

Characterization of a Hypersonic Turbulent Boundary Layer along a Sharp Cone with Smooth and Rough Walls

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Turbulenz, Grenzschicht, Hyperschall, Rauigkeit, H2K

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Charakterisierung einer turbulenten Hyperschallgrenzschicht entlang eines glatten und rauen Kegels

RWTH Aachen University

In dieser Arbeit werden experimentelle Untersuchungen zu turbulenten Hyperschall-Grenzschichten entlang glatter und rauer Wände vorgestellt, die im Hyperschall-Windkanal H2K des Deutschen Zentrums für Luft- und Raumfahrt in Köln durchgeführt wurden. Das generische Modell für die Experimente ist ein scharfer Kegel, der aus drei Segmenten mit austauschbaren Teilen besteht, um glatte und raue Oberflächen zu berücksichtigen. Ein Segment besteht aus einer technischen Rauheitstopologie mit quadratischen Elementen mit einer nominalen Wellenlänge von viermal der Höhe der Elemente. Ein weiteres Segment besteht aus einer sandkornartigen Rauheit unter Verwendung von Carborundum-Körnern. Die Grenzschicht wird mit einem Pitot-Druckrechen und Particle Image Velocimetry (PIV) untersucht. Der konvektive Wärmestrom wird über Infrarot-Thermografie und lokale Sensoren abgeleitet. Zusätzlich werden Schlierenbilder und Oberflächendruckdaten verwendet, um allgemeine Informationen über das Strömungsfeld entlang des Kegelmodells zu erhalten. Die Interpretation der experimentellen Daten wird durch analytische Korrelationen sowie numerischen Simulationen unterstützt, die mit einem Grenzschichtcode und dem Strömungslöser TAU durchgeführt wurden. Im Falle des Grenzschichtcodes wurde ein modifiziertes Rauheitsmodell verwendet, um einen Vergleich mit den Experimenten an rauen Wänden zu ermöglichen.

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Von der Fakultät für Maschinenwesen
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zur Erlangung des akademischen Grades eines Doktors der Ingenieurwissenschaften
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Dominik Neeb

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Preface / Vorwort

Diese Dissertation wäre ohne die Unterstützung und Ermutigung vieler Menschen nicht möglich gewesen.

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Köln, im Juli 2025

Dominik Neeb

Abstract

This work presents experimental studies on hypersonic turbulent boundary layers along smooth and rough walls, performed in the Hypersonic Test Section H2K of the German Aerospace Center in Cologne. The generic model for the experiments is a sharp cone consisting of three segments with exchangeable parts to consider smooth and rough surfaces. One segment consists of a technical roughness topology with square bar elements with a nominal wavelength of four times the height of the elements. Another segment consists of sand grain type roughness utilizing Carborundum grains. The boundary layer is probed with a Pitot pressure rake and Particle Image Velocimetry (PIV). The analysis encompasses the mean and turbulent features of the velocity distribution and large scale coherent structures of the turbulent boundary layer. Further, the convective heat flux is deduced via infrared thermography and local sensors. Additionally, Schlieren images and surface pressure data is used to gain general information about the flow field along the cone model. The interpretation of the experimental data is supported by analytical correlations and simulations conducted with a boundary layer code and computation fluid dynamics code TAU. In case of the boundary layer code, a modified roughness model was used to compare to rough wall experiments.

Übersicht

In dieser Arbeit werden experimentelle Untersuchungen zu turbulenten Hyperschall-Grenzschichten entlang glatter und rauer Wände vorgestellt, die im Hyperschall-Windkanal H2K des Deutschen Zentrums für Luft- und Raumfahrt in Köln durchgeführt wurden. Das generische Modell für die Experimente ist ein scharfer Kegel, der aus drei Segmenten mit austauschbaren Teilen besteht, um glatte und raue Oberflächen zu berücksichtigen. Ein Segment besteht aus einer technischen Rauheitstopologie mit quadratischen Elementen mit einer nominalen Wellenlänge von viermal der Höhe der Elemente. Ein weiteres Segment besteht aus einer sandkornartigen Rauheit unter Verwendung von Carborundum-Körnern. Die Grenzschicht wird mit einem Pitot-Druckrechen und Particle Image Velocimetry (PIV) untersucht. Der konvektive Wärmestrom wird über Infrarot-Thermografie und lokale Sensoren abgeleitet. Zusätzlich werden Schlierenbilder und Oberflächendruckdaten verwendet, um allgemeine Informationen über das Strömungsfeld entlang des Kegelmodells zu erhalten. Die Interpretation der experimentellen Daten wird durch analytische Korrelationen sowie numerischen Simulationen unterstützt, die mit einem Grenzschichtcode und dem Strömungslöser TAU durchgeführt wurden. Im Falle des Grenzschichtcodes wurde ein modifiziertes Rauheitsmodell verwendet, um einen Vergleich mit den Experimenten an rauen Wänden zu ermöglichen.

Contents

Preface / Vorwort	i
Abstract / Übersicht	ii
List of Figures	vi
List of Tables	xviii
Nomenclature	xxi
Abbreviations	xxi
Latin Symbols	xxii
Greek Symbols	xxv
Subscripts	xxvi
Superscripts	xxvii
1 Introduction	1
2 State of the Art	7
2.1 Mean Velocity Scalings	9
2.2 Turbulent Stresses	28
2.3 Coherent Structures	29
2.4 Skin Friction and Heat Flux Augmentations	36
2.5 Roughness Topologies	41
3 Methods	44
3.1 Wind Tunnel and Model Properties	44
3.2 Pressure Measurements	54
3.2.1 Surface Pressure	54
3.2.2 Pitot Pressure	55
3.2.3 Post Processing	57
3.3 Particle Image Velocimetry	60
3.3.1 Set-Up	61
3.3.2 Post Processing	63
3.4 Infrared Thermography	66
3.4.1 PEEK Properties	68
3.4.2 Heat Flux Calculation	68
3.4.2.1 pyHeatflow	69
3.4.3 Post-Processing	72
3.4.4 Normalization	74
3.5 Thermal Sensors	75

3.6	Schlieren System	79
3.7	Cone Flow Approximations	79
3.8	Boundary Layer Code	80
3.9	TAU Code	80
3.9.1	Turbulence Models	81
3.9.2	Particle Module Extensions	81
3.10	Skin Friction Determination	81
3.10.1	Analytical Prediction Methods	82
3.10.2	Fitting Approach	85
3.10.3	Gradient Approach	88
3.10.4	Integral Approach	89
3.11	Stanton Number Approximations	93
3.12	Equivalent Sand Grain Roughness Extraction Approach	95
3.12.1	General Equations	95
3.12.2	Uncertainty Estimation	97
4	Results and Discussion	104
4.1	General Flow Features	104
4.2	Mean flow Data	111
4.2.1	Profiles	111
4.2.2	Friction Velocity and Inner Scaling	116
4.3	Turbulence Data	142
4.3.1	Profiles	142
4.3.2	Diagnostic Plot	152
4.3.3	Two-Point Correlations	155
4.3.4	Uniform Momentum Zones	168
4.4	Heat Flux	171
4.4.1	Smooth Wall	171
4.4.2	Rough Wall	179
4.4.3	Rough-to-Smooth Wall Augmentation	180
4.4.4	Augmentation Comparison to Predictions and Literature Data	190
5	Conclusions	200
6	Bibliography	206
7	Appendix	225
7.1	Additional General Investigations	225
7.1.1	Measurement Uncertainties	225
7.1.2	Condensation Analysis	233
7.1.3	Model Surface Scans	234
7.2	Additional Skin Friction Velocity Investigations	237
7.2.1	Skin Friction Velocity Methods Validation	237
7.2.1.1	Fitting approach	237
7.2.1.2	Integral approach	242
7.2.2	Role of Shift of Origin	246

7.2.3	Shift of Origin Analysis	251
7.3	Additional EQSR Investigations	254
7.3.1	Alternative EQSR Extraction Method Validation	257
7.3.2	Literature EQSR Uncertainty Investigations	257
7.3.3	Role of Law-of-the-Wall Constants	268
7.4	Additional Pitot Pressure Investigations	270
7.4.1	Pitot Sensitivity Studies	273
7.5	Additional PIV Investigations	279
7.5.1	Laser Characterization Studies	279
7.5.2	PIV Sensitivity Studies	280
7.5.2.1	Methodological Sensitivities	280
7.5.2.2	Resulting Sensitivities	302
7.5.3	Synthetic PIV (synPIV)	325
7.5.4	synPIV Sensitivity Studies	325
7.5.5	Particle Sensitivity Studies	336
7.5.6	Digital Filtering Method (DFM)	337
7.6	Additional Heat Flux Investigations	339
7.6.1	PEEK Characterization	339
7.6.2	PEEK Emissivity Analysis	340
7.6.3	CT Parameters	343
7.6.4	pyHeatflow Validation	343
7.6.5	Heat Flux Normalization Study	346
7.6.6	Heat Flux Sensitivity Study	350
7.6.7	ANSYS CT Analysis	362
7.7	Additional Numerical Investigations	370
7.7.1	Modified Krogstad Roughness Model Validation	370
7.7.2	Numerical Sensitivity Studies	375
7.7.3	Particle Module Extensions Validation	380

List of Figures

1.1	Spacecraft surface roughness types with a) discrete and b) distributed roughness according to [109].	2
1.2	Sketch of “inner workings” in a Turbulent Boundary Layer (TBL) in form of a representative pseudo mechanical machine according to [183].	5
2.1	a) Schematic of the different layers, its assumed boundaries and applicable mean velocity laws of a smooth wall TBL (LotW: Law-of-the-Wall; rough LotW with wake function corresponds to equation (2.7) with $\Delta U^+ = 0$) and b) mean velocity profile according to Zhang <i>et al.</i> [296] (VS: Viscous Sublayer, BL: Buffer Layer, LL: Logarithmic Layer, WR: Wake Region).	11
2.2	Schematic of the different layers, its assumed boundaries and applicable mean velocity laws of a rough wall TBL (LotW: Law-of-the-Wall; rough LotW with wake function corresponds to equation (2.7)).	12
2.3	Inner scaled mean velocity profiles along smooth (filled circles) and rough wall (triangles: rods, squares: mesh roughness) according to [134].	15
2.4	Sketches of a) “d-type” and b) “k-type” roughness flow originally proposed by [185]. Figure according to [118].	18
2.5	Roughness function $\Delta U^+ = \Delta U/u_\tau$ against $k^+ = ku_\tau/\nu$ for transverse bars, wire-mesh and sand grain roughness according to [19].	19
2.6	Schlieren images of shock waves along a) sand grain roughness [138] and b) diamond roughness [67] in compressible TBL according (both plots from summary in [32]).	21
2.7	PIV mean velocities in streamwise (left column) and wall-normal direction (right column) along smooth wall a), 3D square bar b) and diamond roughness c) in a compressible TBL according to [69].	26
2.8	EQSR k_s/k against Mach number for TSBR with $\lambda/k \approx 4$ from different authors	27
2.9	a) Schematic of a hairpin eddy attached to the wall, b) signature of the hairpin eddy in the streamwise-wall-normal plane according to [2] (in both cases the arrows indicate fluid flow in a reference frame moving with the local convective velocity).	30
2.10	Concept of hairpins evolving into packets according to [2] (the arrows indicate fluid flow in a reference frame moving with the local convective velocity).	31
2.11	Uniform momentum zones along a smooth wall at $Ma_e = 7.6$, $Re_\tau = 279$ according to [287].	32
2.12	Smoke visualization showing various large scale structures of a TBL according to [2].	33
2.13	Two-point correlations of the streamwise Reynolds stress at various wall-normal distances along smooth and diamond rough wall according to [183].	34
2.14	a) Sketch of the expected probability density function in the plane spanned by the fluctuating velocity components u' and v' (in the plot u and v) according to [2]; b) Quadrant analysis of smooth and diamond rough wall Reynolds shear stress profiles according to [183].	35

2.15	Compressible skin friction and heat flux augmentation from different authors according to [32].	38
2.16	Heat flux augmentation rebuilding by compressible Krogstad model compared to experimental data along flat plate at $Ma = 5.3$ $Re \approx 16 \cdot 10^6$ 1/m according to Merrifield <i>et al.</i> [161].	40
2.17	a) Sketch of distributed roughness topology embedded in a TBL with different geometric definitions and b) sketches of roughness element to show relevant geometric definitions for frontal and plan solidity according to [45].	43
3.1	H2K with a) photo of nozzle, test chamber and diffuser, b) performance map. .	45
3.2	Estimations via correlation by Fenter [71] with $k_s = 1$ mm and $T_w = 340$ K showing a) Roughness Reynolds numbers k_s^+ against axial distance x_a for different inflow conditions and cone angles $\theta = 3^\circ$ and $\theta = 7^\circ$, b) H2K condition map for a 7° cone at $x_a = 0.5$ m at $Ma_\infty = 6$ with the achievable k_s^+ (contours) and k/δ (lines) together with FC1 (square) and FC2 (delta).	49
3.3	Definition of model and technical roughness pattern	50
3.4	Test setups of different test campaigns with a) first PIV campaign, b) Pitot pressure campaign.	52
3.5	Test setups of different test campaigns with a) second PIV campaign, b) Sand grain type roughness configuration in Pitot pressure campaign.	53
3.6	Model instrumentation with view from nozzle to diffuser (P: positions of pressure ports, HFM: positions of HFM sensors, CT: positions of coaxial thermocouples, Pitot rake: Pitot rake position if applicable, dashed boxes: sensor plateaus in case of rough segment).	54
3.7	Pitot rake set-up CAD	56
3.8	Pitot rake main dimensions	57
3.9	Pitot rake photography	57
3.10	Set-up in H2K test section with opened sealed PIV camera box (second campaign). .	63
3.11	Discretization of the model in “pyHeaflow”	71
3.12	IR image without (left) and with reflection correction (right)	73
3.13	Example of post-processing for one IR pixel with temperature, heat flux and Stanton number against time for different filter settings	74
3.14	Photographies of local heat flux sensors used, with a) HFM sensor, Coaxial thermocouples of b) type 1 and c) type 2.	77
3.15	Sensor plateau with a) 3D CAD view at $x_a = 484.58$ mm and $\phi = 90^\circ$ and b) picture in test section with installed CT	78
3.16	Artificially generated TBL profiles for smooth wall (black) and rough wall (blue) by method to reverse the law-of-the-wall in a) terms of U/U_e against y/δ and b) U^+ against y^+ together with terms in equation (3.52).	87
3.17	Integral approach scheme (upper part shows the iteration loop to numerically solve eq. (3.57), lower part sketches the applied momentum balance along the boundary layer and its boundaries for a smooth wall after Kármán [127] and rough wall after Perry <i>et al.</i> [187] with contributing components).	92
3.18	Scheme of MCM EQSR uncertainty analysis approach.	98

3.19	a) Quasi UPC of input uncertainties and b) uncertainties of resulting Π (“Pi”), ΔU^+ (“DU+”) and $\alpha = k_s/k$ (“alpha”) from MCM EQSR uncertainty analysis of data from PIV campaign 1 and 2.	101
3.20	Quasi UPC of resulting a) Π (“Pi”), b) ΔU^+ (“DU+”) and c) $\alpha = k_s/k$ (“alpha”) from MCM EQSR uncertainty analysis of data from PIV campaign 1 and 2. . .	102
3.21	Quasi UMF of resulting a) Π (“Pi”), b) ΔU^+ (“DU+”) and c) $\alpha = k_s/k$ (“alpha”) from MCM EQSR uncertainty analysis of data from PIV campaign 1 and 2. . .	103
4.1	Normalized surface pressure along the smooth cone from measurements (symbols), CFD (solid lines) and Taylor-Maccoll computations (dashed line).	105
4.2	PIV raw images along a smooth wall a) 3° cone and b) 7° cone showing typical features of a TBL (white arrow indicates approximated boundary layer thickness δ , yellow lines indicate a 45° angle to the wall).	106
4.3	PIV data of a) mean streamwise velocity U/U_e , b) wall-normal velocity V/U_e , c) Mach number and d) streamwise rms velocity u_{rms}/U_e above smooth wall. (Run3014, IW 48x48 pix ² with 75 % overlap).	108
4.4	PIV data of a) mean streamwise velocity U/U_e , b) wall-normal velocity V/U_e , c) Mach number and d) streamwise rms velocity u_{rms}/U_e above rough wall. (Run3007, IW 48x48 pix ² with 75 % overlap).	109
4.5	Schlieren images along the technical rough wall (Run2091, upper two images with a zoomed view of the left one on the right) and along the Carborundum rough wall (Run2133, lower two images with a zoomed view of the left one on the right).	110
4.6	Mach profiles derived from Pitot measurements at MS1 and FC2 (Run2032&2033 and 2039&2052).	111
4.7	Outer scaled smooth wall mean velocity profiles for all experimental campaigns including repeatability.	112
4.8	Outer scaled smooth wall mean velocity profiles for selected experimental data at a) MS1 and b) MS2 against CFD from TAU and according to [56].	114
4.9	Outer scaled mean velocity profiles showing the effect of smooth (black lines/symbols) and rough wall (red lines/symbols) for different runs from a) PIV (smooth Run3014, Run3019, rough Run3007, Run3020) and b) Pitot data (smooth Run2032&2033, Run2013&2023, rough Run2090&2091, Run2095-2095).	115
4.10	Streamwise mean velocity data from PIV (IW 512x8 pix ²) at MS1 in viscous sublayer.	119
4.11	Comparison of near wall PIV velocity data points transformed to skin friction velocities via the gradient approach (blues symbols) compared to values from fitting approach (blue solid line for PIV, red solid line for Pitot Run2013&2023) and data from van Driest correlation (black dashed line) and TAU computations (green lines).	121
4.12	Streamwise mean velocity data from PIV (IW 512x8 pix ²) and Pitot pressure in inner (b) and outer scaling (a,c-d) according to Fernholz and Finley [73] (DNS data according to Duan <i>et al.</i> [56]).	122
4.13	Wake parameter $\Delta U_{eff,wake}^+$ against Reynolds number Re_{δ_2} (circles for smooth wall and squares for rough wall results) with data according to Coles (see Fernholz and Finley [73]) and Williams and Smits [289].	123
4.14	Shift of origin determination via different methods described in chapter 7.2.2. . .	125

4.15	Rough wall skin friction velocity u_τ in a) and roughness function ΔU_{eff}^+ in b) from fitting approach against shift of origin ϵ/k for selected rough wall runs. Dashed lines in a) indicate corresponding value range from integral approach including uncertainties.	126
4.16	Inner scaled van Driest transformed mean velocity profiles along smooth (black, grey) and rough wall (colored) together with the law-of-the-wall and DNS data according to Duan <i>et al.</i> [56] and Zhang <i>et al.</i> [296].	127
4.17	Roughness function ΔU^+ against roughness Reynolds number k^+ for square bars of different wavelength close to a value of $\lambda/k = 4$ from different authors (symbols without edge for incompressible and with edge for supersonic data including the edge Mach number in brackets) according to [173], [24], [185], [152], [138], [208], [209], [288], [195], [42], [164], [187], [150], [148], [43], [33].	132
4.18	a) Equivalent sand roughness for various k-type roughness topologies versus the solidity $\lambda_f = k/\lambda$, corrected with empirical drag coefficients. Open symbols, rounded elements (spheres, cones, $c_D = 0.3$; spherical segments, $c_D = 0.13$) from Schlichting [212]. For all others, $c_D = 1.25$. “black delta”, spanwise fences (Schlichting [212]); “black gradient”, spanwise fences (Webb <i>et al.</i> [273]); “plus”, spanwise cylinders (Tani [247]); “x”, spanwise square bars (Bandyopadhyay [19]), spanwise square bars (Berg [24]), 3D blocks (Ekoto [69]), mesh and square bars (Williams <i>et al.</i> [288]). The dashed lines have logarithmic slopes +1 and -2. For the error bars, see text. b) Equivalent sand roughness for d-type roughness topologies versus blockage ratio. delta, Wood and Antonia [290]; circle, Perry <i>et al.</i> [187]; gradient, Bandyopadhyay [19]; square, Tani [247].	133
4.19	a) Inner and b) outer scaled van Driest transformed mean velocity profiles along technical square bar rough wall and sand grain type rough wall (SG) from Pitot and PIV data together with the law-of-the-wall and DNS data according to [296].	134
4.20	Calculated roughness Reynolds numbers k_s^+ along rough cone from BLcode at different EQSR values and assumptions together with data from PIV.	136
4.21	Calculated (lines) and experimental (symbols) a) smooth and rough wall inner-scaled van Driest transformed velocities U^+ and b) difference from log-law against wall-normal distance from BLcode at different EQSR values.	138
4.22	Calculated (lines) and experimental (symbols) a) smooth and rough wall skin friction $c_{f,e}$ and b) skin friction augmentations $c_{f,e,r}/c_{f,e,s}$ along rough cone from BLcode at different EQSR values.	139
4.23	Calculated (lines) and experimental (symbols) a) smooth and rough wall skin friction velocity u_τ and b) skin friction velocity augmentations $u_{\tau,r}/u_{\tau,s}$ along rough cone from BLcode at different EQSR values.	140
4.24	Calculated (lines) and experimental (symbols) rough to smooth wall skin friction augmentation along roughness Reynolds number from different authors (grey/black lines/symbols, all collected in [32]) and data of this thesis (red symbols).	142
4.25	Profiles of outer layer Morkovin scaled velocity fluctuation components in a) streamwise, b) wall-normal direction in comparison to previous experimental ([68], [209], [289], [129]) and CFD data ([56], [57], [296]).	144
4.26	Profiles of outer layer Morkovin scaled velocity fluctuation components in Reynolds shear-stress component in comparison to previous experimental ([68], [209], [289], [129]) and CFD data ([56], [57], [296]).	145

4.27	Streamwise turbulence profiles of smooth (black, grey symbols) and rough wall experiments (colored symbols) in a) outer layer skin-friction velocity scaling and b) outer layer Morkovin scaling together with incompressible and compressible DNS according to [58], [296].	147
4.28	Streamwise turbulence profiles of smooth (black, grey symbols) and rough wall experiments (colored symbols) in a) inner layer Morkovin scaling and b) semilocal Morkovin scaling together with incompressible and compressible DNS according to [58], [296].	148
4.29	Turbulence profiles of smooth (black, grey) and rough wall (colored) experiments in outer layer Morkovin scaling in a) streamwise and b) wall-normal direction together with incompressible and compressible DNS data according to [58], [296].	150
4.30	Turbulence profiles of smooth (black, grey) and rough wall (colored) experiments in outer layer Morkovin scaling in Reynolds shear stress together with incompressible and compressible DNS data according to [58], [296].	151
4.31	Diagnostic plots of a) smooth wall data in smooth scaling together with data according to [6] and b) smooth (black, grey) and rough wall data (colored) in smooth scaling with theoretical correlations according to [39] (smooth) and [146] (rough).	153
4.32	Diagnostic plots of a) smooth (black, grey) and rough wall data (colored) in compressible smooth scaling and b) smooth (black, grey) and rough wall data (colored) in rough scaling ($U' = U + \Delta U$) with theoretical correlations according to [39]	154
4.33	Two-point correlations of streamwise velocity fluctuations above smooth wall (Run3014) at $y/\delta = 0.7, 0.5$ and 0.1 from top to bottom.	156
4.34	Two-point correlations of streamwise velocity fluctuations above rough wall (Run3007) at $y/\delta = 0.7, 0.5$ and 0.1 from top to bottom.	157
4.35	One-dimensional autocorrelation of the streamwise velocity fluctuations R_{uu} at $y/\delta = 0.5$ for smooth wall case 3014 compared to data from [183], [114], [85] and [189].	158
4.36	Dimensions of the two-point correlation of streamwise velocity fluctuations with wall-normal position a) change of upstream, L_x^u , and downstream, L_x^d , extent of different levels (0.2, 0.3 and 0.5), b) change of maximum, L_x^m , extent of different levels (0.2, 0.3 and 0.5). Solid lines show smooth wall data of Run3014 in comparison to incompressible TBL data at $\delta^+ = 2000$ according to Sillero <i>et al.</i> [228].	161
4.37	Wall-normal variation of maximum streamwise dimension L_x^m for two-point correlation contour levels 0.2 to 0.5 of smooth wall Run3014 and rough wall Run3007.	162
4.38	Inclination angle of different levels (0.2 - 0.5) of the two-point correlation of streamwise velocity fluctuations with wall-normal position. Colored symbols show smooth wall data of Run3014 in comparison to incompressible TBL data by Sillero <i>et al.</i> [228], incompressible rough TBL data by Wu and Christensen [292], compressible TBL data by Pirozzoli and Bernardini [189] and hypersonic TBL data by Williams <i>et al.</i> [287].	164

4.39	Inclination angle of the of streamwise velocity fluctuation correlations with wall-normal distance for contour levels between 0.2 and 0.5 for a) smooth wall Run3014 and b) rough wall Run3007. In both plots dashed lines and dashed-dotted lines represent the mean of levels 0.2 to 0.7 and a linear fit through the fitted ellipses, respectively.	165
4.40	a) Averaged max. contour dimension L_x^m against edge Mach number, b) averaged inclination angle θ against edge Mach number in comparison to literature data as noted in the legend (sb: square bar roughness, m: mesh roughness; both followed by the height in millimeters).	167
4.41	Compilation of p.d.f. (upper left) based on an instantaneous velocity field (lower middle) and a comparison of an instantaneous (red line) to the mean velocity profile (black line, upper right) for a) smooth wall (Run3014) and b) rough wall data (Run3007). Boundaries for each UMZ are indicated as black lines of different dash-styles which correspond to each other in all corresponding subplots. TNTI is indicated in all subplots as black dash-dot-dotted contour line.	170
4.42	IR image of smooth wall Run1011 at $t = 15$ s, extracted line information along x -axis for convective heat flux, wall temperature and edge based Stanton number and heat flux along y -axis	172
4.43	Convective heat flux, wall temperature and edge based Stanton number repeatability for smooth wall Run1003 to 1011 and selected smooth wall runs of second PIV campaign.	173
4.44	Smooth wall Stanton number comparison along cone between experimental, analytical and numerical data.	174
4.45	HFM1 and CT3 sensor signals for smooth wall Run 1011.	176
4.46	Convective heat flux a), edge based Stanton number b) and sensor temperature c) over time of HFM and CTC3 sensors for smooth wall Run1011.	178
4.47	Rough wall convective heat flux, wall temperature and edge based Stanton number for Run1012, 1013 and 3006, 3007 compared to smooth run data.	179
4.48	Stanton number (a) and rough wall heating augmentation (b) for different smooth runs (Run1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010) and rough runs (1012, 1013) from IR for all combinations (light blue lines), smooth runs to ensemble averaged smooth values (green lines), rough runs to ensemble averaged smooth values (blue lines), ensemble averaged rough to ensemble averaged smooth values (black and red lines) and mean augmentation spatially averaged within $0.4\text{m} < x_a < 0.5\text{m}$, $0.4\text{m} < x_a < 0.6\text{m}$ and $0.5\text{m} < x_a < 0.6\text{m}$ (symbols from left to right, respectively).	184
4.49	Stanton number and rough wall heating augmentation for different smooth runs (Run3014, 3016, 3019) and rough runs (Run3006, 3007, 3008) from IR for all combinations (light blue lines), smooth runs to ensemble averaged smooth values (green lines), rough runs to ensemble averaged smooth values (blue lines), ensemble averaged rough to ensemble averaged smooth values (black and red lines) and mean augmentation spatially averaged within $0.4\text{ m} < x_a < 0.5\text{ m}$, $0.4\text{ m} < x_a < 0.6\text{ m}$ and $0.5\text{ m} < x_a < 0.6\text{ m}$ (symbols from left to right, respectively).	185
4.50	HFM sensors a) edge Stanton number results and b) heating augmentation for smooth wall Run1011 and rough wall Run1015.	186

4.51	CT sensors heating augmentation for several smooth and rough wall runs at a) mid-sensor CT2 (TE63-3) and b) aft sensor CT3 (101-4).	187
4.52	Heating augmentations for smooth wall runs and SG runs for a) CT2 sensor b) roughened CT2 sensor.	188
4.53	Roughness Stanton number augmentation for different runs from IR (green line from first PIV campaign and red line from second PIV campaign) and local heat flux sensors (symbols).	189
4.54	Calculated Stanton numbers along rough cone from BLcode at different EQSR values.	192
4.55	Rough-to-smooth wall Stanton number augmentations $St_{e,r}/St_{e,s}$ a) along the cone for data with different EQSR from BLcode, analytical predictions from Seidman and Powars as well as IR data (only second PIV campaign) and b) near the roughness section $0.33\text{m} < x_a < 0.65\text{m}$ for data with different EQSR from BLcode and IR data (only second PIV campaign).	193
4.56	Rough wall heat flux augmentation from computations (lines) and current data (symbols).	195
4.57	Rough wall heat flux augmentations from computations (lines) and experimental data of previous and current investigations (symbols).	196
4.58	Rough Reynolds analogy efficiency $(St/St_s)/(c_f/c_{f,s})$ from DNS data for transverse circular rods and square bars with roughness Reynolds number $k_t^+ \approx 40 - 150$ according to [147] and irregular roughness with $k_{rms}^+ \approx 16 - 31$ according to [79]. Plot and data extracted after [45].	199
7.1	Condensation with data from [24], [182], [294]	234
7.2	Exemplary 3D scan of square bar roughness	235
7.3	Exemplary 3D scan of sand grain type roughness	235
7.4	Test data in inner and outer scaling according to [73] with Direct Numerical Simulation (DNS) data according to [296].	238
7.5	Streamwise mean velocity from artificial data in inner and outer scaling according to [73] with full profile in a) and cut by five near wall points in b).	239
7.6	Streamwise mean velocity from artificial data in inner and outer scaling according to [73] with a) $k = 1.27\text{mm}$ and b) $k = 1.65\text{mm}$ transverse square bar roughness profile according to [288].	241
7.7	Ratio of boundary layer thickness to momentum loss thickness along smooth and rough 7° cone from Boundary layer code (BLcode) at FC1	242
7.8	Test of integral approach along 7° cone at FC1 (only every 100th point shown for clarity)	245
7.9	Downsampling to $\Delta y = 128$ pix of velocity profile of TBL along 7° cone at FC1	245
7.10	Effect of the shift of origin ϵ on the theoretical curve from the law-of-the-wall for rough flows according to [110].	246
7.11	Sketch of a TSBR with definition of shift of origin ϵ and melt down height	249
7.12	Shift of origin ϵ/k against wavelength λ/k for TSBR from different authors	251
7.13	Example plots for the extraction of shift of origin ϵ for data of rough wall Run3007 following a) the “Krogstad” (see [135]) and b) the “pseudoPerry” (see [186] or [219]) and “Perry wake” approach (see [185]).	253

7.14 a) Quasi Uncertainty Percentage Contribution (UPC) of input uncertainties and b) uncertainties of resulting Π ("Pi"), ΔU^+ ("DU+") and $\alpha = k_s/k$ ("alpha") from Monte Carlo Method (MCM) Equivalent Sand Grain Roughness (EQSR) uncertainty analysis of literature data according to Berg [24], Ekoto [69], Squire <i>et al.</i> [240], Sahoo <i>et al.</i> [208], Bhatt and McClain [26].	265
7.15 Quasi UPC of resulting a) Π ("Pi"), b) ΔU^+ ("DU+") and c) $\alpha = k_s/k$ ("alpha") from MCM EQSR uncertainty analysis of literature data according to Berg [24], Ekoto [69], Squire <i>et al.</i> [240], Sahoo <i>et al.</i> [208], Bhatt and McClain [26].	266
7.16 Quasi Uncertainty Magnification Factor (UMF) of resulting a) Π ("Pi"), b) ΔU^+ ("DU+") and c) $\alpha = k_s/k$ ("alpha") from MCM EQSR uncertainty analysis of literature data according to Berg [24], Ekoto [69], Squire <i>et al.</i> [240], Sahoo <i>et al.</i> [208], Bhatt and McClain [26].	267
7.17 Roughness function ΔU^+ against k^+ for Nikuradse's sand grain and an exemplary artificial technical roughness with its equivalent sand grain roughness transformation with parameters inspired by [185]	268
7.18 Centerline displacement effect: a) displacement due to averaging [92], b) displacement due to streamline displacement [158].	270
7.19 Viscous and shock correction to measured Pitot pressure in supersonic flow according to [201].	272
7.20 Viscous correction to measured Pitot pressures. a) Wall-normal position y/δ against driving parameter $Re_{2,r} (\rho_2/\rho_1)^{0.5}$ and b) $p_{Pitot,m}/p_{Pitot,i}$ against driving parameter $Re_{2,r} (\rho_2/\rho_1)^{0.5}$ for data from Berg (profile at $x = 39.4$ in and smooth-to-rough case with $k = 0.05$ in according to [24]), CFD code of DLR (DLR) (profile at $x = 0.5$ m smooth surface for FC2) and experimental data (Run 2024&2025) .	273
7.21 Repeatability of Pitot pressure coefficient profiles of smooth runs at FC2 and MS1.	274
7.22 Comparison of a) temperature and b) velocity profiles with nominal Crocco-Busemann velocity-temperature correlation (black symbols) against Walz correlation (red symbols) and the correlation according to [58] for smooth runs at FC2 and MS1.	275
7.23 a) CAD of single Pitot probe and b) Pressure coefficients of Pitot rake vs. single Pitot probe measurements at FC2 and MS2.	276
7.24 Pitot pressure coefficient profiles along rough wall (red symbol, Run2090&2091) and smooth measurement plateau within rough wall (orange symbol, Run2072-2074).	277
7.25 Inner and outer scaled van Driest transformed mean velocity profiles along technical square bar rough wall including data above the smooth measurement plateau within rough wall (orange symbol, Run2072-2074) together with the law-of-the-wall and DNS data according to [296].	278
7.26 Laser pulse photodiode measurement	279
7.27 Laser pulse interval results for a pre-set value of 140 ns with different pulse detection algorithms	279
7.28 Variations of different parameters for a single particle trajectory through the flow field along the cone for particle p_2 in a TAU RSM flow solution at FC1 and $T_w = 360$ K	282

7.29	Relative velocity difference of particles p_1 and p_2 along the 7° cone at FC1 at measurement section MS1 and MS2 (shaded region corresponds to Particle Image Velocimetry (PIV) set-up velocity uncertainty)	283
7.30	Global displacement histograms, illustrating that no peak-locking occurred in PIV results of a) first (Run1007, IW 256x256 pix ²) and b) second PIV campaign (Run3007, IW 64x64 pix ²)	284
7.31	Effect of spatial filtering on synthetic rms velocity compared to theory according to [238].	287
7.32	Effect of spatial filtering on synthetic two-point correlation compared to reference case (red line, for symbols see figure 7.33a).	288
7.33	Effect of spatial filtering on synthetic rms velocity compared to theory according to the method by [238] for unity and non-unity aspect ratio IWs. a) shows legend of symbols used in plots b) to d) with their corresponding combination of $(\Delta x/L_{True})_{eff} = (N_y/n_y N_z/n_z)^{0.5}$, d) $(\Delta x/L_{True})_{eff}$ according to eq. (7.12) (thinner lines show $\pm 10\%$ of theoretical line according to [111])	289
7.34	Effect of spatial filtering on synthetic rms velocity compared to theory according to the method by [238] for unity and non-unity aspect ratio IWs and length scales according to homogenous turbulence. a) shows legend of symbols used in plots b) to d) with their corresponding combination of $N_y/n_y \times N_z/n_z$. Rms velocity plotted against b) $(\Delta x/L_{True})_{eff} = N_y/n_y$ as used [238], c) $(\Delta x/L_{True})_{eff} = (N_y/n_y N_z/n_z)^{0.5}$, d) $(\Delta x/L_{True})_{eff}$ according to eq. (7.12) (thinner lines show $\pm 10\%$ of theoretical line according to [111]).	290
7.35	Longitudinal integral length scales in the TBL according to Peltier [183], Ringuette <i>et al.</i> [204], Ringuette <i>et al.</i> [205].	292
7.36	Morkovin scaled streamwise Reynolds stress in the TBL according to Jiménez <i>et al.</i> [119], Sillero <i>et al.</i> [227] and Duan <i>et al.</i> [56].	295
7.37	Effect of PIV IW filtering on streamwise Reynolds stress in the DNS TBL from Duan <i>et al.</i> [56] and Jiménez <i>et al.</i> [119] according to method of Lee <i>et al.</i> [144].	296
7.38	Variations of Stokes numbers due to turbulence motion across the boundary layer for FC1	298
7.39	a) Histogram of the velocity fluctuations for different particle displacements, b) Estimated turbulent levels as a function of particle displacement for different interrogation window sizes by applying method according to [211].	300
7.40	a) Exemplary two-point correlation of streamwise velocity fluctuations above rough wall (symbols) and a corresponding exponential fit (solid line) and linear fit (dash-dotted line) to points in the vicinity of the coordinate of interest, b) Estimated random noise within the TBL for different IW sizes by applying method according to Williams [285]; both plots for data of Run3007.	301
7.41	IW sub-grid filtering estimates after the approach according to a) Spencer and Hollis [238] and b) Lee <i>et al.</i> [144].	302
7.42	Test of static temperature calculation from velocity via constant temperature assumption according to [241].	303
7.43	Outer scaled mean velocity profiles showing effect of IW size for a) first (Run1007) and b) second PIV campaign (Run3014).	309
7.44	Outer scaled mean velocity profiles showing the effect of smooth (black lines/symbols) and rough wall (red lines/symbols) and repeatability for different runs.	310

7.45	Streamwise mean artificial velocity data for different averaging size and positions for (a) smooth and (b) rough wall (numbers in legend are given in [m]) in inner b) and outer scaling a),c)-d) according to [73].	311
7.46	Streamwise mean velocity data from first PIV (Run1007: FC1, 512x8 pix ²) and second PIV campaign (Run3014: FC1 and Run3015: FC2, both with IW 512x16 pix ²) in inner b) and outer scaling a),c)-d) according to [73] (DNS data according to [56])	312
7.47	Streamwise mean artificial velocity data with (a) full profile and (b) near wall cut profile in inner b) and outer scaling a),c)-d) according to [73].	313
7.48	Streamwise a) and wall-normal b) component of turbulence profiles derived with temperature-velocity model according to Walz compared to artificially sub-grid filtered DNS data (black lines) by [56] and [296]	314
7.49	Streamwise a) and wall-normal b) component of turbulence profiles compared to artificially sub-grid filtered DNS data (grey lines) by [56] and [296] according to the method of [144] for IW 64x64 pix ²	315
7.50	Streamwise a) and wall-normal b) component of turbulence profiles compared to artificially sub-grid filtered DNS data (grey lines) by [56] and [296] according to the method of [144] for IW 512x16 pix ²	316
7.51	Streamwise a) and wall-normal b) component of turbulence profiles compared to artificially sub-grid filtered DNS data (grey lines) by [56] and [296] according to the method of [144]	317
7.52	Impact of IW sizes on two-point correlation of streamwise velocity fluctuation's a) change of downstream scale L_x^d and b) change of inclination angle with wall-normal position from correlation contour level 0.2.	318
7.53	Streamwise length-scale λ_x^u against wall normal position for smooth wall case Run3014 and different calculation approaches compared to data from [183] and [189].	319
7.54	Streamwise length-scale λ_x^u against wall normal position for smooth wall case Run3014 and different IW sizes.	319
7.55	Quadrant plot for a) smooth wall (Run3014) and b) rough wall data (Run3006).	320
7.56	Reynolds stress, conditionally averaged according to their contributions to Q1 to Q4 along smooth (Run3014) and rough wall (Run3006).	321
7.57	Two-point correlation of the wall-normal velocity fluctuations at $y/\delta \approx 0.5$ along a) smooth and b) rough wall.	322
7.58	Compilation of p.d.f. (upper left) based on an instantaneous velocity field (lower middle) and a comparison of an instantaneous (red line) to the mean velocity profile (black line, upper right) for different snapshots of smooth wall Run3014. Boundaries for each Uniform Momentum Zones (UMZ) are indicated as black lines of different dash-styles which correspond to each other in all corresponding subplots. TNTI is indicated in all subplots as black dash-dot-dotted contour line.	323
7.59	Compilation of p.d.f. (upper left) based on an instantaneous velocity field (lower middle) and a comparison of an instantaneous (red line) to the mean velocity profile (black line, upper right) for different snapshots of rough wall Run3007. Boundaries for each UMZ are indicated as black lines of different dash-styles which correspond to each other in all corresponding subplots. TNTI is indicated in all subplots as black dash-dot-dotted contour line.	324
7.60	synPIV synthetic raw particle image comparison. Two on the left from synPIV, right image from experimental first H2K campaign (Run1007).	327

7.61	synPIV sensitivity study for FC1, MS1, s1D, IW 8x512 pix ²	328
7.62	synPIV sensitivity study for FC1, MS1, s2D, IW 8x512 pix ²	330
7.63	Profiles of U , V , u_{rms} , v_{rms} and $\overline{uv}$ from the TAU input (red lines) and extracted from the synthetic flow fields (blue symbols) in pixel shift scaling.	332
7.64	Two-point correlation data of synthetic dataset via synPIV with DFM method with a) R_{uu} , b) x,y -cuts of R_{uu} , c) R_{vv} , d) x,y -cuts of R_{vv} ; red lines and label “generic” shows input.	333
7.65	Two-point correlation data of synthetic dataset from synPIV with a) R_{uu} at $y/\delta = 0.5$ and b) extracted integral length scale for different synthetic particle data sets and IWs.	334
7.66	Two-point correlation fitting results of synthetic dataset from synPIV showing sensitivity of a) L_m^x/δ and b) ϕ for different synthetic particle data sets and IWs.	335
7.67	Particle density simulations and data from first PIV campaign.	337
7.68	Spectral dependency of “TECAPEEK natural” [®] emissivity at room temperature	341
7.69	Temperature dependency of “TECAPEEK natural” [®] emissivity (probe 4)	342
7.70	Emissivity of “TECAPEEK natural” [®]	343
7.71	pyHeatflow cylindrical validation and sensitivity with a) heat flux against time at $x_a \approx 0.5$ m ($r = 0.06$ m) and b) heat flux difference between non-cylindrical assumption and input at $x_a \approx 0.325$ m ($r = 0.015$ m), $x_a \approx 0.5$ m ($r = 0.06$ m) and $x_a \approx 0.733$ m ($r = 0.09$ m).	345
7.72	Heat flux for Stanton number sensitivity study based on boundary layer code simulations along the 7° cone at three different inflow conditions and $T_w = 340$ K if not otherwise indicated: solid lines for fully turbulent and dash-dotted lines for transitional computations	347
7.73	Stanton number definition sensitivity study with a) the adiabatic wall temperature along x -axis via analytical equation ((3.8), dashed), fully turbulent computation with adiabatic wall assumption (solid) and transitional computation with adiabatic wall assumption (dash-dotted); b)-f) Stanton numbers with b) $\rho_{St} = \rho_\infty$, $u_{St} = u_\infty$, $T_{St} = T_0$, c) $\rho_{St} = \rho_e$, $u_{St} = u_e$, $T_{St} = T_{aw}$, d) $\rho_{St} = \rho_\infty$, $u_{St} = u_\infty$, $T_{St} = T_{aw}$, e) $\rho_{St} = \rho_e$, $u_{St} = u_e$, $T_{St} = T_r$, f) $\rho_{St} = \rho_\infty$, $u_{St} = u_\infty$, $T_{St} = T_r$. For b) to e): solid lines for fully turbulent and dash-dotted lines for transitional computations.	349
7.74	Influence of inflow on transition location with a) Convective heat flux, wall temperature and edge based Stanton number seeding influence comparison for Run1003 and Run1011 at $t = 15$ s and b) Stanton numbers for the same runs at different time points, highlighting the influence of seeding on the transition front.	350
7.75	Temporal heat flux distributions based on experimental data of second PIV campaign at FC1 with smooth wall Run3014, 3016 (left column, blue and red symbols, respectively) and rough wall Run3006, 3007 (right column, blue and red symbols, respectively) at $x = 0.5$ m.	351
7.76	Temporal heat flux augmentation distributions based on experimental data of second PIV campaign at FC1 with rough to smooth wall Run3006 to Run3014 and Run3007 to 3016 (blue and red symbols, respectively) at $x = 0.5$ m.	352
7.77	Impact of boundary conditions on a) calculated roughness Reynolds number k_s^+ and b) Stanton number along rough cone from BLcode at different EQSR values.	359
7.78	Rough-to-smooth wall convective heat flux edge based Stanton number from BLcode for different EQSR values at FC1 (solid lines) and FC2 (dashed lines).	360

7.79	Calculated skin friction augmentation against roughness Reynolds numbers k_s^+ along rough cone from BLcode at different EQSR values and assumptions.	360
7.80	Calculated a) turbulent Prandtl number profiles Pr_t at MS1 and b) corresponding Stanton numbers along cone from BLcode with different Prandtl number assumptions. Smooth wall results with black lines and rough wall with colored lines, all with $k_s = 0.5\text{mm}$	361
7.81	ANSYS simulation of CT in PolyEther Ether Ketone (PEEK) setup in a) for convective boundary conditions and in b) for pre-scribed surface temperature conditions.	363
7.82	ANSYS simulation of CT in PEEK with grid in a) and temperature distribution at $t = 20\text{s}$ in b).	364
7.83	CT in PEEK ANSYS simulation results of a) heat flux and b) wall temperatures on PEEK.	366
7.84	CT in PEEK ANSYS simulation results of heat flux on CT.	367
7.85	CT in PEEK ANSYS simulation results of a) wall temperatures and b) backside temperatures on CT.	368
7.86	CT in PEEK ANSYS simulation results of lateral heat flux along the vertical CT-PEEK junction.	369
7.87	a) Inner scaled velocities and b) roughness function ΔU^+ against EQSR Reynolds number k_s^+ from integration of corresponding terms in eq. 7.35 to 7.39 for different formulations according to [260] and [136] and data points corresponding to “circle” Bandyopadhyay1987 $\lambda/k=3.8$, “square” Berg1977(Ma6.0) [24] $\lambda/k=4$, “delta” Betterman1965 [25] $\lambda/k=4.2$, “gradient” Latin2000(Ma2.9) [139] $\lambda/k=3.9$, “left triangle” Liu1966 [152] $\lambda/k=4$, “right triangle” Perry1963 [185] $\lambda/k=4$, “pentagon” Bowersox2001(Ma2.7) [33] $\lambda/k=3.9$, “star” Pritchett2001(Ma0.22) [195] $\lambda/k=3.9$, “plus” Pritchett2001(Ma0.56) [195] $\lambda/k=3.9$, “cross” Leonardi2003 [148] (DNS) $\lambda/k=4$, “hexagon corner up” Moore1951 [185] $\lambda/k=3.9$, “hexagon” Perry1963 [185] $\lambda/k=4$, “1” Perry1969 [187] $\lambda/k=3.6$, “triangle up” Lee2012(DNS) [143] $\lambda/k=4$, “triangle left” Liu1966 [152] $\lambda/k=4$, “triangle right” Leonardi2007(DNS) [150] $\lambda/k=4$; black symbols and gray symbols for compressible and incompressible data, respectively.	373
7.88	Heat flux augmentation calculated by compressible Krogstad model along flat plate at Mach 7 with comparison to data according to [161].	374
7.89	pyHeatflow grid sensitivity ($T_{w,analytical}$)	378
7.90	pyHeatflow grid sensitivity ($T_{w,experiment}$)	379
7.91	TAU particle module validation with data and drag model from Tedeschi <i>et al.</i> [250].	383

List of Tables

2.1	Selected compressible roughness studies; Model Types : FP: flat plate, C: cone, BC: bicone, SC: sphere-cone, TSW: test section wall, OC: ogive-cylinder; Roughness Types : V: v-grooves, S: sand grain (partly grit), H: hemispheres, D: dimples, C: cones, TSB: transverse square bars, TB, transverse bars, 3DB: 3D block, DM: diamonds, M: mesh; Measured Quantities : MF: Mean flow, T: turbulent quantities, (T): temperature fluctuations in case of [24], SF: skin-friction, (SF): total skin friction via force balance, HF: heat flux; Measurement Techniques : PP: Pitot pressure, PIV: Particle Image Velocimetry, LDV: laser doppler velocimetry, HWA: hot wire anemometry, IR: infrared thermography, LS: local sensors, SFB: skin friction balance, FB: force balance; Wall Status : a: adiabatic, na: non-adiabatic; (partly adapted from [32] and [184]).	25
3.1	Nominal flow conditions FC1 and FC2	47
3.2	Nomenclature for the sensors and positioning	55
3.3	Streamwise locations of profile measurement sections (MS) for Pitot and PIV velocity profile measurements	60
3.4	Characteristics of infrared cameras	67
3.5	Setup of infrared cameras (o: outside test section).	68
3.6	Fenter parameters a_i and b_i	84
3.7	Input quantity uncertainties for EQSR MCM uncertainty estimation (see section 7.1.1).	101
4.1	Edge and wall parameters of PIV and Pitot profiles at FC1 and FC2 from fitting approach (s: smooth wall, r: rough wall).	118
4.2	Summary of derived smooth wall skin friction velocities at FC1 and FC2 and MS1 and MS2 (all values in [m/s])	120
4.3	Overview of results from fitting approach (s: smooth wall, r: rough wall).	128
4.4	EQSR results from post-processing via equation (3.75) or equation (3.76)	130
4.5	Skin friction velocities from integral approach of PIV (IW 512x16 pix ²) and Pitot pressure data along rough wall (first and second block for TSB roughness and fourth block for SG roughness (2121 < run < 2131)) (s: smooth wall, r: rough wall).131	
4.6	Reynolds analogy comparison of experimental data (PIV and IR data), numerical (TAU, BLcode) and analytical predictions (White [277], Van Driest [263]) at MS1.175	
7.1	Test matrix (s: smooth wall, r: rough wall, s(m): metallic smooth wall, r(SG): sand grain type Carborundum rough wall).	226
7.2	PIV velocity uncertainties (second PIV campaign)	228
7.3	Input to MCM uncertainty estimation for fitting approach for smooth wall Run3014 and IW 512x16 pix ² (rec.: rectangular distribution, norm.: normal distribution)	230

7.4	Input to MC uncertainty estimation for integral approach for rough wall Run3014 and IW 512x16 pix ² (rec.: rectangular distribution, norm.: normal distribution)	231
7.5	u_τ and Π MCM uncertainty estimations for fitting and integral approach for smooth wall Run3014	232
7.6	Coefficients for Antoine's equation (eq. (7.1)) [41]	233
7.7	Statistics of 3D surface scans of Carborundum sand grain type roughness.	236
7.8	Results of test of fitting approach along 7° cone at FC1 with changing simulated wall normal resolution according to PIV	238
7.9	Validation results for the calculation of u_τ via (3.52) (*: fitted values).	240
7.10	Results of validation cases of integral approach (j: 0 for 2D, 1 for axisymmetric; c: 0 for incompressible, 1 for compressible; r: 0 for smooth, 1 for rough; a: analytical, O: ordinary differential equation (ODE), D: differential quotient).	243
7.11	Results of test of integral approach along 7° cone at FC1 with changing simulated wall normal resolution according to PIV	244
7.12	Shift of origin ϵ/k determination via different methods described in chapter 7.2.2 (all for IW 512x16 pix ² of second PIV campaign).	252
7.13	Selected compressible roughness studies; Superscripts: a: k_s^+ rebuilt via ΔU^+ and eq. (3.75), b: ΔU^+ rebuild from graphs in reference; Subscripts: rb: rebuilt ; Roughness types: TSB: Transverse square bars.	255
7.14	Selected compressible roughness studies, topology parameters; Topology parameters: Λ : Danberg and Sigal [51], λ_{Dvorak} : Dvorak [60], Λ_s : Sigal and Danberg [226], λ_2 : Huang <i>et al.</i> [112]; Roughness types: TSB: Transverse square bars	256
7.15	Comparison of EQSR approaches according to method in section 3.10.2 via eq. (3.74) and eq. (3.76).	257
7.16	Input for MCM EQSR uncertainty analysis with data according to Bhatt and McClain [26] for data originally from Schlichting [212] of plate No. XXV at 20 m/s260	
7.17	Input for MCM EQSR uncertainty analysis with data according to Ekoto [69], Ekoto <i>et al.</i> [66], Ekoto <i>et al.</i> [68], Ekoto <i>et al.</i> [67]	261
7.18	Input for MCM EQSR uncertainty analysis with data according to Squire <i>et al.</i> [240], Squire [239]	262
7.19	Input for MCM EQSR uncertainty analysis with data according to Sahoo <i>et al.</i> [208]	263
7.20	Input for MCM EQSR uncertainty analysis with data according to Berg [24]	264
7.21	Random error [285] and sub-grid filtering [238] approximation of PIV data at different IW sizes at $y/\delta = 1.0$ and $L_{uu}/\delta = 0.5$ according to [183].	293
7.22	Estimated turbulent levels and random errors for different interrogation window sizes by applying method according to [211].	299
7.23	Skin friction velocities from integral approach of PIV data (IW 512x16 pix ²) along rough wall with sensitivity to averaging.	304
7.24	Results of mean number of UMZs (± 1 standard deviation) for smooth run (3014) and b) rough run (3007) for different IW sizes.	308
7.25	Overview of synPIV sensitivity cases (numP - number of particles per 48x48 window, muP and sigma - mean and standard deviation of log-normal particle distribution, s1D - smooth 1D, s2D - smooth 2D field, r2D - rough 2D field; IW field size dimensions switched with wall-normal x streamwise extent).	326
7.26	Coefficients for the material parameters of PEEK	340

7.27	Coefficients for eq. (7.25) to (7.27) of the chromel material parameters according to [107]	343
7.28	IR roughness heat flux augmentation for different spatial and temporal averaging.	353
7.29	Statistics of filtered HFM roughness heat flux augmentation within $t = 3 - 20$ s of rough wall Run1015 to smooth wall Run1011.	354
7.30	CT2 (TE63-3) roughness heat flux augmentation within $t=2.0 - 4.0$ s	356
7.31	CT3 (TE101-4) roughness heat flux augmentation within $t=2.0 - 4.0$ s	357
7.32	Roughened CT2 (TE63-3) roughness heat flux augmentation within $t=2.0 - 4.0$ s	358
7.33	ANSYS CT in PEEK simulation cases	362
7.34	Spatial convergence of boundary layer code grids	376
7.35	Spatial convergence of TAU grids	377

Nomenclature

Abbreviations

ANSYS	commercial FEM program
BLcode	Boundary layer code
CFD	Computational Fluid Dynamics
CI	Confidence Interval
CT	Coaxial Thermocouple
DFM	Digital Filtering Method
DLR	Deutsches Zentrum für Luft- und Raumfahrt e.V.
DNS	Direct Numerical Simulation
EQSR	Equivalent Sand Grain Roughness
ESA	European Space Agency
ExoMars	Exo-biology on Mars (a project to explore Mars by ESA)
FC	Flow Condition
FEM	Finit Element Method
GCI	Grid Convergence Index
H2K	Hypersonic test section Köln (dummy)
HFM	Heat Flux Microsensor
HWA	Hot-Wire Anemometry
IW	Interrogation Window
L2F	Laser 2 Focus
LDV	Laser Doppler Velocimetry
LES	Large Eddy Simulation
MCM	Monte Carlo Method
MSL	Mars Science Laboratory (a project to explore Mars by NASA)
MS	Measurement Section
NASA	National Aeronautics and Space Administration
PANT	Passive Nosedip Technology (a project by NASA)
PEEK	PolyEther Ether Ketone
PIV	Particle Image Velocimetry
RANS	Reynolds Averaged Navier Stokes equations
ROI	Region of Interest
SAE	Spalart Allmaras turbulence model with Edwards modification
SG	Sand Grain type
SNR	Signal-to-Noise-Ratio
TAU	CFD code of DLR
TBL	Turbulent Boundary Layer

TNTI	Turbulent/Non-Turbulent-Interface
TPS	Thermal Protection System
TSB	Transverse Square Bar
TSM	Taylor Series expansion Method
TSW	Test Section Wall
UMF	Uncertainty Magnification Factor
UMZ	Uniform Momentum Zones
UPC	Uncertainty Percentage Contribution
ZPG	Zero Pressure Gradient

Latin Symbols

A_m	factor in temperature-velocity formulation for linear term
a	sonic speed
a_1	parameter in Tedeschi drag model
a_2	parameter in Tedeschi drag model
a_i	parameter in rough wall skin friction correlation according to Fenter and Lyons Jr [72]
A_f	frontal projected area of roughness elements
A_p	plan area of roughness elements
A^+	BLcode turbulence model constant
AR	Aspect Ratio
A_t	total plan area of roughness elements
A_w	total windward wetted roughness surface area
B_m	factor in temperature-velocity formulation for quadratic term
B	law of the wall constant
b_i	parameter in rough wall skin friction correlation according to Fenter and Lyons Jr [72]
B'	modified law-of-the-wall constant for smooth wall Integral approach
B_q	dimensionless wall heat flux
B_r	rough wall law of the wall constant
B'_r	modified law-of-the-wall constant for rough wall Integral approach
C_1	constant in Sutherland's viscosity law
C_2	constant in Sutherland's viscosity law
C_3	constant in Sutherland's viscosity law
C_4	constant in Sutherland's viscosity law
C_5	constant in Sutherland's viscosity law
C_6	constant in Sutherland's viscosity law
$c_{f,e}$	local skin friction coefficient with boundary layer conditions
$C_{h,s}$	smooth wall Stanton number in Stanton number correlation according to Hochrein and Wright [103]
c_v	specific heat capacity at constant volume

c_D	drag coefficient
c_p	pressure coefficient
D	case dimension for Grid Convergence Index (GCI)
D	BLcode turbulence model Van Driest Damping function
D	(model) diameter
ϵ'	parameter in Tedeschi drag model
ES	effective roughness slope
F_C	van Driest skin friction coefficient transfer function
F_θ	van Driest skin friction coefficient transfer function
F_s	safety factor for GCI
H	shape parameter
H	total enthalpy
h	static enthalpy
H_k	kinematic shape parameter
$\bar{h}$	mean roughness elevation
h'	roughness elevation above the mean $\bar{h}$
i	roughness element interval
K	parameter in Tedeschi drag model
k	physical roughness element height
k_1	BLcode turbulence model constant
k_2	BLcode turbulence model constant
k_3	BLcode turbulence model constant
k_4	BLcode turbulence model constant
k_a	average roughness height
k/δ	roughness blockage ratio
Kn	Knudsen number
k_{rms}	root-mean-square roughness height
k_s	equivalent sand grain roughness
k_s/k	normalized equivalent sand grain roughness
k_t	minimum peak-to-trough roughness height
k_z	subsample of minimum peak-to-trough roughness height
L	(model) length
l	mixing length
L_{True}	true integral length scale according to method after Host-Madsen and McCluskey [111]
Ma	Mach number
Ma_1	Mach number upstream normal shock
Ma_e	boundary layer edge Mach number
M_y	field dimension for Digital Filtering Method (DFM)
M_z	field dimension for DFM
N	number of elements for GCI
n_t	parameter in Stanton number correlation according to Hochrein and Wright [103]
n_y	integral length scale in field dimensions for DFM
n_z	integral length scale in field dimensions for DFM

p	order of convergence for GCI
p_0	reservoir pressure
p_{02}	total pressure downstream normal shock
p_1	(static) pressure upstream normal shock
p_{Pitot}	Pitot pressure
p_∞	inflow (static) pressure
Pr_t	turbulent Prandtl number
q	parameter for GCI
$\dot{q}_c$	convective heat flux
q_∞	inflow dynamic pressure
$\dot{q}_r$	radiative heat flux
$\dot{q}_w$	conductive heat flux
R	BLcode turbulence model Van Driest Damping roughness constant
r	result quantity in Monte Carlo uncertainty estimation
Re	Reynolds number
$Re_{\delta 2}$	momentum thickness based Reynolds number ($= \rho_e U_e \theta / \mu_w$)
Re_τ	friction Reynolds number , (Kármán number)
δ^+	Kármán number , (friction Reynolds number)
Re_θ	momentum thickness based Reynolds number ($= \rho_e U_e \theta / \mu_e$)
r_{ji}	refinement ratio between grid i and j for GCI
r_y	half field dimension for DFM
r_z	half field dimension for DFM
S	set of roughness length scales
s	parameter for GCI
s	number of synthetic image pairs for DFM
S'	parameter in Tedeschi drag model
SK	roughness skewness
S_k	Stokes number
S	molecular speed ration in Henderson drag model
St	Stanton number
T_0	reservoir temperature
T_{amb}	ambient temperature of the surrounding
T_{aw}	adiabatic wall temperature
T_{ref}	reference temperature
Tu	turbulence level
T_w	wall temperature
U	mean streamwise velocity
u'	fluctuating streamwise velocity
u	instantaneous streamwise velocity
U_Q	uncertainty of quantity Q
u_{rms}	root-mean-square of streamwise velocity fluctuation
U_e	mean streamwise boundary layer edge velocity
U_{eff}	effective streamwise velocity
u_*	Morkovin scaling (skin friction velocity scaled by the mean density profile)
$\tilde{u}$	filtered velocity for DFM
$\overline{u'u'}$	ensemble averaged streamwise Reynolds stress

U_X	uncertainty of input quantity X in Monte Carlo uncertainty estimation
V	mean wall-normal velocity
v'	fluctuating wall-normal velocity
v	instantaneous wall-normal velocity
W	Coles wake function
w	roughness element width
X	input quantity in Monte Carlo uncertainty estimation
x_a	axial coordinate
x	wall-parallel coordinate
Δx	step size in streamwise direction
ξ	parameter in Tedeschi drag model
y	wall-normal coordinate
y_0	roughness function
y_m	BLcode turbulence model inner layer boundary
y_r	roughness sub-layer thickness

Greek Symbols

δ	boundary layer thickness
δ_v	viscous length scale
ε	emissivity
λ	heat conductivity
σ	standard deviation
σ	Stefan Boltzmann constant
α_λ	absorbance of substance at wavelength λ
α_S	parameter in rough wall Stanton number correlation according to Seidman [223]
α_{VD}	parameter in van Driest skin friction coefficient transfer function
α	normalized equivalent sand grain roughness
β_{VD}	parameter in van Driest skin friction coefficient transfer function
δ_k^*	kinematic displacement thickness
δ^*	displacement thickness
Δx	pixel displacement
$\Delta x'$	pixel displacement fluctuation
Δt	time step size
ΔU_{eff}^+	effective roughness function
ΔU^+	roughness function
ϵ	shift of origin
ϵ_{ji}	relative difference between grid i and j for GCI
$\bar{\gamma}$	BLcode turbulence model intermittency factor
κ	Kármán constant
λ	roughness wavelength

λ_F	parameter in rough wall skin friction correlation according to Fenter and Lyons Jr [72]
λ	wavelength
λ_f	roughness frontal solidity
λ/k	normalized roughness wavelength
λ_p	roughness plan solidity
Λ_s	roughness density by Sigal and Danberg
μ_e	dynamic viscosity at the boundary layer edge
μ_t	BLcode turbulence model eddy viscosity
μ_w	dynamic viscosity at the wall
ν_w	kinematic viscosity at the wall
ω	parameter in exponential viscosity law
ϕ_{GCI}	parameter for GCI
Π	Coles wake parameter
ψ_F	parameter in rough wall skin friction correlation according to Fenter and Lyons Jr [72]
ρ_λ	reflectance of substance at wavelength λ
ρ_e	boundary layer edge density
ρ_w	wall density
σ_F	parameter in rough wall skin friction correlation according to Fenter and Lyons Jr [72]
τ_w	wall shear stress
τ_λ	transmittance of substance at wavelength λ
τ_f	fluid time constant
τ_p	particle relaxation time
θ	momentum loss thickness
θ_k	kinematic momentum thickness
u_τ	skin friction velocity

Subscripts

aw	adiabatic wall
c	compressible
e	boundary layer edge condition
eff	effective
f	fluid
i	inner
K	according to Krogstad
k_s	at sand grain height
o	outer
p	particle
$Pitot$	from Pitot measurements
r	radiative (heat flux)

<i>r</i>	recovery
<i>r</i>	rough wall
<i>ra</i>	Reynolds analogy
<i>rel</i>	relative (e.g. particle to flow velocity)
<i>s</i>	smooth wall
<i>*</i>	Morkovin scaling (skin friction velocity scaled by the mean density profile)
<i>sub</i>	subsonic
<i>c</i>	convective
<i>sup</i>	supersonic
<i>T</i>	according to Tedeschi
<i>trans</i>	transonic
<i>uu</i>	streamwise velocity fluctuations
<i>vp</i>	vapor pressure
<i>VD</i>	according to Van Driest
<i>w</i>	wall condition
<i>x</i>	length-based

Superscripts

+ inner scaling

1 Introduction

Boundary layers are responsible for skin friction drag and convective heat transfer for different types of vehicles, like ships, aircraft, missiles, re-entry vehicles and propulsion systems. Surface roughness influences boundary layers by altering laminar-to-turbulent transition locations and enhancing turbulence, which can further increase skin friction and heat transfer rates significantly [216]. Since all manufactured components of these systems have some level of inherent surface roughness, it is important to quantify how roughness can affect boundary layers to leverage understanding, improve prediction methods and inform design recommendations.

In general, surface roughness may stem from different processes. It can be generated by the manufacturing process or through typical operation. Additionally, it may result from specific mechanisms like corrosion or erosion. Roughness can also be a result of specific depositions like ice accretion on aircraft wings, deposits in gas turbines or combustion chambers and biofouling on ship hulls.

Since Schlichting introduced the concept of EQSR in the 1930s, most of today's predictive roughness models still rely heavily on this concept. Despite over 9 decades of research, still large uncertainties exist for the prediction of the impact of roughness. In 2021, Chung *et al.* [45] estimated the economical impact of these uncertainties on different branches relevant for trade with approx. US\$67 billion per year globally for the shipping industry and approx. 33 % of global profits in 2018 for the aviation industry.

These cases were made for the incompressible to transonic flow regime, for which the existing database is much larger compared to supersonic flow. When operating at high Mach numbers, the boundary layer region is characterized by intense shear stress and heat flux and this causes it to directly influence performance and safety of the vehicle. The understanding of compressible TBL at high Mach numbers is limited due to the lack of available data. This is the case for smooth walls but especially if additional aspects like surface roughness influence the flow. The impact of roughness can become relevant for example for hypersonic gliders [224], airbreathing engines [231], [78], nozzle flows e.g. [265], and reentry vehicles [62], [161], [120], [281], [280]. The predictive limitations pose significant challenges for the design of high-speed vehicles, which often result in large safety margins and in this way ultimately increase weight of the vehicle and cost of the mission. Therefore, collecting and analyzing further experimental data is essential to enhance the physical understanding and enable validation of computational models.

Surface roughness can be categorized into two types, namely isolated and distributed roughness. In case of isolated roughness there are only single, hence isolated, entities of disturbances along a surface whereas for distributed roughness, disturbances form a continuous entity covering a surface. According to Peltier [183], any aircraft typically contains isolated roughness due to rivets, seams or machining imperfections. Distributed roughness can generally be found on vehicles flying at high Mach numbers, due to the necessity of Thermal Protection System (TPS),

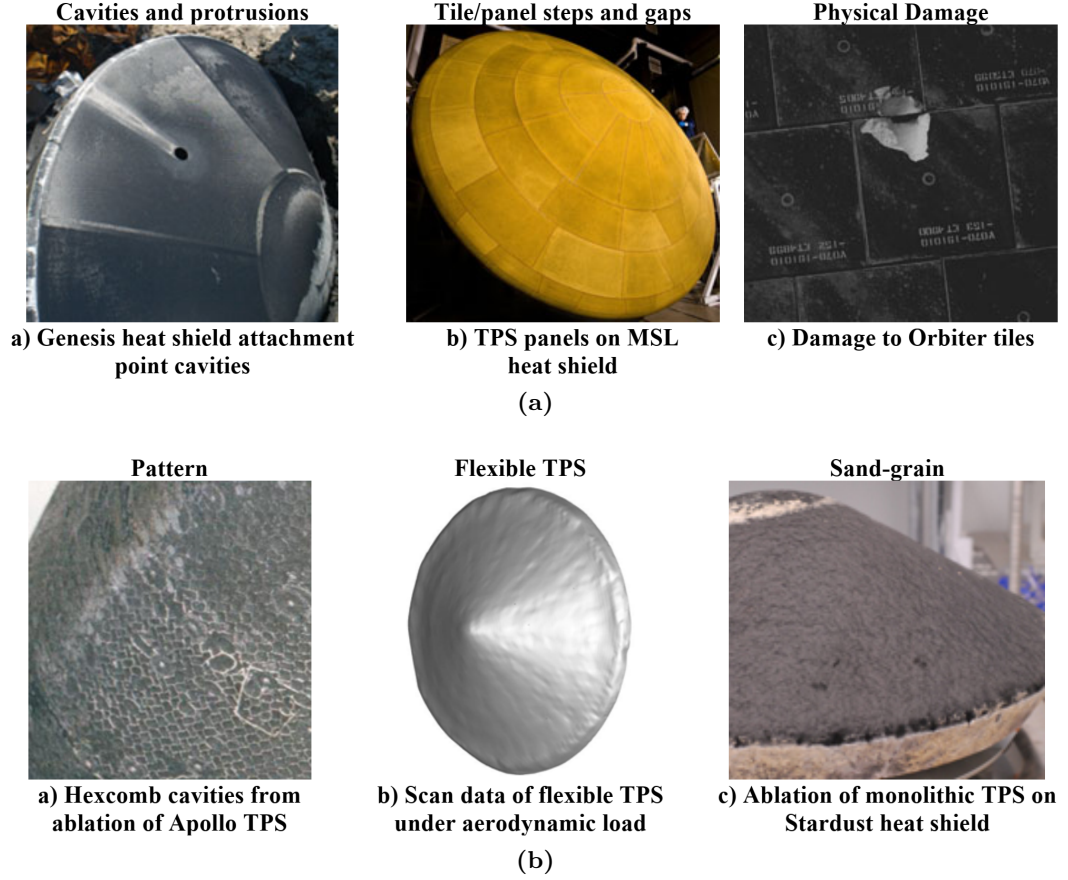


Figure 1.1: Spacecraft surface roughness types with a) discrete and b) distributed roughness according to [109].

protecting the interior from high gas temperatures exposed to in flight. TPS are most often in the form of either tiles or ablatives. The first configuration uses insulating tiles, which are commonly composed of metallic, ceramic, or carbon/carbon components. These tiles are typically arranged most often in a square, diamond, or hexagonal grid along the windward surface of the vehicle. Therefore, seams between the tiles can represent periodic roughness to the turbulent boundary layer. If metallic tiles are used, temperature gradients inside the TPS material can lead to buckling which can even amplify the impact of the TPS. The second TPS configuration uses ablative materials, which are designed to lower the conducted energy content into the vehicle by physio-chemical transformations of outer layers of the material. During flight, surface roughness of ablators can largely be altered. Depending on the type of materials, specific shapes like periodic roughness pattern can develop (see e.g. [245], [183]), which in turn can lead to sizable distortions of the boundary layer structure. Furthermore, random “sand-grain” features can result from the ablation process of a monolithic TPS. Hollis [109] gave an overview of different discrete and distributed surface roughness types, relevant for different spacecraft surfaces, visualized in figure 1.1.

The European Space Agency (ESA) was concerned about the impact of roughness induced heating augmentations on their Schiaparelli entry capsule during its flight through the Martian atmosphere within the ExoMars (Exo-biology on Mars) 2016 mission. In the early phase of the project, potential heat flux augmentations of approx. 1.7 to 3 times the turbulent smooth wall values were speculated [21], [161]. Efforts were made to leverage prediction capabilities via EQSR modeling in Computational Fluid Dynamics (CFD) based on comparisons to heritage and dedicated new experiments. Rebuildings showed partly substantial discrepancies with large over-predictions of heating augmentations but were deemed acceptable for the project due to a conservative approach. One uncertainty at that time was a large scatter in the EQSR value for the tested models, which were taken initially from literature data and later, during the course of the project, from dedicated incompressible tests. This scatter in EQSR resulted in a scatter of predicted heating augmentations between approx. 50 % to 100 %. On system level, roughness induced heating augmentations represented the biggest contributor to the maximum heat flux margins for the design of the Schiaparelli TPS with approx. 80 – 90 % along the front shield and shoulders [120]. The integral heat loads were the second largest contributor to the overall margins with approx. 20 – 30 %, which had a substantial impact on mass margins and therefore loss of scientific payload.

Also during the course of the MSL (Mars Science Laboratory) project by National Aeronautics and Space Administration (NASA), early predictions showed roughness shear stress augmentations up to approx. 40 % and heating augmentations up to approx. 25 % for their entry capsule. This also made the roughness augmentation the largest contributor to the heat flux margins along the leeside shoulder region [62] and was reported to approx. 20 – 30 % of margins overall [30]. Post-flight analysis within the NASA MSL project showed that although signs for significant augmentations due to distributed surface roughness were not observed, local discrepancies were attributed to either shock-layer radiation or roughness augmentation effects as most prominent sources. At this specific location just downstream of the heat shield nose, approx. 50 % heating augmentations were observed. This behavior could be rebuild by EQSR values within a plausible range for protruding gap filler material in the vicinity of the sensor plugs and between PICA tiles [63].

The above examples show the discrepancies between predictions (modeling) and observations (measurements) for different cases. As mentioned before, the majority of predicting models use the concept of EQSR, which is based on specific assumptions about the general structure of the boundary layer and is prone to several different challenges. Therefore, further experimental data is important for the physical understanding and validation of state-of-the-art computations. However, accurate measurements of the compressible turbulent boundary layer in ground-based facilities are very challenging on its own rights due to a multitude of reasons, e.g. limited model dimensions, thin boundary layers, limited useable test times and inflow effects, like turbulence intensity. The choice of measurement technique is also a crucial parameter for the analysis with high impact on spatial and temporal resolution.

This resulted in the following objective and scope of this thesis.

Due to the sparse database on compressible TBL in general but in particular on roughness induced turbulent boundary layer augmentations, a suitable experimental setup was planned with the overarching goal in mind to improve physical understanding of the boundary layer behavior and its imprints.

Based on the experiences made during the ExoMars project, one particular goal was to enable measurements of EQSR and heating augmentation in one experimental setup. Therefore, an experimental setup was designed to enable simultaneous velocity and heat flux measurements. For both quantities, two complementary measurement techniques were chosen to be able to make cross-comparisons additionally to comparisons to predictions and literature data. Velocities were deduced via PIV and Pitot pressure data and heat flux via infrared thermography and local sensors. Due to the observed large scatter in EQSR values from literature, an additional question was posed regarding the accuracy with which this quantity can be extracted from compressible TBL experiments. With this information one could try to narrow down the question if the heating augmentation rebuilding problems were driven by uncertainties in EQSR or the modeling method. If the first would be the case, it could give additional hints toward the question if EQSR might be influenced by compressibility effects. For the second uncertainty source, an additional goal was to make efforts to inspect the evolving compressible TBL and to determine whether so-called outer layer similarity still holds, since most roughness models rely on it. To enable this and to gain insights into the large scale structure of the TBL, PIV data was analyzed. In general, all of the above information can contribute to the validation of state-of-the-art computations.

As a visual wrap up of this section, figure 1.2 shows a simple sketch of the “inner workings” in a TBL in form of a representative pseudo mechanical machine according to Peltier [183]. In the majority of previous investigations, one of the input levers on the far left were switched (e.g. experiments were performed along a smooth wall and then on a rough wall) and corresponding outputs, like Reynolds stress were measured, e.g. via Hot-Wire Anemometry (HWA). Due to limitations of measurement techniques, the part concerning the physical mechanisms were kept mostly hidden. Measurement technique developments in recent years, like PIV, enabled analysis of e.g. coherent structures of the TBL and therefore the inner machinery of underlying mechanics for the outputs like Reynolds and wall shear stress. This thesis aims to contribute to this inner view for the compressible TBL along a specific roughness topology of interest.

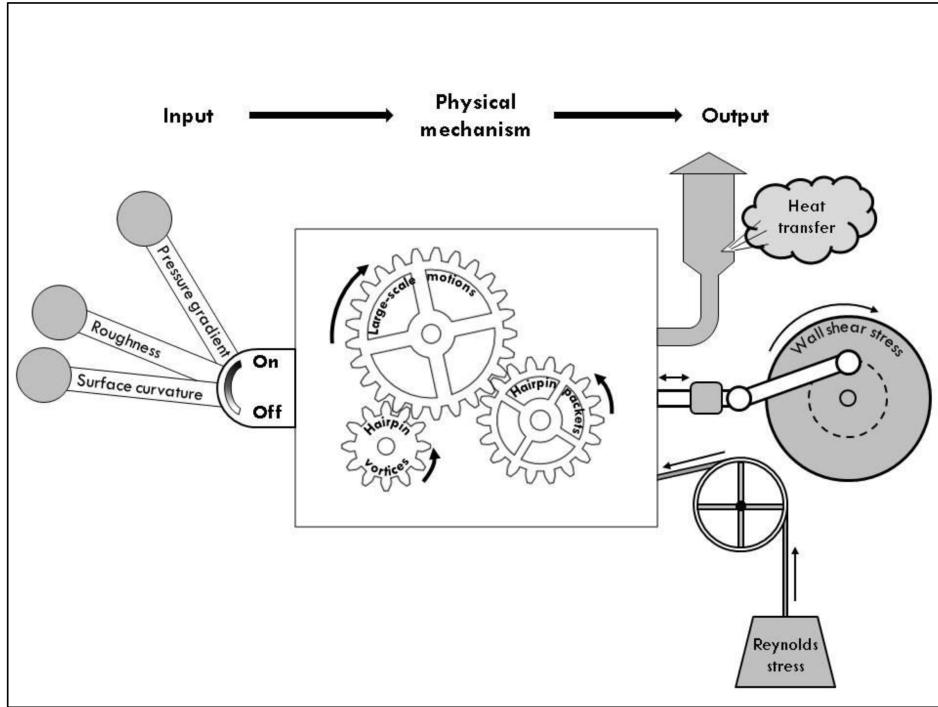


Figure 1.2: Sketch of “inner workings” in a TBL in form of a representative pseudo mechanical machine according to [183].

Author-related pre-published manuscripts and supervised student theses

Parts of this thesis have been published previously in scientific papers for peer-reviewed journals and conferences.

- Neeb, D., Marquardt, P., and Gülhan, A. “Characterization of a hypersonic turbulent boundary layer along a sharp cone with smooth and transverse square-bar roughened wall”, *Experiments in Fluids* 65.9 (2024): 1-21.
- Neeb, D., Marquardt, P., and Gülhan, A. “Heat Flux Roughness Augmentation Along A Sharp Cone at Hypersonic Flow Conditions”, at 1st European Fluid Dynamics Conference (EFDC1) in Aachen, Germany (2024, presentation).
- Neeb, D. Saile, D., and Gülhan, A. “Experiments on a smooth wall hypersonic boundary layer at Mach 6”, *Experiments in Fluids* 59 (2018): 1-21.
- Neeb, D., Gülhan, A., and Merrifield, J. A. “Rough-wall heat flux augmentation analysis within the Exomars project”, *Journal of Spacecraft and Rockets* 53.1 (2016): 121-133.
- Neeb, D., Saile, D., and Gülhan, A. “Experimental flow characterization and heat flux augmentation analysis of a hypersonic turbulent boundary layer along a rough surface”, In *Proceedings of the 8th European Symposium on Aerothermodynamics for Space Vehicles* (2015, No. 89873, pp. 1-15).

- Neeb, D., Gülhan, A., and Merrifield, J. A. “Rough wall heat flux augmentation analysis in the framework of the ExoMars project”, In 11th AIAA/ASME Joint Thermophysics and Heat Transfer Conference (2014, p. 2817).
- Merrifield, J. A., Beck, J. C., Gülhan, A., and Neeb, D. “The prediction of rough wall fluxes to the ExoMars vehicle”, In proceedings of the 7th European Symposium on Aerothermodynamics (2011, Vol. 692, p. 7).

In case of the first six publications, the author of this thesis contributed conceptualization, methodology, test preparations, analysis, post-processing, investigations, original draft preparation as well as editing. In case of the last publication, the author of this thesis contributed experimental data, including test preparations, analysis and post-processing.

The following student theses have been supervised by the author in the framework of the topic of this thesis.

- Dehe, Viola. “Charakterisierung der turbulenten Hyperschallgrenzschicht entlang glatter und rauer Oberflächen”, Master thesis, Aerodynamic Institute at RWTH Aachen University (2015).
- Valero Fort, Vicente. “Verification study for the design of roughness induced heat flux augmentation experiments in hypersonic turbulent flow”, Master thesis, Aerodynamic Institute at RWTH Aachen University (2014).

2 State of the Art

This chapter will give an overview over past investigations on the topic of compressible TBL along rough walls. The study of the dynamics of TBL has been subject of a large number of studies over the past century and therefore this summary is not intended to be an exhaustive overview. Instead, the following sections will briefly review aspects deemed relevant for the current thesis. The reader is referred to the classical literature for further insights (e.g. [255], [214], [277], [190], [202], [118]).

The flow of a fluid along a solid surface creates a thin layer along the surface, where effects of viscosity are important. This boundary layer is formed due to the adhesion of molecules directly at the wall of the solid surfaces. If continuum mechanics apply, within this thin boundary layer the velocity increases from zero at the wall to the value of the outer flow, creating a region of strong gradients. This first definition was given by Ludwig Prandtl in 1904 [194]. The following discussion is focused on boundary layers, evolving freely along a solid wall (e.g. flat plate or cone). Occasionally, examples from pipe or channel flows are used due to similarities in specific aspects, although it should be mentioned that there is ongoing debate about the fundamental similarities and differences between pipe, channel and boundary layer flows which are out of the scope of this thesis (see e.g. [163], [89]).

When inertial forces cause flow instabilities to grow larger than viscous forces which tend to dampen them, instabilities can grow until the flow becomes unstable and evolves to a fully turbulent flow. The ratio between inertial and viscous forces can be given in form of the Reynolds number, defined by

$$Re_x = \frac{\rho U x}{\mu} \quad (2.1)$$

where ρ is the fluid density, U the mean velocity, x is some reference scale like the running length L and μ the dynamic viscosity of the fluid. It can also be interpreted as convective to diffusive momentum transport. For small Reynolds numbers, the boundary layer is typically laminar. Above a certain Reynolds number, the boundary layer transitions to a turbulent state. In addition to the Reynolds number, this transition is influenced by many parameters like inflow disturbances, inflow turbulence intensity, Mach number, surface quality, etc. (see e.g. [214], [277], [8], [216]). Neither laminar boundary layers nor the transition process are focus of this thesis, therefore this chapter just outlines aspects of fully TBL.

The result of turbulence in general is an increase in energy dissipation and mixing which in boundary layers lead to an increase of skin friction (and therefore drag) and heat transfer. The turbulent state is characterized by “apparently random and chaotic three-dimensional vorticity” [89]. It consists of large scale motions that contain the bulk of the turbulent kinetic energy. These large coherent structures then diffuse and transport energy into smaller-scale structures.

This process is often labeled as energy cascade. This process continues until the kinetic energy is converted into thermal energy by the action of viscous forces at the smallest scales of turbulence. Therefore, a large range of velocity and length scales are relevant for turbulent flows, which make it challenging to resolve these flows properly by e.g. experimental measurement techniques or computational grids for CFD codes (see also [190], [89]).

For compressible flows, the Mach number and temperature ratios between the wall and the edge of the boundary layer become additional relevant scaling parameters. Due to the no-slip condition, a subsonic region persists near the wall. With increasing Mach number the sonic line is getting closer and closer to the wall. Furthermore, due to influences of the wall temperature but especially due to high levels of viscous dissipation near the wall, a significant temperature gradient develops across the boundary layer at supersonic speeds. The static-temperature variation can be very large even in an adiabatic flow, resulting in a low-density, high-viscosity region near the wall. This leads to a thicker boundary layer and a region in which viscous effects are somewhat more important than at an equivalent Reynolds number in subsonic flow. This also complicates e.g. the definition of driving parameters for the flow, like the Reynolds number. Besides equation (2.1), for incompressible boundary layer flow, a driving Reynolds number is based on the momentum loss thickness θ as characteristic boundary layer length scale, together with quantities of the free-stream or boundary layer edge conditions like density ρ_e , velocity U_e and viscosity μ_e resulting in $Re_\theta = \rho_e U_e \theta / \mu_e$. Due to the changing density in the compressible boundary layer, this definition might not be sufficient to capture e.g. wall effects. The influence of density is already visible in the compressible momentum loss thickness, defined by $\theta = \int_0^\infty (\rho U) / (\rho_e U_e) [1 - (\rho U) / (\rho_e U_e)] dy$. This led to the definition of additional Reynolds numbers, like $Re_{\delta 2} = \rho_e U_e \theta / \mu_w$, where the viscosity is defined at the wall, instead of the boundary layer edge. This Reynolds number can also be interpreted as the maximum value for the ratio of momentum flux and shear stress (see [73]). Since the wall shear stress is known to dominate the inner regions of the turbulent boundary layer, combined with the fact that it grows in importance with increasing Mach number, this latter definition of Reynolds number is preferred by several authors, e.g. [73], [58], [57].

Walls with roughness can influence the surrounding turbulent flow in several different aspects. It can increase skin friction drag and convective heat transfer above turbulent levels along a surface. Additionally, it modifies the velocity profile and turbulent intensities when compared to the smooth wall case. These effects are the main properties to be detected with appropriate measurement techniques within this thesis. To characterize roughness, relevant parameters need to be defined. Given the large number of parameters thought relevant for a rough-wall TBL (roughness height, roughness Reynolds number, solidity and other geometrical properties, flow compressibility, etc.), a generic model which would describe relevant boundary layer motions over arbitrary rough surfaces is difficult to define. Instead, in the following key findings of past studies are collected, with the goal to emphasize general trends relevant for this thesis. Where possible, the discussion will include effects of surface roughness within compressible TBL. But, since most of the past studies were performed on incompressible TBL, selected studies will be discussed as well. Also, since the smooth wall TBL is the reference, the discussion will include some aspects of incompressible and compressible TBL along undisturbed walls. In hypersonic flow high enthalpy effects can become relevant [8]. In this work, experiments have been performed in a blow down hypersonic facility with a moderate total temperature and therefore focus on the strong compressibility effects without the additional complexity of high temperature phenomena.

The discussion of previous investigations is divided into four different aspects of the TBL along rough walls. First, the mean velocity field along rough walls with corresponding scalings will be discussed. After that, impacts on turbulent stresses will be summarized. Then, studies on the underlying coherent structures of a TBL will be discussed. Afterwards, skin friction and heat flux augmentations as characteristic effects of rough walls will be addressed. Finally, an additional subsection gives an overview about general roughness topologies and some relevant length scales.

If not stated otherwise, the axial, wall-parallel (streamwise) and wall-normal directions are denoted in the following by x , x_a and y , respectively. Velocities denoted by U correspond to the x - and V to the y -direction. Furthermore, mean velocities are denoted by a capital letter (U , V), while instantaneous velocities are the corresponding lowercase letters (u , v). The fluctuating velocities are denoted by an additional bar (u' , v'). An overbar indicates ensemble averaging (e.g., $\overline{u'u'}$).

2.1 Mean Velocity Scalings

First, a general theoretical framework is given, based on collected work of different authors. Afterwards, selected past investigations and their results are summarized, deemed relevant for this thesis.

In the simplest conceptual view, a TBL along a rough surface consists of an inner and an outer layer, comparable to flow along smooth walls (see also figure 2.1 and 2.2). One important assumption typically made is, that the rough wall TBL has essentially the same turbulence structure in the outer layer than a smooth one (see e.g. [202]). In contrast, the inner layer can be directly influenced by the roughness topology. This outer layer independence, sometimes called “outer-layer similarity” or “wall similarity”, was already formulated by Townsend [255] and is the basic assumption on which most roughness modeling rely. As a result, the inner scaled mean velocities and turbulent quantities in the outer layer collapse, independent of the wall topology. Chung *et al.* [45] stated some prerequisites for the outer similarity to hold.

Firstly, the outer layer of a rough wall TBL is limited at the upper bound by the boundary layer thickness δ but at the same time should be much larger than the viscous length scale or any other relevant roughness length scale, so that $\delta \geq y \gg \delta_v, k$, with the roughness height k as representative roughness scale and the viscous length scale defined by

$$\delta_v = \frac{\nu_w}{u_\tau} = \frac{\mu_w}{\rho_w u_\tau} \quad (2.2)$$

with the kinematic viscosity at the wall $\nu_w = \mu_w / \rho_w$. Wall values are used, which is the typical formulation in compressible flow to consider varying mean density. Additionally, the friction velocity u_τ is defined as follows:

$$u_\tau = \sqrt{\frac{\tau_w}{\rho_w}} \quad (2.3)$$

The friction velocity is not a physical velocity but a convenient combination of the relevant driving quantities of wall shear stress $\tau_w = (\mu \partial U / \partial y)_w$ and density ρ_w , again formulated at the wall to include compressible flow.

Secondly, Chung *et al.* [45] stated a need for sufficient viscous scale separation which can be formulated by sufficiently large Reynolds numbers $\delta^+ \gg 1$, where

$$\delta^+ = Re_\tau = \frac{\delta}{\delta_v} = \frac{\delta u_\tau}{\nu_w} \quad (2.4)$$

This Reynolds number δ^+ (also termed friction Reynolds number or Kármán number) represents the separation of length scales of the inner and outer layer for smooth wall flow (see also explanations below).

Thirdly, [45] stated a need for sufficient roughness scale separation with $\delta/k \gg 1$, where the latter is the inverse of the sometimes called roughness blockage with k/δ .

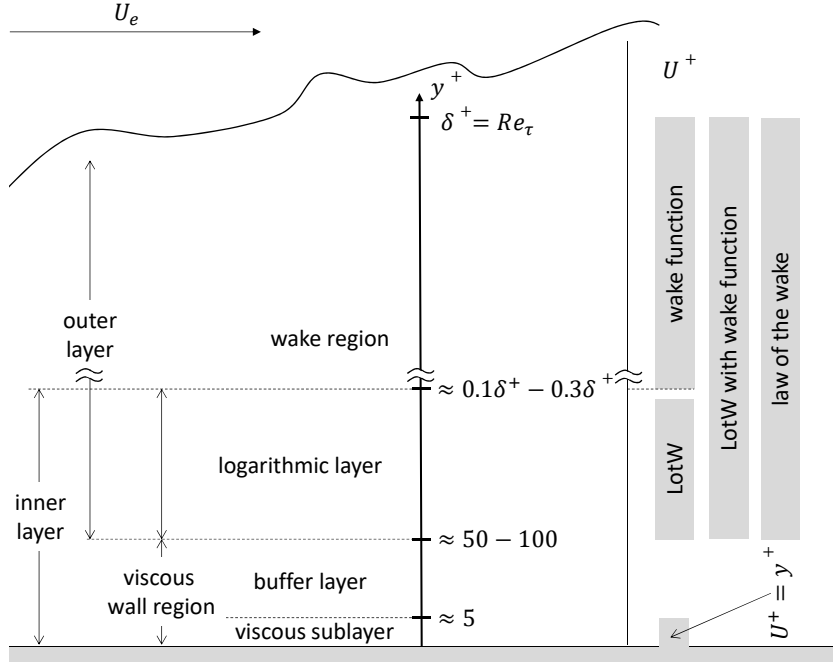
Before giving more details to the inner and outer layer, it should be noted, that this two-layer view, together with the possible interplay between both layers, is still a matter of ongoing debate (see e.g. [202], [77], [45]).

Inner layer:

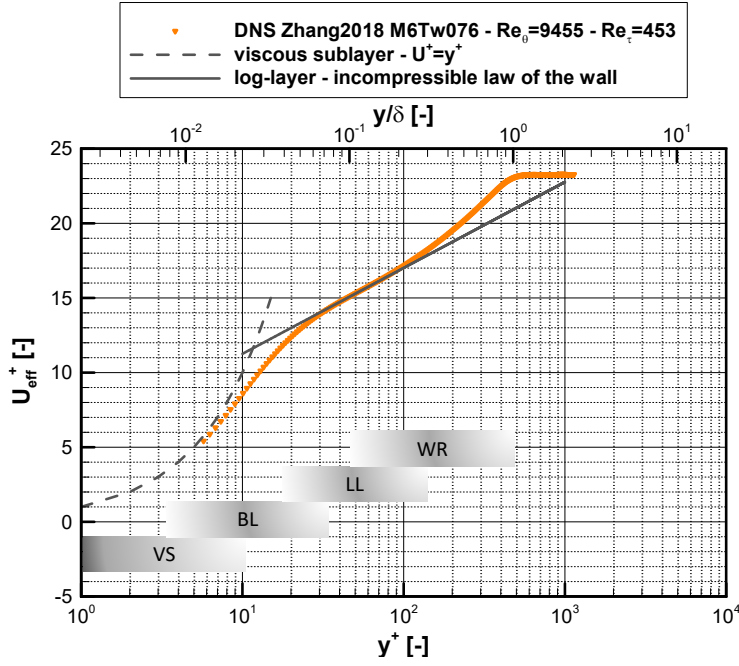
Similar to smooth wall TBL, the relevant velocity and length scales in the roughness inner layer include the friction velocity u_τ and the viscous length scale δ_v . But additionally, all roughness length scales to fully characterize the corresponding topology may become relevant, represented here by S . This set of length scales S strongly depends on the type and topology of roughness of interest (more information will be given in section 2.5). Along a smooth wall, the inner layer of a TBL is thought to consist of a viscous sublayer, where viscous stress dominates, followed by a buffer layer, where turbulent stresses have their maximum but viscous effects still play a role and finally a logarithmic layer, where turbulent stress is dominant and viscosity is negligible (see also figure 2.1 and 2.2). There are discussions for additional layers, like a “mesolayer” (see e.g. [239]).

Along a rough wall TBL, a roughness sublayer exists, which is the counterpart to the viscous sublayer in a smooth wall TBL. Its extent y_r is believed to reach wall distances of $2k$ to $5k$, if k is the only or dominant length scale [202] (examples for specific scale ratios will be given below). Some authors state this extent in terms of the EQSR k_s (will be defined in the following), e.g. $3k_s$ according to Flack *et al.* [76]. Although this definition has the benefit of representing an hydraulic scale (e.g. it includes roughness mechanisms like induced drag), k_s cannot be thought of as a direct geometrical value [45]. The exact extent of the roughness sublayer is still a matter of debate (see e.g. [202],[45]). Roughness is believed to disturb the near-wall cycle in the buffer layer, which is responsible for generating most of the turbulent energy at moderate Reynolds numbers (see [118]). If the scale separation is sufficient, a logarithmic layer, comparable to a smooth wall TBL exists also above the roughness sublayer (see e.g. [202]).

Another peculiarity is that care must be taken by defining the origin of the distance y above a rough wall. Since the roughness displaces the entire flow upwards, a correction needs to be included (see e.g. [202]). This correction is termed in various ways depending on the author,



(a)



(b)

Figure 2.1: a) Schematic of the different layers, its assumed boundaries and applicable mean velocity laws of a smooth wall TBL (LotW: Law-of-the-Wall; rough LotW with wake function corresponds to equation (2.7) with $\Delta U^+ = 0$) and b) mean velocity profile according to Zhang *et al.* [296] (VS: Viscous Sublayer, BL: Buffer Layer, LL: Logarithmic Layer, WR: Wake Region).

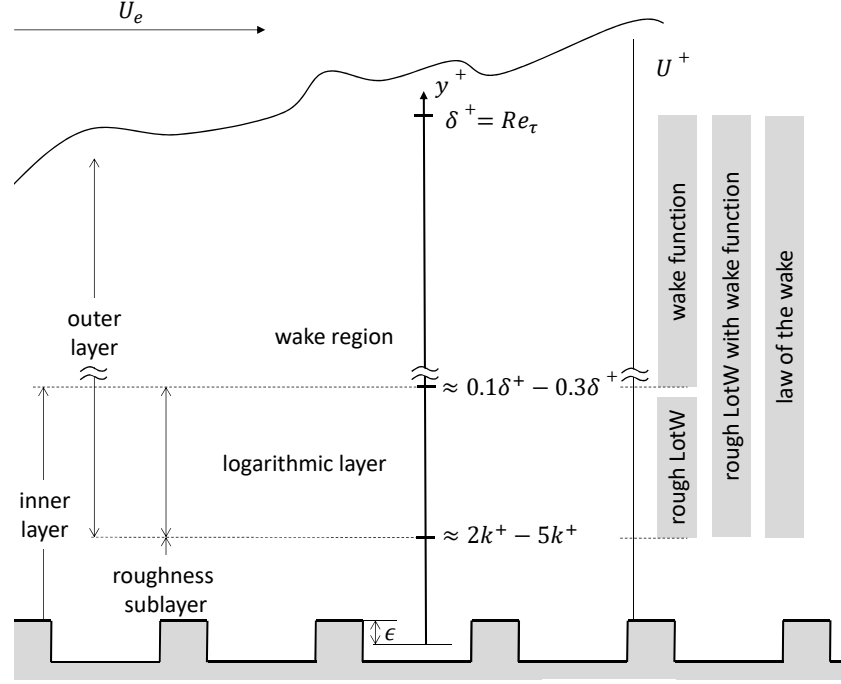


Figure 2.2: Schematic of the different layers, its assumed boundaries and applicable mean velocity laws of a rough wall TBL (LotW: Law-of-the-Wall; rough LotW with wake function corresponds to equation (2.7)).

e.g. “shift of origin”, “zero-plane displacement”, “datum offset”, etc. One of the first to use a so-called “shift of origin” was [164] for a TBL along Transverse Square Bar (TSB) roughness.

In general, the shift of origin ϵ depends on the roughness topology and is a parameter determined by the flow (see [240]). The physical origin and modeling of this displacement is still a matter of on-going discussion (e.g. [202], [45]). Different approaches to extract ϵ are discussed in chapter 3.12.

Outer layer:

In the outer rough turbulent boundary layer u_τ and δ are believed to be the only relevant velocity and length scales, identical with the smooth TBL. Also here, it should be noted that there is on-going debate whether the proper velocity scale in the outer layer is u_τ or U_e (see e.g. [190], [89]).

In between the inner and outer layer, with sufficient scale separation $\delta^+ \gg 1$ a logarithmic layer can exist where the mean velocity profile is only dependent on the distance to the rough wall (including the shift of origin) based on the same argumentation as for TBL along smooth surfaces (e.g. asymptotic matching of the velocity gradient, see [190], [277] or mixing length argumentation, see [260] or “attached eddy hypothesis” after Townsend [255]). As additional condition for rough walls, this region should be located above the roughness sublayer ($y \gg k$)

(see also figure 2.1 and 2.2). Within this overlap layer up to the edge of the boundary layer, the mean velocity profile follows the following function (e.g. after [118]):

$$U^+ = \frac{1}{\kappa} \log \left(\frac{y}{k} \right) + B_r(k^+, S^+) + \frac{\Pi}{\kappa} W \left(\frac{y}{\delta} \right) \quad (2.5)$$

where κ is the von-Kármán constant, B_r a constant offset and W is a function proposed by Coles [50] to include the wake influences in the outer region of a boundary layer, together with a wake parameter Π . The constant B_r is dependent on relevant roughness scales where k^+ represents the roughness Reynolds number according to

$$k^+ = \frac{k}{\delta_v} = \frac{u_\tau k}{\nu_w} = \frac{\rho_w u_\tau k}{\mu_w} \quad (2.6)$$

where the choice for the wall values of density and viscosity accounts for compressible flow. S^+ in eq. 2.5 represents the additional equivalently inner scaled roughness length scales which are potentially relevant.

Equation 2.5 can be re-formulated to (corresponding to “rough LotW with wake function” in figure 2.2)

$$U^+ = \frac{1}{\kappa} \log(y^+) + B + \frac{\Pi}{\kappa} W \left(\frac{y}{\delta} \right) - \Delta U^+ \quad (2.7)$$

which reduces to the smooth wall law-of-the-wall (sometimes labeled log-law) with Coles wake extension if the last term ΔU^+ is set to zero (corresponding to “LotW with wake function” in figure 2.1).

ΔU^+ is the so called roughness function, introduced by Clauser [46] and Hama [96], which represents the momentum deficit due to the surface roughness and therefore is responsible for a reduction of mean velocities at comparable y^+ positions. It can be formulated to

$$\Delta U^+ = \frac{1}{\kappa} \log(k^+) + B - B_r(k^+, S^+) \quad (2.8)$$

Figure 2.1 shows exemplary the inner scaled mean velocity profiles along smooth (filled circles) and rough wall (triangles: rods, squares: mesh roughness) according to [134] where the aforementioned reduction of mean velocity leads to a characteristic parallel shift in the log-layer.

There is a potential third option to re-formulate eq. (2.5), usually used in geophysics or atmospheric flow studies [118]:

$$U^+ = \frac{1}{\kappa} \log \left(\frac{y}{y_0} \right) + \frac{\Pi}{\kappa} W \left(\frac{y}{\delta} \right) \quad (2.9)$$

where y_0 is also called roughness function. It can be shown that the alternative forms to quantify the roughness strength with ΔU^+ and y_0^+ are connected (see e.g. [39]) via:

$$\Delta U^+ = \frac{1}{\kappa} \log(y_0^+) + B \quad (2.10)$$

Coles [50] proposed for the wake function W an equation of the following form, based on an extensive survey of experimental data with:

$$W\left(\frac{y}{\delta}\right) \approx 2 \sin^2\left(\frac{\pi y}{2\delta}\right) \quad (2.11)$$

where W is normalized to zero at the wall and to $W = 2.0$ at $y = \delta$. This term can be used in all of the above formulations (eq. (2.5), (2.7), (2.9)). The law-of-the-wall with Coles' extension, also labeled law-of-the-wake (eq. (2.7)), is an effective composite function to cover both the overlap and the outer layers of a TBL.

An equation for an outer layer formulation of a TBL, sometimes denoted velocity-defect-law, depending on the outer scale y/δ can be given by (here after [68])

$$\frac{(U_{eff} - U_{eff,e})}{U_\tau} = \frac{1}{\kappa} \cdot \log\left(\frac{y}{\delta}\right) - \frac{2\Pi}{\kappa} \left[1 - \sin^2\left(\frac{\pi y}{2\delta}\right)\right] \quad (2.12)$$

in which the wake function according to equation (2.11) was used.

The point to highlight here is that for the above framework, it is assumed that the slope, given by the reciprocal value of the Kármán constant ($1/\kappa$), is the same as for smooth wall flows, independent of the used version of rough wall equation. This corresponds to the statement above, that the general structure of the TBL above the directly influenced roughness sublayer should be the same (outer layer similarity). There is evidence for and against this assumption and it is still a matter of discussion. Several investigations confirm a logarithmic layer with a slope $1/\kappa$, like DNS computations along rough walls (e.g. in turbulent channel flow [251] or TBL [143]) as well as experimental data (e.g. [240]). In contrast, other DNS data (e.g. [149], [295]) and experimental data (e.g. [125], [110]) contradict a constant smooth wall slope of $1/\kappa$. The identification of the slope is strongly coupled with the determination of the shift of origin ϵ , since it has a direct influence on the slope. Since the determination of the shift of origin is also quite challenging (see section 7.2.2) it adds another layer of complexity. Most DNS studies (e.g. [143], [149], [295]) exploit the method of Jackson to derive ϵ but this method is also not free of criticism [45]. See also further discussions in section 7.2.2. For the remainder of this thesis, a universal mean velocity slope $1/\kappa$ is accepted.

ΔU^+ (as well as eq. 2.5) depends on the offset $B_r(k^+, S^+)$ which in turn depends on the relevant inner scaled roughness length scales k^+ and S^+ . In the simplest case, it only depends on k^+ . Even in this case, the previously mentioned sufficient roughness scale separation $\delta/k \gg 1$ is believed to be also a relevant roughness parameter (see e.g. [45], [118]). It can also be given by its reciprocal formulation with $k/\delta \ll 1$ and represents a blockage due to the roughness elements (see e.g. [32]). Jiménez [118] stated a necessary value of $k/\delta < 1/40 = 0.025$ to ensure that roughness effects are constraint to the inner parts of the boundary layer. This limit was based on the assumptions that the roughness sublayer extends up to $y_r = 3k$, the log layer extends up to 0.15δ and that the roughness sublayer ends at half the extent of the log-layer, so that

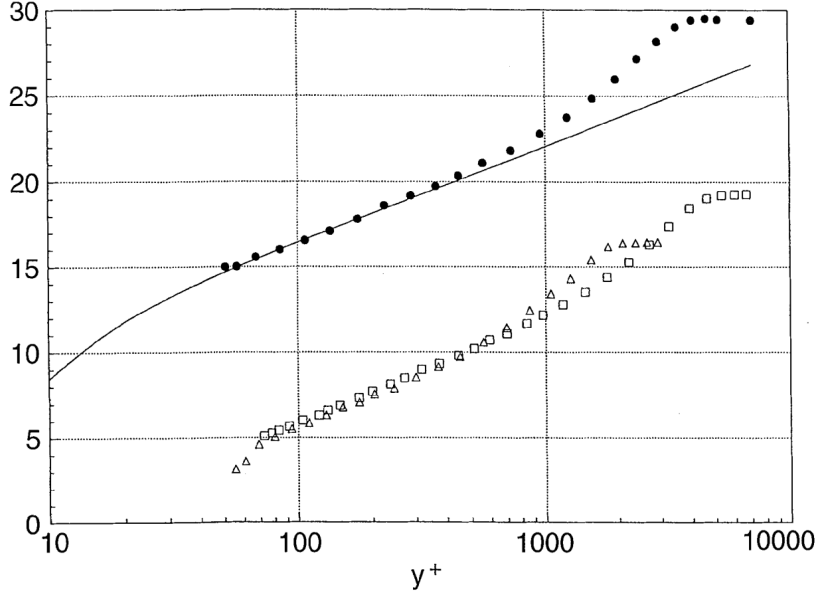


Figure 2.3: Inner scaled mean velocity profiles along smooth (filled circles) and rough wall (triangles: rods, squares: mesh roughness) according to [134].

$3k < 0.5 \cdot 0.15\delta \Leftrightarrow k/\delta < 1/40$. Jiménez argued that if this limit is not kept, the flow is more in the so-called obstacle regime (or cavity flow in case of square bars) with the result of loss of flow universality [118], [45] (e.g. outer layer similarity). In this obstacle regime, the log-law parameters κ , B , B_r are believed to deviate from their “universal” values [45].

Despite the above limit by Jiménez [118], several different studies have shown that outer similarity holds for larger roughness and moderate Reynolds numbers [45]. Distinctions have been made if this applies to only mean flow data or also turbulence data [76],[77]. Furthermore, there is evidence for an influence of roughness shapes [77]. A few examples are given in the following for which different roughness topologies together with its characteristic length scales will be briefly mentioned. An overview of topological parameters will be given in section 2.5).

E.g. Castro [38] observed mean velocity collapse and flow universality of all data from different roughness types (mesh, staggered cube arrays, block, gravel chip roughness) in a TBL up to $k/\delta \leq 0.2$ at $Re_\theta = 1165 - 47787$. Attempts to correlate an impact on Coles wake strength parameter Π was inhibited by measurement uncertainties and therefore scatter. Flack and Schultz [77] showed mean velocity and turbulence data collapse up to $k/\delta < 0.1$ ($\delta/k > 10$) along 3D irregular roughness (sandpaper, packed spheres with grit, honed, and grit blasted surfaces) at $Re_\theta < 13000$. 2D transverse bar roughness with a normalized roughness wavelength $\lambda/k = 8$ (this type of roughness will be examined more in the next paragraphs) showed also mean flow collapse but only up to $k/\delta = 0.03$ at $Re_\theta < 5000$ for which the turbulence data was even more sensitive. Therefore, [77] postulated larger scale separations to be necessary for 2D roughness. Lee and Sung [145] also showed mean velocity collapse in the outer layer for DNS data of TBL over transverse square bar roughness with $\lambda/k = 8$ within a range $k/\delta = 0.045 - 0.125$ ($\delta/k = 8 - 22$) at $Re_\theta = 300 - 1400$. Williams *et al.* [288] confirmed outer layer similarity for

mean and turbulence data (streamwise component) along transverse square bar roughness up to $k/\delta = 0.165$ in a hypersonic TBL flow at edge Mach number $Ma_e \approx 7.2$ and $Re_\tau = 180 - 679$.

The general challenge for experimental setups is to fulfill both of the above criteria with $k^+ \gg 1$ and $k/\delta \ll 1$. Bowersox [32] estimated for a fully rough limit of $k^+ = 70$ (with the assumption that for most roughness topologies $k_s/k \sim \mathcal{O}(1)$) together with Jiménez blockage limit of $k/\delta = 1/40$ that the necessary Reynolds numbers for a fully rough TBL is in the order $\delta^+ = k^+/(k/\delta) \approx 3000$, which is especially challenging for ground-based high-speed flow investigations (see also table 2.1 for an overview of achieved values for past compressible experiments).

In the following selected past investigations will be highlighted, deemed relevant for this thesis. A summary of selected values will be given in the form of a table (table 2.1) and graph (figure 2.8) at the end of the section.

One of the earliest work on roughness influence was performed by Nikuradse [173] on rough pipe flow. He used densely packed sand grains glued along the pipe surface. He was able to scale the effects of roughness based on a roughness Reynolds number k_s^+ , defined by the sand grain diameter as single roughness scale $S = k = k_s$. Nikuradse was able to collapse all his mean velocity profile data from pressure measurements via eq. (2.5) and he found three characteristic regimes of roughness impact. For $k_s^+ < 3.55$, termed hydraulically smooth regime, no roughness influence was detectable since roughness elements are of the same order than the viscous sublayer and any potential additional turbulence is damped by the dominant viscous effects. For $k_s^+ > 67.6$, termed fully rough regime, he found a constant roughness offset $B_{rs}(k_s^+ \rightarrow \infty) = 8.48$, independent of k_s^+ since roughness elements reach into regions of the boundary layer, where no viscous effects play a role anymore and the additional momentum loss is mainly driven by pressure drag. In-between these two regimes, the roughness offset $B_{rs}(k^+, S^+)$ is a function of roughness length scales and viscosity, termed transitionally rough regime. In this regime, the additional momentum loss due to roughness is driven by both viscous and pressure drag (see [45]).

The three regimes with its boundaries are nowadays most often given with (e.g. [19], [32]):

- For $k_s^+ < 5$: hydraulically smooth regime
- For $5 < k_s^+ < 60 - 70$: transitionally rough regime
- For $k_s^+ > 60 - 70$: fully rough regime

Based on the extensive dataset of Nikuradse, Schlichting [212] performed measurements along a variety of technical rough walls within a turbulent channel flow and linked his data to results of Nikuradse via an EQSR k_s . It is an hydraulic scale which represents a hypothetical surface consisting of densely packed sand grain with k_s , which when tested under the same flow condition in the fully rough regime would cause the same drag as the surface roughness of interest. Most often, the EQSR is given normalized based on some relevant physical scale, like the roughness height, with k_s/k . Schlichting invented with the EQSR a single scaling parameter which aims to represent the potential different other scales of technical roughness, and it is still the most widespread used scaling approach, especially in CFD. The procedure will be further detailed in section 3.12.

Multiple different works on technical roughness topologies followed, of which to mention are by Moore [164] ($\lambda/k \approx 3.9$, $k_s/k = 2.7$), Perry and Joubert [185] ($\lambda/k \approx 4$, $k_s/k = 2.5$),

Betterman [25] ($\lambda/k \approx 2.6 - 4.2$, $k_s/k(\lambda/k = 4.2) = 2.7$), Liu *et al.* [152] ($\lambda/k \approx 2 - 96$, $k_s/k(\lambda/k = 4) = 2.6$), Perry *et al.* [187] ($\lambda/k \approx 3.6$, $k_s/k \approx 6$) and Krogstad and Antonia [134] ($\lambda/k = 4$, $k_s/k = 6$) who investigated incompressible flow along transverse square bar roughness. This kind of technical roughness exhibits a topology with consecutive bars with square cross-sections in the streamwise direction (width equals height) spanning the full model dimension in the direction transverse to the main flow. This shape was widely investigated since it is well defined, typically easy to manufacture and can show high roughness impact. Additionally, the previously mentioned potential roughness scales S are believed to be reduced to only the wave-length between consecutive bars λ , which are given in brackets above stated normalized by the bar height k . Additionally stated in brackets are the corresponding EQSR values for $\lambda/k \approx 4$, which is of interest to the current thesis. Despite the fact that all mentioned investigations were performed in incompressible flow and on similar roughness topologies, it should be emphasized how much the corresponding EQSR differ (also visualized in figure 2.8).

Perry and Joubert [185] were one of the first who introduced the categories of “d-type” and “k-type” roughness. Geometries which show a “skimming” of the flow along the roughness topology, characterized by quasi-stable vortices within the cavities and only small disturbances of the layered structure, were categorized as “d-type” (see figure 2.4). His observation was, that in several boundary layer flows along plates with narrow transverse square bar roughness the roughness function was not proportional to the roughness height k but instead to the boundary-layer thickness δ , here labeled by “d” (although Perry *et al.* [187] actually used the Cole’s integral parameter $\Delta_c = \delta^* U_e / u_\tau$ instead of δ). In contrast, for large spacings each element produces a wake that interacts with the following element, which was labeled “k-type” roughness, where the height of the elements k is the relevant length scale. An extreme case of this type is that of an isolated roughness element. Perry found a proportionality of k_s from the shift of origin ϵ based on TBL flow along both d- and k-type data. For k-type roughness ϵ is proportional to k whereas it is not in case of d-type roughness. Some concerns were mentioned by Jiménez since the measurements were performed in an adverse pressure gradient flow and some data points exhibit shift of origins ϵ larger than twice the extent of either the groove width or depth. In case of the d-type roughness Jiménez [118] later reported an approximation for the effective roughness according to $k_s \approx 0.02\delta$, but formulated concern since this approximation was based on limited data. Jiménez [118] also discussed potential arguments for why EQSR values should scale with boundary layer thickness δ instead of k . If one assumes that there are stable vortices in the cavities, the flow in the outer layer experiences a boundary condition that alternates between no slip at the crests and partial slip over the cavities. In contrast to δ , this would suggest the roughness crest width or pitch to be the relevant length scales, which in turn would be proportional to k in case of TSB roughness. In contrast, to bring in δ as relevant scale, it has been proposed by other investigators that groups of cavities occasionally eject vorticity into the outer layer and that these events are triggered by large-scale sweeps originating in the outer flow [255]. The argument for or against a direct coupling between outer flow structures with near-wall events is discussed already for smooth wall flow [118].

Different authors state different parameter ranges to define the transition between d- and k-type roughness. For TSB roughness, e.g. Jiménez [118] formulated a transition based on the bar interval i and defined topologies with $i/k \geq 3 - 4$ as k-type, much like Bandyopadhyay [19] with $i/k \geq 3$. Antonia and Luxton [10] formulated it based on the wavelength λ and defined topologies with $\lambda/k \geq 3 - 4$ as k-type, much like Berg [24] with $\lambda/k \geq 4$. Although firstly only

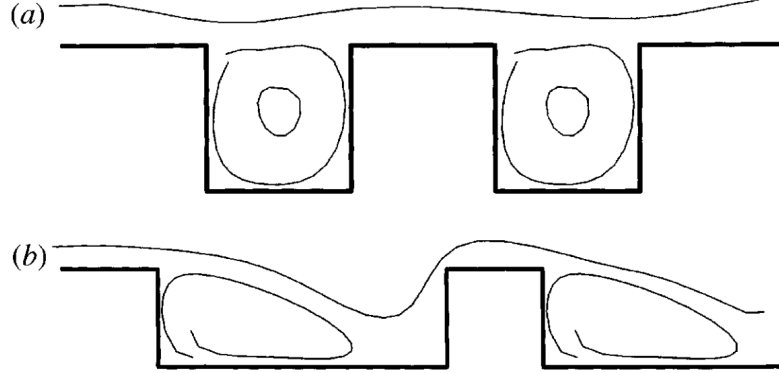


Figure 2.4: Sketches of a) “d-type” and b) “k-type” roughness flow originally proposed by [185]. Figure according to [118].

defined for two-dimensional roughness, debates exist about whether a comparable significant change in flow structure occurs along three-dimensional roughness as well [104].

With results from the variety of technical roughnesses, data showed that the boundaries for the three different roughness regimes stated above are only strictly valid for densely packed sand grain roughness. The boundaries can differ for other roughness topologies, e.g. real surface (distributed) roughness or technical roughness, when scaled by relevant physical roughness scales k . To visualize this together with the concept of EQSR, figure 2.5 shows the roughness function ΔU^+ against k^+ for transverse bars, wire-mesh and sand grain roughness according to [19]. The fully rough flow regime is thought to be universal and is visible with the asymptotic behavior to a slope of $1/\kappa$ for $k^+ \gg 1$, independent of topology (see eq. (2.8)). The visible parallel shift for the different topologies in the fully rough flow regime visualizes the effect of differing EQSR. Here, e.g. transverse bars exhibit for the same k^+ a larger momentum loss ΔU^+ compared to sand grain roughness, which is equivalent to an EQSR k_s/k greater than one. Another way to see this shift is a difference in $B_r(k^+, S^+ \rightarrow \infty)$ previously mentioned in eq. (2.5) and (2.8), where the term $B - B_r(k^+, S^+ \rightarrow \infty)$ acts like an offset in eq. 2.8, since it is the value of the fully rough asymptote at $k^+ = 1$. Additionally, it is visible that the lower limit for fully rough flow, represented by the k^+ where data converges to the asymptotic behavior, changes depending on the topology. For sand grain roughness, the aforementioned limit of $k^+ = k_s^+ \approx 60 - 70$ is visible in the plot, whereas for transverse bars this limit is much lower with $k^+ \approx 8$ for the topology with $\lambda/k = 3.8$ according to [19]. Therefore, different roughness topologies can result in different limits for the fully rough regime (see also review by Kadivar *et al.* [121]). Besides the shift in the fully rough regime, also the behavior in the transitionally rough regime is visibly different for the various types of roughness, since here the momentum loss depends on skin friction and pressure drag and this can be different for every roughness topology.

As it is the case for smooth wall TBL studies, also rough walls in high-speed flows have received considerably less attention. E.g. Bowersox [32] gave an excellent review and discussion about technical challenges which are encountered when studying highly compressible flow fields. As in case of compressible smooth wall flows, also the study of rough wall flows rely on so-called

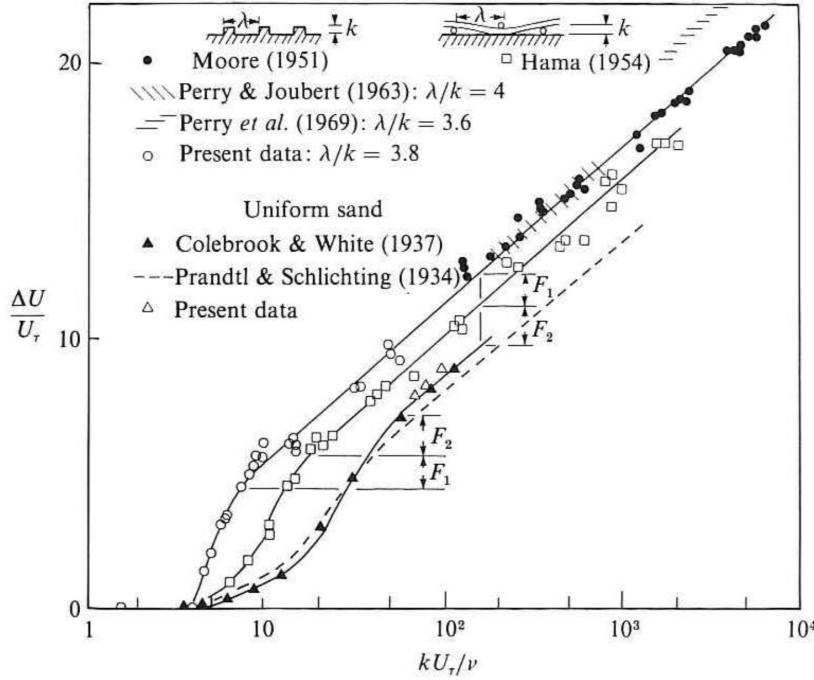


Figure 2.5: Roughness function $\Delta U^+ = \Delta U/u_\tau$ against $k^+ = ku_\tau/\nu$ for transverse bars, wire-mesh and sand grain roughness according to [19].

Morkovin's hypothesis [165]. Morkovin was one of the first to observe along smooth wall zero pressure gradient flat plate flow, that differences in the turbulent statistical properties between supersonic and subsonic cases can be correlated with the thermodynamic property variations across the TBL (e.g. density, viscosity).

Goddard was one of the first authors to show that the incompressible laws for rough wall flows (e.g. (2.5), (2.7)) can be used if compressible data is properly transformed to an effective incompressible mean velocity profile (see [90]). He was able to scale his compressible profiles from Pitot pressure measurements in a range of $Ma = 0.7 - 4.5$ along an adiabatic wall to an effective velocity U_{eff} via the van Driest scaling. This scaling originates from integration of a compressible Prandtl-Mixing-length formulation (see e.g. [259] and [260]) and incorporates the mean wall-normal density profile $\rho(y)$ according to

$$U_{eff} = \int_{U_1}^U \sqrt{\frac{\rho(y)}{\rho_w}} dU \quad (2.13)$$

Here, the suffix 1 indicates the location of the lower end of the validity range of the log law (see also eq. (3.48) in section chapter 3). Goddard found that for a fully rough flow the constant remained $B_{rs}(\infty) \approx 8.5$ just like the incompressible result by Nikuradse [173]. He concluded that within the limits of his setup, no wave drag or other compressibility effect played a role. Additionally,

Goddard could show that the velocity defect law was still applicable if the transformed effective velocity U_{eff} was used.

Fenter [71] extended the experimental analysis of Goddard partly to non-zero pressure gradient flow and to non-adiabatic walls along sand grain and technical roughness, including heat flux measurements. He measured within a range of $Ma = 1.6 - 3.7$ along a flat plate and a cone-cylinder model and used hemispheres, dimples and v-grooves as technical roughness. He confirmed the applicability of the law-of-the-wall and defect law after transformation of the velocity via the van Driest formulation. Apart from the experimental work, he was able to derive analytically and iteratively solvable correlations for compressible boundary layer parameters along rough non-adiabatic surfaces. Based on a mixing-length approach (the author used Kármán's instead of Prandtl's formulation due to better results for the compressible smooth adiabatic wall Zero Pressure Gradient (ZPG) TBL data) and stepwise approximations of complex integrals (comparable to the work of van Driest to derive his widely used transformation [259],[260]), the author derived analytical solutions for the evolution of shape parameter, local and mean skin friction along adiabatic or non-adiabatic, isobaric surfaces with uniform grain-type roughness in compressible flow (see section 3.10.1). For flow along circular cones with zero angle of attack including sand grain roughness, Fenter was able to confirm van Driest results along smooth walls that flat plate approximations for skin friction can be used by using half the running length based Reynolds number along the cone (see eq. 3.39).

Young [294] studied a $Ma = 4.9$ flow along a non-adiabatic flat plate with transverse v-groove roughness ($T_w/T_e = 2.8 - 5.2$ where the latter temperature ratio corresponds to adiabatic wall conditions). With measurements of velocities, skin friction and heat flux augmentation within transitionally and fully rough regime, he supported Goddard's results after van Driest transformation for non-adiabatic conditions.

Berg [24] investigated transverse square bar roughness in a $Ma = 6$ flow along the adiabatic flat wind tunnel test section wall. Berg performed studies of step changes from smooth-to-rough and vice-versa and its impact on the evolving turbulent boundary layer. The roughness was tested at three different heights (0.32, 0.64 and 1.27 mm) corresponding to $k^+ = 7.1, 14.7$ and 33.8, respectively, all with a wavelength $\lambda/k = 4$. A floating element was used to measure the wall shear at two different positions. Additionally, the friction velocity was extracted via fitting velocity profiles to the log-law, with a general good agreement for positions where both was available (except one) [74]. To calculate velocities from Pitot pressures, Berg made use of a Crocco-Busemann-like velocity-temperature-relation but with parameters fit to local temperature profile measurements (see also section 3.12). The Pitot data was corrected for low Reynolds effects via a method described by Ramaswamy [201] (described in section 7.4). The data sets were thoroughly checked and were integrated into a tabulated set of standard cases by Fernholz *et al.*, although some influences from reflecting expansion-compression waves from the nozzle were reported [74]. Berg found a new equilibrium profile for both mean and fluctuating profiles (mass flux and total temperature fluctuations) after $10 - 25\delta$ downstream of the step change from smooth to rough. Berg concluded the largest roughness elements ($k^+ = 33.8$, $k/\delta = 0.046$) to be in the fully rough regime, in agreement with the limits reported in [19]. Based on a resulting inner scaled velocity shift of $\Delta U^+ = 6.5$, utilizing a shift of origin $\epsilon/k = 0.5$, he derived an EQSR of $k_s/k = 1.3$. This value is much lower than results from other authors, mainly derived from incompressible measurements (see also list above) and reasoned with potential compressibility

effect. Different aspects within the current thesis build upon the work and results of Berg, although a critical analysis with respect to his inflow conditions hint at a potential impact of condensation (see section 7.1.2).

Keel [128] and Hill *et al.* [101] extended the study of compressible roughness effects to cone models. Keel investigated sand grain roughness along an internally cooled sharp 5° cone in a $Ma_\infty = 2.5$ ($Ma_e = 2.4$) and $Ma_\infty = 5$ ($Ma_e = 4.7$) flow at non- ($T_w/T_0 = 0.3$) and adiabatic wall conditions ($T_w/T_0 = 1$) [128]. Two different sand grain sizes were tested, both reported to be within the fully rough regime based on an asymptotic heat flux augmentation. The largest roughness at the highest Mach number had a blockage of $k/\delta \approx 0.09$. Keel performed skin friction measurements via a floating element balance, velocity profiles via Pitot pressure measurements and heat flux measurements via slug calorimeter. The slender cone velocity profiles for the smooth, adiabatic wall agreed with flat plate incompressible law-of-the-wall. Hill *et al.* studied sand grain roughness along a sharp and blunt 7° cone in a $Ma = 10$ flow at a non-adiabatic wall condition with $T_w/T_0 = 0.26$ [101]. Three different sand grain sizes were tested with the two largest within the fully rough regime $k_s^+ = 170 - 323$, where the largest roughness had a blockage of $k/\delta \approx 0.06$. Hill *et al.* performed total skin friction measurements via a force balance, velocity profiles via Pitot pressure measurements and heat flux measurements via thermocouples.

Studies at $Ma_e = 2.9$ from Latin [138] and Ekoto *et al.* [66] were among the first compressible investigations to show that outer layer similarity can break down, if the extent of the subsonic region of the compressible boundary layer gets very small and potentially reaches the roughness elements. At that point, the roughness elements can produce shocks and expansion waves which can traverse through the TBL and even into the external flow field. This is visible in figure 2.6 along sand grain and diamond shaped roughness via Schlieren imaging.

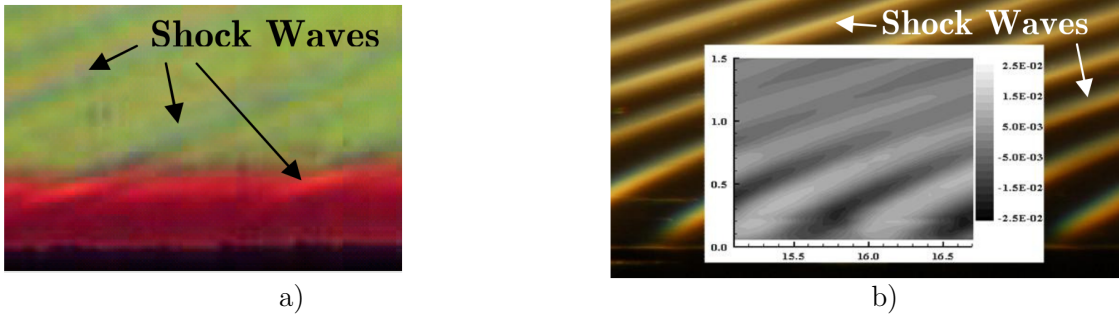


Figure 2.6: Schlieren images of shock waves along a) sand grain roughness [138] and b) diamond roughness [67] in compressible TBL according (both plots from summary in [32]).

