

Comparison of Fuzzy Bayesian Network Methods for maritime applications

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Fuzzy bayesian networks are commonly used to incorporate expert opinions in risk assessment approaches. Various methods exist to aggregate expert statements with fuzzy methods. Yet few efforts have been made to compare these methods and how they affect risk probabilities in fuzzy bayesian networks. This paper aims at filling this gap. We construct a bayesian network for the entry of a ship in an offshore wind farm and analysed it using pythagorean fuzzy weighted geometric, the linear opinion pool, and the similarity aggregation method. We compare results for every node and the network overall. For the calculation of the nodes the normalisation and calculation of failure probability are added or omitted to all three methods and for the similarity aggregation method the beta value is varied. Our paper shows that the choice of fuzzy methods can have a significant impact on the risk probabilities of bayesian networks depending on the combination of answers and weightings. Adding fuzzy failure probabilities has the highest impact on probability calculations in bayesian networks.

Keywords: fuzzy bayesian network, fuzzy set, bayesian network, method comparison, fuzzy aggregation, expert opinion, opinion aggregation.

1. Introduction

Fuzzy Bayesian Networks (FBNs) are commonly used to incorporate expert opinions in risk assessments and situational awareness approaches Ayyildiz et al. (2024), D'Aniello (2023). Fuzzy sets are used to transfer fuzzy statements from experts in a way that they can be handled by systems who expect numerical inputs Zadeh (1965). For systems which need crisp numbers, the fuzzy set has to be transferred with a defuzzification method. As bayesian network (BN) can only handle probabilities in numbers, fuzzy sets with defuzzification are required to translate the qualitative probability statements of experts, for example "very likely", in a numerical value on the interval [0;1]. Examples for the use of FBNs in the maritime area include risk assessments of offshore wind turbine infrastructures and other offshore operations Cheng et al. (2019); Ren et al. (2009).

While various methods exist for aggregating the verbal probability statements of experts, direct comparisons between these methods are often missing Zarei et al. (2019). This raises the question of how different calculation methods affect probabilities. The aim of the paper is to compare the probabilities of several individual nodes and a complete network using different fuzzy methods. To do so, a BN to predict the probability of a ship entering the area of an offshore wind farm (OWF) is constructed drawing on expert insights. OWF are chosen because they are an important energy supply infrastructure. Many OWF are located in busy shipping lanes, which increases the risk of accidents and collisions Gabriel et al. (2022). This BN is used to show that there is a large difference in the probabilities of fuzzy BN depending on the methods and values used, especially when adding fuzzy failure probabilities.

This paper is organised as follows. Section 2.1 briefly describes the BN and the questionnaire. Section 3 presents the fuzzy methods for the aggregation of expert insights and opinions. The results of the comparison of different methods are presented in Section 4 and discussed in Section 5. The study's limitations and directions for future research are outlined in the concluding Section 6.

2. Bayesian Network and Questionnaire

The two sections introduces the bayesian network to create the example for the comparison and the questionnaire to collect expert opinions.

2.1. Bayesian Network for Entry in Offshore Wind Farm

A BN consists of variables with a finite set of states, which are connected by edges to form an acyclic, direct graph. Variables without predecessors are so-called root nodes. These are described with probabilities for different states. A variable with a predecessors, i.e. parent nodes, is described by a table with conditional probabilities. Jensen and Nielsen (2007) The focus of the study is on comparing different fuzzy aggregation methods for BN. It therefore constructs a BN using an expert group within easy reach (see Fig.1), especially experts working on maritime safety and security issues at the authors' institution. The BN tries to determine the probability of whether a ship will enter a wind farm. This helps to recognize and predict potential security risks for early warning and facilitates the initiation of countermeasures by the wind farm operator and the security forces. The BN is constructed to calculate the probability for a specific scenario. It is assumed that the information for the root nodes are given. To describe the scenario, the probabilities for different states are set on one and the probabilities of all other states are set to zero. For the node "ship speed", for example, the probability for "fast" is set on one and the probability for "normal_OR_slow" is set on zero. General probabilities for the root nodes are not provided. The information for ship type, ship speed, wind speed, wave height and position of the ship are specified. They are presented through the nodes "wind force and wave

