disruption management of railway rolling stock. European J. Oper. Res. 220 (2), 496–509.

Odijk, M.A., 1996. A constraint generation algorithm for the construction of periodic railway timetables. Transp. Res. Part B: Methodol. 30 (6), 455–464.

Ortiz-Astorquiza, C., Cordeau, J.-F., Frejinger, E., 2021. The locomotive assignment problem with distributed power at the Canadian National Railway Company. Transp. Sci. 55 (2), 510–531.

Pachl, J., 2018. Railway Operation and Control. VTD Rail Publishing.

Peeters, L., 2003. Cyclic Railway Timetable Optimization (Ph.D. thesis). (EPS-2003-022-LIS), Erasmus Univeriesty Rotterdam.

Pellegrini, P., Marlière, G., Rodriguez, J., 2014. Optimal train routing and scheduling for managing traffic perturbations in complex junctions. Transp. Res. Part B: Methodol. 59, 58–80.

Pellegrini, P., Marlière, G., Rodriguez, J., 2017. RECIFE-SAT: A MILP-based algorithm for the railway saturation problem. J. Rail Transp. Plan. Manag. 7 (1–2), 19–32.

Polinder, G.-J., Cacchiani, V., Schmidt, M., Huisman, D., 2022. An iterative heuristic for passenger-centric train timetabling with integrated adaption times. Comput. Oper. Res. 142, 105740.

Polinder, G.-J., Schmidt, M., Huisman, D., 2021. Timetabling for strategic passenger railway planning. Transp. Res. Part B: Methodol. 146, 111–135.

Quiroga, L.M., Schnieder, E., 2010. A heuristic approach to railway track maintenance scheduling. WIT Trans. Built Environ. 114, 687–699.

Robenek, T., Azadeh, S.S., Maknoon, Y., Bierlaire, M., 2017. Hybrid cyclicity: Combining the benefits of cyclic and non-cyclic timetables. Transp. Res. Part C: Emerg. Technol. 75, 228–253.

Rouillon, S., Desaulniers, G., Soumis, F., 2006. An extended branch-and-bound method for locomotive assignment. Transp. Res. Part B: Methodol. 40 (5), 404–423.

Sartor, G., Mannino, C., Nygreen, T., Bach, L., 2023. A MILP model for quasi-periodic strategic train timetabling. Omega 116, 102798.

Schmidt, M., Weik, N., Zieger, S., Schmeink, A., Nießen, N., 2019. A generalized stochastic Petri net model for performance analysis of trackside infrastructure in Railway Station Areas under uncertainty. In: 2019 IEEE Intelligent Transportation Systems Conference. ITSC, IEEE, pp. 3732–3737.

Schöbel, A., 2017. An eigenmodel for iterative line planning, timetabling and vehicle scheduling in public transportation. Transp. Res. Part C: Emerg. Technol. 74, 348–365.

Schrijver, A., 2002. On the history of the transportation and maximum flow problems. Math. Program. 91, 437–445.

Schrijver, A., Steenbeek, A., 1993. Dienstregelingontwikkeling voor nederlandse spoorwegen NS rapport fase 1. Cent. Voor Wiskd. Inform. (Oktober 1993).

Sedghi, M., Kauppila, O., Bergquist, B., Vanhatalo, E., Kulahci, M., 2021. A taxonomy of railway track maintenance planning and scheduling: A review and research trends. Reliab. Eng. Syst. Saf. 215, 107827.

Sels, P., Vansteenwegen, P., Dewilde, T., Cattrysse, D., Waquet, B., Joubert, A., 2014. The train platforming problem: The infrastructure management company perspective. Transp. Res. Part B: Methodol. 61, 55–72.

Serafini, P., Ukovich, W., 1989. A mathematical model for periodic scheduling problems. SIAM J. Discrete Math. 2 (4), 550–581.

Sousa, J.P., Wolsey, L.A., 1992. A time indexed formulation of non-preemptive single machine scheduling problems. Math. Program. 54, 353–367.

Tan, Y., Li, Y.-X., Wang, R.-X., 2020. Scheduling extra train paths into cyclic timetable based on the genetic algorithm. IEEE Access 8, 102199–102211.

Tan, Y., Xu, W., Jiang, Z., Wang, Z., Sun, B., 2021. Inserting extra train services on high-speed railway. Period. Polytech. Transp. Eng. 49 (1), 16–24.

Tolstoı, A., 1939. Metody ustraneniya neratsional'nykh perevozok pri planirovanii [Russian; methods of removing irrational transportation in planning]. Sotsialisticheskiı Transp. 9, 28–51.

Tréfond, S., Billionnet, A., Elloumi, S., Djellab, H., Guyon, O., 2017. Optimization and simulation for robust railway rolling-stock planning. J. Rail Transp. Plan. Manag. 7 (1–2), 33–49.

Vaidyanathan, B., Ahuja, R.K., Liu, J., Shughart, L.A., 2008a. Real-life locomotive planning: New formulations and computational results. Transp. Res. Part B: Methodol. 42 (2), 147–168.

Vaidyanathan, B., Ahuja, R.K., Orlin, J.B., 2008b. The locomotive routing problem. Transp. Sci. 42 (4), 492–507.

Van Aken, S., Bešinović, N., Goverde, R.M., 2017a. Designing alternative railway timetables under infrastructure maintenance possessions. Transp. Res. Part B: Methodol. 98, 224–238.

Van Aken, S., Bešinović, N., Goverde, R.M., 2017b. Solving large-scale train timetable adjustment problems under infrastructure maintenance possessions. J. Rail Transp. Plan. Manag. 7 (3), 141–156.

van der Aalst, W., Odijk, M., 1995. Analysis of railway stations by means of interval timed coloured Petri nets. Real-Time Syst. 9, 241–263.

Weik, N., Warg, J., Johansson, I., Bohlin, M., Nießen, N., 2020. Extending UIC 406-based capacity analysis—New approaches for railway nodes and network effects. J. Rail Transp. Plan. Manag. 15, 100199.

Xu, X., Li, C.-L., Xu, Z., 2018. Integrated train timetabling and locomotive assignment. Transp. Res. Part B: Methodol. 117, 573–593.

Yan, F., Bešinović, N., Goverde, R.M., 2019. Multi-objective periodic railway timetabling on dense heterogeneous railway corridors. Transp. Res. Part B: Methodol. 125, 52–75.

Yan, F., Goverde, R.M., 2019. Combined line planning and train timetabling for strongly heterogeneous railway lines with direct connections. Transp. Res. Part B: Methodol. 127, 20–46.

Ziarati, K., Soumis, F., Desrosiers, J., Gélinas, S., Saintonge, A., 1997. Locomotive assignment with heterogeneous consists at CN North America. European J. Oper. Res. 97 (2), 281–292.

Ziarati, K., Soumis, F., Desrosiers, J., Solomon, M.M., 1999. A branch-first, cut-second approach for locomotive assignment. Manag. Sci. 45 (8), 1156–1168.

Zwaneveld, P.J., Kroon, L., Romeijn, H.E., Salomon, M., Dauzere-Peres, S., Van Hoesel, S.P., Ambergen, H.W., 1996. Routing trains through railway stations: Model formulation and algorithms. Transp. Sci. 30 (3), 181–194.

Zwaneveld, P.J., Kroon, L., Van Hoesel, S.P., 2001. Routing trains through a railway station based on a node packing model. European J. Oper. Res. 128 (1), 14–33.