Also, Ekoto [69] showed the impact of local disturbances emanating from diamond roughness on mean velocities measured by PIV, whereas along 3D square bar roughness ($w/k \approx 2$, $\lambda_x/k = \lambda_z/k = 4$) only negligible effects were visible (see figure 2.7).

These data were discussed together with other heritage data in the summarizing review by Bowersox [32]. The author summarized that for his reported studies confirmed that the defect law held if the transformed effective velocity U_{eff} was used. An exception was reported for data by Latin [138], for which some signs of roughness topology effects on the relationship between mean

density- and velocity-profiles were reported. These compressibility effects affected the local velocity profiles and wall shear stress, inferred by the near wall turbulence shear stress measurements. In this case, distinct compressibility effects may arise and not only the argumentation for a locally confined effect of roughness may be in question but also the conventional transformations (like van Driest scaling) are in question due to the lack of incompressible counterpart.

Sahoo *et al.* [209], [208] performed PIV measurements on TBL along smooth and rough flat plate models at $Ma_e \approx 7.3$ with square bar roughness of $\lambda/k = 5$ at $Re_\theta \approx 3900$ and 5500 , respectively. In both publications, values are reported with $k^+ = 102, 134$, $k/\delta = 0.14, 0.12$ and $k_s/k = 5.9, 4.5$, respectively. Van Driest mean velocity scaling was confirmed as well.

Peltier [183] investigated the effects of periodic crosshatch roughness ($k^+ = 160$) on a $Ma = 4.9$ turbulent boundary layer ($Re_\theta = 63000$) using PIV. The roughness elements generated a series of alternating shock and expansion waves which protruded the entire boundary layer, consistent with results by Ekoto *et al.* [67]. Van Driest scaled velocity profiles showed the shift below the inner scaled smooth wall values but on a larger level compared to Ekoto *et al.* [67] for which Reynolds number or compressibility effects were speculated. Due to the locality of the flow field, mean velocity profiles showed also small variations depending on the position above the diamond roughness.

Williams *et al.* [287], [288] performed PIV measurements on a TBL at Ma_e between 7.2 and 7.6 and $Re_\tau = 180$ to 679 along smooth and rough flat plate models. This included the investigation of transverse square bar roughness with $\lambda/k = 8.33, 10.63, 5$ which resulted in $k^+ = 30, 70, 112$, $k/\delta = 0.068, 0.098, 0.165$ and $k_s/k = 0.7, 2.3, 3.3$, respectively. Along the smooth wall, the mean and fluctuating streamwise velocity profiles showed strong similarity in the outer layer to incompressible flows at comparable Reynolds numbers when scaled according to van Driest and Morkovin. In case of the rough wall data [288], mean van Driest transformed velocity profiles supported the outer layer similarity hypothesis, although the Reynolds numbers were comparably moderate compared to the usually demanded large Reynolds numbers according to e.g. Chung *et al.* [45]. Additionally, Williams experiments were conducted on roughness large in terms of k/δ compared to the limit according to Jiménez [118].

Additionally, several CFD computations have been performed with the focus on TBL flow in which the rough wall topology is fully resolved by the grid, from which only a few ones exist for compressible flow.

Ikeda and Durbin [115] performed incompressible DNS and Reynolds Averaged Navier Stokes equations (RANS) computations (2D, eddy-viscosity model) on transverse square bars with the interval-to-height ratio of $i/k = 9$ in channel flow. Results confirmed a log-layer above roughness at $y > 2k$ but RANS solution lacked in predicting the correct slope of mean velocity in the log-layer.

Bons *et al.* [29] performed incompressible RANS computations on direct resolved sand grain roughness type in comparison to discrete element method (DEM) and experimental data. Computed skin frictions and Stanton numbers were in acceptable agreement with experiments.

Vijiapurapu and Cui [267] performed incompressible RANS computations (one, two, 7-equation models) and Large Eddy Simulations (LES) compared to experiments along transverse square bars in pipe flow and showed that all RANS turbulence models are at least capable of predicting

mean velocity profiles but do have problems in accurately predicting flow in the cavities due to problems with separation.

Beekman and Martin [22] reported about compressible turbulent DNS boundary layer flow with square bars with wavelength ratio $\lambda/k = 8.3$. The author tried to rebuild the $Ma = 7.3$ case reported in [209], [208]. Unfortunately, the paper gave only preliminary qualitative information, due to problems with numerical stability. Nevertheless, it reported about general similarities to smooth wall flows with appearance of downstream-leaning coherent structures showing resemblance to incompressible hairpin-packet models and characteristic spanwise length scales which are of the same order of magnitude as smooth, incompressible TBL.

Tyson and Sandham [258] performed DNS computations along direct resolved wavy wall roughness in compressible turbulent channel flow ($Ma = 0.3, 1.5, 3.0$). The study reported strong compressibility effects for the transonic and supersonic cases on moderate amplitude wavy walls due to the occurrence and dynamics of shocks along the flow field, visible in the mean flow and turbulence properties. An unusual result of the study was that the van Driest transformed velocity profiles did not collapse even for the smooth wall in contrast to numerous comparable DNS studies like e.g. [56], [58], [57] and [7].

Alvarez [7] performed compressible turbulent DNS computations along 3D bars with wavelength ratio $\lambda_x/k = \lambda_z/k = 12.5$ in a $Ma = 2.5$ and 5 TBL flow. Unfortunately no experimental data could be used for a direct comparison, since a unique roughness topology was chosen, not yet tested at that time. In general, the work supported Morkovin's hypothesis although the Reynolds shear stresses exhibited increased magnitude in regions with local compression. Conversely, the shear stresses decreased in regions with increased expansion.

Modesti *et al.* [162] performed a DNS study in turbulent channel flow within $Ma = 0.3 - 4$ at two different Reynolds numbers $Re_\tau = 500$ and $Re_\tau = 1000$ along cubical bar roughness (3D) with $\lambda/k = 3$ ($k^+ = 41 - 86$) along the upper and lower wall of the channel. He showed that transformations according to van Trettel-Larsson (TL) [256] and Volpiani *et al.* [269] were capable of collapsing smooth mean velocity profiles onto the incompressible theory. In contrast, the transformations failed to capture ΔU_{eff}^+ differences with varying Ma properly. The authors claimed that most previous studies erroneously incorporate compressible effects into the derivations of k_s/k (like in e.g. [24], [101]) and they suggested that a proper compressible transformation of the roughness height k and therefore also for the Reynolds number k^+ is necessary. Suggested transformations are again e.g. via van Trettel-Larsson (TL) and Volpiani or an alternative formulation via

$$k_{*,Modesti}^+ = \frac{k_{*,Modesti} u_\tau}{\nu_w} = \frac{k \frac{\nu_w}{\nu_k} u_\tau}{\nu_w} = \frac{k u_\tau}{\nu_k} \quad (2.14)$$

The motivation behind all these transformations is to account for density variations between the crest and the trough. The rationale of the latter formulation was to ease the use for experimentalists, since transformations according to Trettel-Larsson (TL) can be challenging to use due to near-wall integrations. Nevertheless, estimating the mean kinematic viscosities at the wall and roughness crest k can already be very challenging in experiments. Interestingly, this is

the same transformation which was suggested by Merrifield *et al.* [161] but based on different reasoning.

A selection of past compressible investigations can be found in table 2.1 (partly based on tables in [32] and [184]). Several observations can be made by data included in this table:

- Typically, high Reynolds number flows (δ^+) combined with low blockage (k/δ) were achieved when one Test Section Wall (TSW) of a closed wind tunnel test section was used as model, due to much larger running length of the approaching boundary layer and therefore achieved large thickness δ . The disadvantage of this set-up are so-called history effects, i.e. potential influences that can emanate from the entire passage upstream of the test section (e.g. in the nozzle with corresponding pressure gradients) which influence the state of the boundary layer (see e.g. critical discussion in [73]).
- Wind tunnel tests along e.g. flat plates (FP) have typically lower Reynolds numbers and higher blockage ratios violating corresponding limits suggested by Jiménez [118].
- To the knowledge of the author, there is no investigation yet, in which measured quantities of mean flow, turbulent intensities and heat flux rates are available in a single experimental setup. This thesis aims to close this gap.

Additional parameters for selected compressible roughness investigations along transverse square bars (TSB) close to $\lambda/k = 4$ can be found in table 7.13 and 7.14.

Table 2.1: Selected compressible roughness studies; **Model Types:** FP: flat plate, C: cone, BC: bicone, SC: sphere-cone, TSW: test section wall, OC: ogive-cylinder; **Roughness Types:** V: v-grooves, S: sand grain (partly grit), H: hemispheres, D: dimples, C: cones, TSB: transverse square bars, TB, transverse bars, 3DB: 3D block, DM: diamonds, M: mesh; **Measured Quantities:** MF: Mean flow, T: turbulent quantities, (T): temperature fluctuations in case of [24], SF: skin-friction, (SF): total skin friction via force balance, HF: heat flux; **Measurement Techniques:** PP: Pitot pressure, PIV: Particle Image Velocimetry, LDV: laser doppler velocimetry, HWA: hot wire anemometry, IR: infrared thermography, LS: local sensors, SFB: skin friction balance, FB: force balance; **Wall Status:** a: adiabatic, na: non-adiabatic; (partly adapted from [32] and [184]).

Main investigator	Ma	k_s^+ [-]	k/δ [-]	δ^+ [-]	Wall Status	Model Types	Roughness Types	Measured Quantities	Measurement Techniques
Goddard1957	0.7–4.5	10–1000	0.02–0.07	1000–1500	a	OC	S	MF, (SF)	PP, FB
Young1965	4.9	7–40	-	-	na	FP	V	MF, SF, HF	PP, SFB, LS
Fenter1960	1.6–3.7	-	-	-	na	FP, SC	S, H, D, V	MF, SF, HF	PP, SFB, LS
Berg1977	6	10–45	0.02–0.06	500–750	a	TSW	TSB	MF, (T), SF	PP, SFB
Hill1980	8.1	163	0.06	2800	na	C	S	MF, (SF), HF	FB
Holden1984	2.0–8.0	20–1800	-	-	a	C, BC, SC	S, H, C	HF, SF	LS, SFB
Latin2000/02	2.7	100–570	0.03–0.05	3000–11000	a	TSW	S, TSB, 3DB	MF, T	PP, LDV, HWA
Ekoto2006/07	2.8	50–100	0.05–0.07	1000–1500	a	TSW	3DB, DM	MF, T	PP, PIV
Sahoo2009/10	7.3	37–154	0.06–0.16	566–1089	a	FP	TSB, DM	MF, T	PIV
Peltier2013	4.9	600	0.07	2330	a	TSW	D	MF, T	PIV
Williams2021	7.2–7.6	34–493	0.068–0.165	180–679	na	FP	TSB, M	MF, T	PIV
Present	5–5.4	26–78	0.08–0.12	186–573	na	C	TSB	MF, T, HF	PP, PIV, IR, LS

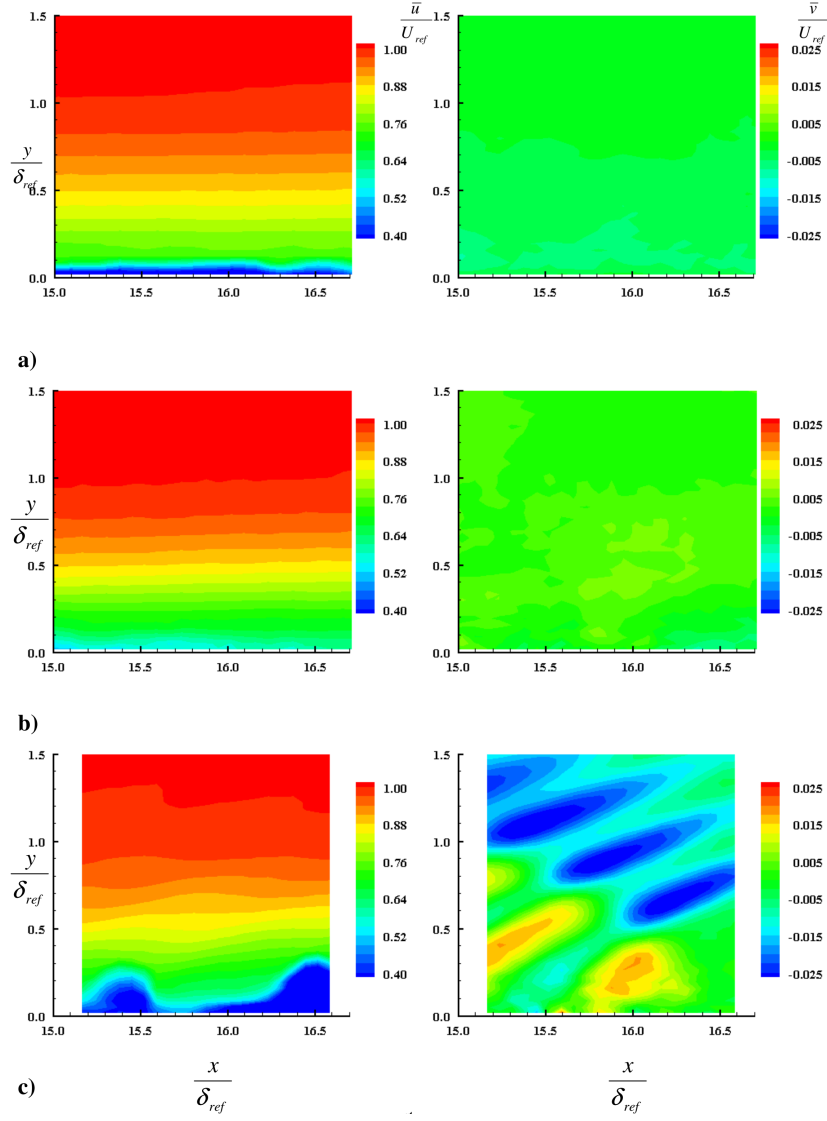


Figure 2.7: PIV mean velocities in streamwise (left column) and wall-normal direction (right column) along smooth wall a), 3D square bar b) and diamond roughness c) in a compressible TBL according to [69].

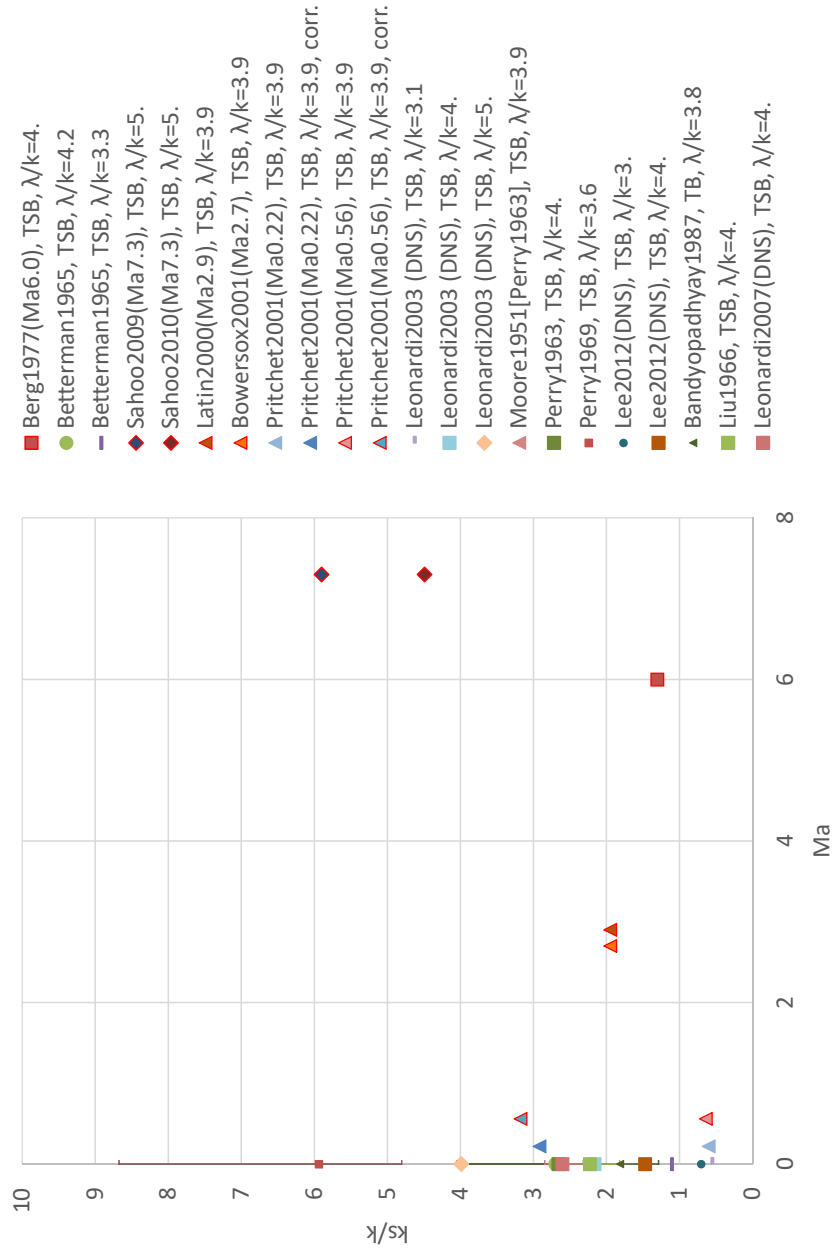


Figure 2.8: EQSR k_s/k against Mach number for TSBR with $\lambda/k \approx 4$ from different authors

2.2 Turbulent Stresses

Typically, TBL are characterized by (apparent) Reynolds stresses that transport mean momentum and produce and dissipate turbulent kinetic energy. These stresses are an additional component to the molecular stresses in a boundary layer flow and stem mathematically from non-linear terms of velocity fluctuations if the flow is decomposed into a mean and fluctuating velocity field. The turbulent stresses are inhomogeneous and corresponding characteristic length scales can vary from a multiple of the viscous length scale δ_v near the wall to a multiple of outer length scales δ (see e.g. [213], [190], [235], [2]).

As is the case for mean flows, also for compressible turbulent stresses typically Morkovin's hypothesis is used who argued that the general structure of turbulent motions are comparable to incompressible flow [165]. He proposed scaling for the turbulence intensities via a density-weighted velocity scale $u_* = \sqrt{\rho/\rho_w} u_\tau$ and showed that the streamwise turbulence intensity collapse in the outer layer for Mach numbers up to his maximum tested value of $Ma = 4.76$. There are different other options to scale compressible Reynolds stresses due to different varying thermodynamic properties in TBL. Williams *et al.* [287] collected compressible data of multiple sources and investigated different options to scale turbulent stresses properly. They found that in comparison to DNS and incompressible TBL data, near wall effects were best scaled by semi-local inner scaling, which allows the local stress and kinematic viscosity to vary with wall distance. Overall, Williams *et al.* showed outer Morkovin scaling of the streamwise turbulent stress to be valid for Mach number up to 7.5.

Studies by Latin [138], Pritchett and Bowersox [195] and Ekoto [69] were among the firsts to include roughness impact on compressible turbulence data via HWA, Laser Doppler Velocimetry (LDV) and PIV. Ekoto [69] showed the impact of local disturbances emanating from diamond roughness. He used near wall turbulence shear stress measurements to infer that the wall shear stress differed by nearly a factor of two, depending on the position above the roughness elements (see also [32]).

Sahoo *et al.* [209], [208] reported that the streamwise and wall-normal inner-scaled fluctuations measured via PIV were damped in presence of the square bar rough wall.

The periodic crosshatch roughness ($k^+ = 160$) in the Mach 4.9 TBL of Peltier [183] created a series of alternating shock and expansion waves that penetrated the entire boundary layer, which caused significant variations, up to +50% and -30%, in the Reynolds shear stress field. Effects in the outer layer were attributed to a potential favorable pressure gradient impact.

Williams *et al.* [288] reported about Reynolds stresses along roughness including TSB roughness in a $Ma_e \approx 7.2$ to 7.6 TBL with $\lambda/k = 8.33, 10.63, 5$. When scaled by the friction velocity, the streamwise Reynolds stress profiles show strong near wall influence of the roughness, but for $y/\delta > 0.3$ all profiles show a high level of agreement to each other and also to smooth wall data. Additionally, Williams *et al.* remarked that in Morkovin scaling the profiles show much more scatter. This was used as argument that this scaling is not appropriate for their own data which showed substantial rough wall effects.

Due to the challenges in experiments to derive quantities like skin friction velocities, Alfredsson and Örlü introduced the so-called diagnostic plot for incompressible flow, which utilizes the

streamwise turbulence profile u_{rms}/U (or u_{rms}/U_e) against the mean velocity profile U/U_e [5], [6]. Castro *et al.* [39] extended this analysis to rough wall data to additionally avoid the delicate determination of a shift of origin ϵ . As an empirical finding, previous investigations showed a linear region in the diagnostic plots which starts in the logarithmic region and spans through most part of the outer wake flow region. With a formulation in the form $u_{rms}/U = a + b U/U_e$ the fitting parameters were found to $a = 0.286$, $b = -0.255$ for smooth walls [39]. Castro *et al.* [39] also extended this approach to rough wall data, utilizing the loss of momentum according to $U' = U + \Delta U_{eff}$ along both axes with u_{rms}/U' against U'/U_e , with which he successfully collapsed different incompressible experimental smooth and rough wall data.

2.3 Coherent Structures

So called coherent structures are nowadays thought of as main underlying contributor to the general structure of TBL, measurable by mean velocities and Reynolds stresses as was discussed in the previous subsections. TBL coherent structures have been the subject of extensive research in the past and summarizing work can be found in e.g. [2]. Here, only some aspects relevant for the current work as well as terminology are presented. Due to technical challenges to experimentally probe compressible boundary layers, this overview will mostly address, although is not entirely limited to, incompressible boundary layers.

In the early days of TBL research, turbulent processes were thought of being purely stochastic in nature. Early work by Theodorsen [252] suggested that Reynolds shear stress resulted from the passage of an inclined vortex loop, described as “horseshoe vortex”. The origin of this vortex structure was speculated to lie in a spanwise vortex filament perturbed in the wall-normal direction. Subsequently, segments of the vortex are momentarily lifted away from the wall [182]. Based on kinematic argumentation, it was suggested that the vortex “head” would experience higher mean velocities in the TBL that would cause the structure to stretch in the streamwise direction. The vortex heads are typically found to be inclined at approx. 45° to the wall. On the other hand, the “legs” as the lower parts of the vortex filament would be drawn together in the lower parts of the TBL and the distance between the legs is typically found to approx. $100\delta_v$. By continually stretching, a vortex structure evolves which resembles a horseshoe or hairpin in shape, giving the structure its name (see figure 2.9).

Due to its shape, the vortex is believed to induce a local flow field of streamwise u' and wall-normal v' fluctuations, whose signs are anti-correlated, hence producing Reynolds shear stress which is negative overall. Due to the sense of rotation, a hairpin vortex would move low-momentum fluid away from the wall, whereas high-momentum fluid is drawn in from higher in the boundary layer. These events were labeled “ejection” and “sweep”, respectively and are visualized in figure 2.9 as Q2 and Q4, respectively (explanation for these labels follows in the next paragraphs). The figure shows the characteristic flow elements that would be seen in a streamwise-wall-normal plane cutting through the hairpin vortex and arrows show local flow velocities when the mean convection velocity is subtracted. Due to the ejection of low momentum fluid, a local region of low momentum appears between the vortex legs (see shaded region close to the wall in figure 2.9).

It was found that these coherent structures can evolve into larger structures, where individual hairpins with approx. the same convection velocities grow into “packets”. A sketch of hairpin

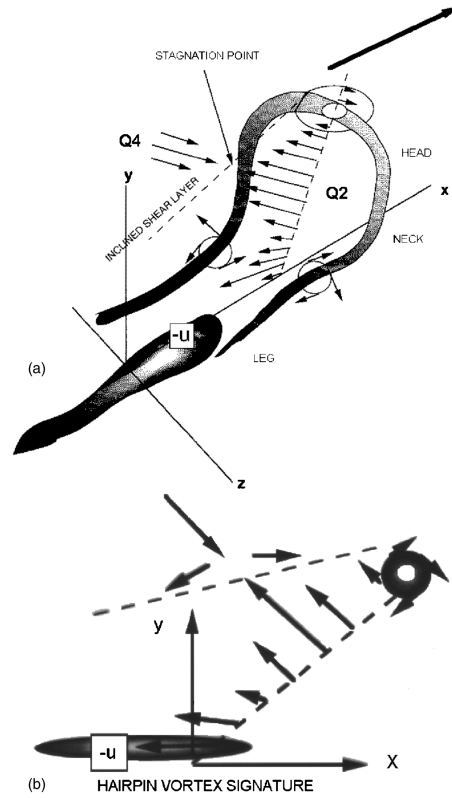


Figure 2.9: a) Schematic of a hairpin eddy attached to the wall, b) signature of the hairpin eddy in the streamwise-wall-normal plane according to [2] (in both cases the arrows indicate fluid flow in a reference frame moving with the local convective velocity).

packets is shown in figure 2.10 according to Adrian [2]. In yellow, individual hairpins are shown which form three packets of different age. Age in this context means that they had different amounts of time to evolve, starting with small near wall hairpins to increasingly larger hairpins, which travel in streamwise direction and its heads move away from the wall. As the packets are lifted away from the wall, the heads align along a line inclined at approx. $15 - 30^\circ$ to the wall. Packets of hairpins are also attributed to the so called bursting process, which is thought of a composition of multiple ejections of increasing amplitudes [182].

This alignment of hairpin packets induce back-flow regions framed by the hairpins of the packet (see e.g. arrows from right to left in figure 2.9b or orange arrows from right to left in figure 2.10), which can evolve to much longer regions than from individual hairpins, also leading to long correlation length of streamwise momentum. The nesting of one packet within another also leads to the creation of multiple zones of different relatively uniform momentum. The heads of the hairpins create regions of large shear, creating boundaries which separate these so-called Uniform Momentum Zones UMZ, which were first observed in several different incompressible studies (see e.g. [3], [229]). In contrast to the term “layer” which is typically used to distinguish between different TBL regions defined by the mean velocities (see paragraphs before), for the uniform momentum the term “zone” is used to distinguish that here the instantaneous velocity is of

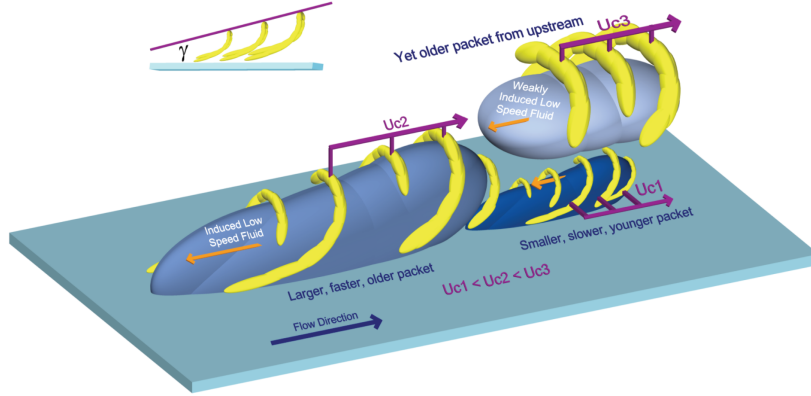


Figure 2.10: Concept of hairpins evolving into packets according to [2] (the arrows indicate fluid flow in a reference frame moving with the local convective velocity).

interest.

The hairpin packet model is also consistent with the so-called “attached eddy hypothesis” after Townsend [255] (see also previous subsection). The hairpin packets can extend beyond the logarithmic layer and in some cases they grow to the edge of the boundary layer. It was suggested that these outer layer packets are responsible for the appearance of so called “bulges” or sometimes labeled large-scale motions (LSM) with length of approx. $2-3 \delta$ (see e.g. [2]). A visualization of these bulges as boundaries between clear and smoke-filled regions near the outer edge in a low Reynolds number TBL is shown in figure 2.12. Apart from bulges also imprints of horseshoe vortices and the near wall low-momentum streaks are visible.

The hairpin vortex model is very significant, since it gives an underlying framework, based on quasi-deterministic vortices, for the seemingly random behavior of the TBL. A limited number of studies exist, investigating coherent structures and their characteristic scales in compressible TBL, of which selected are given in the following.

Early experimental findings based on HWA measurements showed that the streamwise length scales of the outer coherent motions decrease significantly with increasing Mach number, while the spanwise scales remain statistically unchanged. Also, the characteristic angle of inclination of the two-point correlation contours of the streamwise velocity, which are typically used to identify large scale structures, was found to increase with Mach number [234]. In contrast, later measurements based on velocity data by PIV showed that the streamwise length scales only slightly decrease with Mach number (see [287]) or even increase (see [85]). Also, the angle of inclination of the streamwise contours were found relatively insensitive to Mach number (see [287]). A difference in measurement techniques was found as potential reason for these contrasting trends. Since HWA measures mass-flux and therefore is affected by density, it was suggested that there are significant differences in the properties of velocity and density fields at high Mach numbers. Williams *et al.* [287] also detected the appearance of UMZs in a hypersonic TBL with $Ma = 7.6$, $Re_\tau = 279$ along a smooth flat plate (see figure 2.11). A numerical study also confirmed this finding in hypersonic TBL according to Cogo *et al.* [47].

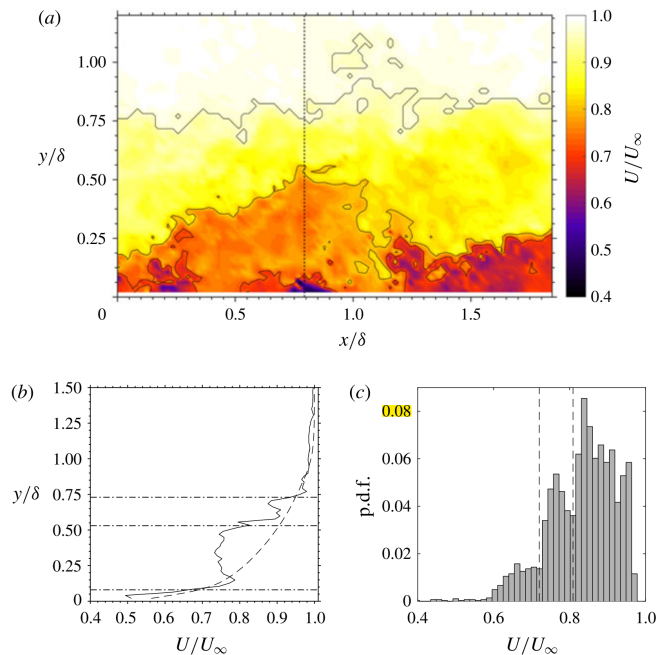


Figure 2.11: Uniform momentum zones along a smooth wall at $Ma_e = 7.6$, $Re_\tau = 279$ according to [287].

The impact of roughness on coherent structure in a compressible TBL flow was investigated in an even smaller number of prior investigations.

Studies by Latin [138], Pritchett and Bowersox [195] and Ekoto [69] were among the firsts to include roughness impact on the general structure of TBL. Peltier [183] reported about a change in the organized motions along their periodic crosshatch roughness in the Mach 4.9 TBL. With the ability to apply Particle Image Velocimetry (PIV) technique on compressible rough wall TBL, also spatial correlations for detection of coherent structures were accessible. Two-point or spatial correlations (see also equation (3.13)) can be exploited to identify the large-scale structures of a TBL as “correlated mass(es) of fluid” [13]. Peltier was able to compare smooth wall to rough wall correlation shape and length scales, as visualized in figure 2.13.

Peltier was also able to visualize the imprints of the ejection and sweep events by a so-called quadrant analysis. It decomposes the contributions of the Reynolds stress component, as measured e.g. by PIV, based on the sign of the fluctuating velocity components u' and v' into four different bins Q1 to Q4. They are typically used, representing the four different regions in a corresponding plane spanned by the fluctuating velocity components u' and v' as shown in figure 2.14b. This is visualized in 2.14a from a publication of [2]. Q2 events are connected to ejection and Q4 to sweep events. This analysis allows evaluations of the different contributions these events have to the total mean values of various quantities such as kinetic energy, dissipation, etc.

In comparison to smooth wall data, Peltier described that hairpin vortices which contribute to the vortex packets or large-scale motions lean farther away from the wall. Additionally, they are

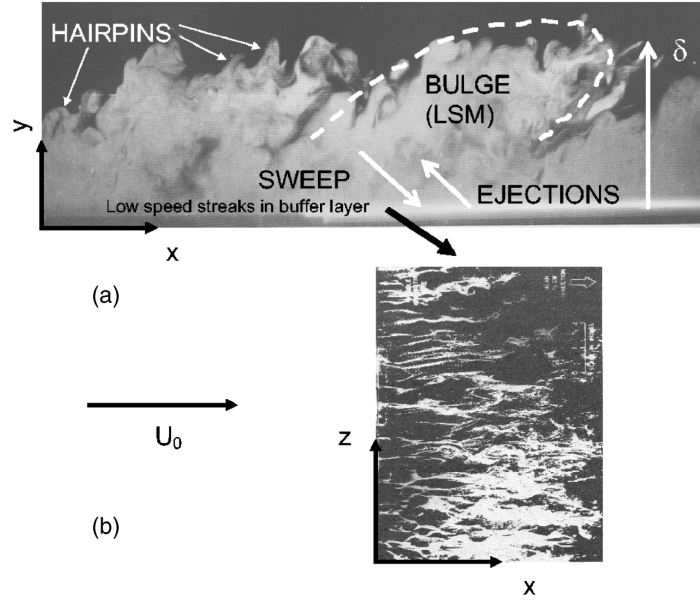


Figure 2.12: Smoke visualization showing various large scale structures of a TBL according to [2].

described to become more spatially compact and also change their populations. In the near wall region, he showed with two-point correlations of the rough wall PIV velocity fluctuations that characteristic length scales decreased clearly to roughly half the smooth ones. As potential reason, an increase in the strength of the hairpins along the rough wall were speculated. Alternatively, the mean arrangement of shock waves and expansion fans as induced by the roughness elements could be made responsible as well.

Williams *et al.* [288] showed that the outer layer length scales as well as structure angles agreed between smooth and rough wall data within the corresponding data scatter, based on two-point correlations of his PIV data. The near wall length scales indicated a reduction above the rough wall, similar to data from Peltier [183]. Only few differences were identified in the outer layer between 2D and 3D roughness elements.

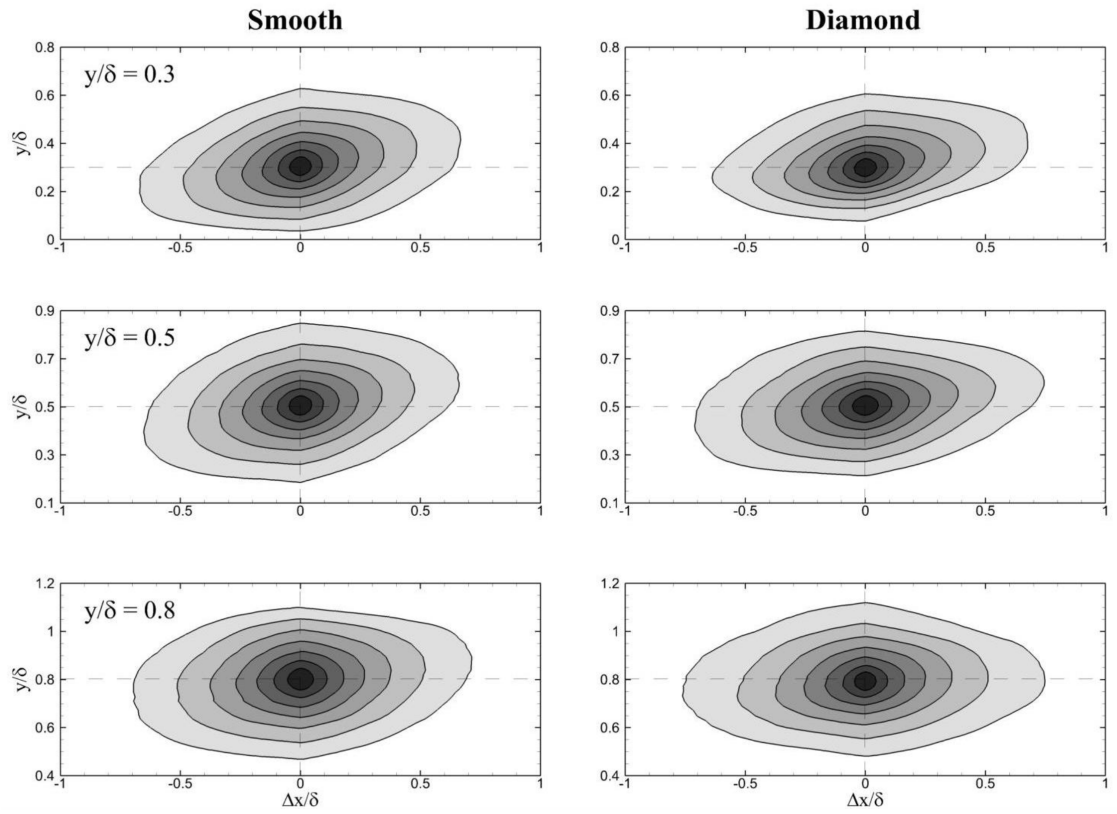


Figure 2.13: Two-point correlations of the streamwise Reynolds stress at various wall-normal distances along smooth and diamond rough wall according to [183].

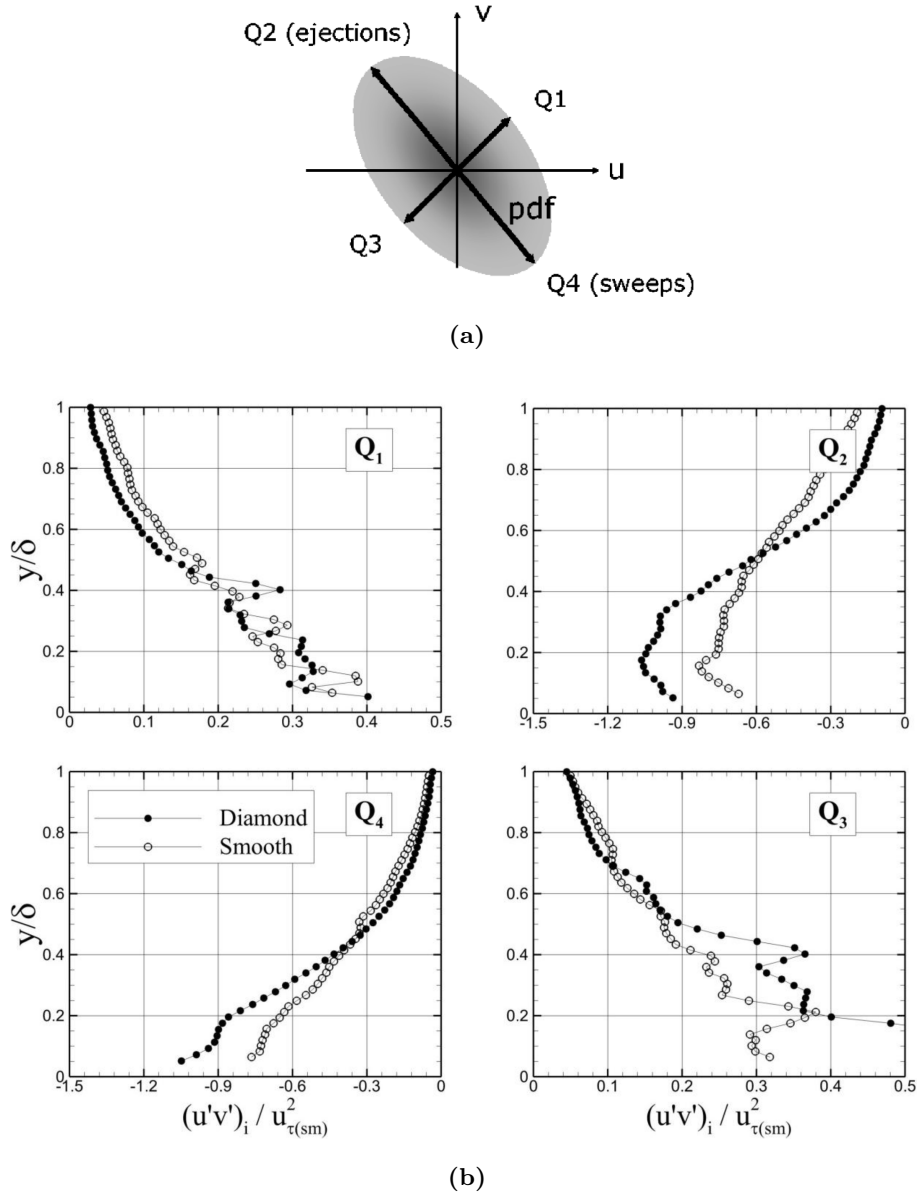


Figure 2.14: a) Sketch of the expected probability density function in the plane spanned by the fluctuating velocity components u' and v' (in the plot u and v) according to [2]; b) Quadrant analysis of smooth and diamond rough wall Reynolds shear stress profiles according to [183].

2.4 Skin Friction and Heat Flux Augmentations

Due to its direct effect on the inner portions of the boundary layer, roughness also has an impact on the skin friction and heat flux along the body.

It typically increases the momentum loss and therefore skin friction above the turbulent level. In the transitionally rough regime, viscous and pressure drag contribute to the momentum loss, whereas, in the fully rough regime, pressure drag dominates. In some cases, special arrangements of regular technical roughness can even lead to a reduction in skin friction under the right circumstances (see e.g. [45]).

Roughness can also augment heat flux above the turbulent level. This increase is typically lower than the corresponding skin friction augmentation by roughness (see also figure 2.15). This is typically explained by the fact that skin friction is mainly augmented by the drag along the roughness elements, whereas the heat transfer augmentation is due to the increased surface area and local secondary flow in the vicinity of the roughness elements, including increased mixing [32]. An alternative explanation is that heat transfer misses an analogue to pressure compared to drag (see [45]). Therefore, it is hard to argue for a direct analogue to scalings via k or the frontal solidity $\lambda_f = k/\lambda$, which is typically used for the form drag. Heat transfer can continue to be sensitive to molecular fluid properties and also dependent on the roughness topology and shape. Sometimes, heat flux augmentations are analyzed as shift of a normalized logarithmic temperature by analogy with the roughness function for the momentum loss (e.g. [181], [45]).

Several studies investigated roughness augmentation effects in incompressible / compressible flow and summaries can be found in e.g. [202], [118], [32], [45].

Fenter [71] performed heat flux measurements via calorimeter for the v-groove roughness at an 90° -angle along a copper cone-cylinder model in a range $Ma = 1.6 - 3.7$. The experimental convective heat flux data $\dot{q}_c$ was presented in the form of Stanton numbers St , which can be derived by

$$St = \frac{\dot{q}_c}{\rho_\infty U_\infty c_p (T_r - T_w)} \quad (2.15)$$

with the inflow density ρ_∞ and velocity U_∞ , the heat capacity of the fluid at constant pressure c_p , the recovery temperature T_r and wall temperature T_w (see also section 3.4.4 for discussion of further formulations). For comparison, Fenter calculated Stanton numbers via Reynolds analogy according to

$$\frac{St}{c_f} = \frac{1}{2} \frac{1}{s_{ra}} \quad (2.16)$$

with s_{ra} as the Reynolds analogy factor (ra), which depends on the state of the boundary layer (see also section 3.11 for further discussion). Fenter assumed a smooth wall Reynolds analogy factor to be still valid (see also section 3.11) and reported about acceptable agreement between experiments and analytical predictions. However, the author admitted that the comparison might be fortuitous due to large uncertainties in the EQSR and Reynolds analogy factor.

For the transverse v-groove roughness in a $Ma = 4.9$ flow by Young [294] ($T_w/T_e = 2.8 - 5.2$ where the upper value corresponds to adiabatic wall conditions), a dependence of the Reynolds analogy factor from the roughness height was found by comparing results from skin friction and heat flux measurements, in contrast to Fenter. For $k/\delta_v = k^+ \lesssim 2$ the Reynolds analogy factor remained the smooth wall value. Above this threshold, in the transitionally rough regime a dependence of $f = (k)$ was reported. For the fully rough regime Young assumed a relatively constant value for large roughness.

Keel [128] reported an asymptotic Stanton number augmentation of $St/St_s = 2.0 - 2.2$ along their sand grain roughness along an internally cooled sharp 5° cone in a $Ma_\infty = 2.5$ ($Ma_e = 2.4$) and $Ma_\infty = 5$ ($Ma_e = 4.7$) flow at non-adiabatic wall condition with $T_w/T_0 = 0.3$. The authors concluded that skin friction coefficients along rough walls exhibit a decisive amount of measurement uncertainty and that the investigated correlations exhibit a varying degree of success of which Fenter's equation looked promising.

Hill *et al.* [101] performed total skin friction measurements via a force balance, velocity profiles via Pitot pressure measurements and heat flux measurements via thermocouples. They used a modification of the Reynolds analogy R_{af} after Owen and Thomson [178] to account for roughness, including a modification for compressibility, to result in heating rates which compared favorably to sand grain roughness data along a sharp and blunt 7° cone in a $Ma = 10$ flow at a non-adiabatic wall condition with $T_w/T_0 = 0.26$. (see section 3.11).

Seidman [223] investigated incompressible and compressible rough wall heat transfer data of different authors (e.g. [71], [294], [128]) and developed an equation to approximate the rough wall Stanton number. It is also based on the Reynolds analogy R_{af} after Owen and Thomson [178] modified for compressible flow and to account for roughness, assuming an exponential viscosity law and neglecting terms only relevant for large values of T_e/T_w (see section 3.11). Based on a partly consistent constant Stanton number shift between experimental data and correlations, they discussed the role of the turbulent Prandtl number and derived a possible correction to include a non-unity and non-constant turbulent Prandtl number throughout the boundary layer.

Holden [104], [105] reported experimental studies in the Calspan shock tunnels on a variety of different smooth and rough wall models and inflow conditions ($Ma_\infty = 6 - 13$, $Re \leq 600 \cdot 10^6$). Overall, they performed local and averaged skin friction (single component strain-gage force balance), heat transfer (silver calorimeter and thin-film platinum gauges) and surface pressure measurements along sharp and blunt wide-angle biconic nosetip models, slender cone models and blunt hemispheric models. They applied sand grain and technical roughness via hemispherical and conical roughness elements to study the effect of roughness height, different shapes and spacings. The inflow conditions and roughness were designed to cover the smooth to fully rough regime. The studies showed that along hemispherical models roughness induced heating augmentation can reach 2 to 4 times the reference smooth wall values, depending on the location of laminar to turbulent transition. Along the slender models the heating augmentation reached close to 2 times the smooth wall for the sand grain roughness. For the technical roughness, the augmentation was largest with maximum 1.5 times the smooth wall value for the most densely packed conical roughness.

An excellent compilation of roughness induced friction and heating augmentation studies in compressible flow was published by Bowersox [32]. Figure 2.15 shows a summary of skin friction

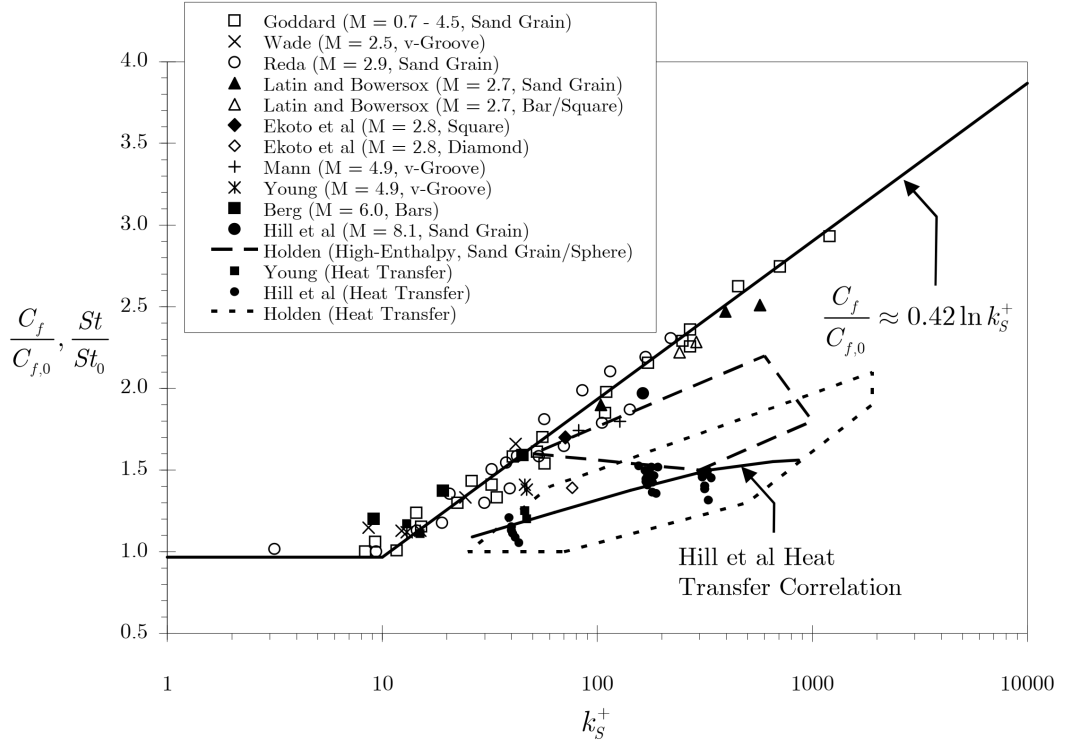


Figure 2.15: Compressible skin friction and heat flux augmentation from different authors according to [32].

($c_{f,0}$ denotes here the smooth reference value) and heat flux augmentation (St_0 denotes here the smooth reference value) against EQSR Reynolds numbers k_s^+ from several authors according to [32].

It is visible that the general trend is a logarithmic increase of friction augmentation with roughness Reynolds number. Most of the data shown in the plot is for adiabatic wall conditions, except for data by Young [294], Hill *et al.* [101] and Holden [104], which encompass heat transfer as well. The adiabatic data seems to be captured by a simple logarithmic approximation. In contrast, cold wall cases typically fall below the prediction, attributed partly to wall-temperature effects, effects of pressure-gradient and entropy swallowing [32], [104], [281]. Another exception are the diamond shape roughness of Ekoto *et al.* [66], [68], which also fall below the skin friction prediction due to its shape and the corresponding effect on compressible shock and expansion wave structure [32].

Additionally, figure 2.15 contains heat flux augmentations from Young [294], Hill *et al.* [101] and Holden [104], [105]. The aforementioned lower augmentation compared to the skin friction is directly visible in the plot. Bowersox [32] used the correlation after Hill *et al.* [101] to rebuild data by Holden [104] with some success.

As introduced in chapter 1, roughness induced heat flux augmentation effects were a concern for the design of the TPS of the Schiaparelli landing capsule within the ESA ExoMars 2016 project.

During previous Mars missions, like the NASA MSL program, typically augmentation factors, so-called bump factors, from empirical correlations (in this case based on Reda *et al.* [203]) were used on smooth wall CFD predictions [62]. Also, during the early phases of the ExoMars project, similar strategies were used, based on the above mentioned Powars correlation (see eq. (3.70) in section 3.11). There were concerns that this approach would utilize empirical correlations far outside its scope of definition in case of a Martian entry. Therefore, it was aimed to move towards CFD capable of including rough-wall effects with the goal to reduce uncertainties in the predictions and with that also TPS margins.

To this end, Fluid Gravity Engineering implemented an EQSR based modified compressible Krogstad model in an algebraic turbulence model [161], [136]. This model, within the TINA CFD code of Fluid Gravity Engineering, was successfully validated within the ExoMars project against heritage data of the PANT (Passive Nostip Technology) program [161]. Additionally, tests were performed at Deutsches Zentrum für Luft- und Raumfahrt e.V. (DLR) to study heating augmentations along different well defined technical roughness topologies [167]. During the high-speed flight phase, boundary layers evolving along capsule surfaces typically have transonic to supersonic boundary layer edge conditions due to the strong bow shock in front of the vehicle (see e.g. [215]). Nevertheless, the boundary layers and therefore also rough wall influences can be prone to compressibility effects like more slender bodies which experience much higher, hypersonic edge Mach numbers [161]. Therefore, with inflow conditions $Ma = 5.3$ at $Re \approx 6 \cdot 10^6 \text{1/m}$ and $Re \approx 16 \cdot 10^6 \text{1/m}$ as well as $Ma = 7$ at $Re \approx 12 \cdot 10^6 \text{1/m}$, Mach number and Reynolds number sensitivities have been tested. Heat flux measurements via infrared thermography were performed and compared to computations utilizing a compressible Krogstad roughness model [161]. The experimental measurements hinted at a compressibility effect with a reduction in heating augmentation with increasing Mach numbers. Numerical rebuildings partly showed substantial discrepancies with large over-predictions which is exemplary visualized in figure 2.16 by the mismatch between experimental symbols and lines from modeling which should agree for corresponding colors representing different roughness topologies. Also the tendencies with Mach number were not captured by the numerical model.

One uncertainty at the time which was discussed as potential reason for this discrepancies, was the EQSR value for the tested models. Since no well-resolved boundary layer data were taken during the hypersonic experiments, EQSR values were taken initially from literature data and later during the course of the project from dedicated incompressible tests [70]. This resulted in a substantial scatter for the EQSR, represented by a value range between approx. $k_s/k = 1$ to 6 for the technical roughness topology with transverse square bars at $\lambda/k = 4$. Already during this phase of the project, possible compressibility effects were discussed with the observation that a lower EQSR, as taken from hypersonic experiments by Berg [24] with $k_s/k = 1.3$, would bring modeling and experiments closer together. But since more confidence was put on the new accompanying incompressible tests which resulted in EQSR toward the upper end of the value spectrum [70] (see also figure 2.8), over-predictions were accepted as additional layer of conservatism, potentially resulting in additional margins for the TPS [167].

More recent work by Hollis [108] and Wilder *et al.* [281], [280] include roughness augmentation investigations on blunt model shapes, enriching the compressible database on actual (re-)entry capsule geometries in different atmospheres. Hollis [108] investigated sand-grain and technical roughness on sphere-cone and hemisphere model geometries at $Ma \approx 6$ wind tunnel flow. Because

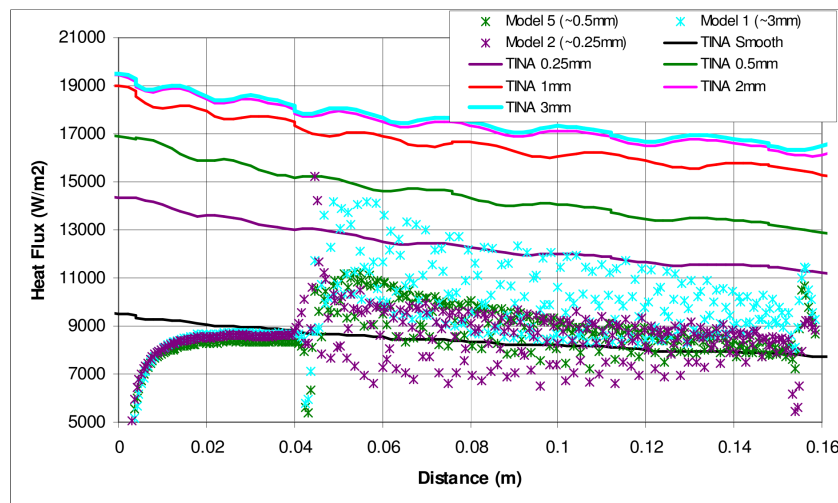


Figure 2.16: Heat flux augmentation rebuilding by compressible Krogstad model compared to experimental data along flat plate at $Ma = 5.3$ $Re \approx 16 \cdot 10^6$ 1/m according to Merrifield *et al.* [161].

of facility limitations, the reference turbulent smooth wall heat fluxes were only derived by CFD computations. The same computations were used to extract the wall viscosity and friction velocity at isothermal wall conditions. The authors chose to define the roughness Reynolds number in terms of the smooth wall conditions, using this extracted data. Augmentation of turbulent levels up to 1.9 were extracted. Although it needs to be mentioned that augmentations were calculated based on the absolute heat flux and not Stanton number levels, so that a potential dependence on wall temperatures persisted. Wilder *et al.* [281] analyzed heat flux augmentation on different sphere-cone front-shield shapes with sand-grain roughness in a free flight ballistic range ($Ma \approx 7.8 - 10$, $Ma_e \approx 1.2 - 1.8$) with air and CO_2 as test gas. Like Hollis, friction velocities were only simulated via CFD along smooth walls, which were then used in the roughness Reynolds number formulation. Although based on absolute heat flux values and scaled by the smooth wall friction velocities, the authors argue that augmentation factors up to 1.8 showed good agreement with the Powars correlation, independent of the test gas (see section 3.10.1). A follow-up study in the same facility and measurement setup extended the investigations to roughness representative of 3D woven heat shield materials [280].

Very limited studies exist which use DNS computations to investigate roughness heat flux augmentations. A DNS study of square bar roughness along one wall of an incompressible turbulent channel flow indicated also a break down of the Reynolds analogy in case of the roughness [147]. The ratio of heat flux to skin friction falls below unity along rough walls due to the increase of form drag. The authors also showed that for the total heat flux, build by the convective part due to the mean flow temperature gradient and the turbulent part by the product of temperature and velocity fluctuations, is dominated by the turbulent part. This turbulent heat flux is enhanced by high-intensity ejections occurring near the leading edge of the roughness elements [147]. The spacing between consecutive elements, described by λ/k , controls the flow reattachment along the channel walls upstream of the next roughness element and with

that the strength of stagnation point along the windward side of this element. This increases the flow ejection and therefore both the turbulent and total heat flux. The maximum within $2 \leq \lambda/k \leq 15$ was detected at $\lambda/k = 7.5$, where also the form drag is largest. It needs to be mentioned, that the blockage factor is non-negligible with $k/h = 0.1$, with h being the channel half-width.

2.5 Roughness Topologies

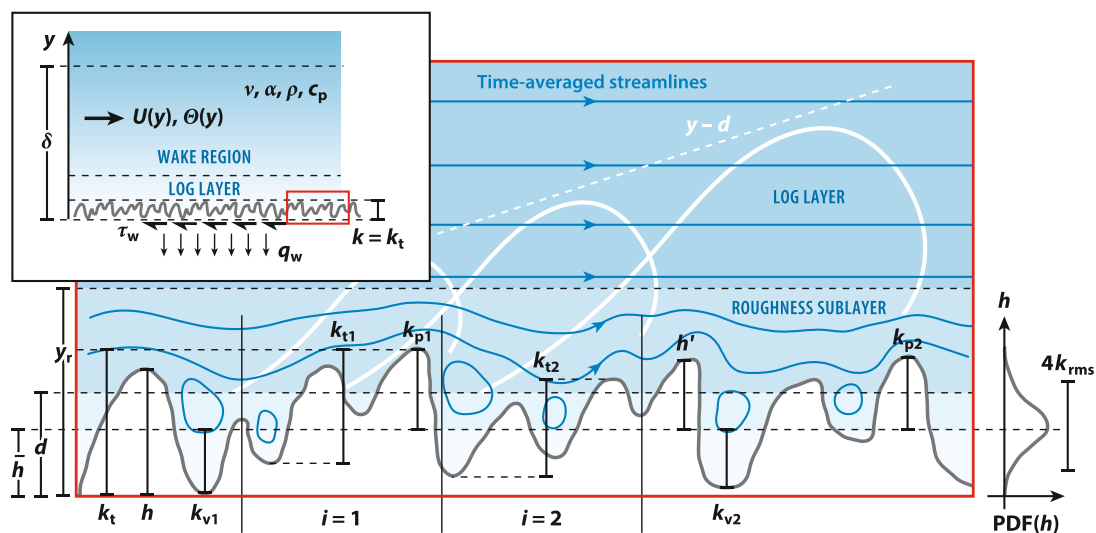
Depending on the type of roughness, the relevant length scales S (see e.g. eq. (2.5) or (2.7)) can represent multiple different parameters. There is no consensus which are the most relevant ones. Chung *et al.* [45] stated three groups of parameters relevant to characterize roughness topologies which represent 1) the roughness height, 2) frontal solidity, effective slope or roughness density parameter and 3) plan solidity, skewness or solid volume fraction. These parameters are introduced in the following.

For the first group 1), the roughness height k is clearly defined for technical roughness but can be challenging for distributed (real) roughness. Possible values are the average roughness height $k_a = 1/A_t \int |h'| dA$ where h' is the variation in roughness elevation about the mean $\bar{h}$ and A_t is the total plan area on which the roughness is located (see figure 2.17a and 2.17b). Another possible value is the root-mean-square roughness height $k_{rms} = \sqrt{1/A_t \int |h'|^2 dA}$ or the minimum peak-to-trough roughness height $k_t = \max(h') - \min(h')$. Instead of deriving the previously stated values for a complete surface, one can extract the mean value for sub-samples of the roughness as k_z . This avoids local outliers in e.g. $\min(h')$ or $\max(h')$ from skewing the value for k_t .

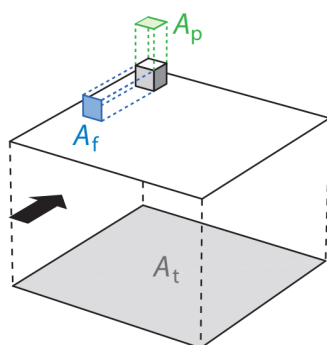
For the second group 2), the frontal solidity $\lambda_f = A_f/A_t$ is defined by the ratio of frontal projected area of roughness elements A_f to the total plan area (see also figure 2.17b). This value characterizes the available area exposed to pressure drag [45]. Based on the frontal solidity different regimes for roughness element densities can be defined. $\lambda_f \rightarrow 0$ define a sparsely packed or isolated roughness element regime where the flow (and with that also ΔU^+) will tend toward smooth conditions. On the other end of the spectrum, large values of λ_f define a densely packed roughness elements regime. In this regime individual elements will experience some sheltering due to their proximity to neighboring elements. Therefore, also for very dense roughness the flow tends toward smooth conditions. In between both regimes for $0.1 \lesssim \lambda_f \lesssim 0.3$ there exists a regime where the influence of roughness peaks. For 2D technical roughness the frontal solidity is simply the ratio of element height to wavelength (or pitch) $\lambda_f = k/\lambda$. Care needs to be taken since some authors use the symbol λ for the reciprocal value of the solidity $1/\lambda_f$ (e.g. [25], [60]). The effective slope $ES = 1/A_t \int |\partial h'/\partial x| dA$ is giving the mean absolute streamwise gradient of the roughness elements. It can be connected to the frontal solidity by $\lambda_f = ES/2$. The roughness density parameter is defined by $\Lambda_s = \lambda_f^{-1} (A_f/A_w)^{-1.6}$ where A_w is the total windward wetted surface area and has been suggested by Sigal and Danberg [226]. It is visible that the frontal solidity, effective slope or roughness density parameter are connected and describe similar characteristics, which is the reason for grouping by Chung *et al.* [45].

For the third group 3), the plan solidity is defined by $\lambda_p = A_p/A_t$ where A_p is the plan area of roughness elements (see also figure 2.17b). It can give an improved measure of the roughness form. For $\lambda_p \rightarrow 1$ roughness topologies are densely packed whereas $\lambda_p \rightarrow 0$ indicate sparse roughness arrangements. For 2D technical roughness the plan solidity is simply the element width to pitch (or wavelength) $\lambda_f = w/\lambda$. The skewness is defined by $SK = (1/A_t \int |h'^3| dA) / k_{rms}^3$. It indicates the asymmetry in surface-elevation distribution. The solid volume fraction can be interpreted as a generalization of the the area-based λ_p to a parameter based on volume and is typically used in literature referring to atmospheric boundary layer studies [45]. For 3D roughness topologies it might be relevant to have information about clustering and sheltering of elements as well.

All of the above stated roughness topology parameters (and more) have been suggested to be decisive for the characterization of roughness. There have been multiple attempts to find the EQSR k_s just based on topology parameters and to find a formulation like $k_s/k = f(S)$ (e.g. [212], [60], [232], [53], [226], [118], [153]) but albeit many suggestions in the last decades, no conclusive options have been found yet.



(a)



(b)

Figure 2.17: a) Sketch of distributed roughness topology embedded in a TBL with different geometric definitions and b) sketches of roughness element to show relevant geometric definitions for frontal and plan solidity according to [45].

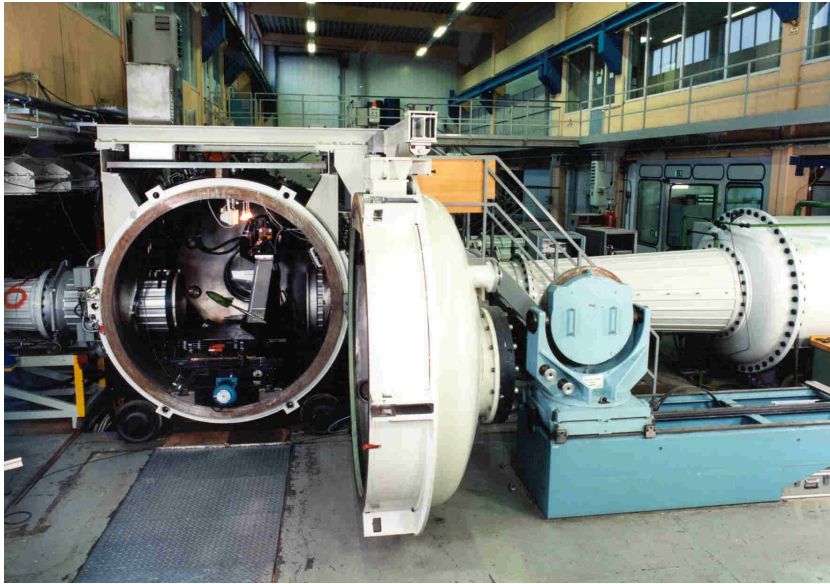
3 Methods

In this section, the different measurement techniques and post-processing techniques are presented. First, the experimental setup with the used wind tunnel and model are described (section 3.1). After that, details about the different measurement techniques are given for pressure measurements (section 3.2), Particle Image Velocimetry (section 3.3), Infrared thermography (section 3.4), Thermal sensors (section 3.5) and Schlieren system (section 3.6). Then, supporting predictions via numerical methods, ie. a boundary layer code (section 3.8) and CFD code (section 3.9), and analytical methods are described (section 3.10, section 3.11). Finally, methods and analysis are reported about the extraction of EQSR. Information about the different measurement uncertainties can be found in section 7.1.1.

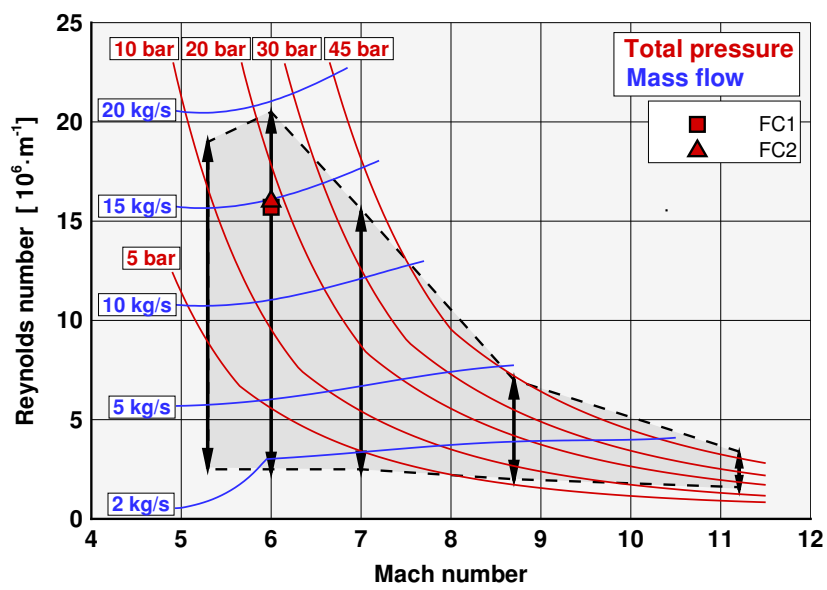
3.1 Wind Tunnel and Model Properties

Wind tunnel:

Experiments have been performed in the DLR hypersonic wind tunnel (Hypersonic test section Köln (H2K)) in Cologne. The facility is an intermittently working blow down tunnel with a free jet test section (see figure 3.1a). Depending on the flow condition, test durations up to 30 seconds can be achieved. The facility can be equipped with five exchangeable contoured nozzles with an exit diameter of 600 mm, i.e. $Ma = 5.3, 6.0, 7.0, 8.7$ and 11.2 . To avoid air condensation as well as to operate the facility at high stagnation temperatures, electrical heaters with a capacity of up to 5 MW are integrated upstream of the nozzle. Unit Reynolds numbers Re between $2.5 \cdot 10^6$ and $16 \cdot 10^6 m^{-1}$ can be set by varying the reservoir pressure p_0 and reservoir temperature T_0 . The range of achievable Reynolds numbers at the five discrete Mach numbers can be seen in the performance map in figure 3.1b. The nominal Mach and Reynolds combinations used in this work are also included in the map.



(a)



(b)

Figure 3.1: H2K with a) photo of nozzle, test chamber and diffuser, b) performance map.

Inflow Conditions and Model:

Previous investigations in H2K within the ExoMars project used a flat plate model (see [161], [170], [167]). To enable transition to a turbulent boundary layer along the model, an angle of attack as well as a tripping device were necessary. Some open questions remained on the amount of influence of the tripping.

To avoid these challenges with the tripping device (see also reports in [285]), a sharp right circular cone was chosen, based on previous experience with successful natural transition in the framework of other projects (see e.g. [283]). A drawback of natural transition is that the transition location is more sensitive to e.g. inflow conditions like Re . Two potential half opening angles of 3° and 7° were initially discussed. To decide between the two model options, initial approximations of the achievable roughness Reynolds numbers k_s^+ were performed with calculations via the analytical model according to Fenter [71] (see section 3.10) with the assumptions of an equivalent sand grain roughness of $k_s/k = 2$ and wall temperature of $T_w = 300\text{K}$. The 7° cone model was chosen, since higher roughness Reynolds numbers could be achieved (see figure 3.2a). Additionally, figure 3.2a contains the results for different H2K inflow conditions and it is visible that an inflow Mach number of $Ma_\infty = 6$ resulted in the highest achievable Reynolds number and was therefore chosen as nominal inflow.

The same analytical model by Fenter [71] was used to guide the choice for reservoir pressure p_0 and temperature T_0 . At a position $x_a = 0.5\text{m}$ (according to PIV Field of View FOV1, see section 3.2 and 3.3) of the 7° cone model, the achievable roughness Reynolds number k_s^+ and blockage ratio k/δ were extracted as relevant output parameters, according to Jiménez [118]. In figure 3.2b a map of the achievable k_s^+ (contours) and k/δ (lines) are shown for the complete operation range of the H2K at $Ma_\infty = 6$. The boundaries of the operating range are defined by minimum reservoir pressure (left), maximum temperatures (upper), maximum heating power (upper right), maximum mass flow (lower right) or risk of condensation of Oxygen / Nitrogen (lower). It is visible that increasing the pressure increases the achievable roughness Reynolds number k_s^+ , whereas increasing the temperature increases the achievable blockage ratio k/δ . Values in the order of $\mathcal{O}(100)$ and $\mathcal{O}(0.1)$ can be achieved for k_s^+ and k/δ , respectively. Since the achievable blockage ratio changes only marginally within the operating range, a nominal Flow Condition (FC) 1 was chosen with $p_0 = 20\text{bar}$ and $T_0 = 500\text{K}$ to maximize the achievable Reynolds numbers, considering heating limitations at that time. Due to infrastructure refurbishments of the H2K facility during the second test campaign, a second flow condition FC2 was defined, aiming at a similar Reynolds number as FC1 with $p_0 = 18.9\text{bar}$ and $T_0 = 475\text{K}$.

Table 3.1 contains both flow conditions FC1 and FC2 used in all wind tunnel campaigns (also included as symbols in figure 3.1b and 3.2b) with details about the Mach number Ma , unit Reynolds number Re , reservoir pressure p_0 , reservoir temperature T_0 and velocity U . The values are reported for the inflow (condition at nozzle exit, upstream the bow shock in front of the cone) and boundary layer edge condition (downstream the bow shock). All tests were performed at an angle of attack of zero. The inflow Mach number is derived from nozzle calibrations, including viscous effects, depending on the Reynolds number. The reservoir conditions were directly measured whereas the static conditions of the inflow were calculated via gas dynamic equations (e.g. [241]). The nominal edge conditions were derived by the Taylor-Maccoll equation (see section 3.7 or e.g. [9]), assuming negligible influence of a thin boundary layer.

Table 3.1: Nominal flow conditions FC1 and FC2

Flow cond.	state	Ma [—]	Re [$10^6 1/m$]	p_0 [bar]	T_0 [K]	U [m/s]
FC1	inflow	6.06	15.7	20.0	500	940
	edge	5.40	20.5	19.8	500	926
FC2	inflow	6.06	16.0	18.9	475	916
	edge	5.40	21.0	18.7	475	903

A complete test matrix of valid runs can be found with table 7.1 in section 7.1.

The model consisted of three exchangeable segments, i.e. a nose-, middle- and aft-segment. The first segment consisted of a sharp metallic nose with a radius in the order of 0.1 mm. The middle and one smooth plus one rough wall aft segment were made of PolyEther-Ether-Ketone (PEEK), which is a colorless organic high-temperature polymer thermoplastic. It is a standard material for heat flux investigations in H2K due to detailed knowledge of the temperature dependent material properties (see section 3.4.1). Additionally, the material has a low thermal conductivity, which reduces lateral heat transfer (see chapter 3.4.2). The material thickness of PEEK was chosen to be at least 10 mm (see corresponding analysis in section 3.4.2). To separate effects of roughness induced transition and roughness induced turbulent augmentation effects, a turbulent boundary layer approaching the roughness elements needed to be realized. Previous test campaigns on 7° sharp cones showed, that at the chosen inflow conditions, the transition front established at a running length based Reynolds number which is reached at the middle segment of the current cone model [283]. Therefore, it was decided to apply the surface roughness only along the aft segment.

One smooth wall aft segment and two different rough wall aft segments were manufactured, one with a technical roughness made from PEEK and one with a sand grain type roughness made from metal. A smooth wall aft segment was used to extract reference data. The technical roughness pattern was based on previous measurements within the ExoMars campaign [167]. It consisted of a transverse square bar pattern (TSB) with a wavelength λ to width w or height k ratio of nominally $\lambda/w = (w + i)/w = (w + i)/k = 4$, with the bar interval i . The nominal bar height k and width w was 0.5 mm (see Fig. 3.3). The top of each square bar was aligned with the smooth part of the model, comparable to studies by e.g. [90], [24], [33]. The roughness pattern begins at $x_a = 0.326$ m ($x_a/L = 0.44$) and ends at $x_a = 0.65$ m ($x_a/L = 0.89$), so that approximately the last 80 mm of the rough wall aft segment exhibits also a smooth wall (originally designed to include local sensors). The total length of the model was $L = 0.73$ m with an end diameter of $D = 0.18$ m.

An additional metallic aft segment with a smooth surface was manufactured and later equipped with silicon carbide (Carborundum) grains to simulate a sand grain type roughness topology. The grains were sieved first with a mesh opening of 0.7 mm and afterwards with 0.5 mm to realize a nominal grain diameter of 0.5 mm. After sieving, a thin glue layer was applied to the model and the grains were applied to achieve a dense roughness layer. The streamwise beginning and end of the grain roughness layer resembled the one of the corresponding technical roughness, whereas only one half of the cone in circumferential direction was covered (see figure 3.5b). After

a proper waiting time, the glue was dried and the grains were fixed to the model and ready for testing.

Surface roughness and outer mold lines of the different models were checked after manufacturing via an optical chromatic distance measurement device MicroSpy[®] Mobile by the company FRT, which has an accuracy typically below $2\ \mu\text{m}$. The roughness of the smooth PEEK parts were taken along multiple line profiles and resulted in an inner scaled roughness of $R_{max}^+ \approx 1.1$, which is well below a value of 5, typically stated as upper limit of an hydraulically smooth surface [212].

For topology measurements, several $20 \times 20\ \text{mm}^2$ sectors were optically measured (see also section 7.1.3) and statistically evaluated (after proper post-processing to consider the conical contour). The sectors were distributed at four circumferential positions at two streamwise locations along the cone, corresponding to the PIV measurement Field of Views of interest FOV1 and 2 (see section 3.2 and 3.3). Additionally, for the TSB aft model segment calibrated PIV images can be exploited to extract the contour. Scans and PIV data suggest that the roughness changes slightly along the axial direction. Along the upstream part (FOV1), values of typically $k = 0.46 \pm 0.03\ \text{mm}$, $b = 0.51 \pm 0.02\ \text{mm}$, $i = 1.49 \pm 0.02\ \text{mm}$, $\lambda = 2.00 \pm 0.02\ \text{mm}$ and $\lambda/k = 4.34 \pm 0.34\ \text{mm}$ were measured from scans. Along the downstream part (FOV2) slightly lower values for k were measured. This tendency was also visible in the PIV images with $k = 0.5 \pm 0.01\ \text{mm}$ at FOV1 and $k = 0.43 \pm 0.01\ \text{mm}$ at FOV2. Although it needs to be mentioned that the reported uncertainties for k are of the same magnitude as the resolution uncertainty of the PIV setup, so that corresponding values might be larger.

In case of the sand grain type rough aft model segment, a multitude of statistical properties of distributed roughness is potentially relevant, as described in section 2.5. In this thesis, parameters according to Bons [28] were chosen, which studied realistic roughness on turbine blades. Bons used the mean height k_z of the minimum peak-to-trough roughness height $k_t = \max(h') - \min(h')$ for several subsets of data as estimate for the mean roughness height k . Here, k_z was derived for every suitable measurement sector and the resulting median of $k_z \approx k = 0.45\ \text{mm}$ was used as representative scale. Additional statistics can be found in section 7.1.3.

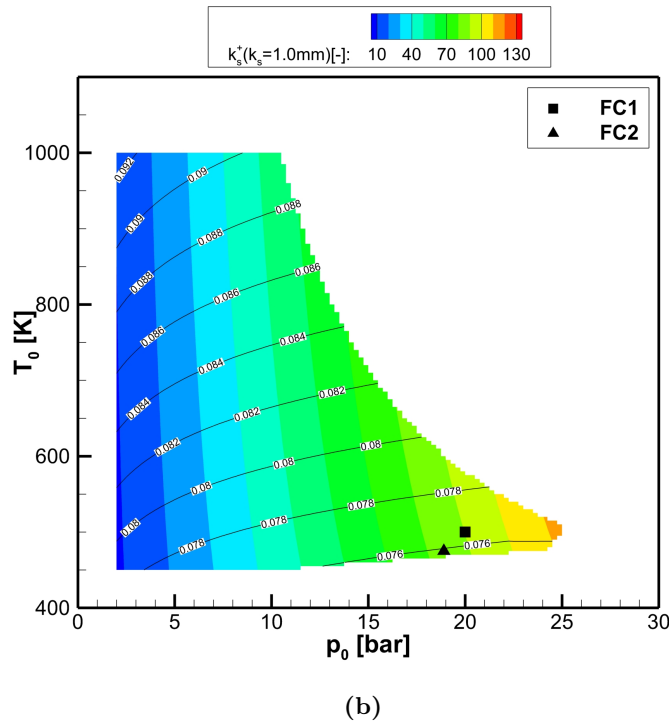
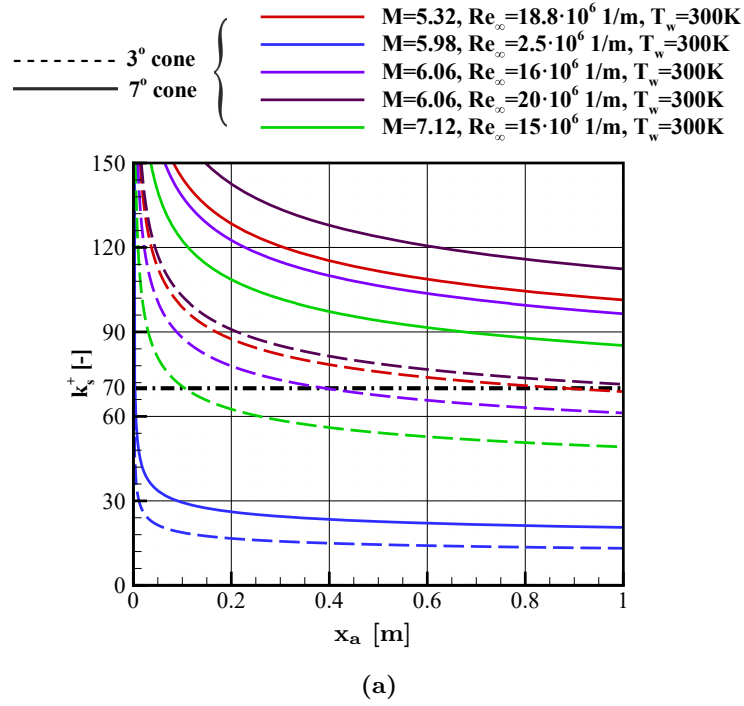


Figure 3.2: Estimations via correlation by Fenter [71] with $k_s = 1 \text{ mm}$ and $T_w = 340 \text{ K}$ showing a) Roughness Reynolds numbers k_s^+ against axial distance x_a for different inflow conditions and cone angles $\theta = 3^\circ$ and $\theta = 7^\circ$, b) H2K condition map for a 7° cone at $x_a = 0.5 \text{ m}$ at $Ma_\infty = 6$ with the achievable k_s^+ (contours) and k/δ (lines) together with FC1 (square) and FC2 (delta).

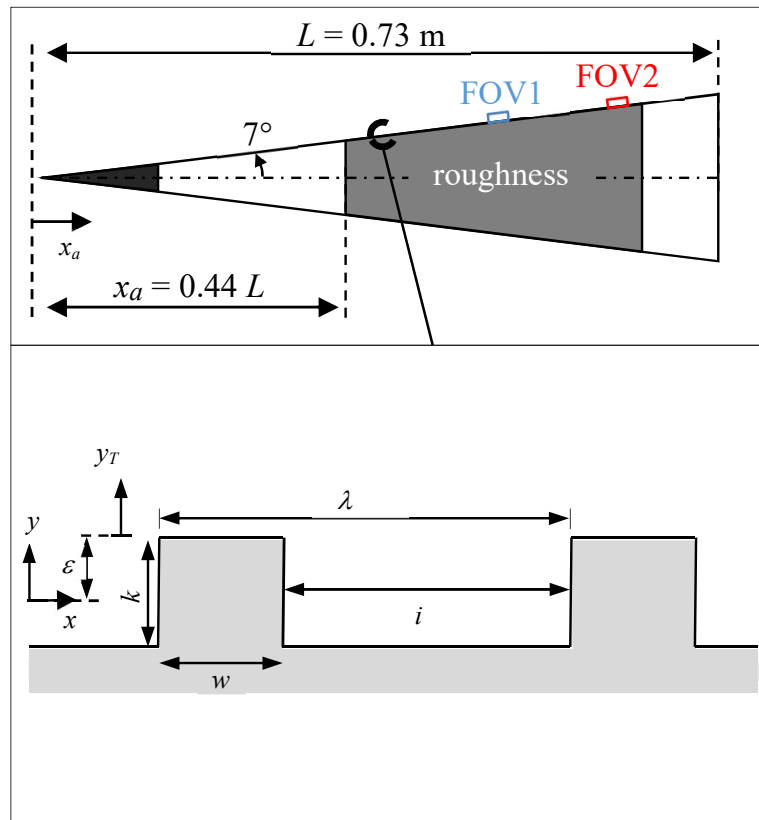
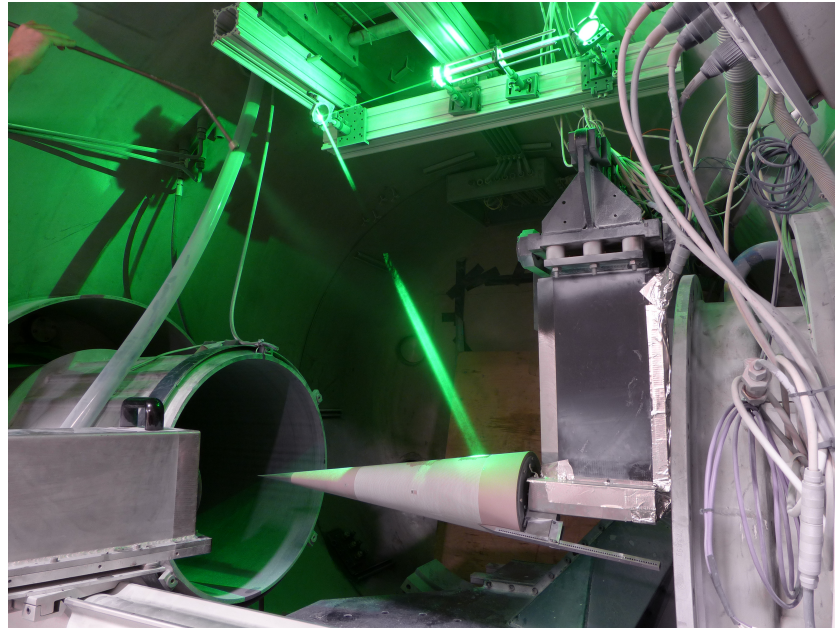


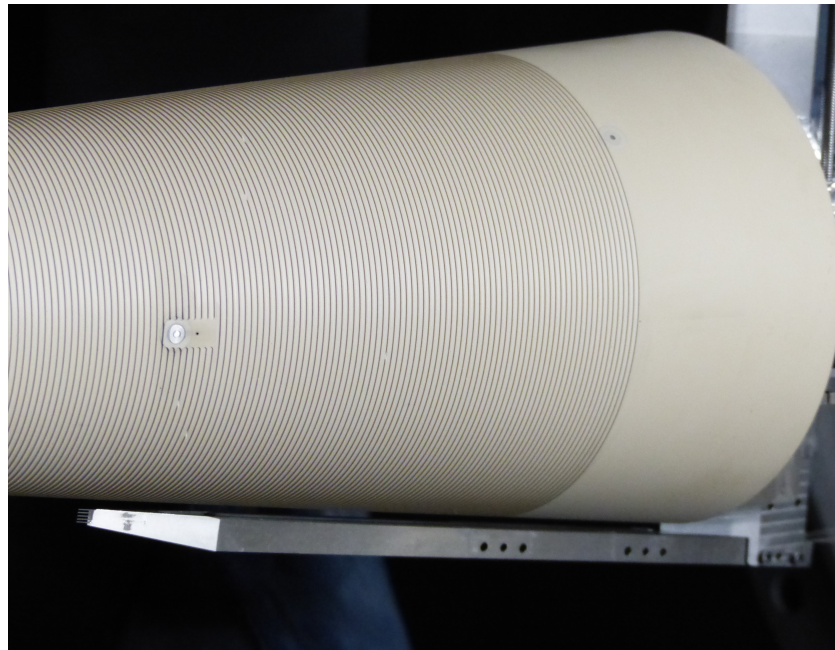
Figure 3.3: Definition of model and technical roughness pattern

Set-Ups:

The model was fixated by a sting configuration and placed in a short distance downstream of the wind tunnel nozzle exit (see figures 3.4 and 3.5). This enabled the positioning of different camera set-ups for the different wind tunnel campaigns. For the first Particle Image Velocimetry campaign the PIV camera was installed in the horizontal plane outside of the test section (see figure 3.4a) and section 3.3). Additionally, two infrared (IR) cameras were installed with one view from top and one from the side (see section 3.4). For the second PIV campaign the PIV cameras were installed in the horizontal plane inside the test section in a sealed box (see figure 3.5 and section 3.3). Also here, two infrared (IR) cameras were installed with one view from top of the nozzle and one from top outside the test section (see section 3.4). The details of the Pitot probe set-up are given in the following section 3.2.

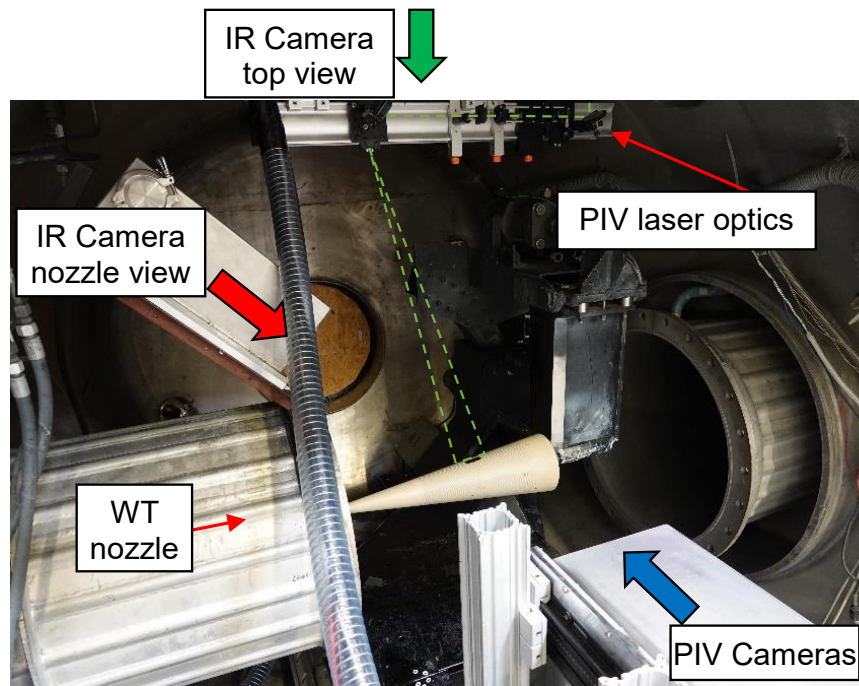


(a)

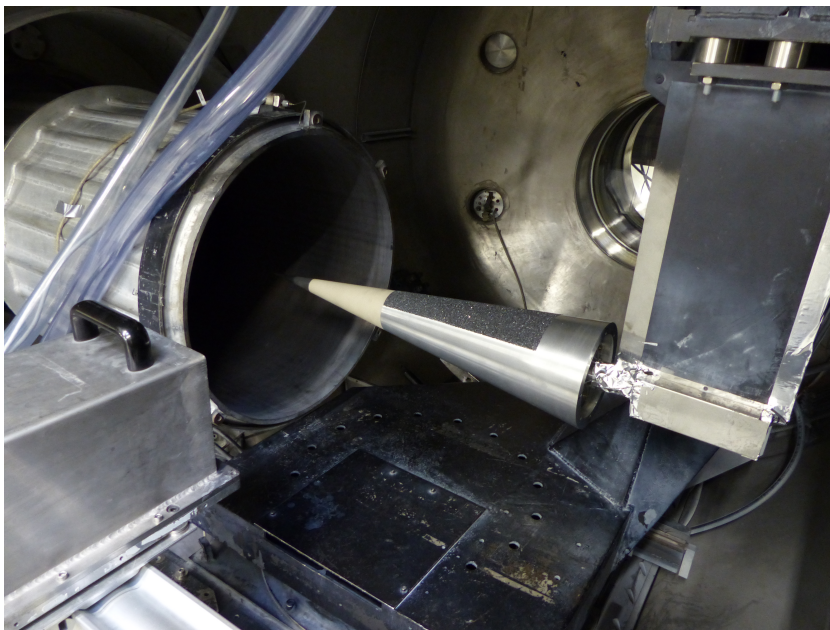


(b)

Figure 3.4: Test setups of different test campaigns with a) first PIV campaign, b) Pitot pressure campaign.



(a)



(b)

Figure 3.5: Test setups of different test campaigns with a) second PIV campaign, b) Sand grain type roughness configuration in Pitot pressure campaign.

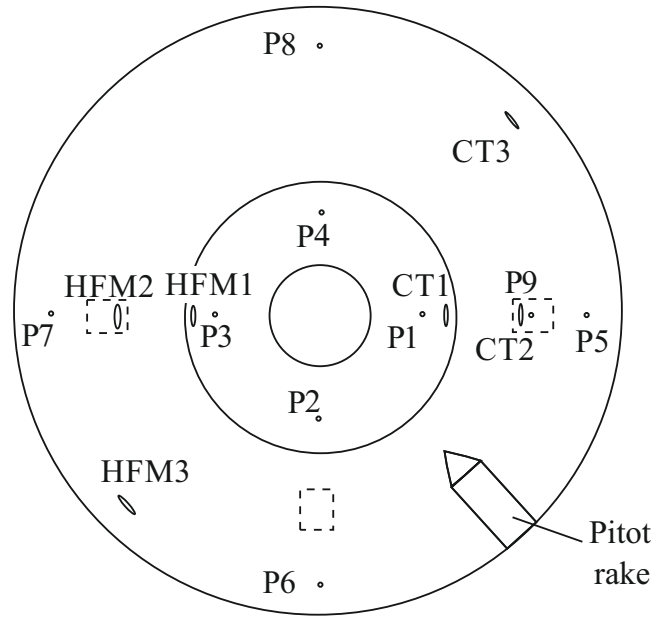


Figure 3.6: Model instrumentation with view from nozzle to diffuser (P: positions of pressure ports, HFM: positions of HFM sensors, CT: positions of coaxial thermocouples, Pitot rake: Pitot rake position if applicable, dashed boxes: sensor plateaus in case of rough segment).

3.2 Pressure Measurements

3.2.1 Surface Pressure

The wind tunnel model was equipped at nine different positions with pressure holes perpendicular to the surface with a diameter of 0.5 mm. These pressure taps were distributed along three different axial sections along the model, at $x_a = 0.25, 0.495$ and 0.651 m. In the first and last section four different taps were positioned with an angular distance of 90° (see figure 3.6 and table 3.2). This set-up enabled the analysis of model alignment and pressure gradients along the surface. The taps were connected via steel and flexible tubing to a miniature Electronic Pressure Scanner (ESP) outside the wind tunnel model. ESP scanners are miniature electronic differential pressure measurement units consisting of an array of silicon piezoresistive pressure sensors. The reference pressure was kept constant in an additional metallic vessel outside the test section. The module had a measurement range of 5 psi (34.5 kPa). In case of the mid-axial location of the rough wall section, small smooth wall plateaus (rectangular cross-section with 8×16 mm²) were foreseen (see figure 3.4b or section 3.5 for more details).

Table 3.2: Nomenclature for the sensors and positioning

Sensor name	Abbreviation	x_a [mm]	ϕ [°]
Pressure tap, middle segment	PT1	250	90
Pressure tap, middle segment	PT2	250	180
Pressure tap, middle segment	PT3	250	270
Pressure tap, middle segment	PT4	250	0
Pressure tap, end segment, within roughness	PT9	495	90
Pressure tap, end segment	PT5	651	90
Pressure tap, end segment	PT6	651	180
Pressure tap, end segment	PT7	651	270
Pressure tap, end segment	PT8	651	0
HFM sensor, non-coated, middle segment	HFM1	300	270
HFM sensor, coated, end segment	HFM2	488.5	270
HFM sensor, coated, end segment	HFM3	654	225
Coaxial thermocouple, middle segment	CT1	300	90
Coaxial thermocouple, end segment	CT2	488.5	90
Coaxial thermocouple, end segment	CT3	654	45

3.2.2 Pitot Pressure

The Pitot pressure was measured with a movable rake, consisting of five tubes, each with an inner and outer diameter of 0.2 mm and 0.4 mm, respectively. The distance between subsequent tubes was 1 mm (see figure 3.8 and 3.9). This multiple tube concept minimized the testing time for a highly resolved boundary layer profile. The mechanics of the rake was designed to realize positions between $x = 494 - 635$ mm parallel to the wall and $y = 0 - 10$ mm normal to the wall. The x -position was fixed during a test run but in wall-normal direction the rake was traversed with a linear actuator *L4118L1804 - T6X1* from Nanotec Electornic GmbH & Co. KG (see figure 3.7). The motor was operated by a Nanotec SMCI33-1 controller which was controlled by a PC running a LabView program. With the program, tailored movement sequences could be programed for each run. The position of the probe was monitored with a Nanotec WEDL5546 encoder. The actuator has a minimal step resolution of $5\text{ }\mu\text{m}$ and a maximal speed of 20 mm/s. These parameters fulfilled the requirement for a design wall normal resolution of $\Delta y = 0.1$ mm to result in approx. 10 points within the logarithmic portion of the boundary layer at nominal inflow conditions and test times. The rake movement during test time was stepwise with corresponding settling times to ensure a sufficient filling of the tube volumes. Pressure and step count were recorded via two different acquisition systems. Pressure data was recorded at 3 Hz and the step count at 40 Hz sampling frequency. In a post processing step both data streams were first synchronized via recorded TTL trigger and afterwards harmonized to 3 Hz. Afterwards, the acceptable test time with constant flow conditions was identified based on the gradient limit of the total pressure signal. This limit was identified based on a test at a constant rake position. A gradient limit was chosen that the corresponding pressure value agrees to 99.9% of the end pressure value at the constant position. Non converged pressure points were omitted. With this procedure typically 10 positions could be realized during one run, which resulted at a step-size of

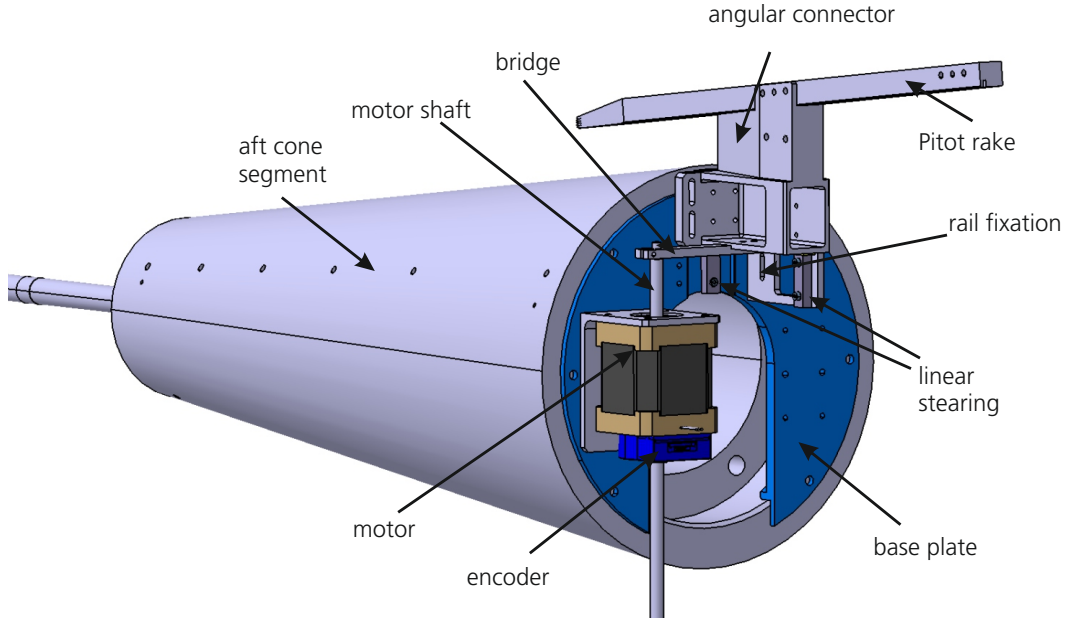


Figure 3.7: Pitot rake set-up CAD

$\Delta y = 0.1$ mm in 1 mm/run. At a typical boundary layer thickness of $\delta \approx 4$ mm a total of at least 5 runs would have been necessary with a single probe device to capture a complete boundary layer including the identification of the boundary layer edge. By applying the Pitot rake usually two runs were sufficient. The impact of potential interaction problems between the different tubes of the rake was tested with measurements via an additional single tube device (see figure 7.23 in section 7.4.1) and the differences were found to fall within measurement uncertainties (see section 7.1.1).

The Pitot tubes were connected via steel and flexible tubing to an ESP scanner with a measurement range of 15 psi (103.4 kPa). Two main measurement sections at $x = 494_a$ mm and $x_a = 635$ mm were defined according to the PIV FOVs (see section 3.3 and 4).

Due to the intrusive nature of a Pitot probe measurement, potential corrections were investigated and influences from viscous effects and curved shock effects were found to be negligible under current test conditions (see Appendix 7.4).

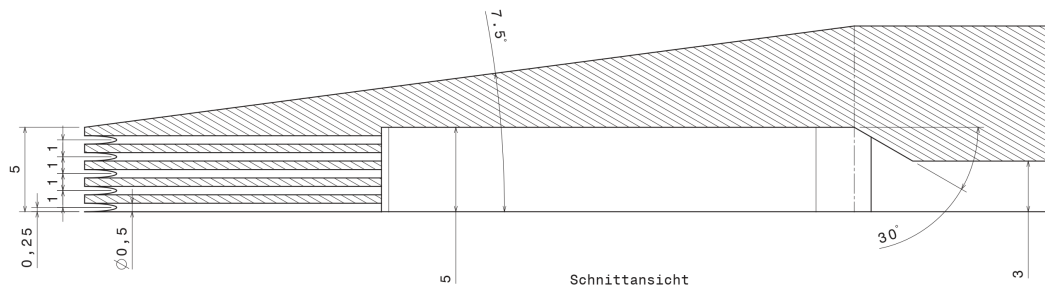


Figure 3.8: Pitot rake main dimensions

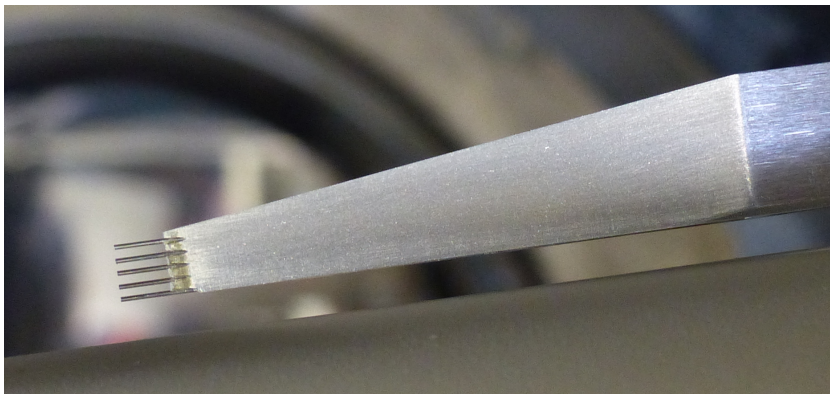


Figure 3.9: Pitot rake photography

3.2.3 Post Processing

Besides the aforementioned lag-time identification, several other post-processing steps were necessary, described in the following. Two wall distance corrections were applied. Firstly, small changes of the tube-to-tube distances in the order of below 0.1 mm due to small deformations were monitored via photographs in front of millimeter paper in between runs and included in the analysis. Secondly, at the first measurement section ($x = 494$ mm) small deviations of the wall normal distance in the order of 0.1 mm were noticed. This was attributed to a small bending of the rake system. Therefore reference measurements with fixated rake were used to identify the amount of Δy and corrected accordingly. Furthermore, to account for small run-to-run inflow differences, the pressure data of each run was first transformed to pressure coefficients via $c_p = (p_{Pitot} - p_\infty)/q_\infty$, with the corresponding inflow values of pressure p_∞ and dynamic pressure q_∞ for each run. Finally, a corresponding p_{Pitot} was re-calculated with the corresponding nominal inflow values of p_∞ and q_∞ and then combined to a complete profile from several runs.

The measured data was transformed from Pitot pressure to velocity profiles applying several assumptions. First, the pressure was transformed to a Mach number profile using the Rayleigh Pitot equation (see e.g. [241]):

$$\frac{p_{02}}{p_1} = \left[\frac{(\gamma + 1)Ma_1^2}{2} \right]^{\frac{\gamma}{\gamma-1}} \left[\frac{(\gamma + 1)}{2\gamma Ma_1^2 - (\gamma - 1)} \right]^{\frac{1}{\gamma-1}} \quad (3.1)$$

This equation assumes adiabatic flow of a perfect gas upstream of the Pitot tubes with 1 and 2 indicating the condition upstream and downstream of the normal shock. In this equation p_{02} corresponds to the measured Pitot pressure p_{Pitot} and p_1 to the corresponding static pressure. The Pitot rake was positioned in such a way, that a surface pressure measurement in close proximity of the Pitot tube was used to account for p_1 . It was assumed that the measured surface pressure is not affected by the bow shock from the nearest tube, and also the pressure throughout the boundary layer was assumed constant. With these assumptions, the Mach number in front of the shock wave, Ma_1 , could be obtained iteratively by applying eq. (3.1).

In order to obtain the velocity U from the Pitot Mach number profile, the Mach number definition can be utilized:

$$U = Ma \cdot a = Ma \cdot \sqrt{\gamma RT} \quad (3.2)$$

where a is the speed of sound in the fluid, R is the specific gas constant and γ is the heat capacity ratio, assumed to be $\gamma = 1.4$. With known Mach number and gas properties, the velocity can be obtained from this equation if a suitable temperature profile through the boundary layer is found.

For this study different temperature correlations were implemented and compared. Firstly, if a constant total temperature throughout the boundary layer is assumed, neglecting any viscous losses, a temperature profile might be calculated from isentropic relations with the Mach number from Pitot pressure (eq. (3.2)) as input from (see e.g. [241]):

$$T = T_0 \left(1 + \frac{(\gamma - 1)}{2} Ma^2 \right)^{-1} \quad (3.3)$$

Secondly, the Crocco-Busemann-temperature distribution was applied, which assumes $Pr = 1$, $dp/dx = 0$ and $c_p = \text{const.}$ for its derivation (see e.g. [277]). Although strictly valid only for laminar boundary layers with zero pressure gradient and constant wall temperature, it proved to transform also compressible turbulent boundary layers (see e.g. [73]) and was already used in a number of different studies including some considering rough walls (see e.g. [24], [57], [71], [44]). A general temperature-velocity formulation can be given with

$$\frac{T}{T_w} = \frac{\rho_w}{\rho} = 1 + A_m \frac{U}{U_e} - B_m \left(\frac{U}{U_e} \right)^2 \quad (3.4)$$

where the parameters A_m and B_m can be defined for the Crocco-Busemann correlation with

$$A_{Crocco} = \left(1 + M_e^2 \frac{\gamma - 1}{2} \right) \frac{T_e}{T_w} - 1 \quad (3.5)$$

$$B_{Crocco} = M_e^2 \frac{\gamma - 1}{2} \frac{T_e}{T_w} \quad (3.6)$$

To approximate a Prandtl number other than unity, Walz [270] introduced the recovery factor to include the effect of dissipative losses in case of adiabatic decelerations in real flows. The recovery factor is defined by

$$r = \frac{T_{aw} - T_e}{T_{0,e} - T_e} \quad (3.7)$$

with the total temperature at the boundary layer edge $T_{0,e}$ and the the adiabatic wall temperature T_{aw} , which was estimated in this thesis by the recovery temperature T_r defined by

$$T_{aw} \approx T_r = T_e \left(1 + r \frac{(\gamma - 1)}{2} Ma^2 \right) \quad (3.8)$$

The recovery factor in case of turbulent flows can be estimated by $r = Pr^{1/3} \approx 0.89$. Incorporating the idea of a recovery factor for the temperature-velocity correlation after Walz, the constants in eq. (3.4) are defined by

$$A_{Walz} = \left(1 + r M_e^2 \frac{\gamma - 1}{2} \right) \frac{T_e}{T_w} - 1 \quad (3.9)$$

$$B_{Walz} = r M_e^2 \frac{\gamma - 1}{2} \frac{T_e}{T_w} \quad (3.10)$$

Lastly, a modified Crocco temperature distribution was implemented which includes a correction term to account for small deviations due to cold wall conditions based on DNS data according to Duan and Martin [58]. The corresponding constants in eq. (3.4) are defined by

$$A_{Duan} = A_{Walz} \cdot 0.8259 \quad (3.11)$$

$$B_{Duan} = B_{Walz} - A_{Walz} \cdot 0.1741 \quad (3.12)$$

where the coefficients were found by a fit to DNS data at $Ma_e = 5$ and within $T_w/T_{aw} = 0.18$ – 0.68 , which are close to the values of this thesis.

For the three latter distributions, an edge temperature is necessary as input. Therefore, the value from the constant total temperature distribution (eq. (3.3)) is used, since only minor viscous effects are expected at the boundary layer edge.

To identify the best distribution for the nominal inflow conditions FC2 and $T_w = 340$ K, the complete transformation from Pitot pressure to velocity, as detailed above, was applied on data from the boundary layer code (see section 3.8), where all values are known. The largest differences

Table 3.3: Streamwise locations of profile measurement sections (MS) for Pitot and PIV velocity profile measurements

Name	Pitot		PIV1		PIV2	
	x_a [mm]	x [mm]	x_a [mm]	x [mm]	x_a [mm]	x [mm]
MS1	492.3	496.0	490.3 - 499.3	493.9 - 503.0	474.9 - 514.6	478.5 - 518.5
MS2	630.2	635.0	623.8 - 632.8	628.5 - 637.6	604.0 - 643.7	608.5 - 648.5

occurred for the constant total temperature assumption close to the wall, as could have been expected. Between the Crocco and modified Crocco distribution only minor deviations were observed (deviations of only $\Delta U/U_e \leq 0.2\%$), since, at our nominal inflow conditions, the wall to adiabatic wall temperature ratio with $T_w/T_{aw} = 0.83$ is close to unity, where the corrections due to cold walls diminish [58]. Based on this comparison the Crocco-Busemann distribution was used to extract a velocity profile from Pitot pressure data in this study.

Pitot profiles were taken at two Measurement Sections (MS) along the cone with corresponding positions summarized in table 3.3. Selected comparisons will be given in the results section 4.2.

For all Pitot data, the edge velocity U_e was determined by averaging the outermost velocity vectors which showed a plateau behavior. The boundary layer edge δ was determined to be the wall-normal location where the velocity reaches 99 % of U_e .

3.3 Particle Image Velocimetry

Particle Image Velocimetry (PIV) is a powerful technique to measure velocity fields in a flow. In contrast to other techniques (e.g. Hot-Wire Anemometry (HWA)) it is minimal intrusive and able to collect data in 2D or even 3D. The general idea consists of seeding a fluid with tracer particles, imaging them by a camera and afterwards correlating the movement between successive exposures. If the time difference between the frames is known, flow velocities can be derived from the pixel shifts. For post-processing, each pair of images is subdivided into smaller Interrogation Windows (IW) for which a cross-correlation is performed to obtain an estimate for the average particle displacement within that region. One mayor assumption of PIV is that the particles follow the flow adequately so that the displacements of particles represent the flow field accurately. Due to the finite size and inertia of the particles, a certain lag exists which can have a filtering effect on local mean but especially on turbulent fluctuating velocities. The impact of particle lag is even enhanced in low density flows, typically occurring in high-speed blow-down wind tunnels. A compromise must be found in the size of the particles to keep the lag as small as possible but at the same time have a sufficient size to scatter enough light. Typically, liquid or solid particles are used. A further advantage of PIV is the ability to measure the velocity directly without being influenced by potential temperature gradients, affecting for example HWA. This is especially important for compressible TBL which are influenced by thermal gradients due to dissipation and differing wall temperatures. The drawbacks of PIV are limitations in resolution and dynamic range. The resolution is defined by the subdivision into finite IWs. The dynamic range is set on the lower end by typical uncertainties in the range of 0.1 pix (pixels) and on the upper end by the maximum displacements (approximately in between 15 to 36 pix in this work).

The size of the IWs do have additional impact on e.g. filtering of fluctuating content of a velocity field. This will be addressed in section 7.5.2. For further details on the PIV technique, the reader is referred to e.g. [4], [198].

3.3.1 Set-Up

Two PIV campaigns with different setups have been performed. In the following both are summarized and presented. In both cases set-ups were used to measure two dimensional, two component (2D2C) flow velocity fields.

First PIV campaign:

An ULTRA CFR Nd:YAG Laser System from Quantel was used as light source. It consisted of two pulse lasers which were superimposed and converted to a wavelength of ± 532 nm. The frequency was set to 15 Hz for a maximum energy of 191 mJ per pulse. The final laser sheet along the model surface had a nominal dimension of 60 x (0.5 – 1.0) mm (streamwise length x thickness). The set-up inside the test section is visible in figure 3.4a. To reduce wall reflections the PEEK model wall was locally painted with red color in the vicinity of the laser sheet impact region. Additionally, the sheet was positioned such that most part of its extent was located on the camera-averted side of the model.

For the acquisition of images, a PCO 1600 camera system was used, with a CCD chip with a maximum resolution of 1600 x 1200 pix². The camera was positioned outside of the test section, at a distance of approximately 2.1 m from the model. A K2 DistaMax long distance microscope from the company Infinity Photo Optical was used as objective. The chosen set-up resulted in a magnification of $M \approx 0.99$. An optical filter (wavelength 532 ± 2 nm) was mounted between the objective and the CCD chip. With this set-up a Region of Interest (ROI) of 9.0 x 12.0 mm² (streamwise x wall-normal distance) was obtained. The location of this ROI is summarized in table 3.3.

For seeding, a Solid Particle Generator (SPG) was used, which was connected to the stilling chamber of the H2K wind tunnel. TiO_2 particles of type 1002 from the company KRONOS INTERNATIONAL INC. were used. The manufacturer states a median diameter of $d_p \approx 0.2 - 0.3 \mu\text{m}$ and a density of $\rho_p \approx 3800 \text{ kg/m}^3$. Before seeding, the particles were sieved and then dehydrated in an oven at 70 °C for $t > 6$ h to 12 h to prevent agglomeration (see e.g. [199]). Investigations resulted in Stokes numbers in a range of $0.4 < S_k < 1.5$ with an expected reduction in streamwise rms turbulence intensity of approximately 8 % to 15 % (see section 7.5.2.1 or [171]). Also, potential deviations for the mean velocities of up to 2% of U_e were approximated. A dedicated sensitivity study of the particle property impact on the response behavior is given in chapter 7.5.2.

The timing of the PIV system was controlled via a LabSmith LC880 8-channel trigger box. The interframing time between two images was set to nominally $t_p = 120$ ns. During testing, a dependence of measured velocities on the laser power level was noticed. The time difference between two subsequent laser pulses was identified as main contributor to this deviation. A dedicated laser characterization with a photo-diode set-up was performed after the test campaign. This characterization resulted in corrected pulse interval values for all pre-set pulse interval and

power level combinations used during the test campaign (see also section 7.5.1). The standard uncertainty of the extracted deviations was in the order of 1 ns or below and were included in the total PIV measurement uncertainty (see section 7.1.1). A comparable influence was also reported in [285]. The described H2K set-up typically resulted in a mean free-stream displacement of $\Delta x \approx 14.5$ pix.

Second PIV campaign:

Due to hard- and software upgrades (especially in terms of the laser system), an enhanced setup was possible for the second PIV campaign. Also, since one lesson learned from the first campaign was, that two-point correlation results are essential to evaluate data quality as well as to enable the analysis of large scale structures, a corresponding larger FOV was targeted.

A SpitLight DPSS 250 PIV Laser system from InnoLas Laser GmbH was used as light source. It has a pulse rate of 100 Hz and a maximum energy of 120 mJ per pulse. It is a diode-pumped solid-state laser (Nd:YAG) at 532 nm and was operated at maximum power with an additional attenuator to control the incident power in the test section. The laser pulse timing was monitored during tests with a photo diode measurement and the uncertainty was estimated to approx. ± 1.5 ns. This was a direct lessons learned from the first PIV campaign, for which laser power influences have been encountered (see section also 7.5.1). Through a flange, equipped with a suitable window for the corresponding laser wavelength, the beam was directed towards the wind tunnel model via an optical set-up. It was fixed to the top of the wind tunnel test section and consisted of three lenses, one of which was a cylindrical one. The final laser sheet along the model surface had a nominal length (in axial direction) of larger than 20 mm and a thickness (in lateral direction) of approx. 1 mm. The set-up inside the test section is visible in figure 3.5a and 3.10.

For the acquisition of images, two LaVison Imager sCMOS cameras with a pixel size of $6.5 \mu\text{m}$ were used. A long-distance microscope K2 Distamax from Infinity Photo-Optical was used on both cameras via a beam splitter. With a CF-1b lens, a magnification of approx. 113 pix/mm was accomplished. The images were de-warped based on the calibration with a final magnification of 128 pix/mm. The images were recorded at a size of 2560×1060 pix² to enable acquisition rates of 50 Hz for each camera. With proper timing, i.e., both cameras recorded images in an alternating order, the laser system could be used at 100 Hz repetition rate since both cameras shared the FOV via the beam splitter. The complete camera setup was installed in a sealed box within the H2K test section to be as close to the target as possible and yet isolated from the vacuum condition in the test section. With this setup a FOV of approx. 20 mm in streamwise and 8 mm in wall-normal direction (approx. $5\delta \times 2\delta$) was recorded. From these two 2D fields, averaged 1D profiles were extracted approximately at the mid of the FOV at $x_a/L = 0.68$ and $x_a/L = 0.85$, called in the following measurement section MS1 and MS2, respectively (see figure 3.3). MS1 is located 170 mm behind the beginning of the rough segment which resulted in approx. 48 times the smooth wall boundary layer height at MS1. This is well above the value of 10 to 25 stated by Berg [24] to be necessary to reach a new boundary layer equilibrium condition downstream a smooth to rough wall topology change.

For seeding, the same generator and TiO₂ particles were used as in the first campaign. The timing of the PIV system was controlled via a Programmable Timing Unit PTU X of LaVision. The triggering was coupled to the wind tunnel data acquisition. The laser pulse timing between two images was set to nominally $\Delta t_p = 250$ ns. A few tests were performed with nominally

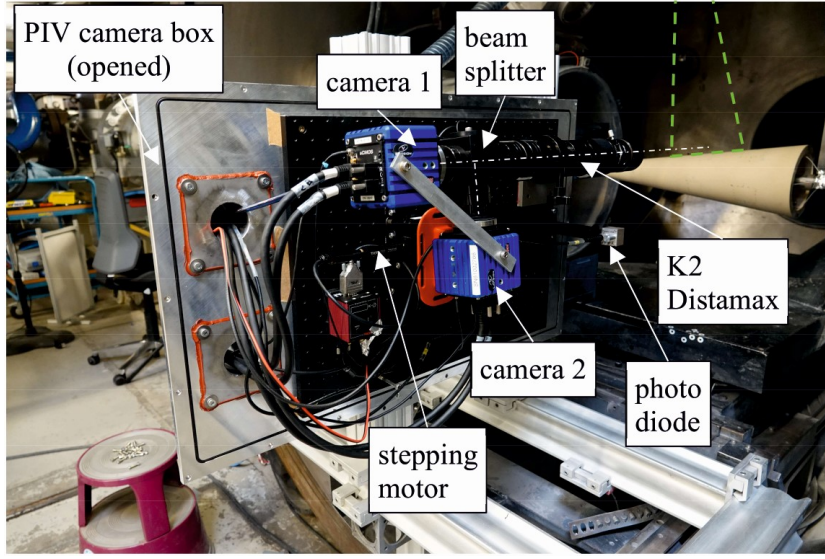


Figure 3.10: Set-up in H2K test section with opened sealed PIV camera box (second campaign).

$\Delta t_p = 150$ ns and $\Delta t_p = 350$ ns to enable random noise and turbulence intensity characterization according to Scharnowski *et al.* [211] (see section 7.5.2). This resulted in a mean pixel shift of $\Delta x \approx 15$ pix, 26 pix, 36 pix for $\Delta t_p = 150$ ns, 250 ns, 350 ns, respectively.

3.3.2 Post Processing

First PIV campaign:

All analyses were performed with the commercial PIVview suite version 3.6.0. Approx. 200 valid PIV image pairs were recorded per run. Several image pre-processing steps were conducted before analysis. Due to small model vibrations, a pattern matching algorithm was used to normalize all recorded images first. The wall was identified based on intensity images, averaged over all images per run. Profiles in the wall normal direction, accumulated in streamwise direction, were used. A small rotation, typically below approx. 1° , was applied to correct for a residual difference between the camera and wall parallel direction. A background image subtraction was omitted since no significant improvement was noticeable.

For the processing of the images several validation criteria were defined and correlated velocity vectors were not accepted if one of the following three criteria's were not met. First, a normalized median filter with a threshold of 8 was applied on each vector along with its eight neighbors. Second, a dynamic mean test with a mean value of 2.0 and variance of 1.0 was applied. The third criterion was a minimum signal-to-noise ratio of each correlation peak of 5.0. An interrogation window size was accepted only if at each vector position at least 90 % of the corresponding images met the above criteria. The remaining single image non-accepted vectors were not included in further analysis (no interpolation was used). Based on these criteria several single-grid and multi-grid interrogation windows were chosen for further analyses (see section 4.1 to 4.3). If

not otherwise noted, an overlap of 50% was used. In all cases an algorithm was applied with standard FFT correlation using Whittaker reconstruction sub-pixel peak fitting. Additionally, the algorithm used a multiple-pass interrogation method, which applied image deformation via cubic B-spline interpolation on all 3 selected steps, to compensate for high shear near the wall. Ensemble correlation analysis was applied to selected data.

For the measurement of mean and turbulence quantities via PIV, different additional random and bias errors may affect the data (see e.g. [4]). Selected impacts, deemed to be most relevant, were investigated and reported in the Appendix (7.5.2). Based on the reported analyses, the interrogation window $512 \times 8 \text{ pix}^2$ was chosen as baseline to perform all mean velocity analyses due to its high resolution in wall-normal direction. For the turbulence data, a size of $512 \times 64 \text{ pix}^2$ was selected for the discussions in section 4.1 to 4.3.

Second PIV campaign:

Approx. 1500 valid PIV image pairs were recorded per camera for each run with LaVision DaVis 10.2.1. For post-processing, the raw images were de-warped and shift / vibration corrected. Finally, a moving average over three subsequent images was used to reduce background influences. Surface reflections were typically limited to near wall regions so that a wall distance below approximately 0.1 mm was masked in the recorded images. Velocity vectors were derived using a multi-grid approach with integer window shift to result in an initial displacement field. After that, the velocity field is refined using an iterative predictor-corrector scheme with sub-pixel accurate image deformation according to the procedure described by Astarita and Cardone [14]. The final corrector step is applied with a Gaussian sub-pixel peak estimator. Typically, for the multi-grid approach two coarsening steps were applied, i.e. a final IW size of $64 \times 64 \text{ pix}^2$ or $512 \times 16 \text{ pix}^2$ started at $256 \times 256 \text{ pix}^2$ or $2048 \times 64 \text{ pix}^2$, respectively. If not otherwise noted an overlap of 50 % was chosen with some selected examples at 75 %. Rectangular and Gaussian window weighting was tested for the iterative PIV evaluation. Between the iterations, outliers in the vector field were detected by different means. A normalized median test (over 5×5 vectors) according to [276] was applied. Furthermore, vectors were neglected outside of a nominal displacement range of $-1 \text{ pix} < \Delta x < 33 \text{ pix}$ and $-2 \text{ pix} < \Delta y < 2 \text{ pix}$. Additionally, maximal displacement difference of 4 pix between neighboring vectors were allowed. Outliers were replaced by interpolated values, based on the neighboring values. A total of five steps of the iterative evaluation were performed, resulting in a typical validation rate of over 80 % in the final dataset. Proper post-processing and validation reduced the valid snapshot count to approx. 1000 images per camera. Additionally, a final post-processing step was used to filter outliers by excluding vectors which lie outside a band of three times the standard deviation around the median value at each vector position, repeating this step for 10 iterations. If not otherwise noted, experimental data points are plotted in this paper only if a minimum of 90 % valid vectors exist at the corresponding position. More details of the methods can be found in [155]. Comparisons between this analysis tool and PIVview were performed and showed only minor differences. Due to the larger flexibilities, for the data of second PIV campaign this tool was used.

As for the first PIV campaign, also for the second campaign selected impacts were investigated and reported in the Appendix (7.5.2). Based on the reported analyses, the interrogation window $64 \times 64 \text{ pix}^2$ was selected as default IW. Selected studies were performed with different IWs and are indicated in the text.

For the discussion of velocity and turbulence profiles, data along the smooth and rough wall of both PIV campaigns were averaged along a streamwise region, which corresponds to the extent of one roughness shape wavelength ($\lambda = 4k \approx 2\delta$). Along the roughness topology, this averaging window is aligned with the beginning at the downstream end of one roughness crest to the next (i.e. at MS1: $x=496$ mm to 498 mm and at MS2: $x=626$ mm to 628 mm). Different averaging alignments and also extents (i.e. two or four wavelengths) were tested with only negligible differences between the results (see figure 7.45 in Appendix 7.5.2).

For all PIV data, the edge velocity U_e was determined by averaging the outermost velocity vectors which showed a plateau behavior. The boundary layer edge δ was determined to be the wall-normal location where the velocity reaches 99 % of U_e . Data of the second PIV campaign was used to extract further statistics in the form of two-point correlations and UMZ for which post-processing steps are summarized in the following.

Two-point correlations:

Two-point correlations can give valuable information about large-scale coherent structures in a TBL. Two-point correlations of the streamwise velocity fluctuations R_{uu} have been extracted applying the following equation according to e.g. [285], [190]:

$$R_{uu}(x, \Delta x, y, y_{ref}) = \frac{\overline{u'(x, y_{ref}) u'(x + \Delta x, y)}}{\sqrt{\overline{u'(x, y_{ref})^2}} \sqrt{\overline{u'(x + \Delta x, y)^2}}} \quad (3.13)$$

Included are the marching step Δx and wall normal coordinate of interest y_{ref} . By repeating the process for different y_{ref} throughout the boundary layer, a 2D correlation map of large scale structures can be deduced. Typically, the maps appear as elliptic correlation levels inclined by a certain angle with respect to the wall plane, called inclination angle ϕ (see e.g. figure 2.13). Overall dimensions of the correlation levels can give information about length scales and the angles about inclination angles ϕ of large scale coherent structures.

To extract characterizing length scales, different approaches are being used in the literature [183], [285], [228]. Of particular interest is the integral length scale, which is defined as the integral below the one-dimensional autocorrelation [190]. This can be challenging to extract from experimental PIV data due to influences like resolution or the FOV dimension. Instead, a typically used representative length scale is the overall dimension of a contour level of the two-point correlations which are also used for direct comparisons between studies. Efforts were made to extract both described length scales from PIV data in this thesis. It was decided to do the main part of the analysis based on the second approach described above, which is less prone to errors and reported in multiple different references as comparison. Some tests have been performed to extract the integral length scale, which are reported in section 7.5.2 (see figure 7.53).

The inclination angle ϕ of the correlation contours can be extracted via the angle of the semi-major axis of an ellipse fitted to the different contour level lines. This was performed via suitable Python scripts based on the results of the corresponding two-point correlations. All of the above post-processing was tested on synthetic datasets (see section 7.5.4).

UMZs:

So-called uniform momentum zones (UMZ), separated by regions of high shear, have been attributed to the general concept of hairpin vortex structures, where a uniformly retarded flow region in each zone is the backflow induced by several hairpins aligned in a coherent pattern in the streamwise direction (see e.g. [3]).