high", "ship speed" and "ship type". The ships to be considered as ship types are, in addition to crew transfer vessels (CTVs), fishing boats and leisure boats, which are already large enough to have Automatic Identification Systems. The nodes "ship moves in the triangle" and "overlap triangle OWF" refer to a triangle spread out before the ship in the direction of the course over ground. The overlap with the triangle can be calculated. As described in Fig.2, the triangle is described as multiples of the ship's length. The triangle dimensions were taken from the International Maritime Organisation specifications on the turning ability of a ship, which specifies a maximum advance of 4.5 ship lengths and a tactical diameter of a maximum of 5 ship lengths International Maritime Organisation (IMO) (2002). A ship with a course towards the OWF could turn away from the area with a 90° turn. When the ships course is in the right angle to the OWF, it would then sail parallel to the area. In Fig.2, for example, the ship would carry out a 90° turn to the left. Since a 90° turn is sufficient for the triangle, the tactical diameter is halved. The node "ship moves in the triangle" refer to the possibility that the ship has a much smaller turning circle then the estimation with the ship length, which are the highest possible values. When the turning circle is smaller the ship could move out of the triangle as it turns at a shorter distance than assumed.

2.2. Structure questionnaire

The questionnaire contains only the conditional probability tables, as the probabilities for the root nodes are assumed to be given (see subsection 2.1). The experts can choose between seven linguistic values for the conditional probabilities, namely very low, low, fairly low, medium, fairly high, high and very high. This selection is based on a range of similar papers presented in Subsection 3.1. Six of the papers have seven linguistic terms, two have nine, and one has five. Going with the majority, this paper uses seven linguistic terms. For an idea of how many experts are included in the survey we looked at 18 papers which use expert opinions. The average number of experts in these papers is seven. Only Li et al.

(2024) and Zarei et al. (2019), who interviewed 34 and 15 experts respectively, interviewed more than ten experts. The questionnaire in this paper was filled out by eight experts. For the calculations,

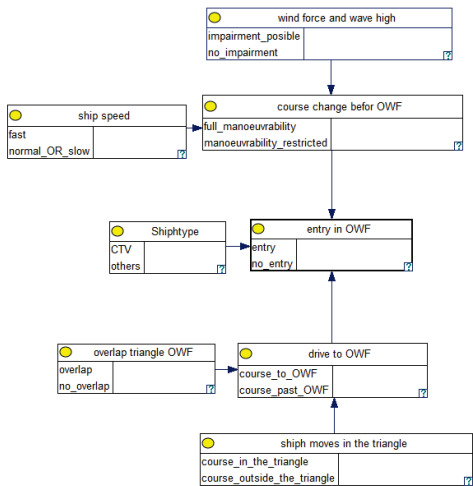


Fig. 1. Bayesian Network for the Entry of a Ship in the Offshore Wind Farm.

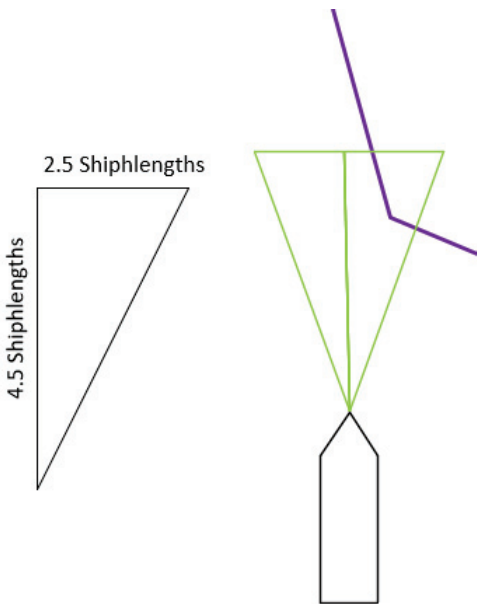


Fig. 2. Explanation of the triangle for the ship course.

the experts get a weight value, which is the sum of the criteria in Table 1 normalised over all expert weightings. The criteria were determined with general parts taken from Lavasani et al. (2012); Ramzali et al. (2015) and completed with a question regarding sea going experience.

