# Mitteilung

# Fachgruppe: Drehflügler

Modal Analysis of Volumetric Flow Fields around a Quadcopter from Experimental Shake-The-Box Results

Alexander Fenk<sup>1</sup>, C. Christian Wolf<sup>2</sup> and Gudrun Thäter<sup>1</sup>

<sup>1</sup>Karlsruhe Institute of Technology, Kaiserstr. 12, 76131 Karlsruhe

<sup>2</sup>German Aerospace Center, Bunsenstr. 10, 37073 Göttingen, Germany alexfenk00@gmail.com, Christian.Wolf@dlr.de, gudrun.thaeter@kit.edu

#### Introduction

Unmanned aerial vehicles experience a rapid rise in various industries as well as in the private sector. This motivates the investigation of their wake in free flight, to allow understanding and development of efficient, safe and silent vehicles. Therefore, a Shake-the-Box experiment of a quadcopter with an overall span of approximately 0.5 m in free flight was conducted at the DLR site in Göttingen, covering a variety of operating conditions, such as flight in ground effect (IGE) and out of ground effect (OGE) [1]. A momentum analysis neglecting the unknown pressure terms was performed on the volumetric, time resolved velocity data from an OGE-case, yielding an estimate of the quadcopter's thrust. The result exhibits strong fluctuations and a systematic deviation to the expected value derived from the vehicle's weight (see Figure 1). Large scale flow structures, going along with pressure gradients, might be responsible and can be identified by a modal analysis. This presentation features the use of a Proper Orthogonal Decomposition (POD) code on the STB-data of both IGE and the OGE cases to investigate the modal structure of the flow around a free flying helicopter.

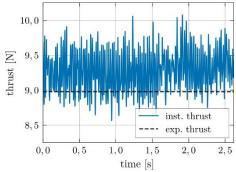


Figure 1: Expected thrust and by momentum analyzation calculated instantaneous thrust.

## **Applied Method**

POD was chosen for a modal analysis due to its optimality with respect to the variance and an assumed stochastic stationarity of the drone centered flow fields. POD in the implemented version decomposes a time and space variant field u(x,t) according to

$$u(x,t) = \overline{u}(x) + \sum_{n=1}^{Z} \alpha_n(t)\phi_n(x),$$

where  $\overline{u}(x)$  is the time averaged mean, Z the number of used time steps,  $\alpha_n(t)$  the time dependent coefficients and  $\phi_n(x)$  the orthogonal and normalized modes, ranked according to their contribution to the overall variance of the flow [2]. The variance can be interpreted as two times the spatially integrated turbulent kinetic energy [3]. For flow fields discretized in time and space, the velocity data for each time step are assembled in a vector  $\boldsymbol{q}^{(i)} = \begin{bmatrix} u'_1^{(i)}, \dots, u'_M^{(i)}, v'_1^{(i)}, \dots v'_M^{(i)}, w'_1^{(i)}, \dots, w'_M^{(i)} \end{bmatrix}^T$  and summarized in a matrix  $\boldsymbol{Q} = [\boldsymbol{q}^{(1)}, \dots, \boldsymbol{q}^{(Z)}]$ . The identified modes are then the solution of the eigenvalue equation

$$C\Phi = \Lambda\Phi$$

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with the spatial covariance  $C = QQ^T$ , the matrix of modes  $\Phi = [\phi_1, ..., \phi_Z]$  and the matrix of eigenvalues  $\Lambda = \operatorname{diag}(\lambda_1, ..., \lambda_Z)$ . The eigenvalues mark the contribution of a mode to the overall variance and thus indicate its energy. To reduce the computational costs, the method of snapshots was applied [3].

## **Results and Discussion**

The spatially continuous experimental STB-data were discretized by evenly spaced regular cartesian grids with a spacing of 1 cm. In the OGE case a drone centered coordinate system was applied, removing a residual position drift of the drone similar to the approach in [1]. The IGE case was investigated from the perspective of a fixed global observer.

The modes 5 and 6 of the OGE case both contributed 1.14% of the overall energy and presented a similar but slightly shifted spatial structure (see Figure 2) indicating travelling structures [4]. Figure 3 shows the result of an FFT analysis of the calculated thrust as well as the coefficients of the modes 5 and 6. In all cases, the most prominent peak is at around 40 Hz. The flow structures represented by the combination of modes 5 and 6 most likely go along with pressure gradients, explaining the fluctuations in the calculated thrust when neglecting the pressure terms.

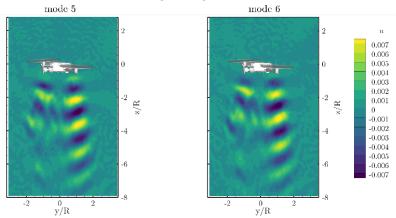


Figure 2: Non-dimensional u-component of modes 5 (left) and 6 (right) of OGE case. Slice through longitudinal axis. Rotor radius R = 0.11 m.

Similarly, for the IGE case, the coefficients of modes 6 and 7, containing 1.10% and 1.09% of the energy respectively, showed related spectra (see Figure 3). They contain a mixture of frequencies, with the most relevant ones being smaller than 20 Hz. Again, the close relation indicates travelling structures.

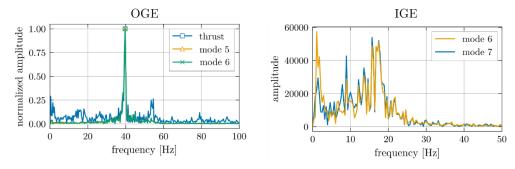


Figure 3: FFT of calculated thrust, modes 5 and 6 OGE case (left) and modes 6 and 7 IGE case (right).

In both the OGE and IGE case the travelling structures are suspected to have aerodynamic reasons such as shear layer instabilities. The other high-ranking modes showed a generally transient behavior, being most likely caused by positional drift and other large-scale effects during the experiments.

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<sup>[1]</sup> C. C. Wolf *et al.*, "Volumetric wake investigation of a free-flying quadcopter using Shake-The-Box Lagrangian particle tracking," *Exp. Fluids*, vol. 65, no. 10, p. 152, Oct. 2024, doi: 10.1007/s00348-024-03880-3.

<sup>[2]</sup> C. Howard, S. Gupta, A. Abbas, T. A. G. Langrish, and D. F. Fletcher, "Proper Orthogonal Decomposition (POD) analysis of CFD data for flow in an axisymmetric sudden expansion," Chem. Eng. Res. Des., vol. 123, pp. 333–346, Jul. 2017, doi: 10.1016/j.cherd.2017.05.017.

<sup>[3]</sup> O. T. Schmidt and T. Colonius, "Guide to Spectral Proper Orthogonal Decomposition," AIAA J., vol. 58, no. 3, pp. 1023–1033, Mar. 2020, doi: 10.2514/1.J058809.

<sup>[4]</sup> K. Taira et al., "Modal Analysis of Fluid Flows: An Overview," AIAA J., vol. 55, no. 12, pp. 4013–4041, Dec. 2017, doi: 10.2514/1.J056060.