UMZ can generally be detected in instantaneous PIV velocity fields [230], [287]. One way to detect them is by visual inspection of proper scaled 2D velocity fields (visible in corresponding plots like figure 4.41 in section 4.3.3). Another way to detect UMZs is to compare instantaneous velocity profiles to a corresponding mean profile at the same position, where UMZ boundaries characterized by increased shear can be observed by large local instantaneous velocity differences. Yet another way to detect UMZs is by identifying local maxima in a probability density histogram (p.d.f.) of the velocities in the instantaneous field according to Silva *et al.* [229]. Before applying the p.d.f., the velocities outside the boundary layer need to be excluded via a criterion for the Turbulent/Non-Turbulent-Interface (TNTI). In this work a criterion according to Silva *et al.* [230] was used. It leverages the iso-surfaces of the local kinetic energy K of the flow in a frame moving with the edge velocity according to $K = \frac{1}{2} (u^2 + v^2)$ with the threshold of $K = c_0 (\frac{1}{2} U_e^2)$ where $c_0 = 1.0e^{-3}$ was used here [230]. Cogo *et al.* [47] suggested the momentum (ρu) instead of the velocities as criterion for the turbulent/non-turbulent interface (TNTI) to include the effect of density fluctuations. Since no information about the latter was available, the formulation from Silva *et al.* was used. The boundaries between UMZs can be distinguished by local minima in the p.d.f.

Although easily detectable, a rigorous quantitative analysis for example with respect to the exact amount of UMZs is more challenging. There are open points with respect to the influence of several parameters to be chosen for the analysis [287]. For example, the proper size of the FOV as well as the resolution of the velocity field has a decisive impact on the detection of UMZs. Also, some parameters concerning the resolution and detection of peaks in the velocity histogram have a certain impact. Therefore, no in-depth but an overall statistical analysis was performed with a certain fixed parameter set in this thesis, listed in the following.

In this work, for the p.d.f. analysis 30 bins were used for the velocity range from $U/U_e = 0.4$ to TNTI. Additionally, a Gaussian kernel-density estimator was used to support identification of local maxima in the p.d.f. The bandwidth was chosen to 0.015 and 0.02 divided by the standard deviation of the streamwise velocity component in each instantaneous field, for the investigated smooth Run3014 and rough Run3007, respectively. With these settings, for the IW 64x64 pix² data at maximum 2890 instantaneous velocity vectors were available for an analysis of 1487 and 1649 valid images for smooth and rough wall data, respectively. Comparisons were made with post-processing of IW 48x48 pix² data on 5290 instantaneous velocity vectors. A maximum possible streamwise extent of $FOV_x^+ \approx 1200 - 1400$ was used, which is lower but close to the suggested value of 2000 according to [229]. Results are reported in section 4.3.4 and some additional data given in section 7.5.2.2.

3.4 Infrared Thermography

Infrared thermography enables the time-resolved measurement of global temperature information along the wind tunnel model surfaces. It was chosen as main measurement technique to determine

heat flux in the context of this thesis. The biggest advantage is to have accurate wall temperature information along complete surfaces in contrast to measurements via local sensors. At the same time, there are different challenges for this technique. For example model surfaces and material properties need to be accurately known as well as influences like reflections need to be considered, only to name a few. Further information can be found in e.g. [151]. In the following the used camera systems, corresponding setups and influences like optical calibration or model properties are addressed.

In total, two different cameras have been applied through the course of the test campaigns. A FLIR ThermaCAM SC3000 with an image rate of 60 frames per second, sensitive to the long-wavelength infrared range ($8 - 9 \mu\text{m}$), was used. It has an GaAs Quantum Well Infrared Photon Detector (QWIP) with a resolution of $320 \times 240 \text{ pix}^2$. Additionally, an Infratec ImageIR8330 with an image rate of 80 frames per second, sensitive to the mid-wavelength infrared range ($2 - 5 \mu\text{m}$), was used. It has an InSb chip with a resolution of $640 \times 512 \text{ pix}^2$. The cameras were well suited for the expected temperature increase with a high sensitivity due to a Stirling cooler system to capture small temperature changes (typically below 30 mK at 300 K) and low uncertainties (below $\pm 2 \text{ K}$). An overview of technical parameters of the cameras is included in table 3.4.

Table 3.4: Characteristics of infrared cameras

Characteristics	InfraTec, ImageIR 8300	Flir, ThermaCam SC3000
Spectral range [μm]	2 - 5	8 - 9
Geometric resolution [Pixel]	630 x 512	320 x 240
Thermal resolution at 30° [K]	0.025	0.025
Aperture angle [$^\circ$]	11 x 8.8	20 x 15
Accuracy	$\pm 1\%$ or $\pm 1\text{K}$	$\pm 2\%$ or $\pm 2\text{K}$

In all IR campaigns, two cameras were installed (see e.g. figure 3.4). The cameras were either installed outside the test section or inside within a sealed box, so that it operated at ambient conditions. Therefore, a proper window, transparent in the IR regime was necessary. Calcium fluoride (CaF_2) windows were used, which required a calibration of the complete optical path to evaluate the accurate transmittance. Contributions from absorption is typically neglected (evacuated test section) and the window transmissivity is assumed to be dominant. The corresponding calibration has been accomplished using a blackbody source with known emissivity. The resulting mean transmittance factors are indicated in table 3.5.

For some tests, one camera (lower resolution FLIR camera) was used to monitor the side of the model whereas the other one recorded the top side of the cone. With the side and top view of the laminar-turbulent transition front, the shape information could be used to realize accurate angle of attack and sideslip positioning of the model, supporting information from surface pressure measurements (see section 3.2). In later tests, only the pressure measurements were used for attitude checks.

Only data of the Infratec ImageIR 8330 camera system, which was positioned to optimize a view of a large portion of the PEEK cone, was used in the post-processing to heat flux values. The different setups in the campaigns are summarized in table 3.5. Besides the positions (view

from top, outside the test section), it includes the different chosen calibrated temperature ranges and effective frame rates. For the heat flux derivation (see section 3.4.2), it was necessary to capture the model temperature before wind tunnel start which typically represented the lowest temperature. At the same time, due to the heating of the model, enabling to record high temperatures with the camera system enlarged useful testing times. Depending on the range, a combination of different integration times were necessary to cover this temperature range, which effectively reduced the recording frequencies. The different frame rates were checked to be sufficient for numerical stability of the explicit formulation for the heat flux calculations (see chapter 3.4.2).

Table 3.5: Setup of infrared cameras (o: outside test section).

Campaign/ Cam	position	lens	res. [pix/mm]	range [°C]	fps [Hz]	τ_{CaF2} [-]
First PIV Infratec	top (o)	25/100mm	0.9	20-100 (smooth) 0-150 (rough)	80 26.7	0.92
Second PIV Infratec	top (o)	25mm	1.1	-10 - 150	26.7	0.94±0.02

3.4.1 PEEK Properties

PEEK was chosen as main model material for the test campaigns (“TECAPEEK natural”[®] from Ensinger). It is a colorless organic high-temperature polymer thermoplastic, which can withstand operating temperatures up to approx. 530 K. It has a high emissivity and low thermal conductivity, which reduces lateral heat dissipation. Additionally, with the density, heat capacity and conductivity, relevant material properties were characterized in the past and are available as temperature dependent polynomials (see section 7.6.1). Additionally, detailed emissivity characterizations were performed with dependencies on material temperature, the emitted wavelength and the angle between the camera and the model (see section 7.6.1). Based on these analyses, a mean emissivity of $\epsilon = 0.95$ was used in this thesis. These properties make PEEK the standard material for heat flux investigations in H2K, where in contrast to short test duration facilities, no isothermal conditions can be assumed.

3.4.2 Heat Flux Calculation

To evaluate the thermal impact of a hypersonic flow on a surface, typically the convective heat flux $\dot{q}_c$ is of interest. In a typical cold blow-down wind tunnel set-up a heat energy balance at the model surface can be formulated to

$$\dot{q}_c = \dot{q}_r + \dot{q}_w \quad (3.14)$$

with the assumption of an equilibrium between the convective heat flux $\dot{q}_c$, the conductive heat flux into the material $\dot{q}_w$ and the radiative heat flux $\dot{q}_r$, emitted by the surface. If the surface temperature T_w is known, the radiative heat flux can be calculated via the Stefan Boltzmann law:

$$\dot{q}_r = \varepsilon \sigma (T_w^4 - T_{amb}^4) \quad (3.15)$$

where ε is the emissivity of the surface, σ the Boltzmann constant with $\sigma = 5.6703 \cdot 10^{-8}$ [W/(m²K⁴)] and T_{amb} the temperature of the surrounding. To evaluate the convective part, the conduction into the material needs to be known. For its determination, a thermal energy balance on a solid volume element can be set up and with the first law of thermodynamics the heat diffusion equation can be given with (see [151]):

$$\rho(T)c_v(T)\frac{\partial T}{\partial t} = \nabla \cdot \lambda(T)\nabla T \quad (3.16)$$

with the density of the material ρ , the heat capacity at constant volume c_v and the thermal conductivity λ . All parameters are kept temperature dependent. In the following the assumption is made that for solids the difference between the heat capacity at constant volume and constant pressure is negligible ($c_v \approx c_p$). If the temperature distribution in the material is known, the conductive part of the heat flux may be derived, which is proportional to the temperature gradient at the wall via Fourier's law:

$$\dot{q}_w = -\lambda(T_w)(\nabla T)_w \quad (3.17)$$

Two different tools were used in this thesis to calculate the conductive heat flux, ANSYS® and “pyHeatflow”.

To enable simulations including lateral heat flux components for selected scenarios, the commercial finite element software ANSYS® was used [116]. A grid sensitivity study is included in chapter 7.7.2 and the model and corresponding results are presented in section 4.4 and 7.6.7.

The description of pyHeatflow is given in the following.

3.4.2.1 pyHeatflow

Blow down facilities with long test duration cause radiation effects and rising wall temperatures, which complicate the derivation of heat flux rates on wind tunnel models. In order to reduce lateral heat transfer, models made of low thermal conductivity materials can be used. The benefit of the low conductivity enables the use of one-dimensional heat flux assumptions. However, the drawback of working with these materials is the need for temperature dependent properties to be taken into account in the analysis. This disables the use of simplifications in the derivation of relevant equations. The difference between using constant and temperature dependent properties of PEEK based on temperature evolution at a typical position along the cone results in approx. 6 % difference in convective heat flux after 20 seconds test time.

Since the heat flux along a cone is of interest in this work, an axisymmetric one-dimensional formulation was used, implemented in a Python script called “pyHeatflow”. The heat equation is solved for each pixel information of the infrared camera. Since the pixel resolution in axial direction is quite high (typically in the order of $\Delta x/L < 0.1\%$), the conical radius change per pixel is negligible, so that the discretized space is approximated by a cylindrical segment per pixel. For each pixel the corresponding radius along the axis was distributed. Comparable investigation showed negligible differences between results derived by a full conical formulation compared to a cylindrical formulation of the heat equation along a cone for points not too close to the nose tip [34].

The formulation of the one-dimensional version of equation 3.16 which covers Cartesian and cylindrical formulations can be given based on e.g. [34] (applying $c_v \approx c_p$):

$$\rho(T)c_p(T)\frac{\partial T}{\partial t} = \frac{1}{r^j}\frac{\partial}{\partial r}\left(r^j\lambda(T)\frac{\partial T}{\partial r}\right) \quad (3.18)$$

with r as the radial distance, perpendicular to the wall and j as the axisymmetry flag. For $j = 0$ and $r = y$ equation (3.18) reduces to the typical one-dimensional formulation.

Equation (3.18) can be transformed to

$$\frac{\partial T}{\partial t} = j\frac{1}{r}\frac{\lambda(T)}{\rho(T)c_p(T)}\frac{\partial T}{\partial r} + \frac{1}{\rho(T)c_p(T)}\frac{\partial \lambda(T)}{\partial T}\left(\frac{\partial T}{\partial r}\right)^2 + \frac{\lambda(T)}{\rho(T)c_p(T)}\frac{\partial^2 T}{\partial r^2} \quad (3.19)$$

For the numerical iteration, equation (3.19) needs to be discretized. The discretization is performed with central differences in space and a forward discretization of first order in time.

$$\frac{\partial T}{\partial t} = \frac{T_{i,k+1} - T_{i,k}}{\Delta t} + O(\Delta t^2) \quad (3.20)$$

$$\left(\frac{\partial T}{\partial r}\right)^2 = \left(\frac{T_{i-1} - T_{i+1}}{2\Delta r}\right)^2 + O(\Delta r^2)^2 \quad (3.21)$$

$$\frac{\partial^2 T}{\partial r^2} = \frac{T_{i-1} - 2T_i + T_{i+1}}{\Delta r^2} + O(\Delta r^2) \quad (3.22)$$

The i variable indicates the depth index while k indicates the time index. Performing these transformations, equation (3.19) can be expressed as:

$$\begin{aligned} \frac{T_{i,k+1} - T_{i,k}}{\Delta t} = & j\frac{1}{r}a(T_{i,k})\left(\frac{T_{i-1,k} - T_{i+1,k}}{2\Delta r}\right) \\ & + b(T_{i,k})\left(\frac{T_{i-1,k} - T_{i+1,k}}{2\Delta r}\right)^2 + a(T_{i,k})\frac{T_{i-1,k} - 2T_{i,k} + T_{i+1,k}}{\Delta r^2} \end{aligned} \quad (3.23)$$

with the thermal diffusivity

$$a(T) = \frac{\lambda(T)}{\rho(T)c_p(T)} \quad (3.24)$$

and

$$b(T) = \frac{1}{\rho(T)c_p(T)} \frac{\partial \lambda(T)}{\partial T} \quad (3.25)$$

In figure 3.11, a sketch of the discretization can be seen. As starting condition, each node of the mesh is set to an initial temperature ($T_{i,k} = T_{initial}$). This initial temperature can be set to an ambient temperature or to the first recorded temperature at the corresponding pixel. In this work, $T_{initial}$ was usually set to the initial measured wall temperature prior to the wind tunnel start. This approach considered small run to run differences of initial model temperatures, due to prior test runs model temperature increases during the day. This procedure was validated during prior test campaigns. The measured infrared data is used as boundary condition at the top surface $i = 0$ ($T_{i=0,k} = T_{IR}(t) = T_w(t)$) for all following time points, whereas an adiabatic condition is used at the back surface of the volume of interest $i = I$ ($T_{i=I,k} = T_{i=I-1,k}$).

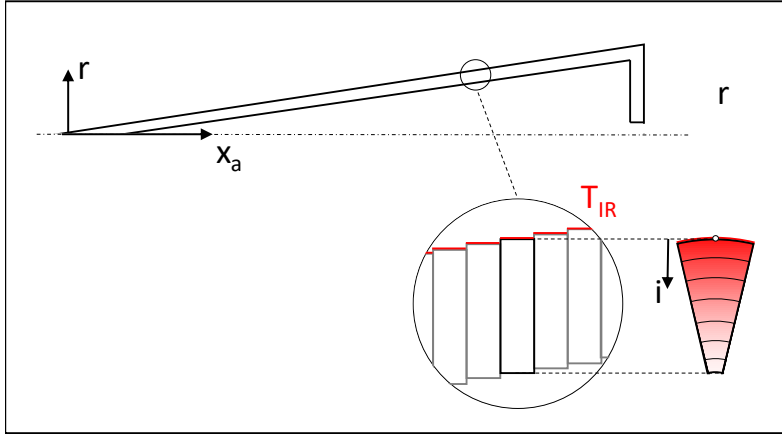


Figure 3.11: Discretization of the model in “pyHeaflow”

Once the temperature distribution within the material is calculated for a given time point, the gradient at the surface can be calculated using a polynomial according to [99], [95]. With this temperature gradient together with the corresponding heat conduction, the conductive heat flux can be calculated according to

$$\dot{q}_w = -\lambda(T_{i=0,k}) \frac{dT}{dr} \Big|_{i=0,k} \quad (3.26)$$

Together with equations (3.14) and (3.15), finally the convective heat flux $\dot{q}_c$ can be derived.

In order to assure numerical stability of this explicit finite-difference procedures, the Courant-Friedrichs-Lewy condition needs to be satisfied. It is postulated as

$$\Delta t \leq \frac{\Delta r^2}{2a(T)} \quad (3.27)$$

Besides stability and since no information on the inside of the PEEK model and therefore backside of our computational domain is known, it needs to be checked that the adiabatic condition is not altering the resulting surface heat flux. One way to minimize a potential impact is to make sure that the heat does not penetrate too far into the material. Therefore, the PEEK segments were designed to ensure that the typical heat penetration depth during nominal testing times is smaller than the material thickness. The penetration depth depends on the duration of the experiment, the material properties, and the wall thickness. It can be approximated via the thermal Fourier number Fo :

$$Fo = \frac{at}{L^2} \quad (3.28)$$

with the thermal diffusivity a , time t and a length L through which the heat conduction occurs. According to [140], [268] a maximum Fourier number of $Fo = 1/4$ should enable one-dimensional analyses with a semi-infinite wall assumption and therefore it is assumed to be sufficient for our adiabatic backside assumption as well. This criterion results in

$$L = \sqrt{4at} \quad (3.29)$$

Since the thermal diffusivity a decreases with increasing wall temperatures, a conservative penetration depth L may be derived, assuming a constant diffusivity a at assumed wall temperature of $T_w = 300$ K before wind tunnel start and with a typical testing time of $t = 20$ s. This results in $L \approx 7.7$ mm for our case. Therefore, the model segments were designed to have a minimum nominal thickness of 10 mm.

The pyHeatflow script was validated with ANSYS computations [116], which are included in section 7.6.4. A grid sensitivity study is included in section 7.7.2 and results are presented in section 4.4.

3.4.3 Post-Processing

Since the calcium fluoride glasses have a transmittance τ_λ below 1, a fraction is either absorbed (α_λ) or reflected (ρ_λ), since according to Kirchhoff's law

$$\alpha_\lambda + \rho_\lambda + \tau_\lambda = 1 \quad (3.30)$$

This detail can become apparent for IR cameras in the form of the so called ‘‘Narcissus effect’’ [217]. When the camera axis is at 90° with respect to the IR window, a cooler spot appears in

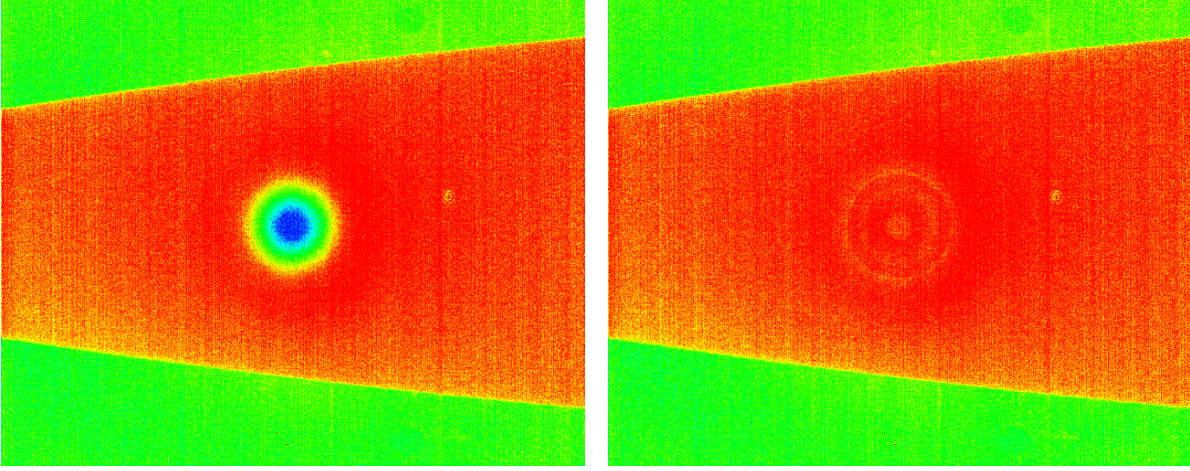


Figure 3.12: IR image without (left) and with reflection correction (right)

the center of the IR image, as visualized in the left half of figure 3.12. To avoid this influence a correction map was used [95]. The result after applying the correction map can be seen on the right side of figure 3.12.

To suppress unphysical heat flux fluctuations caused by small model vibrations a time filter was applied on the IR temperature signal of each pixel before heat flux calculation. Based on experience from previous test campaigns a FIR filter with either a width of 5 time steps and 3 or 25 iteration steps was used. An example for one IR pixel with temperature, heat flux and Stanton number St against time for different filter settings is shown in figure 3.13.

Another influence to address is the impact of IR camera resolution on the measured heat flux values along the wall, since the roughness topology exhibits small scale structures which can be resolved to a varying degree depending on the setup. In this thesis, it is assumed that a camera with lower resolution measures approximately the ensemble average of the pixel information in the same field of view of a corresponding camera with higher resolution. This approximation was confirmed in private communications with the camera manufacturer (Infratec) and by data from the ExoMars campaign for a $Ma_e = 4.2$, $Re = 8.5 \cdot 10^6$ TBL flow along a $\lambda/k = 4$ square bar rough wall topology [168], [167]. Stanton numbers of both resolutions agreed to within approx. $\pm 3\%$ and this will also be confirmed with data in this thesis (see e.g. figure 4.53).

A further point to discuss is that it is speculated that rough wall heat flux augmentations are caused by the increased surface area and local secondary flows around the roughness elements [32]. Also, previous work showed that area-averaged 3D heat flux results needed correction by the wetted-to-planform area ratio if compared to 1D cases [27]. The author argued that conservation of energy applies to the heat transfer rate $Q = \dot{q} \cdot A$ not the heat transfer rate per unit area, or heat flux, $\dot{q}$. Bons worked on an irregular roughness topology where surface-to-camera viewing angles differed by a large range of values so that the effective increase in surface needed to be corrected for if compared to an 1D (projected) modeling. In our case the technical roughness topology is regular and most of the parts visible are the surfaces parallel to the smooth cone model parts (without the sidewalls), and therefore it was assumed that no correction is necessary.

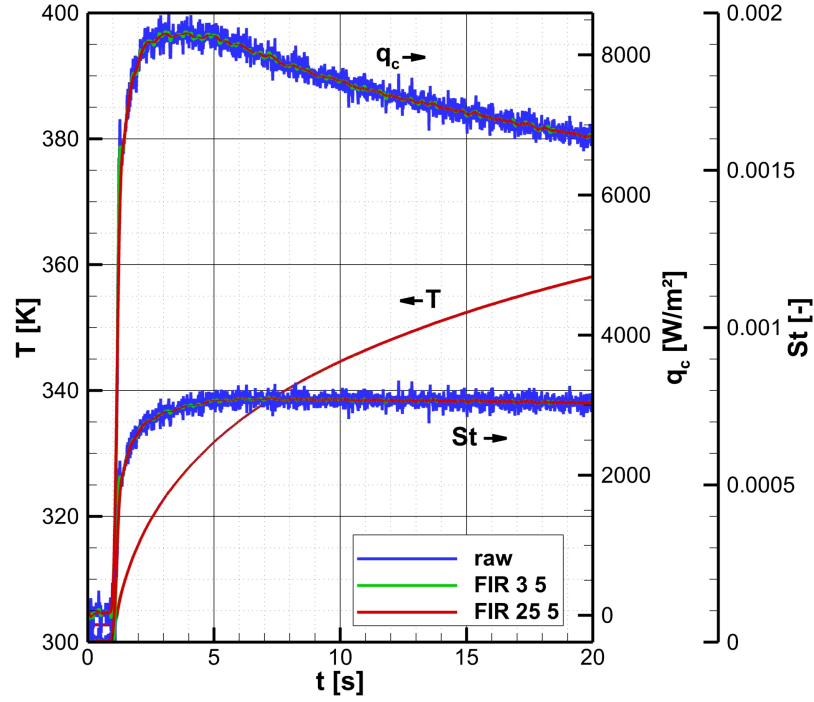


Figure 3.13: Example of post-processing for one IR pixel with temperature, heat flux and Stanton number against time for different filter settings

3.4.4 Normalization

The determined convective heat fluxes were generally normalized and given in form of Stanton number distributions to reduce wall temperature effects and allow comparisons to other measurement campaigns and CFD data. The Stanton number can be seen either as ratio of convective heat flux to a total one carried by the stream [102] or as a heat flux normalized by the mass flow rate and the driving temperature [40].

Different formulations for the Stanton number are being used (see e.g. equation (2.15) in section 2.4). In this work, the following formulation was used, after investigating different other options (see section 7.6.5)

$$St_e = \frac{\dot{q}_c}{\rho_e u_e c_p (T_r - T_w)} \quad (3.31)$$

where the recovery temperature T_r according to equation (3.8) was used to estimate the adiabatic wall temperature T_{aw} . The corresponding edge conditions were derived from the inflow based on the Taylor-Maccoll equations (section 3.7), neglecting viscous interactions along the cone.

3.5 Thermal Sensors

Additionally to infrared thermography, two different local heat flux sensors types were foreseen to be inserted flush into the model surface. This enabled comparisons to values from IR. Although it was expected that the absolute values differ between both techniques (due to e.g. different materials, see e.g. [93]), the corresponding ratios i.e. rough wall heat flux augmentations, were planned to be compared.

Three Heat Flux Microsensors (HFM) from Vatel Corporation were acquired. The sensor is based on a thin-film technology which applies a thermopile construction. Different types for different environmental conditions exist. Two coated and one non-coated low temperature variant HFM-7/L heat flux sensors were chosen. Figure 3.14a shows a photo of a coated HFM sensor. The exact position of the sensors can be found in figure 3.6 and table 3.2.

The HFM sensor performs two separate measurements. First, for direct surface temperature measurements, the probe has a resistance temperature sensing element (RTS), which is used to correct the heat flux signal for temperature dependent probe effects. Secondly, the HFM incorporates a thermopile heat flux sensor (HFS), which occupies most of the surface area on the sensor face, which allows direct heat flux measurements. The sensor head had a total diameter of 6.35 mm.

The HFM sensors are typically calibrated against a radiative heat flux source. Therefore, the typical heat flux values need to be corrected if the convective heat flux is of interest. In case of the coated sensors, the convective heat flux can be derived considering the emissivity of the coating [266]. In case of the uncoated sensor, the convective heat flux can be derived directly. For further information about the HFM-7/L sensor, the reader is referred to the operator's manual [266]. To protect the sensors from damages due to particles from the wind tunnel and PIV seeding, a protective Kapton strip was applied on the sensor heads for all but selected runs.

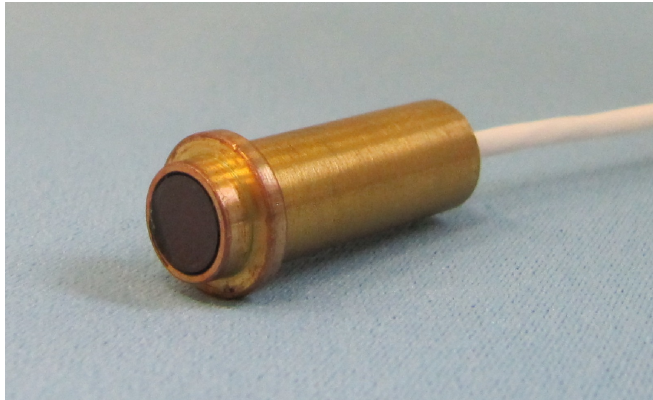
As a second type of local sensors, Coaxial Thermocouples (CT) were foreseen. Due to its design this type of thermocouple enables measurements of local surface temperatures when inserted flush into the model surface. The two thermocouple alloys are arranged coaxially with one alloy as thin wire in the center surrounded by the second alloy in a hollow cylindrical form. Both materials are separated by a thin sheet of insulator and the only connection is allowed by small splinters at the frontal surface. This enables high temporal sensitivities but at the same time also results in a high sensitivity to noise. Different types of sensors were acquired from the Shock Wave Laboratory (SWL) of the University of Aachen.

Two types of CT sensors have been used. The first one is a coaxial thermocouple with 1.9 mm and 1 mm for the outer diameter of the first and second thermocouple element, respectively, together with a length of 12 mm (type 1, see figure 3.14b). The second one is a coaxial thermocouple installed in a housing with 5.5 mm outer diameter and 18 mm length (type 2, see figure 3.14c). Both versions are type E thermocouples with the pairing of chromel and constantan. Typically, these CT are used in short duration facilities but have already demonstrated their ability to deduce long-term heat flux signals (see e.g. [253]). To this end, additionally to the thermocouple junction on the frontal surface the sensors have a second one at the backside of the sensor to monitor heat penetration.

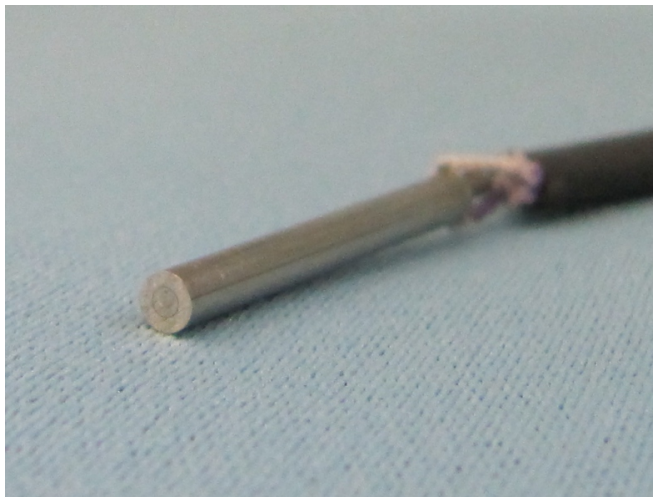
To derive the heat flux from the temperature signals of the CT, the one-dimensional heat equation is solved with the same code as used for IR measurements but in non-axisymmetric mode (see section 3.4.2.1). Before, the temperature signals were filtered with moving average kernel of size 5 and 25 repetitions. According to Hollis [107], a type-E chromel-constantan thermocouple consists primarily of chromel (80 – 90 % by volume). Therefore, and because of uncertainties in thermal properties of constantan, the author recommends to use chromel material parameters for the analysis of CT. The coefficients detailed in table 7.27, according to [107], were used in this work together with equations (7.25) to (7.27).

Due to the cone shape of the model, it is not possible to realize a perfectly flush-mounted flat sensor head and small gaps are inevitable. The sensors were implemented in such a way, that the heads were flush with the surrounding model along a streamwise line, which can be seen as a projected axis into the mid-point of the sensor head. Therefore, parts of the sensor head outside of this line in circumferential direction had a small step up compared to the model. Due to the changing curvature along the axis, this step also changes in size, with the most challenging one represented by the HFM located close to the nose at $x_a = 0.3$ m with a gap of approx. 0.1 mm. This gap was accepted, since no downstream influence was visible in the IR images and also the transition front was located upstream of this position.

The positions of the sensors in the technical roughness segment were the same as in the smooth case. However, since the sensors needed to be flush with the model surface and their diameter was bigger than the roughness elements width, they could not be implemented directly within the rough part. It was decided to include local smooth plateaus at the sensor positions, flush with the height of the roughness elements. The plateaus had a rectangular shape of 16×8 mm², with the longer side in streamwise direction. These dimensions were based on necessary criteria for oil-film interferometry measurements which were discussed in the initial phase of planning. The plateaus began at position $x_a = 484.58$ mm and were located at $\phi = 90^\circ$, $\phi = 180^\circ$ and $\phi = 270^\circ$ (see also figure 3.6). The position $x_a = 484.58$ mm and $\phi = 0^\circ$ was designed without plateau, since it was the side of the model seen by the IR camera (see section 3.4). This ensured a clear view to record the temperatures along the rough segment without a possible influence of a plateau. To ease the manufacturing of the plateaus, the grooves along the longer side edges were finished with rounded edges with a radius of 0.75 mm. It is assumed that this modification had only negligible influence on the measurements of the sensors implemented in the plateaus (see figure 3.15). The HFM and CT sensors were implemented close to the leading edge of the plateaus in streamwise direction at approx. $\approx 2\delta$ downstream the front edge of the plateaus (calculated with a typical rough wall TBL thickness as extracted from PIV at MS1). The objective of this design was to realize that the sensors measure the rough wall effect as best as possible, since it is below a distance for which a new smooth wall equilibrium would be expected (using the estimates from [24] with a distance of 14δ for mean flow and even $20 - 25\delta$ for fluctuating quantities). To hold the sensors in place, glue was used at the backside interface between model and sensor, independent of sensor type (HFM, CT type 1 or 2).



(a)

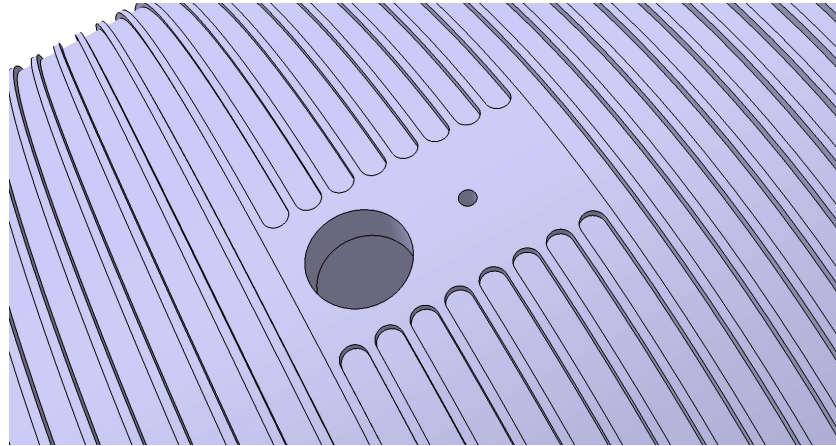


(b)

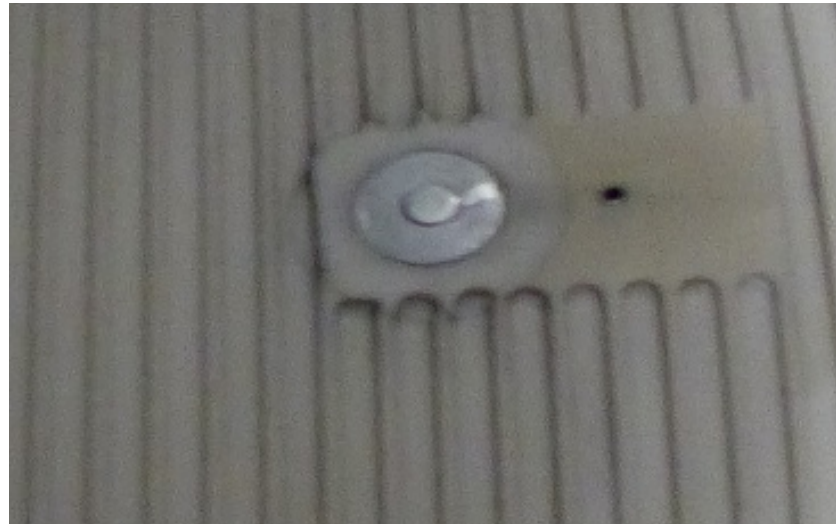


(c)

Figure 3.14: Photographies of local heat flux sensors used, with a) HFM sensor, Coaxial thermocouples of b) type 1 and c) type 2.



(a)



(b)

Figure 3.15: Sensor plateau with a) 3D CAD view at $x_a = 484.58$ mm and $\phi = 90^\circ$ and b) picture in test section with installed CT

3.6 Schlieren System

At the time of the experiments, the hypersonic wind tunnel H2K used a coincident Schlieren optic setup to visualize the flow around the model. This technique with a long optical path increases the sensitivity of the Schlieren technique in flows with relatively low densities and density gradients. With this line-of-sight method, the three-dimensional flow structures are visualized in two dimensions. Density gradients are integrated along the line of sight. The Schlieren technique differs from the shadowgraph technique by the usage of a knife edge. This results in the density gradient being emphasized orthogonal to that edge. The Schlieren setup was implemented only in case of the Pitot pressure campaign, since for both PIV campaigns, the corresponding PIV cameras needed to be placed into the light path of the Schlieren system.

3.7 Cone Flow Approximations

In this work, the focus is on an evolving TBL along a cone model. If the ratio of boundary layer thickness to local cone radius $\delta/r_w \ll 1$, effects due to transversal curvature can be neglected and the equations along a cone and a flat plate are closely related (see e.g. [214]). This is assumed to be the case in this thesis with δ/r_w typically below 0.1, based on data from a boundary layer code (see section 3.8).

Despite the above given ideal assumptions, several things might influence the boundary layer along the cone

- local subsonic region and stagnation point flow at blunted nose instead of perfectly sharp noses
- separated and curved shock in front of blunted nose with corresponding entropy layer
- entropy swallowing, entropy and boundary layer interaction
- pressure gradient along the cone due to viscous interaction
- circumferential curvature effects

Some of these effects are sometimes called second-order effects in the literature, due to its typical contribution of only second order (see e.g. [264]). Selected influences were investigated and are discussed in chapter 4.

Under specific assumptions, for the special case of flow along a sharp cone a solution to the flow equations can be found. In 1933, G. I. Taylor and J. W. Maccoll developed a method to determine the supersonic flow past a cone at zero angle of attack [248]. It is assumed that an attached constant oblique shock is formed upstream the cone (no shock curvature) and therefore the entropy increase across the shock is constant. Furthermore the flow is assumed irrotational, adiabatic and steady with constant total enthalpy along streamlines. The flow field downstream the shock is assumed conical with an applicable spherical coordinate system. Under these assumptions, the authors transformed the conservation equations to ordinary differential equations, which can be solved with a suitable mathematical procedure. This procedure was used in this thesis to calculate edge Mach number and pressure under the assumption of only

small viscous interaction effects. The derived values were used as input parameters for the following analytical and numerical computation methods. Previous investigations compared boundary layer edge values from Taylor-Maccoll equations along a 7° cone at $Ma = 3.5$ with results from DNS computations with an agreement within 1 % (see [137]). Further approaches to transform flat-plate correlations to calculate skin friction and heat transfer along the cone model are reported in section 3.10.

3.8 Boundary Layer Code

A BLcode was used, which had been developed by NASA to solve the laminar, transitional or turbulent two-dimensional or axisymmetric compressible boundary layer equations for perfect gas flows [98]. To solve the boundary layer equations, an iterative three-point implicit finite-element procedure is used. To close the turbulent model, the software includes two options: a two-layer eddy viscosity model or a mixing length model. In this paper only the eddy viscosity model was used with a constant turbulent Prandtl number $Pr_t = 0.9$.

Within the two-layer eddy viscosity model, in the framework of this thesis an extension was implemented to account for flow over rough walls. This model is based on work by Fluid Gravity Engineering within the ExoMars project to include rough wall fluxes along the Mars capsule in the aerothermodynamic database (see [161]). This was achieved by the implementation of an equivalent sand grain based model in a Cebeci-Smith turbulence model, based on the modification of the Van Driest damping function which damps out eddy viscosity near the wall in the inner of the two-layers of the algebraic model. A detailed description of this modified compressible Krogstad model can be found in [161] and [136]. This implementation is a modification of a model, originally proposed by [136] and with ideas from [260]. The implemented modifications to the boundary layer code are summarized in 7.7.1, together with a validation based on data by [161].

The model was used to rebuild friction augmentations (see section 4.2.2) and heating augmentations (see section 4.4) and support the analysis.

3.9 TAU Code

CFD calculations were performed with the DLR TAU code [97], [154]. The TAU code is a finite volume Euler/Navier-Stokes solver, which can use structured, unstructured and hybrid meshes, and has already been applied and validated in studies of various configurations in various flow regimes, including hypersonic flow [97]. Different numerical second order upwind schemes, like AUSMDV with MUSCL extrapolation, Roe or Van Leer can be used. For time discretization, including local time stepping, a three dimensional Runge-Kutta as well as an implicit, approximately factored LU-SGS scheme is implemented. Acceleration techniques like multigrid and explicit residual smoothing are available. Automatic local grid adaptation, accounting for a variety of flow variables, can be used. Furthermore, parallel computing is possible via domain splitting and Message Passing Interface (MPI) communication.

3.9.1 Turbulence Models

Smooth wall laminar and turbulent computations on a hybrid axisymmetric mesh of the 7° cone model were performed with this code. The mesh was pre-refined in areas of large gradients to resolve the corresponding flow phenomena, like the bow shock. A dedicated grid convergence analysis was performed (is reported in chapter 7.7.2). In case of turbulent computations a one-equation Spalart-Allmaras model with Edwards modification (SAE) [64], a two-equation Wilcox- $k - \omega$ model [279] and a seven-equation Reynolds stress model (RSM) with a hybrid Speziale-Sarkar-Gatski / Launder-Reece-Rodi (SSG/LRR- ω) [65] was used. If not otherwise noted, a turbulent Prandtl number of $Pr_t = 0.9$ was assumed in all applied models. This choice is supported by DNS data according to [56].

3.9.2 Particle Module Extensions

To model motion of particles with mass, an analysis module based on a Lagrangian approach already existed for the unstructured Navier-Stokes solver TAU. This module was originally implemented to predict ice accretion on various components of an aircraft, by predicting droplet trajectories in CFD solutions (see [278]). Since this module was designed for sub- to transonic flows, it was necessary to extent the implementation of the equations of motion within this thesis to cover compressible, rarefied conditions. This included modifications to enable variable density and viscosity and also the implementation of suitable drag coefficient models, chosen with the Tedeschi (see [250]) and Henderson model (implemented as given in [254]). Both include the effect of compressibility and variable Knudsen number. In 7.7.3 a short summary of the relevant equations and models is given together with a validation.

With the presented TAU module, single particle trajectories, as well as wall particle impingement points and particle density distributions along the conical flow field could be modeled, extracted and compared to the experimental data. A 1D Python script, originally implemented for validation of the TAU module, was also used to analyze particle response to turbulent fluctuations within the TBL, based on an approach described in [285]. Analyses with this approach are presented in 7.5.2.

3.10 Skin Friction Determination

Skin friction is an important parameter, due to its role in setting a boundary condition for TBL. Therefore, it is used as scaling parameter e.g. in the form of the skin friction velocity u_τ . Several different approaches were used in the framework of this thesis to either approximate a value analytically or estimate a value based on experimental data. These approaches are summarized in the following subsections.

In the initial phases of the thesis, it was discussed if a direct measurement of the wall shear stress would be possible. Typically, so called “floating element” drag balances can be used (see e.g. [257],[18], [239], [128]). There are different challenges to this approach, e.g. proper sealing against secondary flow influences or typical small signal-to-noise ratios. To counteract the latter,

in large scale facilities, balances with increased dimensions can be used to increase the resulting force from small shear forces at the expense of spatial resolution (e.g. a length of 3 m and width of 1 m in case of [18]). Since this option is not possible in case of a H2K setup together with a more challenging necessary setup in case of a cone model, it was decided to neglect this option.

Another approach was applied by Hill *et al.* [101] utilizing a conventional strain-gage balance to measure the complete drag force on a sharp and blunt 7° cone in a $Ma = 10$ flow. They derived the total skin friction via the force balance signals from smooth and rough cone models with three different grain sizes. The same approach was estimated for the 7° cone model in H2K. Initial estimates based on integrating values from analytical correlations for laminar/turbulent/rough wall skin friction contributions (see section 3.10.1) resulted in very low signals for the H2K scaling and conditions (not shown here).

A further method which was discussed in the planning phase was a method based on oilfilm interferometry (see e.g. [218]). Since the dimensions of the technical roughness violated the minimum requirements for this technique, it was omitted.

Eventually, it was decided to use different indirect methods, based on measured velocity data. First, a fitting procedure, based on the law-of-the-wall, is presented. Additionally, the velocity gradient from the wall closest data points of the PIV measurements was exploited. Besides that, a modified integral method to approximate the compressible wall skin friction along the cone model was applied.

But first, analytical models used in this thesis to support the experimental analysis are reported.

3.10.1 Analytical Prediction Methods

In the framework of this thesis, several different analytical methods for laminar and turbulent flows along smooth and rough walls have been used. Most methods are formulated for flat plate 2D boundary layers. In the following the methods are introduced together with their modifications in order to extent their applicability to cones.

The first one is based on the method by Korkegi [132] for compressible flow along a flat plate with heat transfer which was adapted to include Sutherland's viscosity model for laminar flow according to Willems and Gülhan [282]. The skin friction coefficient at the edge of the boundary layer $c_{f,e}$ can be formulated for laminar flows as:

$$c_{f,e} = \frac{0.664}{Re_{x,e}^{1/2} \left(\frac{T_{ref}}{T_e} \right)^{1/4} \left(\frac{T_{ref} + C_1}{T_e + C_1} \right)^{1/2}} \quad (3.32)$$

where C_1 as the Sutherland viscosity model constant (see equation (7.9) and equation (7.9) and [261]) and T_{ref} is the reference temperature [132] obtained from:

$$\frac{T_{ref}}{T_e} = 1 + 0.032Ma_e^2 + 0.58 \left(\frac{T_w}{T_e} - 1 \right) \quad (3.33)$$

The second analytical model is a method based on the work of Van Driest [261] for turbulent compressible flow along a flat plate with heat transfer, which was adapted to include Sutherland's viscosity model by Willems and Gülhan [282]. It can be given with:

$$\frac{0.242}{\sqrt{c_{f,e} F_c}} = 0.41 + \log(c_{f,e} F_\theta Re_{x,e}) \quad (3.34)$$

with the following transfer functions for compressible flow, including the recovery factor r , which was not considered in the original paper but added by Willems and Gülhan [282]:

$$F_C = r \frac{\gamma - 1}{2} \frac{Ma_e^2}{(\arcsin(\alpha_{VD}) + \arcsin(\beta_{VD}))^2} \quad (3.35)$$

$$F_\theta = \frac{T_w + C_1}{T_e + C_1} \left(\frac{T_e}{T_w} \right)^{3/2} = \frac{\mu_e}{\mu_w} \quad (3.36)$$

$$\alpha_{VD} = \frac{2A_{Walz}^2 - B_{Walz}}{\sqrt{B_{Walz}^2 + 4A_{Walz}^2}} \quad (3.37)$$

$$\beta_{VD} = \frac{B_{Walz}}{\sqrt{B_{Walz}^2 + 4A_{Walz}^2}} \quad (3.38)$$

and A_{Walz} and B_{Walz} according to (3.9) and (3.10), respectively.

Van Driest also derived a relation to transform the skin friction derived for a flat plate to a cone (see [262]). This relation is given by the ratio of the Reynolds numbers:

$$\frac{(Re_{x,e})_{cone}}{(Re_{x,e})_{flatplate}} = 2 \quad (3.39)$$

where $Re_{x,e} = \rho_e U_e x / \mu_e$. This was applied to all of the above given flat-plate correlations to calculate values along the cone surface.

A derivation of a rough surface skin friction correlation for a compressible flow along a flat plate with heat transfer based on sand grain roughness was obtained by F. W. Fenter (see [71], [223]). Due to the necessity of some terms being integrated numerically during its derivation, the correlation is valid within $0 \leq Ma_e \leq 10$, $5 \leq U_e^+ \leq 30$, $1 \leq T_w/T_e \leq 21$. The skin friction coefficient can be found by solving the following equations iteratively

$$\frac{\psi_F}{\sqrt{\sigma_F}} \sqrt{\frac{h_e}{H_w}} \frac{1}{\sqrt{c_f}} = 4.13 \log \left(c_f \frac{\rho_e U_e x}{\mu_w} \right) - 1.65 a_i \log k_s^+ - 0.718 b_i + 1.84 \quad (3.40)$$

where

$$u_\tau = U_e \sqrt{\frac{c_{f,e}}{2} \frac{\rho_e}{\rho_w}} \stackrel{p_e = p_w}{=} U_e \sqrt{\frac{c_{f,e}}{2} \frac{T_w}{T_e}} \quad (3.41)$$

$$\sigma_F = \frac{H_e - h_e}{H_w} \quad (3.42)$$

$$\lambda_F = \frac{H_e}{H_w} - 1 \quad (3.43)$$

$$\psi_F = \arcsin \left(\frac{2\sigma_F - \lambda_F}{\sqrt{\lambda_F^2 + 4\sigma_F}} \right) + \arcsin \left(\frac{\lambda_F}{\sqrt{\lambda_F^2 + 4\sigma_F}} \right) \quad (3.44)$$

with the static and total enthalpies h and H , respectively. The model parameters a_i and b_i are given in Table 3.6 with their different values to cover smooth, transient and fully rough regime in case of sand grain roughness

Table 3.6: Fenter parameters a_i and b_i

k_s^+	a_i	b_i
$k_s^+ \leq 5$	0	0
$5 \leq k_s^+ \leq 100$	2.84	-4.58
$100 < k_s^+$	2.5	-3

In principal, the model was designed to cover also other roughness types by adapting the model parameters a_i and b_i correspondingly. The rationale was, that the parameters were included in the derivation of eq. (3.40) in the form of a “universal” roughness function $f = \Delta U^+ = a_i \log(k_s^+) + b_i$. Eq. (3.40) can either be solved with k_s^+ as input or, since $k_s^+ = k_s / \nu_w U_e \sqrt{c_f/2 \cdot \rho_e / \rho_w}$, also only with k_s as input and iterating for the roughness Reynolds number as well.

Additionally, Fenter gave a formulation for the rough wall momentum loss thickness θ with

$$\theta = \frac{\nu_w T_e}{U_e T_w \kappa^2} \left(1 - \frac{2}{\kappa U_e^+} \right) e^{\left[\kappa \frac{U_e^+}{\sqrt{\sigma_F}} \psi_F + \kappa ((a_i \log(k_s^+) + b_i) - B) \right]} \quad (3.45)$$

and a formulation for the boundary layer thickness δ by

$$\delta = \frac{\nu_w}{u_\tau} e^{\left[\kappa \left(\frac{U_e}{u_\tau} \left(\frac{U}{U_e} \right)_{VD,e} - B + \Delta U^+ - 2 \frac{\Pi}{\kappa} \right) \right]} \quad (3.46)$$

with the smooth wall law-of-the-wall constants κ and B and the van Driest transformed velocity velocity ratio at the boundary layer edge $(U/U_e)_{VD,e}$. Additionally, Fenter [71] showed that the transformation by Van Driest from a flat plate to a cone (see equation (3.39)) is also valid for rough walls.

3.10.2 Fitting Approach

The first of the indirect methods to derive skin friction velocities from velocity data is the so-called fitting approach. For this approach, the parameters of the incompressible law-of-the-wall are modified to match a measured mean velocity profile. A variant of this method has been applied in numerous past studies (see e.g. [185], [24], [186], [285]). For smooth wall data, it is sometimes labeled “Clauser chart method” (see e.g. [46], [285]) or in case of rough walls “modified Clauser chart method” (see e.g. [186]). Usually, these methods only include velocity data in the logarithmic layer of the TBL. In contrast, in this work and comparable to e.g. Berg [24] in addition to the logarithmic part, the wake part was also included using a proper wake formulation. On the one hand, including the wake formulation increased the amount of useful data points within the boundary layer, which in turn enhanced the quality of the fitting procedure. On the other hand, this procedure does not rely entirely on the logarithmic part, which might be limited to a narrow y^+ range due to the moderate Reynolds numbers and therefore make the procedure prone to errors. The formulation used in this work can be given e.g. according to Jiménez [118] with the formulations of eq. (2.7) and (2.11), so that the goal is to minimize Φ , with

$$\begin{aligned}\Phi &= \sum_{i=0}^I (U_{i,eff} - U_i^+ u_\tau)^2 \\ &= \sum_{i=0}^I \left(U_{i,eff} - \left[\frac{1}{\kappa} \log(y_i^+) + B - \Delta U_{eff}^+ + \frac{2\Pi}{\kappa} \sin^2\left(\frac{\pi y_i}{2\delta}\right) \right] u_\tau \right)^2\end{aligned}\quad (3.47)$$

where index i represents each individual velocity measurement above the wall, extracted from Pitot rake or PIV data, within all points I of the profile. In compressible flow, inner scaled coordinates are defined with the wall parameters according to $y^+ = y u_\tau / \nu_w$ with $y = (y_T + \epsilon)$ including the wall-normal coordinate with the origin at the crest of the roughness elements y_T and the shift of origin ϵ (see figure 3.3). A detailed discussion of the parameter and its role can be found in section 7.2.2 and results of data from this thesis in section 7.2.3. Additionally, a proper transformation from compressible to an effective incompressible velocity profile U_{eff} and ΔU_{eff}^+ is necessary.

For this transformation different formulations are possible. The most widespread used is the van Driest transformation (see e.g. [261], [24]). It was derived based on the compressible mixing length idea in combination with the Crocco-Busemann equation (eq. (3.4)) postulating that solutions converges to the incompressible version in case of $Ma \rightarrow 0$ and $T_w/T_e \rightarrow 1$, which led to the following analytical approximation to the effective velocity U_{eff} :

$$U_{eff} = \frac{U_e}{\sqrt{B_{Crocco}}} \left[\arcsin\left(\frac{2B_{Crocco} \frac{U}{U_e} - A_{Crocco}}{C_{Crocco}}\right) + \arcsin\left(\frac{A_{Crocco}}{C_{Crocco}}\right) \right] \quad (3.48)$$

where A_{Crocco} and B_{Crocco} are according to eq. (3.5) and (3.6), respectively, and

$$C_{Crocco} = \sqrt{A_{Crocco}^2 + 4B_{Crocco}} \quad (3.49)$$

Although this transformation has proven to be accurate on multiple data sets for flows along adiabatic walls, different other transformations have been proposed to include the effect of heat flux (see e.g. [179], [256]). Since the profiles in this work are close to adiabatic with $T_w/T_{aw} > 0.8$, the van Driest transformation was applied (see also discussions in [171]). Selected comparisons between different transformations will be shown in this thesis (see e.g. section 7.5.2).

For this procedure and all following comparisons, values for the law-of-the-wall constants κ and B need to be defined. The two parameters are assumed to be constant and universal, which is a prerequisite for all typical equivalent sand grain approaches but is still a matter of discussion (e.g. [177], [88]) and deviations and possible dependencies can be found in the literature. E.g. White [277] gave values of $\kappa \approx 0.41$ and $B \approx 5.0$, Berg [24] used $\kappa = 0.4$ and $B = 5.0$ and Fernholz and Finley [73] in their AGARD report used $\kappa = 0.4$ and $B = 5.1$ based on a variety of experimental data. Pope [191] gives a possible value range of $\pm 5\%$ for $\kappa \approx 0.41$ and $B \approx 5.2$. Based on investigations on incompressible and compressible data So *et al.* [236] derived $\kappa \approx 0.35 - 0.52$ and $B \approx 4.0 - 6.82$ depending on velocity range and pressure gradient. Meta studies on carefully selected datasets show scattered values for both parameters [37]. If not otherwise noted, in this work $\kappa = 0.4$ and $B = 5.0$ are assumed, on the one hand to be consistent with other relevant heritage studies e.g. [24], [11], and on the other hand are these values in reasonable agreement with data from the meta study for a Reynolds number range relevant to this work [37].

In case of a smooth wall $\epsilon = 0$ and $\Delta U_{eff}^+ = 0$, so that the term in rectangular brackets in eq. (3.47) reduces to the classical smooth wall law-of-the-wall. In this case, only u_τ and Π need to be fitted, which is typically a stable procedure. Additionally, the fitting approach extracted necessary edge values from the data, like the velocity U_e and temperature T_e , and derived the corresponding Reynolds numbers Re_θ and $Re_{\delta 2}$. The edge temperature was determined based on a constant reservoir temperature assumption for both Pitot and PIV data. This assumption was tested to be valid close to the boundary layer edge (see figure 7.42). Closer to the wall, large deviations are expected due to viscous losses so that within the boundary layer the temperature profile after Crocco-Busemann was used (see section 3.2.3 eq. (3.4)). For selected PIV data, a comparison is made between different temperature models (see section 4.2, 4.3 and 7.5.2).

The general problem of applying a fitting procedure in case of a rough TBL profile is the number of free parameters. Usually, the parameters ΔU^+ , u_τ , ϵ , Π need to be found simultaneously, which makes the fitting procedure very unstable (see e.g. [24]). Further discussion is included in the result section.

Additionally, a modified method was derived and tested in the framework of this thesis. In the literature, there are already different approaches to reduce the number of parameters to ease a fitting procedure, like Krogstad *et al.* [135] who combined the rough wall law-of-the-wall eq. (2.7) formulated at the boundary layer edge and along the complete boundary layer to eliminate the need for the viscosity, B and ΔU^+ . In this thesis, a different approach was found. If the law-of-the-wall in smooth and rough formulation is believed to hold exactly, it can be shown that not all parameters are independent from each other. This was shown already by Berg [24] for the boundary layer thickness. The goal here is to show that also the rough wall friction velocity u_τ is not an independent parameter. One can derive an equation for u_τ starting from formulating the smooth (*s*) and rough version (*r*) of (2.7) at the boundary layer edge $y = \delta$, with:

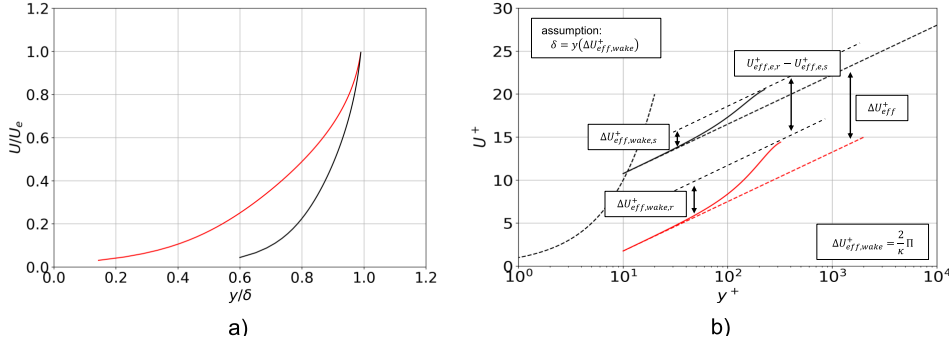


Figure 3.16: Artificially generated TBL profiles for smooth wall (black) and rough wall (blue) by method to reverse the law-of-the-wall in a) terms of U/U_e against y/δ and b) U^+ against y^+ together with terms in equation (3.52).

$$U_{eff,e,s}^+ = \frac{U_{eff,e,s}}{u_{\tau,s}} = \frac{1}{\kappa} \log(\delta_s^+) + B + \frac{\Pi_s}{\kappa} W(1) \quad (3.50)$$

$$U_{eff,e,r}^+ = \frac{U_{eff,e,r}}{u_\tau} = \frac{1}{\kappa} \log(\delta_r^+) + B - \Delta U_{eff}^+ + \frac{\Pi_r}{\kappa} W(1) \quad (3.51)$$

One of the main parameters of interest along rough walls is the roughness function ΔU_{eff}^+ . It is defined as a constant shift in the logarithmic layer of the corresponding TBL profiles. But inspecting eq. (3.51) carefully together with a representation of artificially created profiles in figure 3.10.2, it is clear that the shift is relevant for the complete profiles, including the wake. By subtracting the smooth from the rough equation, (3.51)- (3.50), one gets after some reformulations:

$$u_\tau = U_{eff,e,r}^+ \left[U_{eff,e,s}^+ + \frac{1}{\kappa} \left[\log \left(\frac{\delta_r u_\tau}{\nu_w} \right) - \log(\delta_s^+) \right] - \Delta U_{eff}^+ + \frac{2}{\kappa} (\Pi_r - \Pi_s) \right]^{-1} \quad (3.52)$$

This equation shows a dependency of rough wall u_τ from different parameters, including new necessary input with Π_s , δ_s^+ and $U_{eff,e,s}^+ = U_{eff,s}^+(y = \delta)$, which can be extracted from a corresponding smooth wall profile at comparable flow conditions. This equation can be solved for u_τ either iteratively or by applying the Lambert W function. By utilizing eq. (3.52) together with eq. (3.47) for a fitting procedure, reduces the unknowns by one to only ΔU^+ , ϵ , Π . If ϵ can be found by one of the several methods (see section 7.2.2), fitting of two parameters becomes much more stable than without equation (3.52).

There are some caveats to this approach. First, the exact formulation of the rough law-of-the-wall needs to be accepted, including that κ , B are universal constants. Additionally, the wake formulation $\Delta U_{wake}^+ = \Pi/\kappa W(1)$ is assumed to be located exactly at the boundary layer edge, which is not so straight forward to define in compressible TBL and is also a matter of on-going debate (see e.g. [185], [272], [157]).

For this study, this approach was tested with three different sets of data. First, artificial velocity profiles were generated and the resulting scaled profiles were used to test eq. (3.52). Second, DNS data of incompressible TBL along square bar roughness was used, where all parameters can be independently derived from the data set [145]. Third, DNS data of compressible TBL along 3D block roughness was used [7]. Additionally, the complete fitting procedure, by applying eq. (3.48) and (3.52), was tested on the first two. All results can be found in table 7.9.

To derive the artificial velocity profiles, equation (2.7) together with the van Driest transformation is reversely solved to result in a formulation for the compressible velocity U . This procedure involved using the law-of-the-wall formula (like eq. (2.7)) as starting point by prescribing values for ΔU_{eff}^+ , Π , y^+ , ϵ , δ , ν_w , inserting the transformation according to van Driest (eq. (3.48)) and solving this equation for the absolute velocity U to result in:

$$U = \frac{U_e}{2A_{Crocco}^2} \left[B_{Crocco} + D_{Crocco} \cdot \sin \left(\frac{1}{U_e \kappa} \left[-U_e \kappa \cdot \arcsin \left(\frac{B_{Crocco}}{D_{Crocco}} \right) + u_\tau^2 A_{Crocco}^2 B_\kappa \cdot \log \left(\frac{y u_\tau}{\nu_w} \right) - u_\tau A_{Crocco} \kappa \Delta U^+ + u_\tau A_{Crocco} 2\Pi \cdot \sin \left(\frac{\pi y}{2 \delta} \right)^2 \right] \right] \right] \quad (3.53)$$

This formulation is used to create artificial smooth or rough profiles which can be imported as $U = f(y)$ to the fitting routines for testing purposes. This can be found in section 7.2.1.

3.10.3 Gradient Approach

If the velocity profile in the viscous sublayer close to the wall is available, the wall shear stress $\tau_w = (\mu \partial U / \partial y)_w$ may be approximated by the velocity gradient at the wall, with a differential quotient via

$$\tau_w \approx \mu_w \left(\frac{\Delta U}{\Delta y} \right)_w \quad (3.54)$$

The high wall-normal resolution of the interrogation windows $512 \times 8 \text{ pix}^2$ profiles (given as streamwise x wall-normal dimensions in pixel) of the first PIV campaign, enabled a distance from the wall for the first data point of $iy(1) = 4 \text{ pix}$ which corresponds to $y(1) = 0.03 \text{ mm}$, $y/\delta(1) = 0.0087$, $y^+(1) = 1.79$ which is located well within the viscous sublayer (see figure 4.12). Due to the derivation of the van Driest velocity transformation, its applicability is limited to the logarithmic layer [73], [285]. For the viscous sublayer an alternative velocity transformation can be derived from the shear stress budget in the sublayer with the assumption of negligible turbulent stress contribution, an exponential viscosity law ($\mu = \mu_w (T/T_w)^\omega$) setting the exponent $\omega = 1$, the Crocco temperature distribution, $dp/dx = 0$ and constant wall temperature T_w according to [73] with

$$U_{VS} = U \left(1 + \frac{B_{Crocco}}{2} \left(\frac{U}{U_e} \right) - \frac{A_{Crocco}}{3} \left(\frac{U}{U_e} \right)^2 \right) \quad (3.55)$$

with factors A_{Crocco} and B_{Crocco} according to eq. (3.5) and (3.6), respectively.

For the PIV data in this thesis, the transformation according to equation (3.55) is compared to the nominal van Driest transformation and showed only minor differences in the sublayer, which fell below the corresponding uncertainties (see chapter 4.2.2, figure 4.10). Different developments to derive a scaling which is applicable to both, the viscous sublayer and log-layer, were discussed e.g. by Patel *et al.* [179] and Trettel and Larsson [256]. Trettel and Larsson [256] derived a scaling tested on supersonic channel and boundary layer cases with non-adiabatic wall conditions, including the effect of wall heating. To categorize the impact of wall heating the authors derived a dimensionless parameter with $B_q = \dot{q}_w / (\rho_w c_p u_\tau T_w)$, where $\dot{q}_w$ represents the wall heat flux (negative, if wall is heated). Low absolute values of B_q represent near adiabatic conditions. For boundary layer cases with $|B_q| < 0.069$ only small differences between the introduced and the classical van Driest scaling were visible [256]. For the boundary layer discussed in this paper a value of $|B_q| \approx 0.02$ can be approximated, based on infrared thermography data. This represents a case with only limited heating of the wall ($T_w/T_{aw} \approx 0.8$) which explains the minor differences to the van Driest scaling. Therefore, the van Driest scaled velocities were used in the following analysis.

To estimate the shear stress, the wall closest velocity points were used to estimate the corresponding partial derivative by a difference quotient. The necessary wall viscosity μ_w was calculated via the Sutherland law (extended for low temperatures according to [1], see eq. (7.9)) based on the corresponding wall temperature from infrared thermography. The velocity difference was calculated for all points within $y^+ \lesssim 15$, assuming zero velocity at the wall.

3.10.4 Integral Approach

The integral approach is based on a method originally implemented to approximate values along a rough flat plate wall according to Latin [138]. For this approach the momentum integral equation is solved in between two consecutive positions along the surface by implementing a functional dependence of the kind $c_{f,e} = f(\theta)$. This function can be derived with the law-of-the-wall solved at the boundary layer edge. In case of the rough version of the law-of-the-wall along a flat plate, the resulting momentum integral can be solved analytically [138]. In case of a compressible smooth cone flow, an analytical solution is not possible anymore, which will be shown in the following.

In contrast to the von-Kármán integral, which is already integrated along the wall-normal direction, additional methods have been suggested which use the wall-normal varying terms of the momentum equation and are then numerically integrated afterwards. Partly, ideas of multiple integration of the equations are used, like Fukagata-Iwamoto-Kasagi (FIK) identity, to work out main contributing terms to the skin friction. During the course of the thesis, some methods have been tested but were omitted later, mainly due to the challenges of measuring the Reynolds

shear stress component properly in the hypersonic TBL, which is one of the main contributors to the momentum equation. Additional information can be found in [36], [35], [91], [159].

The so-called von-Kármán-integral [127], which can be derived from the x -momentum equation with the assumption of a thin boundary layer, can be formulated along a flat plate or body of revolution for a compressible boundary layer according to [71]:

$$\frac{d\theta}{dx} + \frac{\theta}{M_e} \left(\frac{H + 2 - M_e}{1 + \frac{\gamma-1}{2} M_e} \right) \frac{dM_e}{dx} + j \cdot \frac{\theta}{r} \frac{dr}{dx} = \frac{c_{f,e}}{2} \quad (3.56)$$

with the edge Mach number M_e , the shape parameter $H = \delta^*/\theta$, radius r , axial distance x and skin friction $c_{f,e}$. The parameter j is 0 in case of two-dimensional flow (boundary layer along a flat plate) and 1 for axially symmetric flow (along a cone). In the latter case, r is the radial coordinate of a body of revolution and x gets the stream-wise running length measured along the surface. The equation is accurate for high Reynolds number flows along flat plates or slender bodies [71].

Several assumptions will be made in the following to simplify equation (3.56). For negligible pressure gradient along the body and negligible viscous interaction between boundary layer and external field, the term $dM_e/dx \approx 0$. Furthermore, along a cone constant edge parameters and wall temperature are assumed and when additionally the transformation $\theta/r \cdot dr/dx = \theta/(\sin(\phi)x) \cdot \sin(\phi) = \theta/x$ is used, equation (3.56) simplifies to

$$\frac{d\theta}{dx} + j \cdot \frac{\theta}{x} = \frac{c_{f,e}}{2} \quad (3.57)$$

With a function in the form $c_{f,e} = f(\theta)$ equation (3.57) is solved analytically or numerically with additional inputs. One option for this type of function can be derived with the law-of-the-wall. Another one is to find suitable skin friction approximations from literature.

We concentrate first on the law-of-the-wall, which is assumed here to be valid throughout the complete TBL. First a formulation is found for smooth wall flows and after that for rough wall flows.

For a compressible turbulent boundary layer along a smooth wall the law can be given by eq. (2.7) and (2.11). To find a value for the skin friction coefficient $c_{f,e}$, eq. (2.7) is formulated at the boundary layer edge and can be given with:

$$\frac{U_{eff,e}}{u_\tau} = \frac{1}{\kappa} \log \left(\frac{\delta u_\tau}{\nu_w} \right) + B + 2 \frac{\Pi}{\kappa} \quad (3.58)$$

The skin friction velocity can be expressed as skin friction coefficient via (3.41) assuming $p_e = p_w$ and the right hand side constants in eq. (3.58) can be combined to one with $B' = B + 2\Pi/\kappa$. Additionally, to transform the boundary layer thickness in eq. (3.58) to the momentum loss thickness it is assumed that $\delta/\theta = const.$, which is an acceptable approximation along a cone, apart from the region close to the nose (see figure 7.7). With the above assumptions eq. (3.58) can be transformed to a functional dependence of $c_{f,e} = f(\theta)$:

$$\sqrt{\frac{2}{c_{f,e}}} \sqrt{\frac{T_e}{T_w}} \frac{U_{eff,e}}{U_e} = \frac{1}{\kappa} \log \left(\frac{\theta U_e \sqrt{\frac{c_{f,e}}{2} \frac{T_w}{T_e}}}{\nu_w} \right) + B' \quad (3.59)$$

Equation (3.59) can either be solved iteratively for $c_{f,e}$ or it may be solved via the Lambert W function. For the latter it may be re-written to a form like:

$$a \cdot x_L = b \cdot \log \left(\frac{c}{x_L} \right) + d \quad (3.60)$$

with $b = 1/\kappa$, $d = B'$, $x_L = \sqrt{2/c_{f,e}}$ and $a = 1$, $c = \theta U_e / \nu_w = Re_{\delta 2}$ for incompressible and $a = \sqrt{\frac{T_e}{T_w}} \frac{U_{eff,e}}{U_e}$, $c = \theta U_e \sqrt{\frac{T_w}{T_e}} / \nu_w = \theta U_e \sqrt{\frac{\rho_e}{\rho_w}} / \nu_w =$ for compressible flow. This form of function can be solved for x_L via the Lambert W function by:

$$x_L = \frac{b}{a} LambertW \left(\frac{c \cdot a}{b} e^{\frac{d}{b}} \right) \quad (3.61)$$

The real part of the solution for x_L can be used to extract the skin friction coefficient via $c_{f,e} = 2/\Re(x_L)^2$.

In case of a fully rough wall, the law-of-the-wall can be given via eq. (2.5), re-formulated at the boundary layer edge to include the skin friction coefficient to:

$$\sqrt{\frac{2}{c_{f,e}}} \sqrt{\frac{T_e}{T_w}} \frac{U_{eff,e}}{u_e} = \frac{1}{\kappa} \log \left(\frac{\delta}{k_s} \right) + B'_r \quad (3.62)$$

with $B'_r = 8.5 + 2\Pi/\kappa$ to become finally

$$c_{f,e} = \frac{2a^2}{\left(\frac{1}{\kappa} \log \left(\frac{\theta}{k_s} \right) + B'_r \right)^2} \quad (3.63)$$

with parameter a and the assumption $\delta/\theta = const.$, as before.

Alternatively, instead of the law-of-the-wall, for $c_{f,e} = f(\theta)$ in eq. (3.57) also the correlations according to Fenter [71] can be used, which can be given for the smooth and rough wall by

$$c_{f,e} = \left[\frac{\sqrt{\sigma_F}}{\psi_F} \sqrt{\frac{T_w}{T_e}} (4.13 \log(Re_{\theta,e}) - \frac{1}{\sqrt{2}} [a_i \log(k_s^+) + b_i] + 4.13\omega \log \left(\frac{T_e}{T_w} \right) + 2.9) \right]^{-2} \quad (3.64)$$

with the parameters and validity range as given in section 3.10.1.

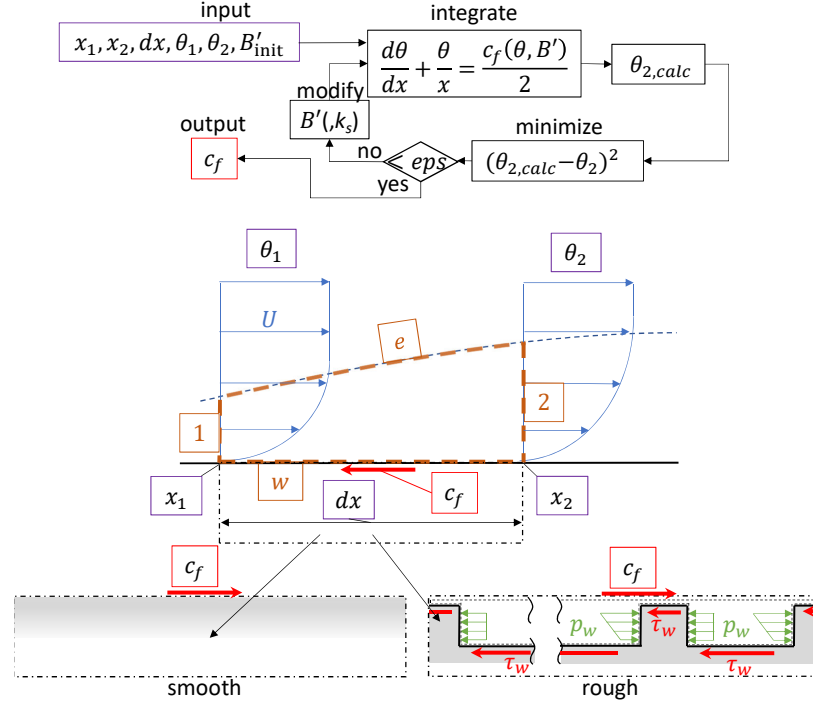


Figure 3.17: Integral approach scheme (upper part shows the iteration loop to numerically solve eq. (3.57), lower part sketches the applied momentum balance along the boundary layer and its boundaries for a smooth wall after Kármán [127] and rough wall after Perry *et al.* [187] with contributing components).

With eq. (3.61) and (3.40) there are two options to close the von-Kármán-integral equation (3.57) in case of a smooth wall. Equivalently, with eq. (3.63) and (3.40) there are two options to close equation (3.57) in case of a rough wall.

To solve in either case equation (3.57) for the unknown $c_{f,e}$, the following procedure is performed (see also figure 3.17 which shows the approach schematically). Equation (3.57) is solved for θ numerically via the solver “vode” and method “bdf” of the Python SciPy package. The independent variable for integration is the running length x in equation (3.57), for which in our case a fixed step-size is defined beforehand. The initial values for x_1 and θ_1 are the values measured at MS1 from either Pitot rake or PIV measurements. Equation (3.57) is numerically integrated until x_2 corresponding to MS2. This integration is iteratively repeated with a modified parameter B' in case of using the law-of-the-wall formulation (eq. (3.61) or (3.64)) or k_s^+ in case of the Fenter formulation (eq. (3.64)), until the resulting θ_2 corresponds to the measured value at MS2 to within a pre-defined accuracy eps . In case of a rough wall, an effective average wall shear stress is derived, which should consider wall shear stress τ_w along the flow parallel wall segments and pressure forces p_w acting on the wall segments perpendicular to the flow direction (see figure 3.17 and Perry *et al.* [187]).

This procedure was successfully cross-checked with the original implementation of Latin [138]. For this cross-check all parameters needed to be forced to their incompressible formulations

(a and c in eq. (3.61)), since compressibility effects were surprisingly not accounted for in the corresponding derivation. The procedure was tested and validated with numerical data along a flat plate and cone derived by the codes described in chapter 3.9 and 3.8 (see section 7.2.1). The resulting deviations of the skin friction coefficient is typically $|\Delta c_{f,e}| \leq 1.2\%$ (see table 7.10 in section 7.2.1). The impact of the resolution of velocity profiles from PIV was estimated with artificially under resolved profile data from CFD. This resulted in typically $|\Delta c_{f,e}| \leq 6\%$ (see figure 7.9 and table 7.11 in section 7.2.1).

The approach was applied on the PIV and Pitot data along the smooth and rough cone and compared to each other and to analytical and numerical data in section 4.2.

3.11 Stanton Number Approximations

One way to approximate the heat flux in terms of the non-dimensional Stanton number is the so called Reynolds analogy. It assumes that wall shear and heat-transfer rates are proportional and can be given according to [263] by:

$$\frac{St_e}{c_{f,e}} = \frac{1}{2} \frac{1}{s_{ra}} \quad (3.65)$$

with s_{ra} as the Reynolds analogy factor, which depends on the state of the boundary layer. For an incompressible laminar boundary layer with a Prandtl number close to unity $s_{ra} = 1$ [277] and equation (3.65) reduces to its simplest form. For incompressible turbulent boundary layers assuming $Pr_t = 1.0$, White [277] gives an approximation for the factor with:

$$s_{ra,W} = 1 + 13 \left(Pr_t^{\frac{2}{3}} - 1 \right) \sqrt{\frac{c_{f,e}}{2}} \quad (3.66)$$

Van Driest [263] derived a compressible turbulent factor for variable turbulent Prandtl numbers with

$$s_{ra,vD} = \left[1 + 5\sqrt{c_{f,e}} \frac{1}{5\kappa} (1 - Pr_t) \left(\frac{\pi^2}{6} + \frac{3}{2} (1 - Pr_t) \right) + \left(\frac{Pr}{Pr_t} - 1 \right) + \log \left(1 + \frac{5}{6} \left(\frac{Pr}{Pr_t} - 1 \right) \right) \right] \quad (3.67)$$

In chapter 4.4 a comparison between selected factors will be presented. If not otherwise noted, in this thesis a turbulent Prandtl number $Pr_t = 0.9$ was used.

Although early investigators assumed the Reynolds analogy still to be applicable in case of rough walls (see e.g. [71]), it is now assumed that it is strictly only valid along smooth walls, since a decoupling of momentum and heat transfer is expected (see e.g. [294], [172], [45], [162]). An

excellent overview of skin friction and Stanton number correlations for rough surfaces is given by Seidman [223].

Nestler *et al.* [172] introduced a Stanton number heating augmentation correlation, based on incompressible rough wall Reynolds analogy ideas after Owen [178], and modified it for compressible flow to result in

$$\frac{St_r}{St_s} = \frac{c_{f,r}}{c_{f,s}} \left[1 + 0.52 \left(\frac{c_{f,r}}{c_{f,s}} \right)^{0.725} \left(\frac{u_{\tau,s}}{U_e} \right) \left(\frac{ku_{\tau,s}}{\nu_w} \right)^{0.45} (Pr)^{0.8} \right]^{-1} \quad (3.68)$$

where the combination of terms $ku_{\tau,s}/\nu_w$ represent the roughness Reynolds number based on the smooth wall skin friction velocity $u_{\tau,s}$. His heat transfer augmentation is dependent on the augmentation of skin friction $c_{f,r}/c_{f,s}$ for which he also proposed an approximation for the skin friction augmentation based on work of Goddard [90] and Young [294] to be applicable on adiabatic and non-adiabatic walls.

Seidman [223] and Hill *et al.* [101] suggested a correlation, based on work by Owen and Thomson [178] for incompressible flow, to derive a modified Reynolds analogy for the rough wall case:

$$St_r = \frac{c_{f,r}}{2} \left[1 + \alpha_S \left(\frac{c_{f,r}}{2} \right)^{0.725} (Re_{k_{s,e}})^{0.45} (Pr)^{0.8} \right]^{-1} \quad (3.69)$$

where α_S is an adjustable constant in the range of 0.4 to 1.3. [223] used $\alpha_S = 0.52$, but in the current thesis a value of $\alpha_S = 0.75$ was used, as in [32]. Since eq. (3.69) needs a value for the rough skin friction $c_{f,r}$, multiple options are possible to calculate a rough Stanton number, either via experimental data or proper skin friction correlations.

Another correlation is the so-called ‘‘Powars’’ or sometimes PANT correlation which is highly used, especially in American literature. The correlation was reported in Hochrein and Wright [103] and some details were given in Wilder *et al.* [281], of which relevant information are repeated in the following. The Passive Nostip Technology Program was conducted in a Mach 5 wind tunnel flow. The correlation was based on calorimeter measurements along surface roughness, generated either by grit blasting or by brazing particles. The measurements were taken along the blunt nosetip of a blunted cone, so that typically $Ma_e < 1$. The conditions were in the range $T_w/T_0 = 0.4 - 0.7$ and $Re_\infty \approx 1.6 - 66 \cdot 10^6 \text{ 1/m}$ (or, based on the nose radius $Re_{rn} = 0.1 - 4.2 \cdot 10^6$). The (equivalent) sandgrain roughness based correlation was formulated according to [103] with

$$\frac{St_r}{St_s} = \begin{cases} 1.0, & \text{if } n_t \leq 10 \\ \frac{2}{3} \log_{10}(n_t) + \frac{1}{3}, & \text{if } 10 < n_t < 10^4 \\ 3.0, & \text{if } n_t > 10^4 \end{cases} \quad (3.70)$$

where

$$n_t = Re_{k_{s,e}} \left(\frac{T_e}{T_w} \right)^{1.3} \sqrt{C_{h,s}} = \frac{\rho_e U_e k_s}{\mu_e} \left(\frac{T_e}{T_w} \right)^{1.3} \sqrt{C_{h,s}} \quad (3.71)$$

where the $C_{h,s}$ indicates the use of the smooth wall Stanton number, based on the difference of total to wall enthalpy $C_{h,s} = \dot{q}_c / (\rho_e U_e (h_t - h_w))$. Wilder *et al.* [281] argued that the correlating parameter n_t was an approximation of the roughness Reynolds number, relating the sublayer thickness to suitable boundary layer edge and surface properties and they set $n_t = k_s u_{\tau,s} / \nu_w$ in their analysis. In this thesis n_t is calculated from eq. (3.71) as in the original paper. Wilder and Prabhu [280] stated a typical uncertainty for the correlation of $\pm 20\%$.

The dependence argued by Wilder *et al.* [281] is in contrast to the typical rough wall correlations (e.g. [173], [212], [90], [203], etc.) where the augmentation is always correlated by the roughness Reynolds number defined with the rough wall friction velocity. Although, rather practical from an engineering perspective, since only smooth wall data are necessary for the prediction of the roughness induced augmentation, it is believed here that the general physical correlation makes more sense when the data is correlated by the rough wall skin friction. This discussion will be continued in section 4.4.

3.12 Equivalent Sand Grain Roughness Extraction Approach

Several different steps are necessary to extract the Equivalent Sand Grain Roughness (EQSR) $\alpha = k_s/k$ from experimental data. The idea for the concept of EQSR and so the methods date back to Schlichting [212]. It represents a hydraulic scale, which represents a hypothetical surface with a densely packed sand grain k_s that would cause the same drag as the surface of interest if exposed to the same flow in the fully rough regime.

3.12.1 General Equations

To find this scale, velocity data must first be scaled according to eq. (2.5) or (2.7). This scaling requires the skin friction velocity u_τ to be known, which is a challenging quantity to be derived for experimental investigations, especially in hypersonic flows. Apart from that, additional quantities need to be known as well, like the shift of origin ϵ or Hama's roughness function ΔU^+ . An additional challenge for compressible flows is that a proper scaling is necessary to enable comparison to the incompressible theories of e.g. the law-of-the-wall. Most of the necessary parameters and strategies to find them was summarized in section 3.10, which addressed the skin friction velocity. In the current section, strategies will be presented to build upon the values found by the methods in section 3.10 and to result in an estimate for EQSR. Additional information will be given about the shift of origin ϵ and the law-of-the-wall constants. Finally, an uncertainty analysis will be reported.

With eq. (2.5), (2.7) and (2.11), the main equations for the extraction of an EQSR value were given. The EQSR is typically found based on data in the fully rough regime, for which the mentioned equation (2.5) results in (now in compressible formulation with U_{eff})

$$U_{eff}^+ = \frac{U_{eff}}{u_\tau} = \frac{1}{\kappa} \log \left(\frac{y}{k_s} \right) + B_{r,s}(\infty) + \frac{\Pi}{\kappa} W \left(\frac{y}{\delta} \right) \quad (3.72)$$

for which $B_{r,s}(\infty) = 8.5$ is assumed just like the incompressible result by [173]. The second version used for EQSR extraction is the law-of-the-wall (eq. (2.7)):

$$U_{eff}^+ = \frac{1}{\kappa} \log(y^+) + B - \Delta U_{eff}^+ + \frac{\Pi}{\kappa} W\left(\frac{y}{\delta}\right) \quad (3.73)$$

If the roughness function ΔU_{eff}^+ is known from TBL data (e.g. by a fitting procedure like described in section 3.10.2) and presuming these data to be in the fully rough regime, an equivalent sand grain roughness EQSR k_s/k can be found by combining eq. (3.72) and (3.73) to

$$k_s^+ = e^{\kappa(B_{r,s}(\infty) - B + \Delta U_{eff}^+)} = e^{0.4(3.5 + \Delta U_{eff}^+)} \quad (3.74)$$

with the necessity to use values of $\kappa = 0.4$ and $B = 5.5$ according to [173] (see section 7.3.3), so that $B_{r,s}(\infty) - B = 8.5 - 5.0 = 3.5$. With a value for k_s^+ one gets the EQSR k_s/k via

$$\frac{k_s}{k} = \frac{k_s^+}{k^+} = k_s^+ \frac{\nu_w}{k u_\tau} = e^{0.4(3.5 + \Delta U_{eff}^+)} \frac{\nu_w}{k u_\tau} \quad (3.75)$$

As stated above, eq. (3.75) is only valid for fully rough flow. In contrast to the sand grain roughness limit of $k_s^+ \approx 60 - 70$ data by Bandyopadhyay [19] on transverse bars ($\lambda/k = 3.8$, $w/k = 0.75$) showed a much lower limit with $k^+ \approx 8$ ($\Delta U^+ \approx 5.5$) (see figure 2.5 and figure 26 in the original reference). The EQSR of Bandyopadhyay's roughness can be extracted to $k_s/k \approx 2.7$ from published data and eq. (3.74) using the largest Reynolds number data at $k^+ \approx 56$ ($k_s^+ \approx 151$). These data already show the clear log-line trend (see figure 2.5) which is also confirmed by data of other investigators like Moore [164] or Perry [185], although on slightly different roughness topologies ($\lambda/k = 3.9$, $w/k = 1$ and $\lambda/k = 4$, $w/k = 1$, respectively). According to Bandyopadhyay's data, the limit for fully rough flow can be approximated with $k^+ \approx 10$ to $k_s^+ \approx 27$ accordingly being much lower than for sand grain roughness.

The transverse square bar roughness of interest in this thesis is a 2D roughness with $\lambda/k \approx 4$, $w/k = 1$, $i/k = 3$ (see also figure 3.3). This is very close to the aforementioned roughness by Bandyopadhyay, Moore and Perry. This topology is on the verge of being d- or k-type roughness (see section 2.1) but since it falls into the category of the latter based on all aforementioned limits for TSB roughness from literature [118], [10], [19], [24], we assume in the following the roughness to be of k-type. Nevertheless, corresponding scalings for both types will be tested on data of this thesis in section 4.2.

Additionally to eq. (3.74), in this thesis a different approach to extract EQSR values was tested, which uses another combination of the relevant equations in the spirit of the previously mentioned methods according to [135] or for eq. (3.52). With a different combination, a formulation can be found, which avoids the need to know the skin friction velocity u_τ which is known to be notoriously difficult to be measured. By combining the above equation as follows $((3.73))/((3.73))_e$, a formulation can be found with:

$$\frac{U^+}{U_e^+} = \frac{U}{U_e} = \frac{\frac{1}{\kappa} \log\left(\frac{y}{k_s}\right) + 8.5 + \frac{\Pi}{\kappa} W\left(\frac{y}{\delta}\right)}{\frac{1}{\kappa} \log\left(\frac{\delta}{k_s}\right) + 8.5 + \frac{\Pi}{\kappa} 2} \quad (3.76)$$

In eq. (3.76) the parameters to be fitted to match a measured velocity profile U/U_e are Π , ϵ and k_s . This approach was successfully tested with artificial velocity profiles according to eq. (3.53) (see section 7.3.1). This second method will be applied on selected data in section 4.

3.12.2 Uncertainty Estimation

As highlighted by several different authors, the extraction of EQSR is intricate (to put it in Schlichting [212] words: “verwickelt”), already for incompressible investigations (e.g. [185], [118]). This is especially true with the additional challenges for measurements of compressible TBLs (see e.g. [24]). Furthermore, due to the challenges of the different methods for a post-processing to extract EQSR (see section 3.10.2 to 3.10.4) only very few authors addressed the corresponding uncertainties of the extracted values. Bhatt and McClain [26] re-evaluated the experiments by Schlichting [212] and is one of the few who included an analysis of the combined uncertainties for the EQSR. They stem from uncertainties in the position and velocity of a profile measurement, the shift of origin and skin friction velocity in an approach based on a Taylor Series expansion Method (TSM) (see also [48] from which the following terminology is borrowed). For this method (sometimes also framed Gaussian uncertainty propagation) one needs to find the partial derivatives of the propagating terms with respect to all input uncertainties. This can be difficult, especially when methods are used which do not have a straightforward functional dependency from input to output, like some iterative methods or numerical integrations. In cases where Bhatt and McClain [26] had no direct functional dependency, i.e. deriving the shift of origin, they used an approximation to the gradient with a finite difference scheme. The advantage of TSM is that a representation for the uncertainty contributions from different input values to the results are directly available with the partial derivative terms. This enables the identification of main uncertainty contributors which can be used to design future measurement setups according to the accuracy of the corresponding measurements.

Instead, in this thesis the EQSR uncertainties were analyzed via MCM, as is the case for the skin friction velocity uncertainties (see section 7.2.1). The general idea of using MCM is sampling uncertainties drawn from assumed distributions for every input quantity and it simulates running the post-processing many times with a different set of sampled uncertainties each time [48]. This is shown schematically in figure 3.18, in which the uncertainty of skin friction velocity U_{u_τ} represents the input uncertainties in the upper left box (including a box plot to show statistics of the input, assumed to be evenly distributed) which is fed into to the fitting approach (see section 3.10.2, which is represented by exemplary data in the graph on the right). After fitting and using the results for ΔU^+ in eq. (3.74) the resulting EQSR values $\alpha = k_s/k$ and its uncertainties U_α from all Monte Carlo runs can be evaluated. Here and in the following, U_Q denotes the uncertainty of quantity Q . From the corresponding uncertainty distribution a percentile distribution can be extracted, which represents the uncertainty U_r of the result quantity r (lower left box, including box plot to show statistics of the output).

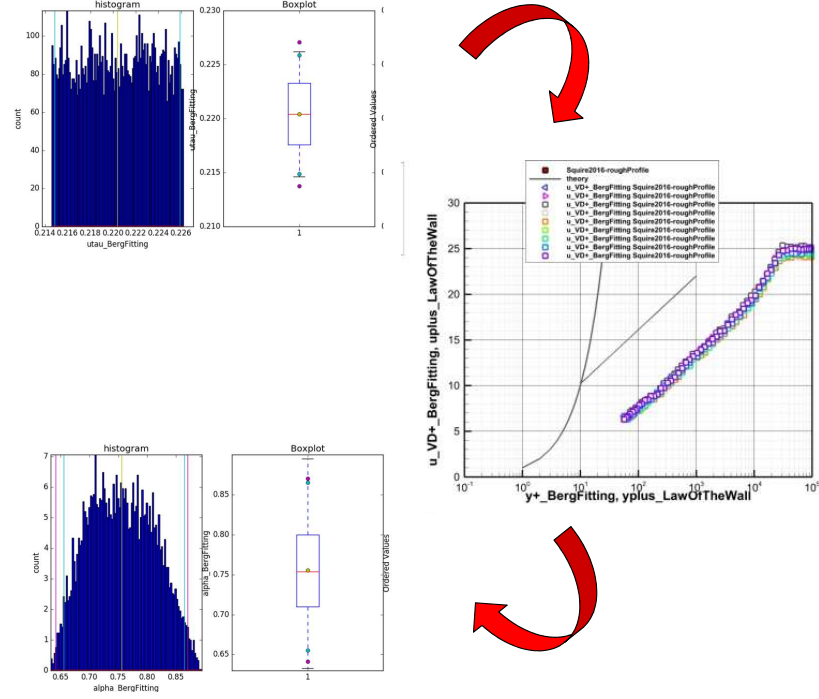


Figure 3.18: Scheme of MCM EQSR uncertainty analysis approach.

Additional to the total uncertainties U_r for each resulting quantity r (r representing either Π , ΔU^+ or α), with the UPC and UMF the data are presented in terms of two different uncertainty parameters to extract further inside into the impact and contributions of different uncertainties U_X of each input quantity X . These two parameters are usually defined based on TSM (see [48]) but were extracted in this thesis based on MCM results, according to the following procedures and definitions. The following equations are given exemplary for the result quantity $r = \alpha = k_s/k$ but the post-processing can be applied equivalently to the other two quantities $r = \Delta U^+$ and Π .

In contrast to the evaluation of total uncertainty U_α , for which all input quantities were biased simultaneously within every post-processing run, now for each input quantity X_i an individual Monte-Carlo simulation was performed, for which all input uncertainties except the one of interest (U_{X_i}/X_i) were set to zero. Here, i ranges from 1 to the total number of input values I , with e.g. $U_{X_1}/X_1 = U_{u_\tau}/u_\tau$ as shown in figure 3.18. Each Monte-Carlo run gave an uncertainty for the resulting quantity α with U_{α_i} . These individual contributions add up as square root of the sum of the squares to the total uncertainty of the result $(U_\alpha/\alpha)_{total}$ (which can be checked by a MCM run with all input uncertainties switched on). The uncertainty percentage contribution (UPC) for each input quantity from MCM was then estimated according to Stephens *et al.* [243] with:

$$UPC_i = \frac{(U_{\alpha_i}/\alpha_i)^2}{(U_\alpha/\alpha)_{total}^2} \cdot 100 = \frac{(U_{\alpha_i}/\alpha_i)^2}{\left(\sqrt{\sum_{k=1}^I (U_{\alpha_k}/\alpha_k)^2}\right)^2} \cdot 100 \quad (3.77)$$

for all input variables I where all uncertainties in equation (3.77) and reported in the following represent the percentage value based on the mean for a Confidence Interval (CI) of 95 %.

The uncertainty magnification factor (UMF) should give an indication about the influence of the corresponding input uncertainty on the overall uncertainty of the result [48] and was estimated based on the MCM results for α with:

$$UMF_i = \frac{(U_{\alpha_i}/\alpha_i)}{\sqrt{\sum_{k=1}^I (U_{X_k}/X_k)^2}} \quad (3.78)$$

An UMF greater than one indicates that the influence of the uncertainty in the variable is magnified as it propagates through the data reduction equation into the result whereas a value below one means the opposite.

The implemented MCM framework was first cross-checked with examples from Coleman and Steele [48] and comparisons to the TSM results by Bhatt and McClain [26] (although he missed to include the uncertainty from the rough wall skin friction velocity, which was added in the analysis of this thesis).

Before it was used on data of this thesis, this method was first used on representative published cases of incompressible and compressible roughness experiments. The resulting uncertainties are not necessarily the “real” uncertainties for the corresponding studies, since on the one hand the analysis framework (fitting approach) of this thesis is used which may differ to the one of the corresponding author and on the other hand input uncertainties needed to be estimated if not fully reported. But it can give an order of magnitude for the comparison of the data in this thesis. The representative literature cases included two incompressible cases [212], [240] as well as a supersonic [69] and two hypersonic cases [208], [24]. The detailed results can be found in section 7.3.2. The main takeaways can be summarized as follows. In general, for compressible data ([24], [69], [208]) both the roughness function ΔU^+ and the EQSR $\alpha = k_s/k$ are prone to higher uncertainties than for incompressible data ([240], [26]), which was expected due to e.g. partially higher uncertainties in measurement quantities as well as additional uncertainties due to e.g. van Driest transformation. For the EQSR up to $U_\alpha \approx 10$ % and up to $U_\alpha \approx 30$ % need to be taken into account for incompressible and compressible investigations, respectively. As already stated, these values need to be considered as order of magnitude results. The results show that for investigations up to supersonic velocities ([240], [26], [69]), the main contributor to the EQSR uncertainty (ΔU^+ or α) is the uncertainty of the skin friction velocity u_τ , whereas for hypersonic cases ([24], [208]) also the velocity, pressure and Mach number uncertainties can play a role. So, for compressible cases, it is most beneficial to increase the accuracy of velocity measurement and skin friction velocity estimation.

This result was taken into account during the course of this work and was incorporated into the transition from the first to the second PIV measurement setup. Input quantity uncertainties from the first and second PIV campaign of the data in this thesis (see measurement uncertainties in section 7.1.1) which were fed into the EQSR uncertainty analysis can be found in table 3.7. The shift of origin uncertainty was conservatively assumed evenly distributed within $U_\epsilon = 0 - 1k$. Results for the EQSR uncertainty analysis can be found in figures 3.19 to 3.21. In general a similar picture is visible compared to the literature data.

In general, by moving from the first to the second PIV campaign, corresponding input (measurement) uncertainties were either kept the same or could be reduced, due to e.g. upgrades in PIV hardware. This is the case e.g. for the velocity and skin friction velocity uncertainty (see figure 3.19a, which shows a quasi UPC of all input uncertainties to see the relative impact).

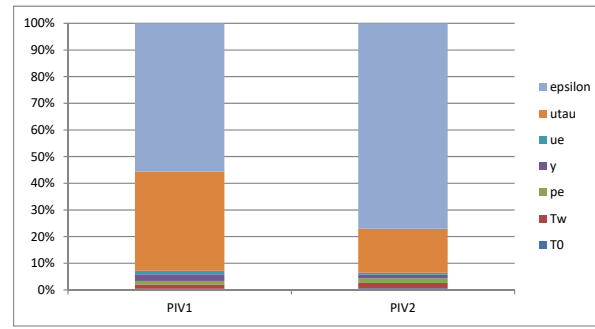
This translated into substantially reduced uncertainties in the resulting EQSR parameters, represented in figure 3.19b by the Coles wake function Π (Pi), the roughness function ΔU^+ (DU+) and the normalized EQSR $\alpha = k_s/k$ (alpha). The resulting EQSR parameters were reduced from $U_\alpha \approx 140\%$ to $U_\alpha \approx 50\%$ and $U_{\Delta U^+} \approx 120\%$ to $U_{\Delta U^+} \approx 30\%$, visible in figure 3.19b. The main contribution came from the uncertainty in skin friction velocity, which could be reduced between the PIV campaigns from above 30% down to $\approx 10\%$, based on the integral approach (see section 3.10.4). Although substantially reduced, the overall uncertainty levels are still slightly above the heritage values (figure 7.14a). One reason for this is the conservative estimate for the shift of origin. For the heritage data a best guess was used from the uncertainty in wall-normal distance measurements U_y , which is most likely too low. As reported, in case of this thesis the full range of $U_\epsilon = 0 - 1k$ was used due to experienced challenges to extract this value from velocity data (as reported in section 4.2.2). If a more narrow uncertainty with $U_\epsilon = 0.2 - 0.5k$, were to be used (based on analysis with the combined fitting and integral approach, as also reported in section 4.2.2), the resulting uncertainty would be reduced to $U_\alpha \approx 36\%$, comparable to the heritage values. For conservative reasons, the larger uncertainties were kept due to the challenges in properly determining this quantity.

The results of the quasi UPC for both campaigns are shown in figure 3.20 and the quasi UMF in figure 3.21. In both cases the EQSR uncertainty results are again represented by the three parameters Π (labeled “Pi”), ΔU^+ (“DU+”) and $\alpha = k_s/k$ (“alpha”). In case of UPCs in figure 3.20, skin friction velocity, shift of origin and velocity uncertainties have the largest contribution to the results, comparable to the heritage data. In case of UMFs in figure 3.21, the velocity and skin friction velocity uncertainties have the largest lever on EQSR uncertainties.

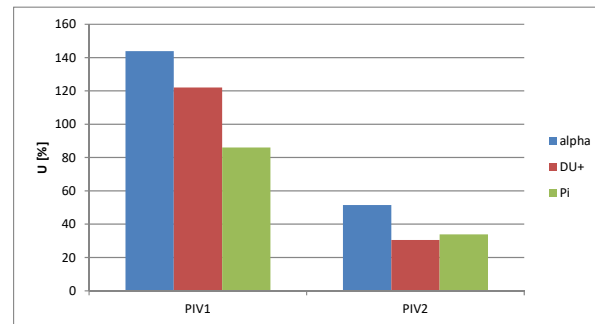
Therefore, by considering the performed uncertainty analysis, especially with respect to the corresponding levers (UMFs), a substantial lowering of EQSR uncertainties could be realized by increasing accuracies for velocity and friction velocity measurements due to PIV hardware upgrades.

Table 3.7: Input quantity uncertainties for EQSR MCM uncertainty estimation (see section 7.1.1).

Input quantity X_i	1st PIV campaign uncertainty U_{X_i} [%]	2nd PIV campaign uncertainty U_{X_i} [%]
Ma	0.6	0.6
p_0	0.33	0.3
T_0	0.4	0.4
T_w	1.4	1.3
p_e	1.2	1.0
y	2.1	0.9
U_e	1.1	0.4
u_τ	31.8	10.2
ϵ	47.5	47.7

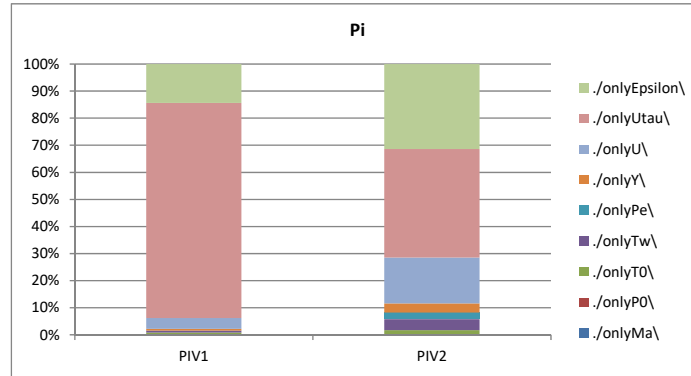


(a)

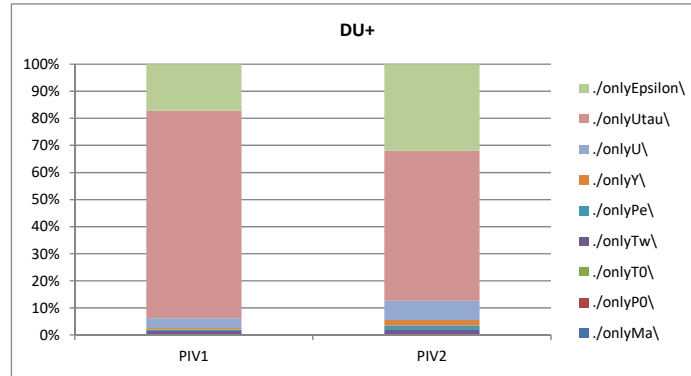


(b)

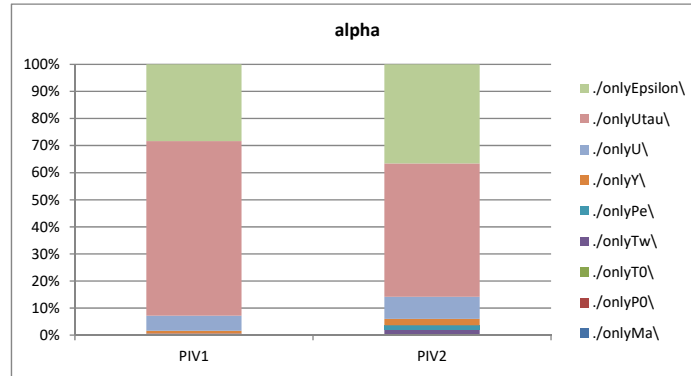
Figure 3.19: a) Quasi UPC of input uncertainties and b) uncertainties of resulting Π (“Pi”), ΔU^+ (“DU+”) and $\alpha = k_s/k$ (“alpha”) from MCM EQSR uncertainty analysis of data from PIV campaign 1 and 2.



(a)

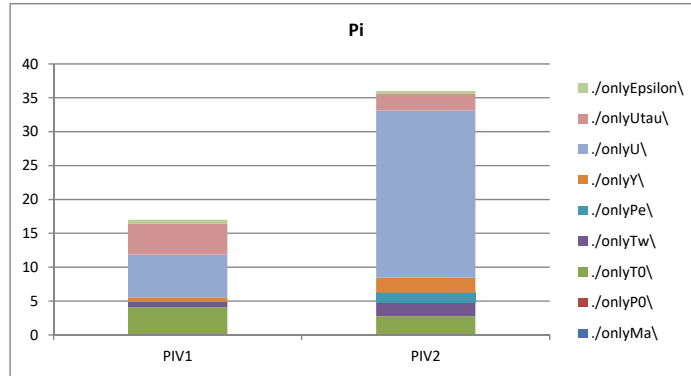


(b)

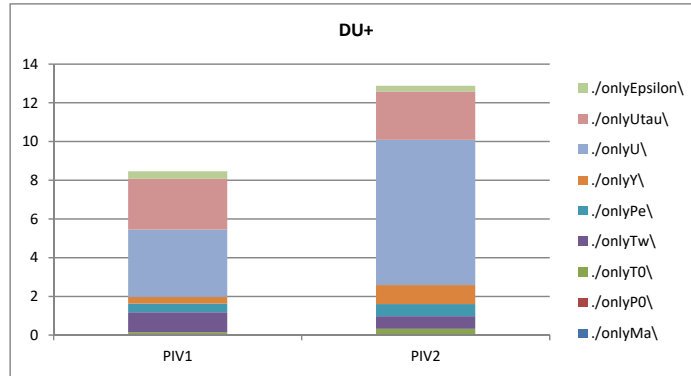


(c)

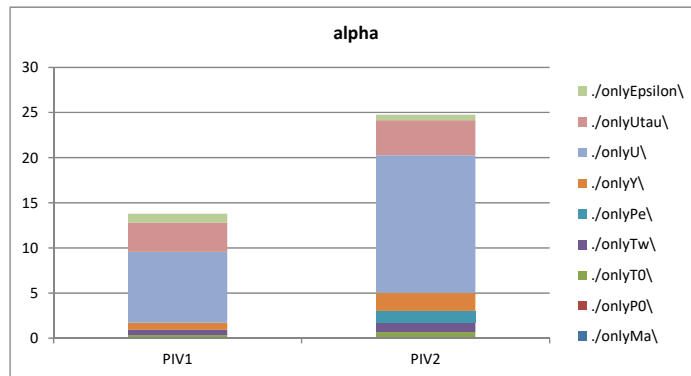
Figure 3.20: Quasi UPC of resulting a) Π ("Pi"), b) ΔU^+ ("DU+") and c) $\alpha = k_s/k$ ("alpha") from MCM EQSR uncertainty analysis of data from PIV campaign 1 and 2.



(a)



(b)



(c)

Figure 3.21: Quasi UMF of resulting a) Π ("Pi"), b) ΔU^+ ("DU+") and c) $\alpha = k_s/k$ ("alpha") from MCM EQSR uncertainty analysis of data from PIV campaign 1 and 2.

4 Results and Discussion

This section contains the main results of this thesis. First an overview of the general flow field features is given. After that, the mean velocity data is discussed. Then, aspects of the turbulent data is reported. Finally, the heat flux data is discussed.

The flow conditions which have been used to derive all discussed results have been introduced in section 3.1. An exhaustive list of performed tests can be found in table 7.1. The numbering of runs in the following is grouped into three blocks with a leading one thousand for the first PIV campaign, two thousand for the Pitot and three thousand for the second PIV campaign. Results presented in this chapter have previously been published in part by Neeb *et al.* [166], [170], [167], [171], [169].

4.1 General Flow Features

In this section, general flow field features are discussed based on surface pressure data, PIV raw images and 2D flow field data and lastly Schlieren images.

As footprint of the smooth wall flow field, Figure 4.1 shows selected normalized surface pressures along the smooth cone from measurements (symbols), CFD (solid lines) and Taylor-Maccoll computations (dashed line). The symbols at the first and last axial position represent four measurements for the different pressure taps in circumferential direction to check the model attitude (see section 3.2). Within corresponding uncertainties, the measurements agree to each other ensuring zero angle of attack and sideslip. The Taylor-Maccoll value and data from inviscid Euler TAU computations show a constant value along the cone, close to the measurements. In contrast, the TAU calculations including friction show the influence of viscous interactions with increased pressures near the nose of the cone, with turbulent data resulting in the largest influence due to the thicker boundary layer. The TAU computation including transition (purple line “TAU trans.”) shows an additional increase in pressure at the location of the pre-set transition ($x_a \approx 0.3\text{m}$ as measured from IR data) due to further local increased boundary layer growth. This curve is also the one in best agreement to the measurements.

Before velocity fields are shown, PIV raw images are discussed in the following. Figure 4.2 shows two exemplary raw images in black and white color scheme, in which particles are visible as brighter (whiter) pixels. Both images show snapshots in time along the evolving smooth wall TBL at similar flow conditions, where figure 4.2a represents data along a 3° cone and figure 4.2b along a 7° cone. As discussed in section 3.1, 3° cone experiments were discussed in the planing phase of this thesis, from which a raw image of initial PIV tests is shown. The reason it is shown here is that a larger FOV was recorded in contrast to the 7° cone tests, with the possibility to capture larger scale features. In both image, the white arrow indicates the approximated

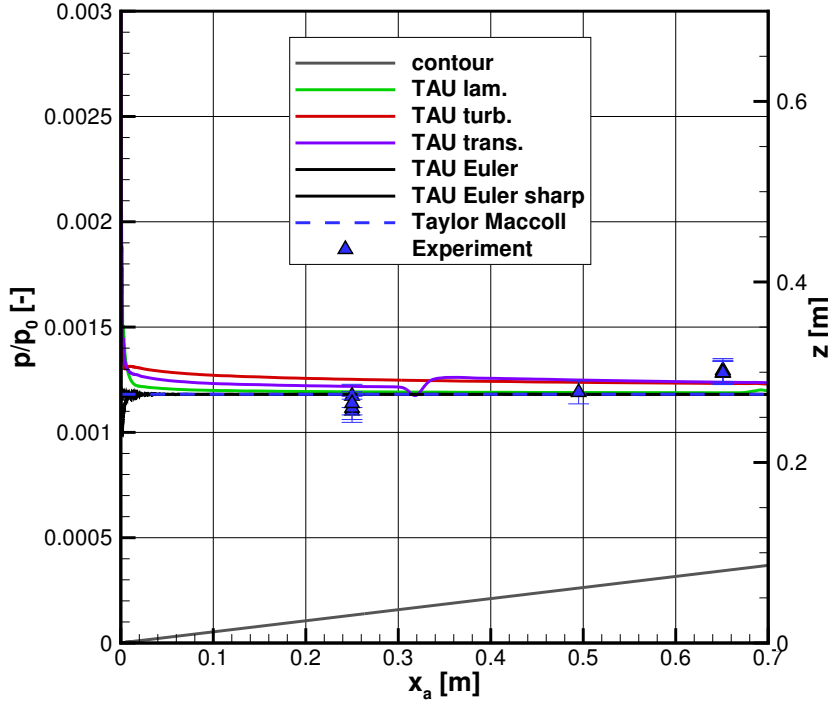


Figure 4.1: Normalized surface pressure along the smooth cone from measurements (symbols), CFD (solid lines) and Taylor-Maccoll computations (dashed line).

boundary layer thickness. It is visible that PIV particles are distributed into denser (bright regions) and less dense clouds (dark regions). Based on these brightness distributions, typical large scale TBL features, like bulges, are visible, when compared to previous investigations like figure 2.12 from smoke visualizations. Additionally, yellow lines indicate angles of 45° to the wall as reference and it is visible that the edges of these large scale structures resemble roughly this orientation, in line with observations from past experiments [2].

To further investigate flow features of the TBL, in the following results of representative PIV measurements along the smooth and rough wall are used, which are depicted in figure 4.3 and 4.4, respectively. In figure 4.3a and 4.4a the mean flow, represented by U/U_e , at FC1 is shown. The streamwise and wall-normal distance were normalized with the boundary layer thickness δ , averaged over the streamwise distance of the FOV. The origin was at the mid-position of FOV1, close to MS1 (see figure 3.3). Only data of the second PIV campaign is shown. To increase the resolution for visible inspection, IWs of $48 \times 48 \text{ pix}^2$ with 75 % overlap were used.

In general, the different streamwise velocity layers of the boundary layer evolving above the walls are clearly visible. The differences of the two configurations for identical inflow conditions manifest in the following flow features. The rough wall boundary layer is with $\delta \approx 4.5 \text{ mm}$ larger compared to the smooth wall counterpart with $\delta \approx 3.6 \text{ mm}$. Although the flow fields are scaled in wall-normal direction accordingly in figures 4.3 and 4.4, lower velocities are visible above the rough wall at comparable scaled wall-distances, representing the well-known flow retardation

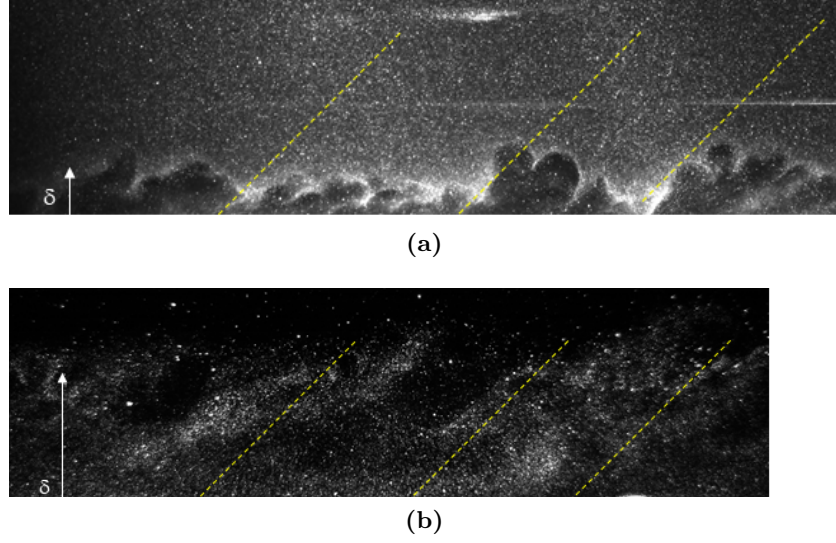


Figure 4.2: PIV raw images along a smooth wall a) 3° cone and b) 7° cone showing typical features of a TBL (white arrow indicates approximated boundary layer thickness δ , yellow lines indicate a 45° angle to the wall).

above rough walls due to momentum losses. Also visible in figures 4.4a, downstream of each roughness crest, shear layers are evolving which are characterized by slightly increasing velocities towards the wall near the downstream end of each cavity. Apart from that, it can be observed that the flow above the roughness crests is evolving rather undisturbed by the roughness topology, visible by only minor changes of the layered structure, also sometimes called *skimming*. This is similar to observations by previous investigators utilizing PIV measurements, e.g. by Williams *et al.* [288] in a $Ma_e \approx 7$ TBL flow along transverse square bar roughness with slightly larger pitch ($\lambda/k = 10.63$) than the roughness in this thesis ($\lambda/k = 4$). Also, Ekoto [69] showed similar behavior in a $Ma_e = 2.9$ TBL flow along 3D block roughness ($w/k \approx 2$, $\lambda_x/k = \lambda_z/k = 4$), in contrast to diamond roughness, which showed large flow field perturbations, as was shown in 2.7 of section 2.1.

The mean wall-normal component V/U_e is shown in figure 4.3b and 4.4b. It is visible that the data are more noisy but it needs to be noted that the magnitude is approx. two orders of magnitude below the streamwise component. Above the smooth wall a region with positive values in the vicinity of the boundary layer edge ($y/\delta \approx 1$) is visible, whereas negative values can be found above this region and close to the surface. This conical flow field behavior, including a sign change in the wall-normal component, was also observed in a numerical rebuilding of a $Ma = 8$ wind tunnel experiment along a 7° half-angle model by Pruett [196]. In case of the rough wall data, positive V values are also visible for $y_T/\delta \gtrsim 0.5$ and negative values close to the surface. Within the cavities, V velocity patterns appear which are typical for a recirculation bubble with positive V values near the upstream and negative values near the downstream end. As was the case for the streamwise component, also the wall-normal component shows no clear disturbances in a region above the roughness crests. This is again qualitatively similar to what was reported in Ekoto [69] along 3D block roughness and visible in 2.7b of section 2.1. Although

it resembles results from other investigators, it needs to be stated that other potential reasons for the undisturbed flow field can be found. One reason might be potential particle lag, which is known to affect more the lower magnitude wall-normal component of a hypersonic TBL [286], [287], [15], [16]. This would make the results less sensitive to any small scale deviations. Another reason for the absence of disturbances might be the magnitude of wall-normal velocities, which are in the same range as the estimated measurement uncertainties (see section 7.1.1), so that data of this component were not used for further in-depth analysis.

Additionally, corresponding Mach number fields were computed based on the Crocco-Busemann temperature-velocity correlation and are shown in figure 4.3c and 4.4c for smooth and rough wall data, respectively. With a white line, the contour at $Ma = 1$ is highlighted. In case of the smooth wall, all recorded mean TBL data are supersonic. In case of the rough wall the $Ma = 1$ contour is located closely above the crests of the roughness elements resembling a slightly wavy pattern. That hints at a purely subsonic mean flow in the vicinity of the crests and below (within cavities).

The scaled streamwise rms velocities u_{rms}/U_e in figure 4.3d and 4.4d show increased values in a layer close to the smooth wall and above the roughness elements in the aforementioned evolving shear layers. Especially in close vicinity of the crests and inside the cavities the highest values are encountered. This will be further addressed in the discussions about the turbulence profile data in section 4.3.1.

Before summarizing the topology results, another inspection of the flow field was made based on recorded Schlieren images. In previous investigations, Schlieren images revealed strong density gradients along rough walls, depending on the roughness topology and test conditions. I.e. strong shocks were reported along diamond roughness by Peltier [183] and the previously mentioned investigations by Ekoto [69], for which an example was shown in figure 2.6 of section 2.1. In contrast, only weak characteristic waves were observed along 3D block roughness in Ekoto [69]. Along three different transverse square bar roughness ($\lambda/k = 5, 8.33, 10.63$), all with slightly larger pitch than the topology of this thesis ($\lambda/k = 4$), Williams *et al.* [288] reported about visible shocks.

Figure 4.1 shows (rotated) Schlieren images along the TSB rough cone (Run2091, upper two images with a zoomed view of the left one on the right) and along the Carborundum Sand Grain type (SG) rough wall (Run2133, lower two images with a zoomed view of the left one on the right). In the experimental data along the technical roughness, the edge of the boundary layer is visible with a sharp change of dark to light region in the contrast enhanced image (indicated by white arrow). The visible strong shock at approx. halfway of the image is due to the Pitot rake at a circumferential position 45° behind the symmetry plane. Besides these features, no clear imprint of the roughness topology is visible in agreement to the mean flow data from PIV (see 4.4a and 4.4b) but in contrast to the previously mentioned TSB roughness investigations by Williams *et al.* [288]. Reasons for this difference might be either the only moderate resolution of the Schlieren images in case of this thesis or the slightly smaller pitch in case of the current topology ($\lambda/k = 4$) compared to the roughness of Williams *et al.* [288] ($\lambda/k = 5, 8.33, 10.63$).

In contrast, corresponding Schlieren along the Carborundum sand-grain-like roughness in the lower part of section 4.1 show various disturbances visible by density variations above the wall and especially outside the boundary layer. This is comparable to observations by Latin [138] in a

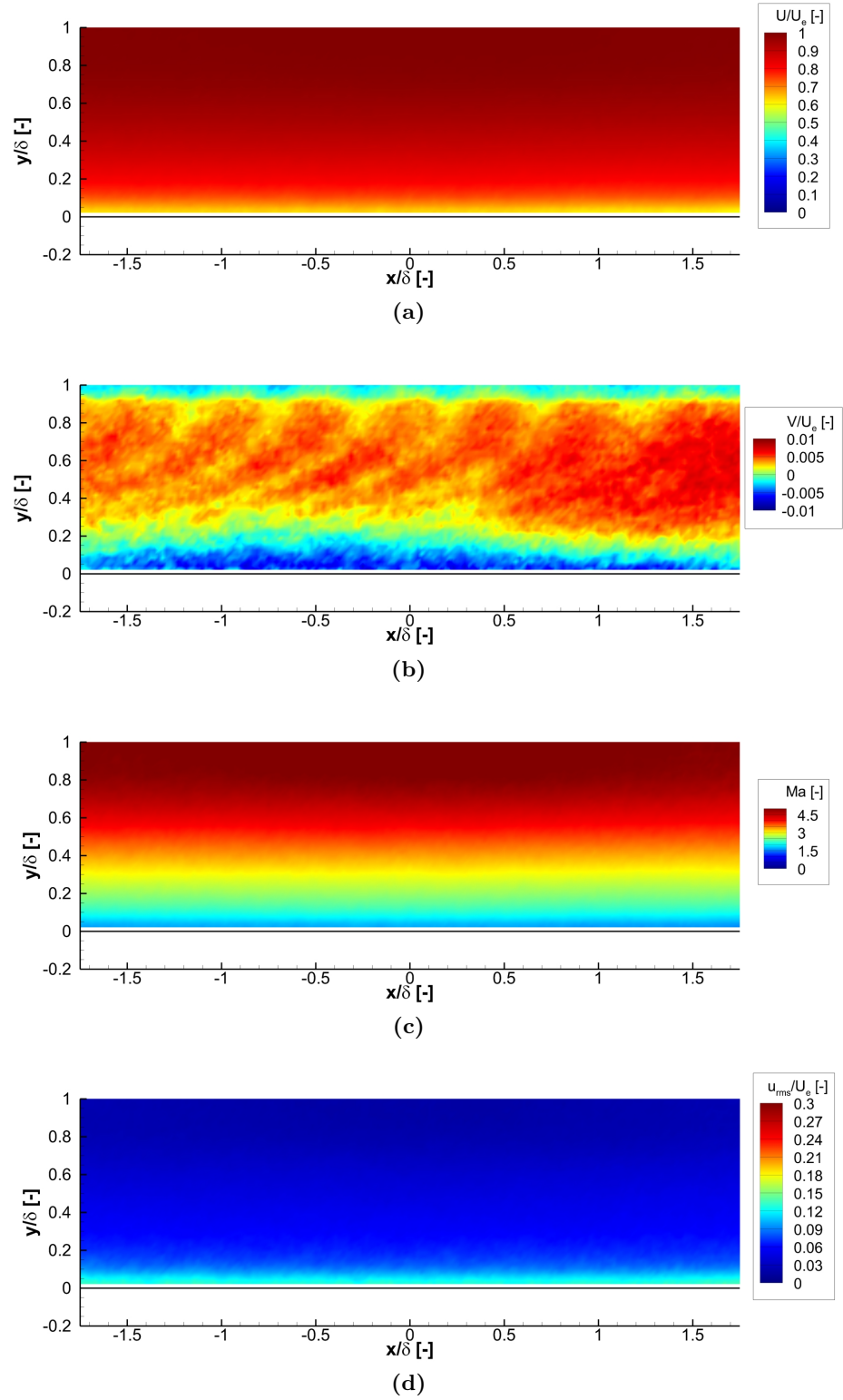


Figure 4.3: PIV data of a) mean streamwise velocity U/U_e , b) wall-normal velocity V/U_e , c) Mach number and d) streamwise rms velocity u_{rms}/U_e above smooth wall. (Run3014, IW 48x48 pix^2 with 75 % overlap).

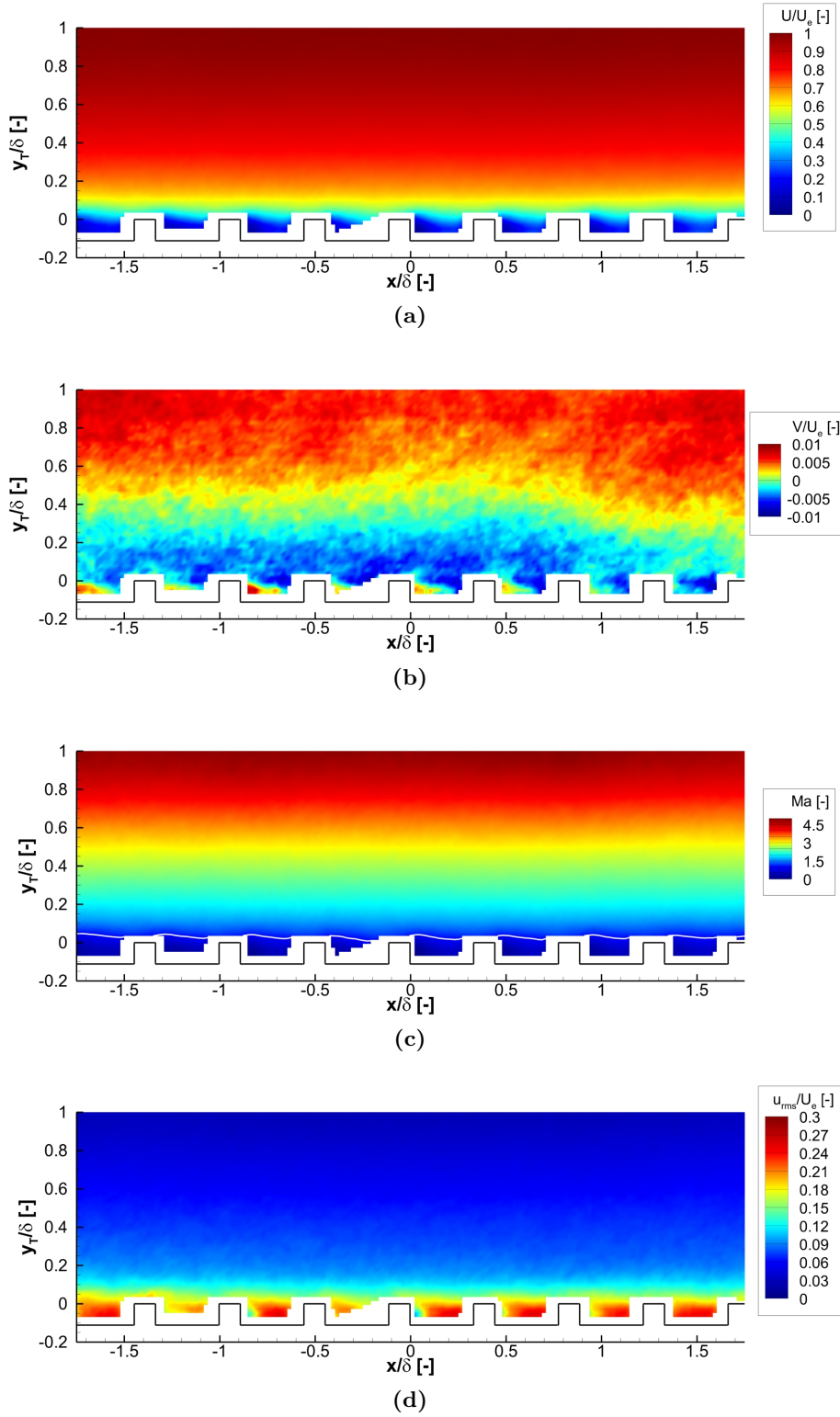


Figure 4.4: PIV data of a) mean streamwise velocity U/U_e , b) wall-normal velocity V/U_e , c) Mach number and d) streamwise rms velocity u_{rms}/U_e above rough wall. (Run3007, IW 48x48 pix^2 with 75 % overlap).