Table 1. Weighting score for experts.

category	weight
professional position	
senior academic	6
Postdoctoral	5
junior academic	4
engineer	3
technician	2
professional experience	
0-5 years	1
6-10 years	2
11-15 years	3
16-20 years	4
21-30 years	5
over 30 years	6
education	
PhD	5
Master	4
Bachelor	3
diploma	4
apprenticeship	1
gone to sea	
yes	5
no	0

3. Fuzzy Methods for the Bayesian Network

This section describes the selection of the methods and presents the three chosen methods, with the fuzzy failure probability as addition to the calculation.

3.1. Method selection

There are different ways to connect fuzzy methods and BN. For example, some papers use fuzzy methods to calculate values in failure tree and to transfer these values into a BN Dong and Wang (2019) while others use fuzzy logics to describe the connection between the nodes of a BN Kumar et al. (2024). There are also different cal-

culation methods for the values of a BN. This paper concentrates on methods which use fuzzy sets to describe the probability of nodes in BN. Our selection of methods is based on nine papers which use FBN mainly for risk assessment (see Table 2). We take the pythagorean fuzzy weighted geometric method in Ayyildiz et al. (2024) because it is the only one that uses a pythagorean fuzzy set (PFS) and not a classic fuzzy set. We also select the similarity aggregation method of Guo et al. (2021) because it is a modification of the similarity aggregation method (SAM), which is used in Hsu and Chen (1996). The third method for the comparison in this paper is the linear opinion pool method (LOP) from Zarei et al. (2019).

3.2. Methods discussion

A_{ij} is the opinion of Expert j ($j = 1, \dots, n$) regarding event i ($i = 1, \dots, m$), where n is the number of experts and m is the number of events. W_j is the weighting of expert j . The fuzzy set is taken from Lin and Wang (1997) and the pythagorean fuzzy sets is taken from Ayyildiz et al. (2024).

3.2.1. Pythagorean fuzzy weighted geometric method

The first method is the pythagorean fuzzy weighted geometric method (PFWG) from Ayyildiz et al. (2024), who use PFS. A PFS has a pair of values $(\mu; \nu)$, where μ describes the degree of membership and ν describes the degree against membership Yager (2014). With the condition that $\mu^2 + \nu^2 \leq 1$ the membership space is grater, then for example for intuitionistic membership grades, where the condition is $\mu^2 + \nu^2 = 1$ Yager (2014). To aggregated the expert opinions we use the PFWG, which is defined as followed with $A_{ij} = (\mu_{ij}; \nu_{ij})$ Yager (2013):

$$\text{PFWG}(A_{i1}, \dots, A_{in}) = (\mu_{agg}; \nu_{agg}), \quad (1)$$

$$\mu_{agg} = \prod_{j=1}^n (\mu_{ij})^{w_j}, \quad (2)$$

$$\nu_{agg} = \sqrt{1 - \prod_{j=1}^n (1 - (\nu_{ij})^2)^{w_j}}. \quad (3)$$

With calculation of the closeness index Zhang (2016)

$$CI_i = \frac{1 - (\nu_{agg})^2}{2 - (\mu_{agg})^2 - (\nu_{agg})^2} \quad (4)$$

the aggregated PFS is transformed in a crisp value.

3.2.2. Linear opinion pool method

The second and third methods use fuzzy sets with trapezoidal form, which are presented as $A_{ij} = (a_{ij1}; a_{ij2}; a_{ij3}; a_{ij4})$. Zarei et al. (2019) uses the LOP

$$M_i = \sum_{j=1}^n W_j A_{ij} \quad (5)$$

for the aggregation of the expert opinions. For the defuzzification of the trapeze fuzzy set Zarei et al. (2019) and Guo et al. (2021) use the Center of Area method, which is calculated as followed

$$X_i^* = \frac{(a_4 + a_3)^2 - a_4 a_3 - (a_1 + a_2)^2 + a_1 a_2}{3(a_4 + a_3 - a_1 - a_2)}. \quad (6)$$