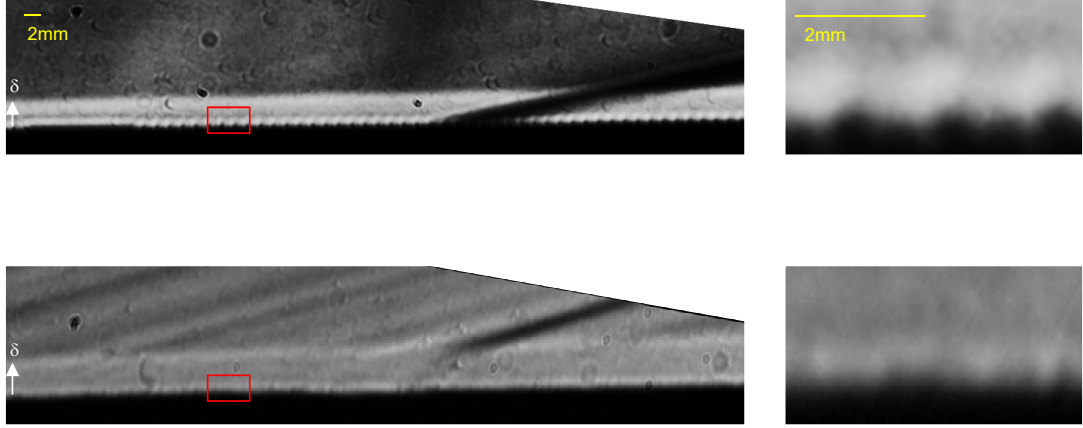


Figure 4.5: Schlieren images along the technical rough wall (Run2091, upper two images with a zoomed view of the left one on the right) and along the Carborundum rough wall (Run2133, lower two images with a zoomed view of the left one on the right).

$Ma_e = 2.9$ TBL flow along sand grain roughness ($k/\delta \approx 0.03 - 0.06$), re-plotted in figure 2.6 and also reported in [32]. Although the Carborundum roughness had approx. the same height k compared to the technical roughness, a reason for the different Schlieren observations might be due to the leveling of the roughness. The Carborundum was applied on top of the smooth wall whereas the technical roughness pattern was designed with the roughness crests being flush to the surrounding smooth wall (see section 3.1). Similar observations depending on the leveling have been made in previous investigations [168].

To summarize observations from PIV and Schlieren data, in a region above the TSB roughness, only negligible streamwise mean flow and Schlieren gradients are visible. This skimming behavior was previously used as argument for a “d-type” roughness by Perry and Joubert [185] or Jiménez [118] (see also section 2.1) although it can be categorized as “k-type” roughness based on the geometrical topology (see section 3.12.1). A potential reason for the absence of any disturbances penetrating the TBL is the observation that roughness crests are submerged within a subsonic layer (see figure 4.4c). Furthermore, the agreement between PIV and Schlieren makes the argument of particle lag as the main reason for the absence of disturbances in the PIV flow fields less likely. This behavior could mainly be driven by the TSB topology with $\lambda/k = 4$, although the only slightly larger pitch TSB roughness investigations by Williams *et al.* [288] ($\lambda/k = 5$) showed shocks in Schlieren images. We will continue this discussion based on results from profile data in the following sections.

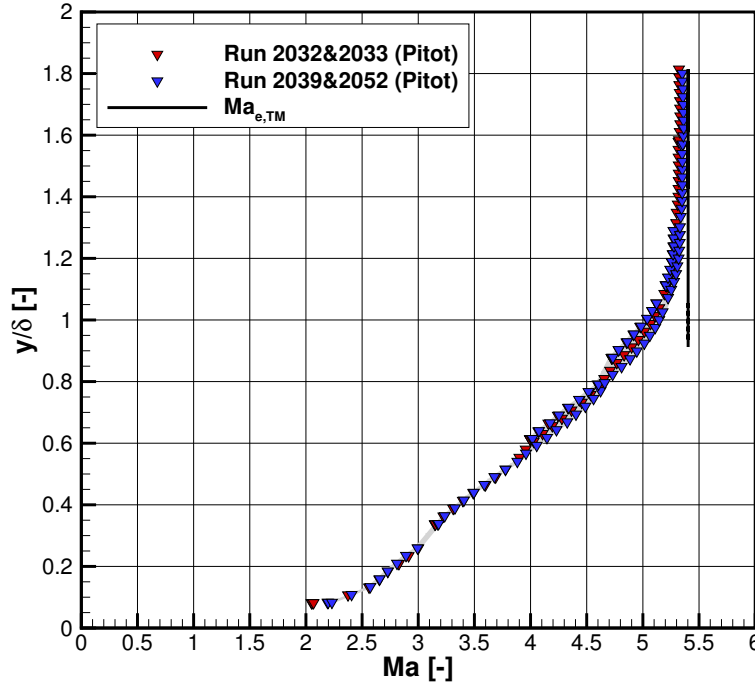


Figure 4.6: Mach profiles derived from Pitot measurements at MS1 and FC2 (Run2032&2033 and 2039&2052).

4.2 Mean flow Data

4.2.1 Profiles

First, for validation purposes several sensitivities and cross-checks based on the profiles along the smooth wall are discussed before a comparison is made to rough wall data. Selected sensitivities are discussed in the following and more exhaustive data are reported in the Appendix 7.4.1, 7.5.2 or 7.5.4. The impact of IW size on the mean velocity profiles was negligible since all data collapse, except for the first point above the wall (see figure 7.43). The PIV profiles were averaged as described in chapter 3. Also different averaging procedures showed only negligible influence (see figure 7.45 in section 7.5.2).

Two Pitot probe profiles at measurement section MS1 and inflow condition FC2 processed to Mach numbers according to the procedure described in chapter 3.2.2 are shown in figure 4.6. The Pitot measurements include a Mach number boundary layer region from approx. $Ma = 2$ to an edge value of $Ma \approx 5.35$, which is in good agreement to the nominal Taylor-Maccoll value for FC2, included as black line (see also table 3.1). It also shows that the largest extent of the boundary layer is supersonic, supporting data from PIV (see figure 4.3).

In figure 4.7a smooth wall velocities of two PIV profiles at FC1 for each PIV campaign (run 1007,

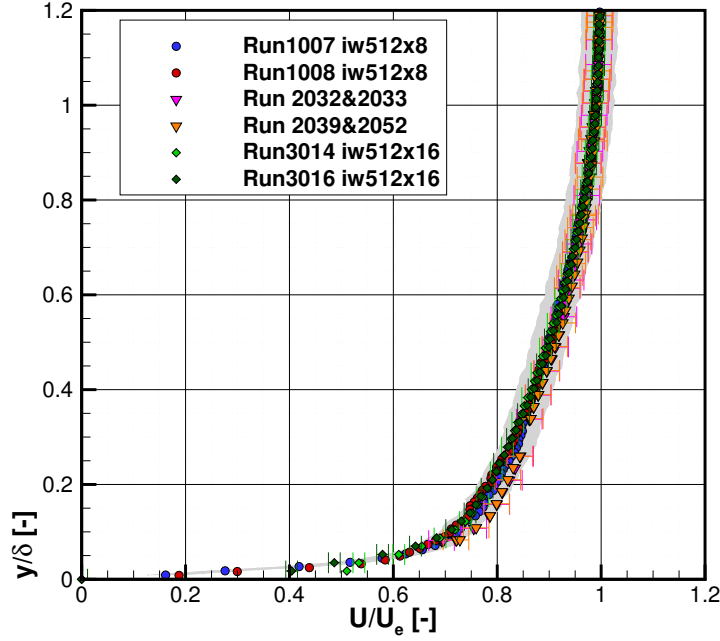


Figure 4.7: Outer scaled smooth wall mean velocity profiles for all experimental campaigns including repeatability.

1008, 3014 and 3016) are directly compared to the velocities derived from two Pitot pressure runs at FC2 (run 2032&2033 and 2039&2052), all extracted at the first measurement section MS1. It is visible that in general all data coincide, except for the first points above the wall. Uncertainties are visualized by a gray area in case of the first PIV campaign and by error bars in case of the Pitot data and second PIV campaign (see section 7.1.1). This comparison enables run-to-run (repeatability) and measurement technique to measurement technique comparisons. The Pitot data as well as PIV data show excellent repeatability behavior within their corresponding uncertainty bands. In the upper parts of the boundary layer, above $y/\delta > 0.3$, both measurement techniques show good agreement. For smaller wall distances, the Pitot data result in slightly higher velocities compared to PIV data, although this differences are still covered by corresponding uncertainties.

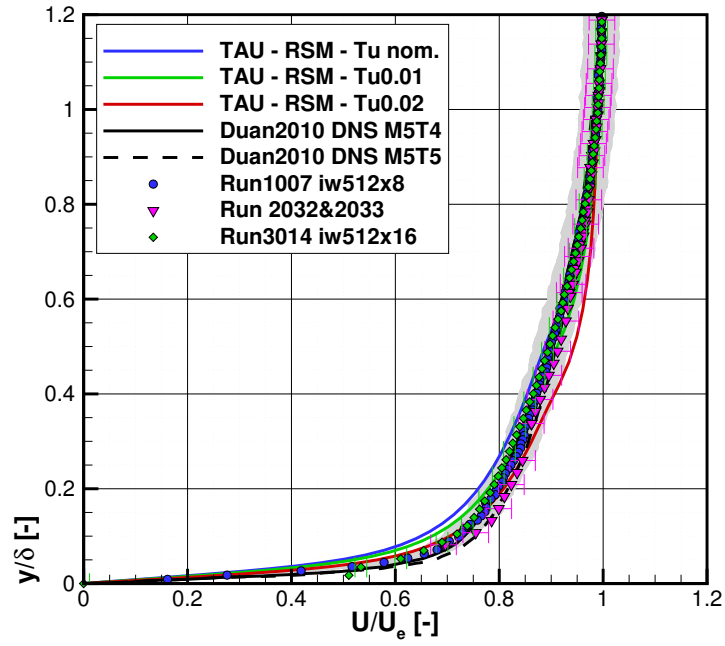
In figure 4.8a different numerical data are included for comparison to one selected profile for each experimental campaign. TAU computations with RSM turbulence model and three different inflow turbulent intensity levels are included, i.e. at levels of 0.1 % (labeled “Tu nom.”, since it corresponds to the nominal value in TAU), 1 % (“Tu0.01”) and 2 % (“Tu0.02”). The latter two values represent a reasonable range for the H2K facility, based on experiences in previous test campaigns and an inflow turbulence characterization campaign with Laser 2 Focus (L2F) [94]. Additionally, DNS data of two turbulent boundary layer profiles along a flat plate at two different wall temperatures are included from literature according to Duan *et al.* [56]. Both DNS profiles share the edge Mach number of $Ma_e = 4.97$ but the Reynolds numbers are $Re_\theta = 3819$ ($Re_{\delta_2} = 1526$) and $Re_\theta = 4840$ ($Re_{\delta_2} = 1537$) for case M5T4 and M5T5,

respectively. The corresponding wall to adiabatic wall temperature ratios are $T_w/T_{aw} \approx 0.7$ and $T_w/T_{aw} = 1.0$. In comparison, the Reynolds numbers extracted from the current experimental data are with $Re_{\delta_2} = 635 - 1053$ slightly lower. The corresponding wall temperature condition is with $T_w/T_{aw} \approx 0.8$ enclosed by the M5T4 and M5T5 data set. The skin friction coefficient of case M5T4 agrees with $c_f = 1.5 \cdot 10^{-3}$ close to the corresponding value of Run1007 (see table 4.1). All data is plotted in outer scaling.

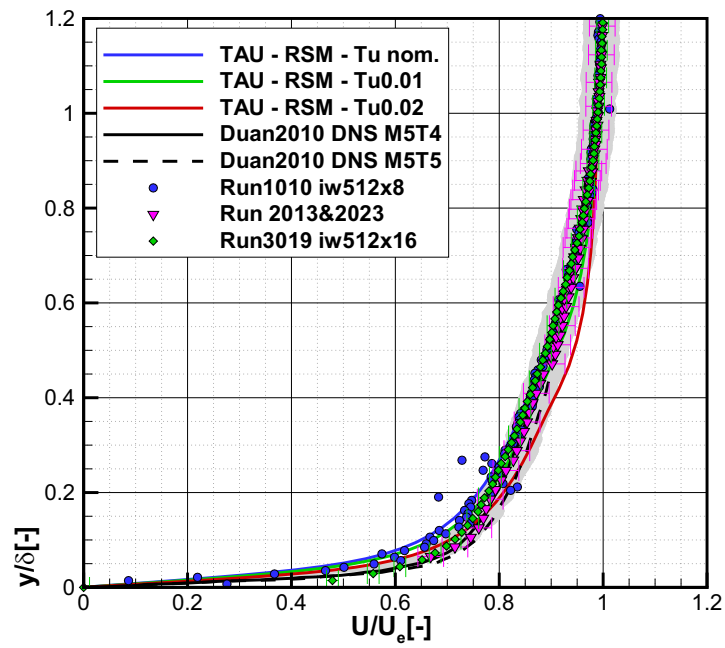
A good agreement is visible for the upper part between experimental data and TAU computations, if corresponding uncertainties are considered. From TAU data, results with increased turbulence level of 1 % (“Tu0.01”) utilizing the RSM turbulence model show best agreement, although deviations above the uncertainty band are visible below $y/\delta \approx 0.1$. Despite the difference in Reynolds number, the DNS and experimental velocity profiles agree within the corresponding measurement uncertainties, except for a region between $0.05 < y/\delta < 0.01$. This difference might be attributed to the difference in Reynolds number, as well as potential particle slip and increased shear close to the wall, which was previously identified as influencing factor for PIV data.

Data of the second measurement section MS2 show the same agreement between PIV and Pitot data (see figure 4.8b). Only the PIV data of the first PIV campaign (Run1010) is more noisy, since for this run only 89 images were suitable for post-processing.

To show the effect of the rough wall on the velocity profiles, figure 4.9 shows different profiles at MS1 (solid lines) and MS2 (dashed-dotted lines) along the smooth (black lines, symbols) and rough wall (red lines, symbols) from PIV (smooth Run3014, Run3019, rough Run3007, Run3020) in figure 4.9a and Pitot data (smooth Run2032&2033, Run2013&2023, rough Run2090&2091, Run2095-2095) in figure 4.9b. The typical flow retardation at the same wall-distance due to the rough surface is clearly visible [202], [118], [32]. Also visible, but only moderately, is the change in velocity due to the increased running length (MS1 vs. MS2). The behavior is very similar between PIV and Pitot data.

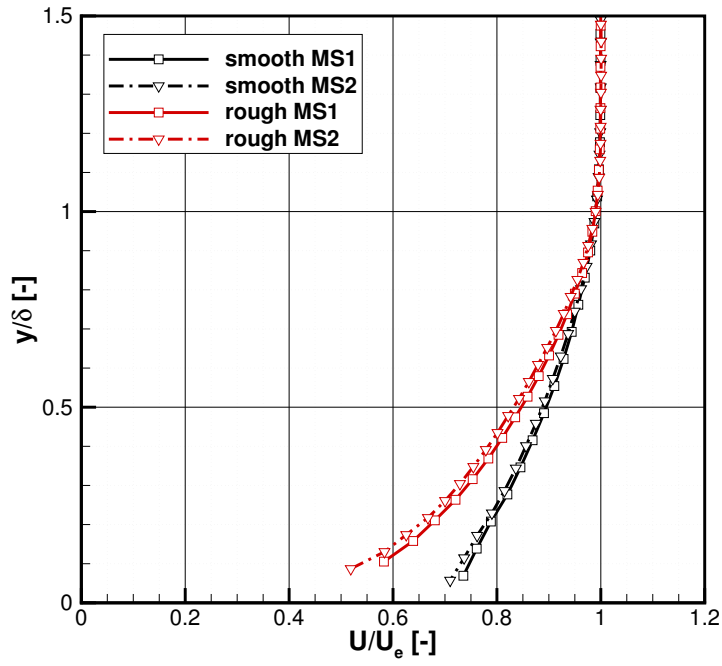


(a)

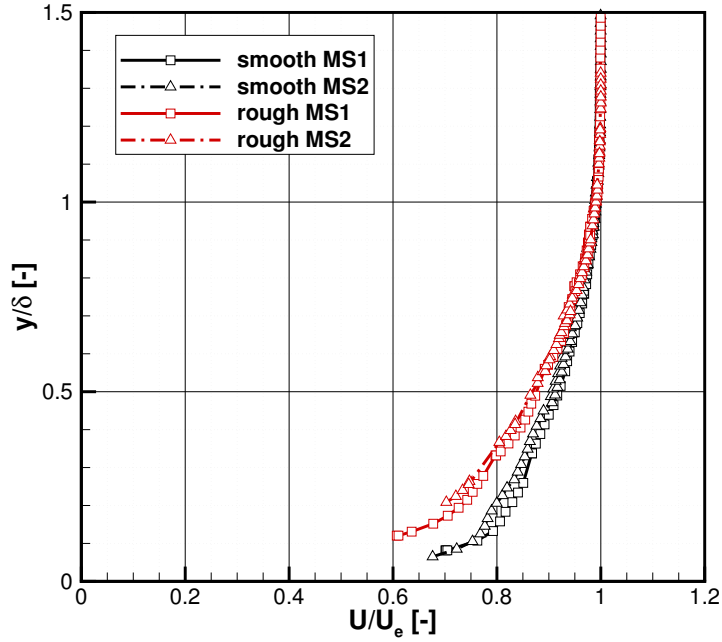


(b)

Figure 4.8: Outer scaled smooth wall mean velocity profiles for selected experimental data at a) MS1 and b) MS2 against CFD from TAU and according to [56].



(a)



(b)

Figure 4.9: Outer scaled mean velocity profiles showing the effect of smooth (black lines/symbols) and rough wall (red lines/symbols) for different runs from a) PIV (smooth Run3014, Run3019, rough Run3007, Run3020) and b) Pitot data (smooth Run2032&2033, Run2013&2023, rough Run2090&2091, Run2095-2095).

In summary, the current pressure and velocity measurements show repeatability, agreement with numerical simulations and well-established correlations/approximations. This provides confidence to investigate the data in depth in the following sections.

4.2.2 Friction Velocity and Inner Scaling

To enable proper inner scaling of the measured velocity profiles, corresponding skin friction velocities need to be extracted. We focus first on the smooth wall data since the extraction approaches are less tedious (as described already in section 3) and comparison to theory and literature data is easier. After that, rough wall data are discussed.

To follow suggested TBL analyses given in the AGARD report 253 ([73]), both, Pitot pressure and PIV velocity profiles, were transformed to inner and outer scaled form. Three approximations, i.e. the fitting, gradient and integral approach, were applied and compared to extract the required skin friction velocity indirectly from the measured mean velocity profiles (see chapter 3.10).

Smooth wall data:

For the skin friction evaluation, different boundary layer edge and wall values were necessary for all approaches. These parameters were extracted either from the velocity data directly or from infrared data and are summarized in table 4.1 for all investigated wind tunnel runs. Table 4.1 contains the run number, the measurement section (MS), the wall conditions smooth (s) or rough (r), the edge Mach number Ma_e and velocity u_e , the edge temperature T_e , the wall to adiabatic temperature ratio T_w/T_{aw} , the boundary layer thickness δ , the momentum loss thickness θ and the Reynolds numbers $Re_\theta = \rho_e u_e \theta / \mu_e$, $Re_{\delta 2} = \rho_e u_e \theta / \mu_w$, $Re_\tau = \delta^+ = \rho_w u_\tau \delta / \mu_w$ and $Re_\tau^* = \rho_e \sqrt{\rho_w / \rho_e} u_\tau \delta / \mu_e$. The latter is the semi-local Reynolds number which was suggested for comparing wall turbulence statistics between cases with substantially different mean density and viscosity profiles [180]. The table includes the edge velocity U_e and corresponding temperature T_e , derived via the Crocco-Busemann correlation. The results of the three approaches to derive the skin friction velocity are summarized and compared in table 4.2. Where possible, the values include corresponding uncertainties (see section 7.1.1), stated in the following paragraphs. In case of the PIV data, only results from the interrogation window $512 \times 8 \text{ pix}^2$ (first campaign) and $512 \times 16 \text{ pix}^2$ (second campaign) are contained.

In general, very consistent skin friction velocities were extracted with an agreement between the different approaches and analytical and numerical data for each measurement section (MS) and flow condition (FC) if the corresponding uncertainties are accounted for (see section 7.1.1). The van Driest correlation was used for the analytical (see chapter 3.10.1) and the TAU RSM results at 1% inflow turbulence intensity (see chapter 3.9) for the numerical comparison.

Fitting approach:

The skin friction velocities for PIV runs 1007 and 1008 show a good repeatability and the expected decrease for an increased running length for Run1010 (see table 4.2). Also data from the second campaign with 3014 show good agreement within corresponding uncertainties. A slight difference in wall temperature between FC1 and FC2 is the reason for a slight difference in the derived skin friction velocities from Pitot data compared to the corresponding PIV results. This is also visible if the corresponding analytical or numerical results are compared. An uncertainty of $\pm 0.4\%$,

$\pm 3.4\%$ and $\pm 1.6\%$ of the mean friction velocity was calculated by a Monte-Carlo analysis for the Pitot and PIV data of the first and second campaign, respectively (see section 7.1.1). The above stated values do include only procedural and no methodical uncertainties for the friction velocity. Methodical uncertainties can only be determined by comparing the results to a reference value of higher accuracy, either from experimental, numerical or analytical prediction. Most often, only cross-checks between different methods or comparisons to analytical predictions are given. For the comparable Clauser chart method, which utilizes the velocity data and its slope only in the logarithmic layer, typical uncertainties of approx. 5% are given in the literature for incompressible investigations (e.g. [197], [236], [274]). For compressible investigations the performance is typically stated within approx. 10% compared to analytical predictions like van Driest (e.g. [24], [236], [285], [184], [289], [287]). This is in agreement with observations in this work (see table 4.2).

Resulting values of the fitting procedure for all valid runs are summarized in table 4.3. To derive the momentum loss thickness θ , the mean density profiles were derived assuming a Crocco–Busemann temperature distribution within the boundary layer (see e.g. [277]) together with the measured surface pressure from the closest tap on the model. Other temperature distributions (e.g. Walz [56]) have been tested and only negligible differences were encountered. The derived edge properties are consistent to each other within the corresponding uncertainties. Resulting smooth wall Reynolds numbers vary within $Re_\theta = 2355 - 4323$ and $Re_{\delta_2} = 635 - 1084$. According to the difference between the inflow condition FC1 and FC2, slightly different edge parameters were derived. This is in agreement with the expected values from Taylor-Maccoll computations for FC2 (see table 3.1). Table 4.3 contains additionally the fitted skin friction coefficient c_f , skin friction velocity u_τ and Coles wake parameter Π .

Gradient approach:

The gradient approach was possible to apply due to the wall-normal resolution of the $512 \times 8 \text{ pix}^2$ profiles of the first PIV campaign with the first point above the wall at $y^+(1) = 1.79$ which is located within the viscous sublayer (see figure 4.12 b). As reported in section 3.10.3, for the viscous sublayer an alternative velocity transformation can be derived (see eq. (3.55)). The PIV data of Run1007 in the sublayer region transformed via classical van Driest and the alternative viscous-sublayer transformation of equation (3.55) (labeled “VS”) is plotted in figure 4.10. Both transformations show only minor differences in the sublayer, which are covered by the corresponding uncertainties. Both data profiles show a slight non-linear behavior. An agreement to the incompressible theory is only visible for data points at the lower and upper end of the viscous sublayer. Since only minor differences were visible between the van Driest velocity scaling and the modified scaling for the viscous sublayer, the van Driest scaled velocities were used in the following.

Taking all uncertainties into account, typically all skin friction velocity values derived by the gradient approach below $y^+ \lesssim 15$ agreed with the corresponding values from the fitting approach. This trend was detected for data from all interrogation windows reported in section 7.5.2. Only the wall nearest point in case of the $512 \times 8 \text{ pix}^2$ profile was biased. This is visualized in figure 4.11 which shows exemplary the resulting gradient approach skin friction velocity values for all wall nearest points of smooth wall Run1010. It is compared to corresponding results from the fitting approach (blue line for PIV and red line for Pitot data), CFD (green lines) and analytical correlation (black dash-dotted line). It is visible that all values correspond closely to each

Table 4.1: Edge and wall parameters of PIV and Pitot profiles at FC1 and FC2 from fitting approach (s: smooth wall, r: rough wall).

Run No. [-]	MS [-]	wall [-]	Ma_e [-]	U_e [m/s]	T_e [K]	T_w/T_{aw} [-]	δ [mm]	θ [mm]	H_k [-]	Re_θ [-]	Re_δ [-]	Re_τ [-]	Re_τ^* [-]
1007	1	s	5.03	916	83	0.79	3.4	0.155	1.5	2355	635	186	1426
1008	1	s	5.59	927	69	0.80	3.7	0.151	1.5	3298	736	201	2002
1010	2	s	4.86	912	88	0.80	4.2	0.204	1.6	2602	743	205	1468
2032&2033	1	s	5.33	901	78	0.79	3.9	0.154	2.7	3198	777	262	1867
2039&2052	1	s	5.36	902	78	0.79	3.9	0.152	2.7	3191	768	262	1873
2013&2023	2	s	5.23	898	82	0.79	4.9	0.207	2.3	4323	1084	329	2087
2024&2025	2	s	5.23	898	82	0.79	4.8	0.202	2.5	4194	1053	320	1966
2072-2074	1	r	5.42	903	80	0.79	4.5	0.194	2.3	4235	1001	377	2433
2084-2086	1	r	5.46	904	76	0.79	4.6	0.199	2.3	4380	1023	371	2755
2090&2091	1	r	5.44	904	76	0.79	4.5	0.192	2.3	4217	989	366	2658
2098&2099	1	r	5.41	903	79	0.79	5.2	0.217	2.8	4710	1118	441	2928
2092-2094	2	r	5.30	900	80	0.79	6.1	0.273	2.1	5925	1454	573	3533
2093&2095	2	r	5.33	901	79	0.79	6.1	0.249	4.0	5430	1320	522	3460
2126-2128	1	r(SG)	5.30	900	81	0.72	4.9	0.224	2.2	4582	1207	481	2813
2129-2131	1	r(SG)	5.36	902	77	0.72	4.9	0.221	2.2	4642	1197	466	2983
2121-2123	2	r(SG)	5.61	907	72	0.72	6.3	0.273	2.0	6310	1506	525	3920
2124&2125	2	r(SG)	5.60	907	72	0.72	6.4	0.278	2.0	6406	1532	510	3857
3014	1	s	5.1	923	81	0.80	3.6	0.17	1.4	3105	807	215	1763
3015	1	s	5.0	896	82	0.80	3.6	0.16	1.5	2706	722	200	1580
3016	1	s	4.9	920	87	0.81	3.6	0.17	1.4	2770	763	212	1596
3019	2	s	5.2	925	80	0.81	4.3	0.20	1.4	4106	1056	265	2221
3006	1	r	4.9	913	86	0.81	4.3	0.21	1.6	3426	954	328	2426
3007	1	r	5.1	918	81	0.83	4.5	0.21	1.6	4044	1027	307	2622
3008	1	r	5.2	928	79	0.82	4.4	0.20	1.7	4832	1201	378	3335
3010	1	r	5.2	947	83	0.81	4.6	0.21	1.7	3797	989	308	2540
3020	2	r	4.9	910	88	0.78	5.5	0.29	1.6	4468	1288	384	2690

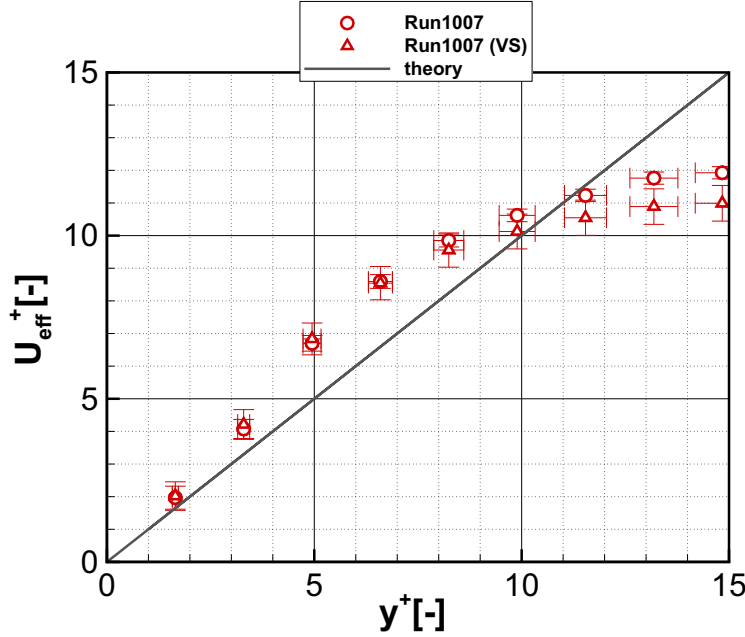


Figure 4.10: Streamwise mean velocity data from PIV (IW 512x8 pix²) at MS1 in viscous sublayer.

other and reported skin friction values in table 4.2 were derived by averaging all values within $8 < y^+ < 15$ (also included in figure 4.11 as dashed blue line). The uncertainties for the gradient approach were evaluated based on linear propagation to $\pm 9.1\%$ based on a mean value.

Integral approach:

For the numerical integration of the von-Kármán equation (eq. (3.57)) a step-size of 0.1% of the difference between x at MS1 and MS2 was set by default. A systematic change of this step-size showed a change of the resulting skin friction coefficient of only $\Delta c_{f,e} < 0.05\%$. Due to repeatability runs, in case of the PIV data and in case of the Pitot data two different and four different combinations were possible for the numerical integration of the von-Kármán equation, respectively. The results are contained in the column named “integral approach” in table 4.2. Although slight variations between the possible combinations at each flow condition are visible, values still agree to each other and to values from the other approaches if the corresponding uncertainties are considered. The combined uncertainties for the integral approach were evaluated to $\pm 5.7\%$, $\pm 33.5\%$ and $\pm 14.5\%$ for Pitot data and the PIV data of the first and second campaign, respectively. The higher uncertainties in case of PIV data can be attributed to higher input uncertainties of edge Mach number, temperature and momentum thickness. In case of the second PIV campaign these uncertainties could be reduced in comparison to the first one due to hardware and setup improvements.

Due to the lower uncertainties and consistent results, it was decided to use the fitting approach for skin friction and Coles wake parameter values to scale and analyze the PIV and Pitot profiles for the following analysis according to suggestions by [73]. Corresponding results of the PIV

Table 4.2: Summary of derived smooth wall skin friction velocities at FC1 and FC2 and MS1 and MS2 (all values in [m/s])

MS	FC	Run no.	fitting approach	gradient approach	integral approach	TAU	van Driest
1	1	1007	53.8 ± 1.8	54.4 ± 4.9		51.6 ± 2.9	52.7
		1008	53.7 ± 1.8	54.2 ± 4.9			
		1007 – 1010			50.7 ± 17.0		
		1008 – 1010			54.5 ± 18.3		
		3014	51.8 ± 0.8			47.8 ± 6.9	
		3014 – 3019					
1	2	2032&2033	52.9 ± 0.2			50.1 ± 2.9	51.2
		2039&2052	52.9 ± 0.2				
		2032&2033 – 2013&2023			52.9 ± 3.0		
		2032&2033 – 2024&2025			51.0 ± 2.9		
		2039&2052 – 2013&2023			53.5 ± 3.1	51.6 ± 3.0	
		2039&2052 – 2024&2025					
2	1	1010	50.5 ± 1.7	51.8 ± 4.7		50.4 ± 2.9	51.4
		1007 – 1010			49.2 ± 16.5		
		1008 – 1010			52.6 ± 17.7		
		3019	49.6 ± 0.8				
		3014 – 3019			46.8 ± 6.9	49.0 ± 2.8	50.0
2	2	2013&2023	49.1 ± 0.2			49.0 ± 2.8	50.0
		2024&2025	49.6 ± 0.2				
		2032&2033 – 2013&2023			51.1 ± 2.9		
		2032&2033 – 2024&2025			49.5 ± 2.8		
		2039&2052 – 2013&2023			51.7 ± 3.0	50.0 ± 2.9	
		2039&2052 – 2024&2025					

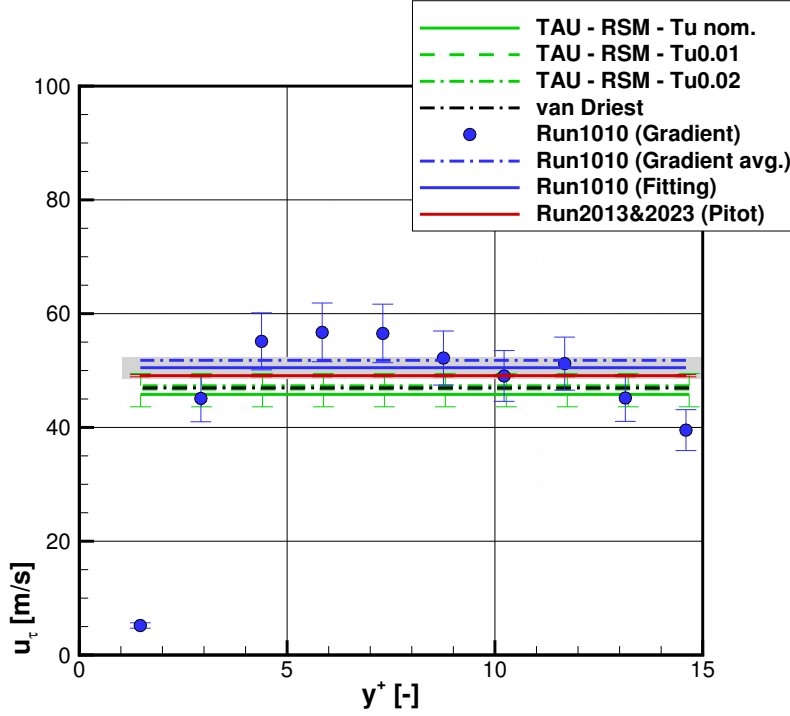


Figure 4.11: Comparison of near wall PIV velocity data points transformed to skin friction velocities via the gradient approach (blues symbols) compared to values from fitting approach (blue solid line for PIV, red solid line for Pitot Run2013&2023) and data from van Driest correlation (black dashed line) and TAU computations (green lines).

data with $512 \times 8 \text{ pix}^2$ of Run1007 at MS1 are included in figure 4.12. The figure contains four different sub-plots. In figure 4.12a the profile is shown as normalized velocity U/U_e against normalized wall-normal distance y/δ for a zoomed region close to the wall. Additionally, a linear line is included which is fitted through data points at $y/\delta < 0.04$. In sub-plot 4.12b the classical law-of-the-wall with the van Driest transformed effective velocities U_{eff}^+ against y^+ is included with one line for the viscous sub-layer ($U_{eff}^+ = y^+$) and one line for the log-layer without wake (see equation (2.7), neglecting the last two terms). Additionally, the two DNS profiles M5T4 and M5T5 according to Duan *et al.* [56] are included in van Driest transformed inner-scaled form. In sub-plot 4.12c an outer scaling is included as $(U_{eff,e} - U_{eff})/U_\tau$ against a wall normal distance y normalized by an integral length scale Δ^* according to [73] and defined by

$$\Delta^* = \delta \int_0^1 \left(\frac{(U_{eff,e} - U_{eff})}{U_\tau} \right) d\left(\frac{y}{\delta}\right). \quad (4.1)$$

The theoretical curve in this sub-plot corresponds to the following equation

$$\frac{(U_{eff,e} - U_{eff})}{U_\tau} = -M \cdot \ln\left(\frac{y}{\Delta^*}\right) - N \quad (4.2)$$

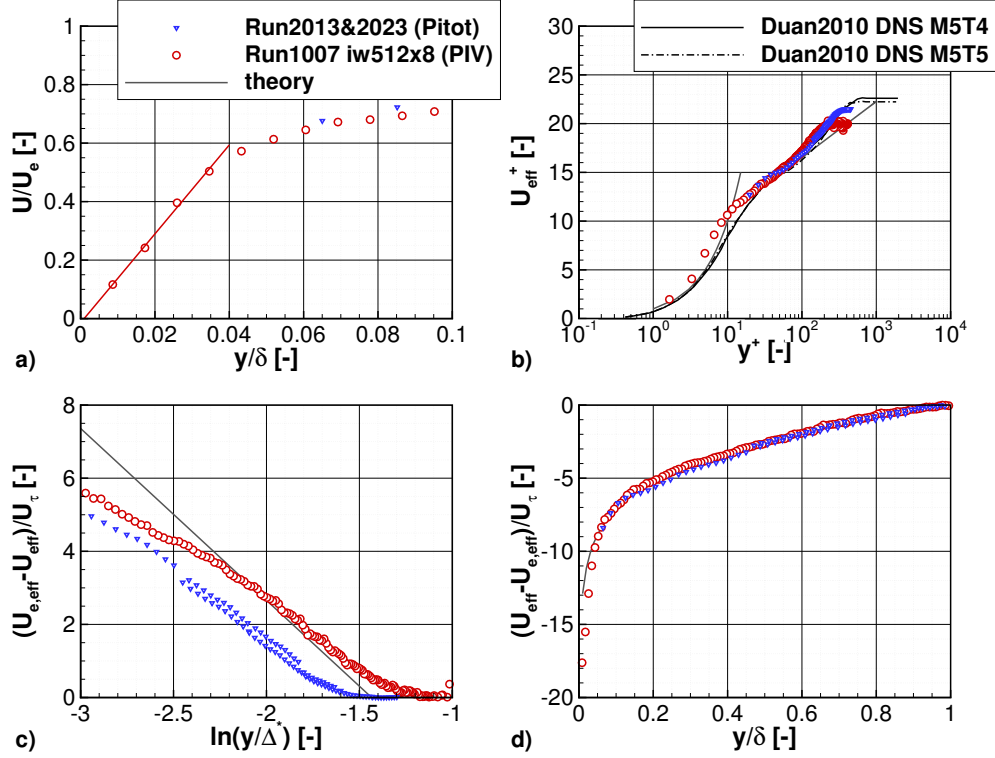


Figure 4.12: Streamwise mean velocity data from PIV (IW 512x8 pix²) and Pitot pressure in inner (b) and outer scaling (a,c-d) according to Fernholz and Finley [73] (DNS data according to Duan *et al.* [56]).

where $M = 4.7$ and $N = 6.74$, which were derived based on experimental data in zero-pressure gradient boundary layers, mainly along adiabatic walls ([73]). The constants are strictly just valid for $1500 < Re_{\delta 2} < 40000$, but [73] showed also consistent analysis for data with Reynolds numbers below this range. In the fourth sub-plot 4.12d, a second outer scaling, represented by $(U_{eff} - U_{eff,e})/U_{\tau}$ against y/δ , is included. The theoretical curve is described, as given in [68] (see eq. (2.12)).

The quality of the fitting approach is directly visible by comparing the data points to the theoretical curves of the law-of-the-wall in figure 4.12b. All characteristic boundary layer parts are visible, like the S-shaped wake above $y^+ \approx 100$, the logarithmic layer for $100 \gtrsim y^+ \gtrsim 10$ and even the viscous sublayer for data points with $y^+ < 10$. The first data point is located well within the viscous sublayer. The linear characteristic of the sublayer is also visible in figure 4.12a comparing the line fit to the wall nearest data points. If compared to the DNS data in figure 4.12b, a typical buffer layer is not visible for the current PIV data. One reason might be the particle slip, which was previously identified to be responsible for locally higher velocities in the near wall region (see paragraph particle slip in section 7.5.2 and figure 7.29). The wake portion of the Run1007 profile was fitted best by a Coles wake parameter of $\Pi = 0.27$ (see table 4.1). Other runs resulted in values $\Pi = 0.27 - 0.47$ (see table 4.3). Due to the low Reynolds number, the wake parameter is lower than $\Pi = 0.55$, which is typically stated for an incompressible zero

pressure gradient flow (see e.g. [277]). This is well in line with data of other compressible flow investigations in a comparable Reynolds number regime as visualized in figure 4.13 with the corresponding scaled velocity shift due to the wake with $\Delta U_{eff,wake}^+ = 2\Pi/\kappa$ according to [73] and [289]. The scatter for the data collection of Coles is also taken from the publication in [73].

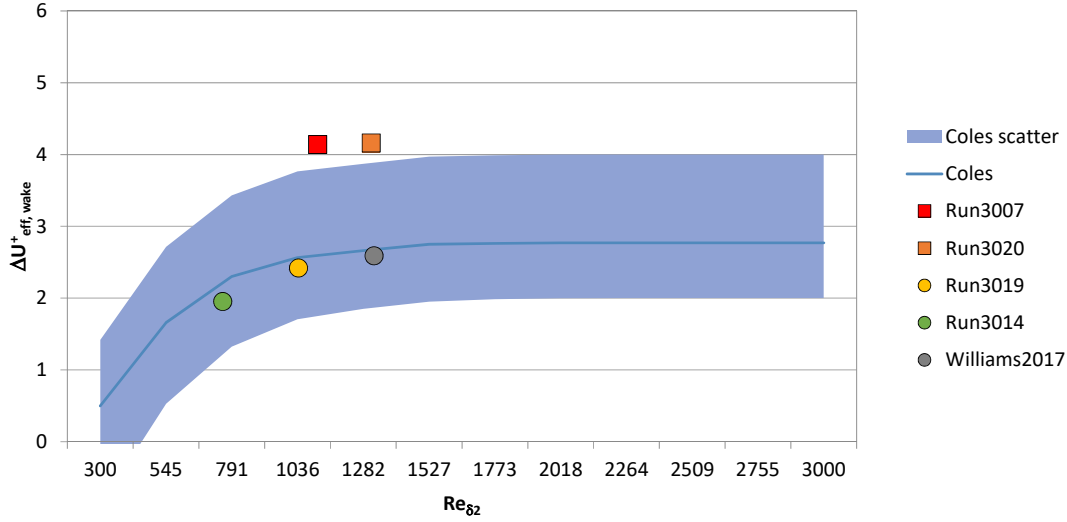


Figure 4.13: Wake parameter $\Delta U_{eff,wake}^+$ against Reynolds number Re_{δ_2} (circles for smooth wall and squares for rough wall results) with data according to Coles (see Fernholz and Finley [73]) and Williams and Smits [289].

The outer scaling results of Run1007 data are contained in figures 4.12c and 4.12d. The theoretical curve in 4.12d also depends on the wake parameter, so that an agreement with experimental data is a further consistency check. An excellent agreement between theory and data is visible down to wall-normal distances of $y/\delta \approx 0.05$, corresponding to the region of the viscous sublayer (see figure 4.12a) for which the theory is not applicable any more. The outer scaling in figure 4.12c shows good agreement between theory and data down to values of $\ln(y/\Delta^*) \approx -2$. For lower wall distances, the data points deviate from theory which was reported for low Reynolds number profiles at $Re_{\delta_2} < 2000$ in [73]. This is consistent with the value range in this paper. To check if the flow is fully turbulent, despite low Reynolds numbers, the kinematic shape factor is a useful indicator, defined as the ratio of displacement and momentum loss thickness with the density set constant to $H_k = \delta_k^*/\theta_k$. The PIV profile of Run1007 with $H_k \approx 1.5$ (see table 4.1) lies in a value range characteristic for fully turbulent boundary layers at low to moderate Reynolds numbers [73].

The same analysis was performed for velocity data derived from Pitot pressures of Run2013&2023, also included in figure 4.12. Technically, Pitot profiles were measured at FC2 in contrast to the PIV profile at FC1 in figure 4.12. Data of the second PIV campaign were used to enable a direct comparison between PIV profiles at FC1 and FC2, which showed only negligible differences (see Appendix 7.5.2 figure 7.46). Although small, the dimension of the Pitot probe restricts the profile to a first wall-normal distance of $y(1) = 0.32$ mm, $y/\delta(1) = 0.082$, $y^+(1) = 19.5$ which is almost one order of magnitude farther away from the wall compared to PIV profiles. This is

directly visible in figure 4.12b, where the profile begins in the logarithmic part of the boundary layer. A small region of locally higher velocity values compared to PIV and theory is visible ($y^+ \approx 300 - 400$). This is most likely connected to near wall effects of the Pitot probe [92], also visible in other investigations like [66] or [209]. Besides this region, the profile shows acceptable agreement with the theory in the logarithmic layer. The outer scaled data in figure 4.12d, which is driven by parameters like y/δ and Π , shows very good agreement with theory. Instead, the outer scaling in figure 4.12c, driven by an integral length scale and therefore by near wall data, is shifted parallel to the theoretical line. This is expected to be mainly driven by the loss of near wall information due to the Pitot probe dimension. To support this argument, a check was performed with an artificial profile, derived by the law-of-the-wall, which was scaled to an edge velocity according to H2K conditions. Before using this artificial profile for the fitting analysis, the profile was intentionally cut below $y/\delta = 0.08$, comparable to the Pitot data. The corresponding fitting results showed the same shift, as visible in figure 4.12c (see discussion in Appendix 7.5.2 and figure 7.47). This is also noticeable with a shape factor of $H_k = 2.3$ (see table 4.1), which is close to the value of a Blasius profile with $H = 2.59$ and would erroneously indicate a laminar profile [277].

Additional comparisons are included in section 7.4.1 and 7.5.2 which show a comparable behavior as the analysis above in figure 4.12. With the reported measurement techniques a total number of approx. 10 – 30 points in the logarithmic layer are available for analysis for Pitot and PIV data. Additionally, the general agreement between the different skin friction velocity approaches and theory and numerics gave confidence for analyzing rough wall data.

Rough wall data:

For the rough wall data, skin friction velocity values and additional parameters were extracted only via the fitting and integral approach. The gradient approach is inconclusive in case of a rough wall, on the one hand due to challenges accomplishing necessary resolutions along the wall but more importantly because the pressure contribution to the momentum loss would not be captured [124], [45].

In case of rough walls, the fitting is much more challenging, as previously described in section 3.10.2. One parameter which was found to have sizable impact in case of the data of this thesis is the shift of origin ϵ , as also reported in previous investigations [240]. There is a multitude of different approaches with the goal to extract this parameter (see e.g. [45]). Different methods to determine the shift of origin are summarized in section 7.2.2 and detailed tests of selected formulations was performed and can be found in section 7.2.3.

Figure 4.14 summarizes the main results, in which different column colors represent selected methods to derive ϵ and the definition of x positions, given behind the corresponding run number labels, represent different axial regions over which the PIV profiles were averaged. For the following analyses, it is only important to recognize, that the results of all methods vary widely and therefore no clear conclusion can be drawn. Therefore, a different approach was used to define ϵ , which is described in the following.

First, a value for the rough wall skin friction velocity u_τ is calculated via the integral approach (see section 3.10.4) utilizing the momentum loss thickness θ of the PIV velocity profiles at MS1 (Run3007) and MS2 (Run3020) as input to solve the von-Kármán-integral. An overview of extracted data via the integral approach are summarized in table 4.5 for PIV and Pitot data.

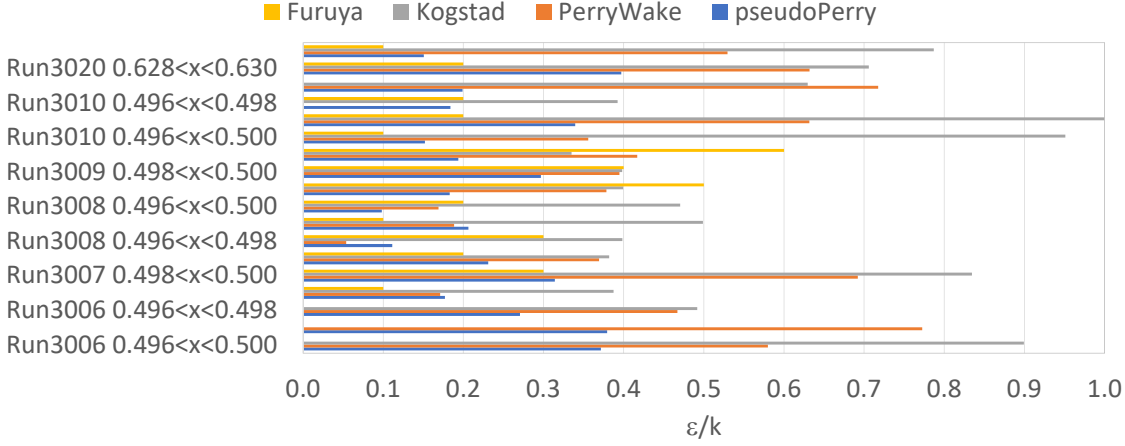


Figure 4.14: Shift of origin determination via different methods described in chapter 7.2.2.

With the corresponding uncertainty band, a most likely value band of u_τ was estimated to $53.4 \text{ m/s} < u_\tau < 71.8 \text{ m/s}$. Then, the fitting approach was applied multiple times, each time with a fixed shift of origin varied in between 0 to k in steps of $0.1 \cdot k$. This ensured that the origin is located between the floor and the crest of the roughness elements, which is an acceptable value range from previous investigations (see e.g. [24], [25], [152]). Figure 4.15a and 4.15b show the results from the fitting procedures for selected rough wall runs represented by the skin friction velocity u_τ and the roughness function ΔU_{eff}^+ , respectively, against normalized shift of origin ϵ/k . The blue and red dashed line highlight the aforementioned estimated most likely value corridor from the integral approach. From the results, a most likely value range of $\epsilon = 0.4 - 0.5k$ can be extracted and a final value of $\epsilon = 0.5k$ was chosen, to be consistent with previous investigations (e.g. [24]). The described procedure was performed on the PIV data of the second campaign and the resulting ϵ was then applied also on the post-processing of data by Pitot pressure measurements.

Besides this result, it is emphasized how much influence ϵ has under the current conditions, most likely due to only moderate values for the blockage ratio k/δ . It underlines the importance of a proper choice for the shift of origin for cases with comparable blockage ratios which is a typical value range for most ground-based rough wall TBL investigations, especially in case of high-speed flows. The resulting fitting parameters are summarized in table 4.3, which additionally contains Coles wake parameter Π , the normalized EQSR k_s/k , the blockage ratio k/δ , the roughness Reynolds number based on the physical roughness height k^+ and based on the EQSR k_s^+ .

Figure 4.16 shows exemplary results of this fitting routine with inner scaled van Driest transformed mean velocity profiles along the smooth and rough wall together with the law-of-the-wall. Additionally, DNS data from [296] at comparable conditions are included. As in the plots before, all smooth data collapse and follow the law-of-the-wall in the mid-part of the semi-logarithmic plot ($20 \lesssim y^+ \lesssim 70$). Corresponding to the larger Reynolds number, the profile at MS2 results at a larger y^+ at the edge of the boundary layer. Also, the wake is more pronounced there according to the higher Reynolds number. The rough wall profiles show the typical shift by

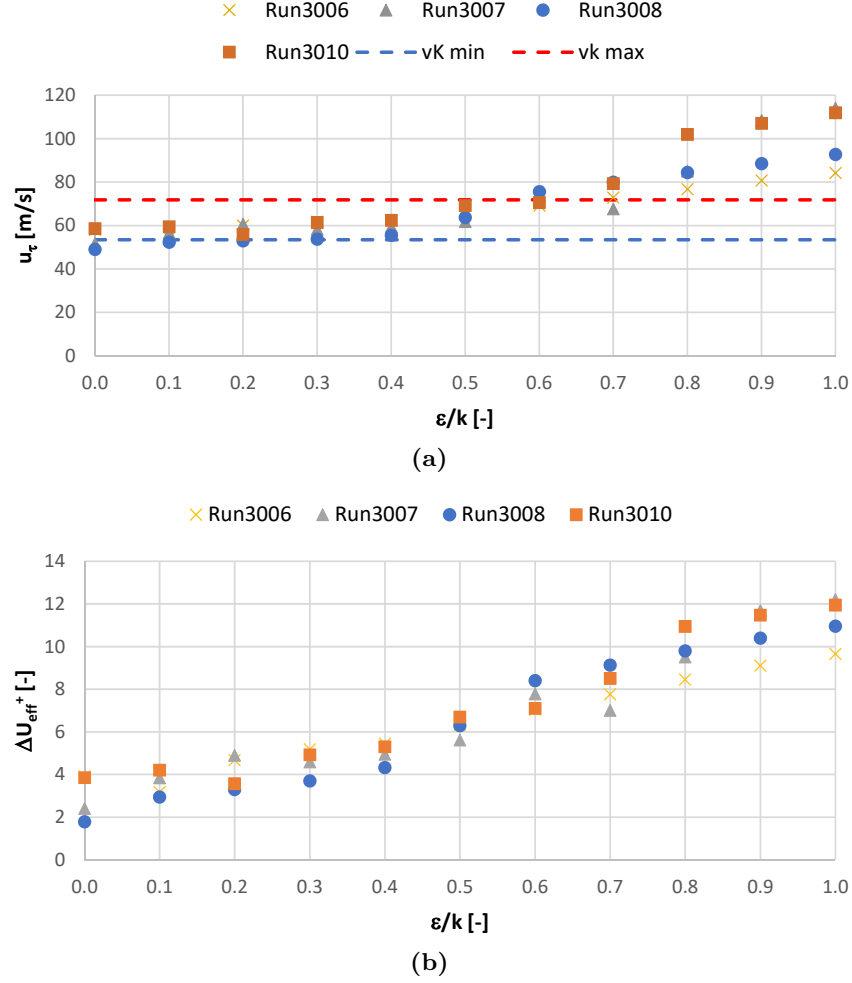


Figure 4.15: Rough wall skin friction velocity u_τ in a) and roughness function ΔU_{eff}^+ in b) from fitting approach against shift of origin ϵ/k for selected rough wall runs. Dashed lines in a) indicate corresponding value range from integral approach including uncertainties.

ΔU_{eff}^+ below the smooth wall profiles. As reference, a log-law line shifted by $\Delta U_{eff}^+ = 6$ is shown in figure 4.16a, which is the order of magnitude derived from all profiles (see Table 4.3). The resulting roughness Reynolds numbers k^+ are in a range of 34 to 38. Previous experimental investigations of incompressible flow along transverse bars with $\lambda/k = 3.8$ ($w/k = 0.75$) showed a value of $k^+ \approx 10$ ($\Delta U^+ \approx 5.5$) to be sufficient for a fully rough TBL according to Bandyopadhyay [19] (see also figure 4.17, in which error bars represent uncertainties as reported in section 3.12.2). Incompressible TBL DNS data along transverse square bars with $\lambda/k = 7$ to 120 showed a value of $k^+ \approx 19$ ($\Delta U^+ \approx 7.5$) to be sufficient for a fully rough flow according to Choi [42]. Also, Berg [24] assumed his compressible data along the transverse square bars with $\lambda/k = 4$ at $k^+ \approx 33.8$ ($\Delta U^+ \approx 6.5$) to be in the fully rough regime. Presuming the TBL data of this work to be fully rough as well, an equivalent sand grain roughness can be found via eq. (3.74), which results in approx. 38 to 59. This would result in $k_s/k = 1.1$ to 1.4, which is closer to the

compressible value by Berg than the incompressible investigations (see also figure 2.8). But it needs to be mentioned that in case of the current, as well as Berg's data ($k_s^+ = 44$), the derived k_s^+ values are slightly below the fully rough limit for sand grain roughness, typically given with $k_s^+ = 60 - 70$. On the other hand, they are above the limit for transverse bar roughness by [19] with $k_s^+ \approx 30$. Nevertheless, transitional roughness effects cannot be excluded completely and the derived k_s/k values need to be taken with some caution.

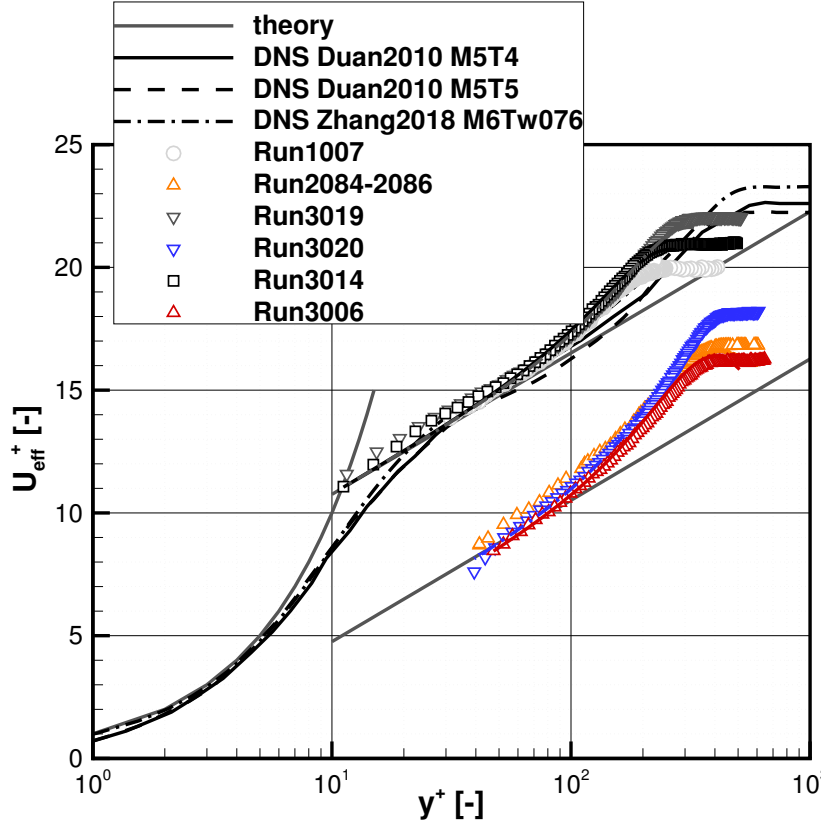


Figure 4.16: Inner scaled van Driest transformed mean velocity profiles along smooth (black, grey) and rough wall (colored) together with the law-of-the-wall and DNS data according to Duan *et al.* [56] and Zhang *et al.* [296].

To further put the extracted EQSR values of the current work into perspective, figure 4.18 shows two plots for k- and d-type roughness topologies based on scalings by Jiménez [118]. The first plot in figure 4.18a shows the EQSR, corrected by the drag coefficient, $k_s/(c_D k)$ against the frontal solidity $\lambda_f = k/\lambda$. As was reported for similar topologies, $c_D = 1.25$ was chosen for the TSB roughness of the current work. The different behaviors of the sparse ($\lambda_f \lesssim 0.15$) and dense roughness regime ($\lambda_f \gtrsim 0.15$) are visible with different slopes. Error bars in case of data by Schlichting indicate re-evaluated values by Coleman *et al.* [49]. Additionally, for comparison super- and hypersonic data points are included with spanwise square bars by Berg [24], 3D blocks by Ekoto [69] as well as mesh and square bars by Williams *et al.* [288]. The current TSB data is shown by the red square symbol together with error bars in both axes, representing uncertainties

Table 4.3: Overview of results from fitting approach (s: smooth wall, r: rough wall).

Run	MS	wall	c_f [-]	u_τ [m/s]	Π [-]	ΔU_{eff}^+ [-]	ϵ [mm]	k_s/k [-]	k/δ [-]	k^+ [-]	k_s^+ [-]
1007	1	s	1.57E-03	53.8	0.27	0	0	0	0	0	0
1008	1	s	1.26E-03	53.7	0.41	0	0	0	0	0	0
1010	2	s	1.48E-03	50.5	0.43	0	0	0	0	0	0
2032&2033	1	s	1.39E-03	52.0	0.19	0	0	0	0	0	0
2039&2052	1	s	1.38E-03	52.0	0.21	0	0	0	0	0	0
2013&2023	2	s	1.24E-03	48.3	0.33	0	0	0	0	0	0
2024&2025	2	s	1.28E-03	48.8	0.27	0	0	0	0	0	0
2072-2074	1	r	1.94E-03	62.3	0.34	4.68	0.25	0.73	0.11	36.2	26.4
2084-2086	1	r	1.98E-03	63.5	0.40	5.29	0.25	0.93	0.11	36.3	33.6
2090&2091	1	r	2.03E-03	64.1	0.34	5.17	0.25	0.87	0.11	36.8	32.1
2092-2094	2	r	2.40E-03	67.9	0.38	7.39	0.25	1.84	0.08	42.4	78.1
2093&2095	2	r	2.06E-03	63.2	0.46	6.42	0.25	1.36	0.08	38.9	52.8
2126-2128	1	r(SG)	2.28E-03	63.1	0.38	6.17	0.25	1.09	0.10	43.8	47.8
2129-2131	1	r(SG)	2.26E-03	63.7	0.34	6.07	0.25	1.06	0.10	43.5	46.0
2121-2123	2	r(SG)	1.75E-03	58.6	0.47	5.30	0.25	0.90	0.08	37.4	33.8
2124&2125	2	r(SG)	1.63E-03	56.4	0.56	5.08	0.25	0.86	0.08	36.0	30.9
3014	1	s	1.38E-03	51.8	0.41	0	0	0	0	0	0
3016	1	s	1.48E-03	52.0	0.40	0	0	0	0	0	0
3019	2	s	1.24E-03	49.6	0.47	0	0	0	0	0	0
3006	1	r	2.45E-03	66.0	0.58	6.34	0.25	1.35	0.12	37.9	51.1
3007	1	r	1.92E-03	61.7	0.76	5.62	0.25	1.12	0.11	34.2	38.4
3008	1	r	1.95E-03	63.6	0.72	6.29	0.25	1.16	0.12	34.3	50.2
3010	1	r	1.52E-03	69.1	0.77	5.70	0.25	1.19	0.11	33.3	39.7
3020	2	r	1.99E-03	58.1	0.83	6.15	0.25	1.35	0.10	35.0	47.4

as estimated in section 3.1 and section 3.12.2. Current data is visibly below incompressible heritage data and are not following the expected trend for k-type roughness but follow the general trend of other super- and hypersonic data points.

The second plot in figure 4.18b shows the EQSR, normalized by the boundary layer thickness, k_s/δ against the blockage ratio k/δ . Heritage data are shown twice, where non-filled symbols indicate originally published data, whereas filled symbols represent data, corrected for the assumed influence of blockage ratio according to Simpson [232], as reported by Jiménez [118]. Corrections after Simpson [232] require unbiased values fulfilling $k/\delta \rightarrow 0$, which is not available for data of this thesis. Therefore only uncorrected data is included. The current TSB data is shown by the red square together with error bars, representing corresponding estimated uncertainties. It is visible that data fall within the spread of incompressible heritage data, but above the speculated scaling with $k_s \approx 0.02\delta$ reported by Perry *et al.* [187] and Jiménez [118].

The two plots could lead to the conclusion that the current roughness results scale neither with k nor with δ i.e. fulfilling neither the criterion for k- nor d-type roughness. But some comments are due at this point. Current data is not the only point which do not scale with $k_s/(c_D k)$ and λ_f , instead the plot hints more towards a general trend for compressible data. Furthermore, the d-type scaling with $k_s \approx 0.02\delta$ is not supported by any of the included data and was already critically discussed by Jiménez [118]. Additionally, it is questionable which impact compressible effects might have on potential d-type behavior. Furthermore, a reverse calculation of corresponding heritage EQSR values from the published plot via $k_s/k = k_s/\delta \cdot k/\delta$ would indicate with $k_s/k \approx 1$ rather low values for data by Perry *et al.* [187] and Bandyopadhyay [19], not corresponding to originally published values. It is not clear how Jiménez rebuilt the heritage data and therefore these points must be used with some caution.

As introduced in section 3.10.2, a new formulation to derive the rough wall skin friction velocity u_τ depending on ΔU_{eff}^+ via eq. (3.52) was tested on selected PIV results. This procedure was developed with the motivation to ease the fitting procedure by reducing the unknown quantities by one (see section 3.10.2). In general, the good agreement for the validation cases can also be confirmed by post-processing data of this thesis, where e.g for Run3006 results of with or without this equation agreed to within approx. 2% for both u_τ and ΔU_{eff}^+ . However, no substantial differences in the stability of the fitting procedure was encountered. Therefore no additional analyses have been performed, since this has been the main motivator.

Additionally, the new method to determine EQSR values was tested, introduced in section 3.12.1. By using eq. (3.76) instead of the “traditional” determination via eq. (3.75), no knowledge about u_τ is required. As a control parameter, also Coles wake parameter Π can be extracted via eq. (3.76) and compared to the value from the fitting approach (eq. (3.47)). For Run3006 this new method resulted in $k_s/k = 1.38$ very close to $k_s/k = 1.35$ via the “traditional” eq. (3.75). And also the wake parameters agree closely with $\Pi = 0.6$ against $\Pi = 0.58$ for eq. (3.76) and eq. (3.47), respectively. Despite this agreement, post-processing of data from other runs showed partly larger scatter, especially in case of the EQSR values of Run3007 and Run3010 (see table 4.4) Also data from other runs were inconclusive and more analysis is needed, so that all following discussions are based on results from the “traditional” eq. (3.75).

The above stated post-processing, combining the fitting and integral approach, gives a best estimate based on the previous choice for ϵ . Due to the range of acceptable u_τ (based on the

Table 4.4: EQSR results from post-processing via equation (3.75) or equation (3.76)

Run No.	(3.75) k_s/k [-]	Π [-]	(3.76) k_s/k [-]	Π [-]
3006	1.35	0.58	1.38	0.60
3007	1.12	0.76	2.18	0.53
3008	1.16	0.72	1.47	0.66
3010	1.19	0.77	2.16	0.71

uncertainties from the integral approach), also the fitting parameters can vary in a certain value range. For example at MS1 (results from four different runs), the accepted range of u_τ between 53.7 and 70.5 would result in ΔU_{eff}^+ to vary from 3.2 to 7.1, therefore k^+ from 34.2 to 37.1, k_s^+ from 14.5 to 69.5 and finally k_s/k from 0.4 to 1.9. This highlights the sensitivity of the analysis, although based on state-of-the-art measurement techniques. This is not necessarily only due to the additional challenges posed by the compressible flow, but can also be seen in incompressible investigations, visible by the large scatter of derived EQSR from different investigators as given in chapter 1 and 3.12.2.

Apart from the previously reported results, rough wall profiles show in general larger wake parameters up to approx. 0.8, similar to previous studies e.g. [135], [186], [38]. The plot in figure 4.13 shows also rough wall data points of this thesis. It is visible that the smooth wall data follow the expected trend with Reynolds number, whereas the rough wall data lies above the scatter of the smooth wall data by Coles. There is ongoing debate if true smooth and rough wall boundary layer universality exists, for which the same wake parameter value range would be expected and different parameters are discussed to have an impact (see e.g. [39]). We continue the discussion about smooth to rough profile similarity based on turbulence data in section 4.3.

Pitot rake profiles:

As mentioned before, the derived ϵ from the data of the second PIV campaign was also applied on the fitting procedure of the Pitot pressure profile data. All values from the fitting approach are summarized in table 4.3, from integral approach in table 4.5 and with data of Run2084-2086 an exemplary result for FC2 at MS1 is also contained in figure 4.16. The plot as well as the results in the tables show that the rough wall parameters in general are in agreement with data from the second PIV campaign. E.g. the values for the roughness function ΔU_{eff}^+ and data for k_s/k have a slightly larger spread compared to the PIV data but cover the same range, especially if corresponding uncertainties are considered. This cross-measurement agreement gives the current result further credibility.

Sand grain type roughness:

Additionally to the technical square bar roughness topology, Pitot pressure profiles were also measured along the model section with Carborundum sand grain type roughness (see section 3.1). The same post-processing chain was used on profile data above this type of roughness (including an assumed $\epsilon/k = 0.5$). Runs in the range 2121 to 2131 in table 4.3 together with figure 4.19 show results from this sand grain type roughness (labeled “SG” for sand grain type in contrast to unlabeled square bar roughness). Due to the metallic substructure, the corresponding

Table 4.5: Skin friction velocities from integral approach of PIV (IW 512x16 pix²) and Pitot pressure data along rough wall (first and second block for TSB roughness and fourth block for SG roughness (2121 < run < 2131)) (s: smooth wall, r: rough wall).

Run MS1	Run MS2	wall	x_{MS1} [mm]	Δx [mm]	θ_{MS1} [mm]	$\Delta\theta$ [mm]	integral $u_{\tau,MS1}$ [m/s]	$u_{\tau,MS2}$ [m/s]	DQ $u_{\tau,MS1}$ [m/s]
3006	3020	r	496	130	0.21	0.07	60.7	58.2	58.8
3007	3020	r	496	130	0.21	0.07	62.2	59.6	60.3
2072-2074	2092-2094	r	496	138	0.19	0.08	64.0	60.8	61.4
2084-2086	2092-2094	r	496	138	0.20	0.07	63.2	60.2	60.9
2090-2091	2092-2094	r	496	138	0.19	0.08	64.6	61.3	61.9
2126-2128	2121-2123	r(SG)	496	138	0.22	0.05	55.0	53.5	54.6
2126-2128	2124-2125	r(SG)	496	138	0.22	0.05	56.5	54.9	55.9
2129-2131	2121-2123	r(SG)	496	138	0.22	0.05	56.2	54.6	55.6
2129-2131	2124-2125	r(SG)	496	138	0.22	0.06	57.7	55.9	56.8

wall temperatures were slightly lower than for experiments along the technical roughness model made from PEEK (see also table 4.1). In figure 4.19 it is visible that SG results agree closely with Pitot and PIV results along the technical roughness in inner and also outer scaling. Also the summarized results in table 4.3 and figure 4.17 show that the parameters from fitting are comparable between SG and TSB. Only the integral values are consistently lower than the values for the technical roughness (see table 4.5). This seems to stem from a lower increase in calculated momentum loss thickness from MS1 towards MS2 compared to data from TSB, although the near wall resolution and behavior seem to be comparable (not shown here). Interestingly, the resulting k_s/k from the fitting approach is almost but not exactly 1, which would be expected for a sand grain roughness. This was already observed by other investigators like Squire *et al.* [240] ($k_s/k \approx 2.2$) and Thakkar *et al.* [251] ($k_s/k \approx 0.9$) with the argumentation that the duplication of the roughness topology according to Nikuradse with respect to density and homogeneity is very challenging.

Also in case of the sand grain type roughness, the k_s^+ values are below the typically stated limits for fully rough conditions and therefore also the resulting k_s/k values need to be used with some caution since viscous influences cannot be excluded completely.

Apart from that, at this point, the comparable behavior between the technical square bar (TSB) and sand grain type roughness (SG) should be emphasized. The similarity in ΔU_{eff}^+ hints at similar momentum loss, despite the different topologies. Based on all made assumptions, this similarity is a further support for the previously extracted EQSR in case of the technical roughness. This agreement is also interesting in context of the Schlieren images (see section 4.1), which showed disturbances throughout the TBL in case of the Carborundum roughness in contrast to the technical roughness. These disturbances have been shown to be a sign for a clearly different behavior in momentum loss and outer layer similarity in case of diamond roughness topologies [69], [32]. This seems to be not the case for the presented data.

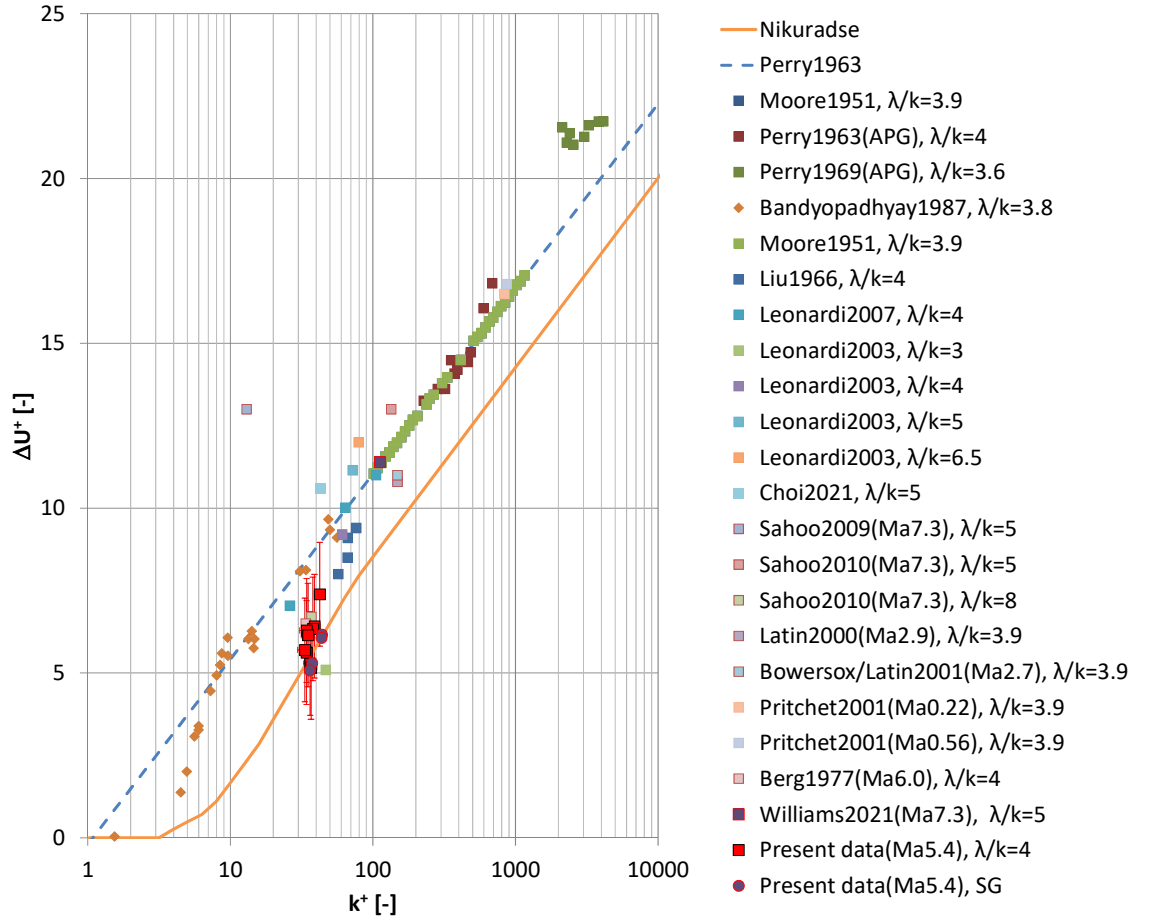
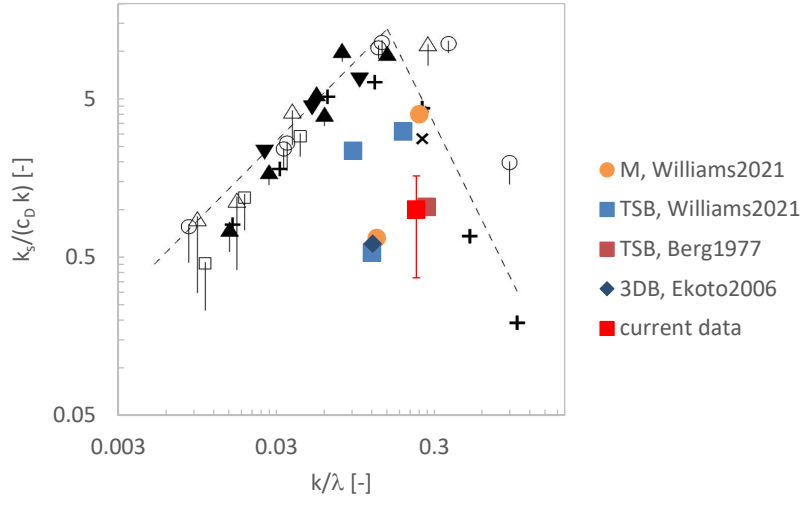
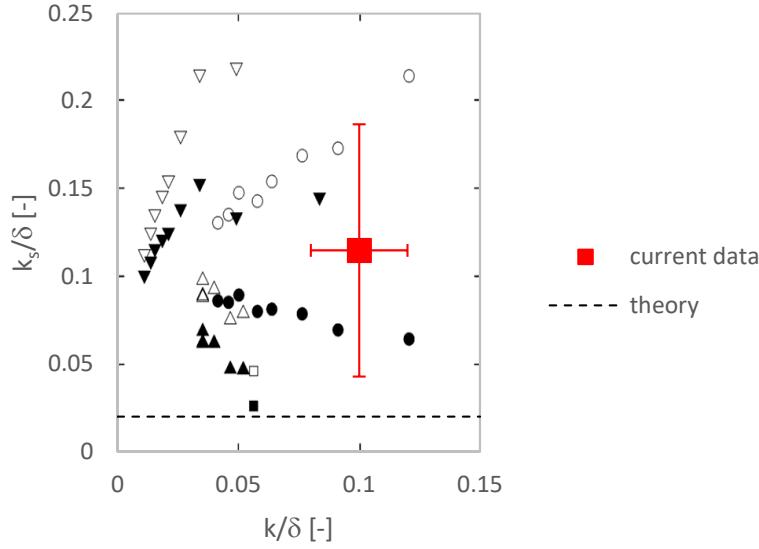


Figure 4.17: Roughness function ΔU^+ against roughness Reynolds number k^+ for square bars of different wavelength close to a value of $\lambda/k = 4$ from different authors (symbols without edge for incompressible and with edge for supersonic data including the edge Mach number in brackets) according to [173], [24], [185], [152], [138], [208], [209], [288], [195], [42], [164], [187], [150], [148], [43], [33].



(a)



(b)

Figure 4.18: a) Equivalent sand roughness for various k-type roughness topologies versus the solidity $\lambda_f = k/\lambda$, corrected with empirical drag coefficients. Open symbols, rounded elements (spheres, cones, $c_D = 0.3$; spherical segments, $c_D = 0.13$) from Schlichting [212]. For all others, $c_D = 1.25$. “black delta”, spanwise fences (Schlichting [212]); “black gradient”, spanwise fences (Webb *et al.* [273]); “plus”, spanwise cylinders (Tani [247]); “x”, spanwise square bars (Bandyopadhyay [19]), spanwise square bars (Berg [24]), 3D blocks (Ekoto [69]), mesh and square bars (Williams *et al.* [288]). The dashed lines have logarithmic slopes +1 and -2. For the error bars, see text. b) Equivalent sand roughness for d-type roughness topologies versus blockage ratio. delta, Wood and Antonia [290]; circle, Perry *et al.* [187]; gradient, Bandyopadhyay [19]; square, Tani [247].

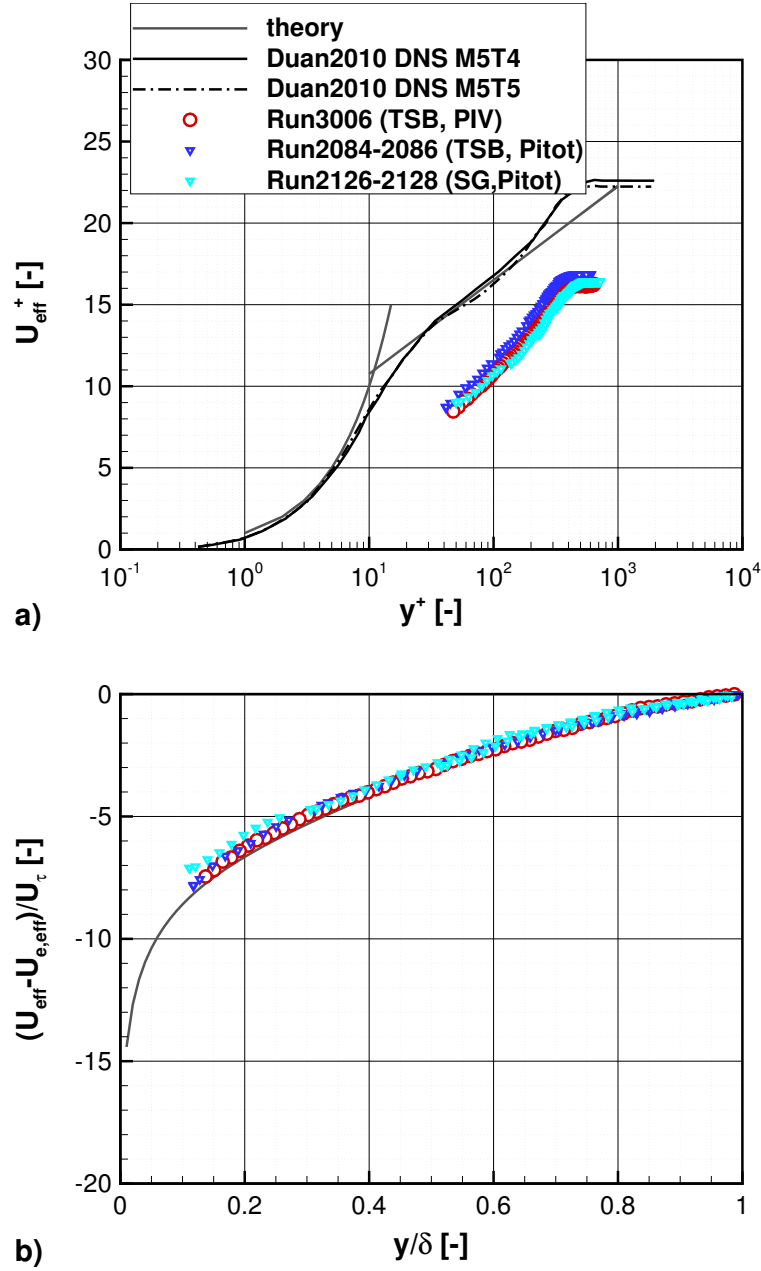


Figure 4.19: a) Inner and b) outer scaled van Driest transformed mean velocity profiles along technical square bar rough wall and sand grain type rough wall (SG) from Pitot and PIV data together with the law-of-the-wall and DNS data according to [296].

Comparison to computed data:

In the following comparisons of experimental data to data from the BLcode with the modified Krogstad roughness model are shown (see section 3.8). Figure 4.20 shows roughness Reynolds numbers k_s^+ along the smooth and rough cone from BLcode at different EQSR values within $k_s = 0.125 - 2.0\text{mm}$ corresponding to $k_s/k = 0.25 - 4.0$, if $k = 0.5\text{ mm}$ is chosen. The impact of using either y^+ , k_s^+ or the compressible formulations y_c^+ , $k_{c,s}^+$ (see eq. (7.37) and (7.39)) is reported in section 7.6.6 (see figure 7.77). If not otherwise noted, in the following only data with the compressible formulation are used. Solid lines represent solutions for a fully rough cone, for which the Krogstad terms are activated for all points downstream the nose of the cone (label “xks=0m”). The dashed lines represent transitional solutions for which the turbulent terms (and with that also the roughness influence) is suppressed upstream of a position $x_a = 0.225\text{ m}$ (label “sst=0.23m”) to enforce a laminar solution and with that a prescribed laminar-to-turbulent transition. The location and length of the transition are designed by the proper parameter choice within the boundary layer code to resemble IR data a-posteriori. The dash-dotted line and dash-dot-dotted line represent the same choice for fully turbulent and transitional computations as before (label “xks=0m” and label “sst=0.23m”, respectively) but additionally, the roughness terms were activated only within $0.33\text{ m} < x_a < 0.65\text{ m}$ (label “xks=0.33-0.65m”), corresponding to the roughness insert of the cone model. Additionally, with long-dashed lines values from analytical calculations according to [71] (see eq. (3.40)) are included which agree very well with fully turbulent / rough BLcode computations for locations some distance downstream of the nose region ($x_a \geq 0.2\text{ m}$). The differences close to the nose are probably an effect of corresponding small δ/k values which have an impact due to the compressible definition in case of the BLcode. Also included are post-processed data from PIV (only second campaign as symbols) showing averaged values with error bars representing run-to-run scatter (of corresponding available runs as summarized in table 4.3).

In general, all roughness Reynolds numbers k_s^+ in figure 4.20 show an increase with increasing EQSR and a decrease with running length due to the increase in TBL thickness along the cone. The prescription of the laminar-turbulent-transition leads to an overshoot immediately downstream the transition location with a relaxation to fully turbulent values downstream distances of approx. 0.4 m . The computations with only segment-wise roughness begin no earlier than at the beginning of the roughness insert and also show an overshoot over the fully rough case (slightly higher in case of the transitional case) but also converge to the latter for distances downstream of approx. 0.5 m . This roughness relaxation was already described by [24] and [104]. In case of small EQSR nearly no differences are visible between the different computational assumptions. Only for $k_s > 1\text{mm}$, which correspond to $k_s/k > 2$, roughness Reynolds numbers are fully rough in terms of $k_s^+ \geq 70$ (black dashed line). The experimental data show a nearly constant trend along the cone (within scatter) and are slightly above the computed k_s^+ profile with $k_s/k = 1$, in line with corresponding values from fitting ($k_s/k \approx 1.1 - 1.4$, see also table 4.3). It must be noticed that at MS2 only one data point was available and therefore no statistical values could be derived.

Figure 4.21a shows calculated and experimental smooth and rough wall inner-scaled van Driest transformed velocities U_{eff}^+ . An offset between computed and experimental smooth wall profiles is visible, which can be expressed by corresponding law-of-the-wall constants with $B = 4.1$ and $B = 5(.0)$ for numerical and experimental data, respectively. The computed rough wall lines

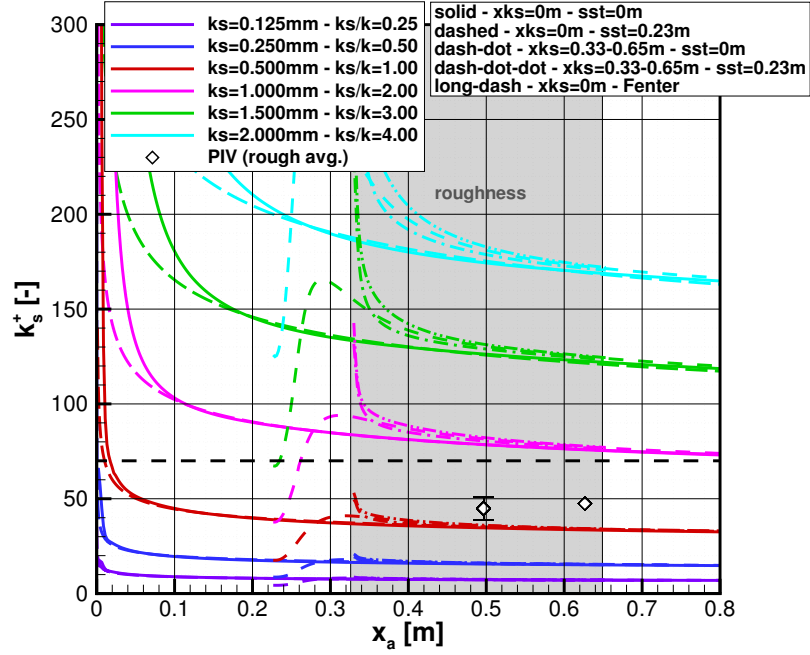


Figure 4.20: Calculated roughness Reynolds numbers k_s^+ along rough cone from BLcode at different EQSR values and assumptions together with data from PIV.

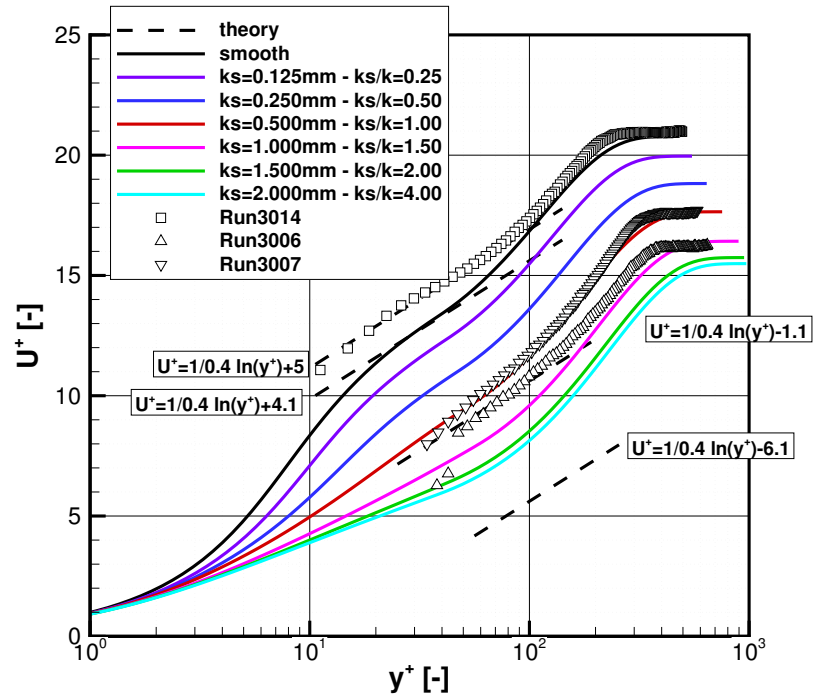
show the expected shift due to roughness below the smooth wall reference. Also the experimental data show this shift and seem to correspond to computed data close to $\Delta U_{eff}^+ \approx 5$ ($B = -1.1$), instead of a value of $\Delta U_{eff}^+ \approx 6$, as extracted from the data directly (see table 4.3). But, since this shift depends on the constant of the reference smooth wall data, also the ΔU_{eff}^+ is affected. Therefore, figure 4.21b shows the difference from the log-law as $U^+ - U_{log}^+$ against wall-normal distance, where for each dataset the corresponding smooth wall constant was chosen. In this plot it is visible that the computed and experimental smooth wall data agree only in a very narrow region around $y^+ \approx 30$. The wake portion is more pronounced in the computed data. In this representation, the rough wall data now correspond more closely to $U^+ - U_{log}^+ \approx \Delta U_{eff}^+ \approx 6$, as extracted from the data directly. Numerical rebuilding agrees best with $k_s/k = 1.0 - 1.5$. In figure 4.21b, it is visible that the slope of the normalized rough wall PIV data is not corresponding exactly to $1/\kappa$ (no plateau visible). This is due to the chosen rough wall post-processing approach, which used skin friction velocity results from the integral approach to guide the choice for a shift of origin ϵ , in contrast to an approach like Perry *et al.*, which uses a shift of origin to match the smooth wall slope. There is on-going debate if the smooth wall slope $1/\kappa$ is necessarily the same above rough walls, as summarized in section 2.1.

In figure 4.22a calculated and experimental smooth and rough wall skin friction values $c_{f,e}$ are shown and corresponding skin friction augmentations $c_{f,e,r}/c_{f,e,s}$ along the rough cone from BLcode computations at different EQSR values are shown in figure 4.22b. In case of the BLcode data, the different assumptions already shown in figure 4.20 are used. As expected, the computed values show an increase of friction due to roughness above smooth wall values. The symbols

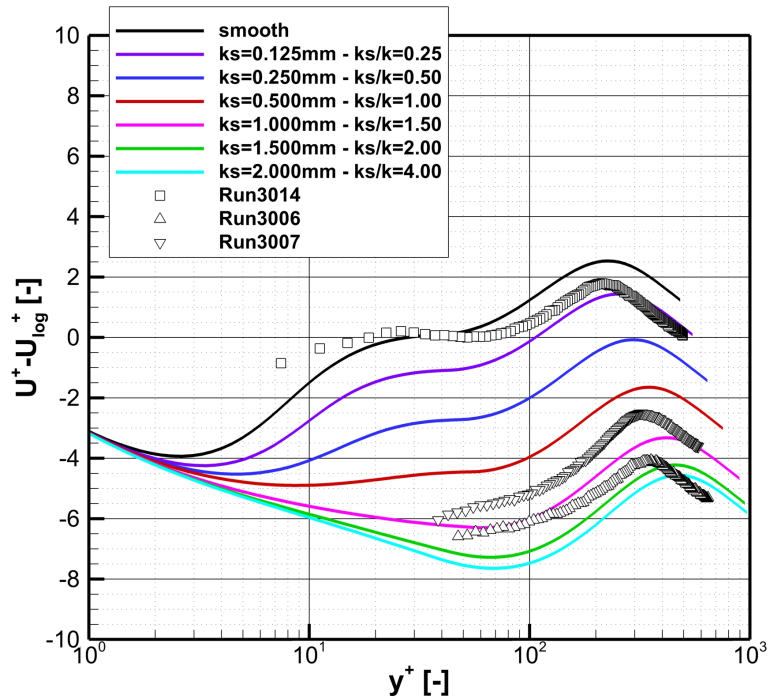
indicate experimentally derived values from PIV and Pitot measurements (see table 4.3), averaged over all available data at each MS and wall condition. The smooth wall experimental data in figure 4.22a correspond close to numerical data, although it is a bit larger in case of MS1, even above the overshoot from the transitional computation (dashed line). The rough wall data are close to computed values with $k_s/k \approx 1 - 2$. Although, data at MS2 show an unexpected behavior with an increase with running length. This might be attributed to the limited data collected at MS2. The corresponding experimental augmentation data in figure 4.22b is close to computed values with $k_s/k \approx 2$ at MS1. But also here, an increase to values at $k_s/k \approx 3 - 4$ is visible at MS2. This is also the case for data from Pitot measurements. This is not fully understood at time of writing and needs further analysis.

Figure 4.23a shows corresponding calculated and experimental smooth and rough wall skin friction velocity u_τ and in figure 4.23b skin friction velocity augmentations $u_{\tau,r}/u_{\tau,s}$ along the cone from BLcode computations at different EQSR values. In this representation, smooth wall experimental data are a bit lower but close to the corresponding computed values in figure 4.23a. Both, numerical and experimental data, show again the expected increase of friction with roughness. The rough wall experimental data in 4.23a are close to computed data with $k_s/k \approx 0.5 - 1$, as extracted from the data directly (see table 4.3). Also, in this representation, data show the expected trend with decreasing friction values with increasing running length, in contrast to the corresponding skin friction coefficient values (see figure 4.22). The corresponding skin friction velocity augmentation $u_{\tau,r}/u_{\tau,s}$, as shown in figure 4.23b, is lower compared to the skin friction coefficient augmentation $c_{f,e,r}/c_{f,e,s}$ (see figure 4.22b). This is due to the fact that for the same inflow conditions and wall temperatures it can be shown that $u_{\tau,r}/u_{\tau,s} = \sqrt{c_{f,e,r}/c_{f,e,s}}$. In this representation, the experimental augmentation levels correspond more closely to computed values with $k_s/k \approx 1 - 2$.

The above results show that numerical rebuilding of the experimental data with the BLcode and modified Krogstad model agrees best when $k_s/k \approx 0.5 - 2$ is used as input, depending on the data representation.



(a)



(b)

Figure 4.21: Calculated (lines) and experimental (symbols) a) smooth and rough wall inner-scaled van Driest transformed velocities U^+ and b) difference from log-law against wall-normal distance from BLcode at different EQSR values.

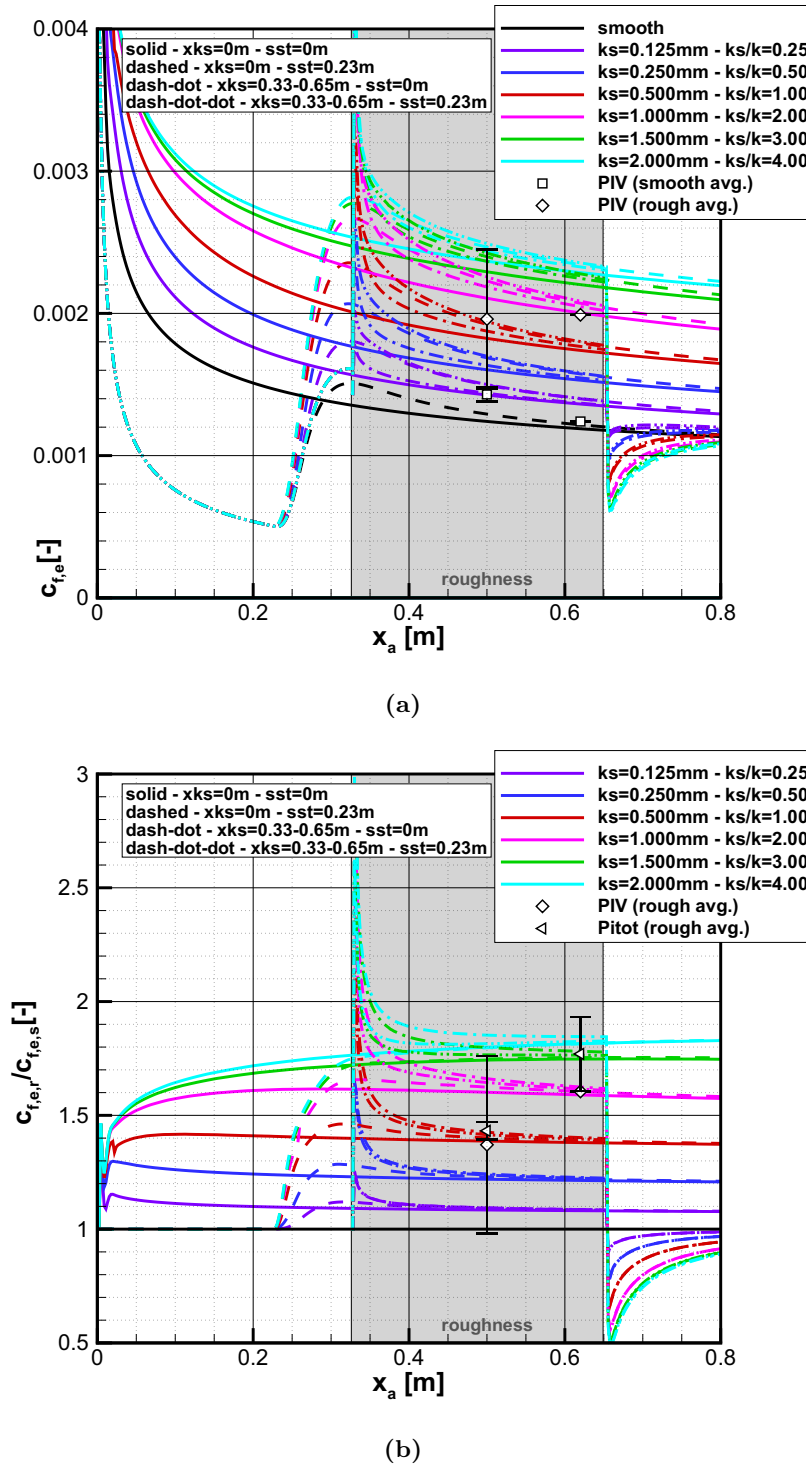
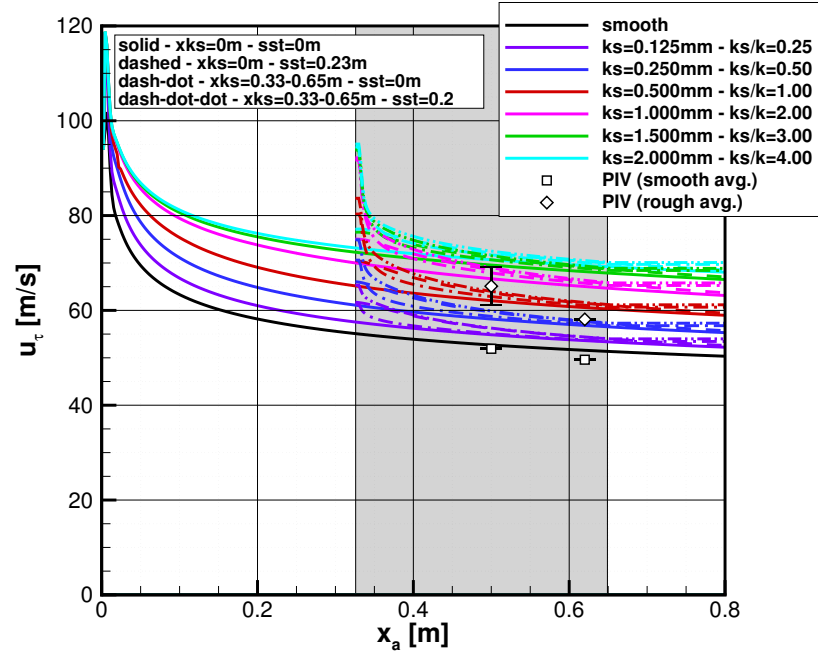
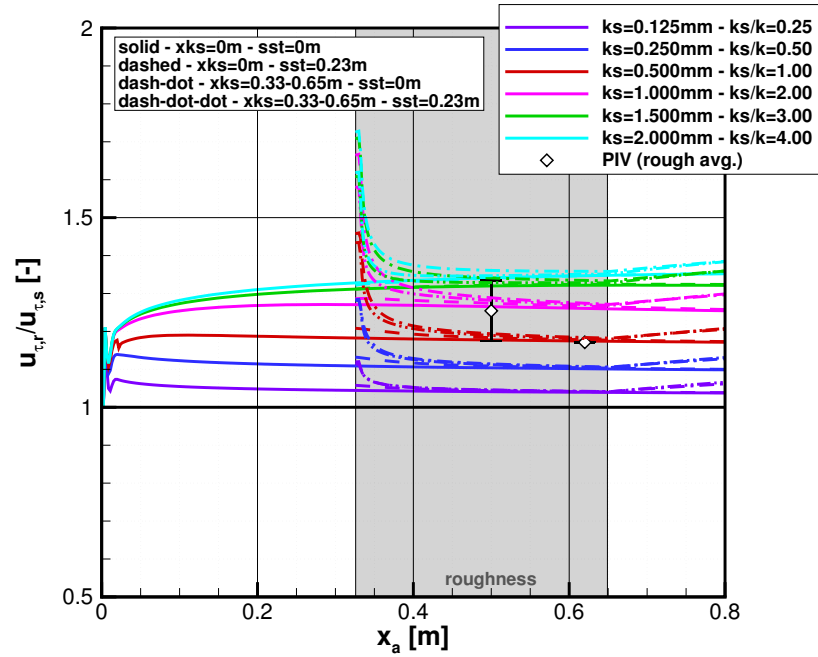


Figure 4.22: Calculated (lines) and experimental (symbols) a) smooth and rough wall skin friction $c_{f,e}$ and b) skin friction augmentations $c_{f,e,r}/c_{f,e,s}$ along rough cone from BLcode at different EQSR values.



(a)



(b)

Figure 4.23: Calculated (lines) and experimental (symbols) a) smooth and rough wall skin friction velocity u_τ and b) skin friction velocity augmentations $u_{\tau,r}/u_{\tau,s}$ along rough cone from BLcode at different EQSR values.

Augmentation summary:

With the information of friction augmentations and roughness Reynolds numbers from EQSR fitting (see section 4.2.2) it is possible to summarize the current data in form of skin friction coefficient augmentation $c_{f,e,r}/c_{f,e,s}$ against k_s^+ and compare it to results from modeling and literature data. Figure 4.24 shows calculated and experimental rough to smooth wall skin friction coefficient augmentations from several different sources (grey/black lines/symbols) together with data from this thesis (red symbols). It is visible that all data show an approximately logarithmic growth of friction augmentation with k_s^+ . Calculated values include data from BLcode computations (label “BLcode Krogstad”) and correlations (labels “Fenter-VD” and “Bowersox”). The line labeled “BLcode Krogstad” corresponds to a fit through data points calculated with the boundary layer code including the modified Krogstad roughness model (see section 3.8) along an artificially extended cone to cover a large k_s^+ regime (see section 7.6.6 and figure 7.79 for discussion). The correlations were introduced in section 3.11 and the line labeled “Fenter-VD” corresponds to a correlation where the rough wall skin friction $c_{f,e,r}$ was calculated via eq. (3.40) and the corresponding smooth wall reference $c_{f,e,s}$ via eq. (3.34). Correlations (Fenter-VD, Bowersox) show indefinite growth whereas numerical predictions with the Krogstad model level off at approx 2.5 (see section 7.6.6 for discussion of blockage ratio effects). Bowersox fitted a curve with $c_{f,e,r}/c_{f,e,s} = 0.42 \log(k_s^+)$ based on data of previous experimental investigations along adiabatic walls, highlighting that a spread of $\pm 20\%$ around this correlation is visible (included as white shape around the correlation line in figure 4.24). He already indicated that cold wall data, like data from Holden [104], [105], [106], Young [294] and Hill *et al.* [101] fall partly below this trend line, but mostly still within the 20 % range (also visible in figure 4.24). Data of this thesis is at near-adiabatic conditions with $T_w/T_r \approx 0.8$. The lower value of the two red symbols in figure 4.24 is the ensemble averaged value of all valid runs at MS1 and the larger at MS2 (see table 4.3). The error bars indicate the minimum and maximum spread of data. It is visible that both averaged values differ by approx. 0.2 but do agree if corresponding spread is considered. Also, the data points fall below the Bowersox trend line but still within the 20 % range. Including the spread, data of this thesis also correspond to the numerical prediction via the BLcode.

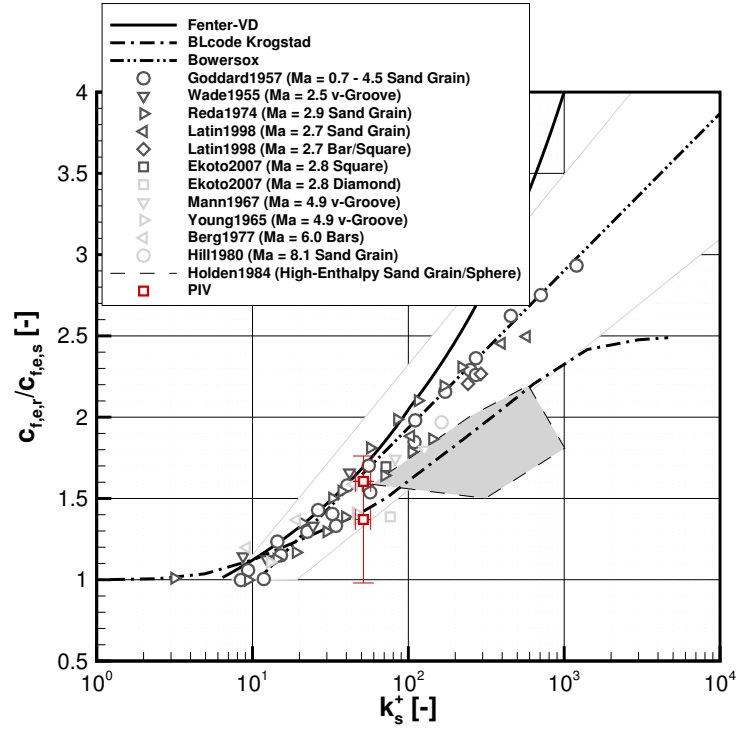


Figure 4.24: Calculated (lines) and experimental (symbols) rough to smooth wall skin friction augmentation along roughness Reynolds number from different authors (grey/black lines/symbols, all collected in [32]) and data of this thesis (red symbols).

4.3 Turbulence Data

In this section, streamwise, wall normal and Reynolds shear-stress velocity fluctuations are discussed in context of literature data, potential sub-grid filtering and the effect of roughness. After that, the diagnostic plot approach is tested on the current data set. Then, two-point correlations of the velocity fields and the presence of uniform momentum zones are analyzed to investigate the structure of the TBL along the smooth and rough wall. This analysis is performed mainly on the PIV data of the second campaign, since it had proper resolution and quality (see discussion in section 7.5.2). Selected comparisons are made to smooth wall data of the first campaign.

4.3.1 Profiles

Smooth wall:

Streamwise, wall normal and Reynolds shear-stress velocity fluctuations were extracted from PIV data. Smooth wall results are shown in figure 4.25 in Morkovin scaling, utilizing $u_* = \sqrt{\rho/\rho_w} u_\tau$, in comparison to CFD data and other experimental data. For the experimental data in this

work, the mean density profiles for the Morkovin scaling were derived assuming a Crocco–Busemann temperature distribution within the boundary layer (see e.g. [277]) together with the measured surface pressure from the closest tap on the model. Other temperature distributions (e.g. according to Walz, see e.g. [56]) have been tested and only negligible differences were encountered (see section 7.5.2). The same spatial averaging as for the mean profiles was applied. The error bars represent the sampling uncertainties according to [23] together with uncertainties for values relevant for Morkovin scaling. For comparison, experimental boundary layer data of other authors were included with $Ma = 2.86$, $Re_\theta \approx 60000$ data according to Ekoto *et al.* [68], $Ma = 7.2$, $Re_\theta \approx 3600$ data according to Sahoo *et al.* [209], $Ma = 7.6$, $Re_\theta = 9337$ data according to Williams and Smits [289] and incompressible experimental data by Klebanoff [129] ($Re_\theta = 6940$). Additionally, DNS data in the plots correspond to Duan *et al.* [56], [58] (in legend M0: $Ma_e = 0.3$, $T_w/T_r = 1$, $Re_\theta = 15148$, $Re_{\delta_2} = 1515$, $Re_\tau = 570$; M5: $Ma_e = 4.9$, $T_w/T_r = 1$, $Re_\theta = 4932$, $Re_{\delta_2} = 1587$, $Re_\tau = 417$; M5T4: $Ma_e = 4.97$, $T_w/T_r = 0.68$, $Re_\theta = 3819$, $Re_{\delta_2} = 1526$, $Re_\tau = 434$; M5T5: $Ma_e = 4.97$, $T_w/T_r = 1$, $Re_\theta = 4841$, $Re_{\delta_2} = 1537$, $Re_\tau = 386$) and Zhang *et al.* [296] (case M6Tw076: $Ma_e = 5.86$, $T_w/T_r = 0.76$, $Re_\theta = 9455$, $Re_{\delta_2} = 1746$, $Re_\tau = 453$).

The streamwise Reynolds stress from this thesis (red symbols in figure 4.25a) corresponds well to CFD data from DNS and TAU within the upper boundary layer. At the boundary layer edge, measured values suggest a value close to, although slightly below, the TAU computation with an inflow turbulence level of 1 %, which corresponds well to the value of 0.8 %, derived by the method according to Scharnowski *et al.* [211] (see section 7.5.5) and values from inflow turbulence characterization campaign with L2F [94]. This cross-measurement agreement gives the current result further credibility. It also agrees with results of the first PIV campaign, shown here with blue circles together with corresponding uncertainty bands as gray areas (see [171]). In the vicinity of the turbulence peak from DNS ($y/\delta = 0.03 - 0.09$, depending on the data set), the current PIV data underpredict the corresponding Reynolds stress. The data only agree to TAU RANS computations, which are known for their lack of resolving the Reynolds stress peak for hypersonic TBL flow (see e.g. [271]). This filtering is visible even more drastically in the other components, namely the wall normal (figure 4.25 b) and Reynolds shear-stress component (figure 4.26), comparable to previous compressible PIV investigations. This effect can be influenced by particle response behavior due to the limited Stokes numbers and also sub-grid filtering. An extended discussion about filtering effects based on the current results is included in section 7.5.2

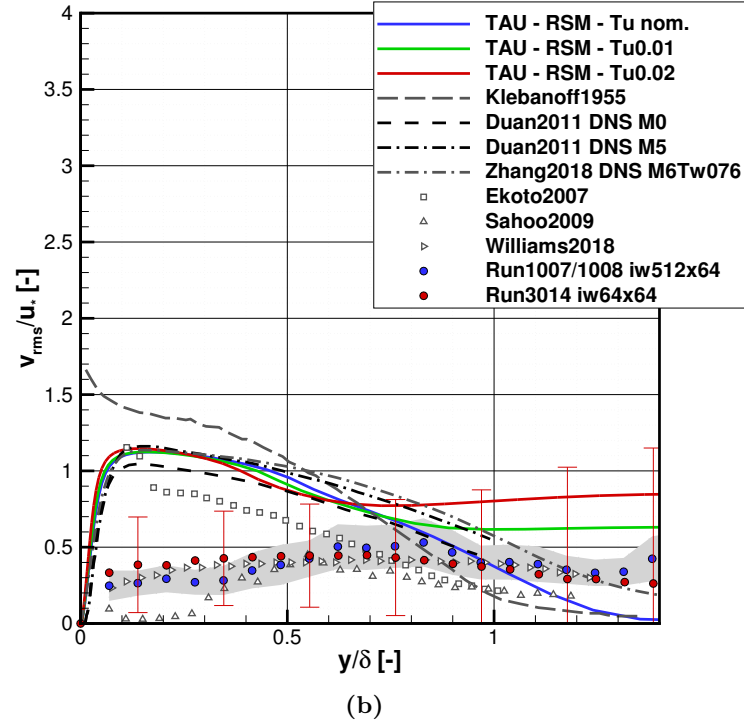
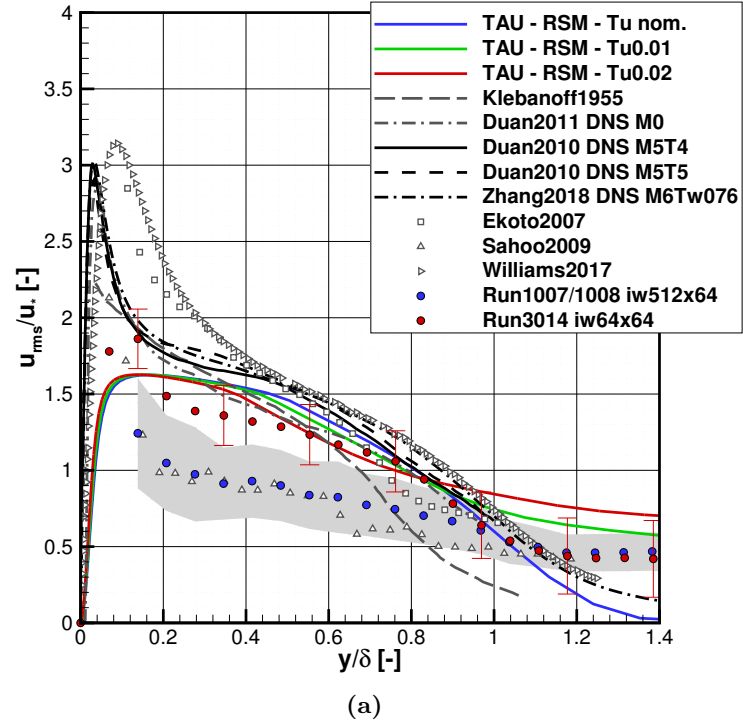


Figure 4.25: Profiles of outer layer Morkovin scaled velocity fluctuation components in a) streamwise, b) wall-normal direction in comparison to previous experimental ([68], [209], [289], [129]) and CFD data ([56], [57], [296]).

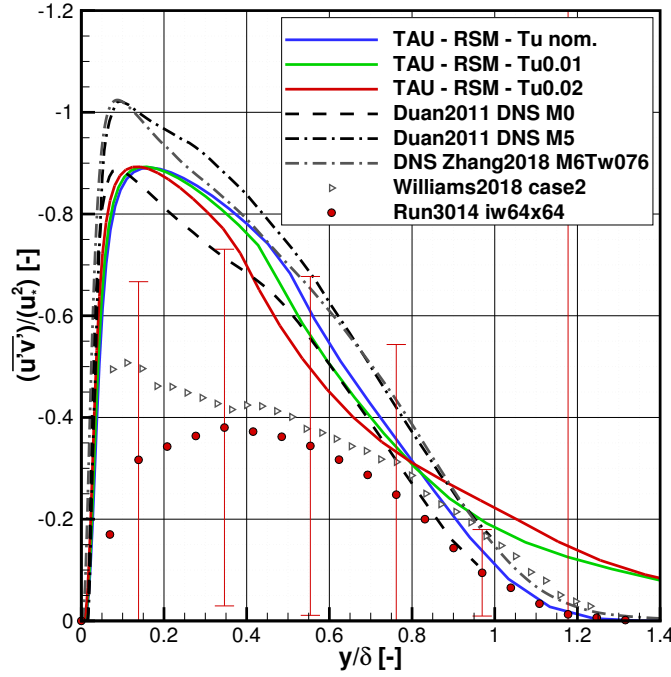


Figure 4.26: Profiles of outer layer Morkovin scaled velocity fluctuation components in Reynolds shear-stress component in comparison to previous experimental ([68], [209], [289], [129]) and CFD data ([56], [57], [296]).

Rough to smooth wall comparison:

Scalings:

There is on-going debate about a proper way to scale varying properties in compressible TBLs including Reynolds stresses. This is because multiple values, deemed relevant for scaling, vary as well with wall-normal distance. Previous work investigated different options and identified encouraging results for a so called semilocal scaling (see e.g. [287]). Figure 4.27 and 4.28 show different options for scaling of the streamwise turbulence profiles along the smooth and rough wall. The sub-plots show in figure 4.27a show data in outer layer skin-friction velocity scaling (u_{rms}/u_τ against y/δ), in figure 4.27b outer layer Morkovin scaling (u_{rms}/u_* against y/δ) and in figure 4.28a inner layer Morkovin scaling (u_{rms}/u_* against y^+) and figure 4.28b semilocal Morkovin scaling (u_{rms}/u_* against $y^* = yu_*/\nu$, letting the local stress and kinematic viscosity vary with wall distance). Additionally, incompressible and compressible DNS according to Duan and Martin [58] and Zhang *et al.* [296] are included for comparison. Although the near wall Reynolds stress peak is not fully resolved in case of the current experimental data, general tendencies can be extracted.

Outer layer skin-friction velocity scaling in figure 4.27a collapses the experimental data with the compressible DNS data, independent if above the smooth or rough wall. This behavior would further support Townsend's hypothesis, for which the outer flow is mostly unaffected by roughness [118]. Only the rough MS2 profile (Run3020) deviates which could also be influenced by slightly

less homogeneous seeding quality in case of this run. The difference between experimental data and compressible DNS outside the boundary layer ($y/\delta > 1$) is most likely driven by differences in inflow turbulence intensities. In this scaling, the incompressible DNS profile does not agree with compressible experimental and numerical data, especially in the mid-parts of the TBL.

The agreement between incompressible and compressible DNS and experimental data is much better in outer layer Morkovin scaling (figure 4.27b), which additionally utilizes the mean density profile to scale, due to $u_* = \sqrt{\frac{\rho}{\rho_w}} u_\tau$. In this scaling, all data collapses within experimental uncertainties in contrast to previous work [287]. Only the MS2 profiles (both smooth and rough) deviate again within $0.2 < y/\delta < 0.9$.

If an inner layer Morkovin scaling is chosen (figure 4.28a), close to the wall all experimental smooth wall data coincide, although not with DNS data. The rough wall near wall streamwise turbulence peak shifts to slightly larger y^+ compared to smooth wall data. As expected, experimental and DNS data do not collapse anymore in the outer layer, which was also seen in other studies (see e.g. [287]).

Lastly, in semilocal Morkovin scaling (figure 4.28b), the locations of near wall Reynolds stress peak move closer together for experiments and DNS, both incompressible and compressible. This is most likely due to small near wall temperature differences being better accounted for (although not completely resolved by experiment). Also in this scaling, outer layers do not collapse as good as in outer scaling (figure 4.27) but a bit better than in inner layer Morkovin scaling.

Based on this comparison, if near wall stresses including the peak values (if resolved) are of interest, the semilocal scaling seems the best choice. If the layers except this near wall region is of interest, best agreement for the outer layer is visible for outer layer Morkovin scaling, which was used for the main part of the discussions in this thesis.

Besides the question for proper scaling, figure 4.27b can be used to estimate general run-to-run scatter from all valid smooth (black, grey symbols) and rough wall data (colored symbols) for the streamwise turbulence intensity profiles. It was visible that all MS1 data agree very well, only the MS2 profiles (grey and blue symbols), show some differences in the mid part of the TBL. This behavior of changing turbulence intensities with running length was also noticed by Kocher *et al.* [131] for PIV profiles along a smooth wall and one with diamond roughness. But they only investigated a running length (measured from the inset of roughness) within $x/\delta \approx 5 - 20$ whereas in the current case, a much larger region with $x/\delta \approx 48 - 85$ was analyzed. Despite these differences, all experimental data agree, independent of the wall status, when the corresponding uncertainties are accounted for. As discussed before for the smooth wall data, also the rough wall data show a considerable amount of filtering in the inner parts of the boundary layer and all fall below the corresponding smooth wall DNS data.

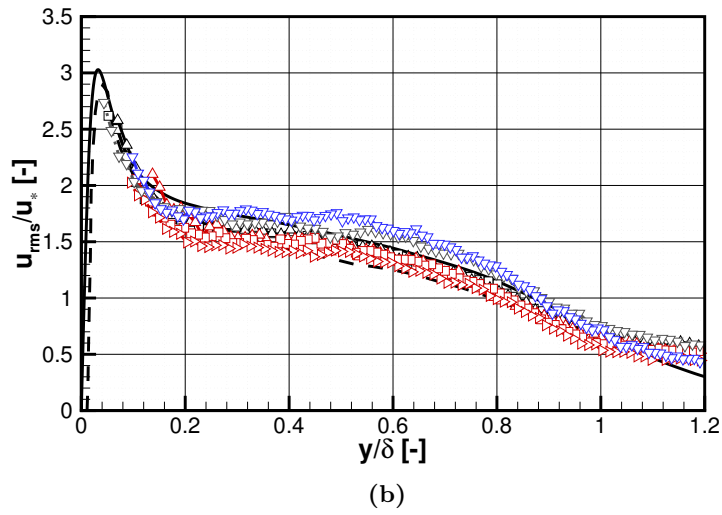
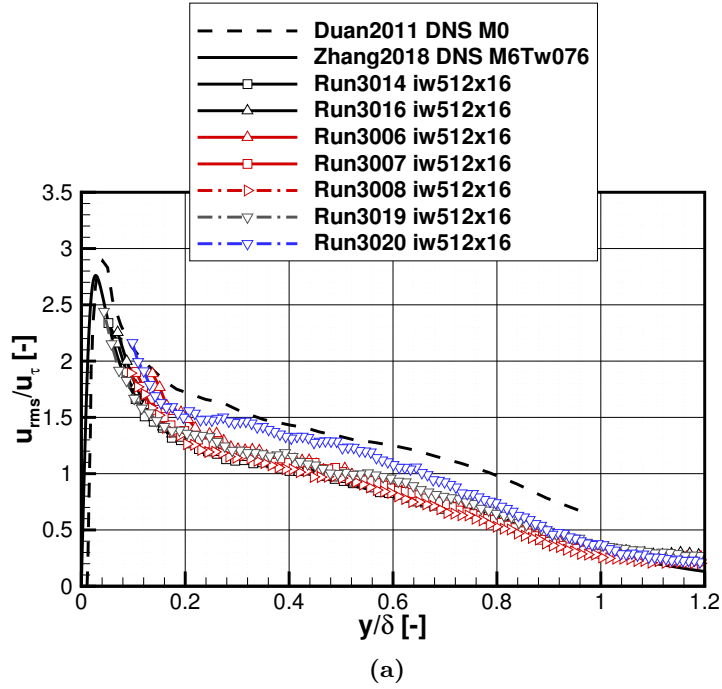


Figure 4.27: Streamwise turbulence profiles of smooth (black, grey symbols) and rough wall experiments (colored symbols) in a) outer layer skin-friction velocity scaling and b) outer layer Morkovin scaling together with incompressible and compressible DNS according to [58], [296].

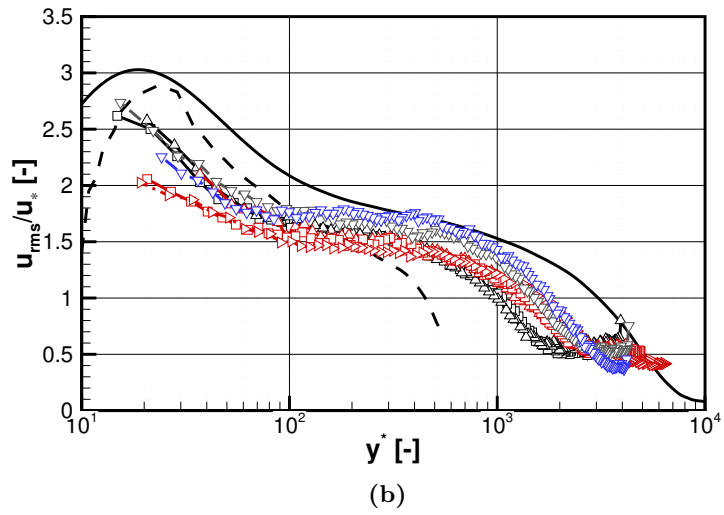
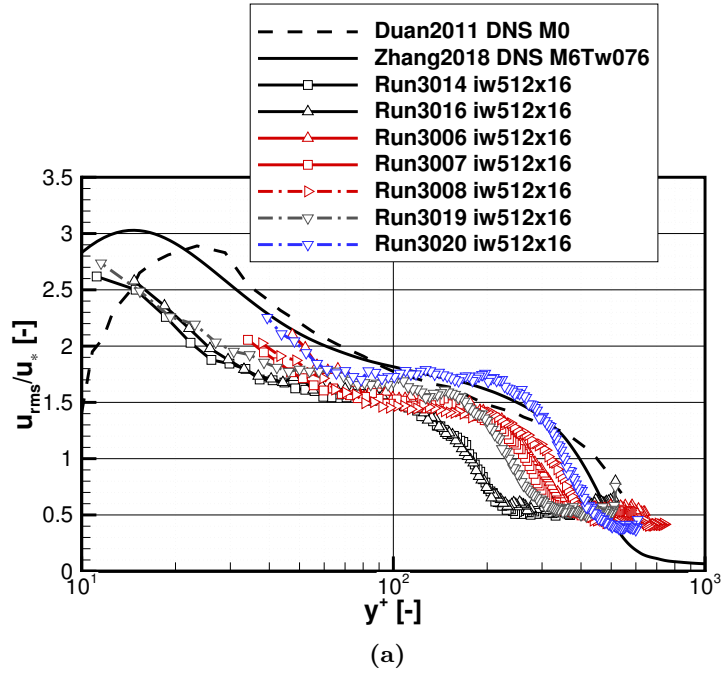


Figure 4.28: Streamwise turbulence profiles of smooth (black, grey symbols) and rough wall experiments (colored symbols) in a) inner layer Morkovin scaling and b) semilocal Morkovin scaling together with incompressible and compressible DNS according to [58], [296].

Roughness Sublayer:

Figure 4.29a shows outer Morkovin scaled turbulence profiles in streamwise, Figure 4.29b in wall-normal direction and figure 4.30 the Reynolds shear stress for smooth and rough wall data together with incompressible and compressible DNS profiles. A representative smooth wall (Run3014) and rough wall run (Run3007) are chosen to enable a direct comparison for all three components. Additionally, two wall normal positions with $y = 5k$ and $y = 3k_s$ are indicated, which have been used in previous investigations to distinguish the height of the roughness sublayer, which is a layer where roughness has a decisive effect on the boundary layer [45]. According to outer layer similarity or Townsend's hypothesis, above this sublayer roughness should have no influence on the outer layer other than defining the boundary conditions like skin friction velocity and boundary layer thickness (see e.g. [45]). This similarity is thought to exist for sufficiently high Reynolds numbers and scale separation ($\delta \gg k$). Despite the only moderate scale separation in this work ($k/\delta \approx 0.1$), in figure 4.29 and 4.30 it is visible that all Reynolds stress components agree to within the scatter of the data (uncertainties plus run-to-run) for the indicated layers, supporting the notion of outer layer similarity. This is surprising since the mean data would suggest otherwise, due to the difference in Coles wake parameter Π (see Table 4.3). Additionally, Flack *et al.* [76] suggested that even higher Reynolds numbers are necessary for two-dimensional transverse square bars in contrast to three-dimensional roughness like cubes.

Castro *et al.* discussed the seemingly contradictory observation of collapsing smooth and rough wall streamwise turbulence profiles (equivalently a collapse in the diagnostic plot, see section 4.3.2) whereas the mean profiles show a difference in Coles wake parameter, which is also the case for the current data. One stated reason was that full universality between smooth and rough TBL simply does not exist. An alternative reason was the general challenge of ensuring that the numerous roughness parameters are sufficiently met, one of the first challenges being that it is not fully known which parameters are actually the most important ones. He suggested two factors to have major influence with the roughness strength y_0^+ and scale ratio δ^+/y_0^+ instead of the, sometimes equivalently used, parameters like k^+ (or k_s^+) and δ/k (e.g. by Jiménez [118]). It can be shown that the alternative forms to quantify the roughness strength with ΔU^+ and y_0^+ are connected via eq. (2.10) (see [39]). Castro *et al.* suggested a necessary parameter range with $y_0^+ \gtrsim 10$ (although $k_s^+ \gtrsim 70$ corresponds already to $y_0^+ \gtrsim 2$) and $\delta^+/y_0^+ \lesssim 300$ to ensure fully rough flow at proper scale separation. In case of the current data $y_0^+ \approx 1.3 - 1.7$ and $\delta^+/y_0^+ \approx 190 - 240$ and $\Delta U_n = \Delta U_{eff}/U_e \approx 0.1$ which are close but fall a bit short of the given boundaries. Therefore, transitional roughness effects cannot be excluded completely. Additionally, outer similarity is still a matter of debate and more data is necessary (see e.g. [76], [39], [12], [220]).

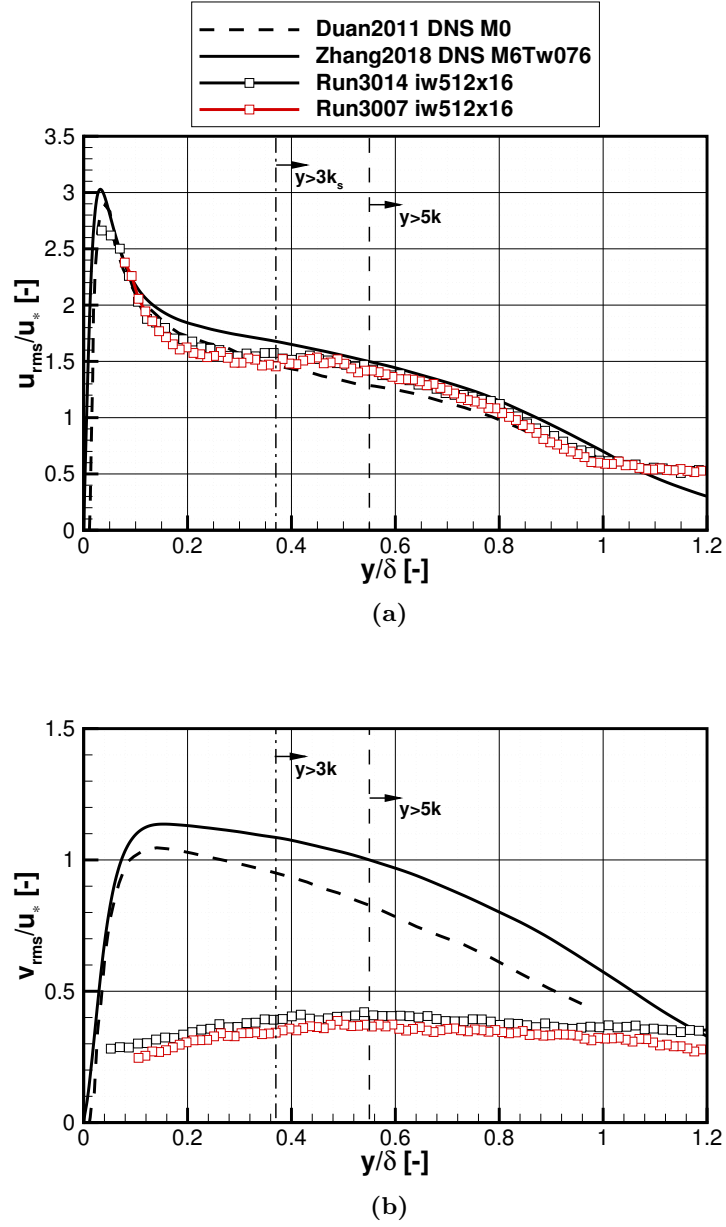


Figure 4.29: Turbulence profiles of smooth (black, grey) and rough wall (colored) experiments in outer layer Morkovin scaling in a) streamwise and b) wall-normal direction together with incompressible and compressible DNS data according to [58], [296].

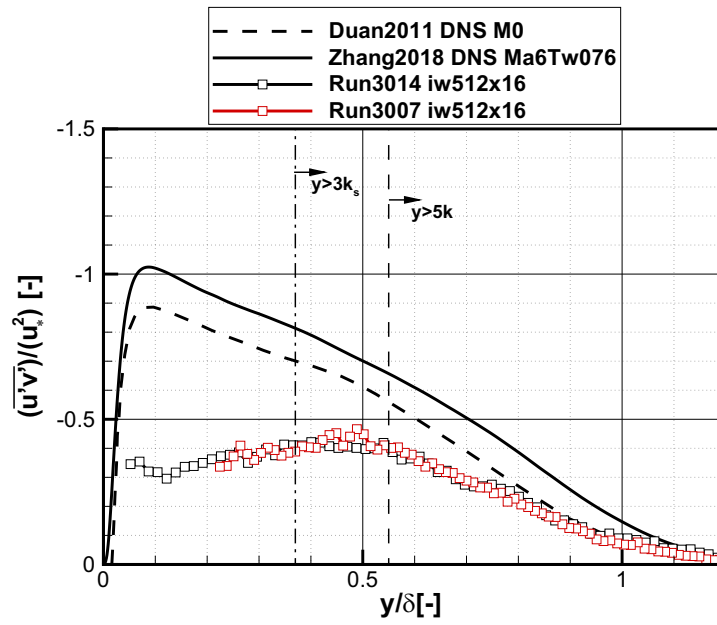


Figure 4.30: Turbulence profiles of smooth (black, grey) and rough wall (colored) experiments in outer layer Morkovin scaling in Reynolds shear stress together with incompressible and compressible DNS data according to [58], [296].

4.3.2 Diagnostic Plot

Due to the challenges in experiments to derive quantities like skin friction velocities, Alfredsson *et al.* [6] introduced the so-called diagnostic plot, summarized in section 2.2. For data in this work, with the diagnostic plot an independent check of parameters previously found by fitting is possible. Figure 4.31 and 4.32 contain data of this work in different variations of the diagnostic plot, together with the empirically found line from smooth wall data according to Castro *et al.* [39].

Figure 4.31a shows only smooth wall data in smooth scaling, meaning the classical u_{rms}/U against U/U_e together with the theoretical correlation according to [39]. The smooth wall profiles of the current work collapse independently of the MS and show the expected linear evolution in close agreement to the theoretical curve. The extent of this linear behavior is limited mainly due to the Reynolds number of this work. This is shown by the different dotted lines from Alfredsson *et al.* [6], representing different Reynolds numbers ranging from $Re_\tau = 250 - 19000$ (arrow in figure indicates increase in Reynolds number).

Figure 4.31b shows both smooth and rough wall data in the same smooth wall scaling as figure 4.31a, with the additional theoretical curve for rough walls according to Léon *et al.* [146]. The colored rough wall data slightly deviates from the smooth wall data but is neither collapsing with the theoretical rough curve. Some of the rough wall data from Castro *et al.* [39] show a similar behavior and this was attributed to transitionally roughness effects in which case the data was not yet in the fully rough regime (transitionally rough with viscosity influences).

The scatter might also be attributed to the fact that the data is compressible and not properly transformed. Therefore, a compressible smooth scaling was tested in figure 4.32a, which shows the same data points but with the mean velocity data in form of the van Driest transformed values U_{eff} and the turbulence data scaled with the mean density profiles, guided by the Morkovin scaling $\sqrt{\rho/\rho_w} u_{rms}$. The experimental data still collapses onto each other in a similar qualitative way as before (figure 4.31b) but show now a different slope than the theoretical correlations so that this compressible scaling does not lead to a better agreement.

Lastly, figure 4.32b shows smooth and rough wall data now in rough scaling, utilizing the loss of momentum according to $U' = U + \Delta U_{eff}$ along both axes, with which [39] successfully collapsed different incompressible experimental smooth and rough wall data. This scaling also collapses the data of this work onto the theoretical smooth wall correlation, although with a certain amount of scatter. The collapse act as an independent check of the roughness function derived from fitting. This gives further confidence in this procedure and corresponding results.

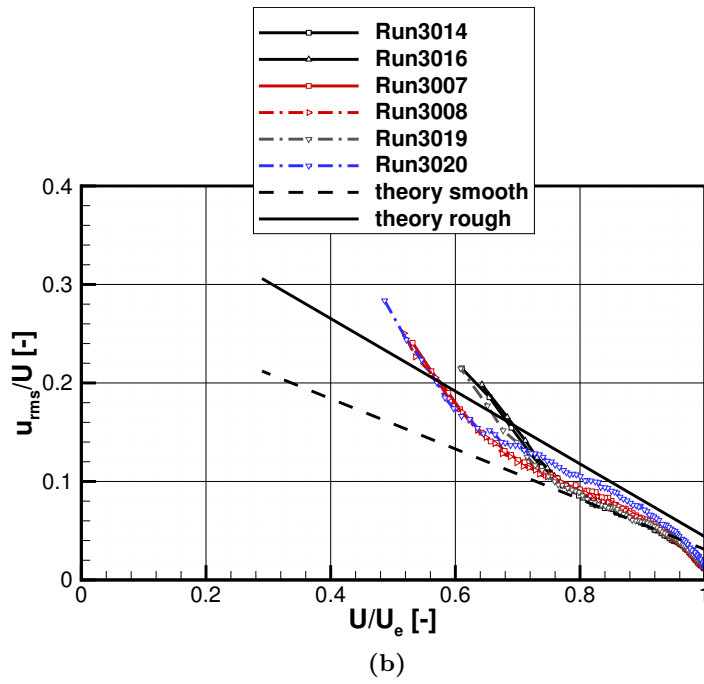
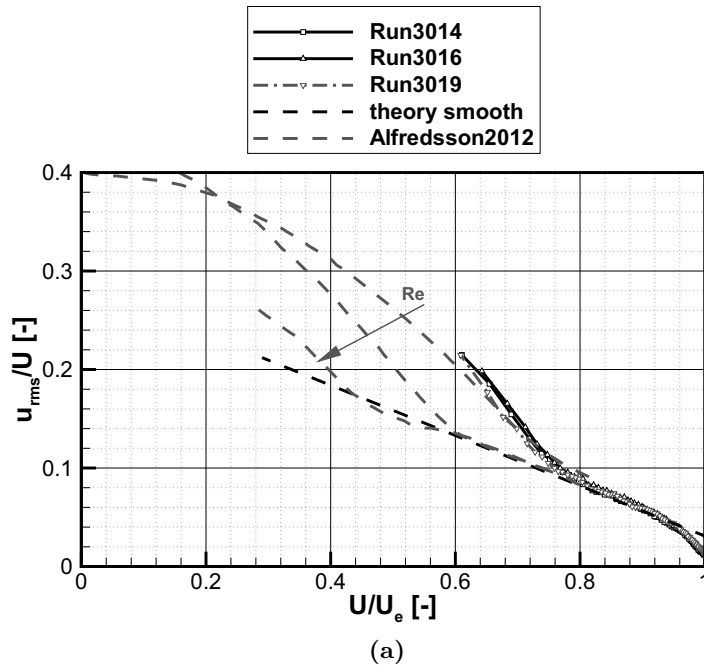


Figure 4.31: Diagnostic plots of a) smooth wall data in smooth scaling together with data according to [6] and b) smooth (black, grey) and rough wall data (colored) in smooth scaling with theoretical correlations according to [39] (smooth) and [146] (rough).

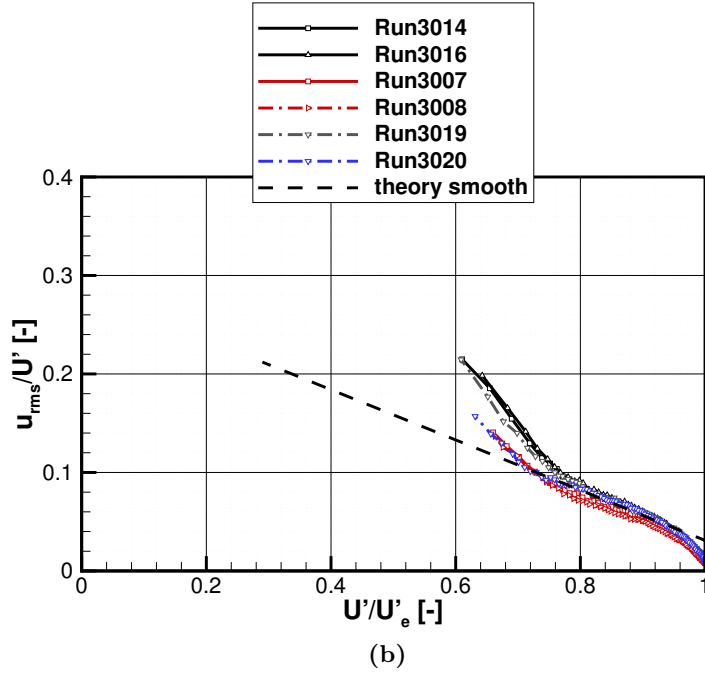
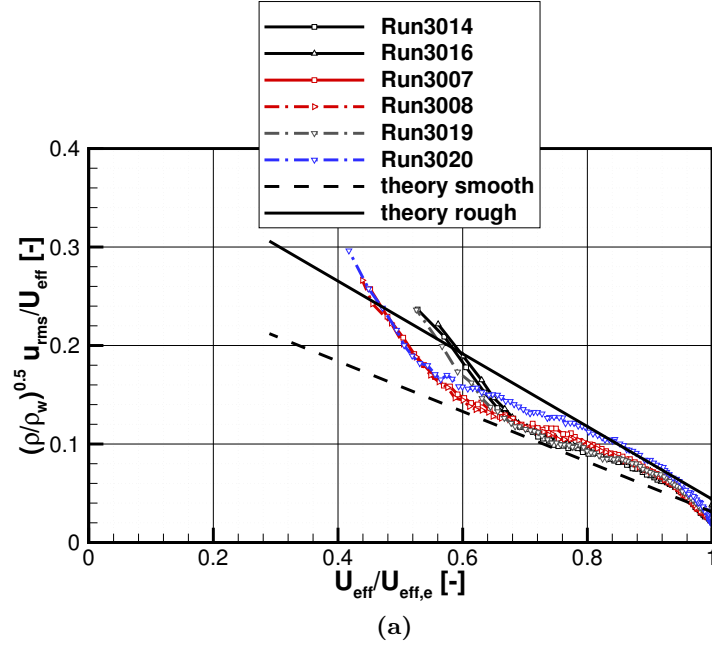
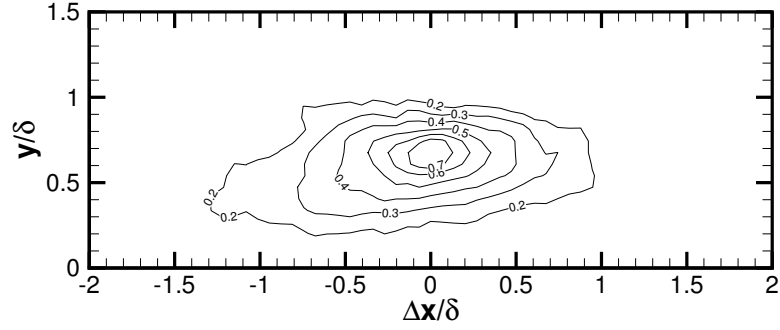


Figure 4.32: Diagnostic plots of a) smooth (black, grey) and rough wall data (colored) in compressible smooth scaling and b) smooth (black, grey) and rough wall data (colored) in rough scaling ($U' = U + \Delta U$) with theoretical correlations according to [39]

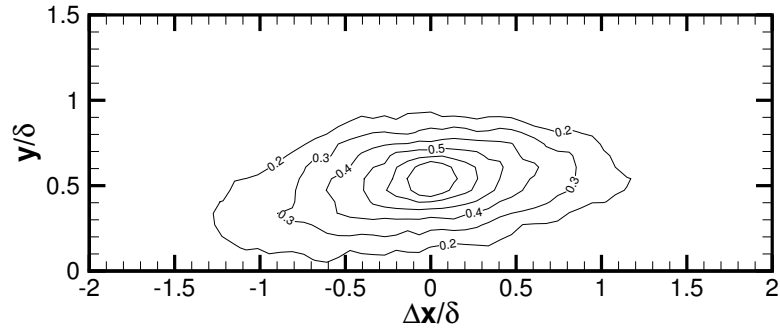
4.3.3 Two-Point Correlations

Two-point correlations, as described by equation (3.13), can be exploited to identify the large-scale structures of a TBL as “correlated mass(es) of fluid” (see e.g. [13]). Figure 4.33 shows two-point correlations of the streamwise velocity fluctuations above the smooth and in figure 4.34 the rough wall at $y/\delta = 0.7, 0.5$ and 0.1 , respectively. The correlations in figure 4.33 are representative for the orientation and distribution of the TBL large-scale motions. These structures are important in characterizing the distribution of Reynolds stresses because most of the turbulent kinetic energy (TKE) in the boundary layer is contained within long-wavelength motions (see e.g. [183]). The correlations are visible as elliptical distributions of R_{uu} at each wall normal location, with a small rotation angle with respect to the wall. The general appearance of both shapes and orientations of the smooth wall correlations are consistent with previous incompressible studies like [114] and supersonic studies like [84], [183] and [287]. The rough wall results are also comparable to previous supersonic studies like [183] and [288]. In general, smooth and rough wall correlations look similar in figure 4.33 and 4.34. Details of dimensions and orientations are extracted and discussed in the following. Before analyzing differences to rough wall data, smooth wall results are compared to data from previous investigations.

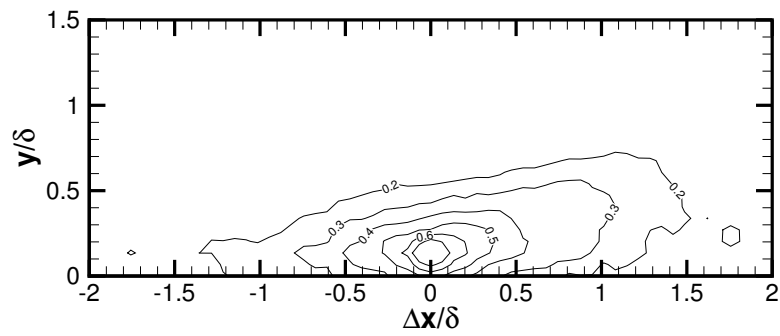
To enable direct comparison to previous studies, figure 4.35 shows the one-dimensional autocorrelation $R_{uu}(y_{ref})$ extracted at $y/\delta = 0.5$ of the two-point correlation, previously shown in figure 4.33. The autocorrelation of the smooth wall Run3014 shows excellent agreement with both incompressible data by Hutchins and Marusic [114] and compressible data of the $Ma = 2$ DNS study of Pirozzoli and Bernardini [189] and the experimental data at $Ma = 4.9$ of Peltier [183]. Only data by Ganapathisubramani *et al.* [85] seem to indicate slower drop of correlation, which hints at larger length scales, previously attributed to the corresponding larger Reynolds number (see [183]).



(a)

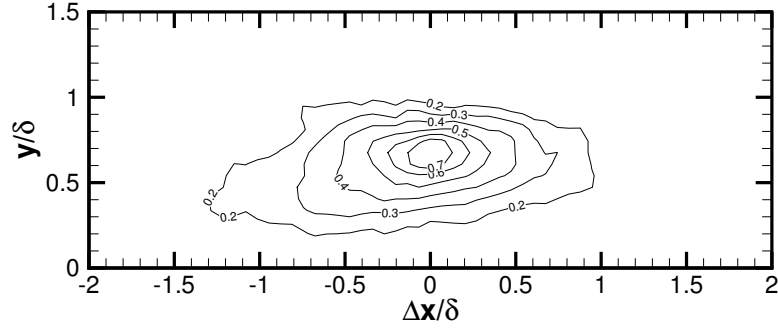


(b)

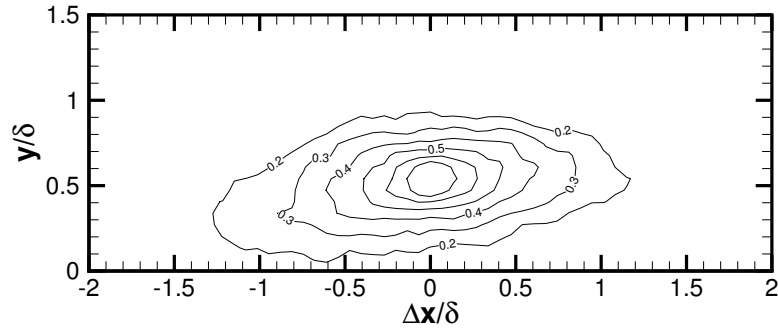


(c)

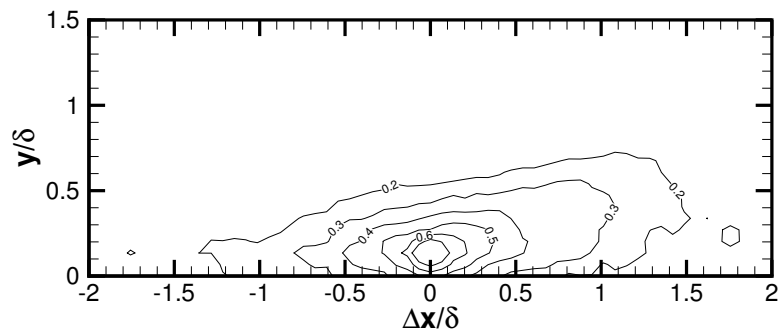
Figure 4.33: Two-point correlations of streamwise velocity fluctuations above smooth wall (Run3014) at $y/\delta = 0.7, 0.5$ and 0.1 from top to bottom.



(a)



(b)



(c)

Figure 4.34: Two-point correlations of streamwise velocity fluctuations above rough wall (Run3007) at $y/\delta = 0.7, 0.5$ and 0.1 from top to bottom.

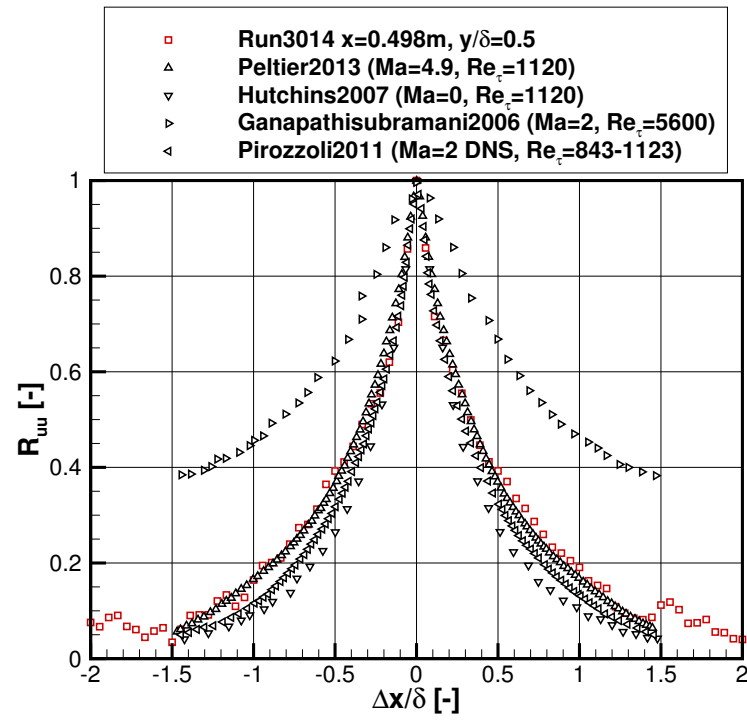


Figure 4.35: One-dimensional autocorrelation of the streamwise velocity fluctuations R_{uu} at $y/\delta = 0.5$ for smooth wall case 3014 compared to data from [183], [114], [85] and [189].

Length scales:

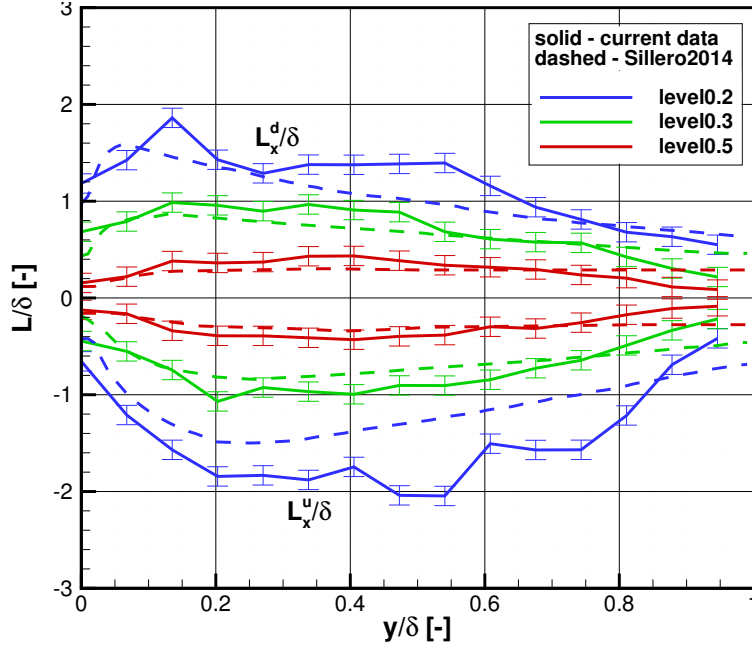
To extract characterizing length scales, different approaches are being used in the literature. In this work the extracted representative length scales are based on the dimension of contour levels of the two-point correlations (see section 3.3.2). Some tests have been performed to extract the integral length scale, which are reported in section 7.5.2 (see figure 7.53).

We follow the nomenclature by Sillero *et al.* [228] with L_x^u being the upstream and L_x^d being the downstream dimension of the corresponding contour level measured from the point of interest ($x, y_{ref} = 0, 0$, where $R_{uu} = 1$). The maximum or total dimension is defined as $L_x^m = |L_x^u| + |L_x^d|$. The impact of IW size on the resulting two-point correlation length scales were found to be minor (see figure 7.52 in section 7.5.2).

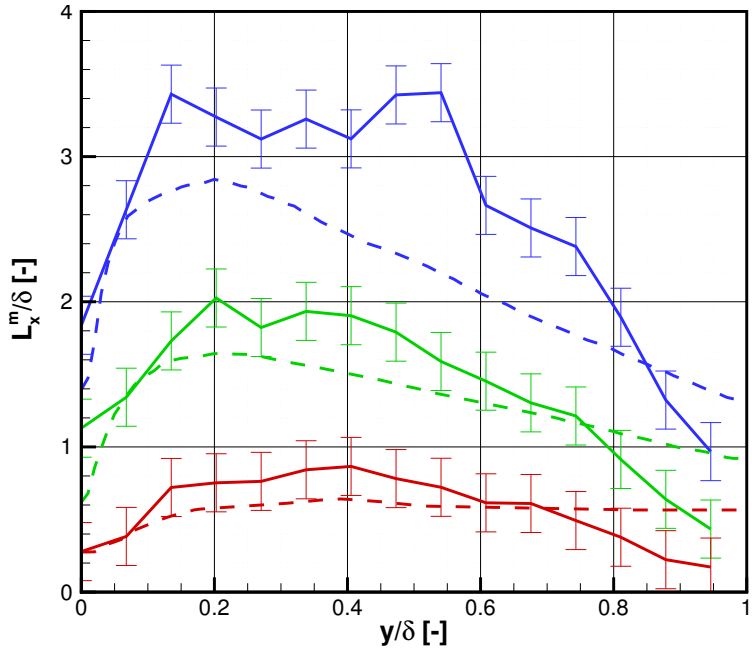
Figure 4.36 shows the dimensions of the three previously defined length scales for the smooth wall Run3014 in comparison to incompressible TBL data of Sillero *et al.* [228] at $\delta^+ = 2000$. Despite the difference in Mach and Reynolds number, good agreement between hypersonic and incompressible data for contour levels 0.2 and 0.3 can be seen (figure 4.36a). Uncertainties were conservatively estimated to $L/\delta \approx \pm 0.1$ (indicated by error bars), based on run-to-run differences and from offsetting the streamwise position for R_{uu} by one IW upstream or downstream. It is visible that the up- and downstream length scales are not symmetrical in case of both data sets, with a slightly larger downstream leg, especially for level 0.2 and 0.3. Some deviations, especially in the mid part of the TBL ($0.1 < y/\delta < 0.7$), are visible for level 0.2. This is also visible in the maximum contour dimension L_x^m (figure 4.36b). Uncertainties for the maximum dimensions were conservatively estimated to $L/\delta \approx \pm 0.2$ (indicated by error bars), based on the same approach as described above (the order of magnitude was confirmed with synthetic data as described in section 7.5.4). Close to the boundary layer edge, current length scales tend to fall below the corresponding values by Sillero *et al.*, which might be attributed to the model being a cone or the seeding quality outside the boundary layer. Different IWs than $64 \times 64 \text{ pix}^2$ were investigated without changing the general trends (see section 7.5.2 figure 7.52).

The comparison of smooth to rough wall maximum streamwise dimensions L_x^m against wall-normal distance is shown in figure 4.37 for the contour levels from 0.2 to 0.5 of smooth wall Run3014 and rough wall Run3007. The length scales increase with distance to the wall to a maximum at a certain location above the wall and taper off again towards the edge of the TBL with a tendency to lower length scales with increasing contour level. This is the case for both smooth and rough wall data, whereas the profiles flatten with increasing contour level. The length scales are in general very comparable between smooth and rough wall data and clear differences are masked by data scatter and uncertainties (indicated by shaded region and bars for the smooth wall and rough wall length scales, respectively). However, for the lowest two contour levels $R_{uu} = 0.2$ and 0.3, close to the wall for $y/\delta \lesssim 0.2$, the rough wall values decrease below the smooth ones by approximately 30 %. Similar observations have been made previously by other investigators. For example Peltier [183] discovered for their $R_{uu} = 0.4$ contours that close to the wall with $y/\delta < 0.2$ in a $Ma = 4.9$ flow, the scales along diamond roughness elements ($k/\delta \approx 0.07$, $k^+ = 161$) decreased clearly to roughly half the smooth ones. This was also observed by Williams *et al.* [288] for the 1.27 mm transverse square bar roughness ($k/\delta \approx 0.1$, $k^+ = 70$) compared to corresponding reference smooth wall data. Peltier [183] connected the near wall behavior to an increase in ejection events via a quadrant analysis of the Reynolds shear stress profiles. If the same analysis is applied to current data, this behavior cannot be reproduced since only

negligible differences between smooth and rough wall data are visible for the different quadrants (see section 7.5.2 figure 7.55). However, current Reynolds shear stress data may be influenced by the observed strong filtering (as visible in e.g. figure 4.29c) and so maybe is the quadrant analysis.



(a)



(b)

Figure 4.36: Dimensions of the two-point correlation of streamwise velocity fluctuations with wall-normal position a) change of upstream, L_x^u , and downstream, L_x^d , extent of different levels (0.2, 0.3 and 0.5), b) change of maximum, L_x^m , extent of different levels (0.2, 0.3 and 0.5). Solid lines show smooth wall data of Run3014 in comparison to incompressible TBL data at $\delta^+ = 2000$ according to Sillero *et al.* [228].

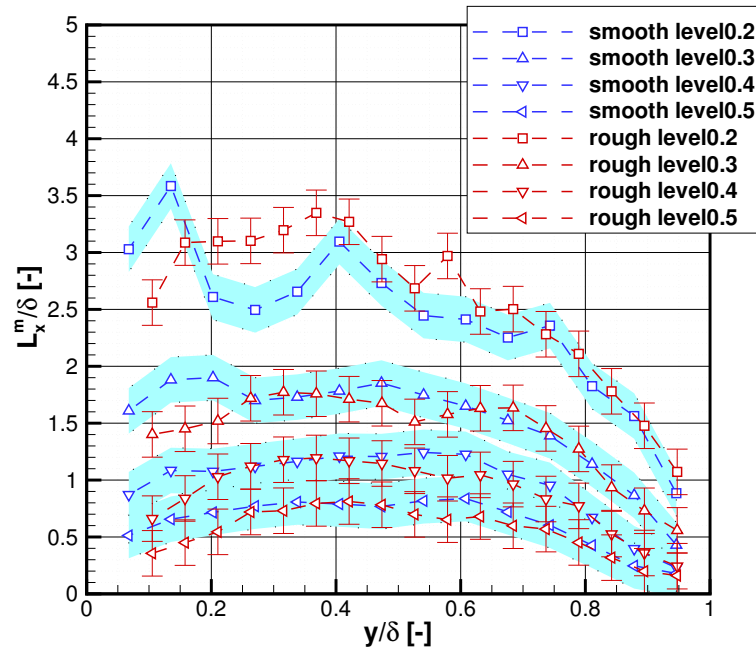


Figure 4.37: Wall-normal variation of maximum streamwise dimension L_x^m for two-point correlation contour levels 0.2 to 0.5 of smooth wall Run3014 and rough wall Run3007.

Inclination angle ϕ :

Another feature of interest is the inclination angle ϕ of the correlation contours, sometimes attributed to the inclination angle of hairpin packets in incompressible TBL (see e.g. [287]). The impact of IW size was analyzed and results showed that, apart from a certain scatter, all data correspond closely to each other, independent of the choice for the IW size (see figure 7.52b in section 7.5.2).

Again, before a comparison to rough wall data is shown, smooth wall data is compared to literature data. Figure 4.38a shows the smooth wall data of Run3014 in comparison to incompressible TBL data by Sillero *et al.* [228], incompressible rough TBL data by Wu and Christensen [292], compressible TBL data by Pirozzoli and Bernardini [189] and hypersonic TBL data by Williams *et al.* [287]. Sillero *et al.* extracted the maximum value of the inclination angle for all correlation levels at a fixed position normal to the wall. Current data, as well as the one by Williams *et al.*, are shown from a range of contour levels (0.2 – 0.5). In general, all data share the trend of an increase of inclination angles with distance to the wall up to a peak value at a certain position within the TBL and a following tapering off towards the boundary layer edge. It is visible that peak inclination angles of $\theta \approx 13^\circ$ have been measured in case of incompressible [228] and supersonic data [189]. Hypersonic data by Williams *et al.* [287] are slightly below but reach comparable levels for the maximum measured values. This is in the typical value range of approx. 12° to 16° reported for incompressible data extracted from estimates of mean log-layer inclination angles, which are thought to be invariant with Reynolds number (see Marusic and Heuer [156]). In case of the H2K data, shallower angles of up to 10° at maximum have been extracted. Also, in case of the current data, the position of peak angles is located near $y/\delta = 0.2 - 0.4$ (depending on the level), whereas in case of Sillero *et al.* [228] and Williams *et al.* [287] it is located more closely to $y/\delta = 0.5$.

Figure 4.39 shows the extracted inclination angles with wall distance for smooth and rough wall data, respectively. The symbols show the angles for different contour levels from 0.2 to 0.5. The dashed line represents the mean angle of the corresponding contour levels. The dashed-dotted line corresponds to angles derived via a linear least-square fit through the fitted ellipses of the two-point correlations according to [189]. It is visible that the angles are in general comparable between the different contour levels and extraction methods, where the second method results locally in slightly smaller angles. Values for $y/\delta \gtrsim 0.8$ are typically blanked, because they tend to be very noisy, again most likely due to the seeding quality. Also for the rough wall data with increasing distance to the wall, the angles increase to a maximum and taper off again towards the edge of the TBL and are roughly constant over a range of $0.1 < y/\delta < 0.5$. The distribution of angles is comparable between smooth and rough wall data with slightly larger angles in case of the rough wall data for $y/\delta \gtrsim 0.2$. This is the opposite to observations reported by Peltier [183] along diamond roughness and Williams *et al.* [288] along square bar roughness, although Williams *et al.* mesh roughness results showed the same trend as in figure 4.39. However, Williams *et al.* admitted that the small angular differences are difficult to interpret due to strong scatter of the data, which is also the case for the current data. Also, analyses of synthetic data indicated uncertainties of the post-processing in the order of $\mathcal{O}(1)$ degrees, but they can only serve as a rough estimate as they cannot cover all relevant influences (see section 7.5.4).

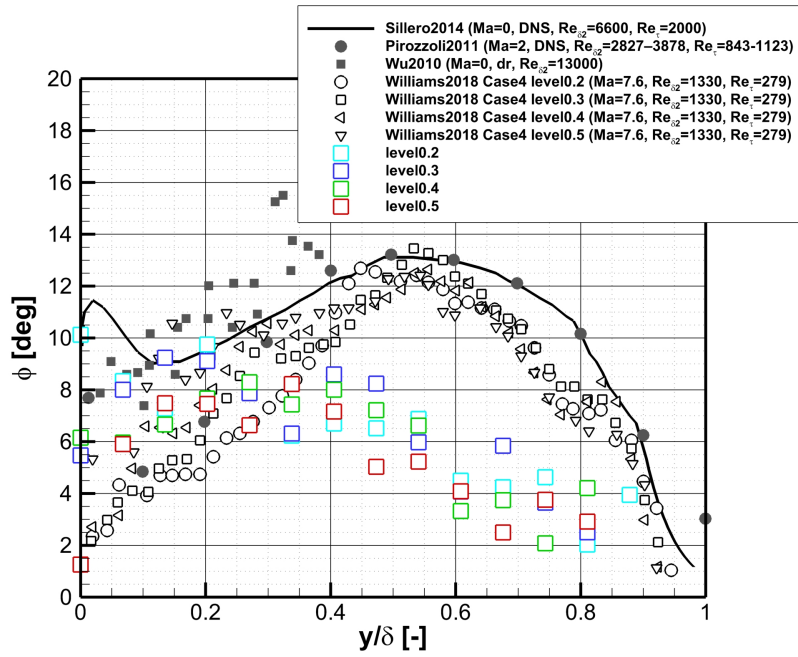
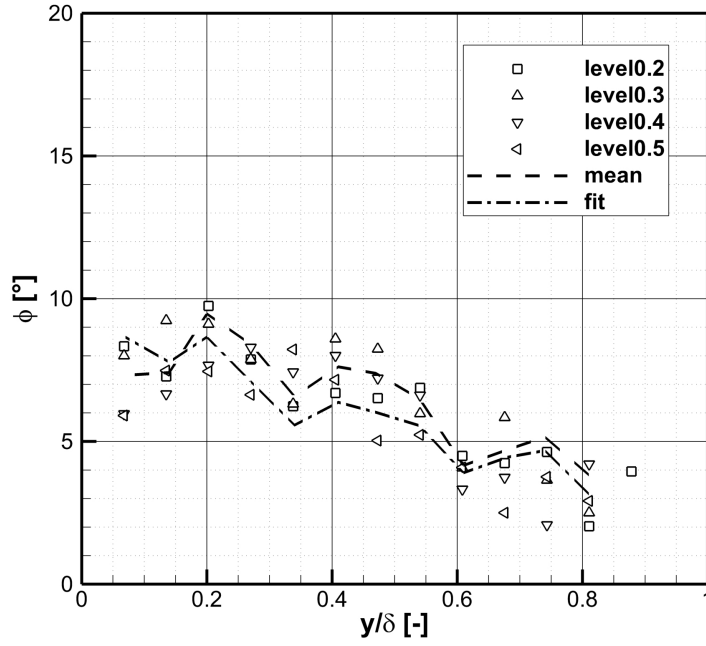
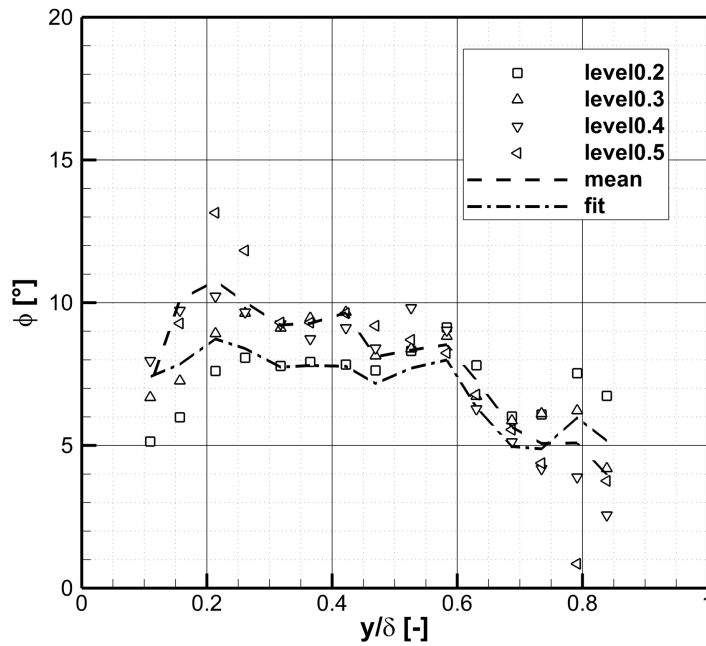


Figure 4.38: Inclination angle of different levels (0.2 - 0.5) of the two-point correlation of streamwise velocity fluctuations with wall-normal position. Colored symbols show smooth wall data of Run3014 in comparison to incompressible TBL data by Sillero *et al.* [228], incompressible rough TBL data by Wu and Christensen [292], compressible TBL data by Pirozzoli and Bernardini [189] and hypersonic TBL data by Williams *et al.* [287].



(a)



(b)

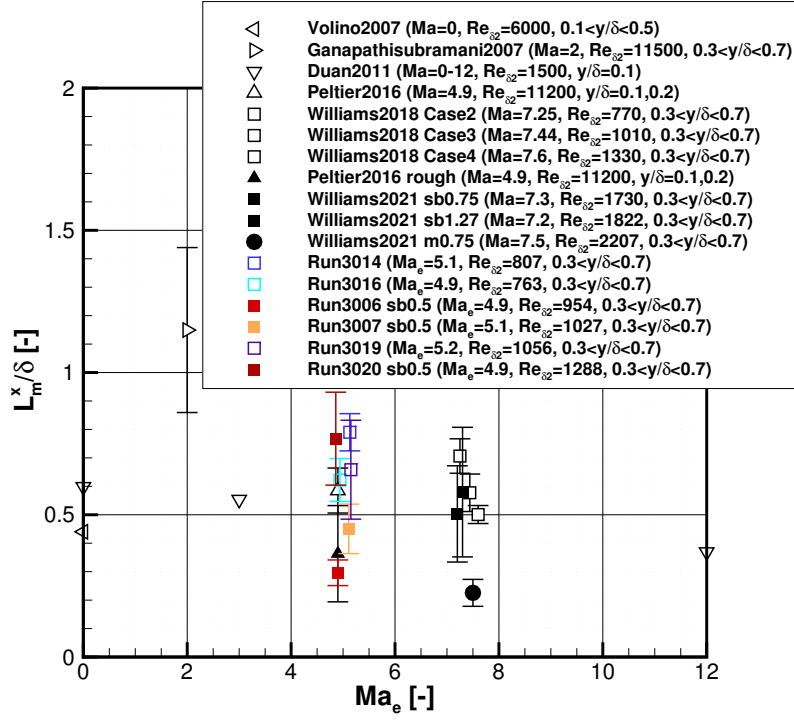
Figure 4.39: Inclination angle of the of streamwise velocity fluctuation correlations with wall-normal distance for contour levels between 0.2 and 0.5 for a) smooth wall Run3014 and b) rough wall Run3007. In both plots dashed lines and dashed-dotted lines represent the mean of levels 0.2 to 0.7 and a linear fit through the fitted ellipses, respectively.

Correlation results summary:

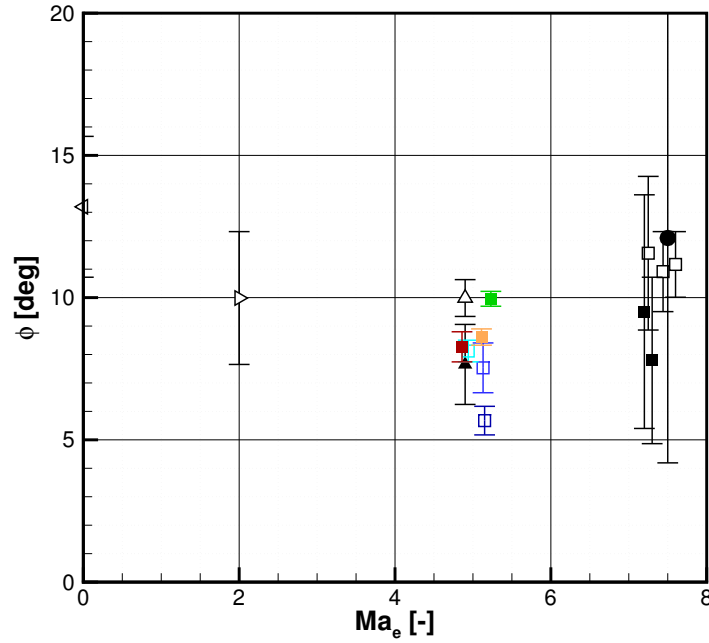
Figure 4.40 puts representative mean values of the length scale L_x^m and inclination angle θ of this work in context with previous investigations plotted against edge Mach numbers Ma_e . The symbols for L_x^m in figure 4.40a typically represent averaged values of the contour level 0.5 in the mid-part of the boundary layer within $0.3 < y/\delta < 0.7$. It is indicated in the legend if other regions are considered for averaging. Bars represent the value variation occurring in the corresponding wall-normal region, if available. Open symbols represent smooth and closed symbols rough wall data.

Typically, values of L_x^m/δ between 0.3 to 0.8 were extracted from data in this thesis, well within the range of previous comparable compressible studies, especially according to Peltier derived at a comparable flow condition. If only the smooth wall data of this study are considered, values are in a range of L_x^m/δ between 0.6 to 0.8. The slightly larger Reynolds number Run3016 shows a larger correlation length than Run3014. This would correspond to an expected trend following Smits and Dussauge [235], who suggested that the integral length scale tends to increase with Reynolds number and decrease with Mach number. In contrast, the value at MS2 (Run3019) with an even larger Reynolds number doesn't follow this trend but falls in between the other two smooth wall data points, although it must be pointed out that it also shows a larger scatter. Typically, rough wall data of this study is below the corresponding smooth wall data, also when data spread is considered. Only the rough wall MS2 value (Run3020) shows a larger or comparable length scale than the smooth wall data, recorded at the largest Reynolds number. In general, the length scale data of this work support the conclusion by Williams *et al.* [287], that correlation lengths are only moderately decreasing with Mach number. This is in contrast to previous conclusions based on data which relied on hot-wire measurements, which are influenced by density [234].

The symbols for the inclination angle θ in figure 4.40b represent averaged values of the two-point correlation contour level 0.5 in the mid-part of the boundary layer to be consistent with previously published comparisons. Typically, this is within $0.3 < y/\delta < 0.7$, again indicated in the legend if a different region was considered. Current data was extracted within $0.3 < y/\delta < 0.5$ where profiles showed a nearly plateau-like behavior (see figure 4.39), although the smooth wall data show an overall nearly linear trend throughout the boundary layer. In general, current data shows mean values within a range of θ between approx. 6° to 10° . As reported above, this is slightly below typical values from incompressible data between approx. 12° to 16° [156]. The inclination angle data of this work supports findings by [287], that inclination angles are slightly decreasing with Mach number, in contrast to the reverse trend suggested by data which relied on hot-wire measurements, influenced by density. The smooth and rough wall data range is in agreement with results from previous comparable compressible studies, especially according to Peltier. Within the scatter of the data, smooth and rough wall data are in agreement, showing no clear tendencies. This would further support the assumption by Townsend, that the outer flow is not drastically affected by roughness.



(a)



(b)

Figure 4.40: a) Averaged max. contour dimension L_x^m against edge Mach number, b) averaged inclination angle θ against edge Mach number in comparison to literature data as noted in the legend (sb: square bar roughness, m: mesh roughness; both followed by the height in millimeters).

4.3.4 Uniform Momentum Zones

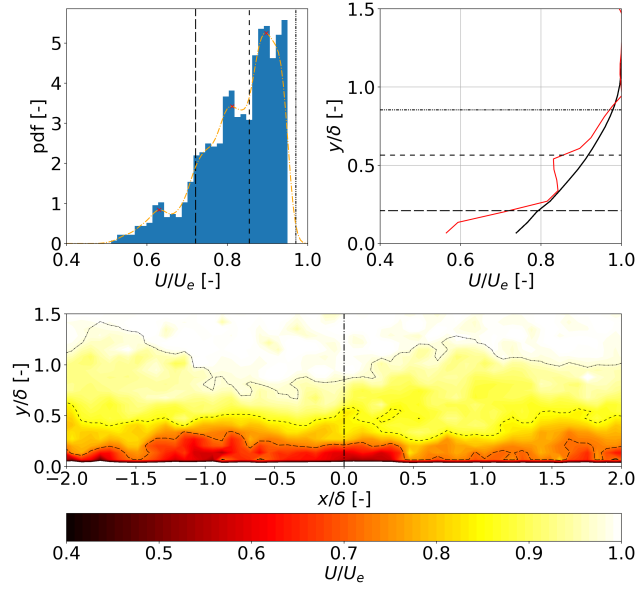
So-called uniform momentum zones (UMZ), which have been attributed to the general concept of hairpin vortices, could be observed in several different previous TBL studies [3], [229], [47]. Therefore, efforts have been made to detect UMZs in the current data, based on methods described in section 3.3.2.

Figure 4.41 shows in each lower subplot exemplary instantaneous velocity fields for both smooth (Run3014, figure 4.41a) and rough wall runs (Run3007, figure 4.41b). UMZs can easily be detected by the occurrence of uniform contour regions (upper dashed-dot-dotted lines indicate TNTI, see section 3.3.2). The length of these UMZs can range up to the length of the FOV with $\delta x/\delta = 4$. At the same time, instantaneous velocity profiles in the upper right subplots (red line, here taken at the middle of the field) are compared to the corresponding mean profile at the same position (black line, based on all images) in which the UMZ boundaries are indicated with horizontal dashed lines and the corresponding increased shear is clearly visible by the large local instantaneous velocity differences. Corresponding p.d.f. of velocities in the instantaneous field are visible in the upper left of figure 4.41a and figure 4.41b with the vertical dashed lines indicating the different UMZ boundaries. The different dash-styles, indicating different UMZ boundaries, correspond to each other in all three subplots.

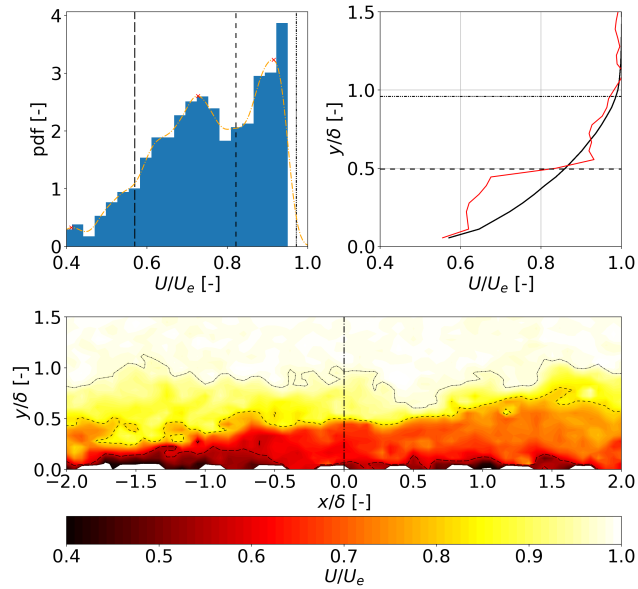
Via the histogram analysis a mean number of zones $\bar{N}_{UMZ} = 2.8 \pm 1.1$ were found for the smooth Run3014 at $Re_\tau \approx 220$. This is comparable to the reported value of $\bar{N}_{UMZ} = 2.9$ according to Williams *et al.* [287] for a smooth wall TBL at $Ma = 7.6$, $Re_\tau = 279$ and also within range of the reported value of $\bar{N}_{UMZ} = 2.5$ UMZs according to Cogo *et al.* [47] for a smooth wall TBL at $Ma = 5.86$, $Re_\tau = 1080 - 1953$. Previous incompressible studies showed a log-linear dependence of the mean number of zones $\bar{N}_{UMZ}$ with Re_τ [229]. According to this empirical curve, the mean number of zones should lie within $\bar{N}_{UMZ} = 2.0 - 2.3$ if a range of Reynolds numbers with $Re_\tau = 200 - 300$ is chosen, corresponding to the current study and the one by Williams *et al.* Therefore, current hypersonic data seem to indicate slightly more zones than predicted from the incompressible correlation, but if the corresponding uncertainties are considered, the numbers still agree. If the semi-local Reynolds number would be taken as characteristic parameter for the hypersonic data with $Re_\tau^* = 1600 - 2700$ (see table 4.1), the incompressible correlation would result in $\bar{N}_{UMZ} = 3.5 - 3.9$, which gives now much higher values than from the above analysis. If the corresponding uncertainties are included, experimental data is covered by both of the above analytical approximations, so it is clear that further analysis is necessary.

The same approach was used on the rough wall data (Run3007 and IW 64x64 pix² in figure 4.41b), which resulted in $\bar{N}_{UMZ} = 2.9 \pm 1.0$. This is in close agreement to the corresponding smooth wall values. Even if the analysis is influenced by the parameter settings, it is assumed that the relative difference between smooth and rough wall data would not change drastically. Sensitivities to IW dimensions have been found to be negligible (see section 7.5.3). Figure 4.41 is only a single snapshot in time, but UMZs are abundant throughout the data set, of which a few more exemplary plots are shown in section 7.5.2. Therefore, this study supports the previous findings of the evidence of the presence of UMZs in hypersonic TBL along smooth walls and extends this finding to rough walls with only negligible differences between both wall conditions in case of the current transverse square bar topology ($\lambda/k = 4$). Due to the assumed connection to hairpin

vortex structures, this would further support the notion that outer layer flow is not drastically affected by the investigated roughness.



(a)



(b)

Figure 4.41: Compilation of p.d.f. (upper left) based on an instantaneous velocity field (lower middle) and a comparison of an instantaneous (red line) to the mean velocity profile (black line, upper right) for a) smooth wall (Run3014) and b) rough wall data (Run3007). Boundaries for each UMZ are indicated as black lines of different dash-styles which correspond to each other in all corresponding subplots. TNTI is indicated in all subplots as black dash-dot-dotted contour line.

4.4 Heat Flux

This section summarizes heat flux results along the smooth and rough cone. Before rough wall data and corresponding heat flux augmentations are discussed, smooth wall data is presented and compared to predictions. For each wall topology, both, results from IR thermography (section 3.4) and from local sensors (section 3.5) are discussed. For the corresponding setups see sections 3.1 and 3.4.

4.4.1 Smooth Wall

IR:

Figure 4.42 contains an exemplary IR image of smooth wall Run1011 at FC1, recorded by the Infratec camera at $t = 15$ s and post-processed to the convective heat flux by the methods described in section 3.4.2. As visible in the 2D image of the figure (upper left), the camera setup allowed to record a large portion of the cone with a streamwise length of approx. 0.5m (approx. 70 %), beginning slightly downstream of the start of the PEEK mid-section up to the end of the aft cone section. The natural transition is clearly visible with a sharp increase in heat flux along the first third of the visible cone part. Additionally, one HFM sensor can be seen in the lower aft part, visible as low heat flux spot. For further discussion, 1D data was extracted from a line along the cone axis as visualized with a black dotted line in the 2D image. The line is not taken along the symmetry plane of the model to avoid influences of the local laser spot in case of runs including PIV measurements. Due to the low angular sensitivities of the PEEK emissivity, the detected temperatures, and with that the heat flux values, do not change much in large portion of the circumferential direction of the cone. This is visualized in the upper right inset of figure 4.42 with heat flux profiles along the image y -axis for a portion of the cone within $500 \text{ pix} < x < 510 \text{ pix}$ (dashed lines in the 2D image). Although by this procedure, the lines cover also a small streamwise region, only marginal streamwise gradients are expected in the covered x_a -region. The resulting lines show an almost flat heat flux plateau (changes typically below 1 %) along the cone portion of the profiles (within $-320 \text{ pix} < y < -160 \text{ pix}$). Therefore the extracted line does represent the typical heat flux distribution along the cone. The extracted data of the surface temperature, the convective heat flux $\dot{q}_c$ and derived edge-based Stanton number St_e is shown along the cone axis in physical distance from the nose x_a in the lower part of the figure. Along these line plots, a behavior characteristic of a laminar-to-turbulent transition is visible with a sharp increase in heat flux at $x \approx 0.25$ m, with a downstream peak value in all three quantities at approx. $x = 0.35$ m, followed by an expected decrease along the cone due to the increasing boundary layer thickness. The distinction between laminar, transitional and turbulent portion of the cone will be discussed further based on numerical data in later plots.

For exemplary run-to-run repeatability (same flow condition) of heat flux measurements via IR thermography figure 4.43 shows the surface temperatures, the convective heat flux $\dot{q}_c$ and derived Stanton numbers St_e for Run1003 to 1011, all at $t = 15$ s. It is visible that the temperature (by approx. 2 % for $x > 0.5$ m) and the heat flux (by approx. 4 % for $x > 0.5$ m) differ slightly. This is mainly influenced by run-to-run differences in averaged total pressure p_0 and temperature T_0 , usually below 1 %. In case of a comparison between results from runs at FC1 and FC2, the heat

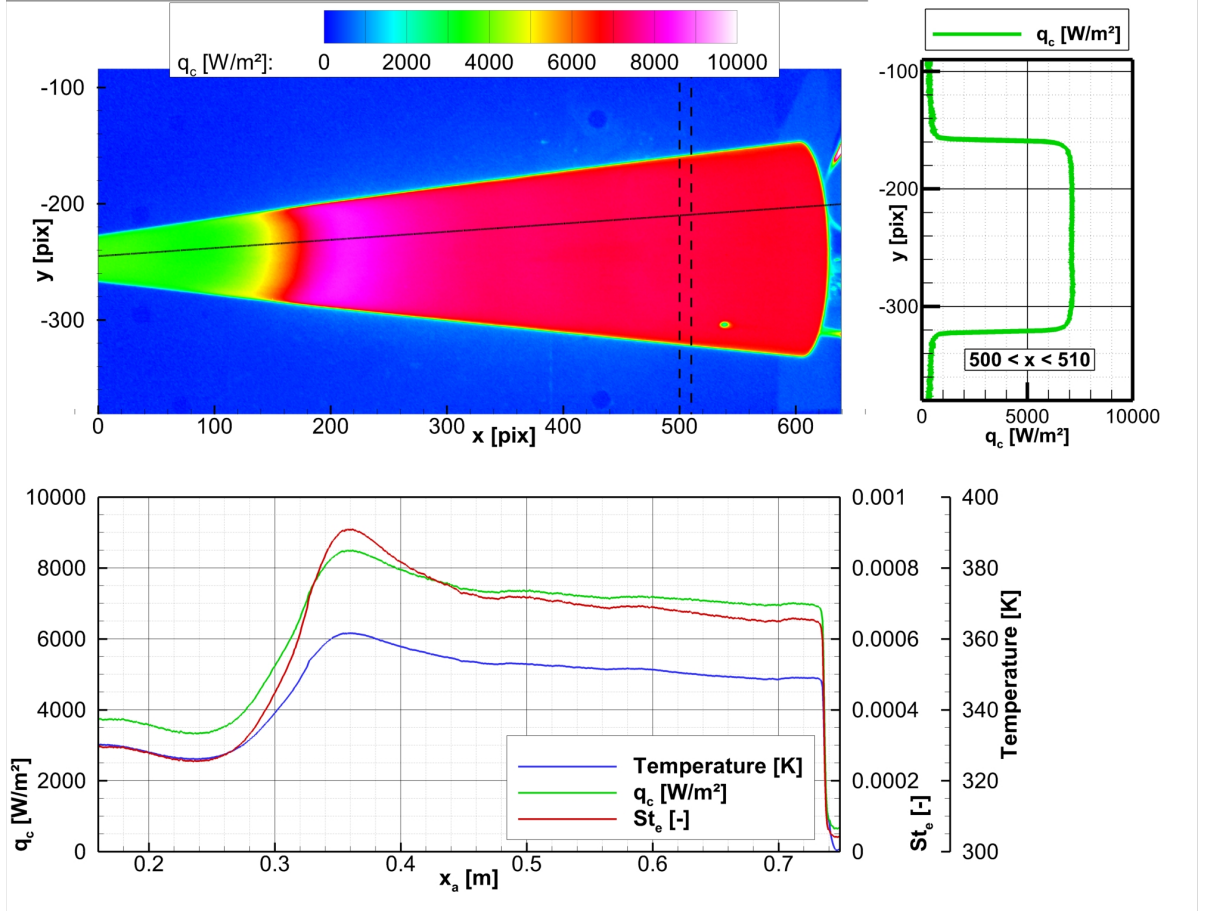


Figure 4.42: IR image of smooth wall Run1011 at $t = 15$ s, extracted line information along x -axis for convective heat flux, wall temperature and edge based Stanton number and heat flux along y -axis

flux differ by approx. 20%. The Stanton number is suitable to normalize the heat flux which agree acceptably between all runs, independent of FC (differing by approx. 4 % for $x > 0.5$ m), well within the indicated uncertainties (see section 7.1.1).

A visible difference between the runs, but especially between Run1004, 1010 and 1011, is the transition location for $x_a \lesssim 0.3$ m, including the start, indicated by the sharp increase of heat flux, and the peak values. This can be attributed to the choice for a natural transition, which may be influenced by changes in parameters like inflow or other initial disturbances. The reason for the observations in figure 4.43 can most likely be attributed to the PIV seeding, as discussed in more detail in section 7.6.6. The disadvantage of not fixing the transition location via tripping was accepted due to the advantages discussed in section 3.1. More important for the goals in this thesis, the turbulent level along the rough part should be reproducible. In figure 4.43 it is visible that the turbulent levels for $x_a \gtrsim 0.4$ m of heat flux do agree closely, independent of the exact location of transition. Therefore, it is assumed that the analysis of turbulent augmentation due to roughness is not influenced by the exact location of the natural transition.

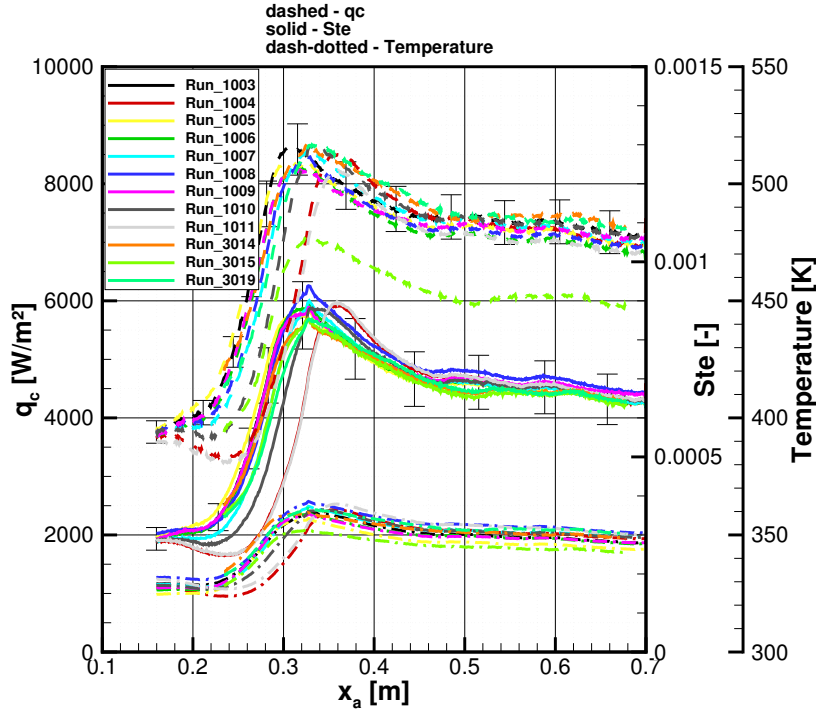


Figure 4.43: Convective heat flux, wall temperature and edge based Stanton number repeatability for smooth wall Run1003 to 1011 and selected smooth wall runs of second PIV campaign.

Additionally, runs from the second PIV campaign are also included in figure 4.43 (run numbers larger than 3000) and excellent agreement between cases at FC1 are visible (e.g. Run1011 and Run3014). With Run3015, also data of a FC2 case is included. It is visible that the heat flux levels (dashed lines) differ between runs at FC1 and FC2, as was expected from computations (see section 7.6.5 figure 7.72). Based on computational data, a sensitivity analysis was performed which Stanton number definition is the most suitable one to compensate different inflow conditions (see section 7.6.5). The edge based Stanton number showed the best agreement between computed heat flux distributions. In figure 4.43, this is confirmed with experimental data of Run3015, which agree closely in form of the edge based Stanton number St_e to all other data (solid lines). Therefore, this Stanton number definition is deemed suitable for further analysis.

Figure 4.44 shows a comparison of smooth wall experimental heat flux data represented by St_e to corresponding predictions. The experimental data (symbols) are represented by two runs for each PIV campaign. Laminar (blue) and turbulent computations (red and green lines) with TAU and the boundary layer code (BLcode) with isothermal wall at $T_w = 340$ K were included. In case of turbulent TAU computations, results for three different turbulence models and three levels of inflow turbulence levels were included. Additionally, analytical correlations by Korkegi (laminar) and van Driest (turbulent) (see section 3.10.1), transformed to Stanton numbers via Reynolds analogy (see section 3.11), were included.

As before, the experimental data show an acceptable run-to-run repeatability if the corresponding

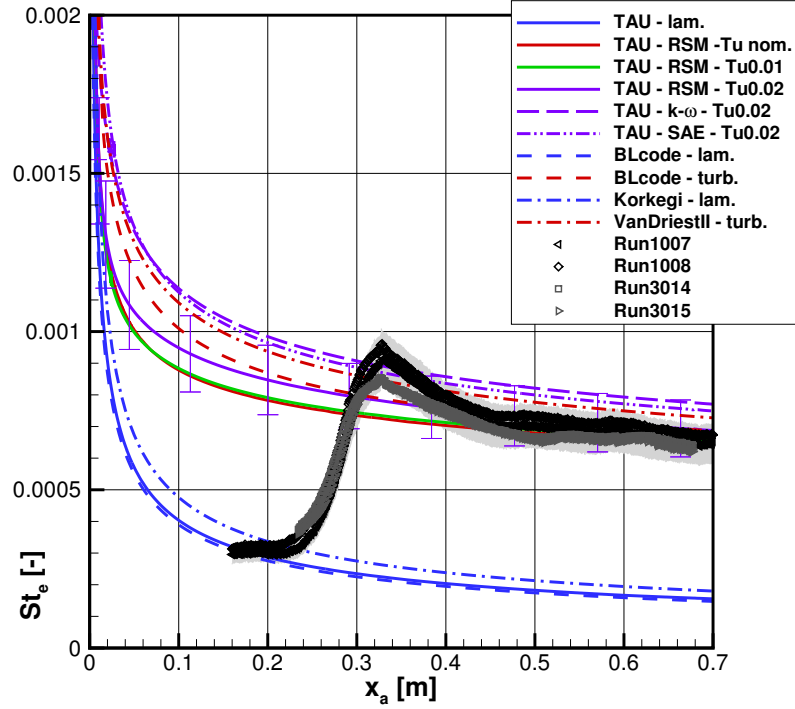


Figure 4.44: Smooth wall Stanton number comparison along cone between experimental, analytical and numerical data.

uncertainties are considered, included as gray areas. The laminar level is best captured by the results of the boundary layer code and TAU. The turbulent level, downstream the transitional overshoot, shows excellent agreement to the RSM TAU computations with an inflow turbulence intensity between $Tu = 1 - 2\%$, well within range of corresponding values from PIV (see section 4.3.1) and previous L2F data [94]. Results from the boundary layer code agree closely to the RSM model at $Tu = 2\%$, whereas the one- and two-equation turbulence models in TAU, as well as the analytical data, over-predict the turbulent Stanton numbers. If the TAU deviations are considered with the corresponding error bars (see section 7.5.2), all computations agree to each other and to the experimental data.

With Stanton numbers from smooth wall infrared thermography and skin friction values from PIV data, an independent consistency check could be performed, if the Reynolds analogy is assumed valid. Representative for the PIV skin friction values, the results derived by the fitting approach were used (see table 4.2). Table 4.6 contains calculated values of the ratio $c_{f,e}/(2St_e)$ from experimental data, TAU and boundary layer code, all extracted at MS1. This ratio should be equal to the approximate Reynolds analogy factor s_{ra} , which is included in the table in form of the incompressible turbulent factor after White [277] (see eq. (3.66) in section 3.11) and compressible turbulent factor after Van Driest [263] (see eq. (3.67) in section 3.11).

In case of the boundary layer code data, the ratio $c_{f,e}/(2St_e)$ is in better agreement with the correlation by van Driest with $s_{ra,vD}$. This was expected since both values were derived with the

Table 4.6: Reynolds analogy comparison of experimental data (PIV and IR data), numerical (TAU, BLcode) and analytical predictions (White [277], Van Driest [263]) at MS1.

case	$c_{f,e}/(2St_e)$ [—]	$s_{ra,W}$ [—]	$s_{ra,vD}$ [—]
BLcode turb.	0.89	0.94	0.86
TAU RSM Tu0.02	0.99 ± 0.03	0.93	0.86
TAU k- ω Tu0.02	0.94 ± 0.03	0.93	0.86
TAU SAE Tu0.02	0.97 ± 0.03	0.93	0.86
Run1007	1.00 ± 0.32	0.93 ± 0.01	0.86 ± 0.005
Run1008	0.79 ± 0.25	0.94 ± 0.01	0.86 ± 0.004
Run1010	1.06 ± 0.34	0.93 ± 0.01	0.86 ± 0.005
Run3014	1.03 ± 0.22	0.93 ± 0.01	0.86 ± 0.005

assumptions of a turbulent Prandtl number of $Pr_t = 0.9$, instead of the White approximation with $Pr_t = 1.0$. For the same reason, the same agreement was expected for the TAU data, but the data of the SAE, k- ω and RSM model are higher than the van Driest predictions and closer to the incompressible prediction of White. Also the corresponding sensitivities of the CFD results cannot compensate the differences (skin friction and heat flux differences are considered with respect to model parameters and spatial convergence, see chapter 7.7.2). Usually, also both theoretical values for the CFD data contain uncertainties due to their dependence on the skin friction (see chapter 3.10.1). This is neglected since the contributions are very small below 0.04% of the given value.

The experimental data of PIV Run1007, 1008, 1010 and 3014 show a comparable value range in between $0.79 \leq c_{f,e}/(2St_e) \leq 1.06$, well within the predicted levels. This confirms the consistency of the experimental data. Unfortunately, if the corresponding experimental uncertainties are considered (derived and combined from the fitting approach and heat flux evaluation, see chapter 7.1.1) no clear statement can be made which of the theoretical Reynolds analogy factors is more applicable in case of the current data.

Local sensors:

Additionally to the IR measurement technique, local heat flux sensors were implemented in the cone model and post-processed, as described in section 3.5.

Figure 4.45 shows typical exemplary results from local heat flux sensors in the smooth cone model. In figure 4.45a for smooth Run1011 the HFS and RTS signal transformed to heat flux and surface temperatures of HFM1 at $x = 0.3$ m are shown. After an approx. 5s transitional phase the heat flux develops a plateau like behavior with slightly decreasing values towards the end of the run. The surface temperature increases monotonically by approx. 10 K. In figure 4.45b the surface and backside temperatures of the CT3 (TE82-3) at $x = 0.654$ m for the same run are shown. The curve labeled “Heatflow” is the 1D computation of the resulting convective heat flux with the shown surface temperature signal as input (as described in section 3.4.2.1). As additional output, Heatflow gives the calculated backside temperatures (labeled “T(i=iend)”) which agree closely to the measured values. It is visible that in case of CT3 the monotonically

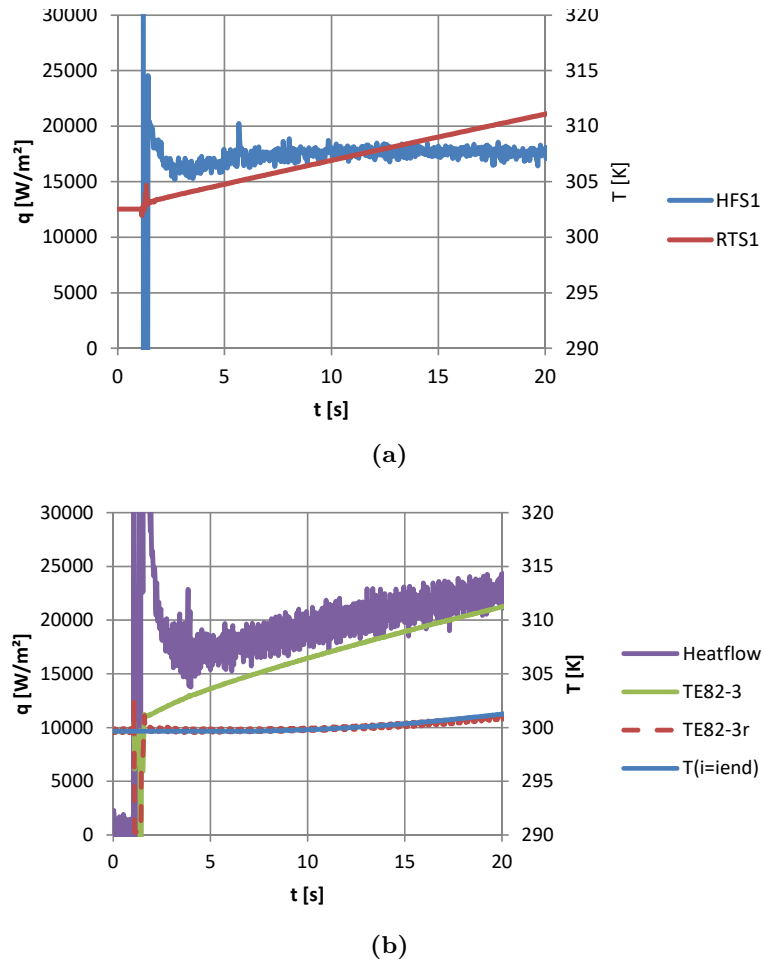


Figure 4.45: HFM1 and CT3 sensor signals for smooth wall Run 1011.

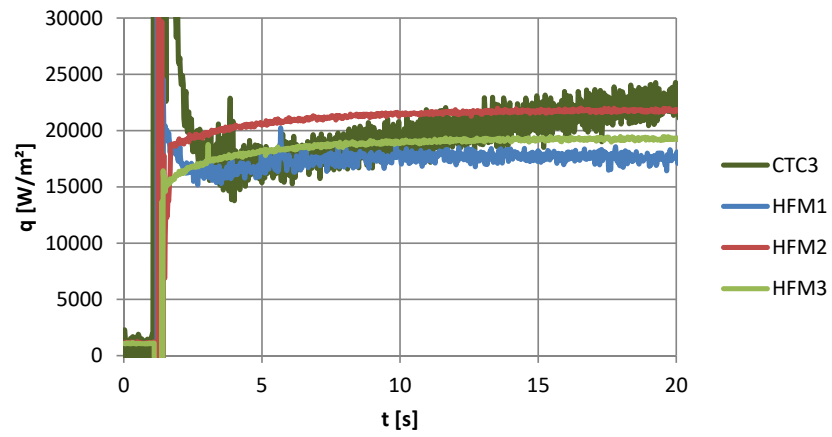
increasing temperature signal results in a heat flux curve characterized by a first overshoot, followed by a minimum after approx. 4 s with a following increase until the end of the run. This is an unexpected behavior since an increase in surface temperature usually lowers the heat flux at constant convective heat transfer (see corresponding signals from IR or HFM with figure 3.13 and figure 4.45a, respectively). This led to a numerical sensitivity study with ANSYS into possible influences (see section 7.6.7). The most probable reason for the observed behavior is an additional lateral heating from the hotter PEEK to the colder CT which results in higher surface temperatures than from convective heating alone. If this increased temperature information is used as input to a method only considering 1D heating, an erroneous higher wall-normal heat flux is calculated which would be necessary to result in this increased surface temperatures. Therefore, it was decided that the most reliable value of the CT heat flux results is the one where the first minimum is reached, before lateral heating effects increase over time. This value is also taken for run-to-run and rough-to-smooth comparisons in the following.

In contrast, the HFM sensor does not show this behavior, most probably since the sensor head is

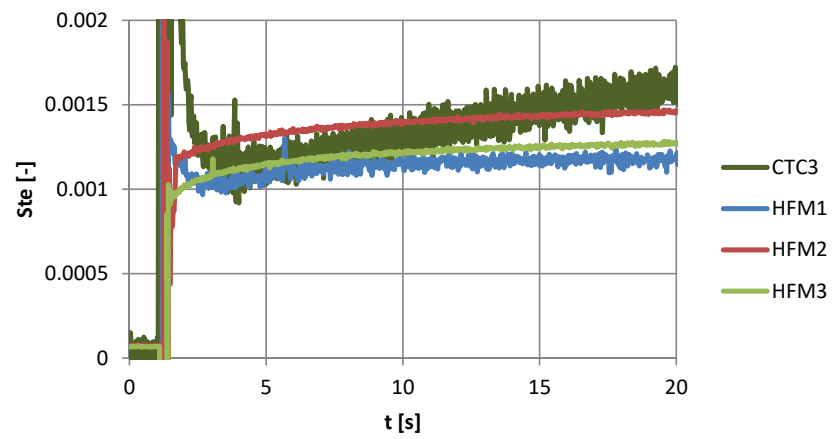
better thermally decoupled by a gap to the housing together with a further gap between housing and PEEK model. In case of the CT sensors, smaller gaps were visible in photographs of the model. Figure 4.46 shows a direct comparison of convective heat flux, Stanton number and sensor temperatures over time between HFM1-3 and CT3 sensor for smooth wall Run1011. Although the general level of heat flux is different, sensor-to-sensor differences between the three HFM sensors are consistent with information from IR. HFM1 ($x_a = 0.3$ m) is located at a streamwise position where laminar-turbulent-transition is visible from IR (see figure 4.44) but still upstream of the maximum peak ($x_a \approx 0.32$ m), therefore it has the lowest heat flux level of the three HFM sensors. HFM2 ($x_a = 0.49$ m) shows the highest heat flux and it is located only approx. 0.2 m downstream of the peak visible from IR. HFM3 ($x_a = 0.65$ m) shows the second largest heat flux since it is located a bit further downstream of HFM2 where the growth of the turbulent boundary layer leads to a lowering of the corresponding heat flux level. The corresponding Stanton numbers based on edge conditions St_e in figure 4.46b show the same trends.

The results from CT3 in figure 4.46 show the already discussed increasing heat flux with testing time, also in terms of Stanton number. CT3 should be comparable to HFM3 which is the case if the result at $t \approx 4 - 5$ s is considered. This is in line with the minimization of potential lateral effects, previously discussed (see also section 7.6.7).

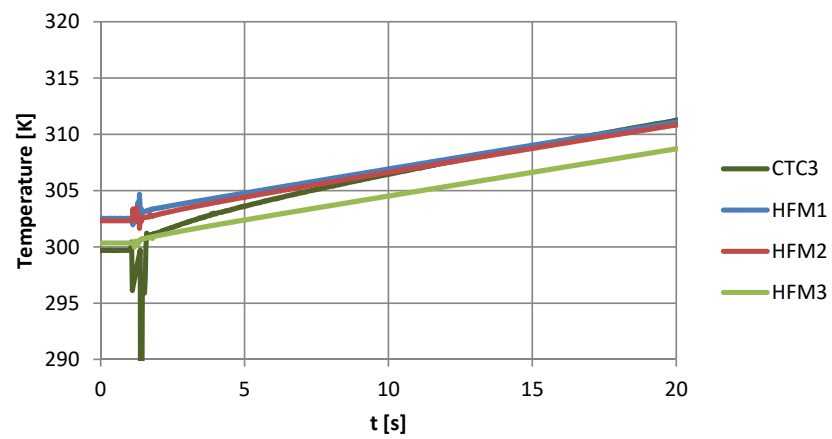
The surface temperatures of all sensors show only moderate increase of approx. 10 K in comparison to approx. 40 K in case of PEEK. Also, the resulting heat flux or Stanton number levels are different between local sensors and via IR technique at comparable model positions (see e.g. figure 4.44). This is mainly driven by the different materials. PEEK was chosen due to its low thermal conductivity, leading to high surface temperatures, whereas thermocouples are made of metallic materials, which have higher thermal conductivity and therefore lower surface temperatures. This is the crux for every heat flux measurement, either in ground based facilities or also in case of flight testing (see discussions in e.g. [20], [93]). One way to address impacts of locally different surface temperatures with computational means was discussed in [93], with simulating the heat flux with enforced temperature distributions from measurements. In this work, rough-to-smooth wall heat flux augmentations are relevant and technique-to-technique differences are assumed to be relaxed since augmentations are calculated for each measurement technique separately.



(a)



(b)



(c)

Figure 4.46: Convective heat flux a), edge based Stanton number b) and sensor temperature c) over time of HFM and CTC3 sensors for smooth wall Run1011.

4.4.2 Rough Wall

IR:

Figure 4.47a shows rough wall convective heat flux, wall temperatures and edge based Stanton numbers for Run1012, 1013 and 3006, 3007 compared to smooth wall data. The transition location and corresponding heat flux increase of all rough runs (colored lines) is comparable to the smooth runs (black, gray lines). There is a clear increase of heat flux over the smooth level after an initial local drop for $x > 0.326$ m, the position where the rough segment begins. The drop is due to the design of the roughness topology which begins with a groove instead of a bump (see section 3.1). The following wavy heat flux data (especially for Run1012, 1013 due to the higher spatial resolution, see section 3.4) appear like noise but actually represent the foot print which resembles the roughness topology of repeated grooves and bumps. A local heat flux peak followed by sharp decrease below the smooth level is visible near the end of the rough section at $x > 0.65$ m. Although at slightly different spatial resolutions, it is visible that data from different campaigns (1000 vs 3000 run numbers) agree within measurement uncertainties, especially in case of the edge based Stanton numbers. At the mid-part of the cone ($x_a \approx 0.5$ m), the data from the last campaign shows slightly smaller Stanton number values than from the first campaign (compare e.g. Run1013 vs 3006). The reason is not fully clear but since this occurs as well in the corresponding smooth wall runs, the impact on the heating augmentations is assumed to be minor (will be addressed below, as visible in figure 4.53).

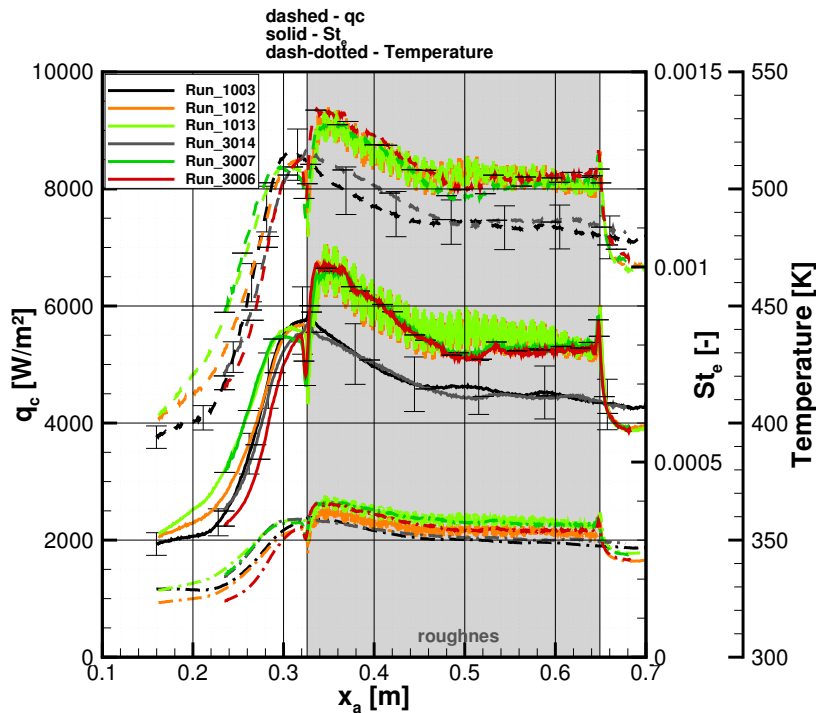


Figure 4.47: Rough wall convective heat flux, wall temperature and edge based Stanton number for Run1012, 1013 and 3006, 3007 compared to smooth run data.

Local sensors:

The local sensors were also used to measure rough wall heat fluxes. Figure 4.50a shows exemplary edge Stanton number results for the HFM sensors of rough wall Run1015. As reference, data of smooth wall Run1011 is also included. It is visible that the heating levels for rough wall data is consistently above the smooth wall level (dashed lines). HFM1 smooth wall data was corrupted so that only the rough wall values are shown. These data as well as data from CT sensors will be discussed further in the following paragraphs addressing rough-to-smooth wall heat flux augmentations.

4.4.3 Rough-to-Smooth Wall Augmentation

With data along the smooth and rough wall cone, augmentations can be calculated, first from the IR data.

IR:

Since small run-to-run and temporal sensitivities exist in both smooth and rough wall data (see discussion in section 7.6.6, e.g. figure 7.75 and 7.76), it was decided to extract statistics within a defined time interval of $10\text{ s} < t < 20\text{ s}$ (see section 7.6.6 and table 7.28 for discussion on the influence of time interval) from different runs. For this, figure 4.48a shows Stanton numbers and rough wall heating augmentations in figure 4.48b for different smooth runs (Run1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010) and rough runs (Run1012, 1013) from IR. The light blue lines correspond to all available data (with a $\Delta t = 1\text{ s}$ within $10\text{ s} < t < 20\text{ s}$) in figure 4.48a and all possible combinations of rough to smooth wall ratios of these available data in figure 4.48b. Green lines correspond to all available smooth wall data to an ensemble averaged smooth wall profile and represent general run-to-run and temporal / spatial scatter. The blue lines correspond to all available rough wall data to the ensemble averaged smooth wall profile. The black and red lines correspond to an ensemble averaged smooth and an ensemble averaged rough wall profile, respectively, in figure 4.48a and the ratio of both as red line in figure 4.48b. Additionally, mean augmentations spatially averaged within $0.4\text{ m} < x_a < 0.5\text{ m}$, $0.4\text{ m} < x_a < 0.6\text{ m}$ and $0.5\text{ m} < x_a < 0.6\text{ m}$ were derived (symbols from left to right, respectively) (see section 7.6.6 and table 7.28 for discussion on the influence of spatial filtering). The corresponding error bars represent minimum and maximum values of the ratio in the corresponding regions.

We focus in the following on a relevant axial distance region within $0.31\text{ m} < x_a < 0.7\text{ m}$, including the rough wall cone section with $0.33\text{ m} < x_a < 0.65\text{ m}$, indicated as gray rectangular area in figure 4.48. Stanton numbers show larger divergences at the beginning of the roughness (see e.g. green lines in b) within $0.31\text{ m} < x_a < 0.4\text{ m}$) mainly due to impact of natural transition upstream of this location. This spread decreases for $x_a > 0.4\text{ m}$. The smooth wall run-to-run differences based on the ensemble averaged smooth wall profile (green lines in figure 4.48b) are within $St_{e,s}/St_{e,s,avg.} \approx \pm 0.04$. The rough wall augmentations in figure 4.48b show first a decrease below one at the onset of roughness. This is again because the roughness topology was arranged with the roughness crests flush with the upstream smooth wall level and so the topology begins with a channel (already described and visible for figure 4.47). Due to IR resolution (approx. 0.9 pix/mm) slightly oscillating ensemble averaged rough to ensemble averaged smooth profile are visible, reflecting the roughness topology. The emerging pattern of the corresponding

$St_{e,r}/St_{e,s,avg.}$ values, which appear like an interference pattern called beat, are most likely due to a combination of the camera resolution and the wavelength of roughness topology. This presumably leads to some pixels resolving peak temperatures along the crests of the roughness better than others, which also lead to an increased heat flux rate. However, the average level is assumed to be less affected, as will be seen in comparison to data from the second PIV campaign. For $x_a > 0.4$ m, an approximately constant mean augmentation of about 20 % is visible. This is also well visible by the different symbols in figure 4.48b, representing spatially averaged values.

The same post-processing was applied on the data of the second PIV campaign which is shown in figure 4.49. In general, smaller smooth wall run-to-run differences ($St_{e,s}/St_{e,s,avg.} \approx \pm 0.02$) are visible. One influence is the amount of runs for ensemble averaging but also the more consistent transition location between runs. Also, the rough wall data has smaller spread compared to the previous plot. The ensemble averaged rough to ensemble averaged smooth profile showing again an approximately constant mean augmentation of about 20 % for $x_a > 0.45$ m, comparable to the data of the first PIV campaign. This supports the assumption of an averaging effect for the IR information over corresponding solid angles due to the different camera resolutions (see section 3.4).

Due to this ensemble averaging, also the uncertainties of heating augmentations are relaxed to $U_{St_{e,r,avg.}/St_{e,s,avg.}} \approx 3.3$ % and 3.6 % for the first and second PIV campaign, respectively.

Local sensors:

As for IR data, heat flux augmentations were also calculated for corresponding local sensors. Figure 4.50b shows HFM heating augmentations for rough wall Run1015 to smooth wall Run1011, corresponding to the previously discussed Stanton number data in figure 4.50a. For the heating augmentation ratios $St_{e,r}/St_{e,s}$, it is visible that due to measurement noise in both smooth and rough wall data also the corresponding augmentation levels are quite noisy. Therefore, the corresponding heat flux data was filtered with a moving average of 50 points before calculating the augmentations, which are additionally included as darker lines in the figure. Clear augmentations above 1 are visible in case of the mid-section sensor HFM2 (red lines) and also for the one directly downstream of the roughness HFM3 (green lines), although at a smaller level.

In figure 4.50b, it is also visible that HFM augmentations slightly decrease with testing time. Therefore, for a comparison with other data, it was decided to use statistics in form of the mean (“mean”), minimum (“min”), maximum (“max”) and standard deviation (“std”) values for a time frame of $t = 3 - 20$ s, excluding wind tunnel start and most of the initial noise in the data and to be consistent with CT signal post-processing (see also section 7.6.6 for discussion about averaging time frame influence). Corresponding data is included in table 7.29 and also as symbols in figure 4.53 in comparison to augmentations from IR data. On average a Stanton number augmentation of approx. 1.13 at MS1 can be observed.

The corresponding plots in case of the CT sensors are shown in figure 4.51a with heating augmentations from heat flux $\dot{q}_r/\dot{q}_{s,avg.}$ and Stanton number $St_{e,r}/St_{e,s,avg.}$ for several smooth and rough wall runs at mid-sensor CT2 (TE63-3) and in figure 4.51b at aft sensor CT3 (101-4). In case of CT signals, only runs from the second wind tunnel campaign at FC2 were useful. Data from boundary layer code computations showed that heating augmentation differences between FC1 and FC2 are negligible in form of edge based Stanton number ratios $St_{e,r}/St_{e,s}$ and conditions in this thesis (see section 7.6.6 figure 7.78), so that it was assumed viable to compare

data from both conditions directly. It is visible for both sensors in figure 4.51, that also the augmentation from CT signals show a temporal behavior. Although, they show an increase with testing time in contrast to the opposite behavior in case of the HFM signals (see figure 4.50). Comparable to the procedure for the HFM sensors, also for the CT sensors statistical values were extracted for comparisons to other data included in tables 7.30 and 7.31 for CT2 and 3, respectively. To reduce the potential impact of lateral heating, a time frame of $t = 2 - 4$ s was chosen, based on discussions above. In case of CT measurements, data was also averaged not only in time but also ensemble averaged over different smooth wall runs to calculate a reference for the rough wall data. This is indicated in the tables with runs labeled “avg.”. Additionally to the rough wall runs, the ratios of $\dot{q}/\dot{q}_{s,avg.}$ and $St_e/St_{e,s,avg.}$ were also calculated for all smooth wall runs. This gives an indication about run-to-run repeatability which fall in the range of approx. ± 4 % in case of all mean values and complete testing time. On average a Stanton number augmentation of approx. 1.1 at MS1 can be observed.

In case of the aft sensor CT3, rough wall heat flux and Stanton number data are in general smaller than corresponding values at CT2, as expected from comparisons to data from IR or HFM. Corresponding data is also included as symbols in figure 4.53 in comparison to augmentations from HFM and IR data, discussed in the following.

Sand grain type roughness:

Efforts were made to extract heating augmentations also along the sand grain (Carborundum) roughness type model. Due to unknowns in emissivity of the Carborundum material and the layer of glue with its thickness and properties relevant for heating (density, heat capacity, heat transfer coefficient), the post-processing used on the IR data was omitted. Alternatively, also the Carborundum model had local heat flux sensors installed at the same positions and with the same smooth wall plateau strategy as the technical roughness model. The same post-processing as reported before was used (see section 3.5). Figure 4.52a shows heat flux augmentations over time along the cone. It was surprising to see that only for $t \gtrsim 10$ s the rough wall data is larger than corresponding smooth wall measurements. However, at these later times, data is probably already prone to lateral heating as discussed before (and see discussions in section 7.6.7). So, based on these data no augmentations could be extracted.

To investigate the impact of the smooth wall plateau around the local sensors, tests have been performed with Carborundum applied along the complete model, including the CT surfaces. Figure 4.52b shows the same plot as before in case of the roughened CTs and this time it is visible that the Stanton number augmentations are above 1 already shortly after wind tunnel start ($t > 2$ s). The corresponding statistics are summarized in table 7.32, which show that the value range is comparable to values along the technical roughness with a mean of approx. 1.1.

It needs to be mentioned, that there are several open questions to this analysis. Although certain aspects are considered in the post-processing, there is a possibility that the agreement between Carborundum and TSB roughness is a coincidence due to combined effects of e.g. lateral heating in case of CT signals or smooth wall plateau effects in case of TSB. Another aspect not considered for the above results but assumed to have an impact on the post-processing of CT signals to heat fluxes is the glue layer in case of roughened CTs. Details with respect to thickness of the layer or thermal parameters of the glue were unknown at the time of writing. Further studies would be necessary to clarify these open points.

If current CT data along the Carborundum roughness is accepted despite these open questions, together with data from Pitot pressure measurements (u_τ or k_s^+), we see that overall results are close to corresponding values along the technical roughness (TSB). This would further give the extracted EQSR values along the TSB roughness credibility.

Augmentation comparison:

Collecting augmentation data from all measurement techniques, figure 4.53 shows the ensemble averaged rough to smooth wall data for the different campaigns from IR (lines) and local heat flux sensors (symbols, as described in the following), represented by $St_{e,r}/St_{e,s}$. Augmentations from IR data are shown from both relevant campaigns with slightly different spatial resolutions. It is visible that after an initial dip below 1, the roughness Stanton number augmentations are consistently above approx. 1.1 for $x_a \gtrsim 0.45$ m. This result is independent of the campaign, especially, if the error bars are considered, representing the corresponding minimum / maximum values from ensemble averaging, previously described. Along the first part of the roughness ($0.33 \text{ m} \lesssim x_a \lesssim 0.45 \text{ m}$), the exact location of the natural transition had the largest impact on the worse agreement between IR data of both campaigns. For a region $x_a \gtrsim 0.45$ m results from IR overlap at an overall level of approx. 20 %. Only the position and extent of the initial dip, as well as the overshoot at the end of the roughness insert, is slightly different between the campaigns, due to the slight difference in spatial resolution. The location $x_a \gtrsim 0.45$ m downstream of which IR data agree closer to each other corresponds to a distance of $\Delta x_a / \delta \gtrsim 35$ when measured from the onset of roughness. Here, the smooth wall boundary layer thickness δ at roughness inset (MS1) from PIV measurements was used. Due to the larger running length at MS1 compared to the onset of roughness, this most likely overpredicts the corresponding thickness slightly which lead to a slight underprediction of this transition length. This relaxation length is slightly larger than the value of 10 – 25 according to Berg [24] as measured from Pitot pressure and temperature fluctuation profiles.

Temporal and ensemble averaged data of the local heat flux sensors are indicated by symbols together with their corresponding minimum/maximum values (within post-processed time range) as error bars. The augmentation results from local sensors fall slightly below the IR data at MS1 whereas at MS2 a closer agreement is visible. In general, HFM sensors show slightly larger values compared to CT sensors. Even if the corresponding data spread is considered, HFM and CT data don't overlap. One reason for the general lower augmentation values from both local sensor types might be attributed to the smooth wall plateaus, which were necessary to include the sensors within the technical roughness. This was also visible in the slightly different fitting results from Pitot measurements in figure 7.25. A potential influence for the difference between HFM and CT results was the difference in post-processing (shorter time interval) in case of the CT sensors to lower potential impact of lateral heating effects. If larger time frames (e.g. $t = 2 - 20$ s) would be used, CT augmentation signals would increase and overlap with HFM signals. Additionally, uncertainties for the local sensors are substantial with $U_{St_{e,r}/St_{e,s}} \approx 36$ % (see section 7.1.1), which would on the one hand lead to an agreement with IR data, but on the other hand would mask any visible measured augmentations above 1. Since uncertainties for the sensors are not exactly clear and $U_{\dot{q}}$ may range from 5 % to 10 % (see discussion in [253]), also corresponding $U_{St_{e,r}/St_{e,s}}$ would vary between approx. 20 % to 36 %. Due to these open points, all following comparisons were mainly based on results based on IR data.

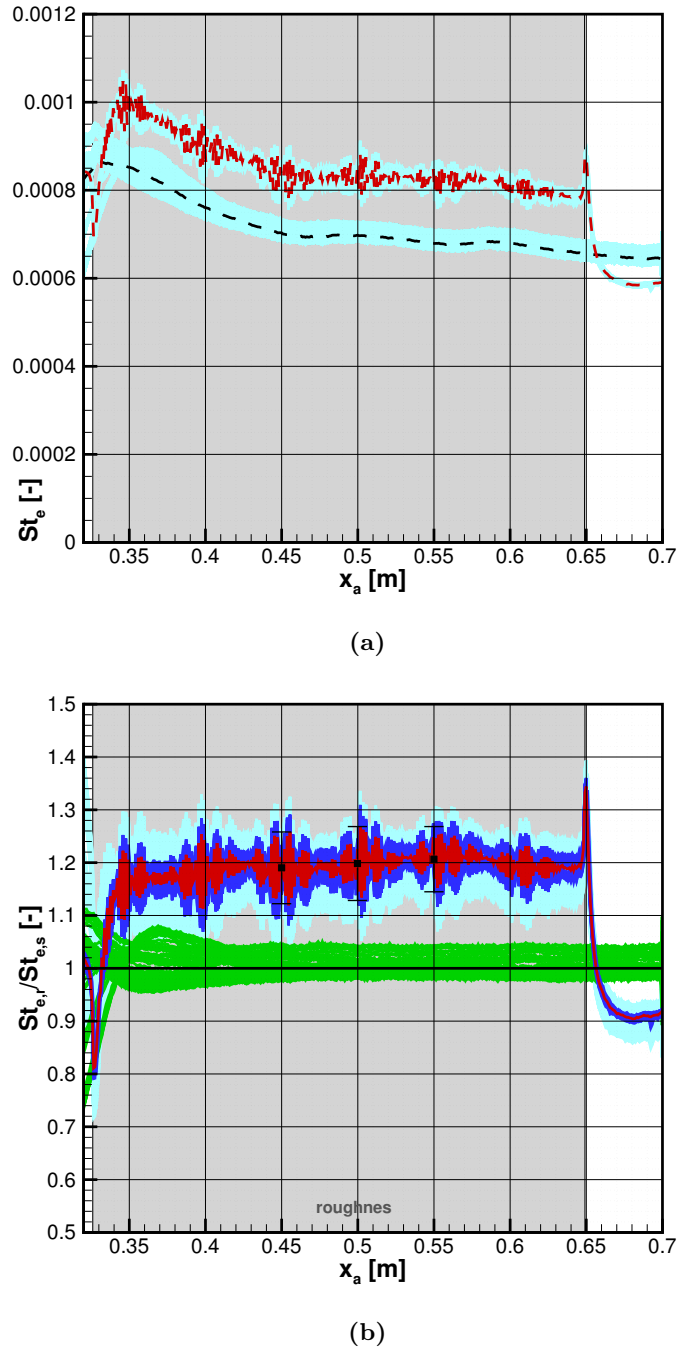
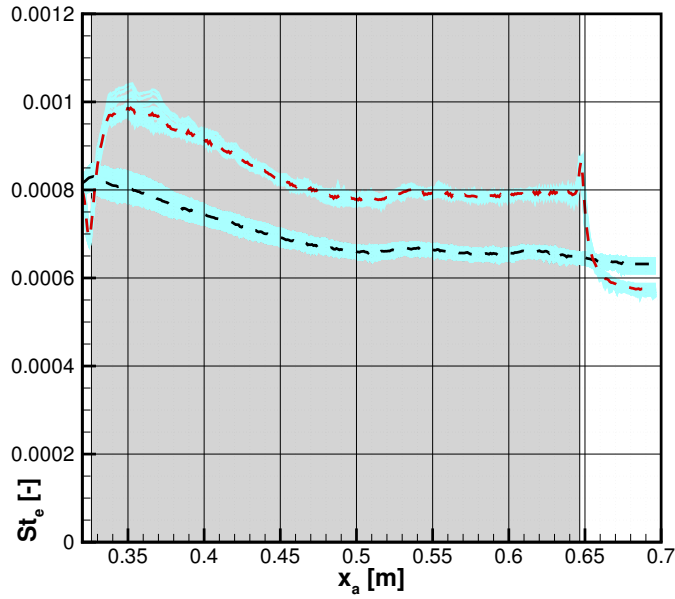
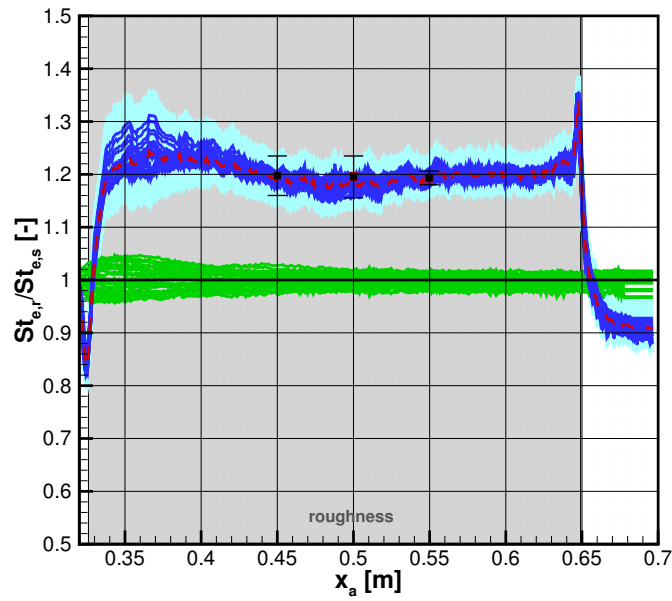


Figure 4.48: Stanton number (a) and rough wall heating augmentation (b) for different smooth runs (Run1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010) and rough runs (1012, 1013) from IR for all combinations (light blue lines), smooth runs to ensemble averaged smooth values (green lines), rough runs to ensemble averaged smooth values (blue lines), ensemble averaged rough to ensemble averaged smooth values (black and red lines) and mean augmentation spatially averaged within $0.4\text{m} < x_a < 0.5\text{m}$, $0.4\text{m} < x_a < 0.6\text{m}$ and $0.5\text{m} < x_a < 0.6\text{m}$ (symbols from left to right, respectively).

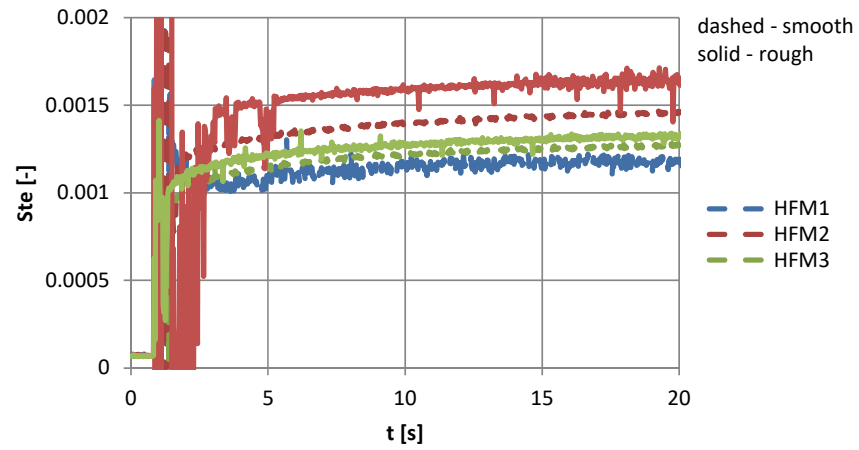


(a)

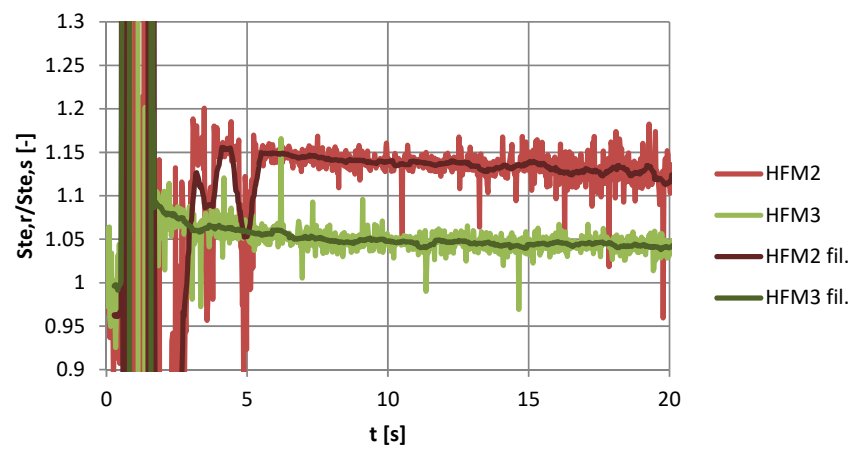


(b)

Figure 4.49: Stanton number and rough wall heating augmentation for different smooth runs (Run3014, 3016, 3019) and rough runs (Run3006, 3007, 3008) from IR for all combinations (light blue lines), smooth runs to ensemble averaged smooth values (green lines), rough runs to ensemble averaged smooth values (blue lines), ensemble averaged rough to ensemble averaged smooth values (black and red lines) and mean augmentation spatially averaged within $0.4 \text{ m} < x_a < 0.5 \text{ m}$, $0.4 \text{ m} < x_a < 0.6 \text{ m}$ and $0.5 \text{ m} < x_a < 0.6 \text{ m}$ (symbols from left to right, respectively).

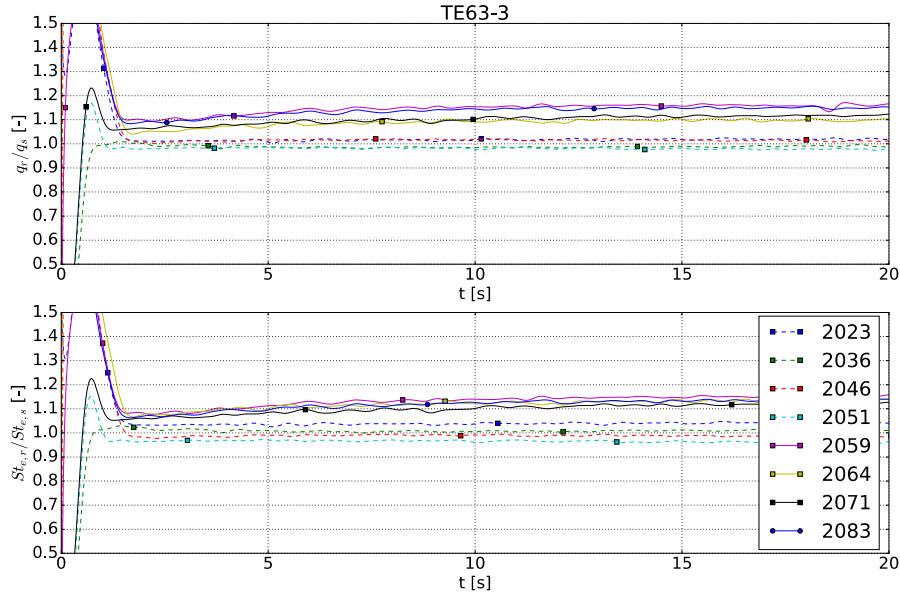


(a)

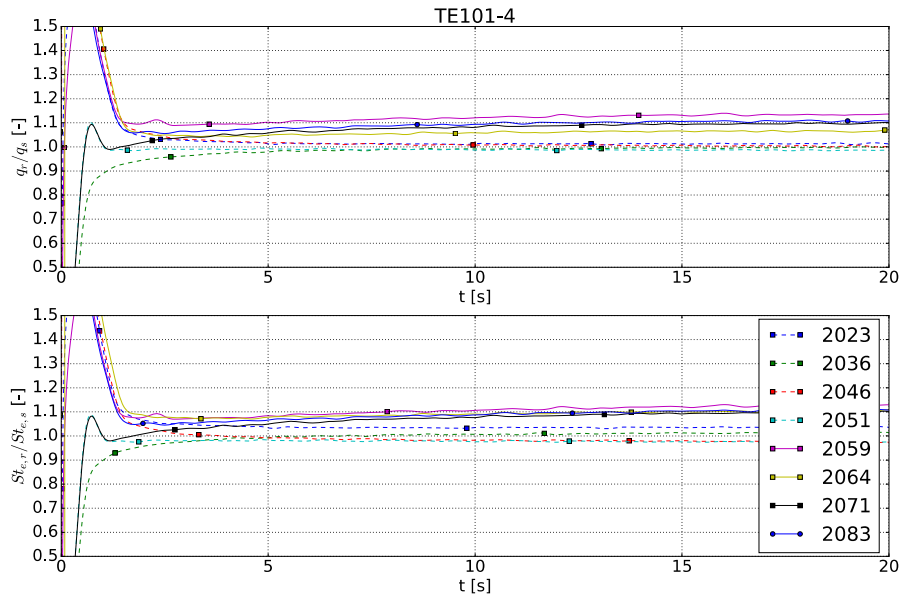


(b)

Figure 4.50: HFM sensors a) edge Stanton number results and b) heating augmentation for smooth wall Run1011 and rough wall Run1015.

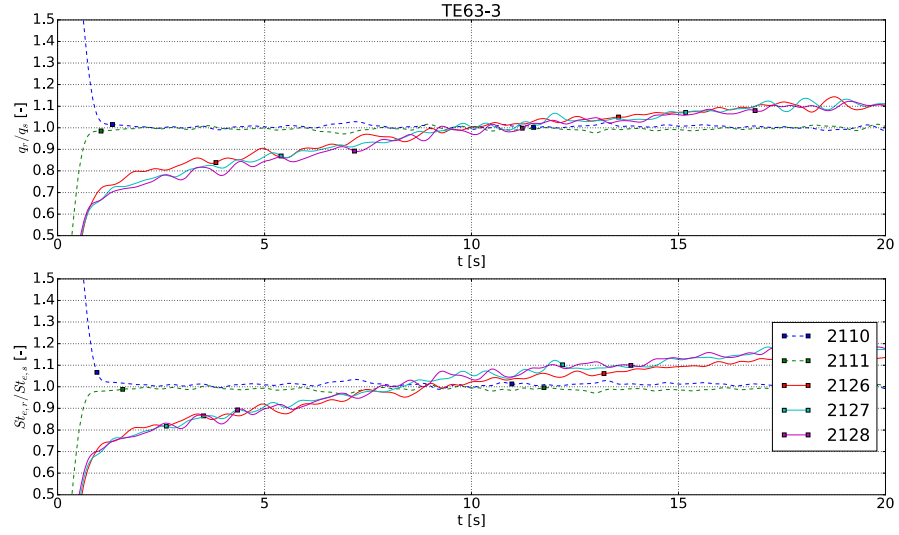


(a)

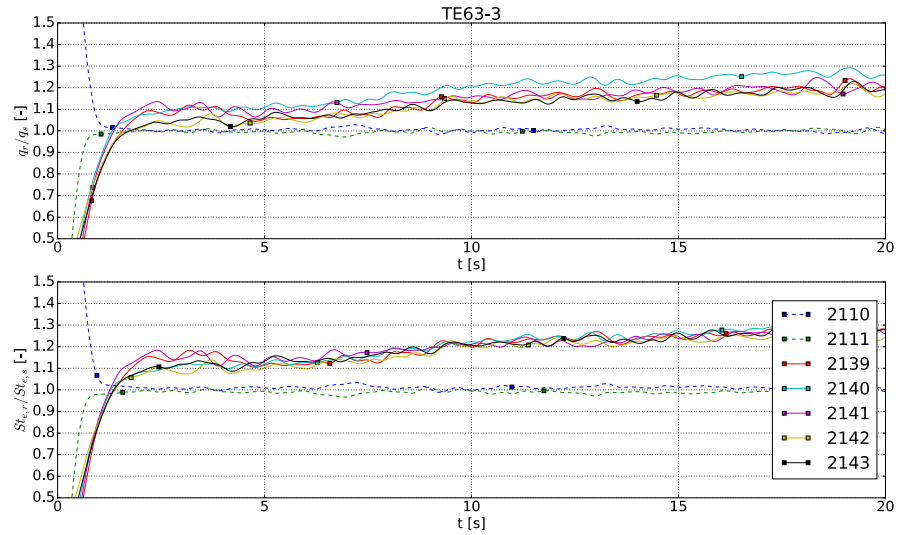


(b)

Figure 4.51: CT sensors heating augmentation for several smooth and rough wall runs at a) mid-sensor CT2 (TE63-3) and b) aft sensor CT3 (101-4).



(a)



(b)

Figure 4.52: Heating augmentations for smooth wall runs and SG runs for a) CT2 sensor b) roughened CT2 sensor.

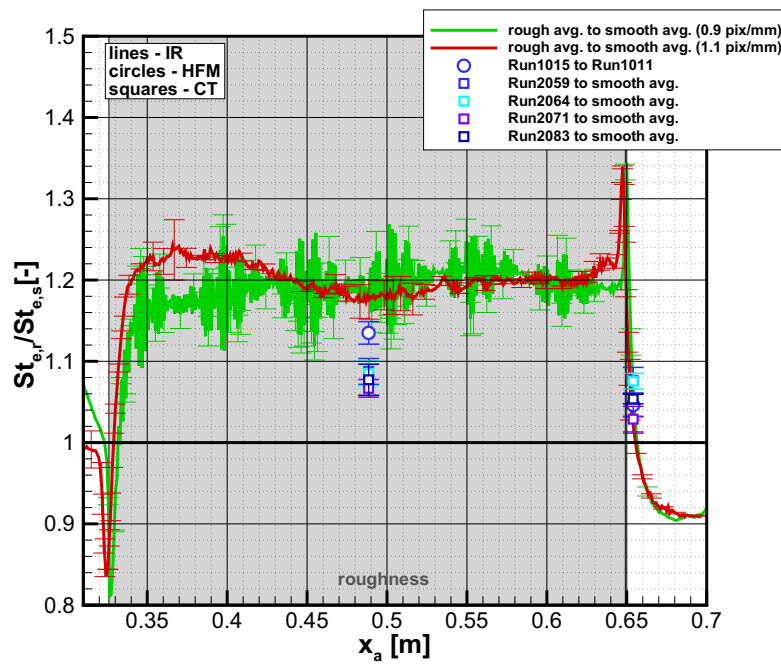


Figure 4.53: Roughness Stanton number augmentation for different runs from IR (green line from first PIV campaign and red line from second PIV campaign) and local heat flux sensors (symbols).

4.4.4 Augmentation Comparison to Predictions and Literature Data

In the following comparisons of experimental data to data from the BLcode with the modified Krogstad roughness model are shown (see section 3.8). Different assumptions were made for the BLcode computations, already discussed in section 4.2.2 (see e.g. figure 4.20). The impact of using either y^+ , k_s^+ or the compressible formulations y_c^+ , $k_{c,s}^+$ (see eq. (7.37) and (7.39)) are reported in section 7.6.6 (see figure 7.77). If not otherwise noted, in the following only data with the compressible formulation are used.

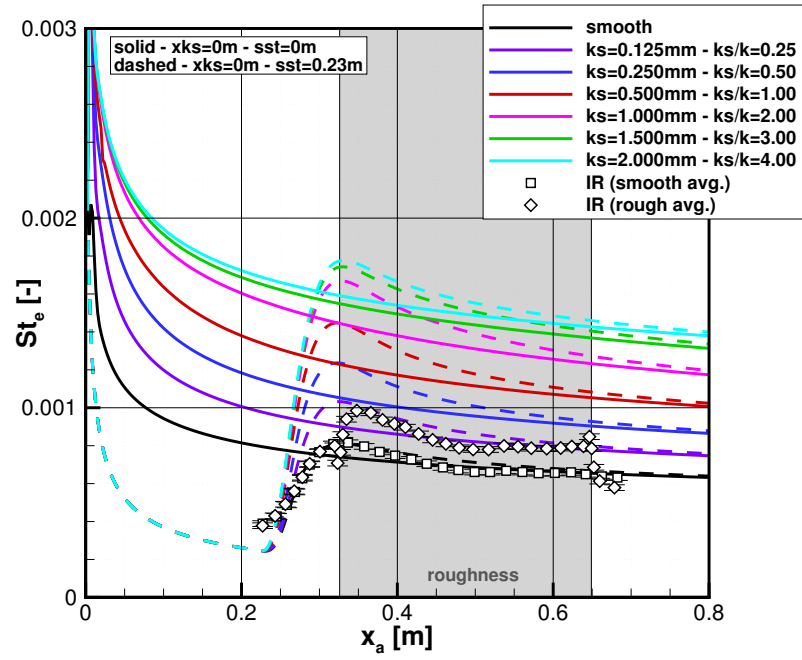
Figure 4.54 shows Stanton numbers St_e along the smooth and rough cone from BLcode at different EQSR values within $k_s = 0.125 - 2.0$ mm corresponding to $k_s/k = 0.25 - 4.0$ if $k = 0.5$ mm is chosen. For better visualization, cases with different assumptions are split in two different figures 4.54a and 4.54b. The calculated Stanton numbers in figure 4.54a show an increase with increasing EQSR with a saturation for higher values. This saturation was enforced by the choice for compressible y_c^+ and $k_{c,s}^+$ (see section 7.6.6). Comparable to k_s^+ (see figure 4.20) also the Stanton numbers overshoot for transitional simulations with a corresponding following relaxation to turbulent levels. The symbols represent ensemble averaged smooth and ensemble averaged rough wall values from IR (see e.g. figure 4.49). The IR data was down-sampled to $\Delta x = 15$ mm to guide visual comparisons. Error bars represent data spread, discussed and shown in figure 4.49. The smooth wall values correspond well to simulated data (as seen before in e.g. figure 4.44) and also the laminar-turbulent transitional overshoot is mimicked well, since all corresponding parameters were set accordingly in the BLcode. The rough wall experimental data show augmentation levels which correspond to simulated data with EQSR of $k_s/k = 0.25$ (purple lines). This is in contrast to the values extracted by fitting directly of $k_s/k \approx 1$ (see table 4.3) and to the rebuilding of the roughness Reynolds numbers (figure 4.20) or skin friction velocity augmentations (figure 4.23) which showed an agreement to PIV data by applying larger EQSR values corresponding to $k_s/k = 1 - 2$.

Also visible in figure 4.54a is that in case of the rough wall IR data the overshoot due to laminar-to-turbulent transition is visible but in contrast to the simulated data, another sharp jump downstream is visible, due to the beginning of roughness. An effort was made to rebuild this effect and figure 4.54b shows the same plots as before, but data of the BLcode were used for which the roughness terms were activated only within $0.33\text{m} < x_a < 0.65\text{m}$ (label “xks=0.33-0.65m”), corresponding to the roughness insert of the cone model (as before to rebuild PIV data in section 4.2.2 e.g. in figure 4.20). The behavior of the heat flux from IR is captured better than before. A slightly better rebuild is possible with the fully turbulent computation (label “sst=0”) immediately after the onset of roughness, whereas downstream towards the end of the roughness inset a better agreement is visible with the laminar-turbulent transitional computations (label “sst=0.23m”). As for rebuilds shown in figure 4.54a, again, the best agreement can be seen with simulations applying EQSR of $k_s/k = 0.25$.

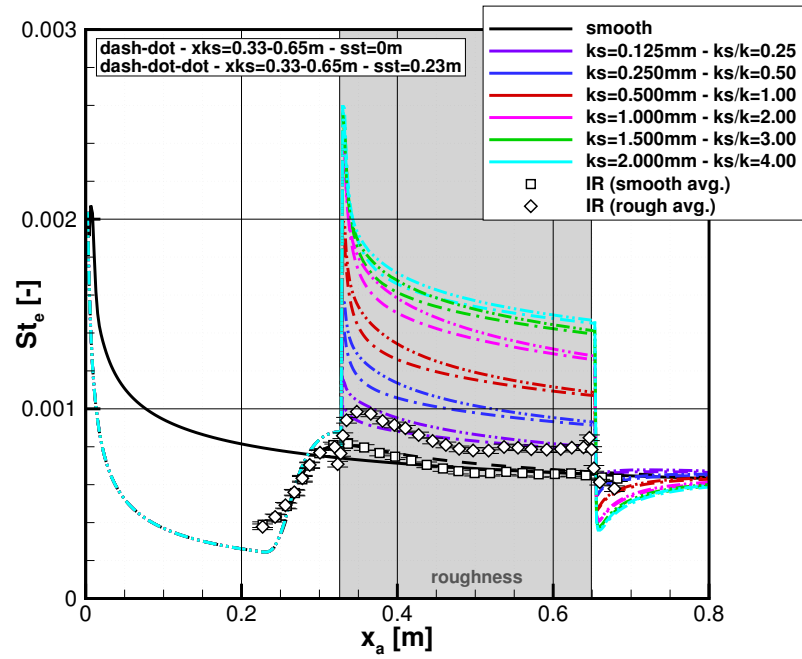
Figure 4.55a shows rough-to-smooth wall Stanton number augmentations represented by $St_{e,r}/St_{e,s}$ for data from BLcode, analytical predictions from Seidman and Powars as well as IR data. The correlations were introduced in section 3.11. The line labeled “Seidman-Fenter-VD” corresponds to a correlation where the rough wall Stanton numbers $St_{e,r}$ were calculated according to [223] via eq. (3.69), using the rough wall skin friction $c_{f,r}$ according to [71] via eq. (3.40). The line labeled “Powars-VD-VD” utilizes the correlation according to [193] via eq. (3.70) for the $St_{e,r}$. In both

cases, the smooth wall reference Stanton numbers $St_{e,s}$ were calculated via skin friction according to [261] (including modifications by [282]) via eq. (3.34) and the Reynolds analogy via eq. (3.65). Since analytical expressions are not able to rebuild any smooth-to-rough or rough-to-smooth transitions, only BLcode data for fully rough computations are shown. In case of IR, only data within $0.34 \text{ m} \lesssim x_a \lesssim 0.62 \text{ m}$ are shown to exclude effects from aforementioned transitions. As before, it is visible that IR data is close to BLcode simulations with $k_s/k = 0.25$. Error bars representing corresponding data spread (as visible in figure 4.49). In contrast, analytical predictions rebuild IR data best applying EQSR of $k_s/k = 2$ and $k_s/k = 0.5$ for methods after Seidman and Powars, respectively. Clearly, a disagreement between the different predictions is visible, already reported about in e.g. [167].

To include the effects of smooth-to-rough and rough-to-smooth wall transitions for augmentations, figure 4.55b shows data with different EQSR from BLcode computations compared to IR data (only second PIV campaign) near the roughness section $0.33 \text{ m} < x_a < 0.65 \text{ m}$. Only BLcode data with localized rough wall influence (label “xks=0.33-0.65m”) are shown. Again, BLcode data with $k_s/k = 0.25$ are closest to the IR data. The first drop in the IR data at the smooth-to-rough wall section is not captured by the BLcode. This is most likely due to the orientation of the technical roughness with the crests of the topology being flush to the smooth wall in experiment. Therefore, the roughness begins with a channel whereas it is modeled as bump first in case of the BLcode. Also not captured is a peak close to the end of roughness, visible in the IR data. In contrast, the drop of augmentations below 1 downstream of the rough wall section ($x_a > 0.65 \text{ m}$) can be rebuild with the simulations and also here, the best agreement is by applying EQSR of $k_s/k = 0.25 - 0.5$.

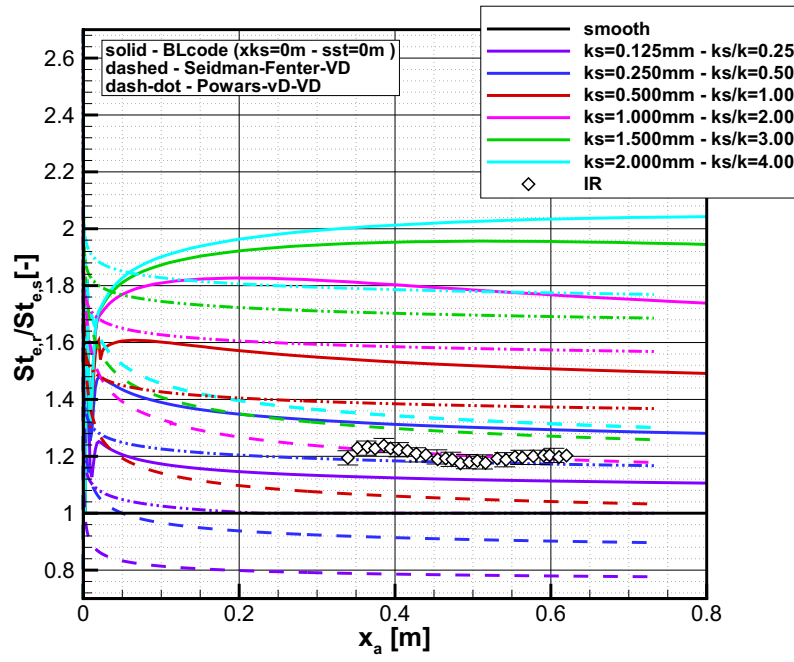


(a)

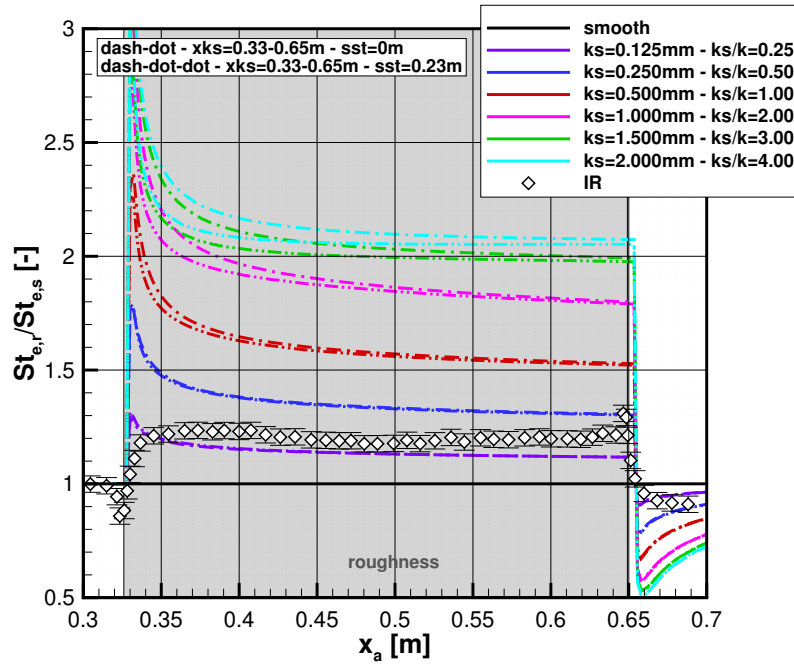


(b)

Figure 4.54: Calculated Stanton numbers along rough cone from BLcode at different EQSR values.



(a)



(b)

Figure 4.55: Rough-to-smooth wall Stanton number augmentations $St_{e,r}/St_{e,s}$ a) along the cone for data with different EQSR from BLcode, analytical predictions from Seidman and Powars as well as IR data (only second PIV campaign) and b) near the roughness section $0.33m < x_a < 0.65m$ for data with different EQSR from BLcode and IR data (only second PIV campaign).