3.2.3. Similarity aggregation method

Guo et al. (2021) use an extension of the SAM from Hsu and Chen (1996). The agreement degree

$$S_i(A_{iu}, A_{iv}) = 1 - \frac{1}{4} \sum_{h=1}^4 |a_{iuh} - a_{ivh}| \quad (7)$$

which is also part of Hsu and Chen (1996) method is extended from Guo et al. (2021) with the weighted (absolute) agreement degree

$$WA_{iu} = \frac{\sum_{j=1; j \neq u}^n W_j S_i(A_{iu}, A_{ij})}{\sum_{j=1; j \neq u}^n W_j} \quad (8)$$

In the next step, with the weighted absolute agreement degree the relative agreement degree

$$RA_{iu} = \frac{WA_{iu}}{\sum_{j=1}^n WA_{ij}} \quad (9)$$

is calculated Guo et al. (2021). The consensus coefficient

$$CC_{iu} = \beta \cdot W_u + (1 - \beta) \cdot RA_{iu} \quad (10)$$

is used to weight the opinions not only with the experience but also with weight for the agreement

Table 2. FBN papers considered for this study.

source	issue	method for opinion aggregation
Aydin et al. (2021)	ship collision in narrow water	similarity aggregation method
Ayyildiz et al. (2024)	occupational health and safety in pharmaceutical warehouses	pythagorean fuzzy weighted geometric
Cheng et al. (2019)	offshore wind turbine installation schedule	frequency of degree of relationship
Guo et al. (2021)	accidents in storage tanks	similarity aggregation method
Li et al. (2020)	road transportation of flammable liquid	linear opinion pool
Li et al. (2024)	maritime autonomous surface ships collisions	arithmetical average
Öztürk (2024)	container loss at sea	similarity aggregation method
Zarei et al. (2019)	comparing FBN and BN	linear opinion pool
Zhang et al. (2024)	dam failure	fuzzy failure probability; combination of expert opinion and monitored data

between the experts. This consensus coefficient is used for the aggregation of the opinions

$$A_i = \sum_{j=1}^n CC_{ij} \times A_{ij}. \quad (11)$$

Finally, the defuzzification is performed with Eq.(6).

3.2.4. Fuzzy failure probability

In an last step of the process, Guo et al. (2021) and Zarei et al. (2019) perform the Fuzzy failure probability on the result:

$$FFP = \begin{cases} \frac{1}{10^K} & X^* \neq 0 \\ 0 & X^* = 0 \end{cases} \quad (12)$$

$$K = \left(\frac{1 - X^*}{X^*} \right)^{\frac{1}{3}} \quad (13)$$

4. Results of the Comparison

For the comparison of the methods, five different analyses are performed. The first four analysis are performed with all three methods. The normalisation of the values and a calculation of the failure probability are added or omitted, while the beta-value of the SAM (subsection 3.2.1) is set on 0.5. Cases without normalisation are not usable for the BN. But this case is also looked at to have an independent comparison, as the result from the calculation with normalisation is dependent on a second value. The fuzzy failure probability with

Eq.(12) is used in some papers to transfer the defuzzified value in a fuzzy failure rate e.g. for human and hardware failure events Lin and Wang (1997). As there are cases where this method can be used and cases where it cannot be used, the fuzzy failure rate calculation is also included in the analysis.

For the first comparison, the methods are applied with normalised values. The second comparison includes the calculation of the fuzzy failure probability after the defuzzification of the value of all three methods. The normalisation is omitted for the third analysis. In the fourth analysis, the values are not normalised and the fuzzy failure probability is added to the calculation. All four analyses have, for most nodes, the smallest difference between the SAM and the LOP. On average, the difference between the methods is less then ten percent of the sum from the differences between all methods (see Table 3). The value of the PFWG is the highest, when the values from the three methods are all smaller then 0.5 and the lowest, when the values are larger then 0.5, for the most cases in the first three analysis. Beside that PFWG is often the lowest or highest value, there is not a specific order of the values, as SAM and LOP are not fix in there order.