Augmentation summary:

With the information of heat flux augmentation (this section) and roughness Reynolds numbers from EQSR fitting (see section 4.2.2) it is possible to summarize the current data in form of Stanton number augmentations $St_{e,r}/St_{e,s}$ against k_s^+ and compare it to results from modeling and literature in figure 4.56. The same analytical correlations as before are shown, with the corresponding k_s^+ values calculated according to [71]. The line labeled “BLcode Krogstad” corresponds to data points calculated with the boundary layer code including the modified Krogstad roughness model (see section 3.8) screening a large range of roughness Reynolds numbers. Data from BLcode show the largest augmentations from all predictions. For the latter also the influence of extracting the augmentations at MS1 and MS2 as well as impact of incompressible or compressible definition for y^+ and k_s^+ are shown. The range of Reynolds numbers in this case were simulated not only by increasing the EQSR value but also by artificially increasing the length of the cone to lower impacts from blockage ratios k/δ (see discussions in section 7.6.6 figure 7.79).

A region of moderate increase of BLcode heating augmentations is visible within $1 \lesssim k_s^+ \lesssim 10$ before the slope approaches a log-linear behavior, closely to the predictions via Powars correlation. At $k_s^+ \approx 65$ the slope of the BLcode log-linear behavior changes, corresponding to the value, at which the Van Driest formulation (see eq. (7.38) in section 7.7.1) is switched to the one by Krogstad (see eq. (7.39) in section 7.7.1). It is visible that augmentations level-off close to $k_s^+ > 100$ to values of approx. 2.6 or 2.7 for compressible and incompressible formulations. IR data of both campaigns with slightly different resolutions are shown (1.1 pix/mm for first PIV campaign and 0.9 pix/mm for second PIV campaign). The symbols represent mean augmentations averaged within $0.5 \text{ m} < x_a < 0.6 \text{ m}$ (see also figures 4.48 and 4.49) where both horizontal and vertical error bars show minimum/maximum variations in mean augmentation and roughness Reynolds numbers, respectively. It is visible, that the IR data points fall in between the different predictions of which the one after Seidman come closest.

To put the current data into context of data from literature, figure 4.57 shows the same plot as before with additional experimental data from Hill *et al.* [101] for $Ma = 8.1$ flow along sand grain roughness, Young [294] for $Ma = 4.9$ flow along v-grooves, and Holden [104] for High-Enthalpy flow along sand grain and sphere roughness. All shown data were extracted from Bowersox [32]. Additionally, data gathered during the ESA ExoMars campaign along square bar roughness with the same topology as the current technical roughness model ($\lambda/k = 4$) in a flat plate model at $Ma_e = 4.2$ $Re = 8.5 \cdot 10^6$ 1/m (FCII, red symbols), $Ma_e = 4.2$ $Re = 23.5 \cdot 10^6$ 1/m (FCIII, green symbols) and $Ma_e = 5.3$ $Re = 17.5 \cdot 10^6$ 1/m (FCV, red symbols) are included (see e.g. [167]). All prediction methods were calculated for the four different flow conditions and are indicated by corresponding line colors in the plot.

In case of the ExoMars data (blue, green and red symbols) two sets of symbols are shown. Here, the filled symbols represent values if a (incompressible) literature value of $k_s/k = 6$ were to be used. This was the initial assumption within the project, to use a conservative EQSR value from previous incompressible investigations. No attempt was made to extract the value directly from measurements. The hollow symbols represent roughness Reynolds numbers if a value of $k_s/k = 1$ were to be used, being closer to values in this thesis and according to Berg [24]. Based on these lower EQSR, ExoMars and current data correspond closer to each other and form, together with other literature data, a broad value band, closely around values calculated with the

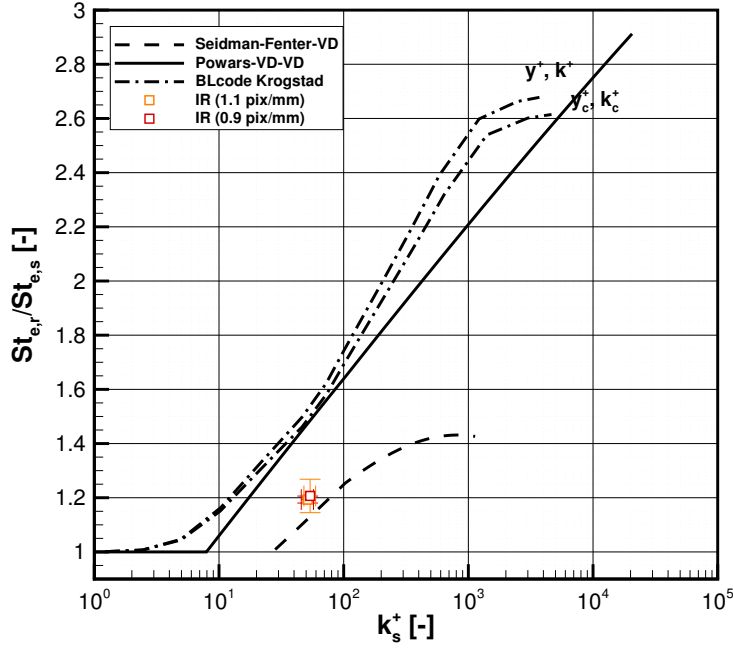


Figure 4.56: Rough wall heat flux augmentation from computations (lines) and current data (symbols).

Seidman correlation but below the widely used Powars correlation. Also, numerical predictions by the BLcode are higher than experimental data, as was discussed before. So it seems that analytical and numerical models tend to over-predict the heating augmentations, except the model according to Seidman [223].

Although the current data points align with heritage data, it needs to be mentioned again that in case of the current data ($k_s^+ \approx 50 - 60$) as well as Berg's data ($k_s^+ = 44$) the roughness Reynolds numbers are slightly below the fully rough limit for sand grain roughness, typically given with $k_s^+ = 60 - 70$. Therefore, transitional roughness effects cannot be excluded completely and the derived k_s/k values, or k_s^+ values via eq. (3.74) for that matter, need to be taken with some caution. However, current values with $k^+ \approx 30 - 40$ and $k_s^+ \approx 50 - 60$ are above the fully rough limit according to previous work by Bandyopadhyay [19] on a similar transverse bar roughness topology in incompressible flow with $k^+ \approx 10$ and $k_s^+ \approx 30$.

Additionally, as discussed in section 2, there have been suggestions for compressible flow to scale not only the roughness function (intrinsically performed via the velocity scaling to an effective velocity ΔU_{eff}^+) but also the roughness Reynolds number k^+ . Modesti *et al.* [162] argued that Berg might erroneously have incorporated compressible effects into the derivation of k_s/k and they suggested that a proper compressible transformation of the roughness height k is necessary before an EQSR is extracted. He suggested transformations via Trettel and Larsson [256], Volpiani *et al.* [269] or an alternative own formulation (see eq. (2.14)) to account for density and viscosity differences between the trough and crest of the roughness. An effort was made to apply this scaling on data of this thesis. But due to the resolution of the IR camera, there is only

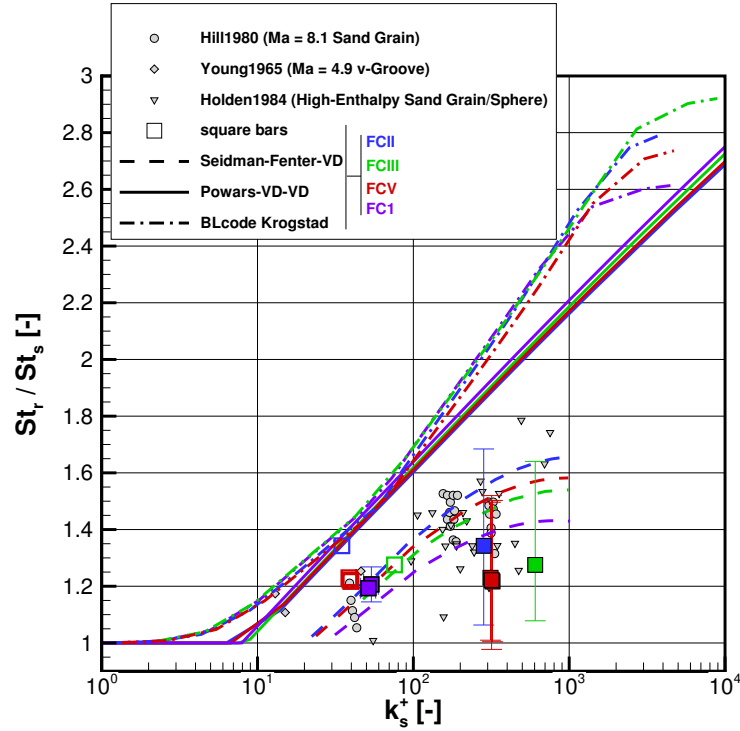


Figure 4.57: Rough wall heat flux augmentations from computations (lines) and experimental data of previous and current investigations (symbols).

one mean wall temperature value, which include information from both the channels (troughs) and crests of the roughness topology. Also, there is only one wall pressure value taken close the roughness topology. This inhibits the extraction of locally resolved density and viscosity at the trough and crest of the roughness. A way to approximate values at a wall distance of k is to use a velocity-temperature relation, like the one by Crocco-Busemann, fed with information of locally resolved PIV velocity profile together with the mean wall temperature from IR. This procedure was performed exemplary with eq. (2.14) on data of Run3006 with an IW $48 \times 48 \text{ pix}^2$ which resulted in $k_{eff}^+ = 36.42$ and therefore with a factor of 0.96 smaller than the value extracted without scaling by Modesti *et al.* (see table 4.3). This would change the EQSR to $k_s/k = 1.4$ instead of $k_s/k = 1.35$ and is therefore not substantial under current conditions.

In contrast to the heat flux rebuilding, it was seen that the BLcode was able to rebuild u_τ or k_s^+ values from PIV closely (see figure 4.20 and 4.23) when EQSR values were used which were in good agreement with values extracted directly from PIV data via eq. (3.74). Several potential influencing factors can be discussed in the following.

On the measurement side, potential reasons for the bad agreement could be connected to either uncertainties for the measured EQSR values or uncertainties for the measured heating augmentations $St_{e,r}/St_{e,s}$.

Addressing the first, even if the very conservatively estimated uncertainties of $U_{k_s/k} = \pm 50\%$

(see section 3.12.2) would be applied to the mean of the extracted EQSR range $k_s/k = 1.1$ to 1.4 , the resulting $k_s/k = 1.25 \pm 0.63$ and therefore at minimum $k_s/k = 0.63$ would still be larger than the $k_s/k \approx 0.25$ necessary for rebuilding. Also, if the full reported spread due to uncertainty of the shift of origin and ultimately from the skin friction uncertainties of the integral approach were accepted (see section 4.2.2), it would result in $k_s/k = 0.4$ to 1.9 and therefore above the rebuilding value.

The second aspect is the uncertainty for the measured heating augmentations which were estimated to $U_{St_{e,r,avg.}/St_{e,s,avg.}} \approx 3.6\%$ (see section 7.1.1). This translates into an uncertainty for the rebuilding EQSR of $k_s/k \approx 0.26$ to 0.36 and is therefore still below the experimental mean value.

Apart from that, there might still be inherent uncertainties for heat flux measurements which were not fully captured. As outlined in this thesis, efforts have been made to carefully consider possible influences along the complete measurement and post-processing chain, but one might argue e.g. that 3D effects, like lateral heating, might influence the 1D post-processing of IR data via the pyHeatflow tool. But, as was discussed for the CT sensors within PEEK material (see section 7.6.7), a positive lateral heat flux typically results in an over-prediction of 1D heating rates which would make the difference between experiments and rebuilding even worse. Instead, negative lateral heating scenarios would lead to an under-prediction of 1D heating but it is hard to imagine scenarios where only negative lateral heatings along the complete roughness topology were unrecognized. Apart from that, measurements with two different resolutions showed an overlap of averaged heating augmentations (see e.g. figure 4.53) due to the effective averaging of the IR technique. This was also supported by data of previous measurements in H2K within the ExoMars campaigns [168].

Additionally, the aforementioned practice of re-scaling 3D heat fluxes with the wetted-to-planform area ratio according to [27] would result in a decrease of heat fluxes and therefore lead to even worse agreement to correlations.. Lastly, if instead of IR, augmentations from local sensors were considered, agreements to correlations would also be worse since they were lower than from IR and also affected by larger uncertainties.

To summarize experimental uncertainties, bringing together the lower worst case scenario from EQSR extraction uncertainty with $k_s/k = 0.4$ and the upper worst case scenario from heating augmentation measurement uncertainties with $k_s/k = 0.36$, would at least bring predictions and observations closer to each other. Nevertheless, the observation that the experimental data align with other heritage data within a band of approx. $\pm 20\%$ (see figure 4.57), would hint more towards modeling uncertainties, which will be discussed next.

On the computational side, there are some modeling uncertainties and insufficiencies already known which have been discussed (see e.g. [32]) with different suggested adaptations (see e.g. [176], [17]). Some aspects to discuss with the focus on the current rebuilding are the following. The strategy of using a BLcode with its approximations might be insufficient, especially in case of rough wall modeling with larger roughness effects which can protrude into the conical flow field above the TBL. Additionally, a two-layer algebraic turbulence model might be insufficient, since it ignores any anisotropic effects due to the Boussinesq approximation with its scalar eddy viscosity. On top of that, the approximations of turbulent heating via constant Prandtl number might be insufficient to capture relevant physics. This was already discussed by Wörz [291]

for incompressible cases of ribbed cooling passages within turbine blades and might also be applicable here.

There are different possible steps to leverage computations. Firstly, a full CFD solver instead of the BLcode would be not restricted to the TBL but also include possible couplings with the outer (conical) flow field. E.g. there are existing roughness models implemented in TAU but none is adapted to compressible flow to the knowledge of the author. Additionally, this step would enable the inclusion of anisotropic Reynolds stresses via RSM models. But, all of the above methods in TAU still typically use a constant turbulent Prandtl number assumption to calculate turbulent heat fluxes. Instead, models are available to calculate varying turbulent Prandtl numbers (see e.g. [291], [17]). Several variable turbulent Prandtl number models have been tested in the BLcode but it had only minor impacts on the results (see section 7.6.6 figure 7.80).

Instead of modeling the roughness topology via a sand-grain roughness approach, there are other options, like so-called Direct Element Methods [249]. Although there are models for compressible flow [75], there are challenges in applying this method on TSB roughness used in this thesis. Another option are direct simulation approaches, in which the actual roughness topology is resolved by the grid, like [115] for an incompressible rough-wall channel flow calculated with a RANS solver. But here, questions persist with respect to the ability of RANS solvers to accurately resolve flow detachments and Reynolds stresses. The ultimate computational approach would be to directly simulate the roughness topology in scale resolving methods like LES or DNS. Although already common practice for some flow regimes (incompressible low to moderate Reynolds number flow, like e.g. [143]), there have been only limited efforts towards high-speed flows in the past (e.g. [22], [162]).

Another point to discuss is that a single scalar parameter like k_s^+ might be not sufficient to capture all physical aspects along a rough wall. This was already discussed based on insufficiencies to capture “only” friction augmentations on different roughness topologies (see e.g. [118], [45]). But this might be even more relevant for heating predictions, if the friction and heating augmentations have different driving physical mechanisms, as outlined in section 2.

One final remark to the discussion on figure 4.57: it is common practice e.g. by NASA for the creation of aerothermal databases (ATDB) to use skin friction velocities derived from smooth wall CFD computations $u_{\tau,s}$ to calculate corresponding $k_{s,s}^+$ and to extract augmentations based on these Reynolds numbers (see e.g. [280]). This introduces a bias towards lower roughness Reynolds numbers at the same augmentation levels. This might lead to a better agreement with e.g. the Powars correlation but is based on non-physical arguments. In case of data in this thesis, such a rescaling would have no large effect and all four cases end up within $30 \lesssim k_s^+ \lesssim 50$. Therefore, corresponding augmentation levels would still fall somewhere between the Powars and Seidman correlation.

As final comparison of data of this thesis, combining both, friction and heating, augmentation data (from figure 4.24 and 4.57) to a rough wall Reynolds analogy efficiency, current data result in $(St/St_s)/(c_f/c_{f,s}) \approx 1.19/1.37 \approx 0.87$ at MS1. Together with the frontal solidity $\lambda_f = 1/\lambda = 0.25$ we can add current data to the plot in figure 4.58 according to [45]. It includes incompressible data from DNS along transverse circular rods and square bars with roughness Reynolds number $k_t^+ \approx 40 - 150$ according to Leonardi *et al.* [147] and along irregular roughness with $k_{rms}^+ \approx 16 - 31$ according to Forooghi *et al.* [79]. DNS data show minima in the range $0.1 < \lambda_f < 0.3$, which

coincide with corresponding maxima in drag. Instead, the current compressible data point lies above the incompressible data, where error bars represent data spread combined from both, PIV and IR data.

Potential reasons for the difference might be e.g. compressibility or transitional roughness effects. As was mentioned before (see e.g. section 4.2.2), data of this thesis might still be influenced by viscous effects in the transitional rough regime, whereas incompressible data are shown for the fully rough regime. Apart from these differences to incompressible data, the compressible correlation after Nestler *et al.* [172] (see eq. (3.68) in section 3.11) results in $(St/St_s)/(c_f/c_{f,s}) \approx 0.88$, based on smooth wall results of Run3014 (see also figure 4.58). This is in close agreement and therefore supports the results of this thesis.

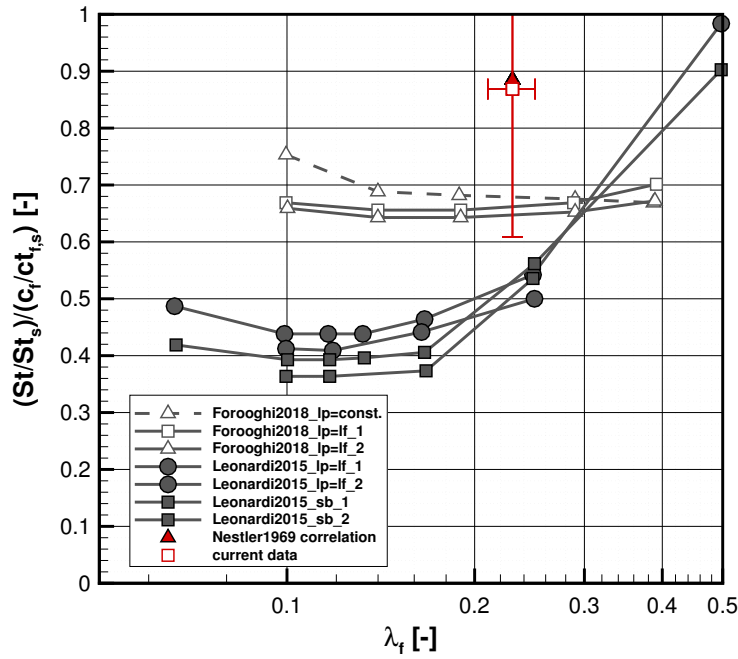


Figure 4.58: Rough Reynolds analogy efficiency $(St/St_s)/(c_f/c_{f,s})$ from DNS data for transverse circular rods and square bars with roughness Reynolds number $k_t^+ \approx 40 - 150$ according to [147] and irregular roughness with $k_{rms}^+ \approx 16 - 31$ according to [79]. Plot and data extracted after [45].

5 Conclusions

In this thesis, measurements along a 7° sharp cone at $Ma_\infty \approx 6$ and $Re_\infty = 16 \cdot 10^6$ 1/m with and without roughness were discussed. The rough wall consisted of two-dimensional transverse square bars with $\lambda/k \approx 4$, $k/\delta \approx 0.1$. Selected measurements were also performed along Carborundum sand-grain type roughness. The corresponding Reynolds numbers varied between $Re_\theta = 2770$ to 6689 , $Re_{\delta_2} = 793$ to 1288 and $Re_\tau = 212$ to 384 . Complementary techniques were used to measure turbulent boundary layer velocity profiles and heat flux augmentations along the cone. For the first, Pitot pressure and PIV measurements were performed. For the heat flux, IR thermography and local sensors were utilized. Additionally, PIV measurements enabled cauterization of turbulent stresses and large scale coherent structures. All measurement techniques and corresponding post-processing have been thoroughly checked and characterized. With this setup the following key aspects were found.

PIV and Pitot pressure measurements were used to extract mean velocity profiles with the assumption of a Crocco-Busemann-temperature model. Initial analysis focused on smooth wall experiments as reference. Both measurement techniques agree well within their uncertainty bands. The mean velocity profiles agree favorably with numerical data of a boundary layer code and CFD. The data was also in excellent agreement to DNS profiles of a flat plate flow at comparable Mach number and slightly higher Reynolds numbers [56], [296]. Three different approaches were applied to extract skin friction velocities to enable inner scaling. First, a fitting approach was used based on the incompressible law-of-the-wall. Furthermore, an integral approach was used which utilized two subsequent profiles and the corresponding loss of momentum. Finally, highly resolved PIV velocity data enabled a gradient approach which approximated the velocity gradient close to the wall. All smooth wall values derived by the different approaches correspond within their uncertainties to each other and to data from analytical, BLcode and CFD computations. The skin friction velocity was used to analyze the velocity profiles in inner and outer scaling according to AGARD suggestions [73]. Applying the van Driest transformation, compressible velocity profiles collapse with the incompressible law-of-the-wall. As in several previous investigations, this result shows further support for the applicability of this scaling in case of current boundary conditions. Also in outer scaling, data collapse in the range of validity with corresponding theories. The kinetic shape factor at the first measurement section lies with $H_k \approx 1.5$ well within the range for a fully turbulent profile after natural transition along the cone [73]. Both, outer scaling and shape factor, are biased in case of the Pitot profiles due to their lack of resolution close to the wall. The same effects could be rebuild with artificially cut theoretical profiles. The resulting Coles wake parameters correspond well to previous investigations and are in a value range expected from fully turbulent profiles in the corresponding Reynolds number range.

In the flow field along the transverse square bar (TSB) roughness topology, only negligible streamwise mean velocity gradients from PIV as well as Schlieren gradients could be observed. A potential reason for the absence of any disturbances penetrating the TBL is the observation

that roughness crests are submerged within the subsonic portion of the TBL (see figure 4.4c). This skimming behavior was previously used as argument for a “d-type” roughness [185], [118] although it can be categorized as “k-type” roughness based on the geometrical topology.

The rough wall velocity profile fitting poses challenges, already recognized by multiple previous investigators. In case of this thesis, it was challenging mainly due to the considerable influence of the shift of origin, most likely due to the k/δ value range of the current data. Methods from literature, typically used to estimate the shift of origin, were largely inconclusive and made a different approach necessary. A combination of the integral and fitting approach was used to bound the shift of origin and at the same time quantify the friction velocity for the rough wall profiles. The combined approach resulted in inner scaled profiles with the characteristic roughness shift ΔU_{eff}^+ in the order of 6 with roughness Reynolds numbers in a range of $k^+ \approx 30 - 40$. Tests along the Carborundum sand-grain type roughness segment showed comparable values, which confirms the design of the model for reaching a comparable roughness effect. At the same time, the agreement gives confidence in the extracted EQSR value for the technical roughness. Although small, the resulting TSB roughness Reynolds numbers k^+ are in a range sufficient for the fully rough flow regime based on incompressible tests by Bandyopadhyay [19]. The resulting EQSR Reynolds numbers with $k_s^+ = 38 - 59$ fall slightly below the fully rough value of $k_s^+ = 60 - 70$ for sand grain roughness, but are above the limit for transverse bar roughness by Bandyopadhyay [19] with $k_s^+ \approx 30$ ($k^+ \approx 10$). If assumed fully rough, EQSR values of $k_s/k = 1.1$ to 1.4 can be extracted. A suggested compressible scaling by Modesti *et al.* [162] was tested on the current results but showed only minor differences for the extracted EQSR. When results of this thesis are scaled and plotted as assumed to be valid for either d-type or k-type roughness, the data show neither of the assumed behaviors of both types. However, since such scaled data agree well to experimental results of other (compressible) studies, questions arise to whether these scalings or concepts are applicable to (compressible) data. In contrast to the smooth wall values, Coles wake parameters were larger along the rough wall, comparable to previous investigations [135], [186], [38]. Rough wall rebuildings via BLcode, utilized with a modified Krogstad roughness model, show the same trend as experimental values for k_s^+ , u_τ and $c_{f,e}$. For these friction augmentation rebuildings, EQSR values of $k_s/k \approx 1 - 2$ were necessary as input to the BLcode, which are in agreement to results from fitting Pitot and PIV data directly.

The smooth wall turbulence profiles in outer Morkovin scaled form agreed to previous experimental data and artificially filtered DNS in case of the streamwise component, if the corresponding resolution of the PIV data for potential sub-grid filtering is considered. Profiles closely agreed to TAU RSM calculations in the upper parts of the boundary layer, if typical turbulence inflow intensities of the H2K wind tunnel between $Tu = 1 - 2\%$ were used as input. This is also in agreement with estimates from PIV random error estimates via a method suggested by Scharnowski *et al.* [211]. The wall-normal component, together with the Reynolds shear component, showed the largest underpredictions compared to DNS data, similar to previous hypersonic TBL investigations [285], [289]. Both components also fall short of artificially filtered DNS data, hinting at other relevant contributions besides sub-grid filtering like particle slip effects [285] or PIV resolution effects [15]. Different scalings of the turbulent stress profiles were tested, due to on-going debates about the most suitable one [287]. Outer Morkovin scaling showed best agreement with DNS for the outer layer, whereas inner mixed scaling agreed best in the vicinity of the buffer layer peak. Overall, the profiles of this thesis are comparable to data

from incompressible boundary layers, providing further experimental evidence that Morkovin scaling is valid for the streamwise turbulence at the tested Mach number.

Turbulence profiles above the smooth and rough wall agree to each other within measurement accuracies, supporting the outer layer similarity argument after Townsend [255], despite the rather large values of k/δ and observed differences of Coles wake parameter in the mean velocity profiles.

The empirical diagnostic plot, typically used to bypass challenges in experimentally finding the skin friction velocity and shift of origin, was successfully applied to the current data. Despite the compressibility of the profiles, it showed good agreement of both smooth and rough wall data with empirical predictions, when the roughness scaling according to Castro *et al.* [39] was used. Since this scaling utilizes the roughness function, extracted in this thesis by the fitting approach, it indirectly gives further confidence in the used procedures and results.

Two-point correlation results of the streamwise velocity fluctuation component showed that the general TBL structure along the smooth wall is comparable to incompressible and compressible data of previous investigations. Data of this thesis support the notion from previous comparable investigations that correlation lengths are only moderately decreasing with Mach number. Also, structure angles show little or no change with Mach number within scatter of the data. Both of the above observations support previous conclusions [287], [288], that correlation lengths and structure angles are only moderately affected by Mach number, in contrast to data which relied on hot-wire measurements, influenced by density. The largest length-scales and structure angles in the outer layer were observed to be similar above the smooth and rough wall within the scatter of the data. This would further support the assumption by Townsend, that the outer flow is not drastically affected by roughness. Only in close vicinity to the rough wall, length-scales decrease below the smooth wall data, comparable to results from other investigators.

Previous evidence for the existence of uniform momentum zones (UMZ) along smooth wall hypersonic turbulent boundary layers, attributed to the general concept of hairpin vortex structures, could be confirmed in this study. To the knowledge of the author, this experimental evidence could be extended to hypersonic TBL along rough walls for the first time. Statistical analysis of the mean number of zones throughout the TBL showed similar values compared to previous investigations and only negligible differences between both wall conditions. Due to the assumed connection to hairpin vortex structures, this would further support the notion that outer layer flow is not drastically affected by the investigated roughness.

All the above observations support the outer layer similarity argument, despite the rather large values of k/δ and low Reynolds numbers in this work. It is important to know the applicability and limits for this argument, since it is a basic assumption on which most roughness modeling rely. A possible explanation for the observation that Coles wake parameters don't agree but turbulence data do collapse in the outer layer was given by Castro *et al.* [39] with potential transitional roughness effects, which cannot be excluded here completely due to moderate roughness Reynolds numbers. To clarify potential impacts of compressibility or roughness regimes and due to on-going debates about the applicability of the outer layer similarity (e.g. [227], [183], [56], [296]), more data is necessary.

Smooth wall heat flux distributions extracted from IR data were in good agreement with numerical data within their corresponding uncertainties, if a proper scaling via the edge based Stanton

number St_e was used. Although the cone represents a generic shape, CFD results differ by $\approx 9\%$ from each other, depending on the applied turbulence model. This underlines the necessity for accurate data to verify computational methods like RANS. Independent measurements of heat flux and indirect assessment of skin friction data enabled a consistency check via the Reynolds analogy and comparison to different approximations from literature. The experimental data agreed with all applied correlations but no clear statement could be made which of the theoretical Reynolds analogy factors is more applicable due to the level of combined uncertainty. Local sensor data showed different heating rates compared to IR, which was expected due to different materials. However, for the following analysis it was assumed that roughness heating augmentations were less impacted, since it was derived for each measurement technique separately.

Rough wall heating augmentations from IR data showed consistent results with approx. 20 % above the smooth wall level, independent of camera resolutions. For augmentations from local sensors slightly lower rates were observed. HFM data augmentations overlap with IR data within the scatter of the data, whereas for CT data this is not the case with resulting augmentations of approx. 10 – 15 %. Potential reasons are relaxation effects of the boundary layer approaching a rough-to-smooth transition, due to the used strategy of implementing the larger sensors into the smaller roughness topology via local smooth wall plateaus. To investigate this argument, mean velocity profiles from Pitot data above a smooth wall plateau were compared to fully rough data. These velocity profiles also show minor differences in inner scaled form whereas direct pressure profiles do overlap. Another potential reason for lower CT signals might be attributed to observed lateral heating rates.

Heating augmentations along the Carborundum sand grain type roughness could only be investigated by local sensor data. Here, heating augmentations were only above one (i.e. above smooth wall values), if the sensor plateaus were fully covered by Carborundum (including sensors). In this case, comparable levels like along the technical roughness of approx. 10 – 15 % could be observed.

To rebuild the IR Stanton number augmentations via the modified Krogstad roughness model, an EQSR of $k_s/k \approx 0.25$ was necessary as input to the BLcode. This is much smaller compared to values necessary to rebuild friction augmentations. Variable turbulent Prandtl number models, typically used to account for observed lower heating augmentations, showed no improvements when implemented in the BLcode. Also, the widespread used Powars correlation showed higher augmentations than experimental data. Only one analytical model via Seidman-Fenter-Nestler showed surprisingly good agreement, despite its derivation and adaptation from simple incompressible arguments. The drawback of the analytical Seidman model like for any analytical models are tunable model constants, which makes it prone to uncertainties.

To summarize the findings, in the following the research questions posed in section 1 are addressed. The intended goal to enable measurements of EQSR and heating augmentation in one experimental setup could be accomplished. Particle Image Velocimetry (PIV) was successfully applied to study the large scale structure of the TBL and the effects of roughness. The goal to enable outer layer similarity detections was accomplished and multiple indications existed that it holds under the current setup so that it should not interfere with typical assumptions for roughness models. The targeted uncertainty analysis for the extraction of EQSR was performed on selected literature cases and for the setup of this thesis. In general, with $U_{k_s/k} \approx 5 - 20\%$ partly substantial uncertainties need to be accepted, even in incompressible experiments. In case of compressible

studies, uncertainties are even larger with $U_{k_s/k} \approx 20 - 50$ %. But this uncertainty range still does not cover the observed scatter in reported values (over 70 % based on a mean of $k_s/k = 3.5$ for a range of $k_s/k = 1 - 6$, see e.g. figure 2.8). With $k_s/k = 1.25 \pm 0.63$ data of this thesis is comparable to other compressible data according to Berg [24] and hint at the lower end of the incompressible EQSR spectrum, reported in the literature. Combining worst case scenarios from estimated uncertainties of EQSR extraction and heating augmentations would bring observations closer to numerical predictions applying the same models as used during the ExoMars campaign. As a consequence, there are indications that EQSR might be influenced by compressibility effects, but it is not completely conclusive. Further data is necessary, also since transitional roughness effects cannot be neglected completely.

As an outlook and suggestion for future investigations, in the following several final remarks are listed. It would be highly desirable to test the same smooth and rough wall model together with the same proper measurement approach for a range of driving parameters, like Ma , Re , k_s^+ , k/δ , etc. Some attempts have already been performed in the past. E.g. testing the same roughness at different low to moderate Mach numbers (see [195], [139], [33]) or testing in incompressible flow at different Re , k_s^+ , k/δ (see e.g. [240]), but bringing together all parts in case of high-speed, highly compressible flow, including heating augmentations, has not yet been performed. There are still open points regarding potential compressibility effects for k_s/k and k_s^+ (see e.g. [32], [258], [162]) and although k_s/k data from the current thesis together with other compressible values (e.g. [24]) lean towards the lower end of the spectrum for this particular roughness topology there is a necessity to test the sensitivity to the complete parameter range.

One of the major challenges in ground-based hypersonic flow testing is simultaneously enabling high Reynolds number turbulent boundary layers (δ^+ , k_s^+) with low blockage ratios (k/δ) along generic models like plates or cones (see e.g. figure 3.2) to fulfill parameter guidelines as suggested by Jiménez [118]. This was previously only achieved when one test section wall was used as model, due to much larger running length of the approaching boundary layer and therefore achievable boundary layer thickness δ . One of the disadvantages of this set-up are so-called “history effects”, i.e. potential influences that can emanate from the entire passage upstream of the test section (e.g. in the nozzle with corresponding pressure gradients) and its effects on the state of the boundary layer (see e.g. critical discussion in [73]).

Efforts should be made to further reduce measurement uncertainties, which would further reduce corresponding uncertainties for derived quantities, like roughness function and EQSR. For PIV measurements, seeding should be optimized for spatial density, homogeneous distribution in space and time and to reduce potential slip effects. This would increase the acceptable count of images per run and therefore statistics of resulting mean and fluctuating velocities. For IR measurements, the resolution along the small scale roughness patterns could be increased to add the capability of estimating potential 3D conduction effects.

Local sensors should be better integrated into the roughness patterns than the smooth wall plateaus used in this thesis. Previous studies along sand grain and technical roughness showed potential to integrate sensors which came at the price of substantial efforts (see e.g. [104], [105], [106]). In our approach, there would be additional challenges due to the used low-conductive material PEEK, which, together with potential glue, would complicate post-processing and increase uncertainties for deduced heat flux rates.

Furthermore, state-of-the-art manufacturing techniques like 3D printing could be used to apply different types of technical roughness directly on a PEEK model. This was already evaluated during the course of the ExoMars project but was discarded at that time due to insufficient resolutions. Due to on-going advancements, better resolutions are possible to be manufactured today and have already been tested in recent investigations (see e.g. [280]).

As a final remark, due to the experiences gained during the course of this work, the general approach to extract EQSR via fitting inner scaled velocity profiles to the theoretical law-of-the-wall is put forward for discussion. A new combined approach, based on profiles at two different positions, was suggested in this thesis which somewhat relaxed the procedure, but still brought burden to the user and made the procedure very delicate and intricate (see the above reported resulting uncertainties). Rebuildings of heritage incompressible data in the framework of the uncertainty estimations (see section 7.3.2) showed that these challenges of the procedure are not limited to compressible data, for which additional steps like velocity transformations are necessary. During the course of this thesis, efforts to rebuild reported values like shift of origin or EQSR from published data of different authors were not successful. Although this effort might be influenced by inaccuracies from scanning data to digital formats or differences in the exact procedures for post-processing, it nevertheless underlined how delicate the procedure is. It could be informative to collect data from multiple different sources and try to streamline post-processing with a consistent approach to enable better comparability and consolidate huge data scatter. An attempt to at least collect data was initiated by the University of Southampton [237]. Additionally, if it would be possible to substantially lower uncertainties for the small valued wall-normal velocity fluctuations in hypersonic TBL measurements and therefore enhance detections of Reynolds shear stress profiles (see e.g. figure 4.25 and [287], [15]), additional methods would be available. For example methods to exploit the full momentum equation based on two profiles [36], [35], [91] or only one profile [159] could be used to extract u_τ based on Reynolds shear stress measurements. The optimum would be a direct measurement of the skin friction and recent developments in the field of MEMS (Micro-Electro-Mechanical Systems) could be an option for the hypersonic TBL research [81], [174].

The experimental data in this work was collected at low Reynolds numbers, which makes it accessible for DNS calculations. The applied experimental set-up with its capabilities and uncertainties was characterized and reported based on existing data. It would be interesting to have further comparison and insights based on data from DNS and therefore being able to extract further physical understanding of hypersonic turbulent boundary layers above smooth and rough walls.

6 Bibliography

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7 Appendix

7.1 Additional General Investigations

Test Matrix:

Table 7.1 contains detailed information for every reported run in this thesis. The reservoir conditions of the conducted runs typically differ by less than approx. 1% from the nominal values. Only runs with sufficient quality with respect to inflow conditions and boundary layer measurements were chosen and included in the analysis. The table contains the actual inflow Mach number Ma_∞ , Reynolds number Re_∞ , reservoir pressure $p_{0,\infty}$ and temperature $T_{0,\infty}$, static pressure p_∞ and temperature T_∞ and velocity U_∞ . Additionally, the streamwise location for boundary layer profile measurements is given with measurement section MS1 or MS2 for runs of corresponding measurement techniques (MT). Both sections are defined in table 3.3 with the axial and the surface running length x_a and x , respectively, for both applied measurement techniques. The numbering of runs is grouped into three blocks with a leading one thousand for the first PIV campaign, two thousand for the Pitot and three thousand for the second PIV campaign.

7.1.1 Measurement Uncertainties

In this section measurement uncertainties of all relevant parameters are summarized. First, general parameters of the wind tunnel inflow are listed. Then, details about pressure and heat flux uncertainties are given. Finally, velocity and skin friction velocity uncertainties are reported, which, in contrast to the other uncertainties, differed between the campaigns. If not otherwise noted, for combined uncertainties linear propagation of uncertainties is assumed. This uncertainty value represents a maximum level considering the most unfavorable and improbable case when all independent influencing variables reach their minimum or maximum uncertainty value.

General:

The uncertainties of the inflow conditions are based on calibrations of the corresponding instrumentation and can be given with $U_{p_0} = \pm 0.1\%$ (full scale of $p_0 = 70$ bar), $U_{T_0} = \pm 1.1^\circ\text{C}$ or $\pm 0.4\%^\circ\text{C}$ (whichever is higher) and $U_{Ma} = \pm 0.04$.

Pressure:

In case of the surface and Pitot pressure measurements, uncertainties were derived by calibration of the corresponding ESP modules (see chapter 3.2). For the wall pressure, calibration of a 5 PSI module (345 mbar) resulted in a maximal uncertainty of $U_{p_w} = \pm 0.28$ mbar. For the Pitot pressure, calibration of a 15 PSI module (1034 mbar) up to max. pressures of 900 mbar resulted

Table 7.1: Test matrix (s: smooth wall, r: rough wall, s(m): metallic smooth wall, r(SG): sand grain type Carborundum rough wall).

Run No.	Ma_∞ [-]	Re_∞ [10^6 l/m]	$p_{0,\infty}$ [bar]	$T_{0,\infty}$ [m/s]	p_∞ [bar]	T_∞ [K]	U_∞ [m/s]	MS [-]	MT [-]	wall [-]
1003	6.06	15.7	20.0	500	0.0119	60	940	1	PIV	s
1004	6.06	15.6	19.9	500	0.0118	60	940	1	PIV	s
1005	6.06	15.8	19.9	495	0.0118	59	935	1	PIV	s
1006	6.06	16.0	20.1	494	0.0120	59	934	1	PIV	s
1007	6.06	15.7	20.1	500	0.0120	60	941	1	PIV	s
1008	6.06	15.8	20.0	497	0.0119	59	937	1	PIV	s
1009	6.06	15.9	20.0	494	0.0119	59	935	2	PIV	s
1010	6.06	15.6	20.0	501	0.0119	60	942	2	PIV	s
1011	6.06	15.7	19.9	498	0.0119	60	938	2	PIV	s
1012	6.06	15.7	19.8	497	0.0118	60	937	1	PIV	r
1013	6.06	15.7	19.9	499	0.0119	60	940	1	PIV	r
1014	6.06	15.3	20.1	511	0.0120	61	951	1	PIV	r
1015	6.06	15.6	20.1	501	0.0119	60	942	1	PIV	r
1016	6.06	15.1	20.0	511	0.0119	61	951	2	PIV	r
2013	6.06	15.9	19.5	486	0.0116	58	927	2	Pitot	s
2023	6.06	16.0	19.0	475	0.0113	57	917	2	Pitot	s
2024	6.06	16.0	19.0	477	0.0113	57	918	2	Pitot	s
2025	6.06	16.0	18.7	471	0.0111	56	913	2	Pitot	s
2032	6.06	16.0	19.0	475	0.0113	57	917	1	Pitot	s
2033	6.06	15.8	18.8	477	0.0112	57	918	1	Pitot	s
2039	6.06	16.0	18.9	475	0.0113	57	916	1	Pitot	s
2052	6.06	15.9	18.8	475	0.0112	57	916	1	Pitot	s
2072	6.06	15.9	19.1	479	0.0113	57	920	1	Pitot	r
2073	6.06	15.7	18.9	481	0.0113	58	922	1	Pitot	r
2074	6.06	15.9	19.0	478	0.0113	57	919	1	Pitot	r
2085	6.06	16.2	18.6	466	0.0110	56	907	1	Pitot	r
2086	6.06	16.2	18.7	466	0.0111	56	908	1	Pitot	r
2087	6.06	15.7	18.7	476	0.0111	57	917	1	Pitot	r
2091	6.06	15.8	18.8	477	0.0112	57	918	1	Pitot	r
2092	6.06	15.8	18.7	474	0.0111	57	915	2	Pitot	r
2093	6.06	16.0	18.8	474	0.0112	57	915	2	Pitot	r
2094	6.06	15.8	18.7	476	0.0111	57	917	2	Pitot	r
2095	6.06	15.7	18.7	476	0.0111	57	918	2	Pitot	r
2116	6.06	15.7	18.7	469	0.0111	56	911	2	Pitot	s(m)
2117	6.06	15.7	18.6	463	0.0111	55	905	2	Pitot	s(m)
2118	6.06	15.7	18.5	460	0.0110	55	902	1	Pitot	r(SG)
2119	6.06	15.7	18.7	453	0.0111	54	895	1	Pitot	r(SG)
2120	6.06	15.7	18.6	468	0.0111	56	910	1	Pitot	r(SG)
3006	6.06	15.6	20.0	501	0.0119	60	942	1	PIV	r
3007	6.06	15.8	20.2	500	0.0120	60	941	1	PIV	r
3008	6.06	15.2	19.9	507	0.0118	61	947	1	PIV	r
3009	6.06	15.8	18.9	479	0.0112	57	920	1	PIV	r
3010	6.06	15.4	19.8	502	0.0118	60	942	1	PIV	r
3014	6.06	15.5	20.1	504	0.0119	60	945	1	PIV	s
3015	6.06	15.6	18.8	481	0.0112	58	922	1	PIV	s
3016	6.06	15.4	20.2	508	0.0120	61	948	1	PIV	s
3019	6.06	15.5	20.1	506	0.0120	61	946	2	PIV	s
3020	6.06	15.7	20.1	500	0.0119	60	940	2	PIV	r
3023	6.06	15.9	18.9	477	0.0112	57	918	2	PIV	r

in a maximal uncertainty of $U_{p_{\text{Pitot}}} = \pm 1.12 \text{ mbar}$. Due to the necessity of c_p normalization for the combination of multi-run Pitot profiles, the final Pitot pressure uncertainty increased to $U_{p_{\text{Pitot}}} = \pm 15.82 \text{ mbar}$ due to the propagation of inflow parameter uncertainties.

Heat flux:

The uncertainty of the IR measurement according to the manufacturer as input to the heat flux calculation according to chapter 3.4.2 was $U_{T_w} = \pm 1 \text{ K}$. The uncertainties of the absolute convective heat flux via IR thermography were derived from calculations using the numerical method described in chapter 3.4.2 with perturbed input parameters within their uncertainty limits (e.g. the temperature dependent material parameters of PEEK within $U_{c_p, \text{PEEK}} = \pm 2.5 \%$, $U_{\rho, \text{PEEK}} = \pm 1.0 \%$ and $U_{\lambda, \text{PEEK}} = \pm 5.0 \%$). To determine the maximum absolute uncertainty of the Stanton number, a linear propagation of uncertainties was applied. The influencing parameter to consider for the Stanton number were the convective heat flux $\dot{q}$, the upstream density ρ_∞ and velocity V_∞ , which are calculated with p_0 , T_0 and gas dynamic equations, and the wall temperature T_w . The uncertainties of these parameters were defined by the accuracy level of the corresponding sensors. For the applied flow conditions and test environment this resulted in a final uncertainty of $U_{\dot{q}} = \pm 5.1 \%$ and $U_{St} = \pm 9.9 \%$. Single run heating augmentation uncertainties are then $U_{St_{e,r}/St_{e,s}} \approx 19.8 \%$. Due to ensemble averaging for the augmentation extraction, the corresponding uncertainties are relaxed to $U_{St_{e,r,avg}/St_{e,s,avg}} \approx 3.3 \%$ and 3.6% for the first and second PIV campaign, respectively.

In case of the local heat flux microsensors (HFM), the manufacturer Vatel states a measurement uncertainty of 5% , combining repeatability and calibration accuracy. In contrast, several past wind tunnel tests showed an increased uncertainty of up to $U_{\dot{q}} = \pm 10 \%$ (see [253]). Therefore, this more conservative uncertainty was also used in this thesis. In case of the CT sensors, an uncertainty of up to $U_{\dot{q}} = \pm 10 \%$ was assumed, due to prior experiences (see [253]). For both local sensor types this resulted in $U_{St} = \pm 16 \%$.

Velocities:

The relevant uncertainties for the flow velocities derived by PIV differed slightly between the campaigns due to changes in the corresponding hardware (see section 3.3). Also the velocity uncertainties derived from the Pitot pressure are reported.

First PIV campaign:

In case of flow velocities derived by PIV, uncertainties resulting from typical set-up and analysis, lens distortion effects, laser pulse timing influences and calibration image were included with the general procedure aligned to [141]. A typical set-up uncertainty (su) is approximated by $U_{su} = \pm 0.1 \text{ pix}$, as usually given in the literature e.g. [275]. The lens distortion influence was approximated with analysis of synthetic images via synPIV (see chapter 7.5.3). For this purpose nominal synthetic images were artificially modified via pincushion distortion (di) to result in a displacement of approx. 10 pix close to the image corners, described as typical values for the used lens in [123]. This result in a particle displacement difference of $U_{di} = \pm 0.2 \text{ pix}$. Furthermore, the laser pulse timing influence (lp) was based on the standard deviation derived by the laser characterization $S_{lp} = \pm 1.1 \text{ ns}$ described in section 7.5.1. This resulted in $U_{lp} = K \cdot S_{lp} = \pm 2.2 \text{ ns}$ for a 95% confidence interval. Additionally, a typical reading error of $\pm 5 \text{ pix}$ for the calibration images was used, which resulted in a magnification uncertainty (ma) of $U_{ma} = \pm 0.625 \text{ pix/mm}$ for

Table 7.2: PIV velocity uncertainties (second PIV campaign)

parameter	nominal	uncertainty	comment
l	25 mm	0.01 mm	from PIV calibration
L	2560 pix	1.5 pix	from PIV calibration
Δt	250 ns	1.5 ns	from diode
dx	26 pix	0.1 pix	general setup
U (averaging)	0 – 926 m/s	7.4 m/s	using value after [23] at profile position of where $\overline{u'u'}$ is maximum
U (E0.95)	0 – 926 m/s	9.9 m/s (IW 512x16 pix ²), 8.1 m/s (IW 64x64 pix ²)	95% CI

our set-up. The uncertainties for averaging over the available instantaneous velocity vectors were based on the work of [23]. The total uncertainty resulted in $U_U = 22.85$ m/s or $U_U = \pm 2.47\%$ based on the reference Taylor-Maccoll edge velocity at FCI of $U = 926$ m/s. Although the laser pulse deviation seems to be quite small, with $U_{lp} = \pm 1.8\%$ the laser pulse uncertainty has the biggest impact on the total uncertainty U_U , which is a great challenge in hypersonic high velocity flows.

In case of the the detailed analysis of viscous sublayer velocities (see e.g. figure 4.10) an additional uncertainty of $U_{U_{vs}} \approx 0.5\%$ was added due to the effects of non-constant wall temperature during the wind tunnel runs. Outside this region this effect was neglected (see chapter 7.5.2).

Second PIV campaign:

In case of the second campaign, uncertainties resulting from typical set-up and analysis, calibration quality and laser pulse timing influences were included with the general procedure aligned to [141] (lens distortions were covered by the corresponding optical calibration procedure). Again, a typical set-up uncertainty is approximated by $U_{su} = \pm 0.1$ pix, as usually given in the literature e.g. [198]. The calibration quality resulted from the rms deviation of calibration points from DaVis to $U_{cal} = 1.5$ pix. Furthermore, the laser pulse timing influence was estimated to $U_{lp} = \pm 1.5$ ns based on the readings from the diode measurements. The uncertainties for averaging over the available instantaneous velocity vectors were based on the work of [23]. The total estimated uncertainty with 95% confidence interval, derived by propagation of all above given deviations, resulted in $U_U = 9.9$ m/s or $U_U = \pm 1.1\%$ based on the reference edge velocity. A representative breakdown and summary of the relevant uncertainties is given in table 7.2.

Pitot campaign:

The uncertainty of the velocity derived from Pitot pressure was estimated at two different wall-normal positions based on linear propagation. Uncertainties for Pitot and wall pressure as well as wall temperature were included. The first point was located at $y = 0.57$ mm ($y^+ = 35$) in the log-layer and one outside the TBL at $y = 6.8$ mm ($y^+ = 414$). A difference of $\Delta p_{Pitot} = 15.82$ mbar translated into $\Delta Ma = 2.56\%$ and 0.9% or $\Delta U = 2.25\%$ and 0.25% for the lower and upper wall-normal position, respectively.

Velocity fluctuations:

In both PIV campaigns, the uncertainties for the turbulent fluctuations $U_{u'u'}$ and $U_{v'v'}$ were based on the work of [23]. Three different approaches were applied and compared. Two approaches were based on statistical assumptions for either normally or arbitrarily distributed fluctuations for all images. The third approach applied a re-sampling algorithm via the bootstrap method. The resulting uncertainties are directly given in the corresponding graphs in section 4. The uncertainties of the values in scaled form were derived by linear propagation and are stated in the corresponding chapters.

Skin friction velocity:

In the following the uncertainties of skin friction velocity u_τ are listed, derived by the different applied methods i.e. the fitting approach (see chapter 3.10.2), the gradient approach (see chapter 3.10.3), the integral approach (see chapter 3.10.4).

To determine u_τ and Π via the fitting approach, the equations need to be solved iteratively. In case of the integral approach, differential equations need to be integrated numerically. Therefore, classical uncertainty propagation methods, based on Taylor series expansion (TSM), are not applicable. Therefore, in case of both approaches, the uncertainties were determined by a Monte-Carlo method (MCM) (see e.g. [48]). This method samples errors drawn from assumed distributions for every input parameter (measurement) and simulates running the complete analysis process chain many times, each time with a different set of sampled errors. The advantage of MCM is that it is not restricted to analytical functions in the process chain and to small input uncertainties, since no linearization like in case of TSM is necessary. The challenge for MCM is that the distribution for each input uncertainty and eventual inter-dependencies between different input uncertainties need to be known and considered (see [48]).

Fitting approach, first PIV campaign:

For the fitting approach, additionally to u_τ also the uncertainty of the second fitting parameter, the Coles wake parameter Π , is reported. In case of the PIV data, input parameters were the reservoir temperature T_0 and pressure p_0 , inflow Mach number Ma_∞ , wall temperature T_w , wall pressure p_w (equivalent to edge pressure by assuming constant pressure throughout the boundary layer), the wall-normal distance y and mean velocities U . These parameters were intentionally biased by their corresponding uncertainties (according to the previous paragraphs) and seeded for $N = 8000$ samples applying a uniform distribution. This is a conservative approach, since no statistical information is given e.g. in the corresponding sensor data sheets. For the wall temperature T_w instead of the IR camera specification an increased uncertainty of $U_{T_w} = \pm 5$ K was used to cover the differences in mean wall temperatures from run to run and run to numerical data. For the wall-normal distance y , in case of PIV the error was estimated to ± 4 pix, resulting in our set-up to $U_y = \pm 0.02998$ mm. The resulting values of the Monte-Carlo analysis can have arbitrary distributions. The representative uncertainty of each resulting value is extracted to represent a 95.4% confidence interval (CI) of the distribution, equivalent to a 2σ uncertainty, if the data would be normally distributed (which is the case for selected parameters). This resulted in $U_{u_\tau} = \pm 3.4\%$ and $U_\Pi = \pm 10.2\%$ based on mean values for run 1007 of $u_\tau = 53.6$ m/s and $\Pi = 0.28$, respectively. To show convergence of the MCM, the number of samples were stepwise reduced to 4000, 2000, 1000 which changed the mean of u_τ by only 0.02%. The resulting 95.4% uncertainty of the skin friction velocity changed by 0.8%, showing a clear convergence with only

Table 7.3: Input to MCM uncertainty estimation for fitting approach for smooth wall Run3014 and IW 512x16 pix² (rec.: rectangular distribution, norm.: normal distribution)

parameter	uncertainty	comment
Ma_∞	0.04	from nozzle calibration, rec.
p_0	7000 Pa	0.4% FS, rec.
T_0	2 K	0.1% FS, rec.
T_w	5 K	rec.
p_w	28 Pa	rec.
y	1.3e-02 mm	from DaVis calibration, rec.
U	5.1 m/s	1 standard deviation, norm.

a 0.07% change between 4000 and 8000. Therefore, 8000 samples were assumed to be sufficient to extract reliable statistics from the data.

Fitting approach, Pitot campaign:

In case of the Pitot data, the input parameters were the same as for the PIV data, except the velocity is exchanged by the Pitot pressure p_{Pitot} and its corresponding uncertainty together with a different $U_y = \pm 0.05$ mm, based on reference measurements [52]. The same MCM approach with $N = 8000$ samples and equally distributed input uncertainties lead for a 95.4% CI to $U_{u_\tau} = \pm 0.4\%$ and $U_\Pi = \pm 11.5\%$ based on mean values for Pitot runs 2032&2033 of $u_\tau = 52.9$ m/s and $\Pi = 0.18$, respectively.

Fitting approach, second PIV campaign:

For the second PIV campaign, most of the above stated input uncertainties apply as well, so only the differences are mentioned in the following. Besides the different values for U_U as stated above, the uncertainty of wall-normal distances U_y was estimated from DaVis calibration, resulting in our set-up to $U_y = \pm 0.013$ mm. An exemplary table with the input is shown in table 7.3. MCM analysis resulted for IW 64x64 pix² in $U_{u_\tau} = \pm 1.4\%$ and $U_\Pi = \pm 4.6\%$ based on mean values for Run3014 of $u_\tau = 51.6$ m/s and $\Pi = 0.39$, respectively. For IW 512x16 pix² this resulted in $U_{u_\tau} = \pm 1.6\%$ and $U_\Pi = \pm 5.3\%$.

Gradient approach, first PIV campaign:

The maximum uncertainty of the gradient approach skin friction velocity is derived by linear propagation of uncertainties with the following influencing input parameter uncertainties for wall-distance U_y , edge pressure U_{p_e} , wall temperature U_{T_w} , wall density U_{ρ_w} , wall viscosity U_{μ_w} and velocity gradient close to the wall $U_{(\frac{\partial U}{\partial y})_w}$. This resulted in $U_{u_\tau} = \pm 9.1\%$.

Integral approach, first PIV campaign:

In both cases, Pitot and PIV data, the biased input parameters were the edge temperature T_e , velocity U_e and Mach number Ma_e , wall temperature T_w , boundary layer momentum thickness θ and distance in between the two input profiles Δx_a . The uncertainties of the wall and edge temperature, velocity, Mach number and momentum thickness were taken from the results of the Monte-Carlo uncertainty analysis of the corresponding fitting procedure. The distance

Table 7.4: Input to MC uncertainty estimation for integral approach for rough wall Run3014 and IW 512x16 pix² (rec.: rectangular distribution, norm.: normal distribution)

parameter	uncertainty	comment
U_e	5.1 m/s	1 standard deviation, norm.
T_e	4.8 K	1 standard deviation, norm.
θ	6.4e-3 mm	1 standard deviation, norm.
T_w	5 K	rec.
Δs	1.3e-02 mm	from DaVis calibration, rec.

was biased by uniformly distributed uncertainties for 8000 samples within their corresponding uncertainties.

In case of the PIV data the analysis resulted for a 95.4% confidence interval in $U_{u_\tau} = \pm 33.5\%$ based on mean values for run 1007 of $u_\tau = 51.2$ m/s. The uncertainty for the difference quotient approach resulted in $U_{u_\tau, DQ} = \pm 28.9\%$.

Integral approach, Pitot campaign:

In case of the Pitot data Monte-Carlo analysis resulted for a 95.4% confidence interval in $U_{u_\tau} = \pm 5.7\%$ and $U_{u_\tau, DQ} = \pm 4.6\%$ based on mean values for run 2032&2033 of $u_\tau = 51.4$ m/s. The higher uncertainty values in case of the PIV results can be attributed to higher input uncertainties (from the fitting procedure) of edge Mach number, temperature and momentum thickness with $U_{Ma_e} = \pm 6.5\%$, $U_{T_e} = \pm 41.7\%$ and $U_\theta = \pm 5.7\%$, respectively. In contrast, the same input values in case of the Pitot data are much smaller with $U_{Ma_e} = \pm 0.6\%$, $U_{T_e} = \pm 1.2\%$ and $U_\theta = \pm 3.9\%$.

Integral approach, second PIV campaign:

For the second PIV campaign, the input uncertainties changed slightly according to the hardware. An exemplary table with the input is shown in table 7.4. In case of the IW 64x64 pix² data the analysis resulted in $U_{u_\tau} = \pm 11\%$ based on mean values for Run3014 of $u_\tau = 48$ m/s. The uncertainty for the difference quotient approach resulted in $U_{u_\tau, DQ} = \pm 9.5\%$. In case of the IW 512x16 pix² data the same analysis resulted in $U_{u_\tau} = \pm 15\%$ based on mean values for Run3014 of $u_\tau = 47.8$ m/s. The uncertainty for the difference quotient approach resulted in $U_{u_\tau, DQ} = \pm 12.4\%$. An overview of uncertainties for the second PIV campaign is given in table 7.5.

A few additional comments to the MCM uncertainty analysis: The fitting approach MCM can be performed either with independent or dependent uncertainties for y and U (i.e. seeded uncertainties for each wall-normal position). Here, independent seeding typically leads to lower uncertainties, including U_{u_τ} , except for II. Typically, the more conservative values are reported.

To wrap up the skin friction velocity uncertainty analysis, the above stated values do include only procedural and no methodical uncertainties for the friction velocity. Methodical uncertainties can only be determined by comparing the results to a reference value of higher accuracy, either from experimental, numerical or analytical prediction. Most often, only cross-checks between different methods or comparison to analytical predictions are given in the literature. For the comparable Clauser chart method, which utilizes the velocity data and its slope only in the logarithmic layer, typical uncertainties of approximately 5% are given in the literature for incompressible

Table 7.5: u_τ and Π MCM uncertainty estimations for fitting and integral approach for smooth wall Run3014

avg $\pm$ 95%CI	u_τ	integral		Π
	fitting MS1	MS1	MS2	fitting MS1
iw64x64	51.6 ± 0.7	48.0 ± 5.3	47.0 ± 4.8	0.39 ± 0.02
iw512x16	51.8 ± 0.8	47.8 ± 6.9	46.9 ± 6.3	0.41 ± 0.02
95%CI [%]				
iw64x64	1.41	11.04	10.20	4.59
iw512x16	1.64	14.51	13.46	5.30

investigations (e.g. [197], [236], [274]). Squire *et al.* [240] states that the incompressible smooth wall Clauser fit approach can be conservatively estimated to have $< 8\%$ uncertainty and his own data shows deviations within $< 2.5\%$ in direct comparison to oil-film interferometry. For compressible investigations, the performance is typically stated within approximately 10% compared to analytical predictions like van Driest (e.g., [24], [236], [184], [289], [287]). This is in agreement with observations in this thesis.

A further check to approximate methodical uncertainties at the test conditions of this thesis can be made by applying the same analyses tools on computationally derived data from CFD. For this purpose, smooth wall data for the FCI at $T_w = 360$ K using the $k-\omega$ turbulence model were used. Keeping the wall-normal resolution as calculated by TAU, the fitting approach resulted in deviations of $\Delta u_\tau = 0.6\%$ compared to the friction derived by the solver. The integral method led to deviations of $\Delta u_\tau = -1.5\%$ and $\Delta u_{\tau,DQ} = -0.8\%$. All of the above values are within the reported methodical uncertainties in the literature. Further checks have been performed (e.g. wall-normal resolution) and are reported in section 7.2.1.

Wall temperature:

Due to the choice for PEEK as wind tunnel model with its low thermal conductivity, the wall temperature changed during a wind tunnel run by approximately 3–10% , corresponding to a viscosity change of approximately 2–9% .The corresponding impact on TBL velocity measurements were based on numerical simulations, described in [171]. For larger wall-normal distances, including the logarithmic layer, the difference is in the order of $\Delta U/U \approx 0.02\%$ and was neglected.

Particle slip:

The impact of particle lag is addressed in section 7.5.2 (see also [171]) for the current flow conditions. Possible deviations on the mean velocity up to 2% of U_e were approximated. For the velocity fluctuations Stokes numbers in the order of 0.1 to 1 were estimated.

Model:

Uncertainties due to model manufacturing uncertainties were already addressed in section 3.1.

Table 7.6: Coefficients for Antoine's equation (eq. (7.1)) [41]

	Oxygen	Nitrogen
Validity	$54.00K < T < 154.15K$	$54.15K < T < 125.72K$
Coefficient A_{vp}	$-3.4087E + 00$	$-4.5048E + 00$
Coefficient B_{vp}	$-1.0984E + 03$	$-9.7605E + 02$
Coefficient C_{vp}	$3.1801E + 01$	$3.6323E + 01$
Coefficient D_{vp}	$4.1906E - 05$	$8.3635E - 05$

7.1.2 Condensation Analysis

Despite checks by other authors, a thorough check of Berg's wind tunnel inflow conditions revealed that there was an increased risk for local condensation in the flow. According to Berg [24], the nominal inflow conditions were set to $p_0 = 15.7$ bar and $T_0 = 428$ K. Via 1D isentropic equations the resulting static pressure and temperature in the test section were calculated to $p_\infty = 0.009$ bar and $T_\infty = 51.2$ K, respectively. This state is included in figure 7.1 together with different lines representing critical conditions for either Nitrogen (N_2) or Oxygen (O_2) condensation. Limits for both constituents are included via the vapor pressure curves from the Chemical Engineering & Material Research Information Center (CHERIC) [41]. The curves were derived by an Antoine equation with four coefficients according to

$$\log(p_{vp}) = A_{vp} \log(T) + B_{vp}T + C_{vp} + D_{vp}T^2 \quad (7.1)$$

and the coefficients listed in table 7.6.

Additionally, in figure 7.1 an additional is included, representing a condensation boundary typically used for H2K tests, which combines the onset of condensation for different constituents. Additionally to the conditions of Berg, the test conditions within this thesis (RC2014: FC1, RC2015: FC2) and test conditions of Young [294] and Peltier *et al.* [182] are included in figure 7.1. It is visible that Berg's conditions are past the theoretical limit for Nitrogen condensation.

These vapor pressure curves are applicable if the flow conditions are in thermodynamic equilibrium. This is not necessarily the case for high Mach numbers in Laval nozzles, where gases can occur in a supersaturated state [175]. Here, the flow velocity counteracts droplet formation rates, which are usually required as condensation nuclei. Therefore, in general the vapor pressure curve is a conservative approach for the determination of the onset of condensation. In case of Berg's experiments, condensation might not have occurred but the risk was increased due to the choice of inflow parameters.

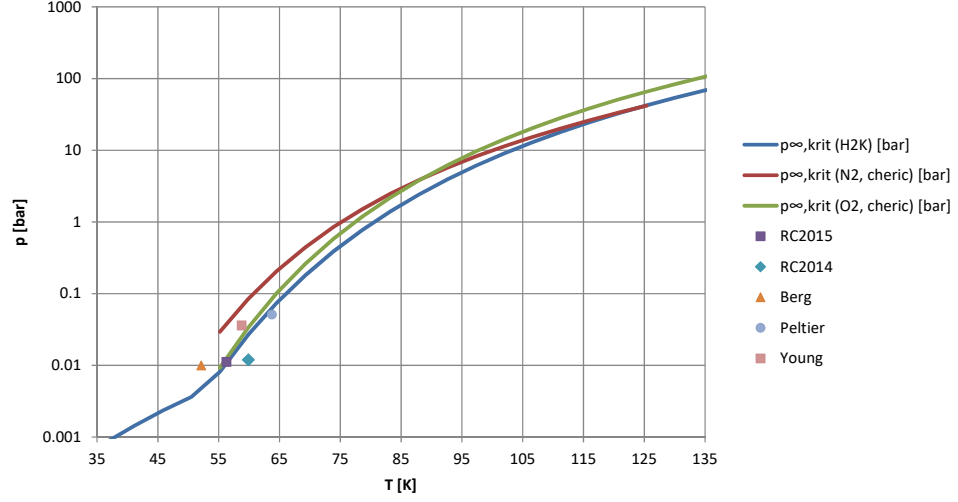


Figure 7.1: Condensation with data from [24], [182], [294]

7.1.3 Model Surface Scans

The model parts (smooth wall, technical roughness and Carborundum roughness) were measured via an optical chromatic distance measurement device (see section 3.1). The 20x20mm² measurement sectors were distributed at four circumferential positions at two streamwise positions, corresponding to FOV1 and 2.

Figure 7.2 shows an exemplary 20x20mm² measurement scan of the technical square bar roughness results.

Figure 7.3 shows an exemplary 20x20mm² measurement scan of the Carborundum sand grain type roughness results.

The conical scans were post-processed to consider the conical contour and after that statistically evaluated. Table 7.7 summarizes statistics of relevant parameters for the Carborundum sand grain type roughness according to Bons [28]. Here, the nomenclature according to Chung *et al.* [45] was used with additional information about corresponding names according to Bons [28] in brackets. The table contains the total number of available points N , the mean $\bar{y}$ [-], the median $\text{med}(y)$ and the standard deviation $\text{std}(y)$ of measured wall-distances y , the average roughness height k_a (Ra in [28]), root-mean-square roughness height k_{rms} (Rq in [28]), maximum peak-to-trough roughness height k_t (Rt in [28]), the mean of the peak-to-trough roughness height k_z (Rz in [28]), the skewness SK and the kurtosis KU .

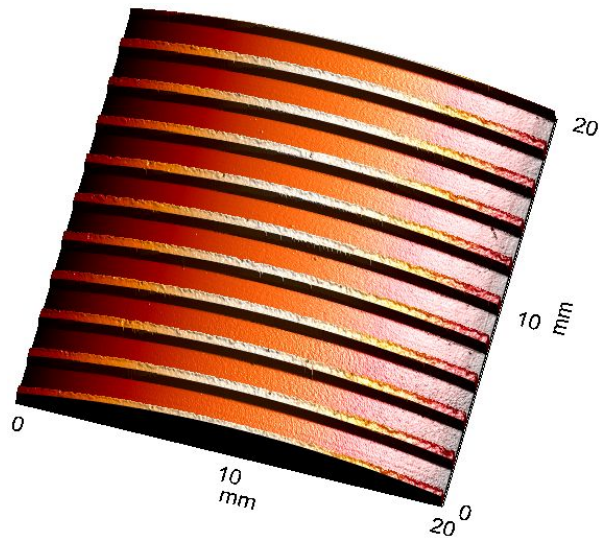


Figure 7.2: Exemplary 3D scan of square bar roughness

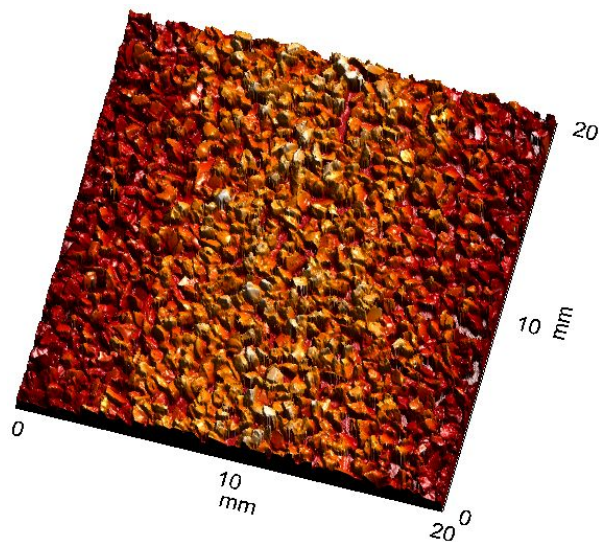


Figure 7.3: Exemplary 3D scan of sand grain type roughness

Table 7.7: Statistics of 3D surface scans of Carborundum sand grain type roughness.

Field	1	4	5	8
N [-]	117712	119429	113548	111750
$\bar{y}$ [-]	0.461	0.319	0.436	0.365
$\text{med}(y)$ [-]	0.469	0.349	0.444	0.385
$\text{std}(y)$ [-]	0.193	0.236	0.185	0.202
k_a [-]	0.461	0.319	0.436	0.365
k_{rms} [-]	0.500	0.397	0.473	0.417
k_t [-]	0.454	0.458	0.411	0.454
k_z [-]	1.160	1.017	1.005	0.938
SK [-]	3.078	4.690	3.336	4.171
KU [-]	4.904	9.023	5.434	7.506

7.2 Additional Skin Friction Velocity Investigations

7.2.1 Skin Friction Velocity Methods Validation

The different methods to derive skin friction velocities (see section 3.10.2 to 3.10.4) were validated against different data which is summarized in the following.

7.2.1.1 Fitting approach

The fitting approach was tested with heritage data and artificial velocity profiles. First, the fitting approach utilizing eq. (3.47) is addressed. After that, the second approach utilizing eq. (3.52) is analyzed.

First, eq. (3.47) was tested on smooth wall data ($\epsilon = 0\text{mm}$, $\Delta U^+ = 0$). Figure 7.4 shows results from the fitting approach on DNS data according to [296] (M6Tw076 condition, which is close to the condition tested in this thesis). The results of this procedure are shown in the different inner and outer scaling plots and list of parameter values given in the inset in the upper left corner. In general, all fitted data agree closely to the theoretical curves (see also description in section 4.2.2). The skin friction velocity $u_\tau = 45.37$ is very close to the value $u_\tau = 45.1$ from DNS. The resulting $\Pi = 0.36$ cannot be compared directly to input from DNS but a good agreement of the fit (red line) to the data (symbols) is visible in the inset in the inner scaled plot (upper right, which shows a zoomed view with reduced data points).

Second, artificial velocity data of representative smooth and rough wall profiles in the form $U = f(y)$ via eq. (3.53) were created for FC1 with input of $Ma = 6.06$, $p_0 = 20\text{bar}$, $T_0 = 500\text{K}$, $T_w = 360\text{K}$, $p_w = 2362\text{Pa}$, $\delta = 4\text{mm}$, $\epsilon = 0\text{mm}$ and $\Delta U^+ = 9$. The impact of Pitot and PIV resolution near the wall was estimated with artificially under resolved profiles from the artificial data. Figure 7.5 shows exemplary artificial data in inner and outer scaling according to [73] of a full profile in figure 7.5a and cut by five near wall points ($y^+(1) = 65$) in figure 7.5b. Table 7.5 contains a summary of this resolution sensitivity study. It is visible that the method is very stable, even for poor near wall resolution (tested up to $y^+(1) = 187$), since it takes data in the wake part into account.

Furthermore, the fitting approach was used on the $k = 1.27\text{mm}$ and $k = 1.65\text{mm}$ transverse square bar roughness data according to [288] at $Ma_e \approx 7.2$ and $Re_\tau = 715$. The results for $k = 1.27\text{mm}$ are visible in figure 7.6a with $u_\tau = 88.2\text{m/s}$, $\Pi = 0.53$ and $\Delta U^+ = 9.26$. The reported data was $u_\tau = 92\text{m/s}$, $\Pi = 0.38$ and $\Delta U^+ = 9.3$. It is visible that the velocity shift is well captured whereas the skin friction velocity and wake component values differ slightly. As a counter example in figure 7.6b it is visible that the reported values with $u_\tau = 105\text{m/s}$, $\Pi = 0.38$ and $\Delta U^+ = 11.4$ are not well captured by the fitting approach with $u_\tau = 71\text{m/s}$, $\Pi = 0.98$ and $\Delta U^+ = 8.5$ on the $k = 1.65\text{mm}$ roughness. One observation for the experimental data of the latter case is a non-monotonic and non-continuous slope of the profile, which complicates the analysis with any chosen method, so that a clear result is not expected.

An additional comment to the above comparisons to data by Williams. In both of the above cases the shift of origin ϵ was kept as reported by Williams *et al.* [288], based on a method by Perry

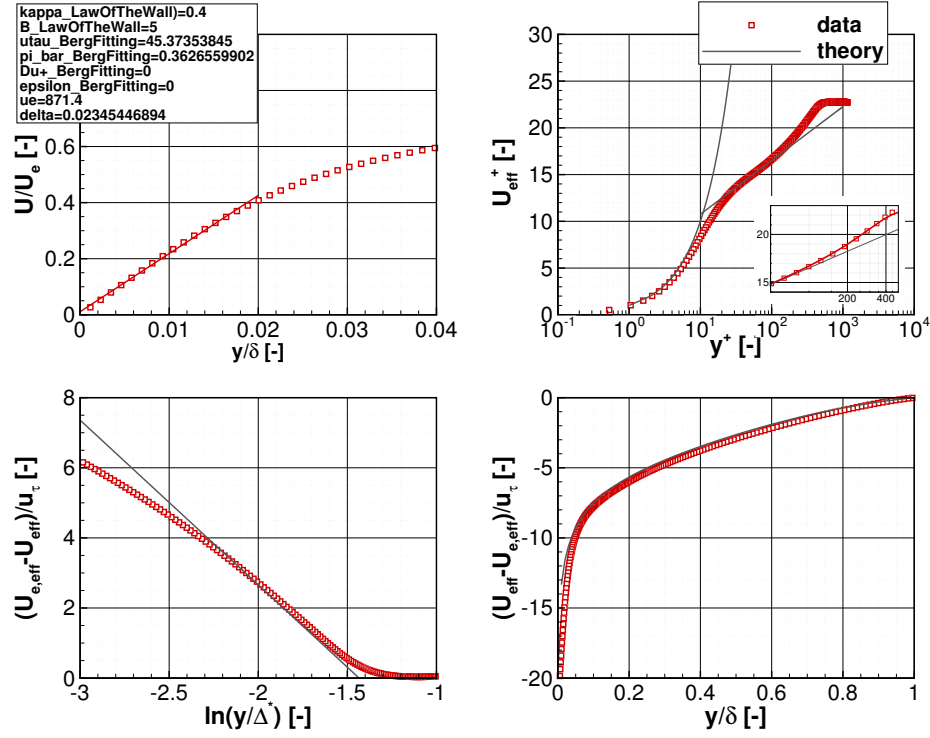


Figure 7.4: Test data in inner and outer scaling according to [73] with DNS data according to [296].

Table 7.8: Results of test of fitting approach along 7° cone at FC1 with changing simulated wall normal resolution according to PIV

$y^+(1)$ [-]	u_τ [m/s]	Π [-]	ΔU^+ [-]
input	74.49	0.80	9.00
25	74.17	0.80	8.94
34	74.45	0.80	8.99
41	74.18	0.80	8.94
49	74.18	0.80	8.94
57	74.15	0.80	8.94
65	74.01	0.80	8.90
110	75.05	0.81	9.20
148	74.52	0.81	9.06
187	74.33	0.81	9.01

and Joubert [185] (see section 7.2.2). It needs to be mentioned that with their reported negative values for ϵ , the resulting origin would lie above the roughness crests. Although not uncommon, this is in contrast to the acceptable value range of the majority of previous investigations, for which the origin is located between the floor and the crest of the roughness elements (see e.g.

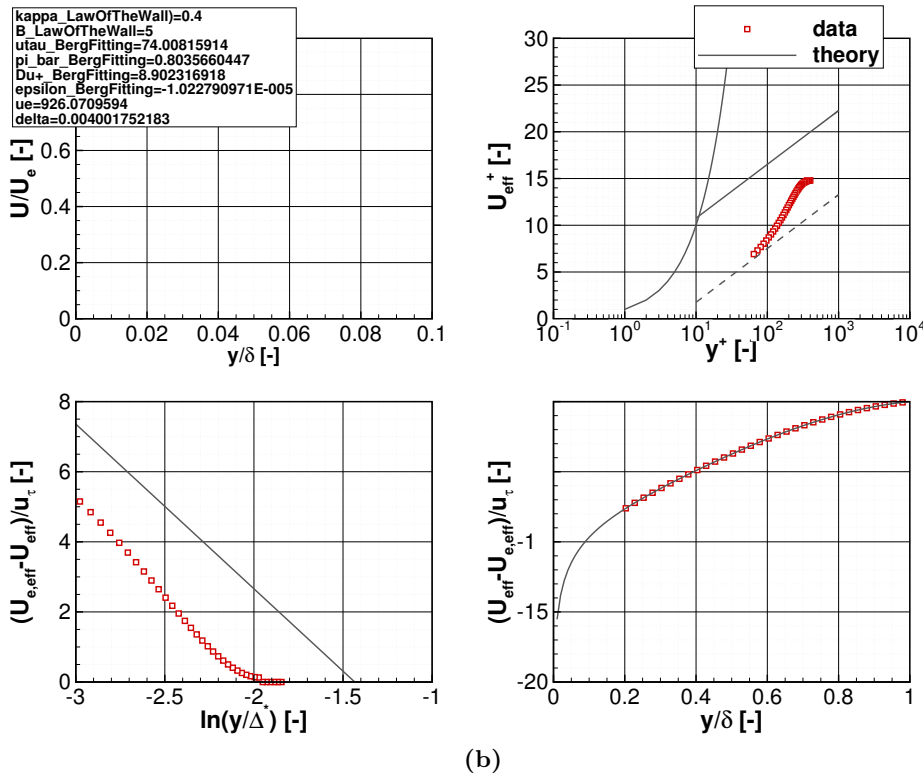
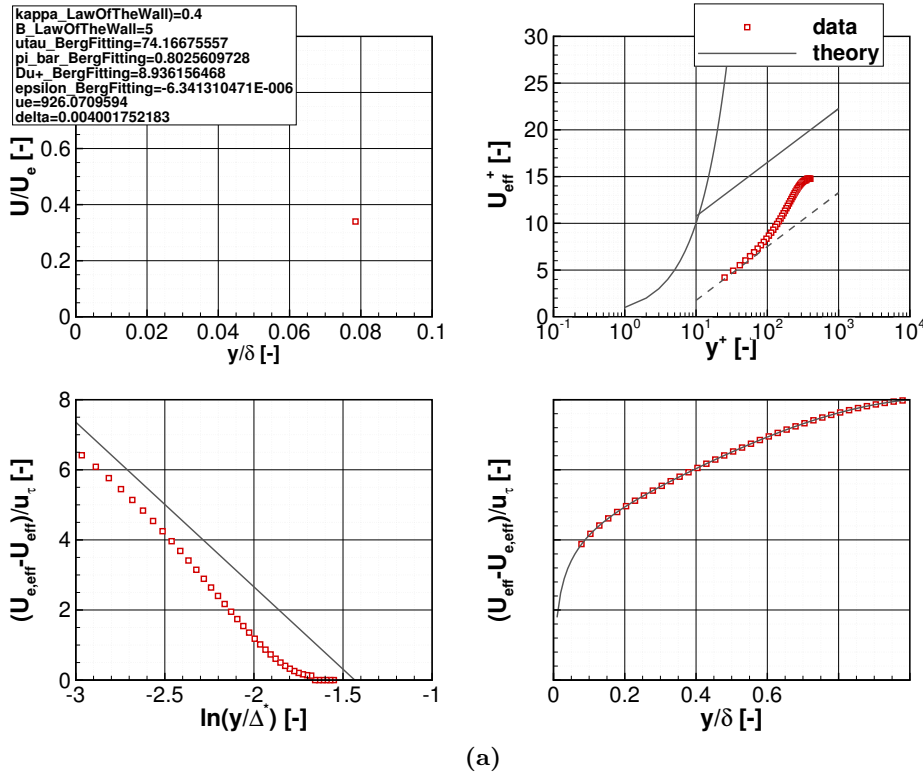


Figure 7.5: Streamwise mean velocity from artificial data in inner and outer scaling according to [73] with full profile in a) and cut by five near wall points in b).

Table 7.9: Validation results for the calculation of u_τ via (3.52) (*: fitted values).

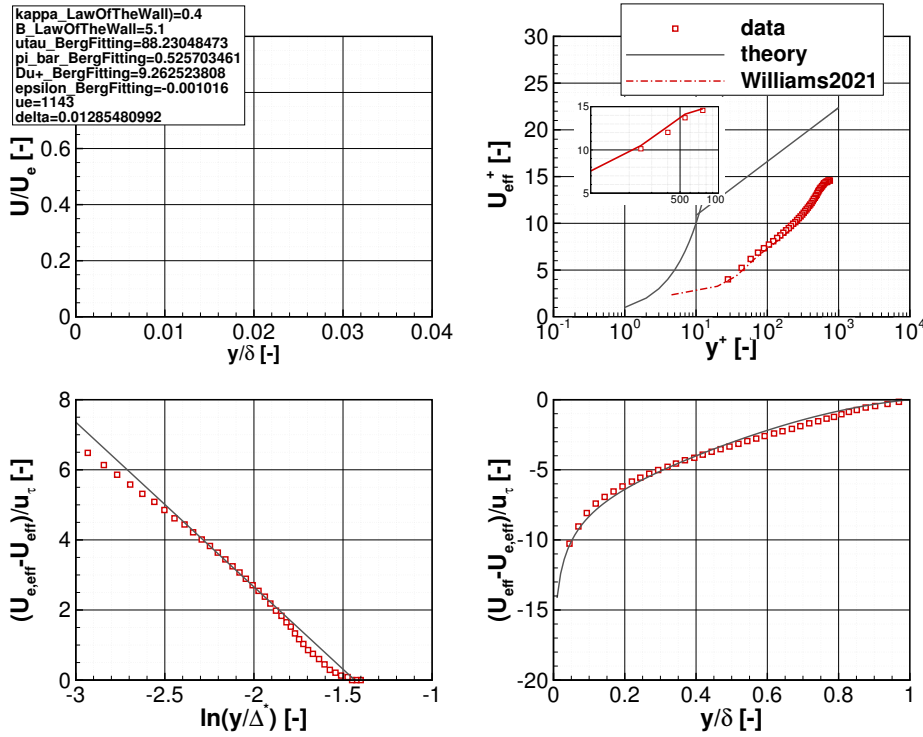
	δ^+	$U_{eff,e}^+$	Π	ΔU^+	u_τ	u_τ (3.52)
	[-]	[-]	[-]	[m/s]	[m/s]	[m/s]
artificial profiles						
smooth	226	20.6	0.4	0	52.3	
rough	322	14.4	0.8*	9	74.49	74.49
after [145]						
smooth	378	22.6	0.58*	0	0.0446	
rough			0.33*	9.86	0.0737	0.0766
after [7] Ma2.5						
smooth	361	19.9	0.13*	0	35.04	
rough	689		-	4.7	40.94	40.71
after [7] Ma5.0						
smooth	269	19.2	0.14*	0	80.77	
rough	500		-	2	84.03	83.63

[24], [25], [152]). An own assessment would probably end up in a different ϵ which would most likely change also the resulting roughness parameters. Unfortunately, Williams didn't report a second profile downstream from the first, so that the reported approach in this thesis cannot be applied directly.

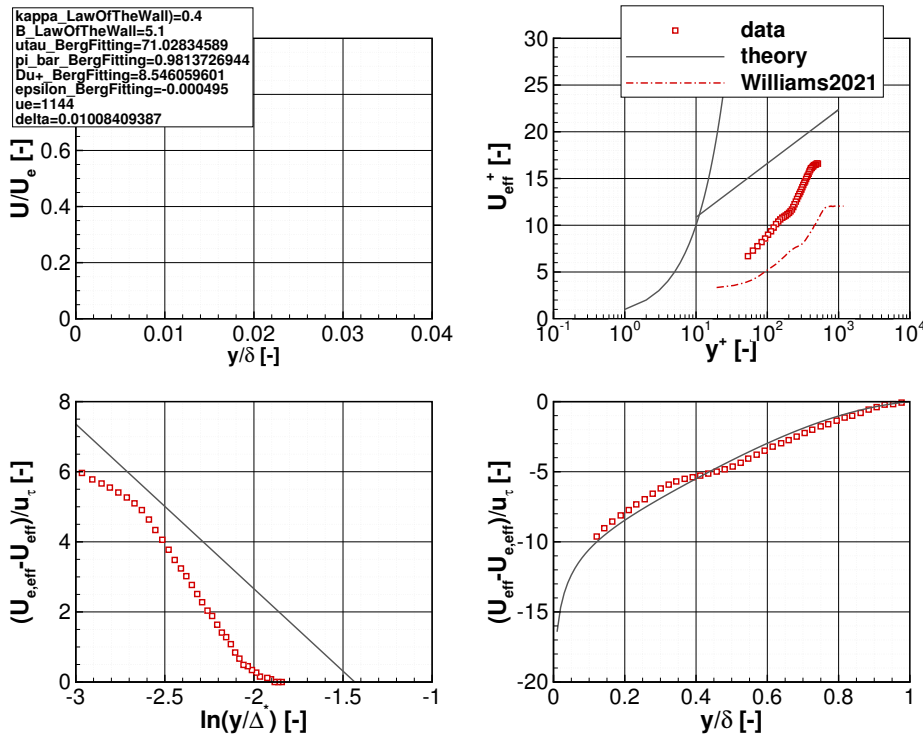
As a general comment, it needs to be mentioned, that the above observation, that some data can be rebuild with good agreement whereas others of the same author might not, was not an uncommon experience and should highlight how delicate the analysis is.

The second, new fitting approach utilizing the formulation for u_τ via eq. (3.52) was tested with three different sets of data. First, artificial velocity profiles were used to test eq. (3.52). Second, DNS data of incompressible TBL along square bar roughness was used, where all parameters can be independently derived from the data set [145]. Third, DNS data of compressible TBL along 3D block roughness was used [7]. Additionally, the complete fitting procedure, by applying (3.48) and (3.52), was tested on the first two. All results can be found in Table 7.9.

For the artificial velocity profiles, equation (2.7) was used to create representative profiles with input of $Ma = 6.06$, $p_0 = 20bar$, $T_0 = 500K$, $T_w = 360K$, $p_w = 2362Pa$, $\delta = 4mm$ and varying ΔU^+ (see section 3.10.2 for $\Delta U_{eff}^+ = 9$). Table 7.9 shows relevant parameters of the profiles and the Asterisk in the column for the wake parameter Π indicates that the value was fitted and was not independently given in the publication. The resulting u_τ from Eq. (3.52) is in excellent agreement to the input value. This is also the case for the incompressible TBL data along TSBR with $\lambda/k = 8$ according to [145] and compressible TBL along (non-staggered) block roughness ($k/L=0.16$, $k/W=0.16$) with $\lambda/k = 6.25$ according to [7].



(a)



(b)

Figure 7.6: Streamwise mean velocity from artificial data in inner and outer scaling according to [73] with a) $k = 1.27\text{mm}$ and b) $k = 1.65\text{mm}$ transverse square bar roughness profile according to [288].

7.2.1.2 Integral approach

Within the derivation of equations for the integral approach, it was assumed that $\delta/\theta = \text{const.}$, which was expected to be an acceptable approximation along a cone, apart from the region close to the nose. This is visible in Fig. 7.7 which shows BLcode calculations of smooth and rough 7° cone simulations at FC1, where the differences in the region of interest (MS1 to MS2 marked with orange rectangular) is below 1%.

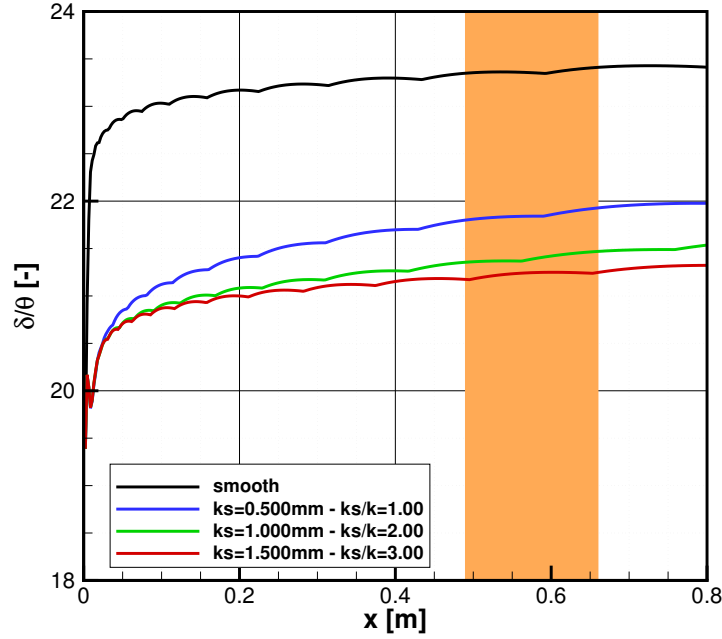


Figure 7.7: Ratio of boundary layer thickness to momentum loss thickness along smooth and rough 7° cone from BLcode at FC1

The integral approach was tested and validated with numerical data along a flat plate (second term in equation (3.57) is omitted) and cone derived by the codes described in chapter 3.9 and 3.8. Lines in figure 7.8 show results for the BLcode data along a 7° cone at FC1 with a smooth wall and three different EQSR values. Symbols represent the corresponding calculations via integral approach, which used the θ_1 at x_1 from the BLcode as input and stopped at x_2 , where x_1 and x_2 correspond to MS1 and MS2 as in the experimental setup. For these test cases the resulting percentage deviations of the skin friction coefficient is typically $|\Delta c_{f,e}| \leq 1.2\%$ (see table 7.10). Additionally heritage data from other investigators were used as test cases, which partly show larger deviations (see table 7.10).

The impact of PIV resolution was estimated with artificially under resolved profile data from CFD to result in typically $|\Delta c_{f,e}| \leq 6\%$ for IWs of 64 pix or less (see table 7.11).

Table 7.10: Results of validation cases of integral approach (j: 0 for 2D, 1 for axisymmetric; c: 0 for incompressible, 1 for compressible; r: 0 for smooth, 1 for rough; a: analytical, O: ordinary differential equation (ODE), D: differential quotient).

case	j	c	r	x ₁	Δx	θ_1	$\Delta \theta$	a	Mae	T _e	U _e	T _w	k _s	$ \Delta c_{f,a} $	$ \Delta c_{f,O} $	$ \Delta c_{f,D} $
[-]	[-]	[-]	[-]	[m]	[m]	[mm]	[mm]	[-]	[-]	[K]	[m/s]	[K]	[mm]	[%]	[%]	[%]
Bowersox-Latin rough model: 2D [138]	0	0	1	0	0.54	0.2	1.12	1	2.73	119	597	274	0.56	-0.58	-0.49	14.27
Bowersox-Latin rough model: 3D [138]	0	0	1	0	0.54	0.2	1.09	1	2.73	119	597	273	0.56	-0.23	-0.13	14.36
Bowersox-Latin rough model: 80grit [138]	0	0	1	0	0.54	0.2	0.91	1	2.73	119	597	273	0.53	-0.93	-0.83	11.6
Bowersox-Latin rough model: 20grit [138]	0	0	1	0	0.54	0.2	1.25	1	2.7	120	594	273	0.83	-0.2	-0.11	16.03
Bowersox-Latin smooth model [138]	0	0	0	0	0.54	0.2	0.6	1	2.75	118	598	276	0.00	-	29.33	39.76
Pimenta rough model: $U_{inf} = 130$ [188]	0	0	1	1.5748	0.3048	4.37	0.76	1	0	0	0	0	0.79	7.12	7.23	9.17
Pimenta rough model: $U_{inf} = 89$ [188]	0	0	1	0.6604	0.3048	2.16	0.79	1	0	0	0	0	0.79	-1.12	-1.02	2.51
Berg flat plate data: smooth [24]	0	1	0	0.69596	0.508	0.9	0.12	0.23	6.05	51	864	342	1.27	-	-33.78	-33.18
Berg flat plate data: S->R k=0.05in [24]	0	1	1	1.05156	0.1524	0.9	0.14	0.23	6.05	51	864	342	1.27	42.65	42.79	45.69
BLcode smooth flat plate data: FCII	0	1	0	0.15	0.1	0.18	0.09	0.41	4.2	110	885	340	0.00	-	1.53	5.23
BLcode smooth flat plate data: FCV	0	1	0	0.15	0.1	0.13	0.06	0.31	5.58	84	1028	340	0.00	-	1.16	4.82
BLcode rough flat plate data: FCII ks=0.5mm	0	1	1	0.15	0.1	0.27	0.12	0.41	4.2	119	919	355	0.50	1.11	1.11	5.93
BLcode rough flat plate data: FCII ks=1.0mm	0	1	1	0.15	0.1	0.29	0.14	0.41	4.2	119	919	355	1.00	0.5	0.5	5.59
BLcode rough flat plate data: FCV ks=0.5mm	0	1	1	0.15	0.1	0.2	0.09	0.32	5.23	103	1067	409	0.50	-0.01	-0.01	4.79
BLcode rough flat plate data: FCV ks=1.0mm	0	1	1	0.15	0.1	0.22	0.11	0.32	5.23	103	1067	409	1.00	0.14	0.14	5.5
TAU smooth flat plate data: FCII	0	1	0	0.16787	0.1	0.19	0.08	0.41	4.2	110	885	340	0.00	-	-0.62	2.85
BLcode Krogstad cone data: ks=0.5mm	1	1	1	0.4994	0.1298	0.27	0.05	0.3	5.35	74	925	340	0.50	-	0.24	2.44
BLcode Krogstad cone data: ks=1.0mm	1	1	1	0.4994	0.1298	0.31	0.06	0.3	5.35	74	925	340	1.00	-	0.03	2.47
BLcode Krogstad cone data: ks=1.5mm	1	1	1	0.4994	0.1298	0.32	0.07	0.3	5.35	74	925	340	1.50	-	-0.71	1.92
BLcode Krogstad cone data: smooth	1	1	0	0.4994	0.1298	0.18	0.04	0.29	5.38	73	926	340	0.00	-	0.44	2.11
TAU cone data: smooth	1	1	0	0.4994	0.1298	0.17	0.04	0.29	5.4	73	926	340	0.00	-	0.1	1.79
TAU cone data: smooth Tn0.02_muer354	1	1	0	0.4994	0.1298	0.18	0.04	0.29	5.4	73	926	340	0.00	-	-0.32	1.47
Keel cone data: Tw=0.35*Taw k=0.043in [128]	1	1	1	0.59663	0.53154	1.27	0.86	0.96	2.42	138	570	99	1.09	-	0.76	7.09
Keel cone data: Tw=Taw k=0.043in [128]	1	1	1	0.59663	0.40576	0.99	0.63	0.6	2.42	138	570	282	1.09	-	21.49	30.14
Keel cone data: Tw=0.35*Taw k=0.023in [128]	1	1	1	0.59663	0.58586	0.66	0.5	0.53	4.45	61	694	96	0.58	-	12.94	20.24
Keel cone data: Tw=Taw k=0.023in [128]	1	1	1	0.59663	0.29729	0.44	0.23	0.32	4.45	61	694	274	0.58	-	16.62	23.54

Table 7.11: Results of test of integral approach along 7° cone at FC1 with changing simulated wall normal resolution according to PIV

Δy [pix]	Δy [mm]	Δy^+ [-]	θ_1 [mm]	θ_2 [mm]	$c_{f1,ODE}$ [-]	$c_{f1,Fenter}$ [-]	$\Delta c_{f1,ODE}$ [%]	$\Delta c_{f1,Fenter}$ [%]	B' [-]	h_s^+ [-]	$U_{\tau,1}$ [m/s]	$ \Delta c_{f1,ODE} $ [%]	$ \Delta c_{f1,Fenter} $ [%]
0	0.000E+00	0	0.177	0.213	1.217E-03	1.214E-03	0.41	0.17	15.39	5.96	49.19	0.41	0.17
8	7.080E-05	4	0.172	0.207	1.188E-03	1.185E-03	-1.97	-2.19	15.75	5.30	48.61	1.97	2.19
16	1.416E-04	8	0.171	0.206	1.185E-03	1.182E-03	-2.18	-2.41	15.79	5.24	48.55	2.18	2.41
32	2.832E-04	17	0.168	0.203	1.175E-03	1.174E-03	-3.00	-3.14	15.93	4.82	48.37	3.00	3.14
64	5.664E-04	34	0.159	0.194	1.148E-03	1.185E-03	-5.29	-2.19	16.33	4.85	48.61	5.29	2.19
128	1.133E-03	67	0.141	0.176	1.082E-03	1.212E-03	-10.69	0.06	17.30	4.92	49.16	10.69	0.06
256	2.265E-03	134	0.104	0.139	9.555E-04	1.281E-03	-21.14	5.70	19.49	1.70	50.53	21.14	5.70

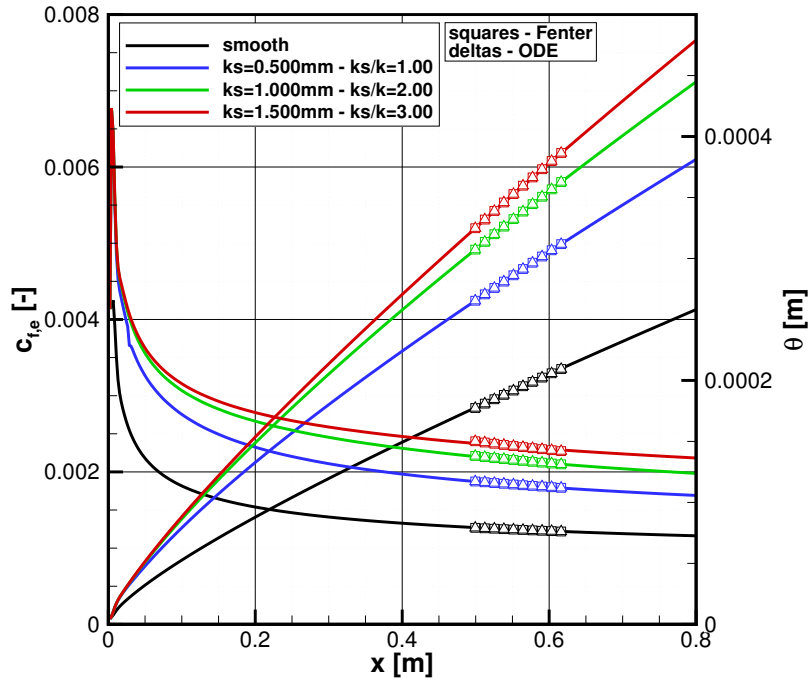


Figure 7.8: Test of integral approach along 7° cone at FC1 (only every 100th point shown for clarity)

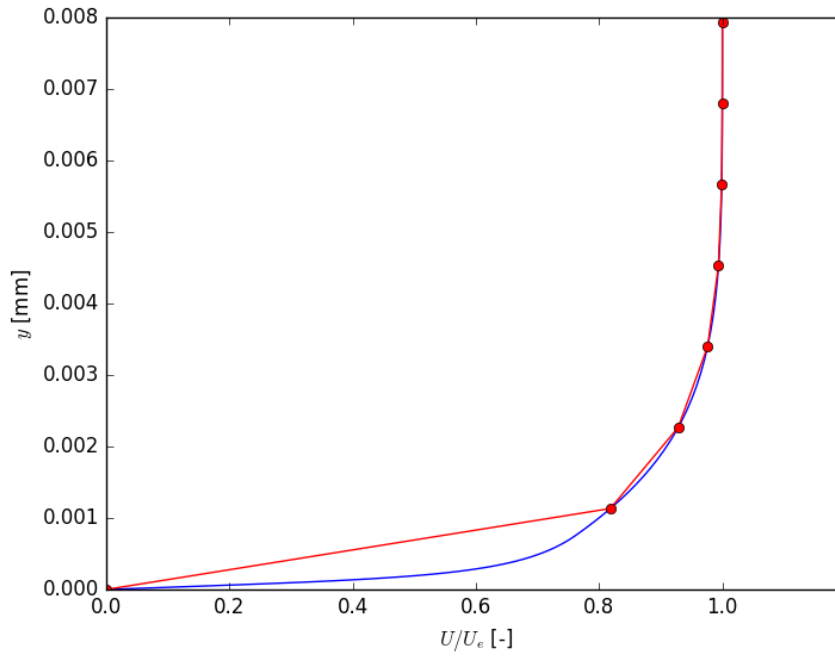


Figure 7.9: Downsampling to $\Delta y = 128$ pix of velocity profile of TBL along 7° cone at FC1

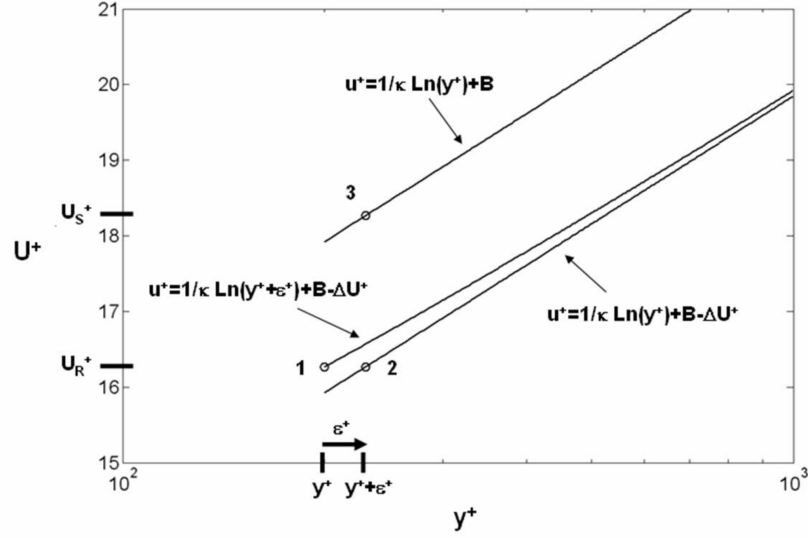


Figure 7.10: Effect of the shift of origin ϵ on the theoretical curve from the law-of-the-wall for rough flows according to [110].

7.2.2 Role of Shift of Origin

The shift of origin is sometimes denoted with ϵ , d , $d0$ or other entities. Also the definition may change, depending on the author. E.g. the top of the elements can be used as reference, especially if the general origin or bed on which the roughness elements are mounted cannot be determined [45]. Both options are visualized for the case of a transverse square bar rough surface in figure 7.11 with ϵ as the shift of origin as measured from the top of the elements and $d0$ as the one from the ground level or bed of roughness. If not otherwise noted, ϵ denoted in this thesis refers to the definition shown in figure 7.11. In the following and in the general context of rough wall TBL equations (like e.g. eq. (3.73)) y indicates $y = y_T + \epsilon$ and $\delta = \delta_T + \epsilon$ where the subscript T indicates the top (crest) of the roughness element (see also figure 3.3, figure 7.11 in case of transverse square bar roughness). Especially the second condition to scale also the boundary layer thickness is not indicated by every author and might lead to erroneous results, especially once k and therefore ϵ is a relevant portion of δ . Typically, it is assumed that $0 \leq \epsilon/k \leq 1$.

To visualize the general effect of ϵ on the inner scaled velocity profile, figure 7.10 shows the logarithmic portion of the smooth wall (eq. (3.73) with $\Delta U^+ = 0$) and rough wall law-of-the-wall (eq. (3.73)) where the latter is plotted with and without the effect of introducing ϵ . It is visible that points of constant velocity are shifted along the y^+ axis by the amount ϵ^+ (see point 2 to 1 in figure 7.10). The effect of introducing ϵ for the complete profile is visible with the departure of the slope from the smooth wall value $1/\kappa$ for the curve including ϵ compared to the one without (here the nominal slope is encountered for $\epsilon = 0$ whereas along real roughness topologies it would be the other way around). Apart from this effect, ϵ also enters the wake formulation via the term y/δ and therefore effects also the profile outside of the log-region (see eq. (3.73)).

There are a multitude of different approaches to extract ϵ . A selection of methods is discussed in

the following, which are categorized in different groups.

mean velocity fitting type:

One group is termed mean velocity fitting type, since it exploits the measured mean velocity profile to fit a certain theoretical slope or distribution in specific boundary layer regions via the necessary parameters, including ϵ . This group can be further sub-divided according to the layer from which the velocity profile is exploited. All of the fitting approaches do need further parameters like the skin friction velocity u_τ . If there is no independent measurement, this parameter need also to be fitted. Therefore, the search for ϵ has some overlap with the search for u_τ (see chapter 3.10.2). The most widely used approach exploits the mean velocity in the logarithmic portion of the TBL. One of the first to use this method was Moore [164] for TBL along TSBR, where he was able to rebuild the smooth wall $1/\kappa$ slope if he added $\epsilon = 2/3k$ to his data (some authors remark the rather small values of δ/k [110]). This approach is also sometimes labeled modified Clauser method by Perry and Li [186], refined from an earlier version in [185]. Some authors also term it error in origin method [185], [10]. The underlying intention of modifying ϵ is to maximize the extent of the logarithmic region of the mean streamwise velocity profile. This method is applied in some variations by e.g. [219], [221], [209]. All these methods assume a constant smooth wall $1/\kappa$ slope. One challenge of this method is the ambiguity of results for a fitting of several different parameters at the same time. Another drawback is to identify beginning and end of the logarithmic portion of the profile and the corresponding data points. Even for smooth walls the limits of the log-layer are blurry and at the same time, both u_τ and ϵ modify the corresponding y^+ values of a data point and therefore may change its position to different layers. This approach was tested on selected data of this thesis and is labeled “pseudoPerry” (see section 4.2.2).

Apart from the logarithmic layer, there are methods to fit the outer layer mean velocity via the velocity-defect law [135], [35], [83], [19]. This method usually has the advantage of exploiting more data points of a larger portion of the TBL and typically measurements of mean velocity in the outer layer are less demanding than in the very near-wall logarithmic layer. Another advantage is that due to the formulation of the defect law, less parameters than in the log-region need to be fitted. This can be seen by formulating the equation according to [135] starting from equation (2.7) to get

$$\begin{aligned}
 U_e^+ - U^+ &= \\
 &\left(\frac{1}{\kappa} \log(\delta^+) + B + \frac{\Pi}{\kappa} W(1) - \Delta U^+ \right) \\
 &- \left(\frac{1}{\kappa} \log(y^+) + B + \frac{\Pi}{\kappa} W\left(\frac{y}{\delta}\right) - \Delta U^+ \right) = \\
 &\frac{1}{\kappa} (\log(\delta^+) - \log(y^+)) + \frac{\Pi}{\kappa} \left(W(1) - W\left(\frac{y}{\delta}\right) \right) = \\
 &\frac{\Pi}{\kappa} \left(W(1) - W\left(\frac{y}{\delta}\right) \right) - \frac{1}{\kappa} \log\left(\frac{y}{\delta}\right)
 \end{aligned} \tag{7.2}$$

where $W(y/\delta)$ is again the (smooth wall) wake function of choice (eq. (2.11)). As visible, by formulating the difference for the velocity-defect formulation, on the right hand side of eq. (7.2)

the viscosity, B , ΔU^+ are eliminated. In contrast to fitting eq. (3.73) a functional dependence of $f = (y, U, \delta, u_\tau, \Pi, \epsilon, B, \kappa, \nu_w, \Delta U^+)$ is reduced to $f = (y, U, \delta, u_\tau, \Pi, \epsilon, \kappa, \nu_w)$. This is a reduction of only two parameters but may ease the challenges for multi-parameter fitting. This approach was tested on selected data of this thesis and is labeled “Krogstad” (see section 4.2.2).

A special kind of the outer layer fitting is the approach introduced by [83] and also used e.g. in [19]. It used a graphical method via the velocity defect profiles but plotting it against a term $y_T u_\tau / (\delta_T^* U_e)$, where δ_T^* represents the displacement thickness in a coordinate system with origin at the top of the elements (see e.g. figure 3.3). This plot is overlaid with the following expression

$$U_e^+ - U^+ = -\frac{1}{\kappa} \log \left(\frac{y_T u_\tau}{\delta_T^* U_e} \right) - \frac{1}{\kappa} \log \left[1 + \left(\frac{\epsilon u_\tau}{\delta_T^* U_e} \right) \left(\frac{y_T u_\tau}{\delta_T^* U_e} \right)^{-1} \right] + \left[\frac{1}{\kappa} \log \left(1 + \frac{\delta_\epsilon^*}{\delta_T^*} \right) - 0.6 \right] \quad (7.3)$$

where $y = y_T + \epsilon$ and $\delta^* = \delta_T^* + \delta_\epsilon^*$, so that δ_ϵ^* is the difference in displacement thickness in the y and y_T coordinate system. By tuning the parameters ϵ is found if both eq. (7.3) and the data points agree in the described plot. One obvious challenge of this method is the necessary near-wall resolution of data points due to the integration for the displacement thickness. This approach was tested on selected data of this thesis and is labeled “Furuya” (see section 4.2.2).

Apart from the above, there are methods to fit (only) the wake portion of the velocity profile. E.g. [185] describes an approach where the wake function and its value of $f(y/\delta) = 2$ is exploited but which is only recommended for a large wake strength $\Pi > 1$. This approach was tested on selected data of the current thesis and is labeled “Perry wake” (see section 4.2.2).

volume aspect type:

Another group is termed volume aspect type, since it tries to keep the volume of flow the same between smooth and rough wall. It is based on the early work by Schlichting [212], which argued that the shift ϵ reflects a height a hypothetical smooth wall should have as origin to bound the same fluid volume than the rough surface. Another explanation argues with the height which the surface would possess if the material would be melted at constant volume, therefore sometimes called melt down height. This approach can be applied easily on technical rough surfaces with a defined topology. Figure 7.11 shows a sketch of a TSBR with definition of the shift of origin ϵ and two areas AI and AII , necessary to define the melt down height.

With AI and AII one can derive ϵ based on geometrical grounds via

$$\begin{aligned} A_{rough} &= AI + AII = (\lambda \cdot h) + (i \cdot k) \\ A_{smooth} &= (\lambda \cdot (h + \epsilon)) \\ A_{rough} &\stackrel{!}{=} A_{smooth} \Leftrightarrow \\ \epsilon &= \frac{ik}{\lambda} = \frac{ik}{i + w} \end{aligned} \quad (7.4)$$

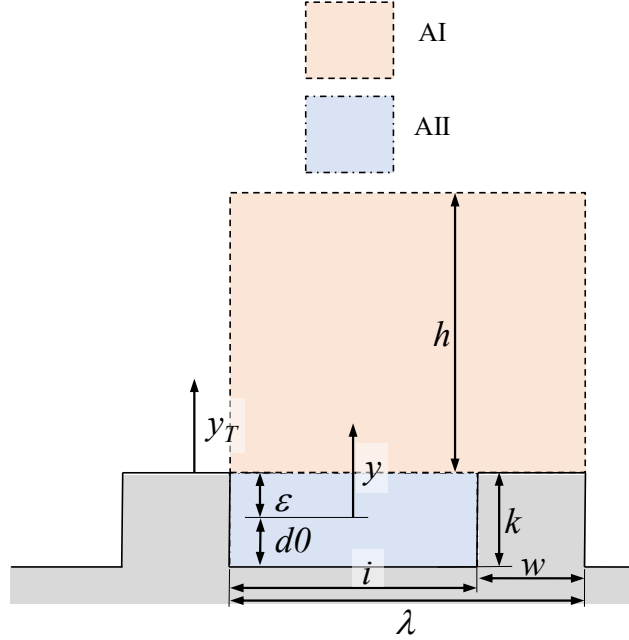


Figure 7.11: Sketch of a TSBR with definition of shift of origin ϵ and melt down height

Coleman *et al.* [49] noted that Schlichting's displacement height is logical but criticized that it is unrelated to any characteristics of the rough wall velocity profile. Apart from that, there have been suggestions for extensions to include flow separation regions [87], [110], [244]. The idea behind the extensions were that besides the volume of the roughness topology also the added volume of separated regions near the roughness elements needed to be included. Stewart and George achieved fairly good agreement with the estimation of ϵ using the idea of the mean separated flow region acting as an approximate mean roughness height for determining ϵ .

pressure contribution type:

Another group is termed pressure contribution type, since it exploits forces and moments (pressure and friction) acting on the roughness elements. Jackson [117] argued that the shift represents the mean height of momentum absorption by the surface and therefore the centroid of the drag profile is necessary to evaluate ϵ . This is only possible if the complete skin friction and pressure distribution along the rough surface is known. This is available in computational analysis but seldom in experimental studies. Therefore, several computational investigations applied this analysis, e.g. using DNS [148], [150], [145], [143]. If roughness elements allow the implementation of pressure taps along relevant surfaces of the elements, authors have derived u_τ for single roughness elements with the assumption of negligible skin friction contribution, based on an

integral momentum balance (e.g. [187], [10], [83]). The assumption of a dominant pressure drag is equivalent to assuming fully rough flow. Although the skin friction is not dominant in this case, it will have some amount of contribution and therefore this assumption will also affect the derived u_τ [125]. Once a pressure distribution is known and the assumption of negligible friction is accepted, one can also apply Jackson's method to experimental data [125], [10]. Experimental comparisons between ϵ extracted by the method of Jackson against a fitting procedure resulted in a difference of $\approx 30\%$. In contrast Leonardi *et al.* [150] compared the same methods along TSBR in turbulent channel flow computed via DNS and reported that results of both methods agree for small values of $\lambda/k = 1/\lambda_f$ but larger differences occur when $\lambda/k = 1/\lambda_f \gtrsim 6$. Although widely applied, especially in numerical studies, the method according to Jackson is not free of criticism (e.g. [45]). Chung criticizes that the method typically assumes zero pressure gradient and fully developed flow, which seldom are perfectly satisfied in either TBL or internal pressure-driven flows like pipe or channel flow [45].

mixing length type:

Another group is termed mixing length type, since it uses a modified mixing length model, defined by a characteristic length scale of roughness and ϵ [110]. This approach lends from the classical smooth wall turbulent mixing-length idea of Prandtl and introduce a characteristic roughness length scale, like k , comparable to other approaches, e.g. [260], [71], [136].

zero velocity type:

Another group is termed zero velocity type, since it uses the location where the mean velocity is zero to define ϵ . This was applied e.g. in [222] along sand grain and sprayed epoxy surface. The challenge for this method is not only to measure the velocity profiles below the elements crest reliably to identify possible zero velocities but also that $U(y_T + \epsilon)$ is not necessarily zero above rough walls [45].

alternative type:

Another group is termed alternative type, since it makes use of alternative scaling laws. E.g. Huang[112] proposed an alternate scaling for the mean velocity in the roughness sublayer with $U^- = U^+ - U_w^+$ and $y^- = y/\epsilon$, where U_w^+ is the spatially averaged (temporal) mean slip velocity at the wall distance corresponding to the top (crest) of the roughness elements, which is typically non-zero, depending on the roughness configuration [112]. Based on this formulation, the author is able to scale the velocity profiles of a variety of computational and experimental studies along different technical roughness topologies (2D and 3D). This scaling includes the roughness sublayer in the immediate vicinity of the roughness elements, additionally to the log-layer.

To wrap up the different types of methods to extract ϵ , Hopkins [110] compiled a comparison between some of these different methods. They performed LDV measurements in a TBL along sand grain, grid and hemispherical roughness topologies and tested some fitting and the mixing length approaches. In general all fitting approaches were evaluated problematic. The outer layer fitting according to Krogstad *et al.* [135] was deemed insufficient due to unrealistic values. The mixing length method gives realistic values except for the sand grain roughness. Other than that, the resulting values were not further evaluated.

For the special case of TSBR, figure 7.12 gathers selected shift of origins ϵ/k and show a plot against $\lambda/k = 1/\lambda_f$. It includes values via the method by Jackson from surface pressure data

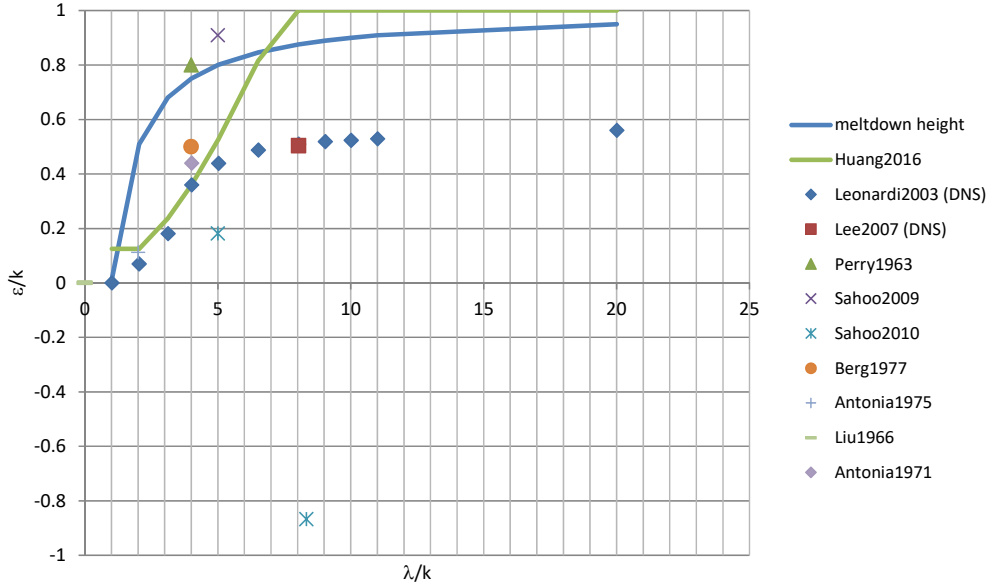


Figure 7.12: Shift of origin ϵ/k against wavelength λ/k for TSBR from different authors

reported in [10] and by the method of Huang *et al.* [112]. On the one hand it shows a general tendency of increasing ϵ with increasing wavelength of elements $\lambda/k = 1/\lambda_p$. This is anticipated since for increasing wavelength the single elements exhibit more and more of the external flow and tend toward the isolated flow regime. On the other hand, huge scatter is visible. This is true for different methods e.g. between values derived by the Jackson method (Leonardi2003 (DNS) data points), meltdown height (corresponding label) and Huang’s method. And it is also true for different datasets at the same wavelength. E.g. at λ/k equal 4 or 5 a scatter in between $\epsilon/k = 0.2 - 0.9$ is visible. This huge range of values was even extracted along the same roughness (TSBR) from the same author with the same methodology [209], [208]. This gives an indication of the inconclusive situation about the shift of origin. The major challenge is that the shift of origin parameter depends on the flow and roughness topology, as the equivalent sand grain roughness [45]. Besides the already challenging endeavor for incompressible flows, in compressible flow even further challenges exist (see chapter 4.2.2).

7.2.3 Shift of Origin Analysis

Selected methods of the previously described methods to derive the shift of origin ϵ (see section 7.2.2) were tested on data of this thesis.

Exemplary results are shown in figure 7.13 for data of run 3007. In figure 7.13a the profile data (symbols) are shown to be fitted with good agreement to the outer layer formulation according to Krogstad *et al.* [135] (line labeled “Krogstad”). In figure 7.13b the data is fitted in the log-region

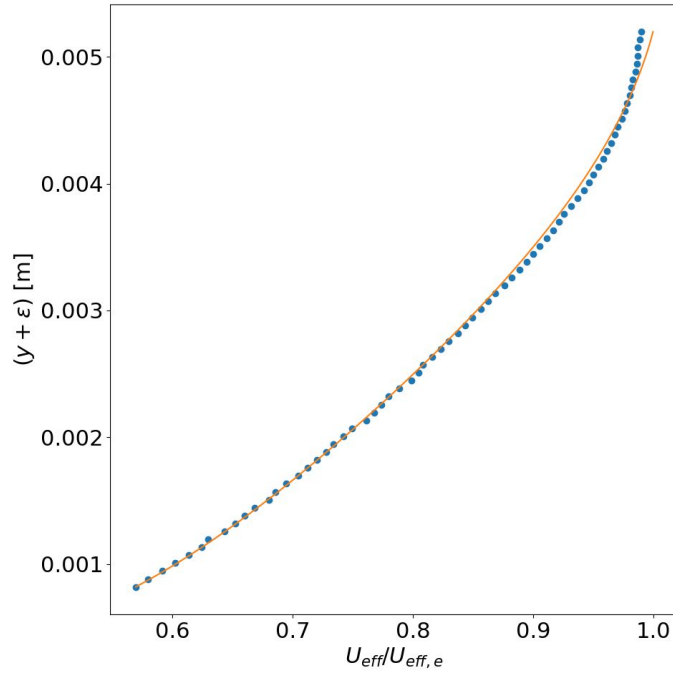
according to the method labeled “pseudoPerry” (after Perry and Li [186] or Schultz and Flack [219], see section 3.12) and in the wake part to “Perry wake” (after Perry and Joubert [185], see section 3.12).

Also in this plot, the agreement of the log-fit (blue dashed line) as well as the wake fit (not directly visible) is in good agreement to expected behavior. Other profile data show comparably good visual agreements between correlation and data. Despite this apparent success of the approaches, the derived values for ϵ showed large scatter, if results from different selected extraction methods and different runs / different averaging regions for the profiles were compared. This was presented visually in figure 4.14 of section 4.2.2 and can also be found in the following table 7.12, which contains extracted shift of origin ϵ/k (all for IW 512x16 pix² of second PIV campaign).

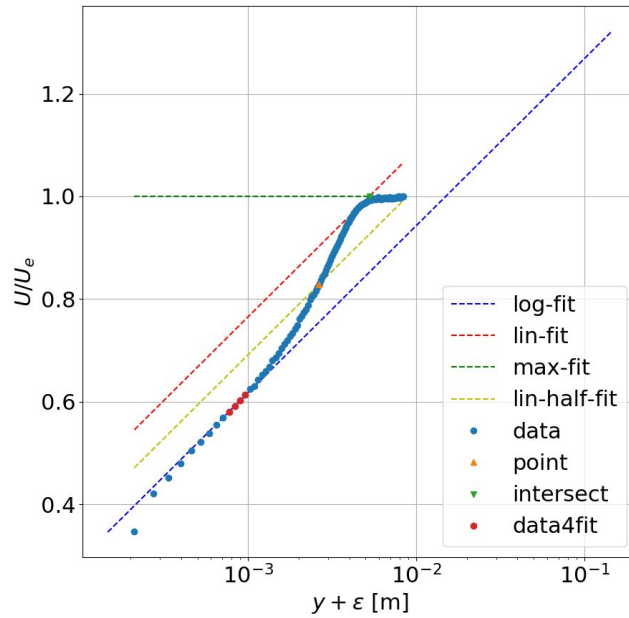
Table 7.12: Shift of origin ϵ/k determination via different methods described in chapter 7.2.2 (all for IW 512x16 pix² of second PIV campaign).

run	avg. [m]	pseudoPerry [-]	Perry wake [-]	Krogstad [-]	Furuya [-]
3006	0.496<x<0.500	0.4	0.6	0.9	0.0
3006	0.498<x<0.500	0.4	0.8	0.0	0.0
3006	0.496<x<0.498	0.3	0.5	0.5	0.0
3007	0.496<x<0.498	0.2	0.2	0.4	0.1
3007	0.498<x<0.500	0.3	0.7	0.8	0.3
3007	0.496<x<0.500	0.2	0.4	0.4	0.2
3008	0.496<x<0.498	0.1	0.1	0.4	0.3
3008	0.498<x<0.500	0.2	0.2	0.5	0.1
3008	0.496<x<0.500	0.1	0.2	0.5	0.2
3009	0.496<x<0.500	0.2	0.4	0.4	0.5
3009	0.498<x<0.500	0.3	0.4	0.4	0.4
3009	0.496<x<0.498	0.2	0.4	0.3	0.6
3010	0.496<x<0.500	0.2	0.4	1.0	0.1
3010	0.498<x<0.500	0.3	0.6	1.0	0.2
3010	0.496<x<0.498	0.2	0.0	0.4	0.2
3020	0.626<x<0.628	0.2	0.7	0.6	0.0
3020	0.628<x<0.630	0.4	0.6	0.7	0.2
3020	0.626<x<0.630	0.2	0.5	0.8	0.1

As visible by the spread of resulting ϵ , run-to-run and also the range and size of the mean velocity averaging had a sizable effect without a converging answer. Therefore, a different approach was used to define ϵ was used in this thesis, described and corresponding results reported in section 4.2.2.



(a)



(b)

Figure 7.13: Example plots for the extraction of shift of origin ϵ for data of rough wall Run3007 following a) the “Krogstad” (see [135]) and b) the “pseudoPerry” (see [186] or [219]) and “Perry wake” approach (see [185]).

7.3 Additional EQSR Investigations

Table 7.13 and 7.14 summarizes additional parameters for selected roughness topologies.

Table 7.13: Selected compressible roughness studies; Superscripts: a: k_s^+ rebuilt via ΔU^+ and eq. (3.75), b: ΔU^+ rebuild from graphs in reference; Subscripts: rb: rebuilt ; Roughness types: TSB: Transverse square bars.