In a fifth analysis the β -value is varied in steps of 0.1 starting with zero and ending with one in Eq.(10), to show how the β -value influences the outcome of the SAM. When β is set on one

Table 3. Overview of the average results from the first four analysis.

method	average value	percentage of sum in %
analysis 1		
PFWG – SAM	0.0630	49.68
PFWG – LOP	0.0528	41.64
SAM – LOP	0.0110	8.68
analysis 2		
PFWG – SAM	0.0763	49.35
PFWG – LOP	0.0649	41.98
SAM – LOP	0.0134	8.67
analysis 3		
PFWG – SAM	0.0744	48.25
PFWG – LOP	0.0722	46.82
SAM – LOP	0.0076	4.93
analysis 4		
PFWG – SAM	0.0095	48.97
PFWG – LOP	0.0094	48.45
SAM – LOP	0.0005	2.58

the second part of Eq.(10) is set on zero and the equation is the same as Eq.(5) from the LOP. When the values are not too near to 0.5 there could be made a clear distinction between the order of the values. For values lower than 0.5 the values increase with increasing β and for values higher than 0.5 the values decrease with increasing β . For all nodes the difference to the LOP is decreasing with increasing β . That is the same for the PFWG with exception of two nodes, where the gap is increasing with increasing β .

After the analysis, the values were inserted in the BN. That is only possible for the variations where the values are normalised, as the BN require normalised values. Between the three methods, the difference for the probability of the entry in the OWF is less than one percent. But comparing the results with and without fuzzy failure probability leads to a difference from over ten percent for all three methods. For the BN the probability of the ship entering the OWF is 44.69% with LOP but without fuzzy failure probability (see Fig.3). Including fuzzy failure probability leads to a probability of 57.57% (see Fig.4).

5. Discussion

The answers of the eight experts is only a small selection of all combinations of possible answers with seven options in sixteen nodes, which leads to a total of

$$n^k = 7^8 = 5764801 \quad (14)$$

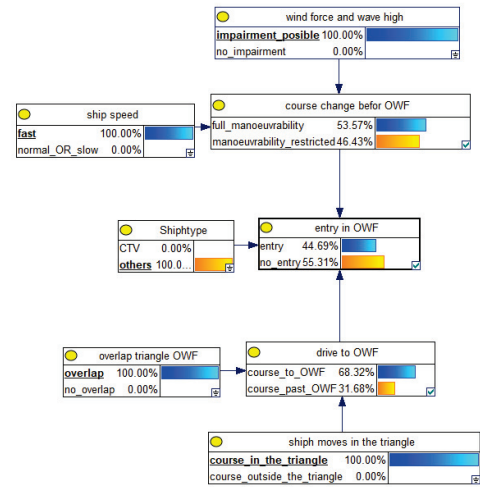


Fig. 3. Bayesian Network with the values from the calculation with the LOP without fuzzy failure probability.

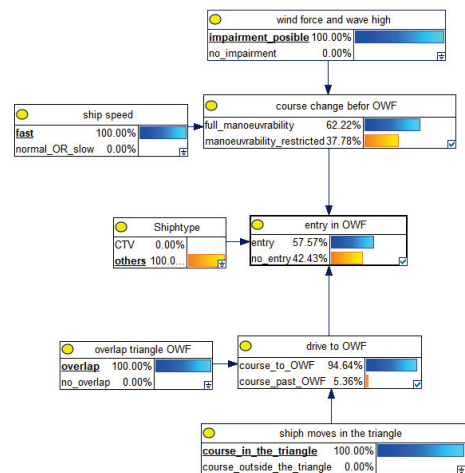


Fig. 4. Bayesian Network with the values from the calculation with the LOP with fuzzy failure probability.

combinations. That means that the results from the comparison cannot be generalised for all possible combinations. But they show that the selection of methods can make a probability difference of up to 0.1745. This translates into a very significant difference of more than 17 percent for probability values of less than one. General statements about the behavior of the methods are difficult, even for a relatively simple calculations such as the LOP, because there are many different options to weight values. Another variable which influences the result are fuzzy sets with different options for both the form and the values. Further research should therefore investigate if input sequences behave like the BN in this paper, so that for example the difference between the SAM and the LOP are lowest. To see if the statements about the different behavior could be widened to a larger group or if they are specific for the example values used in this paper.

6. Conclusion

The paper has demonstrated that the selection of fuzzy methods can have a significant impact on probability calculations, depending on answers and weighting. The study does not allow us to make a general statement about how different method influence the results. It indicates, however, that scholars should calculate probabilities with more than one method and take into account how these methods affect results. More broadly, scholars need to consider this insight when interpreting FBNs.

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