Main investigator	Roughness type	λ/k [-]	Ma_∞ [-]	δ^+ [-]	k_s/k [-]	k^+ [-]	k_s^+ [-]	ΔU^+ [-]	k/δ [-]	κ [-]	B [-]	$(k_s^+)^{rb}$ [-]	ΔU_{rb}^+ [-]	$(k_s/k)^{rb}$ [-]	Reference
Berg1977(Ma6.0)	TSB	4.0	6.00	845	1.3	33.8	43.9	6.5	0.04	0.40	5.0	44.7	6.5	1.3	[24]
Sahoo2009(Ma7.3)	TSB	5.0	7.30	729	5.9	102.0	601.8 ^a	13.0	0.14	0.40	5.1	601.8	13.0	5.9	[209]
Sahoo2010(Ma7.3)	TSB	5.0	7.30	1099	4.5	134.0	601.8 ^a	13.0	0.12	0.40	5.1	601.8	13.0	4.5	[208]
Sahoo2010(Ma7.3)	TSB	8.3	7.30	566	1.3	37.0	48.4 ^{a,b}	6.7	0.07	0.40	5.1	48.4	6.7	1.3	[208]
Latin2000(Ma2.9)	TSB	3.9	2.90	4454	1.9	148.5	289.0	10.8	0.03	0.41	5.5	249.6	11.2	1.7	[139]
Bowersox2001(Ma2.7)	TSB	3.9	2.70	4454	1.9	148.5	289.0	11.0	0.03	0.41	5.5	270.4	11.2	1.8	[33]
Pritchett2001(Ma0.22)	TSB	3.9	0.22	-	0.6	837.0	501.0 ^b	16.5	-	0.41	5.5	2440.6	12.5	2.9	[195]
Pritchett2001(Ma0.56)	TSB	3.9	0.56	-	0.6	867.0	552.0 ^b	16.8	-	0.41	5.5	2751.8	12.8	3.2	[195]

Table 7.14: Selected compressible roughness studies, topology parameters; Topology parameters: Λ : Danberg and Sigal [51], λ_{Dvorak} : Dvorak [60], Λ_s : Sigal and Danberg [226], λ_2 : Huang *et al.* [112]; Roughness types: TSB: Transverse square bars

Main investigator	Roughness type	$\lambda/k = 1/\lambda_p = 1/\lambda_f$ [—]	k [mm]	w [mm]	i [mm]	λ_{Dvorak} [—]	Λ [—]	Λ_s [—]	λ_2 [—]	D_{Dvorak} [—]	$D_{Danberg}$ [—]	Reference
Berg1977(Ma6.0)	TSB	4.0	1.27	1.27	3.81	4.00	4.00	4.00	0.19	-0.38	-0.28	[24]
Sahoo2009(Ma7.3)	TSB	5.0	1.65	1.65	6.60	5.00	5.00	5.00	0.14	2.36	2.20	[209]
Sahoo2010(Ma7.3)	TSB	5.0	1.65	1.65	6.60	5.00	5.00	5.00	0.14	2.36	2.20	[208]
Sahoo2010(Ma7.3)	TSB	8.3	0.75	0.75	5.50	8.33	8.33	8.33	0.08	-0.09	2.20	[208]
Latin2000(Ma2.9)	TSB	3.9	0.56	0.56	1.62	3.89	3.89	3.89	0.20	-0.71	-0.62	[139]
Pritch2001(Ma0.22)	TSB	3.9	0.56	0.56	1.62	3.89	3.89	3.89	0.20	-0.71	-0.62	[195]

7.3.1 Alternative EQSR Extraction Method Validation

Table 7.15 shows relevant parameters of the profiles and the resulting fitted values of the equivalent sand grain roughness $k_{s,fit}$ which show deviations below 0.2 % to the values calculated via eq. (3.74) based on given input ΔU^+ , u_τ , ν_w and k_s^+ .

Table 7.15: Comparison of EQSR approaches according to method in section 3.10.2 via eq. (3.74) and eq. (3.76).

wall	δ^+ [-]	$U_{eff,e}^+$ [-]	Π [-]	ΔU_{eff}^+ [m/s]	u_τ [m/s]	k_s [mm]	$k_{s,fit}$ [mm]
smooth	226	20.6	0.4	0	52.3	-	-
rough	236	19.7	0.8	3	54.7	0.2280	0.2285
rough	273	17.0	0.8	6	63.2	0.6554	0.6563
rough	322	14.4	0.8	9	74.5	1.8453	1.8461

7.3.2 Literature EQSR Uncertainty Investigations

This chapter contains the EQSR uncertainty estimates of selected literature cases, based on the described method in section 3.12.2. The representative literature cases include the already mentioned case from Bhatt and McClain [26] for technical roughness data originally from Schlichting [212] of plate No. XXV at 20 m/s. This represents an incompressible case with the benefit to enable a comparison to derived uncertainties from [26]. Table 7.16 contains the assumptions for the corresponding input uncertainties. Another incompressible case is analyzed with the high Reynolds number hot-wire anemometry data along sand-grain roughness for case 1 (x=21.7m, U=30.4m/s) data by Squire *et al.* [240] with the corresponding assumed input uncertainties in table 7.18. A supersonic case is studied with the first PIV profile measured along the square roughness from [69] with the corresponding assumed uncertainties in table 7.17. A hypersonic case is studied with the PIV data along the k=0.75mm square bar roughness from Sahoo *et al.* [208] with the corresponding assumed input uncertainties in table 7.19. Finally, a further hypersonic case is studied with the profile for the smooth-to-rough case at x=47.4 inch along the k=0.05 inch square bar roughness from Pitot data by Berg [24] with the corresponding input uncertainties in table 7.20.

If available, the input value uncertainties were taken as published or an educated guess was made otherwise (marked in the comments column). The profile data are either digitized from plots or from tabulated data of the corresponding references (e.g. Berg's data from [74]). The digitization is a source for additional error, which is accepted here. The input uncertainties are sometimes reported as absolute sometimes as percentage values. Before fed into the calculation for UPCs and UMFs, the percentage input uncertainties were re-calculated based on relevant scales. This included e.g. the wall-normal uncertainty U_y being based on 0.3δ , representative of largest log-layer extent (see section 1) instead of the mean value. Also, the shift of origin uncertainty U_ϵ was based on corresponding roughness heights k . The latter was especially necessary, since for some authors the assumed nominal ϵ was zero. A different choice for these values would change the percentage partition between the inputs and so an optimal choice is a matter of discussion.

During data collection, several discrepancies were encountered. E.g. in Ekoto [69], [66], [68] and [67] different inner scalings and roughness parameters are reported, although it seems as though the same data sets are discussed. This is also the case for data reported in Sahoo *et al.* [209], [208]. These differences give already a hint about the sensitivities of the EQSR extraction process.

First, the percentage partition of the input uncertainties are shown in figure 7.14a. For that, all input percentage uncertainties are re-scaled to add up to 100%, i.e. for inflow Mach number, reservoir pressure and temperature, wall temperature and edge pressure, (edge) velocity, wall-normal position, skin friction velocity and shift of origin. It should give a qualitative reference about the different contribution of input uncertainties. It is visible that usually the main contributions come from uncertainties in y , u_τ and in case of compressible cases also p_e , which are important due to their role in the van Driest transformation. It should be mentioned that in Berg's case [24], with approx 1 % a rather small uncertainty is reported for the skin friction velocity measurements (measured by a floating balance).

In 7.14b, the final EQSR extraction result uncertainties are given, represented by the Coles wake function Π ("Pi"), the roughness function ΔU^+ ("DU+") and the normalized EQSR $\alpha = k_s/k$ ("alpha"). All three values come out of the analysis framework, presented in 3.10.2 and 3.12. In case of Bhatt and McClain [26] no value for Π is given since it is based on Schlichting's channel flow data. In general, the percentage error for the normalized EQSR α (alpha) is larger than for the corresponding roughness function ΔU^+ (DU+) uncertainties, since the mean α values are smaller (approx. 0.8 – 4.0) than the mean ΔU^+ values (8.6 – 12.9). In general, for compressible data ([24], [69], [208]) both the roughness function and the EQSR are prone to higher uncertainties than for incompressible data ([240], [26]). This was expected due to e.g. partially higher uncertainties in measurement quantities as well as additional uncertainties due to e.g. van Driest transformation. For the EQSR up to $U_{\alpha=k_s/k} \approx 10$ % and up to $U_{\alpha=k_s/k} \approx 30$ % need to be taken into account for incompressible and compressible investigations, respectively. As already stated, these values do not represent exact uncertainty predictions for the heritage data but can act as an order of magnitude reference for data in this thesis.

The results of the quasi UPC for all literature cases are shown in figure 7.15 and the quasi UMF in figure 7.16. In both cases the EQSR uncertainty results are again represented by the three parameters Π ("Pi"), ΔU^+ ("DU+") and $\alpha = k_s/k$ ("alpha"). It is visible in figure 7.15 that for investigations up to supersonic velocities ([240], [26], [69]), the main contributor to the EQSR uncertainty (ΔU^+ or α) is the uncertainty of the skin friction velocity u_τ . For hypersonic cases ([24], [208]) also the velocity, pressure and Mach number uncertainties can play a role. There are differences in between data by Berg [24] and Sahoo *et al.* [208], since the first of both is based on Pitot pressure whereas the second is based on PIV data. Therefore, slightly different post-processing is necessary (see sections 3.2 and 3.3) which can be prone to different uncertainties.

The UMFs in figure 7.16 show that for the incompressible case by Squire *et al.* [240] the uncertainties in the wall-normal distance U_y has the largest lever on the resulting EQSR uncertainties. This is because the corresponding input uncertainty is quite small (due to the large boundary layer thickness of the high Reynolds number wind tunnel) with a value of ≈ 0.02 %. For the other cases, the velocity uncertainty U_U has a considerable lever together with the skin friction velocity U_{u_τ} . For the Pitot pressure analysis ([24]), also the edge pressure U_{p_e} has a sizable lever.

So, for compressible cases, it is most beneficial to increase the accuracy of velocity and skin friction velocity measurements. This was the goal in this thesis by transitioning from the first to the second PIV campaign.

Table 7.16: Input for MCM EQSR uncertainty analysis with data according to Bhatt and McClain [26] for data originally from Schlichting [212] of plate No. XXV at 20 m/s

measured variable	nominal	U	U [%]	comment
Δh [mm]		2.00		Assuming to be measured with scale (Table 1)
Δx [mm]		1.00		Assuming to be measured with scale (Table 1)
$u_{\tau,s}$ [m/s]			5.00	Assuming standard uncertainty due to lack of information (Table 1)
w [mm]	170.00	1.00	0.59	channel test section spanwise width (just stated in text, Appendix F)
b [mm]	38.80	1.00	2.58	channel test section height (just stated in text, Appendix F, although [212] reports about a minimum requirement of $Db=1/20$ mm to full-fill accurate static pressure measurements which he succeeded to accomplish, so $Db \leq 1/20=0.05$ mm)
Δh_{Pitot} [mm]		0.25		Assumed to be measured with slanted manometer (Table 1)
y [mm]		0.03		just stated in Appendix G (very small!! $\tilde{0}.0254$ mm)
ϵ [mm]				
k [mm]	3.75	?		[26] used example data for Schlichtings XXV Kegehrauhigkeit, in [26] a fixed value is used and also [212] does not comment
κ [-]	0.40	?		in [26] a fixed value of $1/\kappa \ln(10)=5.75$ with $\kappa=0.4$ is used
$u_{\tau,s}$ [m/s]	0.22			[26]
T_0 [K]		1.00		nominal value guessed via excel. UT0 estimated for typical high-quality TC
p [bar]			1.00	nominal value guessed via excel. Up estimated from Omega dyne PX309-005A5V spreadsheet
Pitot				
δ [mm]	22.96			rebuild
k/δ [-]	0.16			
δ/k [-]	6.12			

Table 7.17: Input for MCM EQSR uncertainty analysis with data according to Ekoto [69], Ekoto *et al.* [66], Ekoto *et al.* [68], Ekoto *et al.* [67]

measured variable	nominal	U	U [%]	comment
$u_{\tau,s}$ [m/s]	23.00	1.38	6.00	95% CI ([66]), τ_w calc. with tabulated u_τ and tabulated $p_e = p_w$, T_w
y [mm]			0.10	
k [mm]	0.79			nothing stated
$u_{\tau,r}$ [m/s]	30.00	1.80	6.00	95% CI ([66]), τ_w calc. with tabulated u_τ and tabulated $p_e = p_w$, T_w
$\tau_{w,r}$ [N/m ²]	262.67	31.52	12.00	95% CI ([69]), τ_w calc. with tabulated u_τ and tabulated $p_e = p_w$, T_w
Ma [-]	2.86	0.03	1.00	
U [m/s]				
p_0 [bar]	6.90	0.35	5.00	different in text and table 2, I used text version
T_0 [K]	313.00	10.96	3.50	different in text and table 2, I used text version
p [bar]		0.00		
Pitot				
p_{pitot} [bar]		0.02		
p_{pitot}/p_0 [-]				
U [m/s]			0.40	dU/Ue, depending on the edge velocity
U_{eff}^+ [m/s]			0.60	
U_{eff}^+ [m/s]	625.87	3.76	6.10	
U_e [m/s]			0.60	nominal Ue calculated from Ma, T0, since Ekoto does not state clearly nominal Ue
$U_{eff,e}^+$ [m/s]	20.00	1.22	6.10	
δ [mm]	11.60	0.70	6.00	
x [mm]			0.40	
PIV				
U [m/s]			0.50	
U_{eff}^+ [m/s]	20.00	1.20	6.00	
δ [mm]	11.60	0.58	5.00	
p_w [Pa]	24626.00	2634.98	10.70	mentioned in [69], but also 1.5% given w/o thermodynamic fluctuations, but not sure what of both is valid, repeated in [69]
k/δ [-]	0.07			
δ/k [-]	14.68			

Table 7.18: Input for MCM EQSR uncertainty analysis with data according to Squire *et al.* [240], Squire [239]

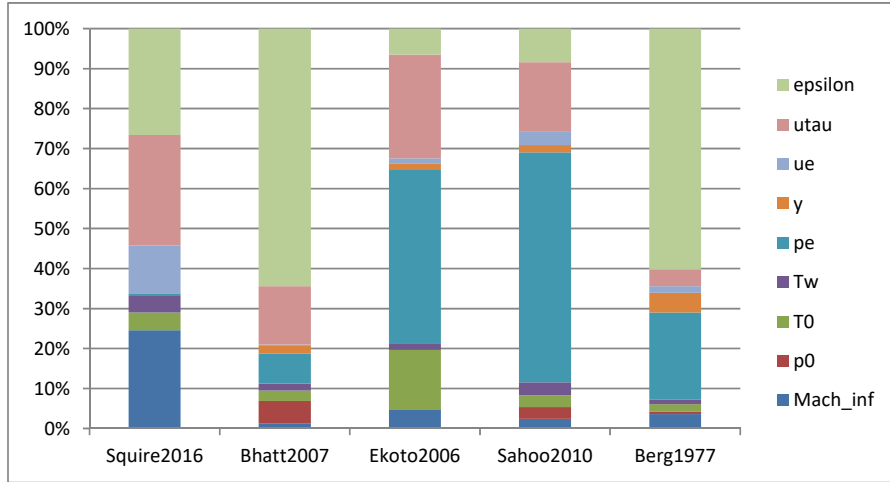
measured variable	nominal	U	U [%]	comment
$u_{\tau,s}$ [m/s]	0.90	0.02	2.30	only for the profiles where direct drag balance measurements are available, for other profiles indirect modified Clauser fitting with $\pm 5\%$ and total shear stress method $\pm 2\%$ are used (here data at $x=21.7\text{m}$ used)
y [mm]		0.02		due to aluminium tooling plate, probably larger
ϵ [mm]		0.02		same as y , probably larger
k [mm]	0.90	0.15	16.67	sand-grain roughness, nothing stated directly about uncertainty, but at least krms given with 0.15mm, but difficult to assess since sand-grain
κ [-]	0.39			
B [-]	4.30			rather low, but I have checked it against data points
$u_{\tau,r}$ [m/s]	1.23	0.03	2.30	only for the profiles where direct drag balance measurements are available, for other profiles indirect modified Clauser fitting with $\pm 5\%$ and total shear stress method $\pm 2\%$ are used (here data at $x=21.7\text{m}$ used)
$\tau_{w,r}$ [N/m ²]	2.41	0.11	4.60	estimated
Ma [-]	0.09	0.00	2.11	estimated
U [m/s]	30.40	0.15	0.50	
p_0 [bar]	1.37	0.00	0.03	nominal value estimated from [240], and uncert. for a 30psid (2bar) ESP DTC scanner
T_0 [K]	300.00	1.10	4.10	nominal value estimated from [240], and uncert. for TC Special Limits of Error: $\pm 1.1^\circ\text{C}$ or 0.4%
HWA				
U [m/s]			1.00	[246]
U_e [m/s]	30.40	0.30	1.00	Talluru2014
$U_{eff,e}^+$ [m/s]	24.72	0.62	2.51	estimated from combined U_{u_τ} uncertainties
δ [mm]	388.00			
k/δ [-]	0.00			
δ/k [-]	431.11			

Table 7.19: Input for MCM EQSR uncertainty analysis with data according to Sahoo *et al.* [208]

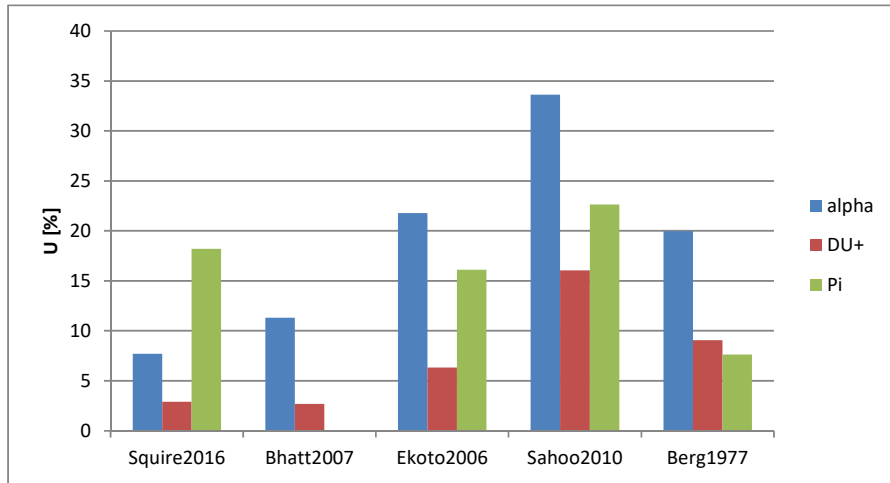
measured variable	nominal	U	U [%]	comment
y [mm]		0.04		"2 pixels which corresponds to about 0.036 mm or 0.4% of the boundary layer thickness"
ϵ [mm]	-0.65	0.04	-5.54	Upsilon estimated, based on Uy
k [mm]	0.75			
κ [-]	0.40			
B [-]	5.10			
$u_{\tau,r}$ [m/s]	75.00		10.00	estimated
Ma [-]	7.30	0.10	1.37	table2
U [m/s]	1150.00			nominal modified from 1181 to fit Te and Mae
p_0 [bar]	79.29	1.35	1.70	
T_0 [K]	720.00	12.24	1.70	
PIV				
U [m/s]		23.00	2.00	guess for realistic supersonic PIV set-up
U_e [m/s]	1150.00	23.00	2.00	guess for moderate supersonic PIV set-up
$U_{eff,e}^{++}$ [m/s]	16.20			
δ [mm]	11.50			
p_w [Pa]	1465.66	510.00	34.80	nominal guessed via excel. Up estimated from Omegadyne PX309-005A5V spread-sheet
T_w [K]	360.00	7.20	2.00	table2
k/δ [-]	0.07			
δ/k [-]	15.33			

Table 7.20: Input for MCM EQSR uncertainty analysis with data according to Berg [24]

measured variable	nominal	U	U [%]	comment
y [mm]		0.13		"Estimates of the probe position (with respect to the flat plate) accuracy under tunnel operating conditions vary from ± 0.002 inch at the aft end of the test section to ± 0.005 inch at the maximum probe extension of 28 inches"
ϵ [mm]	0.64	0.13	20.00	U_ϵ estimated, based on U_y
k [mm]	1.27			
κ [-]	0.40			
B [-]	5.00			
$u_{\tau,r}$ [m/s]	53.60	0.63	1.17	AGARD-AG-263 [73] run 770.S0412, U_{u_τ} [24] p.14/19, seems optimistic
$\tau_{w,r}$ [N/m ²]	28.26	0.66	2.33	estimated
Ma [-]	6.02	0.06	1.00	AGARD-AG-263 [73] run 770.S0412, U_{Ma} [24] p.23
U [m/s]	865.40			AGARD-AG-263 [73] run 770.S0412
p_0 [bar]	1600160.26	3200.32	0.20	AGARD-AG-263 [73] run 770.S0413
T_0 [K]	423.94	2.12	0.50	AGARD-AG-263 [73] run 770.S0415, U_{T_0} as stated in [24] p.8
p [bar]	979.60			AGARD-AG-263 [73] run 770.S0415, no U_p given, since not measured, only calculated based on $p = f(Ma, p_0)$
Pitot				
p_{pitot} [bar]	46224.39	462.24	1.00	[24] p.11
p_{pitot}/p_0 [-]	0.03			reconstructed from [73]
U_{eff}^+ [m/s]				
U_e [m/s]	865.40			
$U_{eff,e}^+$ [m/s]	16.15			
δ [mm]	27.84			reconstructed from [73], but it is $y = f(U = U_e)$ instead of $0.99U_e$!
x [mm]	1.20			
p_w [Pa]	979.60			no U_p given, since not measured, only calculated based on $p = f(Ma, p_0)$
T_w [K]	347.00	1.10	0.32	UT0 estimated for typical high-quality TC
k/δ [-]	0.05			
δ/k [-]	21.92			

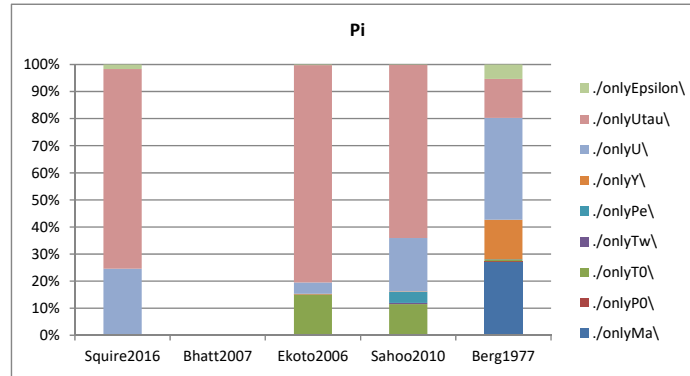


(a)

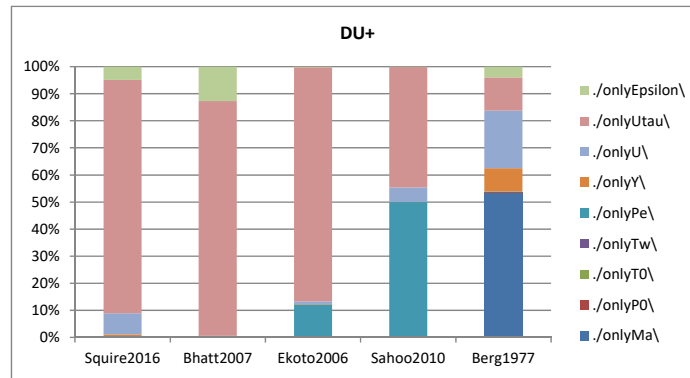


(b)

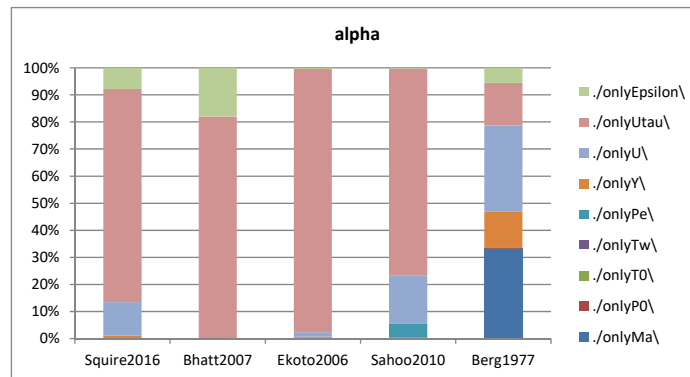
Figure 7.14: a) Quasi UPC of input uncertainties and b) uncertainties of resulting Π (“Pi”), ΔU^+ (“DU+”) and $\alpha = k_s/k$ (“alpha”) from MCM EQSR uncertainty analysis of literature data according to Berg [24], Ekoto [69], Squire *et al.* [240], Sahoo *et al.* [208], Bhatt and McClain [26].



(a)

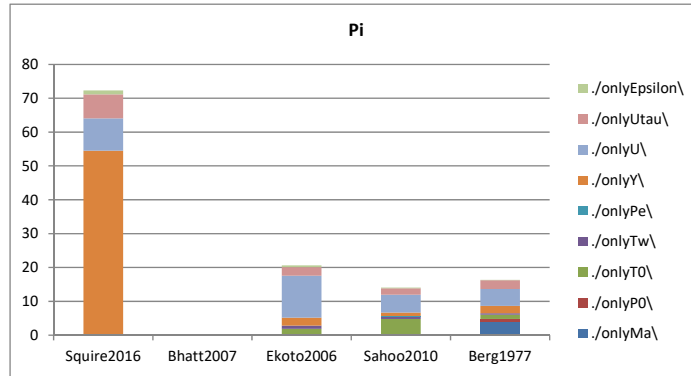


(b)

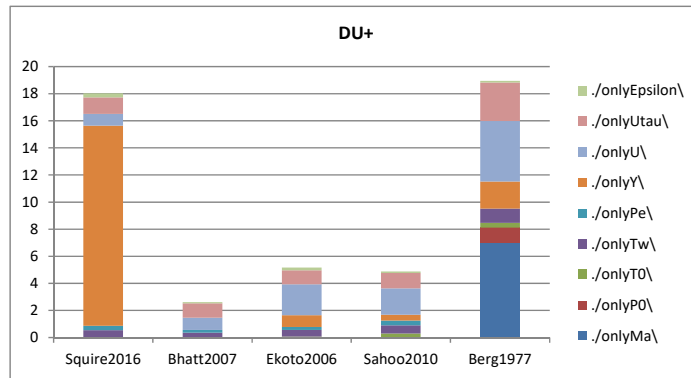


(c)

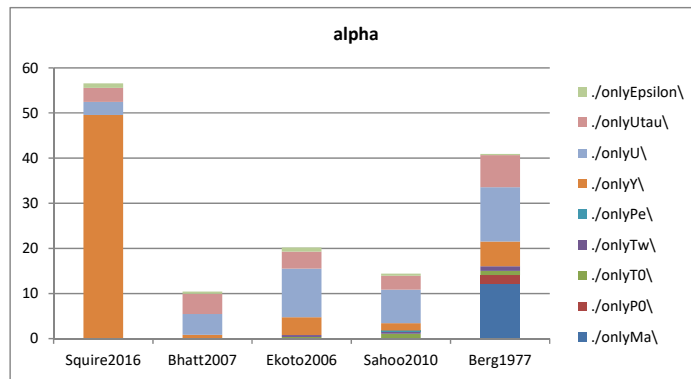
Figure 7.15: Quasi UPC of resulting a) Π (“Pi”), b) ΔU^+ (“DU+”) and c) $\alpha = k_s/k$ (“alpha”) from MCM EQSR uncertainty analysis of literature data according to Berg [24], Ekoto [69], Squire *et al.* [240], Sahoo *et al.* [208], Bhatt and McClain [26].



(a)



(b)



(c)

Figure 7.16: Quasi UMF of resulting a) Π ("Pi"), b) ΔU^+ ("DU+") and c) $\alpha = k_s/k$ ("alpha") from MCM EQSR uncertainty analysis of literature data according to Berg [24], Ekoto [69], Squire *et al.* [240], Sahoo *et al.* [208], Bhatt and McClain [26].

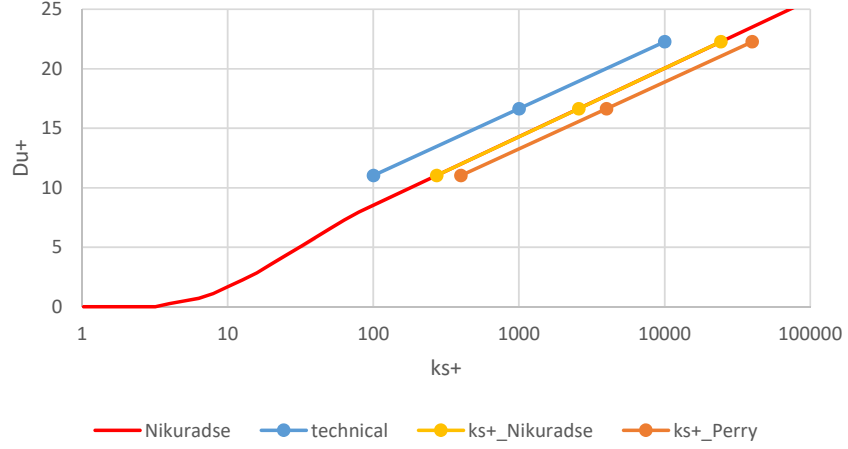


Figure 7.17: Roughness function ΔU^+ against k^+ for Nikuradse's sand grain and an exemplary artificial technical roughness with its equivalent sand grain roughness transformation with parameters inspired by [185]

7.3.3 Role of Law-of-the-Wall Constants

To be fully consistent with the data by Nikuradse for the derivation of k_s/k , the parameters κ and B in eq. (3.75) need to be chosen according to the original work of Nikuradse to $\kappa = 0.4$ and $B = 5.5$ [173]. In the following, own estimates are reported on which this argument was based on, together with similar findings from previous authors.

From the two mentioned parameters, especially the slope of the roughness function in the fully rough regime with $1/\kappa$ is decisive. With a different choice for κ than $\kappa = 0.4$, applying eq. (3.75) to a measured ΔU^+ would result in a biased k_s/k . This is shown in figure 7.17 for an exemplary artificial technical roughness which possesses a $B_r = 5$ and was created via $\kappa = 0.41$ and $B = 4.9$, $B_r = 5.1$ ($\Delta U^+ = 1/\kappa \cdot \log(k^+) + B - B_r = 1/0.41 \cdot \log(k^+) - 0.2$), inspired by the data and parameters in Perry and Joubert [185]. The red line indicates the sand grain data by Nikuradse [173]. The yellow and orange line indicate a calculation of k_s^+ according to eq. (3.75) by using the parameters of Nikuradse ($\kappa = 0.4$ and $B = 5.5$) and Perry ($\kappa = 0.41$ and $B = 4.9$), respectively. It is visible that only the parameters according to Nikradse project the artificial roughness data points successfully onto the sand grain roughness curve. If the parameters according to Perry would be chosen in eq. (3.75), the projected data would visibly result in an offset to the sand grain curve. This offset is even increasing with increasing k^+ (hardly visible), due to the difference in slope $1/\kappa$. Additionally, even with the correct choice for κ and B in eq. (3.75), the resulting $k_s/k = k_s^+/k^+$ is not constant for the current example, as usually assumed, but changes with k^+ . This is because the artificial technical roughness data points exhibit a $1/0.41$ slope instead of $1/0.4$ (see [185]). In the shown example, inspired by the data points of Perry and Joubert [185], the change is in the order of $\Delta k_s/k \approx 10\%$ within $100 < k^+ < 10000$, typical for a fully rough flow in a laboratory setup (atmospheric flows can be orders of magnitude larger). For the actual data points in Perry and Joubert [185] the difference even ranges up to $\Delta k_s/k \approx 50\%$.

Own estimates show that with a choice for $\kappa = 0.39$ or $\kappa = 0.41$, a range which can typically be

found in the literature, the ΔU^+ differs by $\approx 3\%$ for the range $100 \leq k^+ \leq 10000$. The resulting error in k_s/k due to this difference in ΔU^+ would be up to $\pm 27\%$ within this range of k^+ . One possible work around would be to modify the $B_{r,s}$ parameter according to a different κ based on the data by Nikuradse. This means, that you would assume Nikuradse would have used another fit to his data points, according to a different κ (which is obviously less accurate but can be done). This would result in $B_{r,s} \approx 8.3$ and $B_{r,s} \approx 8.7$ for $\kappa = 0.39$ and $\kappa = 0.41$, respectively. This would reduce the error in k_s/k to $\pm 7\%$.

Nikuradse's data was re-evaluated by different authors in the past. Tani [247] performed a re-evaluation of Nikuradse's data and he came to the same conclusion as outlined above, to modify $B_{r,s}$ according to the choice for parameters κ and B . Based on his choice for $\kappa = 0.41$ and $B = 5.0$, Tani used another approach where he used the formulation of the log-law together with an own definition for the wake component and the mean pipe velocity to find an analytical equation to compute the $B_{r,s} = f(\kappa, B)$ based on tabulated measurements of Nikuradse. With this approach he did not need to include a potential shift of origin and resulted in $B_{r,s} = 8.6$ for his choice of parameters. With the same equation one can also compute the resulting $B_{r,s}$ with the above mentioned parameter combinations, which result in $B_{r,s} \approx 8.45$ and $B_{r,s} \approx 8.55$ for $\kappa = 0.39$ and $\kappa = 0.41$. This parameter span is close to the own estimates but is much closer to the unbiased value of Nikuradse with $B_{r,s} \approx 8.5$ for $\kappa = 0.4$. This is because our own estimation above was performed for a range of $k_s^+ \lesssim 10000$, whereas the approach by Tani [247] is restricted to data points with $Re \leq 3e^5$ or $k_s^+ \lesssim 2200$, where Nikuradse performed direct measurements of the mean pipe velocity.

A more recent DNS study on a realistic grit-blasted surface mimicking Nikuradse's sand roughness, showed excellent agreement to Nikuradse's data in the transitionally rough regime [251]. Unfortunately, not enough data points were available to reliably extract the slope of the fully rough regime $\Delta U^+ = f(\kappa)$ from the DNS data.

So, the question for a "true" value of κ for Nikuradse's sand grain roughness is a challenging or even ill-posed one. It would be interesting to evaluate the uncertainties for κ and ultimately the roughness constant $B_{r,s}$ for Nikuradse's data based on estimates for the measurement uncertainties of the initially measured quantities, comparable to [126], [26]. Either way, besides the fact that $B_{r,s} = f(\kappa)$, what must be kept in mind is that the choice for $B_{r,s} = f(\kappa)$ is not only decisive in the derivation of k_s/k from the measurement of ΔU^+ (as shown above) but also for the later use of the corresponding k_s/k or k_s . If the EQSR k_s is used e.g. as input to a calculation based on the sand grain roughness approach (analytical, CFD, etc.), the corresponding $B_{r,s}$ in the applied computational method would need to correspond to the modified parameter according to $B_{r,s} = f(\kappa)$ as well. Otherwise there would be a mismatch of slope and therefore a roughness Reynolds number dependent EQSR $k_s/k = f(k_s^+)$ effect in the corresponding results of the computation.

To add further complexity, a large variety of values exist in the literature for the law-of-the-wall parameters κ and B , assumed to be universal, and a discussion was included in section 2.1 and 3.12.1. Even the same authors may choose different values in different publications (see e.g. Perry and Joubert [185] $\kappa = 0.41$, $B = 4.9$ for $\lambda/k = 4.0$ TSBR and Perry *et al.* [187] $\kappa = 0.4$, $B = 5.1$ for $\lambda/k = 3.6$ TSBR, both probably in the same experimental setup). This choice is purely data driven, to fit the experimental data as best as possible. Even the resulting roughness functions $\Delta U^+ = f(k^+)$ were presented with the differing slopes, corresponding to the respective

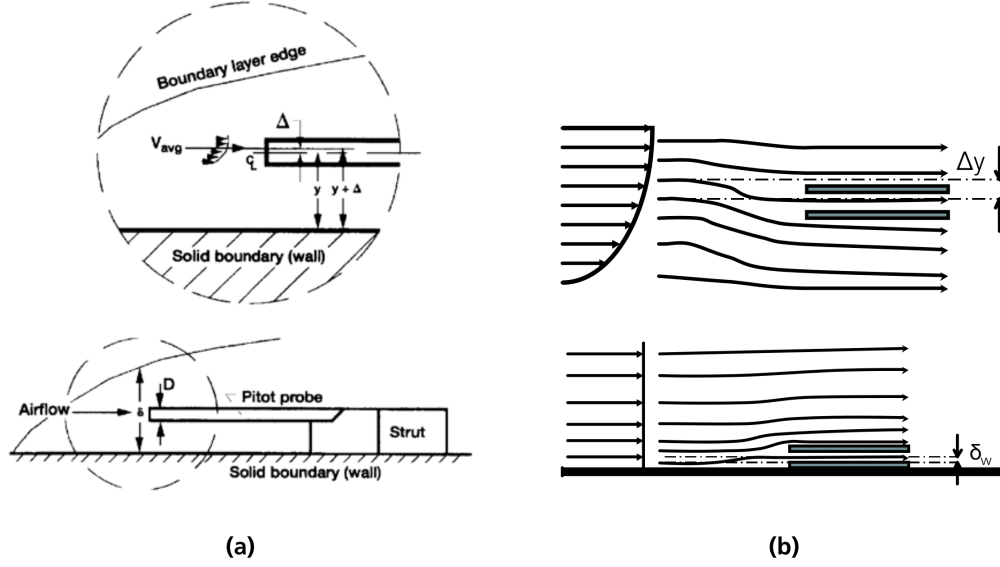


Figure 7.18: Centerline displacement effect: a) displacement due to averaging [92], b) displacement due to streamline displacement [158].

κ used. Figure 22 of Perry *et al.* [187] compares the results of both investigations together with a corresponding suggested log-fit of $\Delta U^+ = 1/0.41 \cdot \log(k^+) - 0.2$ and $\Delta U^+ = 1/0.4 \cdot \log(k^+) + 1.2$ for Perry and Joubert [185] and Perry *et al.* [187], respectively.

A further challenge is the typically small range of k^+ which can be accomplished in a single measurement campaign and therefore the choice for κ within $\Delta U^+ = f(k^+)$ may become inconclusive or subject to large uncertainties. At least within a single publication, the κ for plotting/correlating $U^+ = f(y^+)$ and $\Delta U^+ = f(k^+)$ need to be the same, otherwise the use of equations (3.72), (3.73) and (2.8) and its combinations in the derivation are inconsistent. If it would be the case that the extracted ΔU^+ is showing a different slope than the used κ in the plot via eq. (2.5) or (2.7), then the parameter B_r is not independent of k^+ and therefore would not show Reynolds independence, which is typically assumed for fully rough regime.

All challenges listed above contribute to the reason why the analysis of roughness results in general and especially the comparison between different authors is a delicate endeavor.

7.4 Additional Pitot Pressure Investigations

Various effects can influence the measured pressure from a Pitot probe which have been encountered in both subsonic and supersonic flows. E.g. the “centerline displacement” effect, which describes the displacement of the centerline of the Pitot probe. This occurs due to the violation of the assumption that only a single streamline approaches the probe [257]. This causes two different effects, which are shown in figure 7.18.

First, since the diameter of a Pitot probe cannot be infinitely small, the measured pressure is an average over the corresponding cross-section of the pressure profile. Since the pressure in the boundary layer is typically non-linear, the averaged value over a finite section does not exactly correspond to the value in the middle of this section, but is somewhat higher (see figure 7.18a after Grosser [92]). Second, due to the finite size of the probe, the stagnation point line is shifted, as shown in figure 7.18b after McKeon *et al.* [158]. Grosser [92] was able to identify the Mach number gradient $\partial Ma/\partial y$ and the inflow Mach number Ma_∞ as main influencing parameters and to determine a correlation for the displacement Δ depending on the external outer diameter D of the probe. She also found that the displacement Δ/D was insensitive to the boundary layer thickness δ , the probe diameter D/δ , and the Reynolds number Re . In subsonic conditions, this shift $\delta/D < 1$. The range of validity for their correlation is within $0.031/\text{mm} < \partial Ma/\partial y < 0.11/\text{mm}$, whereas in the present study corresponding Mach number gradients in the order of $1.21/\text{mm}$ occur. Therefore, this relationship cannot be used for the present measurements.

Moreover, in turbulent flow, an increase in the measured total pressure can occur due additional contributions of the velocity fluctuations. However, this effect can be ignored for a turbulence intensity $\sqrt{u'^2}/\bar{U} < 10\%$ according to [257]. This is applicable for the H2K flow at $Ma_\infty = 6$ with a turbulence intensity of $\sqrt{u'^2}/\bar{U} \approx 1\%$ (see e.g. [171], [94]).

Additional aspects were considered by Berg [24], who performed Pitot pressure measurements in a comparable flow regime to this work. He applied corrections according to Ramaswamy [201], who established an own empirical correction for supersonic and subsonic conditions based on Sherman [225] and Potter and Bailey [192] to correct for viscous effects and curved shock effects within a Mach number range of $0 \leq Ma \leq 6$. Literature review of more recent publications show that data by Sherman [225] is still baseline for corrections today [31], [257]. This correlation is split into a part considering the viscous subsonic portion of the flow behind the shock and a supersonic part considering the effect of a curved shock. The subsonic part is formulated with a dependency on Reynolds number based on the probe diameter. This should include the effect of viscous flow for which the Bernoulli equation is not valid anymore. In the latter case the stagnation streamsurface may extend forward of the probe tip which can lead to significantly different pressures, known from other investigations (see e.g. [257]).

Ramaswamy [201] reformulated the corrections as additive terms. For axisymmetric open-ended probes it can be given with the measured pitot pressure $p_{Pitot,m}$, the “ideal” pitot pressure $p_{Pitot,i}$, as calculated by e.g. eq. (3.1) and the Reynolds number $Re_{2,r}$, based on the probe radius, where all parameters with the suffix 1 and 2 indicate the condition upstream and downstream of a normal shock in front of the Pitot tube, respectively, to

$$\left(\frac{p_{Pitot,m}}{p_{Pitot,i}} - 1 \right)_{sub} = \frac{\frac{1}{2}\rho_2 u_2^2}{p_{Pitot,i}} \left[\frac{3.14}{Re_{2,r} \left(\frac{\rho_2}{\rho_1} \right)^{0.5}} \left(\frac{\rho_2}{\rho_1} \right)^{0.5} - 0.1136 \right] \quad (7.5)$$

$$\left(\frac{p_{Pitot,m}}{p_{Pitot,i}} - 1 \right)_{shock} = - \left[\frac{F}{80 + 0.536F^{2.1336}} \right] \quad (7.6)$$

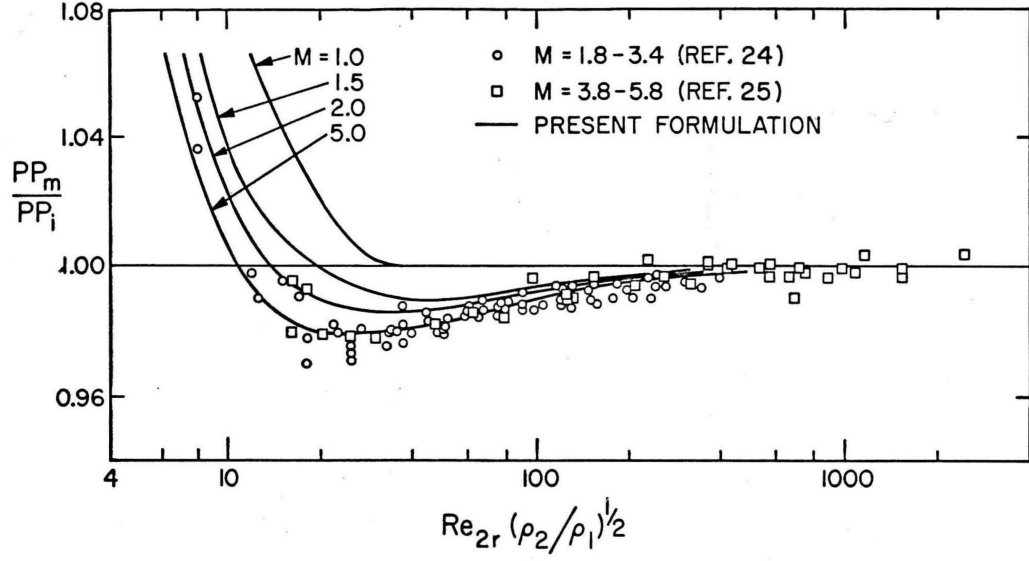


Figure 7.19: Viscous and shock correction to measured Pitot pressure in supersonic flow according to [201].

$$F = \left(\frac{\rho_2}{\rho_1} \right)^{0.5} \frac{Ma_1^2}{(Ma_1^2 - 1)} \quad (7.7)$$

$$\left(\frac{p_{Pitot,m}}{p_{Pitot,i}} - 1 \right)_{total} = \left(\frac{p_{Pitot,m}}{p_{Pitot,i}} - 1 \right)_{sub} + \left(\frac{p_{Pitot,m}}{p_{Pitot,i}} - 1 \right)_{shock} \quad (7.8)$$

Some authors suggest to use the inner instead of the outer Pitot tube radius (or diameter). In this work the inner radius was used, since it is conservative with respect to the lowest encountered Reynolds numbers for viscous corrections. In line with Berg, also in this work, only the viscous part of the correction was applied (eq. (7.5)) (see [24], [74]).

Figure 7.20 shows relevant plots for the viscous correction of measured Pitot pressures. In figure 7.20a the wall-normal position y/δ against the driving parameter for the correction with $Re_{2,r} (\rho_2/\rho_1)^{0.5}$ is shown for data from Berg (profile at $x = 39.4$ in and smooth-to rough case with $k = 0.05$ in according to [24]), TAU (profile at $x = 0.5$ m smooth surface for FC2) and experimental data (Run 2024&2025). The region below $Re_{2,r} (\rho_2/\rho_1)^{0.5} < 100$ is highlighted, since it is the value regime where viscous corrections start to become relevant (see figure 7.19). It is visible that approx. 60 % of the the experimental boundary layer profile by Berg falls within this region. This is different for data of this work, where only the wall nearest 10% of the CFD profile is in the relevant value range for correction. Due to the finite Pitot tube dimensions, the experimental data is only above the range deemed relevant for correction.

This is also visible in figure 7.20b, with actual amount of correction as calculated by the correlation of Ramaswamy, represented by $p_{Pitot,m}/p_{Pitot,i}$. In case of Berg's data, the correction peaks

at almost 10%, whereas for the current experimental data no correction is necessary. This is mainly influenced by the difference in density and tube dimensions. Berg used $T_0 \approx 420$ K and $p_0 \approx 16$ bar with an inner tube opening height (used here as diameter) of 0.004 in ≈ 0.1 mm, whereas in H2K it is $T_0 \approx 475$ K and $p_0 \approx 18.9$ bar with an inner tube diameter of 0.2 mm. This resulted in larger Reynolds numbers not prone to relevant viscous effects.

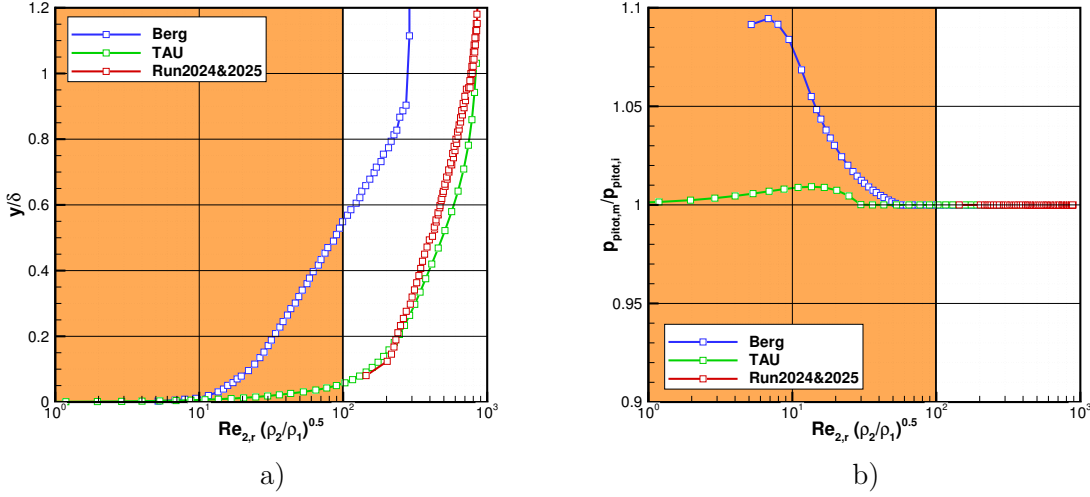


Figure 7.20: Viscous correction to measured Pitot pressures. a) Wall-normal position y/δ against driving parameter $Re_{2,r} (\rho_2/\rho_1)^{0.5}$ and b) $p_{Pitot,m}/p_{Pitot,i}$ against driving parameter $Re_{2,r} (\rho_2/\rho_1)^{0.5}$ for data from Berg (profile at $x = 39.4$ in and smooth-to-rough case with $k = 0.05$ in according to [24]), TAU (profile at $x = 0.5$ m smooth surface for FC2) and experimental data (Run 2024&2025)

7.4.1 Pitot Sensitivity Studies

In the following several different additional sensitivity analysis based on the Pitot pressure data are reported.

Run-to-run repeatability:

In general measured Pitot profiles showed good run-to-run repeatability, represented in figure 7.21 with the agreement of two smooth wall datasets at FC2 and MS2.

Velocity-temperature correlations:

To study the impact of considered velocity-temperature correlations (see section 3.2.3), figure 7.22a shows comparisons of temperature and in figure 7.22b velocity profiles with the nominal Crocco-Busemann velocity-temperature correlation (black symbols) against Walz correlation (red symbols) and the correlation according to Duan and Martin [58] for smooth runs at FC2 and MS1. Only small differences, all within corresponding uncertainties, are visible between the different temperature correlations.

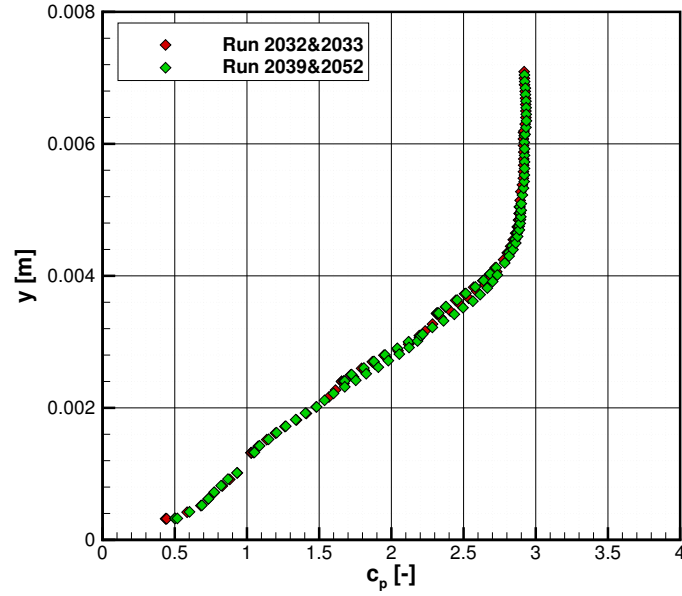


Figure 7.21: Repeatability of Pitot pressure coefficient profiles of smooth runs at FC2 and MS1.

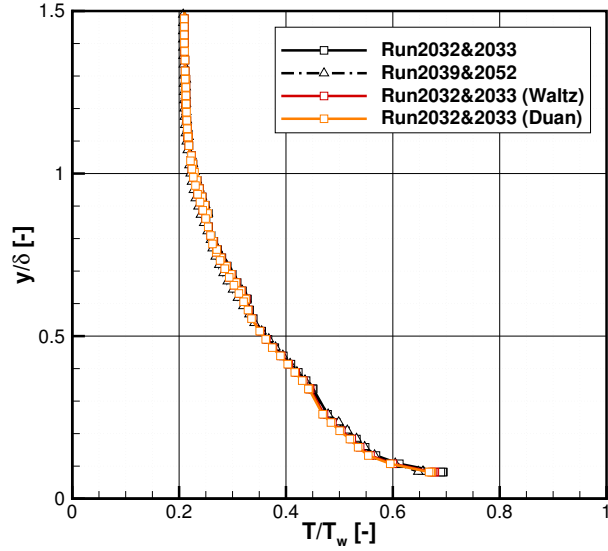
Pitot rake tube influence:

Additionally to the nominally used five tube Pitot rake (see section 3.2.2 figure 3.8) a single tube Pitot probe was manufactured. Only the boom of the Pitot measurement setup needed to be exchanged with a single probe device (see CAD in figure 7.23a). Selected tests were performed to cross-check with the Pitot rake results. Figure 7.23b shows a comparison of Pitot profile results from both measurement devices at FC2 and MS2 above the sand grain type roughness. It is visible that only negligible influences are visible, if all uncertainties are taken into account.

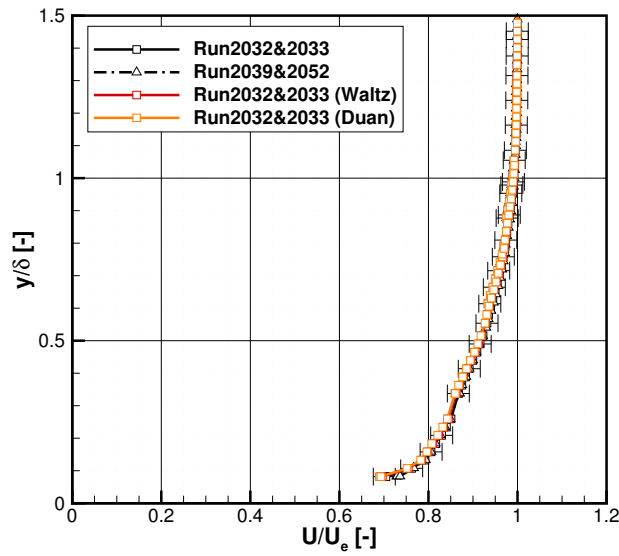
Measurement plateau influence:

Selected tests were performed with the Pitot rake positioned at half the streamwise extent of one of the smooth wall measurement plateaus within the technical rough wall (see section 3.2 figure 3.6 and figure 3.15). Figure 7.24 shows Pitot pressure coefficient profiles above the plateau (orange symbol, Run2072-2074) in comparison to a typical profile above the technical roughness (red symbol, Run2090&2091). The plateau has only negligible influence on the Pitot profile as visible by the agreement between the red and orange symbols.

If the data above the plateau is processed to inner scaled profiles, in general comparable behavior as corresponding profiles above the roughness (Run2084-2086 and Run2090&2091) can be observed in figure 7.25. Although, if closely inspected, small deviations for the wall nearest points are visible in inner scaling in contrast to the small differences in Pitot pressures (figure 7.24). This resulted also in small differences in the corresponding fitting results with e.g. a smaller ΔU^+ , visible in table 4.3. But a statement about an influence of the plateau is challenging since these differences still fall within corresponding uncertainties.

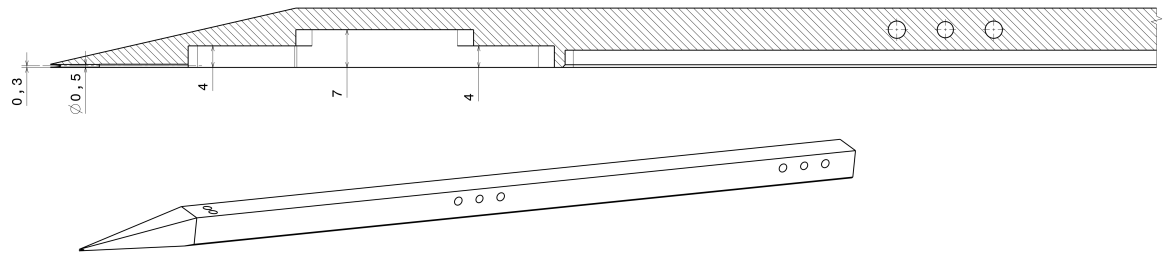


(a)

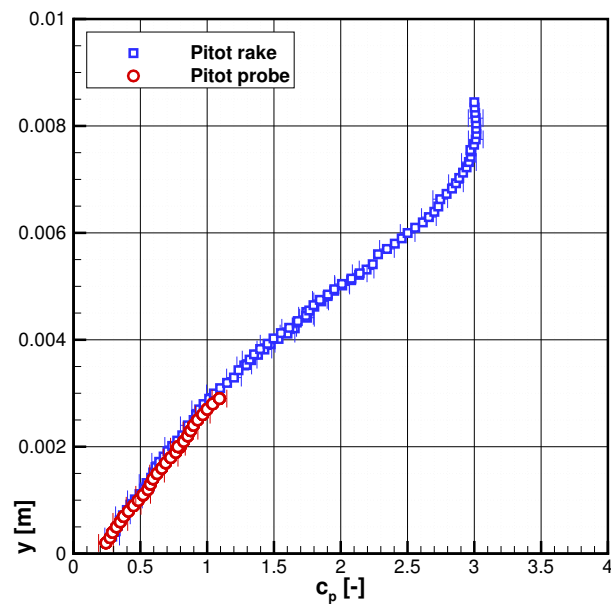


(b)

Figure 7.22: Comparison of a) temperature and b) velocity profiles with nominal Crocco-Busemann velocity-temperature correlation (black symbols) against Walz correlation (red symbols) and the correlation according to [58] for smooth runs at FC2 and MS1.



(a)



(b)

Figure 7.23: a) CAD of single Pitot probe and b) Pressure coefficients of Pitot rake vs. single Pitot probe measurements at FC2 and MS2.

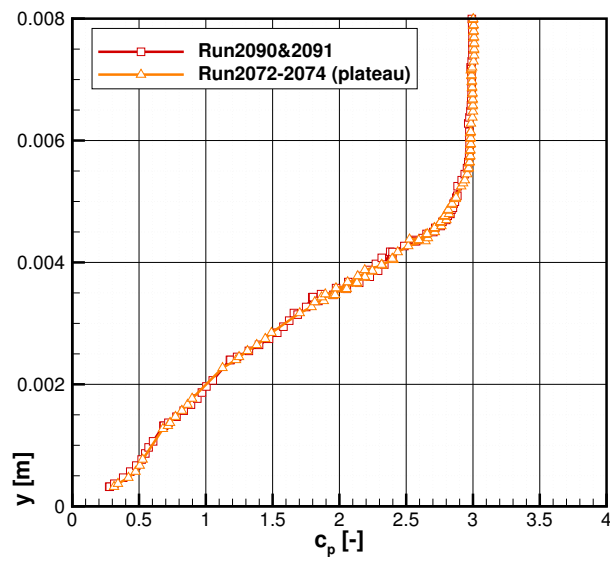


Figure 7.24: Pitot pressure coefficient profiles along rough wall (red symbol, Run2090&2091) and smooth measurement plateau within rough wall (orange symbol, Run2072-2074).

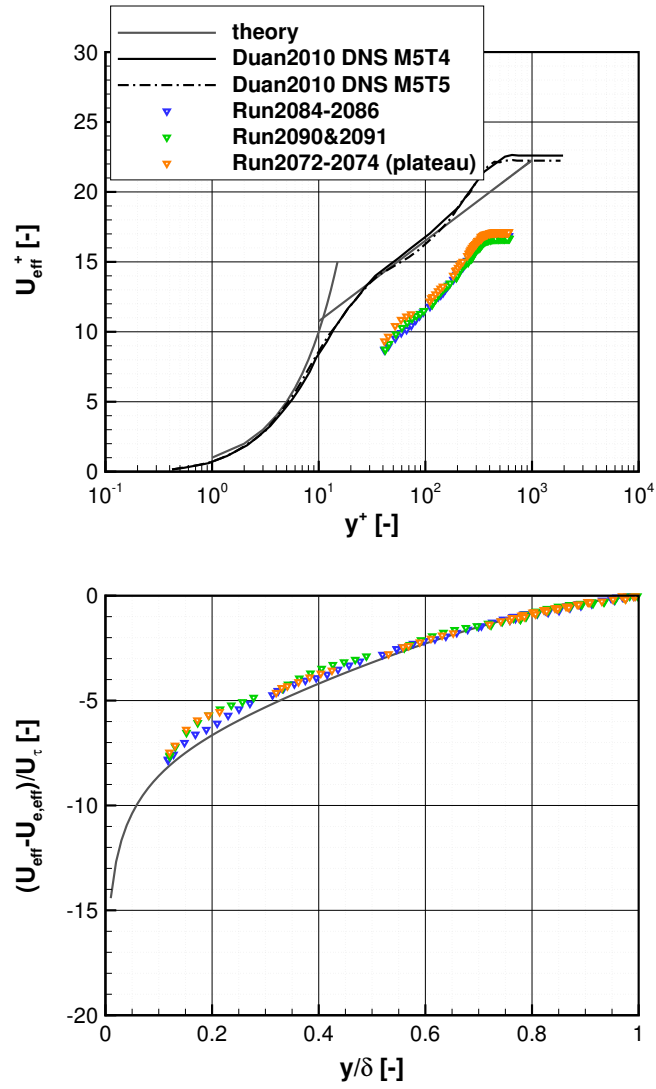


Figure 7.25: Inner and outer scaled van Driest transformed mean velocity profiles along technical square bar rough wall including data above the smooth measurement plateau within rough wall (orange symbol, Run2072-2074) together with the law-of-the-wall and DNS data according to [296].

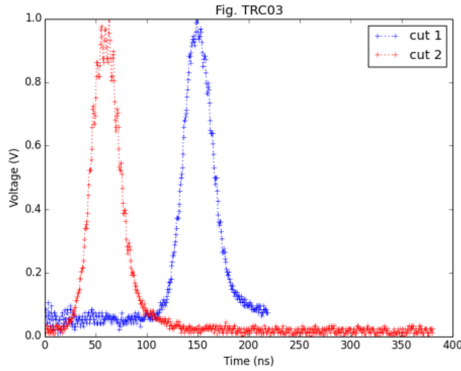


Figure 7.26: Laser pulse photodiode measurement

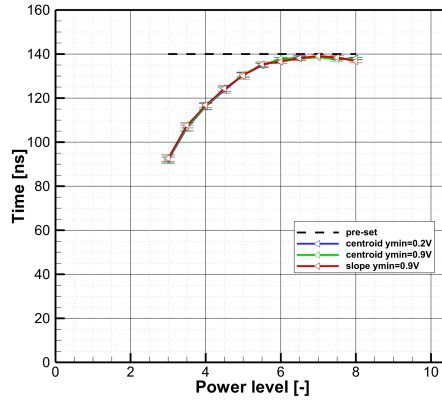


Figure 7.27: Laser pulse interval results for a pre-set value of 140 ns with different pulse detection algorithms

7.5 Additional PIV Investigations

7.5.1 Laser Characterization Studies

During the first PIV test campaign, a difference in pre-set and actual laser pulse time interval with laser power level was noticed. A comparable influence was also reported in [285]. A dedicated laser characterization was performed after the test campaign. The hardware (laser system plus trigger unit) was the same as during the test campaign but with an additional photo-diode (pin photodiode Thorlabs 201/579 – 7227) to measure the actual laser pulse time interval. The power levels of the laser were changed between 2 and 10 and the pre-set pulse interval between 130 and 4000 ns, which covered the setting conditions during the measurement campaign. A series of 20 tests was performed for each combination to enable statistical characterization. Due to the shape of the pulse peaks without a well defined maximum (see figure 7.26), the laser pulse interval was determined with several different detection algorithms. This included a Gaussian fit to each peak, determining the centroid of each peak above a normalized voltage and determining the first slope of each peak above a normalized voltage. The results of all algorithms overlapped within the uncertainties of the measurements (see figure 7.27).

A pre-set pulse interval of 140 ns resulted in a deviation in between 5 to 20 ns by changing the laser power level in between 4 to 6. This characterization resulted in corrected pulse intervals for all combinations of pre-set pulse interval and power level used during the test campaign. The standard uncertainty of the extracted deviations was in the order of 1 ns or below.

7.5.2 PIV Sensitivity Studies

7.5.2.1 Methodological Sensitivities

In the following several sensitivities for the PIV methods and post-processing are reported. Results presented in this chapter have previously been published in part by Neeb *et al.* [171], [169].

For the measurement of mean and turbulence quantities via PIV, different additional random and bias errors may affect the data (see e.g. [4]). Selected impacts, deemed to be most relevant, are investigated and reported in the following for both PIV campaign setups. The measurement of velocity fluctuations, or turbulent intensities, is sensitive to the choice of IW size. Choosing the size too small for a given data quality increases the random noise whereas choosing it too large increases the amount of sub-grid filtering. Both influences can compensate each other so that care must be taken for the choice of a proper IW. In this study, both influences were addressed based on different IW sizes.

First PIV campaign:

First, selected impacts on the mean velocity results were analyzed.

interrogation window:

Sensitivity studies were performed with synthetic PIV images based on TAU RSM data of the cone flow to test the influence of the interrogation window size on the mean velocity profiles (see section 7.5.4). Especially the impact of stretched, high aspect ratio, interrogation windows was of interest. This type of window was selected to increase the wall normal resolution but at the same time ensure an acceptable amount of particles per interrogation window. This window shape is supported by the nature of the boundary layer with a much higher streamwise than wall-normal velocity magnitude. Interrogation window sizes were selected according to the analysis of the experimental data with $512 \times 64 \text{ pix}^2$, $512 \times 32 \text{ pix}^2$, $512 \times 16 \text{ pix}^2$ and $512 \times 8 \text{ pix}^2$ and multi-grid analysis with 256×256 to $96 \times 64 \text{ pix}^2$ (published in [170]). Valid results could be extracted from the synthetic images for all tested interrogation windows. Only in the immediate vicinity to the wall ($y/\delta \lesssim 0.1$ or $y^+ \lesssim 20$), differences of pixel shift above $\Delta dx = 0.1 \text{ pix}$ (corresponding to 0.7% of the edge velocity) were detected which were mainly caused by increased velocity gradients in this regions (see e.g. figure 7.62). The drawback of using the aforementioned high aspect ratio interrogation windows is a limited resolution in streamwise direction. But estimations based on TAU simulations resulted in an uncertainty below $\Delta U/U_e \approx 0.3\%$.

After utilizing synthetic images, the same interrogation windows were applied on the experimental data of run 1007. Profiles in wall-normal direction were extracted at the mid points in streamwise direction of the corresponding 2D vector fields. All experimental profiles nearly collapsed, independent of interrogation window size, and the differences fell well below the corresponding uncertainties (see figure 7.43a). Therefore, the interrogation window $512 \times 8 \text{ pix}^2$ was chosen to perform all mean velocity analyses due to its high resolution in wall-normal direction (see section 4.2).

wall temperature:

Due to the chosen wind tunnel model material PEEK, the wall temperature rose during a wind tunnel run by approx. 3 – 10 %, corresponding to a density change and a viscosity change of approx. 2 – 9 %. The dynamic viscosity in this thesis was calculated via Sutherland's law, extended for low temperatures according to [1], with

$$\mu = \begin{cases} C_2 \frac{T}{C_1}, & \text{if } T < 120\text{K} \\ C_3 \frac{C_4}{T+C_5} \left(\frac{T}{C_6}\right)^{1.5}, & \text{otherwise} \end{cases} \quad (7.9)$$

with $C_1 = 120\text{ K}$, $C_2 = 8.3310^{-6}\text{ kg/(m s)}$, $C_3 = 17.8910^{-6}\text{ kg/(m s)}$, $C_4 = 398\text{ K}$, $C_5 = 110\text{ K}$, $C_6 = 288\text{ K}$.

This influence was analyzed based on numerical calculations with changing wall temperatures, according to the wind tunnel tests. For this, the temperature profiles $T_w = f(x_a)$ of N time points with a frequency of approx. 27 Hz from IR data were used as input to simulate N cases with the BLcode. The boundary layer profile at MS1 of all N simulations was then used for further post-processing. An ensemble averaged velocity field from all temperature computations was compared to the velocity from a single calculation with a mean wall temperature. This resulted in an estimated maximal difference at MS1 of $\Delta U/U \approx 0.5\%$, limited to a region close to the wall for approx. $y^+ \lesssim 20$ ($y/\delta \lesssim 0.1$). This difference was included in the uncertainty for the detailed analysis of viscous sublayer velocities (gradient approach for the friction velocity). For larger wall normal distances, including the logarithmic layer (relevant e.g for the fitting for the friction velocity, see chapter 4.2), the difference dropped below $\Delta U/U \approx 0.02\%$ and was therefore neglected.

particle slip:

The finite particle response can have a major influence on velocities measured via PIV, especially in low density, hypersonic flows [199], [285]. Simulations were performed with the particle module described in chapter 3.9.2 to approximate the impact on the velocity profiles in the boundary layer under given experimental conditions. Since only manufacturer data for the used particles and no in-situ measurements were available, sensitivity studies were performed to investigate the impact of these parameters on the particle response. On the one hand the manufacturer values of the applied KRONOS 1002 particles for diameter and density were taken with $d_{p1} = 0.23\text{ }\mu\text{m}$ and $\rho_{p1} = 3800\text{ kg/m}^3$, as representative values for a state without any agglomeration. On the other hand approximating an agglomerated state, particle parameters with $d_{p2} = 2.0\text{ }\mu\text{m}$ and $\rho_{p2} = 800\text{ kg/m}^3$ were used according to Williams [285], who derived these values experimentally for comparable TiO_2 particles at comparable particle Mach and Reynolds numbers.

The particles were seeded into a TAU RSM flow solution at FC1 and $T_w = 360\text{ K}$ (see chapter 7.7.2). Figure 7.28 contains variations of different parameters for a single particle trajectory through the flow field along the cone, exemplary chosen with p_2 . It is visible that upstream of the oblique shock in front of the cone (for the particle at $x \approx 0.22\text{m}$), all parameters except the Knudsen number and the scaled drag coefficient are zero, since particles were seeded into the free-stream without any relative velocity. This assumption neglects particle slip throughout the nozzle flow which was unknown at the time of writing. The estimated Knudsen numbers with $Kn \approx 0.3$ are non-negligible, so that the calculated drag coefficients are only approx. half the corresponding value from Stokes (see drag model descriptions in section 7.7.3). Immediately

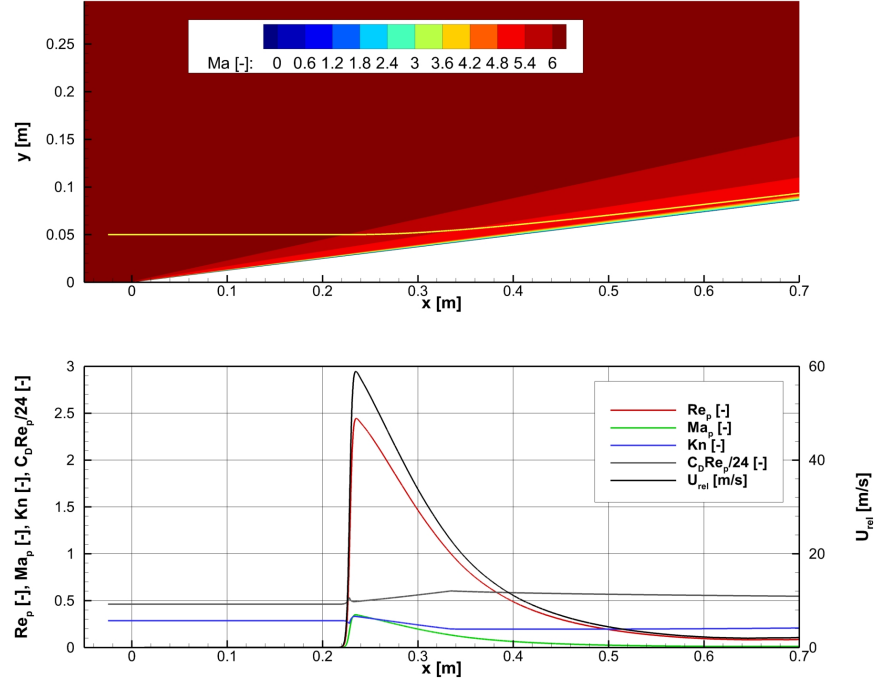


Figure 7.28: Variations of different parameters for a single particle trajectory through the flow field along the cone for particle p_2 in a TAU RSM flow solution at FC1 and $T_w = 360$ K

downstream of the shock, the relative particle velocity increases but the corresponding relative Mach number stays below approx. 0.4. Therefore, particles are in subsonic regime but may be affected already slightly by compressible effects. As the particle flows further along the cone, it is visible that the relative values like velocity decrease again, so that the particle is slowly adapting to the surrounding flow field (which is also changing along the trajectory).

The same CFD flow field was used to seed a number of particles (with identical properties for density and size) which were then caught in a plane, corresponding to MS1. With this procedure, particle velocity profiles can be extracted and compared to flow velocity profiles at MS1. Figure 7.29 contains the modulus of all relative particle velocity components $|\vec{V}_{rel}| = (U_{rel}^2 + V_{rel}^2)^{0.5}$ (where $U_{rel} = U_p - U$), based on the streamwise edge velocity component U_e to $|\vec{V}_{rel}|/U_e$ for each caught particle p_1 and p_2 against the wall normal distance, normalized by the local boundary layer thickness δ . The collected particles show negligible particle slip outside and near the boundary layer edge, whereas an increase towards the wall is visible. The maximum values are reached with $|\vec{V}_{rel}|/U_e \approx 1\%$ for particle p_1 and $|\vec{V}_{rel}|/U_e \approx 2.1\%$ for particle p_2 . The difference in velocity is mainly driven by only weak successive deceleration of high-speed particles through the low-density boundary layer region. Although first decelerated by passing the bow shock in front of the cone, high Knudsen numbers and inertia effects lead to a persisting higher velocity than the surrounding fluid. This difference decreases with increasing running length, visible in figure 7.29 for the second measurement section MS2. Here the maximum relative

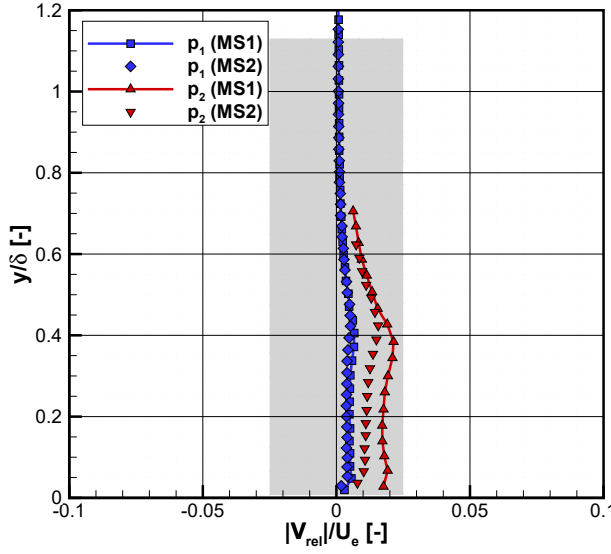


Figure 7.29: Relative velocity difference of particles p_1 and p_2 along the 7° cone at FC1 at measurement section MS1 and MS2 (shaded region corresponds to PIV set-up velocity uncertainty)

velocity drops to $|\vec{V}_{rel}|/U_e \approx 1.5\%$ for particle p_2 .

Although on a noticeable level, the difference due to slip is not included in the general PIV uncertainty but kept in mind for later discussions. This decision was mainly based on the fact that only sensitivity values for the particle parameters were available and so only a tendency could be estimated. Additionally, previous PIV measurements using comparable particles in a $Ma \approx 7$ boundary layer flow showed no signs of slip influence on the mean velocity profiles [286], [287].

image number convergence:

To check the convergence of the mean velocities, the cumulative mean of the streamwise velocity for different amounts of images N was analyzed at three positions outside the boundary layer. The extracted data between $y/\delta = 1.59 - 1.72$, including its uncertainties, showed an agreement with the corresponding Taylor-Maccoll edge velocity as reference for image numbers approx. $N > 25$ and nearly constant behavior above $N \gtrsim 150$.

In the following, selected impacts on the velocity fluctuation results are reported. Several different influences can affect the extraction of turbulence intensities via PIV measurements (see e.g. [4]). The influence of peak-locking, random errors, spatial resolution, particle lag filtering and convergence with image number was analyzed and is discussed in the following.

peak-locking:

Peak-locking is another potential uncertainty for PIV results, where the displacement vectors are biased towards integer values. This phenomenon has only limited effect on mean properties but

can have sizable impact on first order variances [285]. According to Williams [285] peak-locking in hypersonic PIV can be attributed to three different origins. One common reason is when particle images are too small and the sub-pixel accuracy is not accomplished by typical post-processing routines. To counteract, the particle image size should exceed two pixels. A second reason might be through pixel saturation leading to a top-hat profile of the correlation peak and which also corrupts sub-pixel accuracy. CCD cameras are known to suffer from pixel “blooming” which means that charge can bleed from saturated to neighboring pixels. The third reason stated is high shear in the flow field. Locally high shear can cause particles to move at different speeds within one IW. This can lead to a broadening of the corresponding correlation peak. If this broadening is wider than expected by the post-processing algorithm, this can lead to peak-locking. Figure 7.30a shows an exemplary pdf of a smooth wall run (Run 1007), which shows no signs of peak-locking.

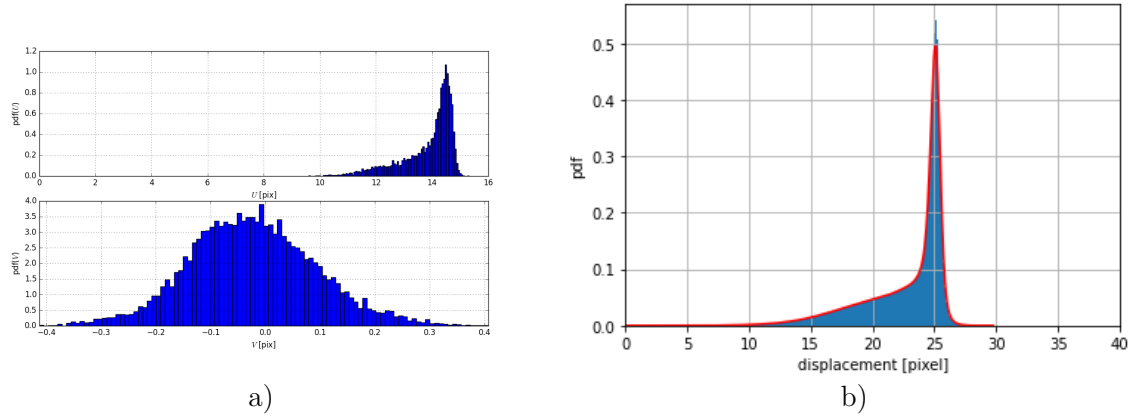


Figure 7.30: Global displacement histograms, illustrating that no peak-locking occurred in PIV results of a) first (Run1007, IW 256x256 pix²) and b) second PIV campaign (Run3007, IW 64x64 pix²)

random error:

There are different options to characterize PIV data with respect to random errors, of which two were chosen for this thesis. One is based on the two point-correlation results and the other by changing the laser pulse time interval. Only the first one was applied for data of the first PIV campaign, whereas both were applied for data of the second campaign. The first approach will be introduced in the following, whereas the second one will be introduced in the description of the second campaign.

The first approach utilizes two-point correlations in the streamwise direction derived from the instantaneous velocity vectors of all recorded images according to Williams [285], originally based on a work by Discetti *et al.* [54].

The two-point correlation for the streamwise velocity fluctuations R_{uu} can be defined by eq. (3.13). The approach is based on the assumption that random PIV measurement noise is statistically uncorrelated between non-overlapping interrogation windows. Therefore, information from the correlation curve of R_{uu} with a distance Δx greater than the overlap can be used to extrapolate

an unbiased value at zero distance. For this, in equation (3.13) y is set to y_{ref} to reduce it to the one-dimensional correlation. Typically, the correlation curves describe a parabola shape for small distances and transitions into an exponential decay for larger distances (see e.g. [190]). For highly resolved data, a parabola fit to the data could be used to identify random noise. Instead, in this work a linear fit was used according to Williams [285], which tends to underestimate the error but is a viable alternative to a parabolic fit in case it is not feasible due to spatial resolution.

The corresponding *Signal – to – Noise – Ratio*(SNR) can be derived via

$$SNR = \frac{R_{uu,lin}(0,0)}{1 - R_{uu,lin}(0,0)} \quad (7.10)$$

where $R_{uu,lin}$ corresponds to the linear regression of the nearest, non-overlapping data points to the origin position at $\Delta x/\delta = 0$. An exemplary fit to data of Run1007 with IW 128x128 pix² from the second PIV campaign is shown in figure 7.40a.

Unfortunately no random error could be extracted directly for the high aspect ratio interrogation windows with a streamwise extent of 512 pixel, since no non-overlapping points are available for the two-point correlation within the current ROI. Instead, a multi-grid approach starting from 512x512 pix² to a final size of 256x*iy* pix², with *iy* for different wall-normal dimensions, was used to approximate the error. Due to non-consistent two-point correlation curves, the results for windows with 256x8 pix² and therefore also 512x8 pix² were discarded for further turbulent analyses.

At the boundary layer edge, windows with 256x16 pix², 256x32 pix² and 256x64 pix² resulted in random errors of approx. 27%, 28% and 12% of the local turbulence intensity, respectively. For the same interrogation windows, the analysis for points in the boundary layer resulted in random errors of approx. 27%, 19%, 13% at $y/\delta \approx 0.8$ and 65%, 67%, 27% at $y/\delta \approx 0.5$. For a reference value, a large interrogation window with final size of 256x256 pix² was used, restricted to the boundary layer edge due to its size. This window resulted in a random error of 5%. This decrease of error with increasing window size could imply that the amount and distribution of particles in the flow was only sufficient for a moderate quality of turbulence data. Before a final decision for a suitable window size was made, the sub-grid filtering was also analyzed.

sub-grid filtering:

The drawback of using larger interrogation windows to reduce random errors is an increase of sub-grid filtering. There are different approaches to address this effect, i.e. making filtering estimations based on the IW sizes [284], [238], based on the IW and FOV size [80], based on frequency spectrum analysis (for channels [160], [61]) or deriving parallels based on filtering of different measurement techniques, like Hot Wire Anemometer (HWA) [287]. We chose two approaches for the data of this thesis. First an approach by Spencer[238], who used the IW size on synthetic and experimental data to derive estimates for the amount of filtering. And second by Lee *et al.* [144], who created a database approach based on DNS data of an incompressible turbulent boundary layer which uses the inner-scaled interrogation window size and the laser sheet thickness as input.

Spencer approach:

In the first approach, Spencer and Hollis [238], performed analyses with synthetic and experimental data and compared the results to theoretical curves by Host-Madsen and McCluskey [111], who based their work on the assumption of homogeneous isotropic turbulence for particles within a single interrogation window. Both works investigated the impact of interrogation window size, but exclusively for an aspect ratio $AR = 1$. Since in this paper stretched windows with an aspect ratio $AR \neq 1$ were used, the approach needed some extensions. For this, the same synthetic data approach by Spencer and Hollis [238] was used, as described in the following.

Synthetic velocity fields with pre-defined mean velocities, Reynolds stresses and two-point correlations were created by applying digital filtering based on a technique introduced by Klein *et al.* [130] (see section 7.5.6). The sub-grid filtering is then approximated by spatial averaging the synthetic velocity field, where the corresponding spatial box represents a simplified approximation of an interrogation window. This approach just simulates the filter characteristics without being influenced by typical additional factors of PIV analysis like e.g. particle density or size distribution. See [238] and [130] for further details of this approach.

At this point, the symmetrical spatial box filter was extended to an asymmetrical box, approximating interrogation windows with different aspect ratios. The approach was first validated for windows with $AR = 1$ with data by Spencer and Hollis [238].

For the validation, Case 2 point A from [238] was used which intended to simulate homogenous (isotropic) turbulence with an exponential two-point correlation incorporating prescribed integral length scales and assumed constant mean and fluctuating velocities throughout the field with $u'/U = 0.2705$ (Table 1 in the corresponding publication). The synthetic field in our case had a size of $M_y \times M_z = 30 \times 40 \text{ pix}^2$ and the integral length was fixed to $n_y = n_z = 5 \text{ pix}$. Additionally, values for mean velocities, Reynolds stresses, field size and number of fields were varied and the influence on the results was found to be negligible.

To derive the synthetic fields, the kernel to derive the filtered velocities $\tilde{u}$ was defined by

$$\tilde{u}_i(y, z) = \frac{1}{(N_y N_z)} \sum_{y-r_y}^{y+r_y} \sum_{z-r_z}^{z+r_z} u_i(y, z) \quad (7.11)$$

with the size $N_\alpha = 2r_\alpha + 1$ and half lengths r_α with $\alpha = y, z$. The local averaging applied by Spencer and Hollis was rebuild with $N_y = N_z$ from 1 (unfiltered) to 17, which corresponds to $r_y = r_z$ ranging from 0 to 8. We extended the synthetic data by using the same filtering Kernel (eq. 7.11) but let $r_y \neq r_z$, so that each varied from 0 to 8, to simulate IW with non-unity aspect ratios. The ensemble average mean and rms velocity as well as integral length scale are evaluated based on $s = 2000$ images with the above filters applied. This was sufficient to derive reliable statistics.

Figure 7.31 shows the expected effect of spatial filtering on the synthetic rms velocity compared to theory according to the original publication by Spencer and Hollis [238]. Here “HMN” means Hoest-Madsen and Nielsen and corresponds to theoretical work by the same authors in [111]. “Synth” means the synthetic data Spencer derived with the above described DFM. “Integral” and “Taylor” corresponds to the two different characteristic turbulent length scales and “Expon” and

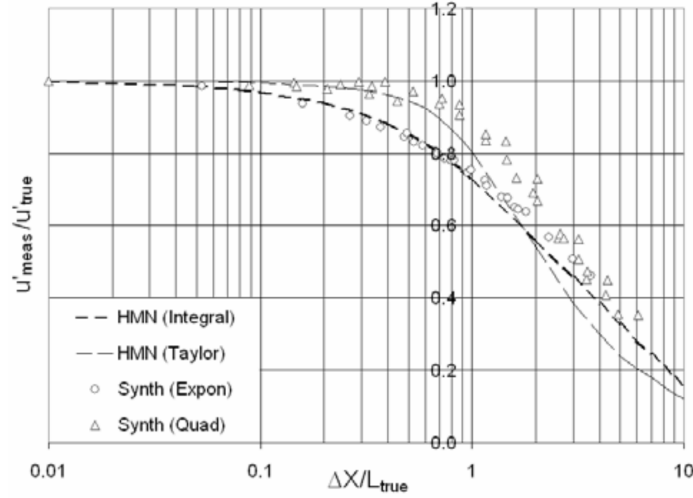


Figure 7.31: Effect of spatial filtering on synthetic rms velocity compared to theory according to [238].

“Quad” (exponential and quadratic, respectively) to the shape of the corresponding correlation functions, which are intended to artificially approximate the former. The plot shows the general trend that increasing IW size (Δx) leads to increasing filtering of the velocity fluctuations (here represented by the rms velocity u').

Generally, the analyzes by Host-Madsen and McCluskey [111] and Spencer and Hollis [238] assume homogenous turbulence for the scales of one IW. In this case, it can be assumed that integral length scales in the lateral direction is half the value of the one in longitudinal direction [191]. Nevertheless it needs to be mentioned here that Spencer used anisotropic length scales only for the quadratic form of the correlation for his synthetic data (page 18 in [238]). Therefore, in case of his synthetic data from the exponential correlation form, the results do not actually correspond to the assumption of homogeneous flow. First, for validation purposes, synthetic data was analyzed based on his strategy with $n_y = n_z$. After that, the impact of assuming homogenous turbulence is analyzed ($n_y \neq n_z$).

Figure 7.32 shows exemplary two-point correlations of the derived synthetic data in this thesis and visualizes general effect of spatial filtering. Here, the red line shows the reference input correlation that was fed into the DFM framework. The symbols correspond to the different combinations of filter widths and heights and are summarized as $N_y/n_y \times N_z/n_z$ plot in figure 7.33a with a value range of 0.2 to 3.0. The corresponding IWs with $AR = 1$ are indicated with larger symbols (diagonal in figure 7.33a). As a sanity check, in figure 7.32 the blue line with sphere symbols shows the extracted two-point correlation from the synthetic data without filtering ($r_y = r_z = 0$, $N_y = N_z = 1$, $N_y/n_y = N_z/n_z = 0.2$). A good agreement with the input correlation can be realized. By increasing the size of the filter window, the width of the correlation peak widens, as visible in figure 7.32. This leads to an erroneous increase of the extracted integral length scale, previously noted by [238].

This trend is also visible in figure 7.33b, which plots the resulting effect of spatial filtering on the

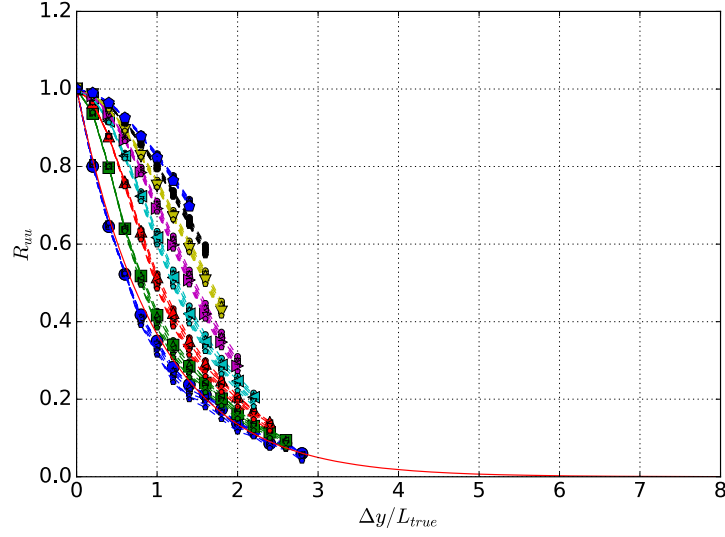


Figure 7.32: Effect of spatial filtering on synthetic two-point correlation compared to reference case (red line, for symbols see figure 7.33a).

extracted rms velocity of the synthetic data. It is represented as ratio of filtered to true value, marked as u'_{meas}/u'_{true} and is compared to theory by Host-Madsen and McCluskey [111] for both unity and non-unity aspect ratio IWs. The x -axis uses the same scaling as Spencer and Hollis [238] with $(\Delta x/L_{True})_{eff} = N_y/n_y$. The larger symbols, reflecting $AR = 1$, follow closely the theoretical curve by Host-Madsen and McCluskey [111], in agreement to the corresponding plot in Spencer's publication (replotted in figure 7.31). This confirmed that the implemented DFM framework worked nominally.

If now data of non-unity AR IWs is investigated in figure 7.33b, it is visible that the data deviates from the theoretical curve. This deviation can also be interpreted as failing of the scaling according to Spencer and Hollis [238] with $(\Delta x/L_{True})_{eff} = N_y/n_y$. If instead an alternative scaling, including both filter scales N_y and N_z , with $(\Delta x/L_{True})_{eff} = (N_y/n_y \cdot N_z/n_z)^{0.5}$ is assumed, figure 7.33c shows that the non-unity AR collapse much better to the theoretical curve. However, for larger $AR \gtrsim 5$ the agreement gets worse. Therefore, another scaling is suggested which still includes both length scales but added vectorially together with an extra term which emphasizes the impact of non-unity AR according to

$$(\Delta x/L_{True})_{eff} = \frac{1}{\sqrt{2}} \left(\left(\frac{N_y}{n_y} \right)^2 + \left(\frac{N_z}{n_z} \right)^2 \right)^{0.5} AR^{-0.15} \quad (7.12)$$

where $AR = \min(N_y/N_z, N_z/N_y)$. Here the factor $1/\sqrt{2}$ ensures that eq. (7.12) reduces to the formulation by Spencer and Hollis [238] with N_y/n_y in case of $N_y = N_z$, $n_y = n_z$ and $AR = 1$. With this scaling the investigated data collapse to the theoretical curve within $\pm 10\%$ (indicated

with thinner lines) as visible in figure 7.33d and the theoretical curve by Host-Madsen and McCluskey [111] is still applicable.

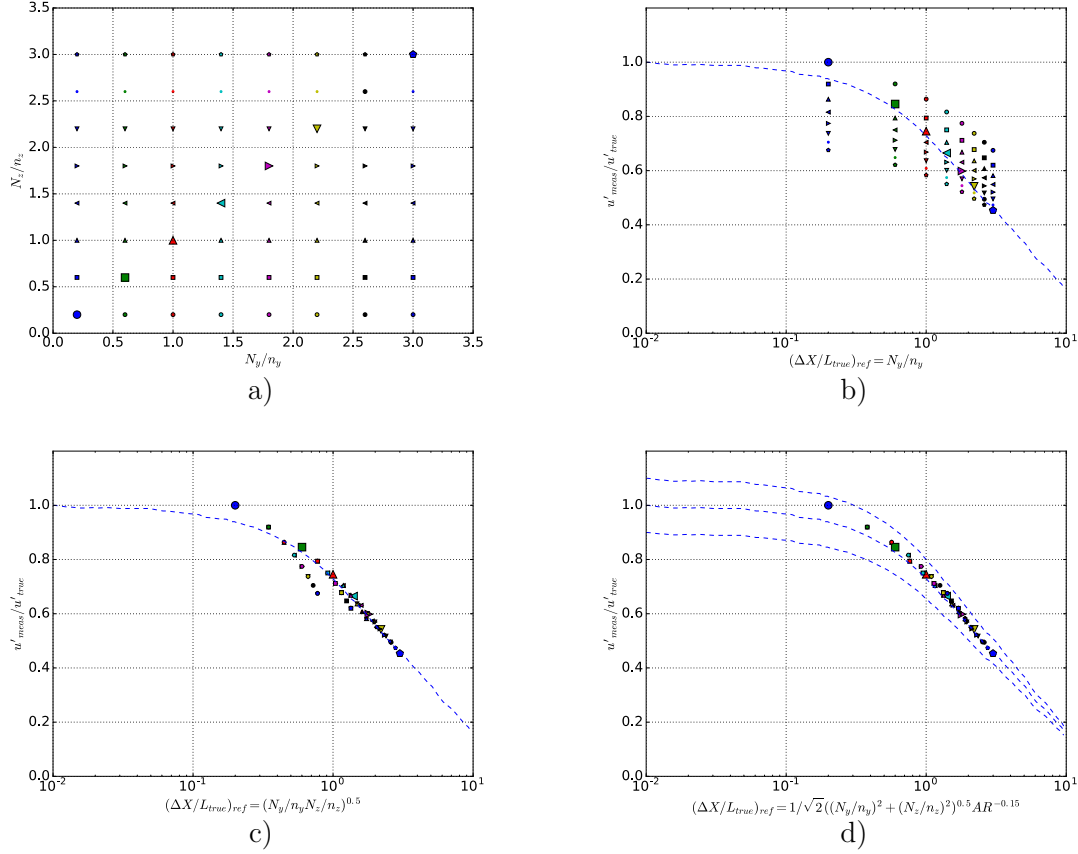


Figure 7.33: Effect of spatial filtering on synthetic rms velocity compared to theory according to the method by [238] for unity and non-unity aspect ratio IWs. a) shows legend of symbols used in plots b) to d) with their corresponding combination of $(\Delta x/L_{True})_{eff} = (N_y/n_y N_z/n_z)^{0.5}$, d) $(\Delta x/L_{True})_{eff}$ according to eq. (7.12) (thinner lines show $\pm 10\%$ of theoretical line according to [111])

To analyze the impact of assuming homogenous turbulence (see discussion above), Spencer's DFM approach was repeated with an exponential correlation form but with a lateral to longitudinal integral length scale ratio of ${}^yL_{uu}/{}^zL_{uu} = 0.5$, i.e. with $n_y = 10$ and $n_z = 5$. The results with the same scalings applied as before are contained in figure 7.34. Additional data with r_y and r_z from 0 to 16 are included, resulting in the same N_y/n_y range as before and extending the N_z/n_z range up to above 6.

To compare to the approach by Spencer, we can first investigate the unity AR IW data in figure 7.34b (large symbols) applying the same scaling as Spencer with $(\Delta x/L_{True})_{eff} = N_y/n_y$. It is visible that the filtering impact is now larger than the theoretical curve for increasing filter width N_y/n_y . This is due to the additional impact of the now relevant ratio of N_z/n_z , not

reflected in Spencer's scaling. This shows that with the scaling according to Spencer the amount of rms velocity filtering of truly homogenous synthetic data is underpredicted. Figure 7.34c and figure 7.34d show the alternative scalings previously described. It is visible that the scaling including both filter and length scales according to eq. (7.12) are capable of collapsing the data in case of homogenous turbulence assumption to the theoretical curve by Hoest within $\pm 10\%$.

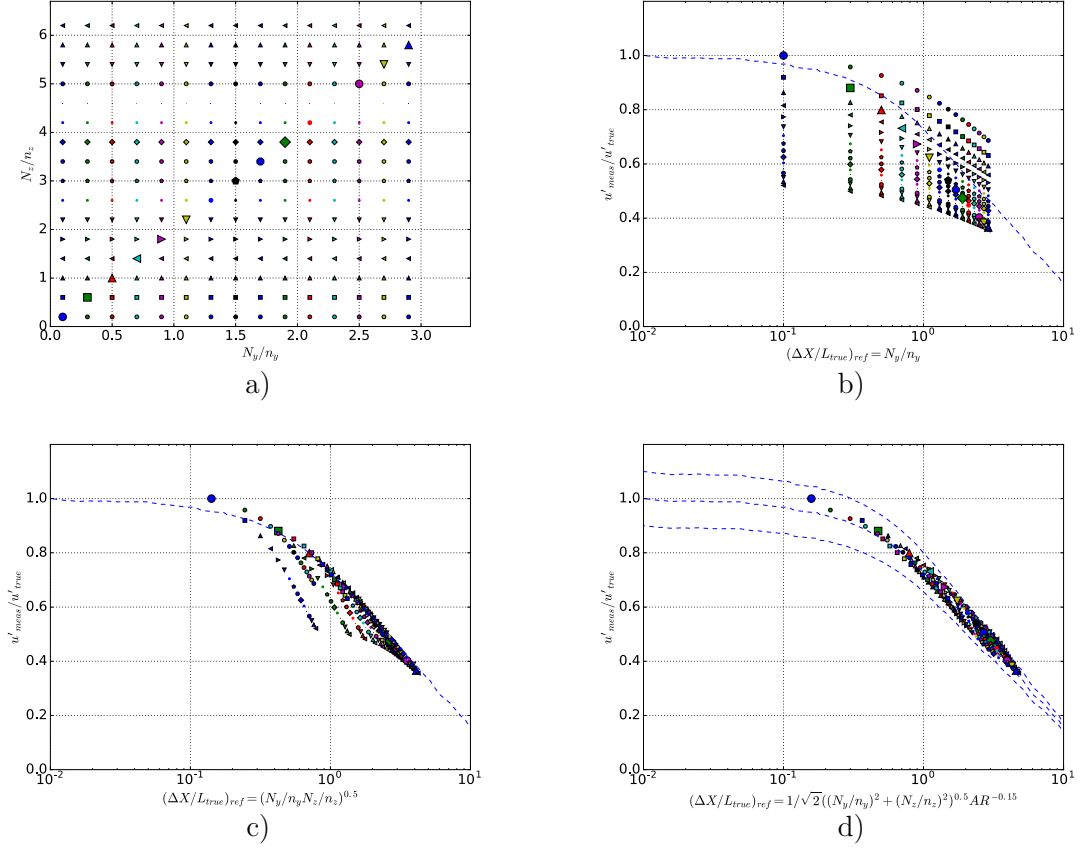


Figure 7.34: Effect of spatial filtering on synthetic rms velocity compared to theory according to the method by [238] for unity and non-unity aspect ratio IWs and length scales according to homogenous turbulence. a) shows legend of symbols used in plots b) to d) with their corresponding combination of $N_y/n_y \times N_z/n_z$. Rms velocity plotted against b) $(\Delta x/L_{True})_{eff} = N_y/n_y$ as used [238], c) $(\Delta x/L_{True})_{eff} = (N_y/n_y N_z/n_z)^{0.5}$, d) $(\Delta x/L_{True})_{eff}$ according to eq. (7.12) (thinner lines show $\pm 10\%$ of theoretical line according to [111]).

To check the very large AR used in the analysis of the H2K data ($512 \times 8 \text{ pix}^2$, $512 \times 16 \text{ pix}^2$, $512 \times 32 \text{ pix}^2$ and $512 \times 64 \text{ pix}^2$) corresponding to AR up to 64, additional synthetic data was created with the described DFM approach, since the above data set to rebuild Spencer's results included only AR up to 29. This made additional DFM fields necessary with $M_y \times M_z$ of 140×64 and integral length scales of $r_y \times r_z$ of 32×32 and $r_y \times r_z$ of 32×16 for the non- and homogenous turbulence case, respectively. It could be shown that the scaling according to eq. (7.11) still holds for these large AR data.

With the above described framework it was possible to approximate the sub-grid filtering effect in case of the H2K setup. One missing piece was the necessary true integral length scale L_{True} to calculate $(\Delta x/L_{True})_{eff}$ together with the known size of the IWs for Δx . A value for this length was based on literature data, since the limited PIV ROI and resolution of the first PIV campaign data prohibited an extraction from the data directly. In general, an extraction of a “true” value is challenging in both experimental and numerical work. Experimentally derived values may suffer from combined temporal and / or spatial filtering and random errors. Numerically derived values may suffer from a limited domain size due to computational costs. This has an impact especially on large scale structures known to be existent in the outer layers of the TBL (see e.g. [2]). Therefore, it was decided to use a lower and upper estimate to investigate a range of possible values for the integral length scale in this work.

First, a lower value with $L_{uu} = 0.5\delta$ was assumed, based on data by Peltier [183]. The author investigated a $Ma = 4.9$ turbulent boundary layer via PIV measurements at a higher Reynolds number $Re_\theta = 23000$ ($Re_{\delta_2} = 5300$) than H2K data but comparable wall temperature with $T_w/T_{aw} = 0.93$. The author approximated the area below the two-sided two-point correlation curve assuming an exponential behavior and extracted the length where this curve falls below a value of $1/e$. Second, an upper value with $L_{u'u'} = 3\delta$ was used, based on data by Ringuette *et al.* [204]. The authors investigated turbulent boundary layers from $Ma = 3 - 5$ at Reynolds numbers ranging from $Re_\theta = 2390 - 6225$ and adiabatic wall temperatures via DNS computations. The integral lengths for all Mach numbers are reported throughout the boundary layer, extracted by integrating the corresponding two-point velocity correlation curves. At the boundary layer edge $L_{uu} \approx 1.5\delta$ and $L_{uu} \approx 1.25\delta$ can be extracted for $Ma = 3$ and $Ma = 5$, respectively. For the lower Mach number case, the authors investigated the influence of domain size by increasing the streamwise extent by a factor of 2.6, which resulted in a similar increase of the corresponding integral length scale. This indicates a length scale truncation due to limited region for integration. Therefore the latter $Ma = 3$ value from the increased domain was chosen as upper value, although a moderate decrease of length scale with increasing Mach number is expected (as visible for the nominal domain size in [204], discussed in [59] and based on data of this thesis with figure 4.40). The extracted length scales used as input are shown in figure 7.35.

With values to approximate L_{True} to calculate $(\Delta x/L_{True})_{eff}$, the sub-grid filtering analysis was first performed at the boundary layer edge. Here a rough estimate for a reference turbulence intensity (without filtering) value could be used for comparison, based on the L2F measurements, previously mentioned.

Based on the lower estimate for a length scale of $L_{uu} = 0.5\delta$, the filtering analysis resulted for the investigated interrogation window sizes of $512 \times 16 \text{ pix}^2$, $512 \times 32 \text{ pix}^2$ and $512 \times 64 \text{ pix}^2$ in a resolvable content of 77 %, 72 % and 71 % of the local turbulence intensity at the boundary layer edge, respectively (see also table 7.21). If these interrogation windows were characterized based on the original square window data by Spencer and Hollis [238], the resolvable content due to filtering would erroneously result in 53% for all above stated windows, since the streamwise extent of 512 pixel would be the only decisive parameter. A control interrogation window with final size of $256 \times 256 \text{ pix}^2$ resulted in resolvable content due to sub-grid filtering of 72 %. If instead the upper estimate for the integral length scale with $L_{uu} = 3\delta$ is used, all above stated interrogation windows (except the $256 \times 256 \text{ pix}^2$ window) resulted in 94 – 95 % resolvable content.

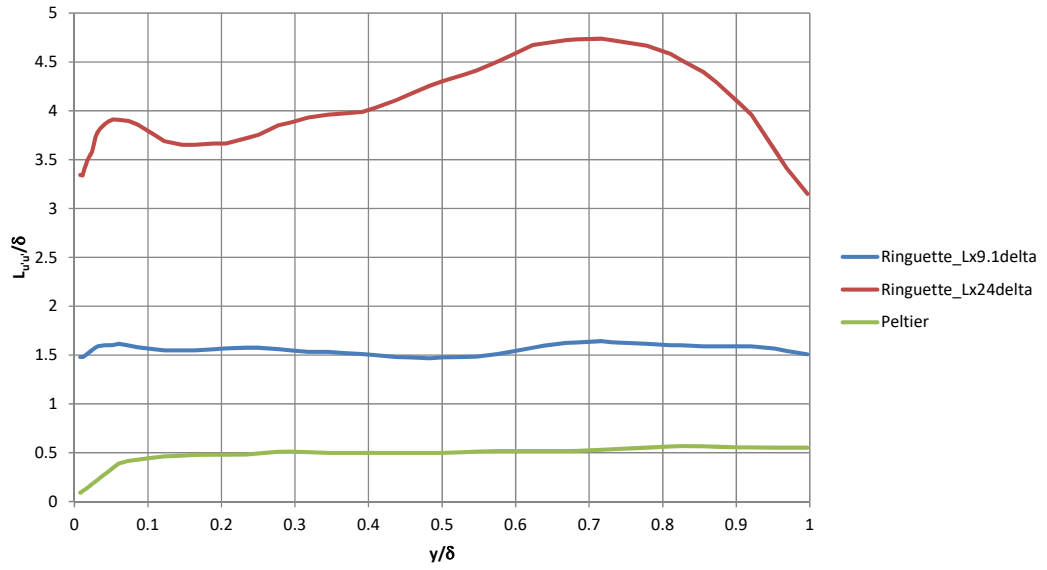


Figure 7.35: Longitudinal integral length scales in the TBL according to Peltier [183], Ringuette *et al.* [204].

So far, the analysis was restricted to the boundary layer edge and therefore most likely underestimate the sub-grid filtering within the boundary layer. If the length scale distribution throughout the boundary layer according to Peltier [183] is used in the modified Spencer approach, as described above, a difference of only 5 % compared to the edge value would be determined for $y/\delta > 0.1$. This is mainly due to only small changes in the length scale data, which is most likely also biased since the corresponding values were derived from PIV investigations, which are also affected by resolution and filtering effects.

Since the method by Host-Madsen and McCluskey [111] and Spencer and Hollis [238] makes the assumption of homogenous turbulence, the question arises whether this holds true for the largest investigated IW dimensions. Therefore, another method is also applied on data of this thesis, described in the following.

Table 7.21: Random error [285] and sub-grid filtering [238] approximation of PIV data at different IW sizes at $y/\delta = 1.0$ and $L_{uu}/\delta = 0.5$ according to [183].

iw _x [pix]	iw _y [pix]	$\overline{u'_{meas}}$ [m/s]	U_w^-		$(\overline{u'_{meas}}/\overline{u'_{True}})$				$(\overline{u'_{meas}}/\overline{u'_{True}})$			
			random	filter	random	filter	filter,square	filter	filter,square	filter	filter,square	filter
			[m/s]	[m/s]	[%]	[%]	[%]	[%]	[%]	[%]	[%]	[%]
16	256	17.54	4.17	3.09	23.76	17.60	41.69	85.03	70.58	72.31	70.58	49.81
16	512	15.26	4.17	5.18	27.30	33.96	87.54	74.65	53.32	55.72	53.32	28.43
32	256	14.57	4.26	2.90	29.24	19.91	41.69	83.40	70.58	69.55	70.58	49.81
32	512	14.98	4.26	5.74	28.43	38.33	87.54	72.29	53.32	52.26	53.32	28.43
64	256	13.33	1.85	3.05	13.89	22.90	41.69	81.37	70.58	66.21	70.58	49.81
64	512	14.89	1.85	6.47	12.44	43.44	87.54	69.72	53.32	48.60	53.32	28.43
96	96	15.89	4.44	2.22	27.97	13.97	13.97	87.74	87.74	76.99	87.74	76.99
128	256	12.65	1.02	3.56	8.05	28.11	41.69	78.06	70.58	60.93	70.58	49.81
128	128	16.82	3.33	3.19	19.82	18.96	18.96	84.06	84.06	70.66	84.06	70.66
256	256	14.95	0.74	6.23	4.96	41.69	41.69	70.58	70.58	49.81	70.58	49.81

Lee approach:

The second approach to estimate sub-grid filtering is a method according to Lee *et al.* [144]. The authors derived a database for the attenuation of the streamwise and wall-normal Reynolds stress, which uses the inner-scaled interrogation window size and the laser sheet thickness as input. This database is based on reference DNS data of an incompressible turbulent boundary layer at $Re_\tau = u_\tau \delta / \nu_w = 1500$ according to Sillero *et al.* [227]. The idea behind this approach was to use the attenuation database (e.g. $\Delta \overline{u'u'}^+$ for the streamwise stress) derived from a “low” Reynolds DNS solution to reduce the resolved content of a higher Reynolds number DNS solution which can then be compared to a corresponding PIV dataset. This way, DNS data are reduced by the small-scale energy which the PIV is not able to resolve.

Unfortunately, the database is limited to a streamwise interrogation window size which corresponds to 238 pix under H2K set-up conditions. Therefore a window of 238x64 pix² was used to estimate the streamwise Reynolds stress attenuation according to the procedure described in [144]. Since the database only contains attenuation in form of a difference term (e.g. $\Delta \overline{u'u'}^+$) to calculate a percentage filtering like $\overline{u'u'}_{meas} / \overline{u'u'}_{true} = (\overline{u'u'}_{true} - \Delta \overline{u'u'}^+) / \overline{u'u'}_{true}$, another reference value for $\overline{u'u'}_{true}$ e.g. from DNS need to be found. Here, the $Ma = 4.97$ turbulent boundary layer DNS data according to Duan *et al.* [56] was used. For this approximation inner-scaling of the wall normal distance and Morkovin’s scaling of the Reynolds stress of the Duan data was applied. The following assumptions need to be accepted in order to apply this method:

- General structure of turbulence and coherent structures are comparable between incompressible and compressible flow, including decisive length scales, although a dependence on Mach number of the latter is a matter of discussion ([59], [287])
- Morkovin’s scaling is accepted to hold in order to bring the data from the incompressible database together with compressible DNS data
- Inner scaling of the wall normal distance is accepted to hold in order to bring e.g. stress peak locations together, although still a matter of discussion if other scalings are more appropriate (e.g. [287], [256])

Figure 7.36 shows the corresponding unbiased DNS stresses plotted against inner scaled wall-normal distance y^+ from Duan *et al.* [56] together with two incompressible datasets by Jiménez *et al.* [119] and Sillero *et al.* [227]. The corresponding Reynolds numbers Re_τ of the data are given in the legend with $Re_\tau = 434$, $Re_\tau = 386$ for the *M5T4* and *M5T5* case of Duan, respectively, and $Re_\tau = 580$, $Re_\tau = 1500$ for the incompressible cases of [119], [227], respectively. In general, it is visible that the stresses do collapse to a certain degree. Especially the peaks in the buffer layer do not collapse exactly in position and level. The difference in level is known to be an effect of Reynolds number (see e.g. [113]). The difference in position is known to be an effect of proper compressible-to-incompressible scaling, where different scalings are discussed (e.g. [256]). Another Reynolds number influence is visible in figure 7.36 with the increase of turbulent energy in the outer layer in case of the incompressible higher Reynolds number case of Sillero, compared to the other lower Reynolds number cases. This difference will become important for the filtering analysis and it is visible with the data by Jimenez that it is independent of compressibility effects.

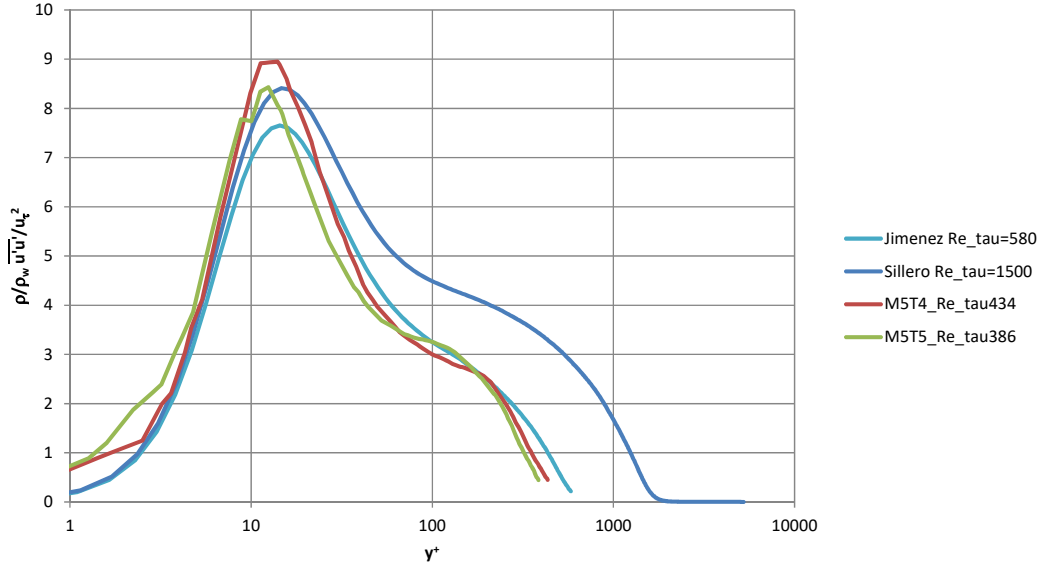


Figure 7.36: Morkovin scaled streamwise Reynolds stress in the TBL according to Jiménez *et al.* [119], Sillero *et al.* [22]

If the above three assumptions are accepted, sub-grid filtering was estimated by the method of Lee *et al.* [144] based on the following procedure. First, a filter kernel was estimated with an inner-scaled interrogation window of 238×64 and a laser sheet thickness of 1.0 mm (and additionally 0.5 mm to show a sensitivity) to $(x^+, y^+, z^+) = (100, 27, 56)$. The resulting $\Delta \overline{u'u'}^+$ from Lee's database was used to calculate $\overline{u'u'}_{meas}/\overline{u'u'}_{true} = ((\rho/\rho_w \overline{u'u'})_{Duan} - \Delta \overline{u'u'}_{Lee}^+(100, 56, 27))/((\rho/\rho_w \overline{u'u'})_{Duan})$. The results of measured to true streamwise Reynolds stress are plotted in figure 7.37. The curves are all plotted in outer scaled wall distances y/δ and as reference, the corresponding y^+ for case *M5T5* is shown as well.

In general, an expected increase of filtering with decreasing wall distance is visible for all cases. For values $y^+ \lesssim 14$ no data is available since it is below half the size of the filter kernel. Therefore, also the effect on the Reynolds stress peak is masked, since it is located close to this y^+ region. An expected increase of filtering with increasing laser sheet thickness is visible (by 35 % at maximum). The largest loss of resolvable content due to filtering of $< 40\%$ is visible for the M5T5 case with 1 mm laser sheet at $y^+ \approx 20$. Somewhat unexpected, but not of great concern here, is the apparent increased filtering towards the boundary layer edge (for $y^+ \gtrsim 200$). Usually, at this position larger length scales and therefore less filtering would be expected. The effect is most likely attributed to the lower Reynolds number of the Duan data ($Re_\tau \approx 400$) compared to the filter database ($Re_\tau \approx 1500$). As was seen before, larger Re_τ lead to higher Reynolds stress levels in the outer layer, independent of compressibility (see unbiased Reynolds stress comparison in figure 7.36 for Sillero and Jimenez data). Therefore, the level of filtering can reach the order of the unfiltered stress levels and is not representative anymore. To highlight that this is not attributed to compressibility effects (or a breakdown of corresponding distance scaling), the graph in figure 7.37 also shows the filtered incompressible data by Jimenez (only

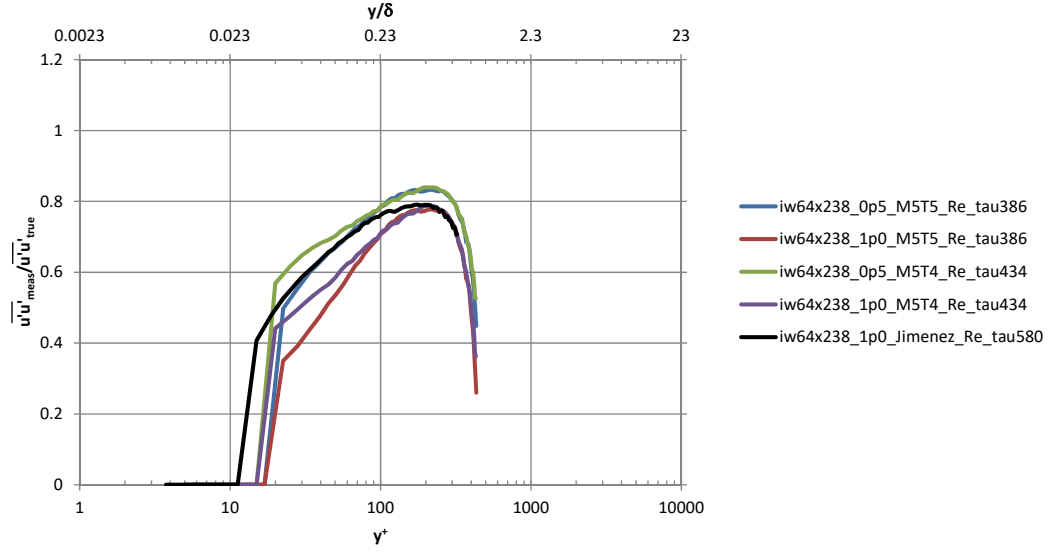


Figure 7.37: Effect of PIV IW filtering on streamwise Reynolds stress in the DNS TBL from Duan *et al.* [56] and Jiménez *et al.* [119] according to method of Lee *et al.* [144].

for a laser sheet thickness of 1 mm). The same increase of filtering close to the boundary layer edge is visible for the incompressible low Reynolds number data set (see line with label “iw64x238_1p0_Jimenez_Re_tau50”).

Additionally, the method according to Lee enables an approximation of the impact of laser sheet thickness on the sub-grid filtering. A maximum further decrease of resolvable rms velocity content of approx. 19% was estimated for a laser-sheet thickness of 1.0 mm compared to 0.5 mm in the H2K set-up. To summarize, close to the boundary layer edge, a resolvable content of approx. 80% can be estimated by the method of Lee. This is a bit larger than previously estimated by the method of Spencer, but for Lee’s approach values were derived based on the Duan DNS data and an interrogation window of $238 \times 64 \text{ pix}^2$ which therefore represent most likely only a lower limit for the larger $512 \times 64 \text{ pix}^2$ windows.

Please note that the filtering approximation according to Lee *et al.* [144] is based on inner (e.g. interrogation window sizes) and the one according to Spencer and Hollis [238] was used with outer scaled relevant length scales (e.g. integral length scales). Therefore, a direct comparison between results of both methods is challenging, if the Reynolds numbers Re_τ are not comparable for data sets, from which relevant length scales are used.

particle slip:

Independent of the post-processing, the measurement of turbulence intensities is influenced by particle slip for tracer based techniques. This slip can have a bigger impact on the turbulent part

than on mean velocity results (see e.g. [285]). To estimate this impact in H2K conditions, representative Stokes numbers S_k were approximated, defined by $S_k = \tau_p/\tau_f$, with the characteristic flow time constant τ_f and particle time constant τ_p .

To estimate the particle time constant in super- and hypersonic flows the response through an oblique shock is typically investigated (e.g. [199]). However, Williams showed that results from these tests may overestimate the capability of particles to follow turbulent disturbances [285]. Instead, Williams performed simulations, based on the equation of particle motions, for pre-defined wall normal positions and their corresponding local flow parameters in the boundary layer. The equation of motion of a particle in a fluid can be simplified based on assumptions like high particle to fluid density ratio and small, spherical particle dimensions (see section 7.7.3 or [285]). The resulting equation may be integrated numerically, assuming suitable drag models for the corresponding flow regime.

In this thesis, William's approach was adopted and the drag models stated in chapter 3.9.2 and 7.7.3 were implemented (dubbed "1D approach" based on a Python script). The numerical integration was typically performed with a step size Δt of 1 % of the corresponding Stokes relaxation time. Representative particle relaxation times τ_p for all applied drag models were extracted where the normalized velocity ratio drops to $1/e$ [285]. The integration tool was validated with literature data. The required input parameters like mean velocity, density, temperature and viscosity were extracted from a TAU RSM boundary layer profile at $T_w = 360$ K (MS1, see table 3.3). To be comparable with Williams [285], a simulation with a representative turbulence level of $0.05U_e$ for 40 equidistant wall-normal locations was performed for both particles p_1 and p_2 , defined above. Compared to actual PIV results, this turbulence level should give a reasonable estimate.

The fluid time constant τ_f is typically chosen to represent a characteristic length scale, e.g. the large eddies in the turbulent boundary layer. For the current analysis, a value with $\tau_f = 10\delta/U_e$ was used according to [286] or [287].

The results for particle p_1 and p_2 are contained in figure 7.38 as Stokes numbers along the wall-normal distance in the boundary layer. The corresponding particle relaxation times τ_p were derived by the described simulations for the Stokes and Tedeschi/Henderson drag model (see chapter 3.9.2). For the latter two a mean value is given and the error bars represent the differences between both. For the Stokes drag model, figure 7.38 contains only one representative curve for particle p_1 , just to show the difference in slope and level, erroneously resulting in $S_k < 0.1$, if no Mach and Knudsen effects are considered. If compressible and rarefied influences are included the results for the Tedeschi/Henderson models span a range between $0.4 < S_k < 1.5$, depending on the particle type. These values violate the limit of approx. $S_k \leq 0.1$ which is considered to be adequate not to filter large scale turbulence based on LDV measurements according to [210]. Based on the estimated Stokes numbers, with data from the same authors an estimate for the amount of turbulent intensity filtering is possible. This results in a streamwise rms turbulence intensity reduction of approx. 8 % and 15 % of a reference velocity U_e , for particle p_1 and p_2 , respectively. The latter value is similar to predictions published for comparable particle and Stokes numbers by Williams *et al.* [287]. In contrast, although experimental data by Williams indicate a clear particle lag effect for the wall-normal turbulence intensity only limited effects are visible for the streamwise counterpart (see also discussions and comparisons in chapter 4.3.1).

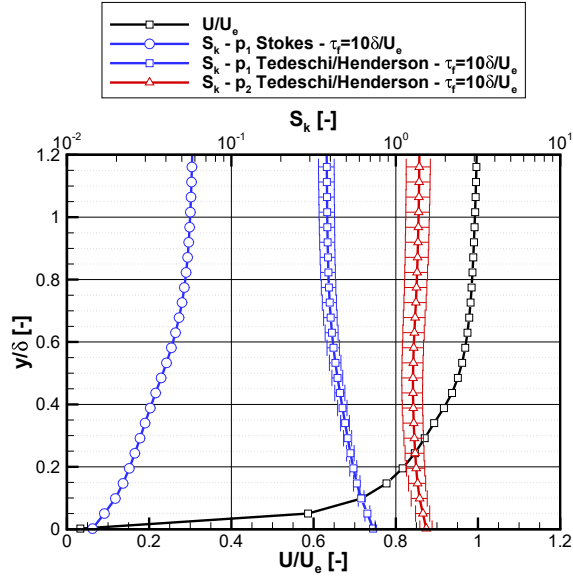


Figure 7.38: Variations of Stokes numbers due to turbulence motion across the boundary layer for FC1

image number convergence:

The uncertainties of the turbulent fluctuations due to convergence followed a procedure by Peltier [183] which used a re-sampling algorithm via the bootstrap method according to Benedict and Gould [23]. Due to the limited number of approx. 200 images per run, data of run 1007 and 1008 were combined by applying a weighting based on the corresponding amount of images per run, according to Sahoo *et al.* [209]. The estimated convergence resulted in approx. 8 % of the local streamwise turbulence intensity for the total of 429 images, independent of the used interrogation window.

Summary first campaign:

Based on the analysis above, IW 512x8 pix² was deemed acceptable for the mean velocity analysis. Based on the collected data for random error and sub-grid filtering, IW 512x64 pix² was selected for the turbulence intensity analysis of the first PIV campaign (see section 4.3.1).

Additional influences to the above discussed may affect the measurement of turbulence intensities.

Second PIV campaign:

Also for the second campaign several different influences potentially affecting the extraction of turbulence intensities via PIV, like random errors, spatial resolution, particle lag filtering and convergence with image number were analyzed and are discussed in the following.

random error:

As mentioned before, additionally to the two-point correlation method, during the second PIV campaign runs were foreseen to enable an approach according to Scharnowski *et al.* [211]. This approach incorporates tests with different laser pulse intervals Δt_p at the same flow conditions. Longer pulse intervals result in larger pixel displacements Δx and therefore larger signal-to-noise levels, if a fixed random error is assumed. Presumptions are that out-of-lane motions and flow gradients lead to only small amounts of loss-of-correlations. The measured mean pixel displacement fluctuations ($\overline{\Delta x_m'^2}$) can be formulated dependent on the turbulence level Tu (based here only on the streamwise component) and the standard deviation $\sigma_{\Delta x}^2$ via

$$\frac{\overline{\Delta x_m'^2}}{(\Delta x)^2} = Tu^2 + \frac{\sigma_{\Delta x}^2}{(\Delta x)^2} \quad (7.13)$$

with the mean pixel displacement ($\overline{\Delta x}$). It assumes that the random noise can be assumed Gaussian distributed which is shown to be valid for data of this thesis (figure 7.39a). With Run3010, 3007 and 3008 mean pixel shifts of $\Delta x \approx 15, 26, 36$ pixel have been realized with $\Delta t_p = 150$ ns, 250 ns, 350 ns, respectively. All data were used to fit equation (7.13) by optimizing the unknowns Tu and $\sigma_{\Delta x}^2$. For this, data at $y/\delta \approx 1$ and within $-1 < x/\delta < 1$ have been used. The results are shown in figure 7.39b and table 7.22. The last column in table 7.22 shows the percentage error based on the edge velocity in the nominal setup with $\Delta t_p = 250$ ns.

The figure shows the expected trend of increasing total measured fluctuation levels with decreasing mean pixel displacement and / or smaller IW sizes. For increasing mean pixel displacements and / or larger IW sizes, an asymptotic behavior towards a value of $Tu = 0.8 \pm 0.1\%$ is visible. This value is well within the expected range for H2K wind tunnel inflow turbulence intensities of approx. $Tu = 0.83 \pm 0.11\%$ (95% CI) from Laser 2 Focus measurements [94]. Random errors of 0.41 ± 0.01 pix, 0.44 ± 0.01 pix and 0.55 ± 0.01 pix for an IW of 64×64 pix², 48×48 pix² and 512×16 pix² (corresponding to inner scaled dimensions of $r^+ \approx 30 \times 30, 22 \times 22, 237 \times 7$) can be estimated, respectively. For the nominal setup with $\Delta t_p = 250$ ns this translated into 1.6 %, 1.7 % and 2.2 % of U_e , respectively.

Table 7.22: Estimated turbulent levels and random errors for different interrogation window sizes by applying method according to [211].

IW	Tu [%]	$\sigma_{\Delta x}$ [pix]	$\sigma_{\Delta x}$ [%]
iw128x128	$0.8 \pm 0.1\%$	0.31 ± 0.01	1.2 ± 0.04
iw96x96	$0.8 \pm 0.1\%$	0.34 ± 0.01	1.3 ± 0.04
iw64x64	$0.8 \pm 0.1\%$	0.41 ± 0.01	1.6 ± 0.04
iw48x48	$0.8 \pm 0.1\%$	0.44 ± 0.01	1.7 ± 0.04
iw512x32	$0.8 \pm 0.1\%$	0.44 ± 0.01	1.7 ± 0.04
iw512x16	$0.8 \pm 0.1\%$	0.55 ± 0.01	2.2 ± 0.04

As for the first PIV campaign, also random errors via the two-point correlation method have been estimated, which are shown in figure 7.40b. By comparing values from figure 7.40b and table 7.22 it is visible that the values extracted by the correlation approach are below the values (at $y/\delta \approx 1$). This was expected due the linear approximation of non-overlapping IWs [285].

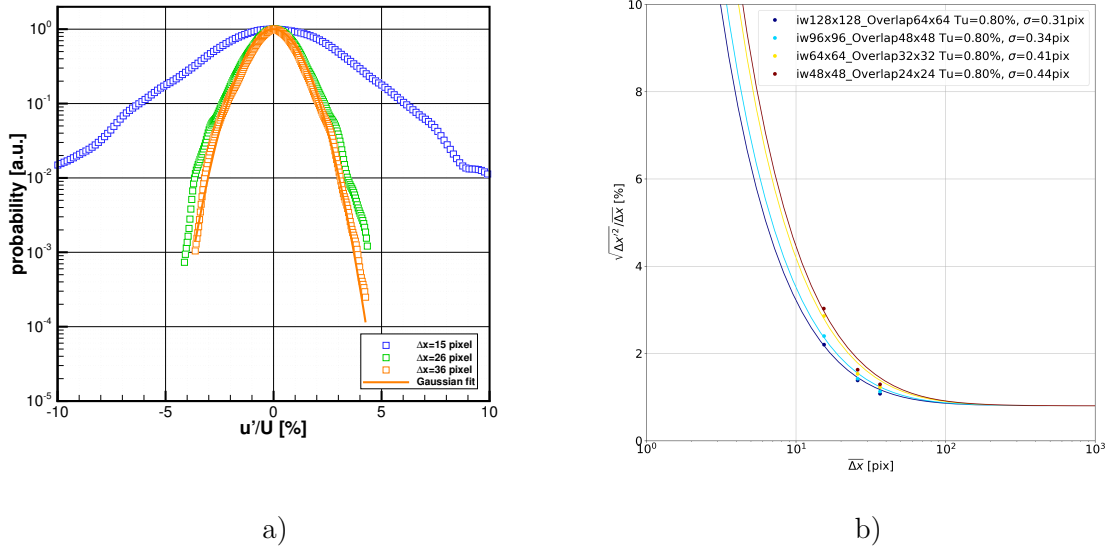


Figure 7.39: a) Histogram of the velocity fluctuations for different particle displacements, b) Estimated turbulent levels as a function of particle displacement for different interrogation window sizes by applying method according to [211].

sub-grid filtering:

Sub-grid filtering was again estimated based on the work by Spencer and Hollis [238] and Lee *et al.* [144]. Results of both approaches are summarized in figure 7.41. Figure 7.41a shows results by the modified method of Spencer (described above), where the left and right columns correspond to results based on the lower estimate for integral length scales by Peltier and upper estimate by Ringuette, respectively (see also corresponding paragraphs for the first PIV campaign). In both cases three different wall normal positions are investigated at $y/\delta = 1, 0.8, 0.5$. Since Peltier's length scales do not differ much with wall-normal height (see figure 7.35), also the filtering is almost constant. In contrast, in case of Ringuette's scales always data at $y/\delta = 0.5$ closest to the wall shows the highest filtering due to the decrease of length scales (see again figure 7.35). Figure 7.41b shows results by the method of Lee, where the resulting level of streamwise Reynolds stress attenuation at a representative wall-normal distance of $y/\delta = 0.5$ is in the order of approx. 10 % for all IWs in this work with 8.5 %, 7.6 % and 10.7 % for an IW of 64x64 pix², 48x48 pix² and 512x16 pix², respectively.

wall temperature:

See corresponding section for first PIV campaign.

particle slip:

See corresponding section for first PIV campaign.

image number convergence:

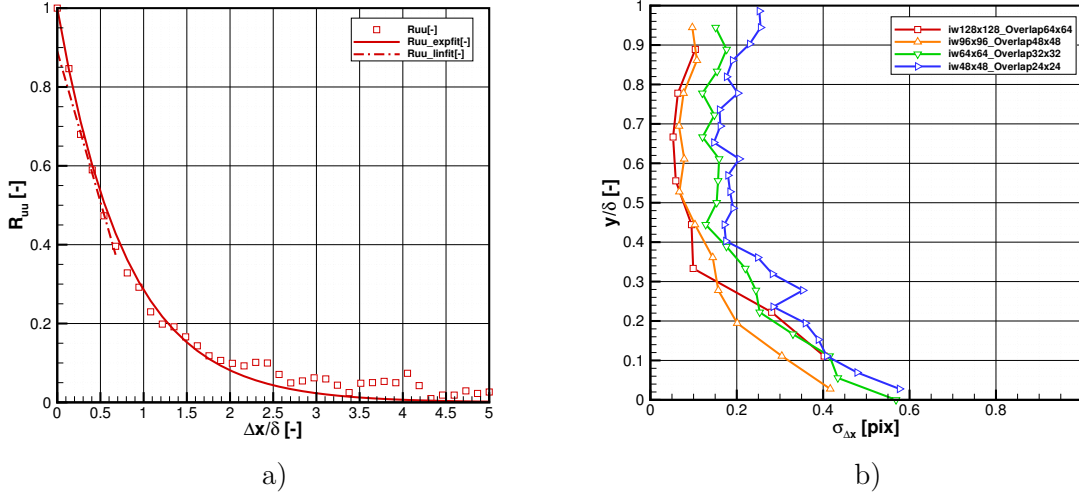


Figure 7.40: a) Exemplary two-point correlation of streamwise velocity fluctuations above rough wall (symbols) and a corresponding exponential fit (solid line) and linear fit (dash-dotted line) to points in the vicinity of the coordinate of interest, b) Estimated random noise within the TBL for different IW sizes by applying method according to Williams [285]; both plots for data of Run3007.

Also for data of the second campaign, a re-sampling algorithm via the bootstrap method according to Benedict and Gould [23] was used.

peak-locking:

Figure 7.30b shows an exemplary pdf of a smooth wall run which shows that peak-locking effects can be neglected.

Summary second campaign:

Based on the above analysis, the IW 64x64 pix² (sometimes labeled in the graphs with “iw64x64_Overlap32x32”) were chosen as main IW for mean and turbulence data due to a good tradeoff between random errors and filtering. Some selected analysis have been performed on data with stretched, high aspect ratio, IW 512x16 pix² (“iw512x16_Overlap256x8”) windows. These latter IWs were used to increase the wall-normal resolution and at the same time ensure an acceptable number of particles per IW. This increased resolution leveraged the extraction of EQSR, where data points of the mean velocity down to the logarithmic region is beneficial (see section 4.2.2). Comparisons of the mean velocity profiles for different IWs showed only negligible differences, except for the corresponding wall-nearest point (see figure 7.43). Also, previous investigations on synthetic and experimental data showed acceptable results with comparable IWs (see [171])

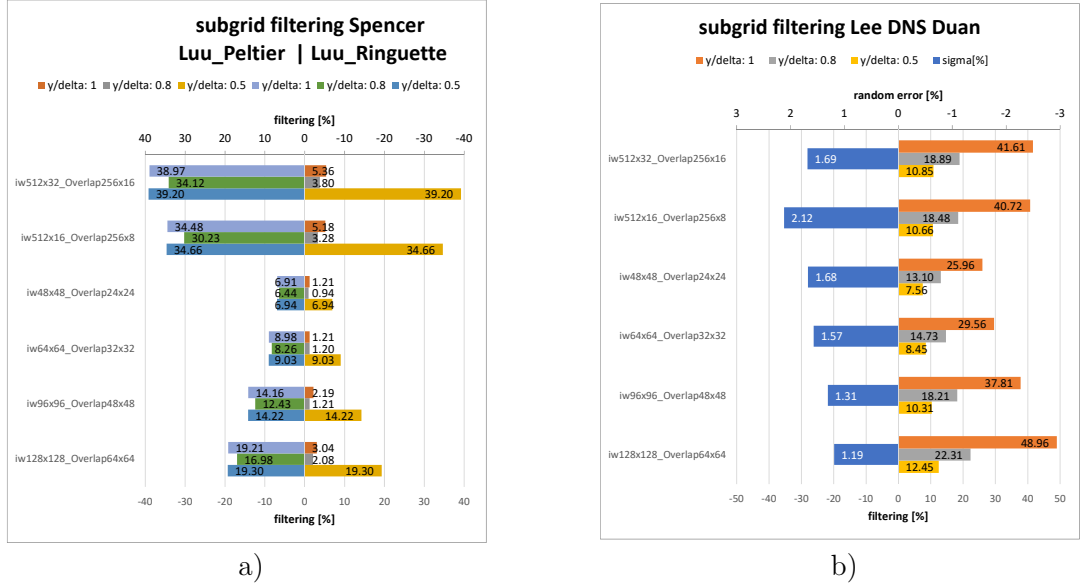


Figure 7.41: IW sub-grid filtering estimates after the approach according to a) Spencer and Hollis [238] and

7.5.2.2 Resulting Sensitivities

In the following several sensitivities for the resulting quantities are reported.

Edge Temperature calculation:

Edge temperatures, necessary for the calculation of parameters like eq. (3.5), (3.6), were derived based on the assumption of a constant total temperature T_0 from eq. (3.3) and (7.14) according to [241] in case of Pitot pressure and PIV data, respectively.

$$T = T_0 \left[1 - \frac{\gamma - 1}{2} \frac{U^2}{\gamma R T_0} \right] \quad (7.14)$$

This assumption was tested to be valid down to the boundary layer edge (see figure 7.42). Closer to the wall, larger deviations are visible and expected due to viscous losses so that within the boundary layer the temperature profile after Crocco-Busemann was used (see section 3.2.3 eq. (3.4)).

Effect of IW size on mean velocity profile:

A check was made to see the impact of IW size on the mean velocity profiles. Figure 7.43a and figure 7.43b include outer scaled mean velocity profiles showing effect of IW size for first and second PIV campaign, respectively. It is visible that all data collapse except for the first point above the wall in case of both PIV campaigns. Therefore, IW effects were assumed negligible in case of mean velocities.

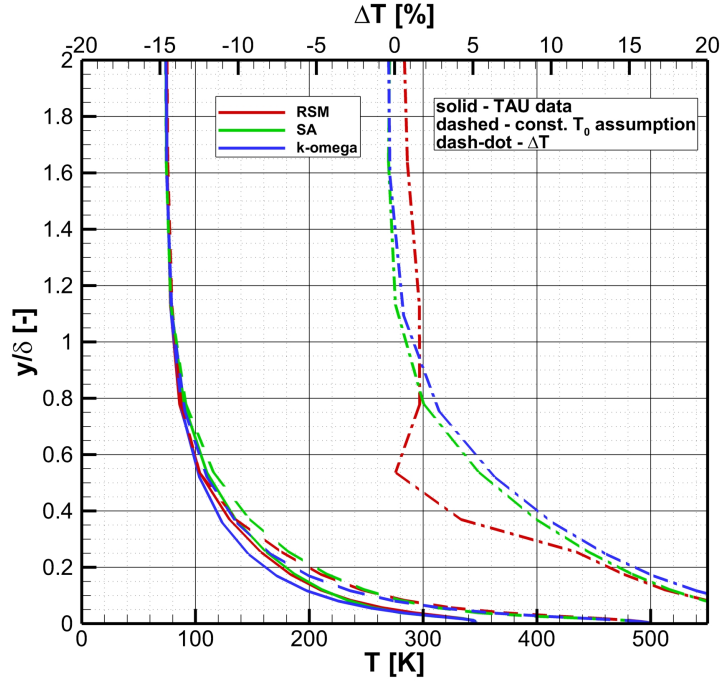


Figure 7.42: Test of static temperature calculation from velocity via constant temperature assumption according to [241].

Run-to-run repeatability:

Figure 7.44 contains outer scaled mean velocity profiles above the smooth (black lines/symbols) and rough wall (red lines/symbols) for different runs. The run-to-run repeatability is very good, visible here for example with the agreement between profiles from Run3006 compared to 3007. Therefore, run-to-run effects were assumed negligible and results to be independent from individual runs.

Effect of averaging size and position:

For the discussion of velocity and turbulence profiles, data along the smooth and rough wall of both PIV campaigns was averaged along a streamwise region, which corresponds to the extent of one roughness shape wavelength ($\lambda = 4k \approx 2\delta$). Along the roughness topology, this averaging window is aligned with the beginning at the downstream end of one roughness crest to the next (i.e. at MS1: $s=496$ mm to 498 mm and at MS2: $s=626$ mm to 628 mm). Different averaging alignments and also extents (i.e. two or four wavelengths) were tested with only negligible differences between the results as visible in figure 7.45.

Table 7.23 shows the impact of averaging on the skin friction velocities from integral approach of PIV data (IW 512x16 pix²) along the rough wall. The difference in taking into account different ranges and length for averaging (e.g. at MS1: $x=496$ mm to 498 mm or $x=498$ mm to 500 mm) result in slightly different positions for the first profile x_{MS1} and distance to MS2 Δx . It is visible

Table 7.23: Skin friction velocities from integral approach of PIV data (IW 512x16 pix²) along rough wall with sensitivity to averaging.

Run MS1	Run MS2	x_{MS1} [mm]	Δx [mm]	θ_{MS1} [mm]	$\Delta\theta$ [mm]	integral $u_{\tau,MS1}$ [m/s]	$u_{\tau,MS2}$ [m/s]	DQ $u_{\tau,MS1}$ [m/s]
3006	3020	496	130	0.21	0.07	60.7	58.2	58.8
3006	3020	496	132	0.21	0.08	61.6	58.8	59.4
3006	3020	496	131	0.21	0.08	61.2	58.5	59.2
3006	3020	498	128	0.21	0.07	61.4	58.8	59.4
3006	3020	498	130	0.21	0.08	62.3	59.4	60.0
3006	3020	498	129	0.21	0.08	61.9	59.2	59.8
3006	3020	497	129	0.21	0.07	61.0	58.4	59.1
3006	3020	497	131	0.21	0.08	61.9	59.1	59.7
3006	3020	497	130	0.21	0.08	61.5	58.8	59.4
3007	3020	496	130	0.21	0.07	62.2	59.6	60.3
3007	3020	496	132	0.21	0.08	63.1	60.2	60.8
3007	3020	496	131	0.21	0.08	62.7	59.9	60.6
3007	3020	498	128	0.21	0.08	64.3	61.3	61.9
3007	3020	498	130	0.21	0.09	65.2	61.9	62.5
3007	3020	498	129	0.21	0.08	64.8	61.6	62.2
3007	3020	497	129	0.21	0.08	63.0	60.3	60.9
3007	3020	497	131	0.21	0.08	63.9	60.9	61.5
3007	3020	497	130	0.21	0.08	63.5	60.6	61.3

that these difference have only minor impact on the resulting u_{τ} with variations below ± 3 m/s, well below corresponding uncertainties.

Effect of flow condition on velocity profiles:

Figure 7.46 shows a plot after the analysis according to Fernholz and Finley [73] for a comparison of data from first PIV (IW 512x8 pix²) and second PIV campaign (IW 512x16 pix²) for FC1 (Run1007, 3014) and FC2 (3015). It is visible that only marginal differences exist between both campaigns and that negligible differences are visible between FC1 and FC2.

Effect of near wall resolution on integral plot according to [73]:

In figure 4.12 it was visible that by plotting Pitot data based on an integral length scale Δ^* according to [73] (see eq. (4.1)), a parallel offset in comparison to PIV data and theory was visible. Figure 7.47 shows artificial profiles analyzed by the same approach, derived by the method described in section 3.10 (see eq. (3.52)) with the input equivalently to FC1 with $Ma_e = 5.4$, $T_e = 73.12\text{K}$, $T_w = 340\text{K}$, $u_{\tau} = 50\text{m/s}$, $\Pi = 0.4$, $Re_{\delta 2} = 1077$. The artificial profile was created by enforcing a viscous sublayer for $y^+ < 10$ ($U^+ = y^+$), without trying to mimic a buffer layer behavior. One set of data was intentionally cut below $y = 3\text{mm}$ ($y^+ = 19$) to simulate the near wall resolution of Pitot pressure measurements. To guide the analysis, the different layers are colored by blue (viscous sublayer), green (log-region) and red (wake). In figure 7.47a it is visible that the full artificial profile follows all theoretical curves in inner and outer scaling. In figure 7.47b it is visible that the underresolved artificial profile follows all theoretical curves, except the integral one according to Fernholz and Finley [73]. Therefore, it is assumed that

the parallel shift of the Pitot data, visible in figure 4.12, is mainly influenced by the near-wall resolution.

Temperature-velocity model impact on turbulent profiles:

To test the impact of different temperature-velocity models, figure 7.48 shows the streamwise and wall-normal turbulence intensity profiles for different runs with the temperature model according to Walz (see e.g. [56]). The differences to profiles with the Crocco temperature model are negligible, by comparing to 4.29, so that it has no impact on the analysis.

IW dimension impact on sub-grid filtering:

The effect of sub-grid filtering is further investigated in figure 7.49 and 7.50, where data of smooth Run3014 with two different PIV resolutions, i.e. IW $64 \times 64 \text{ pix}^2$ and $512 \times 16 \text{ pix}^2$, are shown in comparison to artificially filtered DNS data by Duan *et al.* [56] and Zhang *et al.* [296]. The amount of sub-grid filtering for the streamwise and wall-normal component of the DNS data was estimated based on an approach according to [144] (see chapter 3). With the grey lines, representing the attenuated profiles of the DNS data in figure 7.49 and 7.50, it is visible that a measurable amount of the corresponding Reynolds stress profile is reduced, especially in the near wall peak region. As depicted in figure 7.49a, the corresponding attenuated DNS data agrees to the IW $64 \times 64 \text{ pix}^2$ PIV data within the corresponding uncertainties, supporting the notion that sub-grid filtering is a major contributor to the underprediction of Reynolds stress, already visible in figure 4.25 and 4.26. In case of the IW $512 \times 16 \text{ pix}^2$ PIV data (figure 7.50a), the streamwise component agrees already with the unfiltered (un-attenuated) DNS data, especially in the peak region close to the wall. At first, this is not to be expected due to the larger streamwise extent of the interrogation windows, which would lead to an increase in sub-grid filtering. But it might be influenced by an increased amount of random error due to the smaller wall-normal dimensions. Additionally, the agreement shows no effect of particle slip which would be expected from the approximated Stokes numbers (see chapter 3). This absence of slip effects on the streamwise component was already noticed in previous investigations (see e.g. [287]).

In case of the wall-normal component (figure 7.49b and 7.50b), the PIV data is clearly lower than the DNS data. It even falls below the attenuated DNS data in large portion of the boundary layer. Only the IW $512 \times 16 \text{ pix}^2$ PIV data corresponds to the attenuated DNS if the upper bounds of the uncertainties are considered (figure 7.50b). For the latter case, as explained in section 7.5.2, it needs to be noticed that the filter kernel needed to be reduced to $256 \times 16 \text{ pix}^2$ instead of $512 \times 16 \text{ pix}^2$ due to limits in the attenuation data base (see chapter 3) and therefore this most likely underpredicts the real filtering.

In general, data in figure 7.49b and 7.50b support the observation of a more pronounced reduction of wall-normal and Reynolds shear-stress component in case of PIV data for high-speed flows, most often attributed to particle response behavior, either by experimental evidence (see e.g. [287], [183], [288]) or synthetically seeded particles in DNS data (see [15]). At the same time, numerical results suggest that the resolution has a higher impact on the wall-normal component than particle lag. This was attributed mainly to the energy scales, which drop much slower in comparison to streamwise scales and are therefore more prone to filtering effects [15]. In contrast to this argumentation, the higher resolved experimental data in figure 7.50b show no large difference to the lower resolved data in figure 7.49b. But it needs to be considered that the resolution was not increased in both dimensions (only in wall-normal direction). It was tested

also a higher resolution in both directions (IW 48x48 pix², see section 7.5.2) which resulted in $\approx 20\%$ higher wall-normal stress levels in the mid-parts of the TBL ($0.2 < y/\delta < 0.8$) compared to IW 64x64 pix², but still fall short in comparison to correspondingly filtered DNS data and showed increased levels of uncertainty.

Figure 7.51 shows additional sensitivities for sub-grid filtering of the streamwise and wall-normal Reynolds stress components in case of an IW 48x48 pix² (75 % overlap). The streamwise Reynolds stress component shows comparable filtering effects as the results with IW 64x64 pix² (see figure 7.49). In contrast, figure 7.51b shows that the analysis resulted in $\approx 20\%$ higher wall-normal stress levels in the mid-parts of the TBL ($0.2 < y/\delta < 0.8$) compared to IW 64x64 pix² (see figure 7.49, in section 4.3.1) but still fall short in comparison to correspondingly filtered DNS data by the method according to [144]. Additionally, the corresponding uncertainties increase drastically and would hide any clear trend.

IW dimension impact on two-point correlation data:

Figure 7.52a shows the impact of IW size on the downstream length scale L_x^d . It is visible that, apart from a certain scatter, all data correspond closely to each other, independent of the choice for the IW size. The same statement can be made for the inclination angle ϕ , visualized in figure 7.52b. Therefore, it is assumed that results shown in section 4.3.3 are representative for data in this thesis.

Integral length scale extraction:

Although more challenging, it was tested to extract also the integral length scales from PIV data since the extent of the current FOV is with $\Delta x/\delta \approx 6$ larger than e.g. data by [183] ($\Delta x/\delta \approx 2$) who performed a comparable analysis. To extract the streamwise length scale λ_x^u from autocorrelation, different approaches were tested. Firstly, a threshold of $1/e$ for the two-sided autocorrelation data was used as representative value for the length scale, according to [183] (“avg.” in the legend of figure 7.53). Secondly, the same threshold was used, but on an exponential fit through the data points of the autocorrelation (“fit.” in the legend). And lastly, the exponential fit through the autocorrelation data points was used for a numerical integration. The boundaries for the integration were set to the positions where the fitted autocorrelation crosses the x -axis below zero (“int.” in the legend). To reduce noise, all extracted length scales were averaged over the streamwise extent of the FOV ($\Delta x \approx 5.5\text{mm}$). The impact of IW size on the integral length-scales was tested and showed very comparable results (see figure 7.52 in section 7.5.2).

In figure 7.53 it is visible that the three different approaches to calculate λ_x^u agree closely. The data of this study shows typical length scales of above $\approx 1.2\delta$ for a large portion of the boundary layer ($0.1 < y/\delta < 0.6$). The length scales near the boundary layer edge at $y/\delta > 0.7 - 0.8$ are not reliable most likely due to the seeding quality in this region. All data share a rapid near wall increase of scales from $y/\delta \approx 0.1$ to 0.3 , consistent with Townsend’s attached eddy hypothesis [255], where the slope in case of our data is more comparable to results from Pirozzoli and Bernardini than those from Peltier. In the mid-part of the boundary layer, current data are slightly larger than corresponding length scales from the $Ma = 4.9$ study of Peltier [183], which used the aforementioned approach of $1/e$ threshold to derive λ_x^u . It is comparable to the maximum value of the $Ma = 2$ DNS data of Pirozzoli and Bernardini [189], which used the aforementioned definition of the numerical integration to derive λ_x^u . In case of results from

Pirozzoli and Bernardini a more rapid decrease of length scales with increasing wall distance is visible ($y/\delta \gtrsim 0.2$), which is also visible for our data but for larger wall distances ($y/\delta \approx 0.6$). In case of data from Peltier no decrease is visible.

Apart from the Mach and Reynolds numbers, also the FOV is different between the studies with 2δ in case of Peltier, 6δ in this work and $> 100\delta$ in case of Pirozzoli and Bernardini. This might still influence the ability to extract proper integral length scales and might be part of the reason for the described differences.

IW dimension impact on integral length-scales:

The sensitivity of the extracted streamwise length scales λ_x^u from the IW size is shown in figure 7.54. This is shown only for one of the extraction options (“fit”, see section 4.3.3) and it is visible that in general the values for λ_x^u are very comparable between the different IWs. This is also in line with analysis based on synthetic data (see section 7.5.4). Only the largest windows with $128 \times 128 \text{ pix}^2$ show larger deviations with much larger length scales. This is in line with analysis according to [238], which showed that insufficient resolution tends to overestimations of integral length scales. In contrast to λ_x^u , it was visible in figure 7.52 that the dimension of contour levels as length scale is more robust with respect to IW size.

Quadrant analysis:

Different investigators utilize a quadrant analysis of the velocity fluctuation data to gain further insight into coherent structures and to study what is creating Reynolds shear stress and shape its form, meaning what specific eddy motions cause the u' and v' components to be anti-correlated [183] (see also figure 2.14).

Figure 7.55a and figure 7.55b show a quadrant plot for smooth wall and rough wall data, respectively, where each symbol represents a u' and v' pair of a valid vector for all collected images at a position close to the boundary layer edge at $y/\delta \approx 1$. It is visible that more data points are located in Q2 and Q4, as expected, where smooth and rough wall data show similar appearances.

Figure 7.56 shows profiles of Reynolds shear stress, conditionally averaged according to their contributions to Q1 to Q4. Comparing the absolute values from all four quadrants for example at a position $y/\delta \approx 0.2$, it is visible that Q2 and Q4 have the largest contributions. Within scatter of data, smooth and rough wall data show similar profiles, independent of quadrant. This is in contrast to findings by Peltier, who identified differences between smooth and diamond rough wall data (see figure 2.14 or 2.13 in [183]). Therefore, and due to the substantial filtering of this component, no difference on sweep or ejection events can be made for the rough wall data.

Two-point correlations of wall-normal velocity fluctuations:

Selected two-point correlations of the wall-normal velocity fluctuations were analyzed. Figure 7.57a and figure 7.57b show the corresponding correlations along the smooth and rough wall, respectively. It is visible that the shape of the correlation contours are more compact in streamwise and elongated in wall-normal direction, compared to their streamwise counterparts (see e.g. figure 4.33). This observation is consistent with observations by Jimenez, that “the structures of u are long, and those of v are tall” [189].

This general elongated appearance is similar to incompressible data at larger $Re_\tau = 1800$ according to Ganapathisubramani and Schultz [86]. Also DNS data of a $Ma = 2$, $Re_\tau = 1116$ TBL flow according to Pirozzoli and Bernardini [189] show comparable correlation appearance at $y/\delta = 0.3$. One difference is the correlation length of this work are slightly larger in streamwise extent with approx. 0.5δ for the $R_{vv} = 0.5$ contour at $y/\delta = 0.5$ compared to approx. 0.2δ according to the incompressible data by Ganapathisubramani and Schultz [86] and compressible data by Pirozzoli and Bernardini [189].

If current smooth and rough wall data are compared (figure 7.57a vs. figure 7.57b), the general shape and extent look similar, just like the streamwise counterparts (see figure 4.33 and 4.34). Due to the small magnitude of the wall-normal component no further investigations were performed.

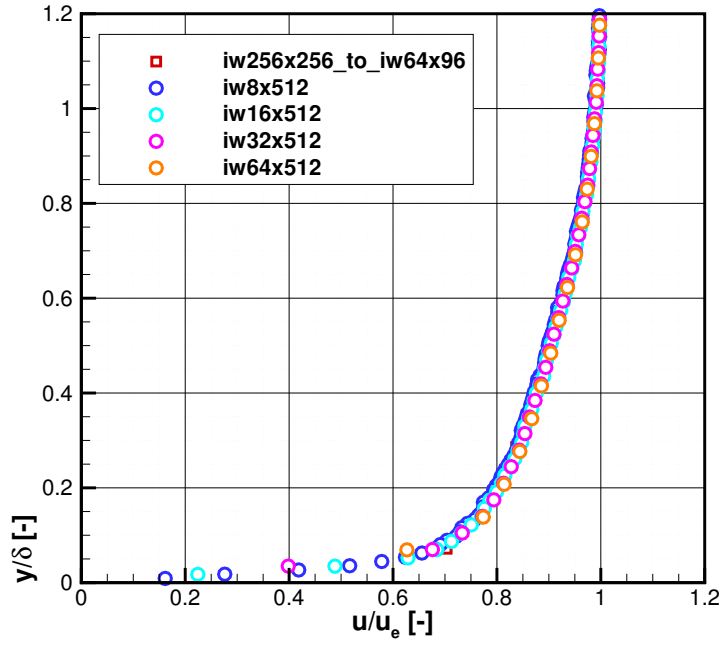
Extended Uniform Momentum Zone Examples:

In section 4.3.4 exemplary flow field snapshots were shown and analyzed to show the appearance of UMZ. Figure 7.58 and figure 7.59 show further examples for smooth wall Run3014 and rough wall Run3007, respectively.

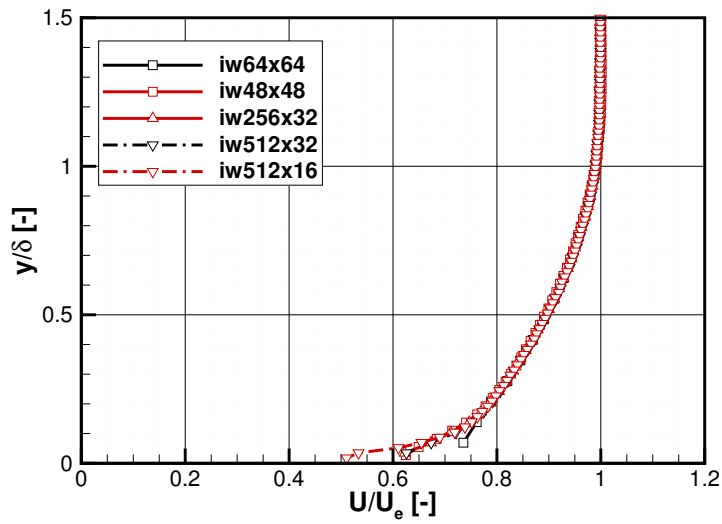
The impact of IW dimensions on the resulting UMZ statistics was studied with post-processing of data from IW of $48 \times 48 \text{ pix}^2$. Smooth wall Run3014 data resulted in 2.9 ± 1.1 on 5290 instantaneous velocity vectors, which is close to the reported 2.8 ± 1.1 from the nominal IW of $64 \times 64 \text{ pix}^2$, reported in section 4.3.4. Rough wall Run3007 data resulted in $\bar{N}_{UMZ} = 2.5 \pm 1.0$ from 5290 instantaneous vectors, which is close to the reported 2.9 ± 1.0 from the nominal IW of $64 \times 64 \text{ pix}^2$. A direct comparison is included in table 7.24 which shows that sensitivities to IW dimensions have been found negligible within corresponding uncertainties.

Table 7.24: Results of mean number of UMZs (± 1 standard deviation) for smooth run (3014) and b) rough run (3007) for different IW sizes.

Wall type	Run	IW	$\bar{N}_{UMZ}$	$N_{max,hist}$	N_{images}
smooth	3014	64x64	2.8 ± 1.1	2890	1487
smooth	3014	48x48	2.9 ± 1.1	5290	1487
rough	3007	64x64	2.9 ± 1.0	2890	1649
rough	3007	48x48	2.5 ± 1.0	5290	1649



(a)



(b)

Figure 7.43: Outer scaled mean velocity profiles showing effect of IW size for a) first (Run1007) and b) second PIV campaign (Run3014).

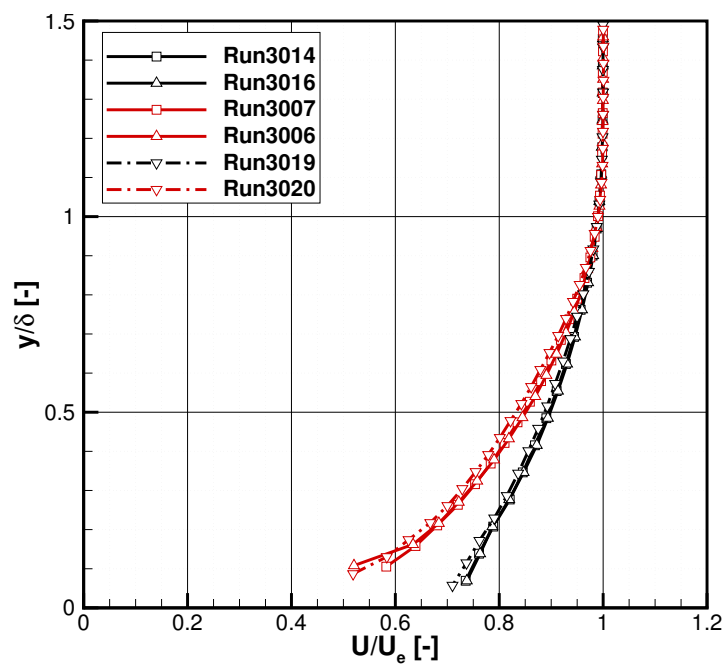


Figure 7.44: Outer scaled mean velocity profiles showing the effect of smooth (black lines/symbols) and rough wall (red lines/symbols) and repeatability for different runs.

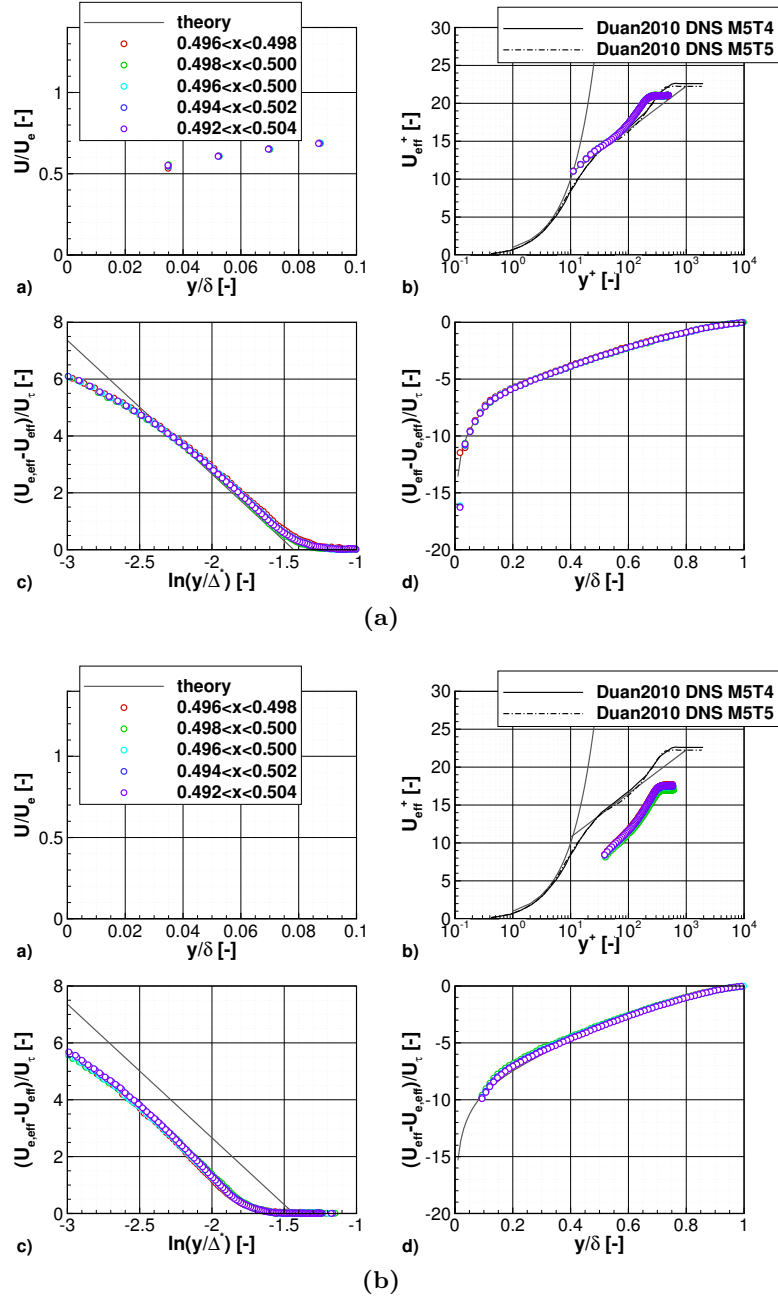


Figure 7.45: Streamwise mean artificial velocity data for different averaging size and positions for (a) smooth and (b) rough wall (numbers in legend are given in [m]) in inner b) and outer scaling a),c)-d) according to [73].

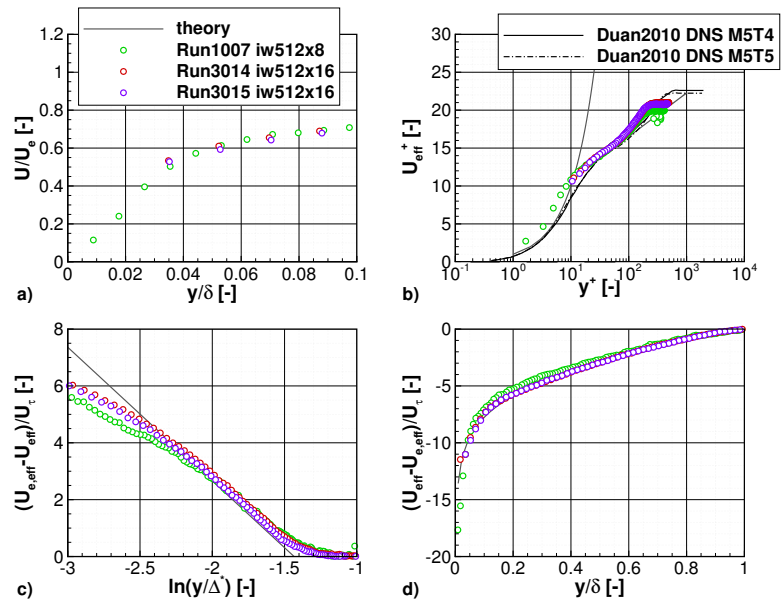


Figure 7.46: Streamwise mean velocity data from first PIV (Run1007: FC1, 512x8 pix²) and second PIV campaign (Run3014: FC1 and Run3015: FC2, both with IW 512x16 pix²) in inner b) and outer scaling a),c)-d) according to [73] (DNS data according to [56])

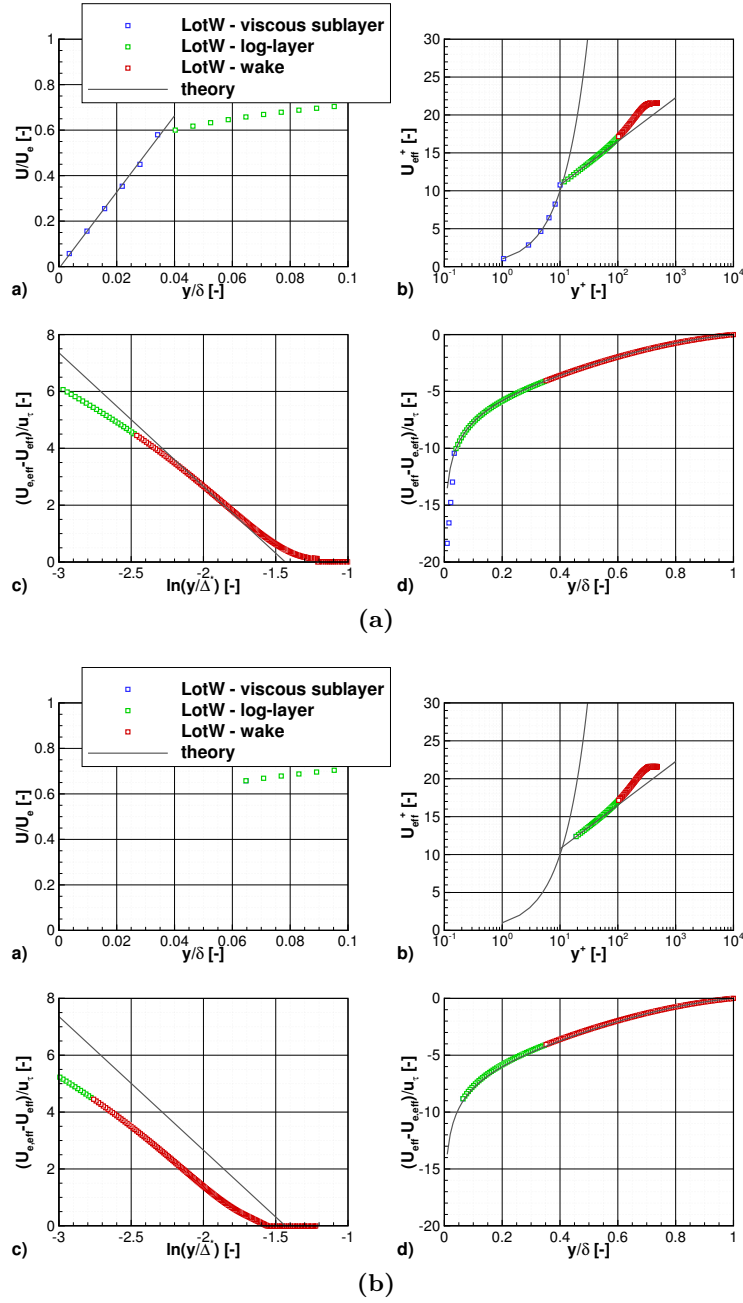


Figure 7.47: Streamwise mean artificial velocity data with (a) full profile and (b) near wall cut profile in inner b) and outer scaling a),c)-d) according to [73].

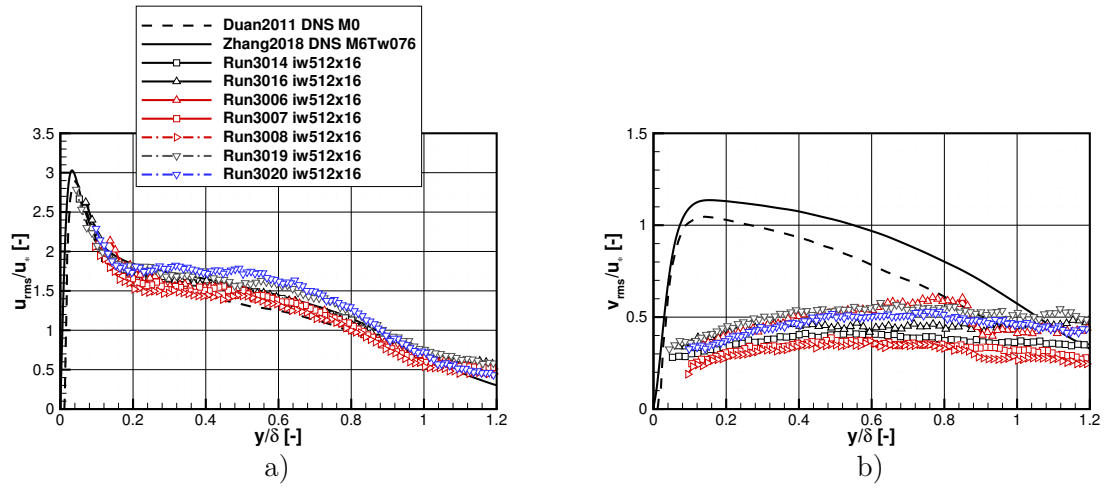
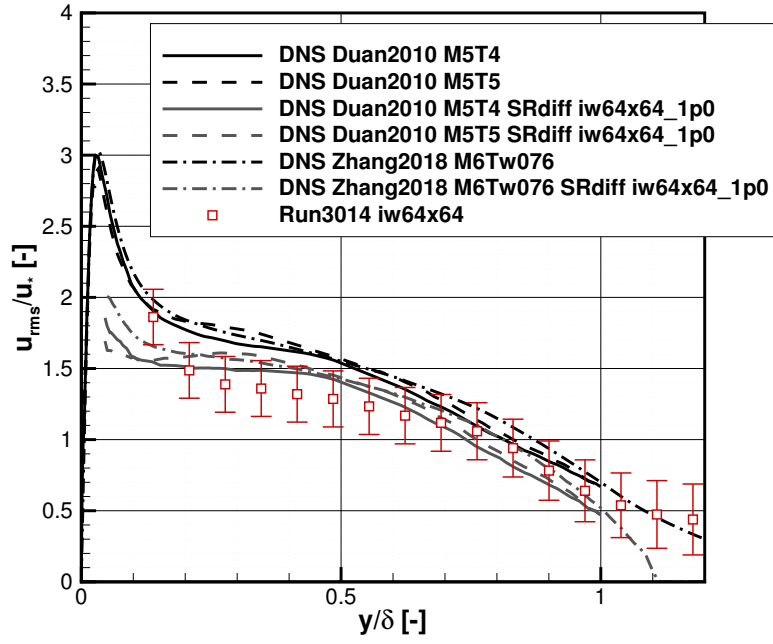
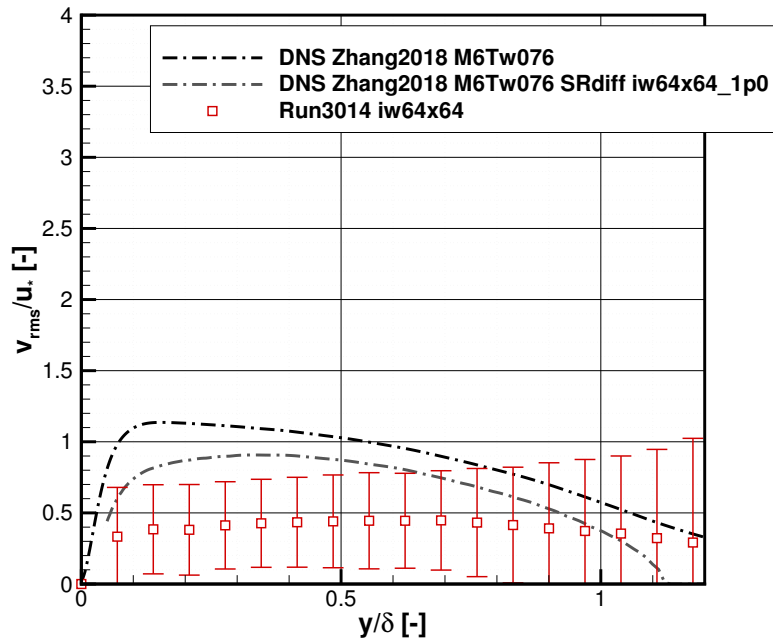


Figure 7.48: Streamwise a) and wall-normal b) component of turbulence profiles derived with temperature-velocity model according to Walz compared to artificially sub-grid filtered DNS data (black lines) by [56] and [296]

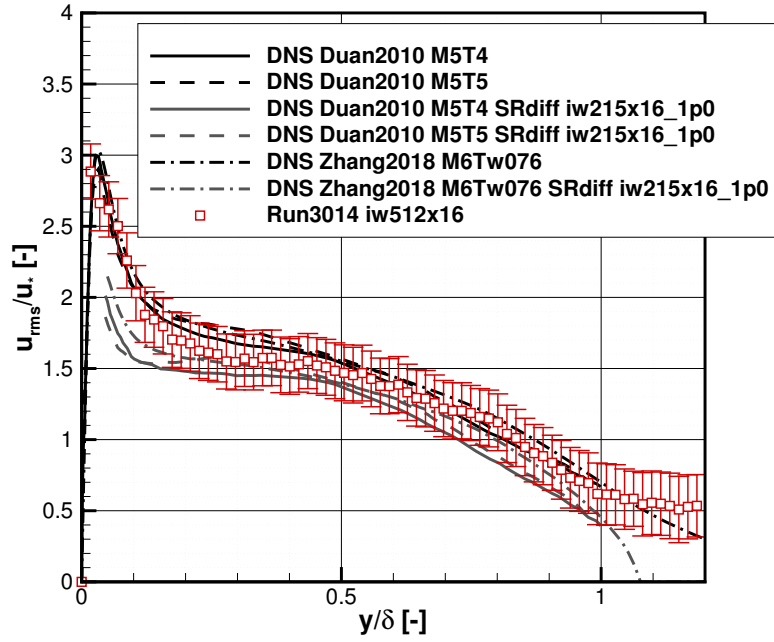


(a)

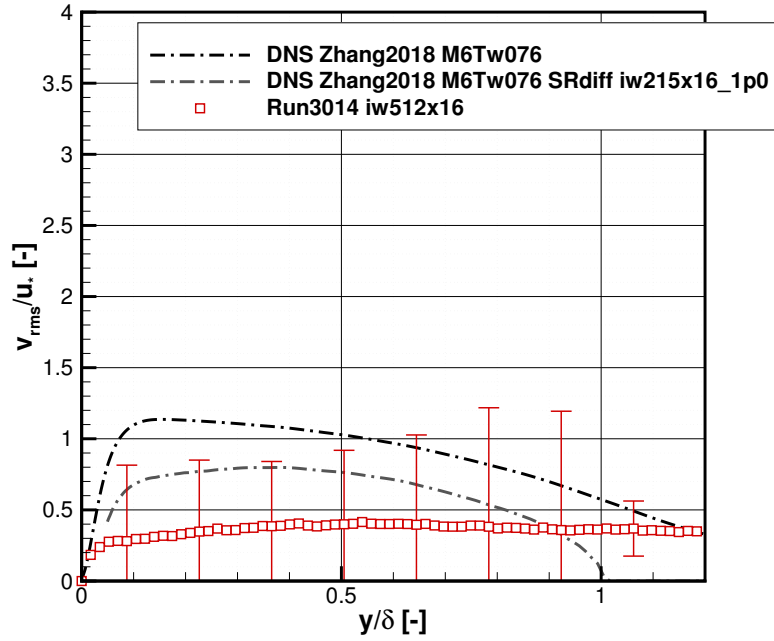


(b)

Figure 7.49: Streamwise a) and wall-normal b) component of turbulence profiles compared to artificially sub-grid filtered DNS data (grey lines) by [56] and [296] according to the method of [144] for IW 64x64 pix².

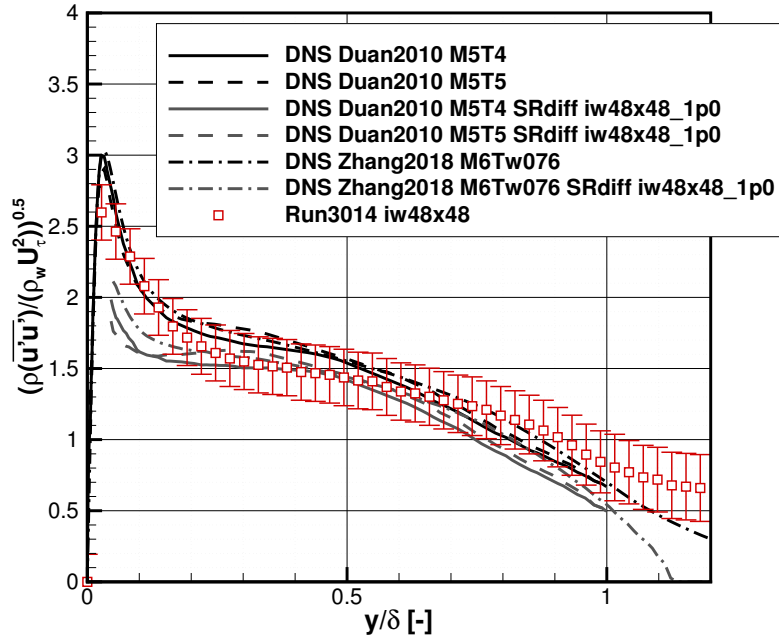


(a)

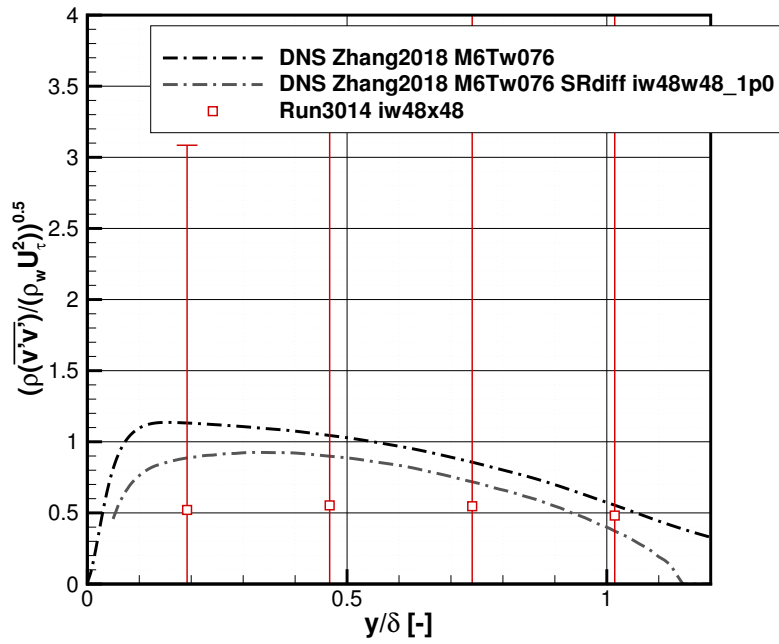


(b)

Figure 7.50: Streamwise a) and wall-normal b) component of turbulence profiles compared to artificially sub-grid filtered DNS data (grey lines) by [56] and [296] according to the method of [144] for IW 512x16 pix².

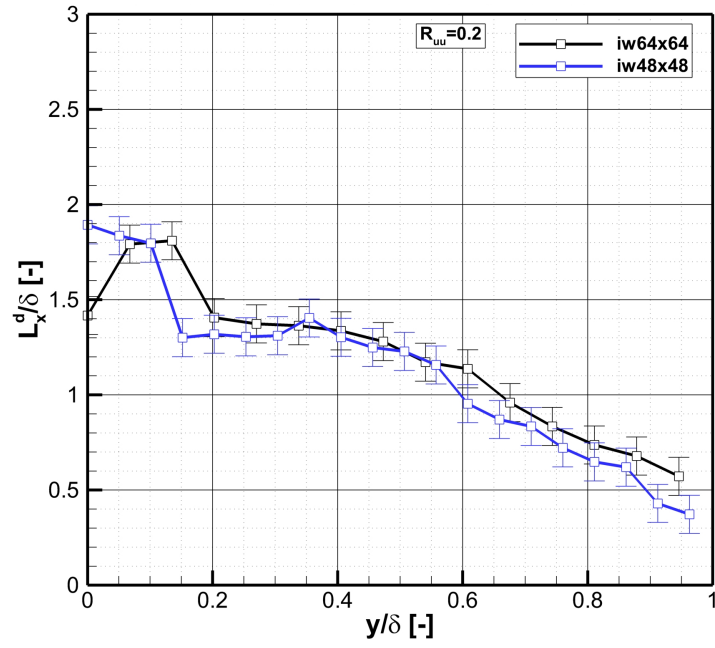


(a)

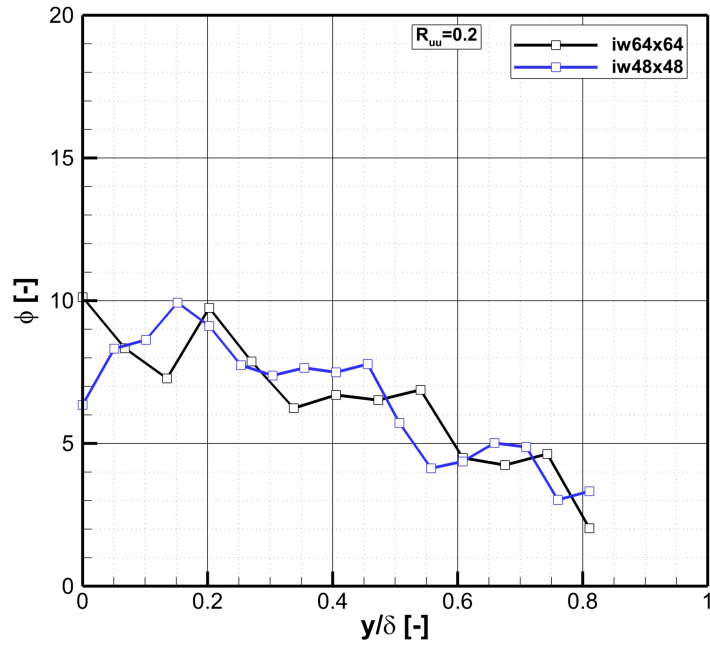


(b)

Figure 7.51: Streamwise a) and wall-normal b) component of turbulence profiles compared to artificially sub-grid filtered DNS data (grey lines) by [56] and [296] according to the method of [144]



(a)



(b)

Figure 7.52: Impact of IW sizes on two-point correlation of streamwise velocity fluctuation's a) change of downstream scale L_x^d and b) change of inclination angle with wall-normal position from correlation contour level 0.2.

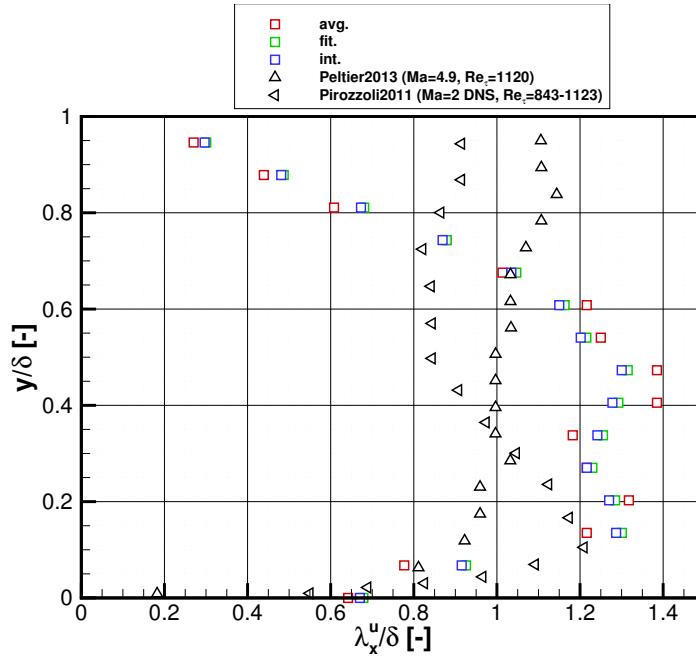


Figure 7.53: Streamwise length-scale λ_x^u against wall normal position for smooth wall case Run3014 and different calculation approaches compared to data from [183] and [189].

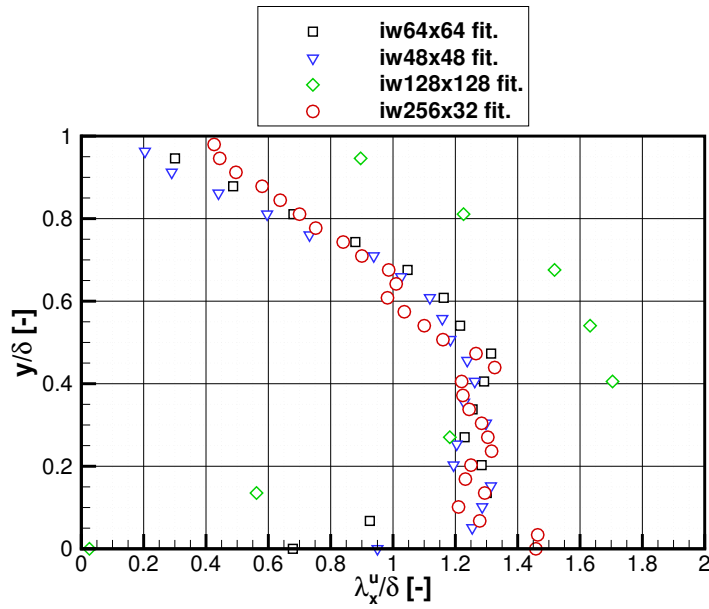
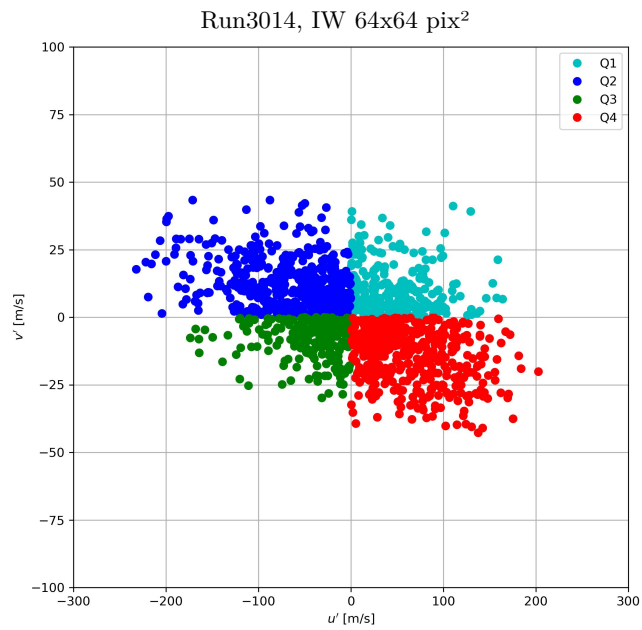
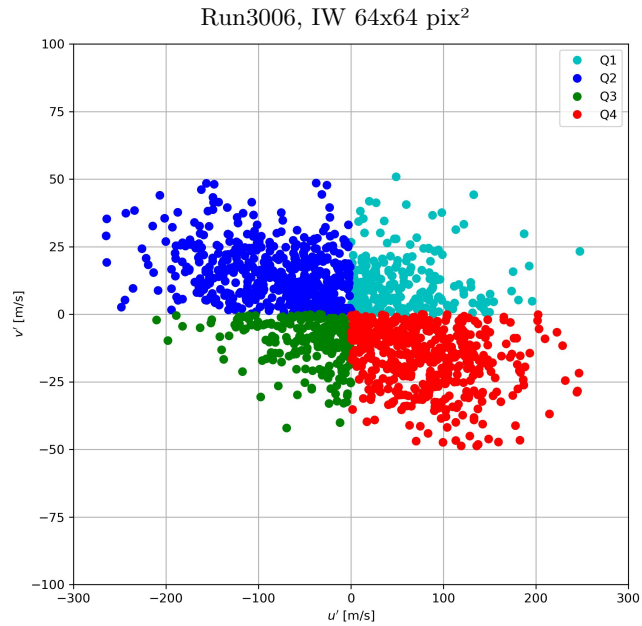


Figure 7.54: Streamwise length-scale λ_x^u against wall normal position for smooth wall case Run3014 and different IW sizes.



(a)



(b)

Figure 7.55: Quadrant plot for a) smooth wall (Run3014) and b) rough wall data (Run3006).

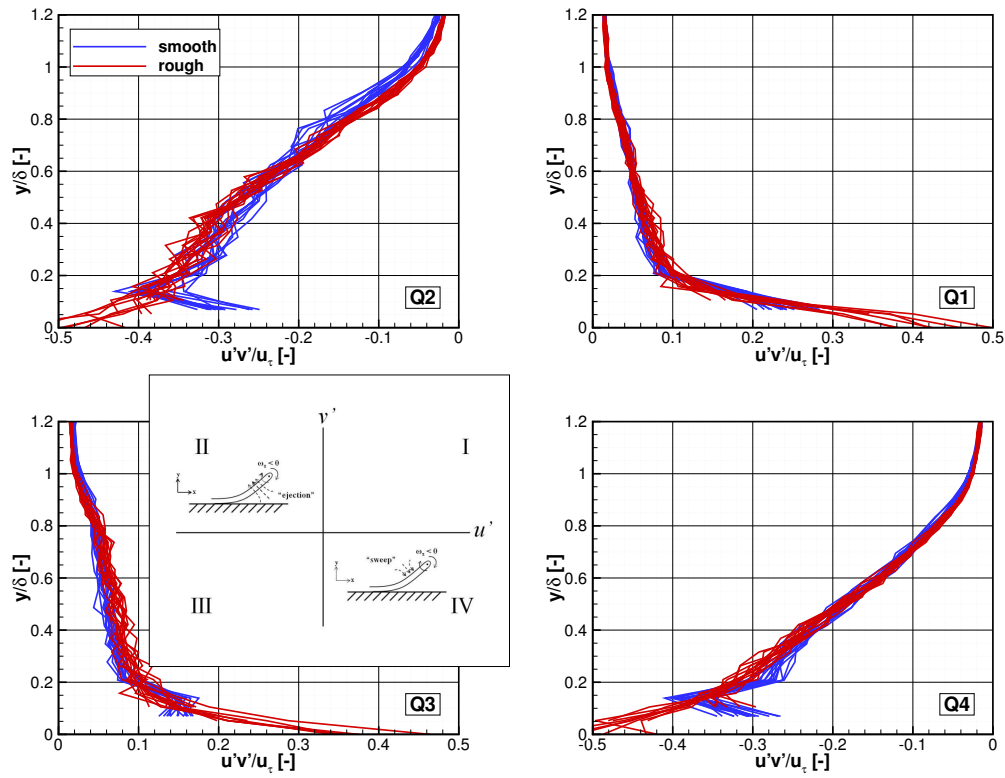
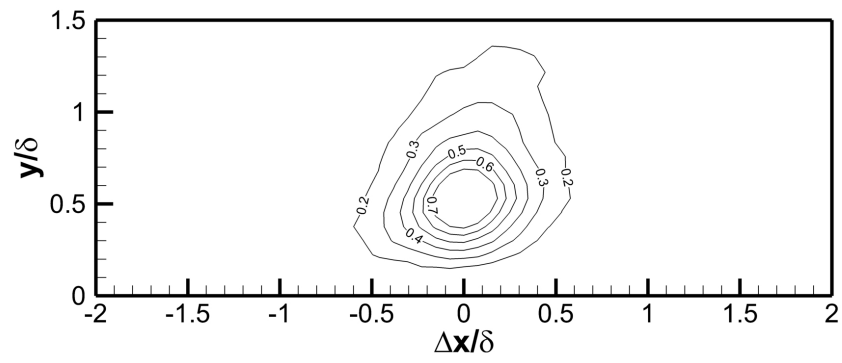
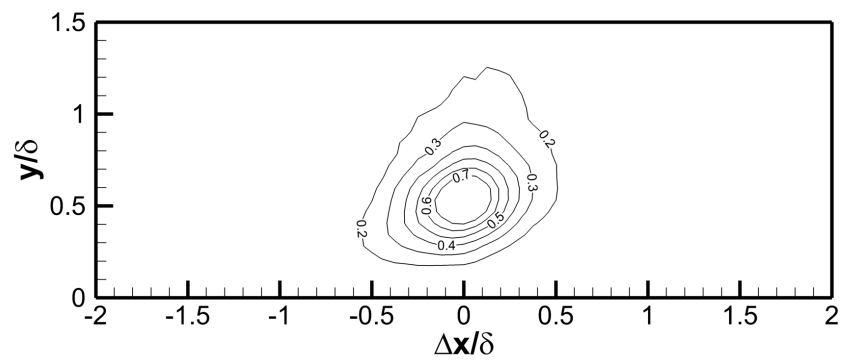


Figure 7.56: Reynolds stress, conditionally averaged according to their contributions to Q1 to Q4 along smooth (Run3014) and rough wall (Run3006).



(a)



(b)

Figure 7.57: Two-point correlation of the wall-normal velocity fluctuations at $y/\delta \approx 0.5$ along a) smooth and b) rough wall.

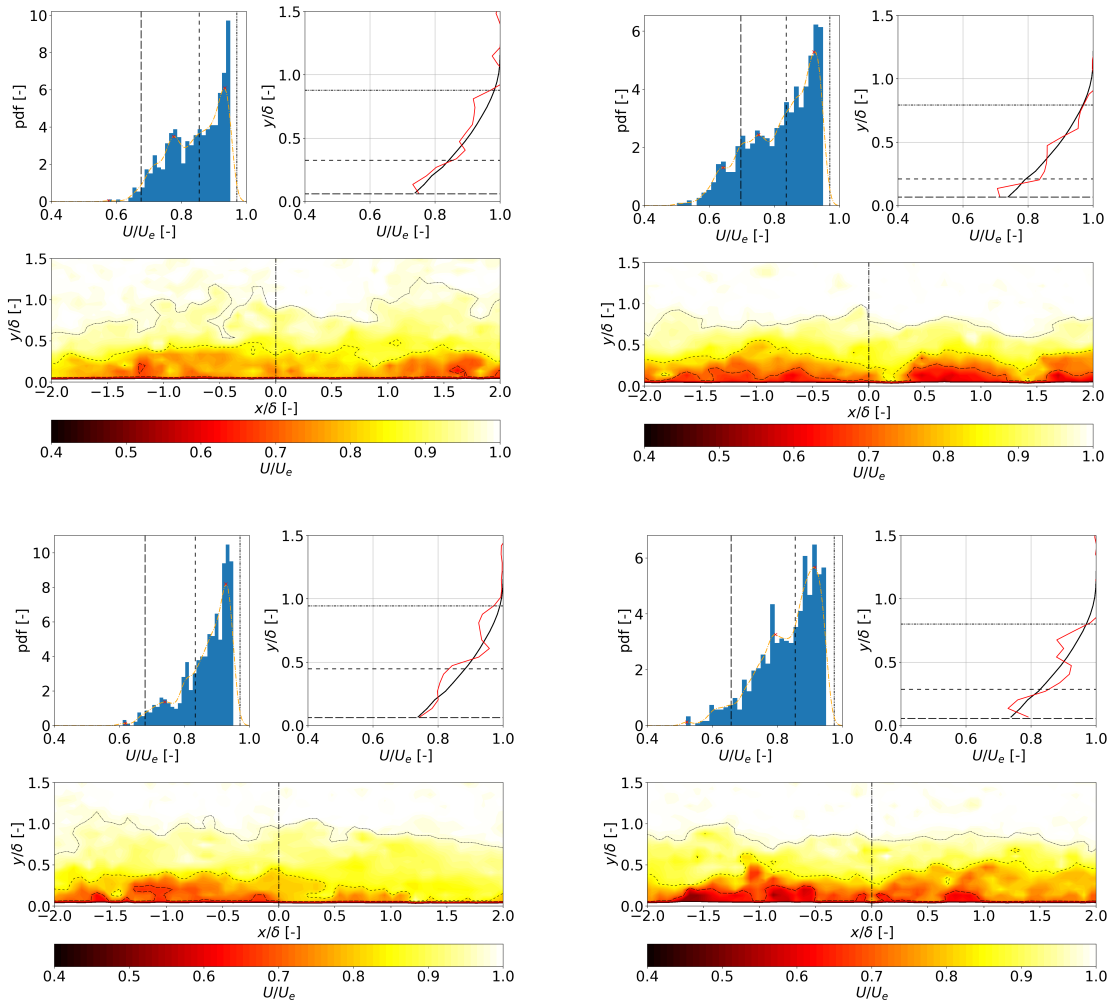


Figure 7.58: Compilation of p.d.f. (upper left) based on an instantaneous velocity field (lower middle) and a comparison of an instantaneous (red line) to the mean velocity profile (black line, upper right) for different snapshots of smooth wall Run3014. Boundaries for each UMZ are indicated as black lines of different dash-styles which correspond to each other in all corresponding subplots. TNTI is indicated in all subplots as black dash-dot-dotted contour line.

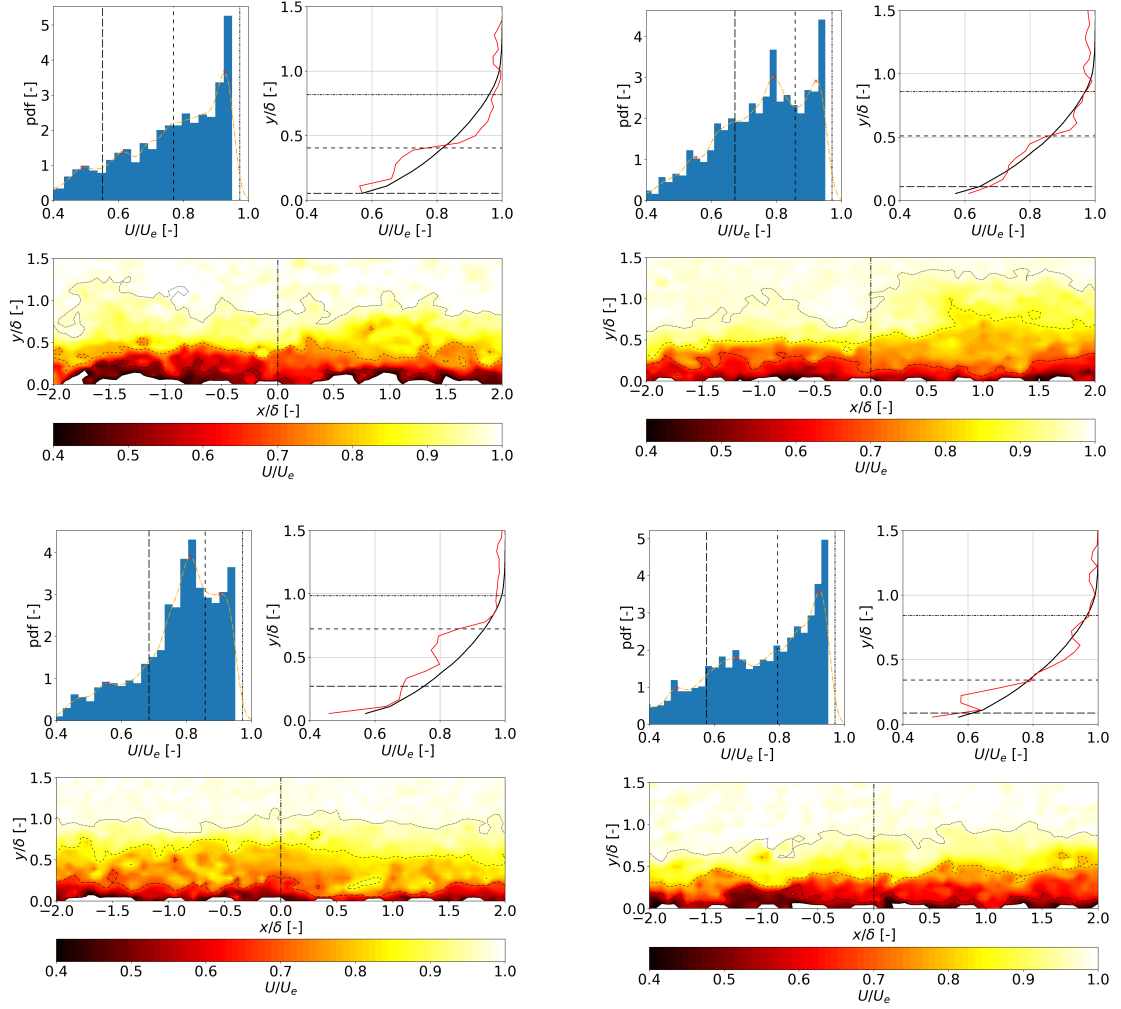


Figure 7.59: Compilation of p.d.f. (upper left) based on an instantaneous velocity field (lower middle) and a comparison of an instantaneous (red line) to the mean velocity profile (black line, upper right) for different snapshots of rough wall Run3007. Boundaries for each UMZ are indicated as black lines of different dash-styles which correspond to each other in all corresponding subplots. TNTI is indicated in all subplots as black dash-dot-dotted contour line.

7.5.3 Synthetic PIV (synPIV)

To study selected influences a tool was implemented to create synthetic PIV images. The focus of this tool was to rebuild the H2K PIV images as simple and ideal as possible to enable clear reference results via the tools of the PIV analysis tools. Furthermore, the tool should enable a defined modification of specific parameters without the necessity to include the complete chain e.g. laser set-up like the SIG package [142]. The tool of this study, implemented in python V2.7.4 including several extra packages like NumPy, SciPy and OpenCV version 2.4.6 (hereafter called synPIV), was able to create double images representative of the applied 2D2C set-up with a defined image size. For the particle seeding the first image is divided into pre-defined sub-windows. In each sub-window particles are created as white circles with a Gaussian intensity distribution. The particle size can be seeded either with a rectangular or a normal distribution. In the first case the amount and size of the particles are defined with a minimal and maximal diameter. In case of a normal distribution the amount, a mean and standard deviation of the particle diameter is defined. The intensity of the particles can be distributed linearly between a pre-defined minimum and maximum value corresponding to the pre-defined minimum and maximum diameter of the particle. The positions of the corresponding particles are uniformly distributed in each sub-window. The generated particles can be shifted in the second of the double images by a constant amount of pixels, a constant gradient or based on CFD data. No extra treatment is performed at the image edges, which means, no new particles emerge if not already created in the first image. A pre-defined signal-to-noise ratio can be simulated in the images, either with normally or uniformly distributed white noise. In the first case, Gaussian noise with zero mean intensity and a standard deviation of $\sigma_{SNR} = I_{hist,max}/SNR_{synPIV}$ was added with $I_{hist,max}$ as maximum intensity of the image histogram and SNR_{synPIV} as the pre-defined signal-to-noise ratio [122]. In the second case uniform noise with zero mean intensity and a maximal intensity with $s_{SNR} = I_{hist,max}/SNR_{synPIV}$ is added. Before saving, the images can also be uniformly blurred by a defined amount of kernel width and height with a Gaussian blur filter. Finally, the images are saved as 8-bit grayscale Tagged Image File Format (TIFF), which is compatible with the used PIV software.

Validation and sensitivity studies are included in the following section.

7.5.4 synPIV Sensitivity Studies

The synPIV tool, as described in section 7.5.3, was used to enable a sensitivity study for the PIV tools, especially with respect to the necessary elongated windows and their impact on the results. It was successfully validated with a simple case and results were compared to results of the program “simpiv” version 3.6.0.a, contained in the commercial PIVview package (not shown here).

To study the impact of elongated IWs, selected TAU solution fields were used to create synthetic particle images via the synPIV tool. Table 7.25 gives an overview of all sensitivity cases performed. In the following only some selected ones are presented and a synthesis is given.

TAU simulations with the RSM turbulence model at FC1 and 2% inflow turbulence intensity with an isothermal wall temperature of $T_w = 360$ K at highest mesh resolution was used as input.

Table 7.25: Overview of synPIV sensitivity cases (numP - number of particles per 48x48 window, muP and sigma - mean and standard deviation of log-normal particle distribution, s1D - smooth 1D, s2D – smooth 2D field, r2D – rough 2D field; IW field size dimensions switched with wall-normal x streamwise extent).

numP	muP [pix]	sigma [pix]	noise	cases	IW [pix ²]
1	10	2	0	s1D, s2D, r2D,	256 to 32x32, 256 to 64x96, 8x512
3	10	2	0, 5	s1D, s2D, r2D	256 to 32x32, 256 to 64x96, 8x512
5	10	2	0	s1D, s2D, r2D,	256 to 32x32, 256 to 64x96, 8x512
5	4	1	0	s1D, s2D, r2D	256 to 8x8, 256 to 32x32, 256 to 64x96, 8x512

For 1D profiles the data was extracted at $x = 0.5$ m corresponding to MS1. This data was used for the complete particle field along the streamwise direction, so no gradient in this direction exists (s1D). In case of 2D field a corresponding 2D solution was extracted from the CFD solution, interpolated on a IJ-ordered grid with the resolution necessary for the synthetic particle field. This way, also gradients along the streamwise direction were considered (s2D). The same strategy was used for a case with the rough wall fully resolved (r2D).

Several different interrogation windows (IW) were tested to represent corresponding cases used on the actual experimental data. IW of size 256 to 32x32 pix² (initial square IW with size 256x256 pix² with a final size of 32x32 pix²) were used as maximum resolved case (minimum final IW size), especially for rough cases. IW 256 to 64x96 pix² were used to represent a sensible medium resolution case and as reference to data published in [170]. Finally, IW of 8x512 pix² represent the high wall-normal resolved case. IW field size dimensions within this subsection are switched with wall-normal x streamwise extent.

The particle size distribution was set to represent the experimental data qualitatively. Figure 7.60 shows a comparison of two synthetic images (two left images) to an experimental raw image (right image). Here, a log-normal distribution with a mean of 10 pixel and a sigma of 2 pixel for 5 particles per 48x48 pix² window was used. Additionally, the most left image was created with a Gaussian noise with a signal-to-noise ratio of 5. This setup resembles the general feature of a typical H2K PIV raw image, although occasional highly dense regions are not considered (lower third of the most right image). The setup was not intended to duplicate the experimental conditions exactly, but to create sensible synthetic images on which the PIV algorithm can reliably work without being too detached from the real conditions.

The influence of differences in particle size distributions and noise was considered by parameter variations. Here, the above mentioned log-normal parameters were used as reference. With values of number of particles of 1, 3, 5 per 48x48 pix² window a seeding density sensitivity was addressed. A particle size sensitivity was studied with a mean particle size of 4 pix and a sigma of 1 pix, resulting in images deemed very suitable for PIV with most particle image sizes around 2 pixel (see e.g. [4]). Additionally, signal-to-noise ratio snr of 0, 5 were used to address a sensitivity.

Figure 7.61 shows an overview of results for s1D and 8x512 pix² windows with a sensitivity with respect to particle density (numP) and particle size (muP, sigma). All results are plotted in terms of pixel shift dx pix against wall-normal position in pixel iy pix. In general each plot shows the reference CFD streamwise and wall-normal pixel shift dx and dy , respectively, as lines, the corresponding PIV results as empty symbols and the corresponding difference between both as filled symbols as Δdx and Δdy , respectively. Additionally, for the latter, lines at ± 0.1 pixel are included as reference, representing an often stated typical uncertainty of a PIV setup. In the



Figure 7.60: synPIV synthetic raw particle image comparison. Two on the left from synPIV, right image from experimental first H2K campaign (Run1007).

current setup ± 0.1 pixel translate to ± 6 m/s or $\pm 0.7\%U_e$. If the results for the case with best PIV conditions (numP3, muP4-sigma1) are considered, it is visible that both the streamwise and wall-normal pixel shifts follow the reference very closely with deviations of clearly below 0.1 pixel. This is especially remarkable in case of the wall-normal shift dy due to its very small absolute values of typically below 0.2 pixels. This gives confidence with respect to the used method for sub-pixel shift accuracy (Whittaker reconstruction). Larger particles at same density (numP3, muP10-sigma2) show noisier results with slightly larger deviations for the wall-normal shift dy but still in the order of ± 0.1 pixel. The streamwise shift is still captured accurately. The picture changes not much if for the larger particles the density is changed (numP1 to numP5 per 64x64 window).

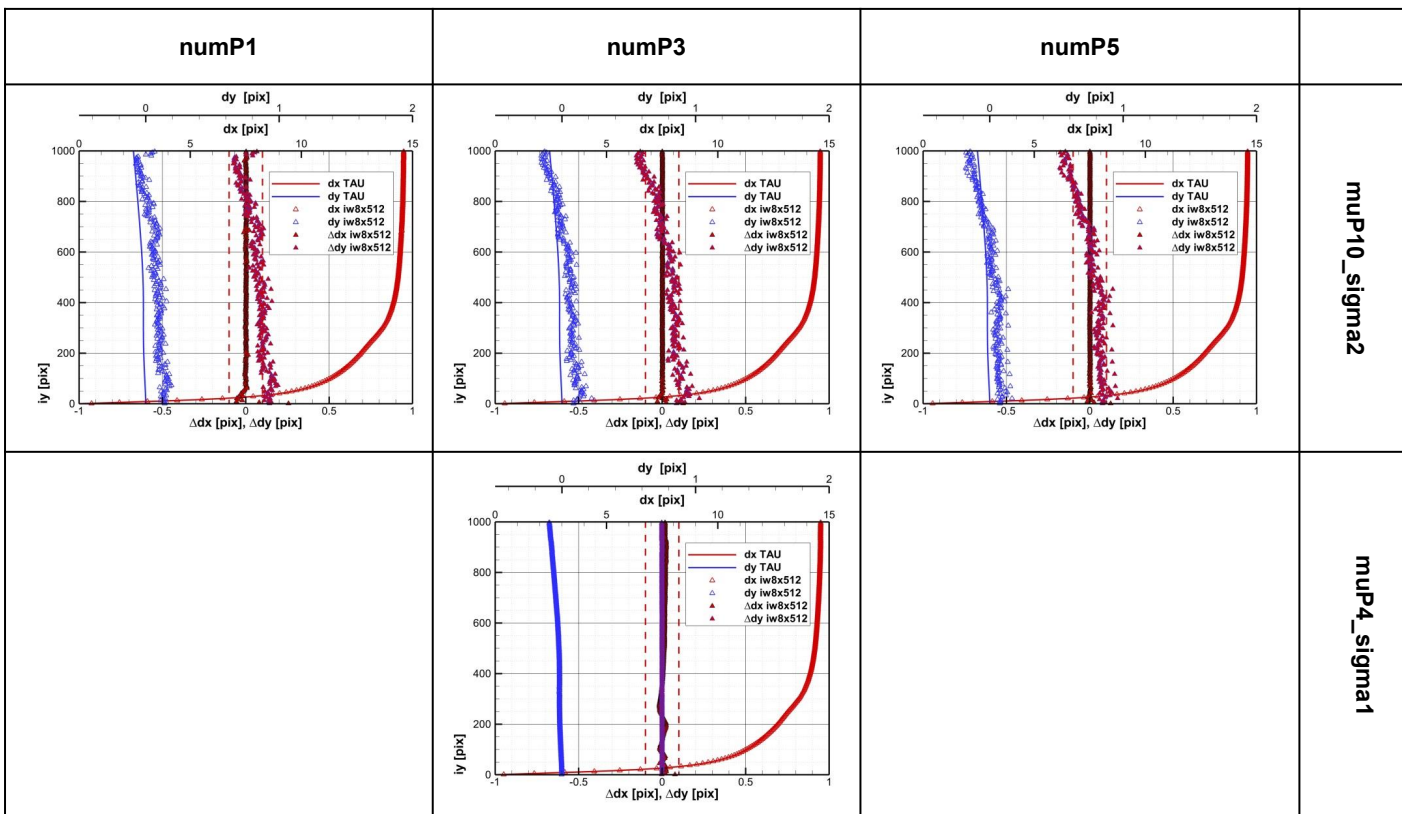


Figure 7.61: synPIV sensitivity study for FC1, MS1, s1D, IW 8x512 pix².

Figure 7.62 show the same plots in case of s2D. In general the results look similar to the one for s1D, but in the immediate vicinity of the wall, corresponding to $y/\delta \lesssim 0.1$ or $y^+ \lesssim 20$, differences of pixel shift above $\Delta dx = 0.1$ pix can be observed (corresponding to 0.7% of the edge velocity). Since this near wall deviations were not observed in the s1D case, the highly elongated IWs may be influenced by small streamwise gradients. For the fitting approach (see section 3.10.2), it is interesting to check whether the largest deviations occur in the region of the logarithmic part of the boundary layer profile. Therefore, the extent of the log-layer region is approximated starting at $y_{l,i}^+ \approx 30$ and ending at $y_{l,e}/\delta \approx 0.3$, corresponding to $y_{l,i} \approx 62.4$ pixel and $y_{l,e} \approx 193.4$ pixel, respectively. It is visible that the deviations are below the marked ± 0.1 pixel within this region.

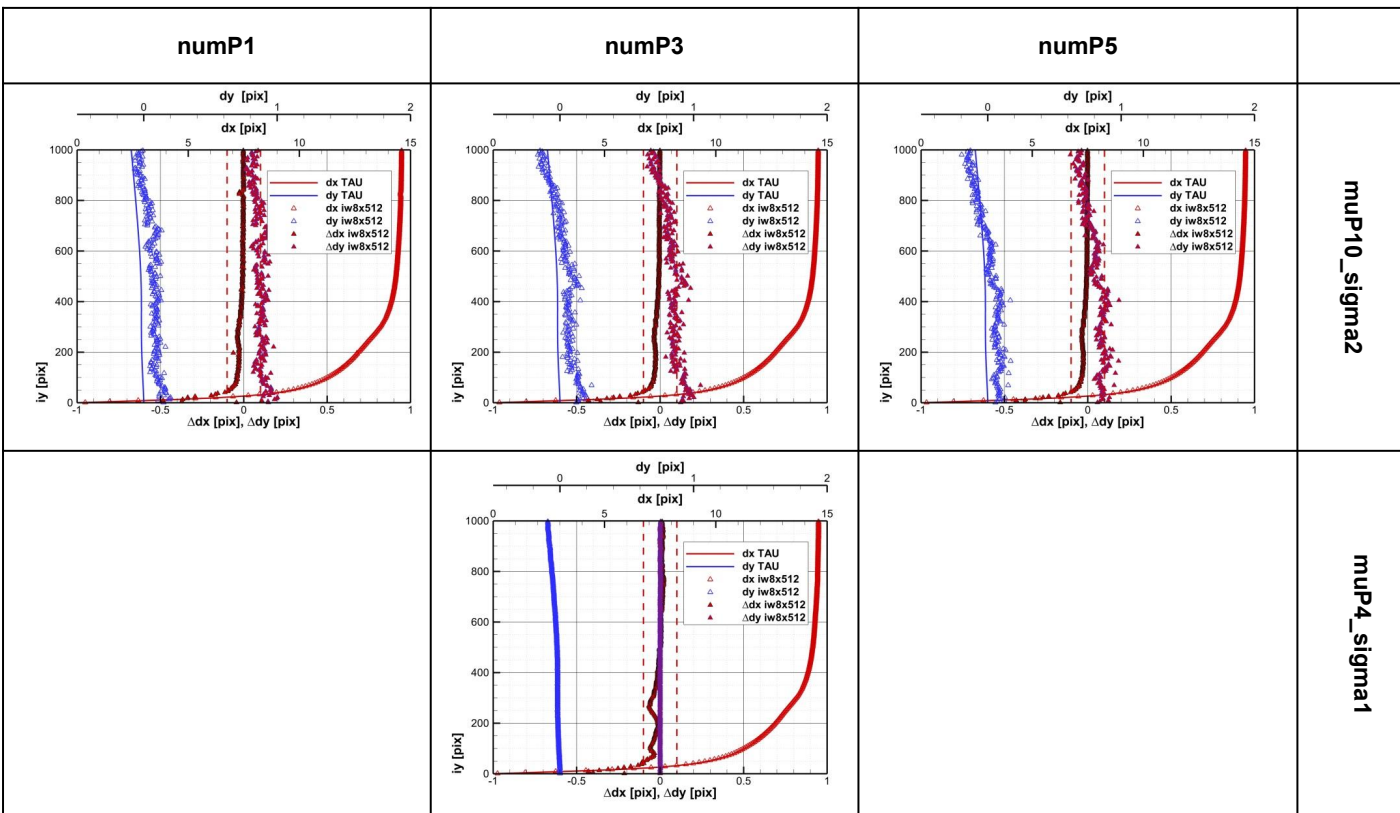


Figure 7.62: synPIV sensitivity study for FC1, MS1, s2D, IW 8x512 pix²

In case of the other investigated IWs apart from the 8×512 pix², the streamwise deviations are typically below ± 0.1 pixel, also in the relevant log-layer region, except for only the very first point above the wall.

Additional tests with increased image noise, also lead to noisier pixel shift profiles but not a change in the general order of magnitude in deviations (not shown here).

Different additional settings and sensitivities were tested with synPIV, like the cropping of images, leveling of images, etc., not reported here. Selected validation cases were also post-processed by the PIV tool used in the second PIV campaign, which showed comparable results as reported above.

Additionally, synPIV was used on selected cases with pre-defined two-point correlations via DFM (see section 7.5.6) to study the impact of IW size on extracted length scales. Also, the applicability of both used PIV codes to extract two-point correlations were checked with case B (synthetic PIV from DNS of turbulent open channel flow) from the second PIV challenge in 2003 [242].

A few exemplary results are shown in the following for data from synPIV based on flow field data from DFM. As input, integral length scales from TBL DNS at $\delta^+ \approx 2000$ of Sillero *et al.* [228] were used with $\Lambda_{u,x}/\delta \approx 1$, $\Lambda_{u,y}/\delta = \Lambda_{u,z}/\delta \approx 0.25$ and $\Lambda_{v,x}/\delta = \Lambda_{v,y}/\delta = \Lambda_{v,z}/\delta \approx 0.25$ (see figure 10 in the original publication [228]). The synthetic flow fields were created by the digital filtering method described in section 7.5.6 based on mean and turbulence quantities from the TAU solution corresponding to s2D dataset described above. Before synthetic PIV images were created from these synthetic flow fields, a check was made for the prescribed and extracted U , V , u_{rms} , v_{rms} and $\overline{uv}$ from the TAU input (red lines) and extracted from the synthetic flow fields (blue symbols) in pixel shift scaling.

Additionally, two point correlations were prescribed in the flow field with an exponential form based on the above given length scales. Figure 7.64 shows exemplary checks of these two-point correlations with figure 7.64a R_{uu} , figure 7.64b x,y -cuts of R_{uu} , figure 7.64c R_{vv} , figure 7.64d x,y -cuts of R_{vv} .

From these synthetic flow fields via synPIV 1000 synthetic particle image pairs were created with two different particle densities. With numP3, muP10-sigma2 (see explanation of parameters above) particle images which closely resemble conditions of the first PIV campaign were created. Additionally, with numP10, muP2-sigma0.02 a set of particle images was created with larger particle density and diameters close to 2 pixel which represent near optimal PIV images. In the following only selected plots are shown to check the ability to extract proper two-point correlations via the processing chain used in this thesis in section 4.3.3. Selected IWs, used in this thesis, were applied on both particle image qualities. Figure 7.65 shows two-point correlation data of synthetic dataset from synPIV with figure 7.65a R_{uu} at $y/\delta = 0.5$ and figure 7.65b extracted integral length scale, both for different synthetic particle data sets and IWs as indicated in the legends. In general no substantial differences can be observed for both plotted representations with varying particle image quality and IW sizes. The extracted length scale λ_{uu}/δ , representing the integral length scale from fitting an exponential curve to the autocorrelation at the corresponding wall distance (see section 7.5.2.2, shows good agreement to the pre-set value of 1 down to approx. $y/\delta \approx 0.2$. Below, wall influences skew the extracted length scales.

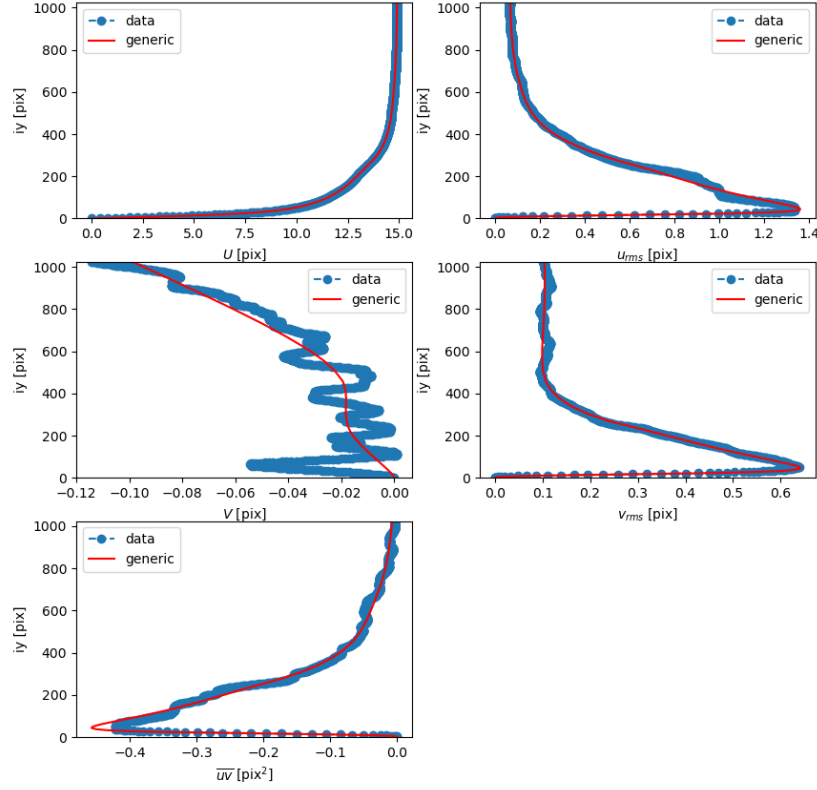


Figure 7.63: Profiles of U , V , u_{rms} , v_{rms} and $\overline{uv}$ from the TAU input (red lines) and extracted from the synthetic flow fields (blue symbols) in pixel shift scaling.

Figure 7.66 shows the impact of the different synthetic particle data sets and IW dimensions on results derived from ellipse fitting to the correlation levels, used in section 4.3.3 to derive maximum dimensions L_m^x/δ of different representative correlation levels and inclination angles ϕ . Figure 7.66a shows the extracted maximum dimensions L_m^x/δ of different correlation levels and no large impact of seeding quality and IW dimension is visible. Also the extracted inclination angles in figure 7.66b show no large impact of IW dimensions. In both cases the DFM input values are included as “generic” and deviations can be used to estimate uncertainties of post-processing. Although it can only serve as a rough approximation, as it is highly dependent on the quality of the PIV images and the synthetic versions were only parametrically approximated. Estimated uncertainties for $U_{L_m^x/\delta} \approx \pm 0.2$, as described in section 4.3.3, are included as error bars and it is visible that most deviations are captured, despite single near wall points for larger IWs. In case of the inclination angle deviations up to $U_\phi \approx 4^\circ$ are visible, whereas data spread in the real PIV data showed smaller deviations.

To summarize all synPIV sensitivity analysis, the results showed that the applied processing

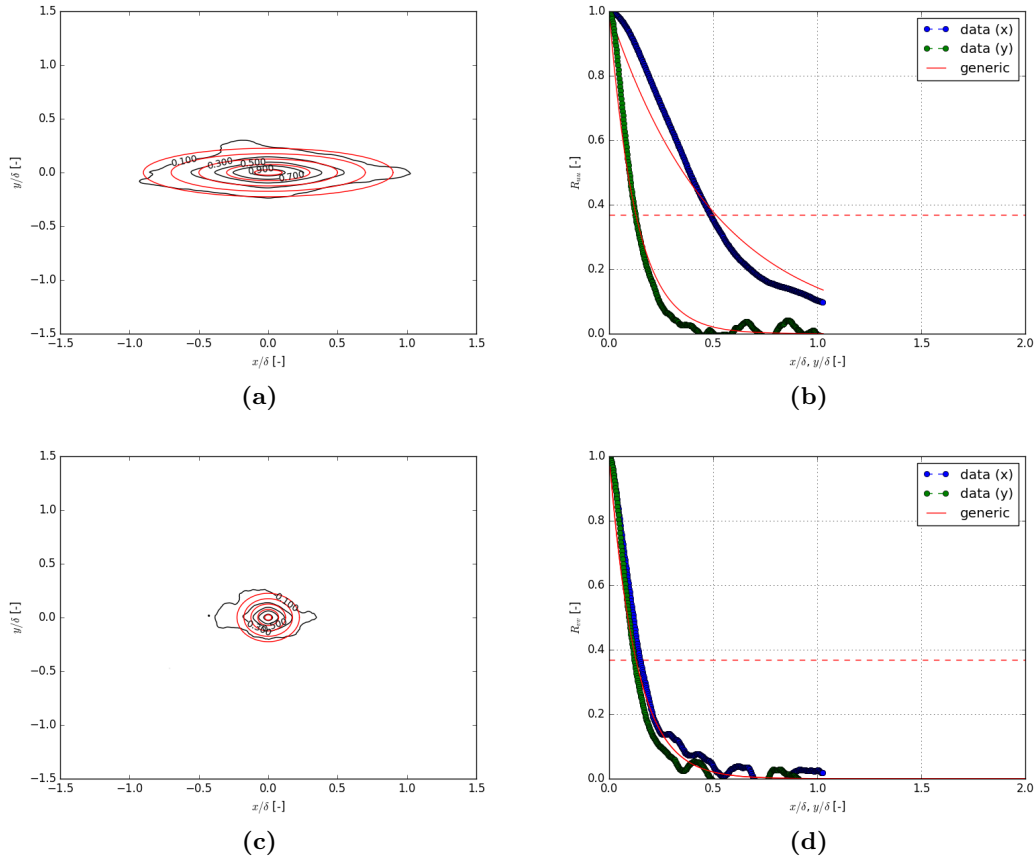
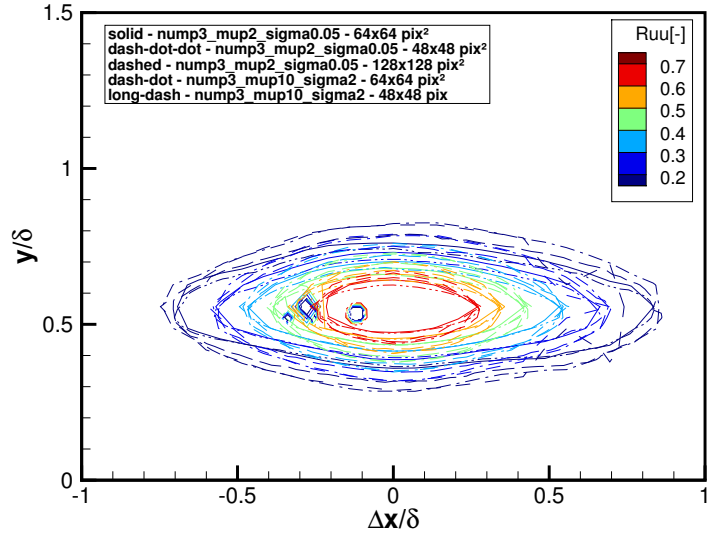
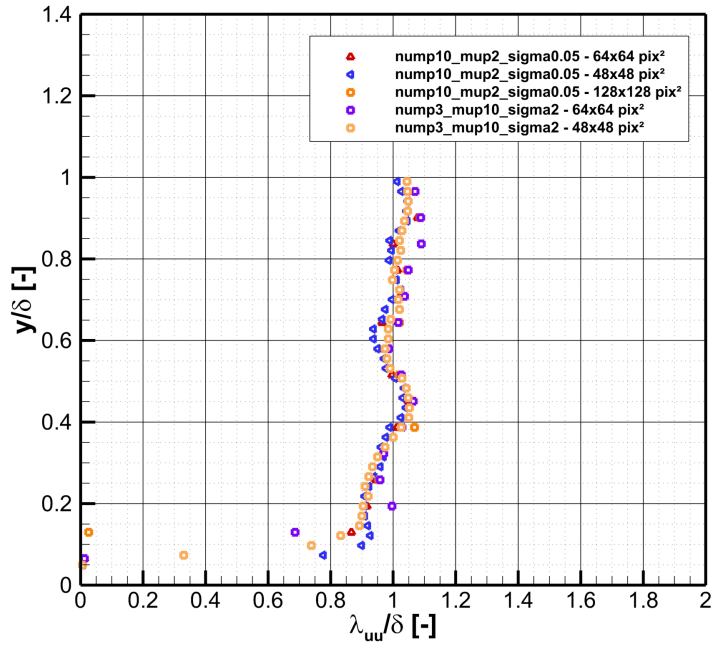


Figure 7.64: Two-point correlation data of synthetic dataset via synPIV with DFM method with a) R_{uu} , b) x, y -cuts of R_{uu} , c) R_{vv} , d) x, y -cuts of R_{vv} ; red lines and label “generic” shows input.

chain of this thesis can give accurate results for particle velocities (also for the highly elongated IWs), turbulence quantities as well as two-point correlations and length scales. This gave further confidence in the results of this thesis.

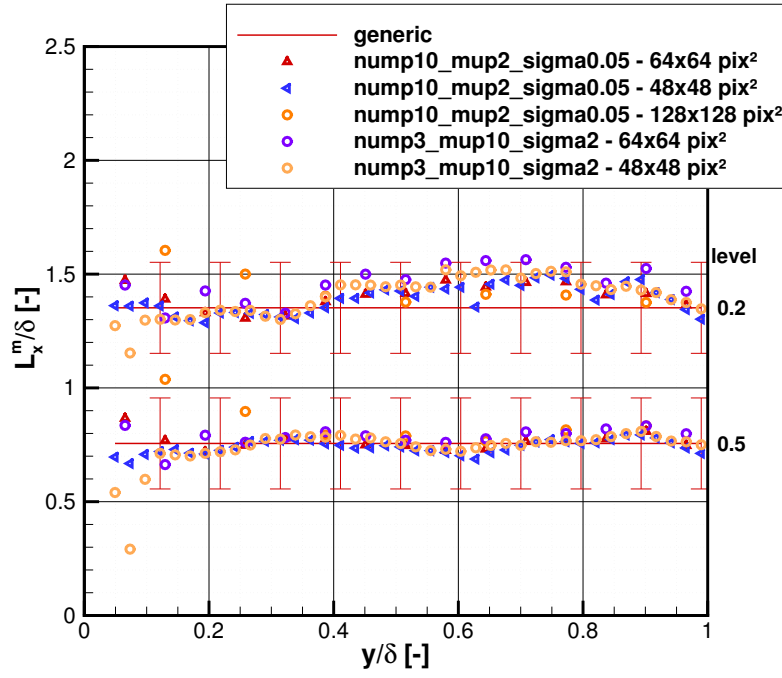


(a)

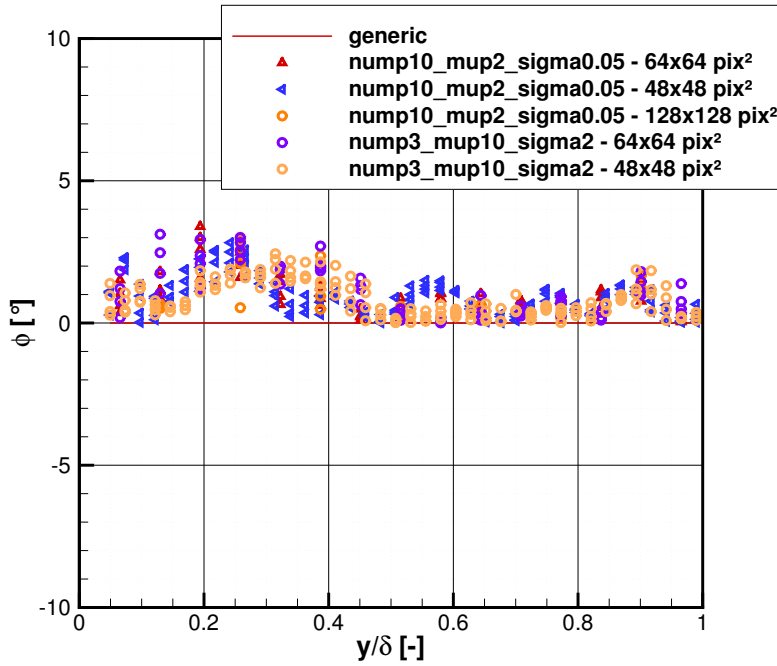


(b)

Figure 7.65: Two-point correlation data of synthetic dataset from synPIV with a) R_{uu} at $y/\delta = 0.5$ and b) extracted integral length scale for different synthetic particle data sets and IWs.



(a)



(b)

Figure 7.66: Two-point correlation fitting results of synthetic dataset from synPIV showing sensitivity of a) L_x^m/δ and b) ϕ for different synthetic particle data sets and IWs.

7.5.5 Particle Sensitivity Studies

With the TAU particle module (see section 3.9.2) an interesting additional aspect could be analyzed. In PIV raw images a slightly higher particle density at a certain region above the wall could be observed (see e.g. figure 7.60 or 4.2). Since the particle module is also able to extract particle densities (called “local catch efficiency” in TAU), simulations were used to try to rebuild this observation.

For this, a CFD solution along the cone at FC1 was seeded with particles and these particles were caught at a pre-defined catching plane corresponding to MS1 or MS2 (see also section 7.7.3 or 7.5.2.1). Here, the density “beta” is extracted, which is defined as the local ratio of inter-particle trajectory distances from the catching plane to the initiation plane (close to the inflow boundary). The drag model according to Tedeschi was used (see section 7.7.3). Figure 7.67 shows the calculated particle densities for p_1 ($d_p = 0.23\mu\text{m}$, $\rho_p = 3800\text{kg/m}^3$) and p_2 ($d_p = 2\mu\text{m}$, $\rho_p = 800\text{kg/m}^3$) (see also section 7.5.2.1), together with data from PIV of the first campaign (symbols). The latter were extracted by averaging the image intensities (normalized via the image histogram) along the streamwise direction. This is just a qualitative analysis and therefore no units are used. The plot is arranged in such a way, that the peak levels from CFD and PIV roughly coincide. Additionally, for reference, the mean velocity profile from CFD is plotted.

It is interesting to note, that for both experimental and CFD data, two peaks in the particle density are visible. One is located at the wall and a secondary peak is with $y/\delta \approx 0.75 - 1$ close to the boundary layer edge in case of experiments, whereas the CFD data show this peak at $y/\delta \approx 0.5$, independent of particle sizes and densities. The wall distance of this peak slightly increases with running length as visible by the difference between MS1 and MS2 results, but stays close to the boundary layer edge.

Previous data suggested a certain velocity lag of particles as they move along the conical flow field (see figure 7.28). It seems that this drift in combination with particles which hit the cone model (especially in the nose part, where the oblique shock is closer to the surface) lead to an aggregation of particles at a certain distance to the wall. However, it needs to be mentioned that in the current version of particle simulations no particle reflections are considered. As soon as a particle hits the wall it is removed from the simulation, so that this effect is not accounted for. Other drag models (Henderson) were tested as well and showed only minor differences (not shown here).

Although the analysis can only provide qualitative results, it nevertheless gives an indication of potential reasons for the observed decrease of particle quality in the vicinity of the boundary layer edge, reported about in the framework of correlation results in section 4.3.3 and visual appearances in figure 4.2.

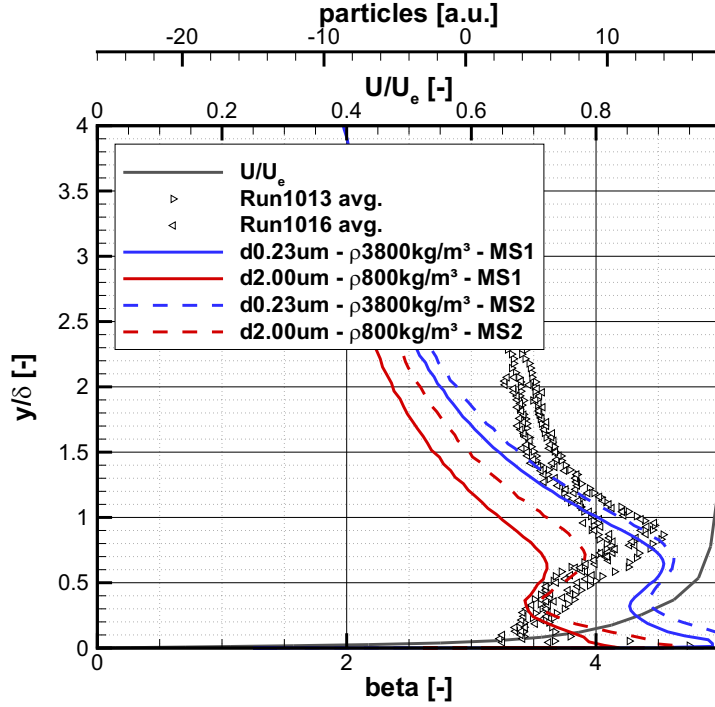


Figure 7.67: Particle density simulations and data from first PIV campaign.

7.5.6 Digital Filtering Method (DFM)

Synthetic velocity fields were created by applying the digital filtering method (DFM) [130], [293]. With this procedure, an artificial velocity field can be created which possess certain pre-defined statistical properties, like mean values, fluctuations, cross correlations, higher order moments and length and time scales. The nomenclature was kept from the corresponding literature, meaning that our plane of interest (PIV image plane) is denoted by the coordinates y and z (representing x and y in our system, respectively).

To imprint a defined cross correlation, a suitable functional form needs to be chosen. It can be shown that an exponential form is very convenient for DFM according to

$$R(r,0,0) = \exp\left(-\frac{\pi r}{2L}\right) \quad (7.15)$$

with L as a pre-defined length scale (e.g. turbulent integral length scale) and r the distance to the origin of the two-dimensional correlation. Compared to a realistic two-point correlation in a turbulent flow, eq. 7.15 oversimplifies and introduces errors in the behavior near $r = 0$ but shows realistic large-scale behavior.

For simplicity, the following steps are defined for one-dimensional data. With n representing the number of grid steps of a defined step size Δx and the length scale as $L = n\Delta x$, the exponential correlation may be re-written to

$$\frac{\overline{u_m u_{m+k}}}{\overline{u_m u_m}} = R_{uu}(k\Delta x) = \exp\left(-\frac{\pi|k|}{2n}\right) \quad (7.16)$$

A filter function can be defined as

$$u_m = \sum_{j=-N}^N b_j r_{m+j} \quad (7.17)$$

where r_m is a series of random data with $\overline{r_m} = 0$, $\overline{r_m r_m} = 1$, b_j are filter coefficients and N is connected to the length scale of the filter, where $N \geq 2n$ was used in this work. The mean value is $\overline{u_m} = 0$, but because $\overline{r_m r_n} \neq 0$ for $m \neq n$, one obtains

$$\overline{u_m u_{m+k}} = \sum_{j=-N+k}^N b_j b_{j-k} \quad (7.18)$$

For an exponential correlation, a simple but approximate solution to eq. 7.18 can be found with the filter coefficients defined as

$$b_k = \tilde{b}_k / \left(\sum_{j=-N}^N \tilde{b}_j^2 \right)^{0.5} \quad (7.19)$$

where

$$\tilde{b}_k \approx \exp\left(-\frac{\pi|k|}{n}\right) \quad (7.20)$$

To extend the above approach to two-dimensional data, a corresponding two-dimensional filter can be obtained by

$$b_{jk} = b_j b_k \quad (7.21)$$

The following recipe was followed in this work to derive two-dimensional velocity fields, which possess pre-defined mean velocity fields, Reynolds stress distributions and cross correlations based on a given length-scale L :

1. Choose for each of the two coordinate directions a length scale with $L_y = n_y \Delta y$ and $L_z = n_z \Delta z$ and a filter width defined by $N_\alpha \geq 2n_\alpha, \alpha = y, z$.

2. Initialize and store two two-dimensional random fields $\mathfrak{R}_\alpha, \alpha = y, z$ with dimensions $[-N_y + 1 : M_y + N_y, -N_z + 1 : M_z + N_z, s]$ where $M_y \times M_z$ are the dimension of the grid in the plane of interest and s the number of images, representing the number of PIV images (since the time between two double-pulses of the laser with a frequency of maximal $f \approx 100$ Hz was large in comparison to typical convection times in the high-speed boundary layer, no correlation in the time domain was applied with the assumption that the fields are uncorrelated).
3. Calculate the filter coefficients $b(i, j)$ according to eq. 7.19, 7.20 and 7.21
4. Apply the following filter operation for $j = 1, \dots, M_y, k = 1, \dots, M_z$ and every image s

$$\mathcal{U}_\alpha(j, k, s) = \sum_{j'=-N_y}^{N_y} \sum_{k'=-N_z}^{N_z} b(j', k') \mathfrak{R}_\alpha(j + j', k + k', s) \quad (7.22)$$

5. To include a defined mean field $\overline{u_n}$ and Reynolds stress tensor R_{nm} for the field in the corresponding $n = y, z$ direction, the final velocity field may be derived by a further transformation according to

$$u_n = \overline{u_n} + a_{nm} \cdot \mathcal{U}_m \quad (7.23)$$

with

$$(a_{nm}) = \begin{pmatrix} (R_{11})^{0.5} & 0 \\ R_{21}/a_{11} & (R_{22} - a_{21}^2)^{0.5} \end{pmatrix} \quad (7.24)$$

where the final s fields u_n and $\mathcal{U}_m$ have the dimensions (j, k) and also the one mean field and Reynolds tensor components have the dimensions (j, k) .

For further details of DFM, the reader is referred to [130] and [293].

The above described method was used to derive synthetic data fields according to the approach by Spencer and Hollis [238] (see chapter 7.5.2) and to enforce defined two-point correlations to selected synPIV data (see section 7.5.4).

7.6 Additional Heat Flux Investigations

7.6.1 PEEK Characterization

In this section, the emissivity properties of PEEK are detailed, followed by the material characteristics, relevant for thermal analyses.

To assess the surface temperature via infrared thermography properly, the emissivity of the material of interest need to be known. This emissivity is usually dependent on a variety of

properties, e.g. on the material temperature, the emitted wavelength and the angle between the camera and the model. The aforementioned dependencies were characterized for different probes of “TECAPEEK natural”[®] externally, at the Bavarian Center for Applied Energy Research, ZAE Bayern. The detailed results can be found in section 7.6.2. Based on this data for this thesis, a typical mean emissivity of $\epsilon = 0.95$ within a viewing angle of $\phi = 0 - 40^\circ$ was assumed.

The polynomial correlations for temperature dependent material properties for density ρ , heat capacity c_p and thermal conductivity λ are given in the following, based on measurements by the Institute of material sciences at DLR (see [95]).

$$\rho_{\text{PEEK}} = A_\rho + B_\rho T \quad (7.25)$$

$$c_{p,\text{PEEK}} = A_{cp} + B_{cp} T \quad (7.26)$$

$$\lambda_{\text{PEEK}} = A_\lambda + B_\lambda T + C_\lambda T^2 + D_\lambda T^3 \quad (7.27)$$

$$\frac{\partial \lambda_{\text{PEEK}}}{\partial T} = B_\lambda + 2C_\lambda T + 3D_\lambda T^2 \quad (7.28)$$

The corresponding coefficients are detailed in table 7.26.

Table 7.26: Coefficients for the material parameters of PEEK [95]

	ρ_{PEEK}	$c_{p,\text{PEEK}}$	λ_{PEEK}
A_x	1382	6.09998	$-2.98173 \cdot 10^{-1}$
B_x	-0.29	3.50000	$4.13700 \cdot 10^{-3}$
C_x	-	-	$-1.00939 \cdot 10^{-5}$
D_x	-	-	$8.16379 \cdot 10^{-9}$

7.6.2 PEEK Emissivity Analysis

To assess the surface temperature via infrared thermography properly, the emissivity of the material of interest needs to be known. This emissivity is usually dependent on a variety of properties, e.g. on the material temperature, the emitted wavelength and the angle between the camera and the model. The aforementioned dependencies were characterized for different probes of “TECAPEEK natural”[®] externally, at the Bavarian Center for Applied Energy Research, ZAE Bayern. The spectrally-resolved emissivity was measured via Fourier-transform infrared spectroscopy. Based on the typical testing conditions in H2K, the material probes were characterized at room temperature, 50° C, 100° C, 150° C and 200° C. At temperatures equal and above 50° C the emissivity was measured for viewing angles of 0°, 40°, 50°, 55°, 60° and 70°.

Two different PEEK material probes were investigated. Probe 2 represents a surface condition after extensive testing in the wind tunnel environment. Probe 4 represents a completely new surface condition after milling off approx a millimeter of a probe equivalent to probe 4. A further probe (probe 1) was additionally investigated at room temperature, representing a surface condition after moderate testing. Since the latter probe showed nearly the same results as probe 2 (see figure 7.68), only probe 2 and 4 were used for the full emissivity characterization. In the following, only representative results of probe 4 are shown.

Figure 7.68 shows the spectrally-resolved emissivity $\epsilon(\lambda)$ at room temperature. Additionally, the default value based on experience with $\epsilon = 0.95$ is included as reference. Furthermore, the spectral operating ranges of the potential IR cameras is included. It is visible that within the camera spectral ranges, in total the emissivity varies between $\epsilon \approx 0.6 - 0.98$.

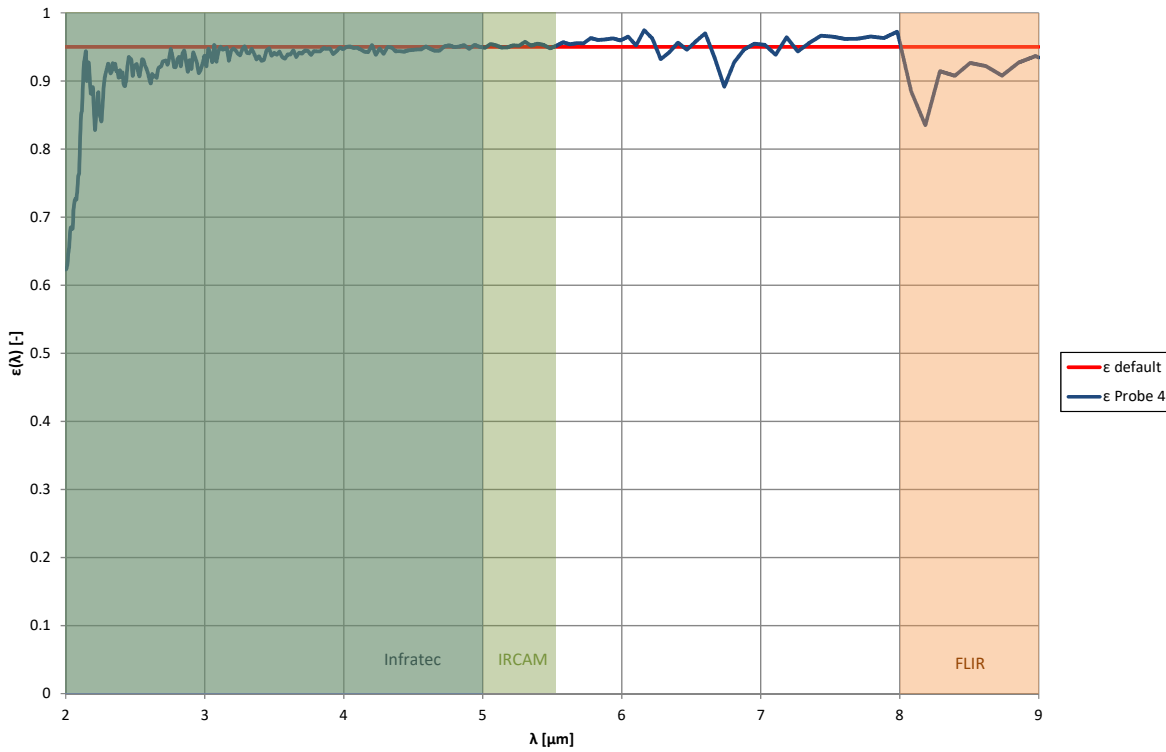


Figure 7.68: Spectral dependency of “TECAPEEK natural”[®] emissivity at room temperature

From the spectrally-resolved emissivity data an average value was extracted via integration over the wavelength for both camera ranges, as visible in figure 7.69. Additionally, estimated uncertainties are included with $U_\epsilon = 0.02$ via error bars. Based on this data, a global emissivity of $\epsilon = 0.95$ was used for the measurements via the Infratec camera. The data of the FLIR camera was only used qualitatively. But the importance of including spectral information is clearly visible in figure 7.69, since a clear difference in averaged emissivities is visible between the short-/ mid-wavelength ($\lambda = 2 - 5 \mu\text{m}$) and long-wavelength infrared regime ($\lambda = 8 - 9 \mu\text{m}$).

Figure 7.70 shows the emissivity dependency on the viewing angle. Here, the angle ϕ is defined as angle between the camera axis and the local normal vector on the probe surface. In general,

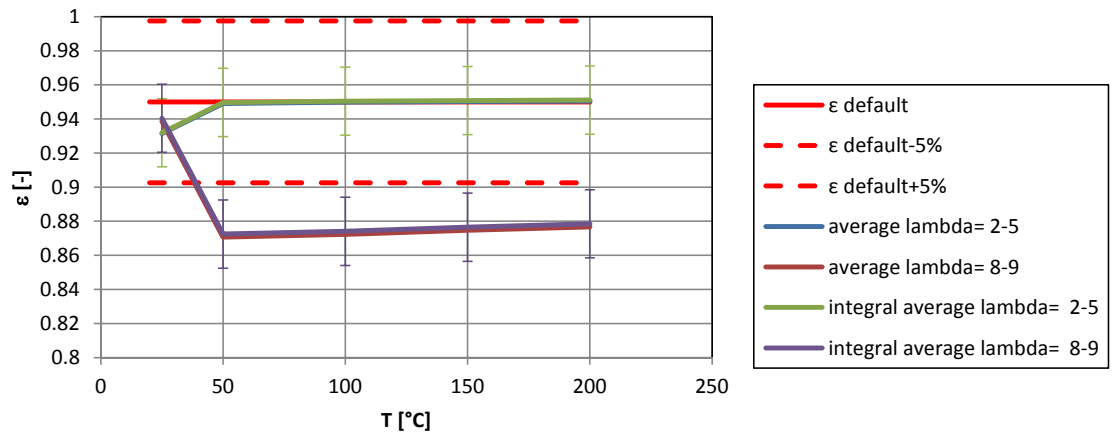


Figure 7.69: Temperature dependency of “TECAPEEK natural”[®] emissivity (probe 4)

the typical behavior of a dielectric material is visible with relatively constant emissivities at low angles and a sharp decrease at higher angles. For PEEK, the change of emissivity stays within 1% up to an angle of 40°.

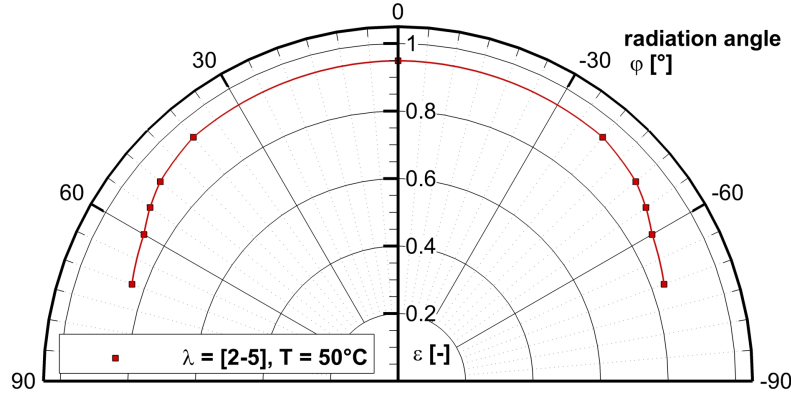


Figure 7.70: Emissivity of “TECAPEEK natural”[®].

7.6.3 CT Parameters

According to Hollis [107] chromel material parameters were used for the analysis of CT. The coefficients detailed in table 7.27, according to [107], were used in this work together with equations (7.25) to (7.27).

Table 7.27: Coefficients for eq. (7.25) to (7.27) of the chromel material parameters according to [107]

	ρ_{CHROMEL}	$c_{p,\text{CHROMEL}}$	λ_{CHROMEL}
A_x	8714	386.25	11.845
B_x	-	0.23981	$1.9132 \cdot 10^{-2}$
C_x	-	-	-
D_x	-	-	-

7.6.4 pyHeatflow Validation

The implementation of the axisymmetric formulation of the heat balance for “pyHeatflow” (see section 3.4.2) was checked via ANSYS. For this, a generic case with a cylindrical one-dimensional element was simulated in ANSYS by applying a constant conductive heat flux of 10kW/m^2 on the surface of the element, representative for a typical heat load during H2K tests. A typical test time for evaluation of the IR results is simulated with 15 seconds. The element was designed to

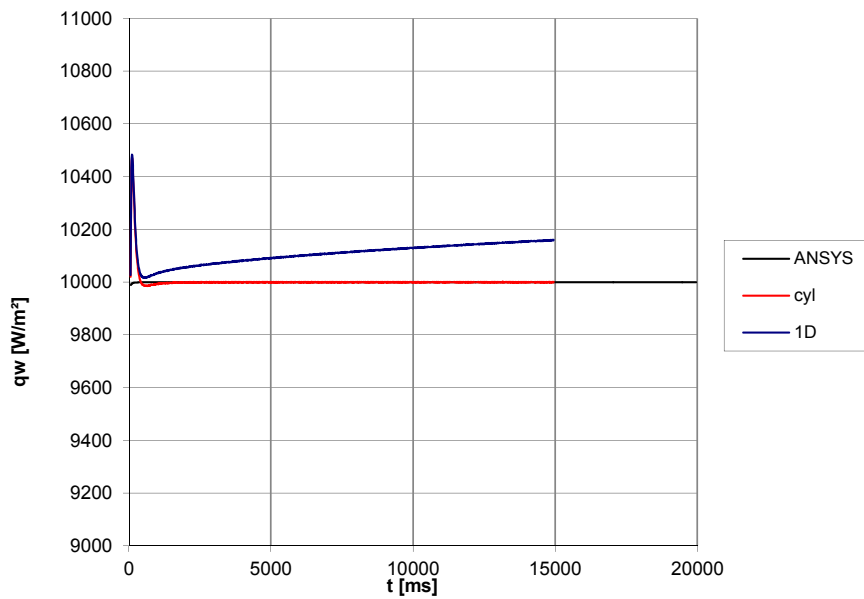
have a radius of $r = 0.06$ m, which corresponds to an axial distance of approx. 0.5 m along a 7° cone, according to the typical position of boundary layer measurements in this thesis (MS1).

The resulting surface temperatures over time were extracted from ANSYS and used as input to “pyHeatflow” (see chapter 3.4.2.1). The plot in figure 7.71a includes a comparison between the calculated heat flux from “pyHeatFlow” in cylindrical mode ($r = 1$ in equation (3.23), labeled “cyl”) and in non-cylindrical mode ($r = 0$ in equation (3.23), labeled “1D”) and the input value from ANSYS against time.

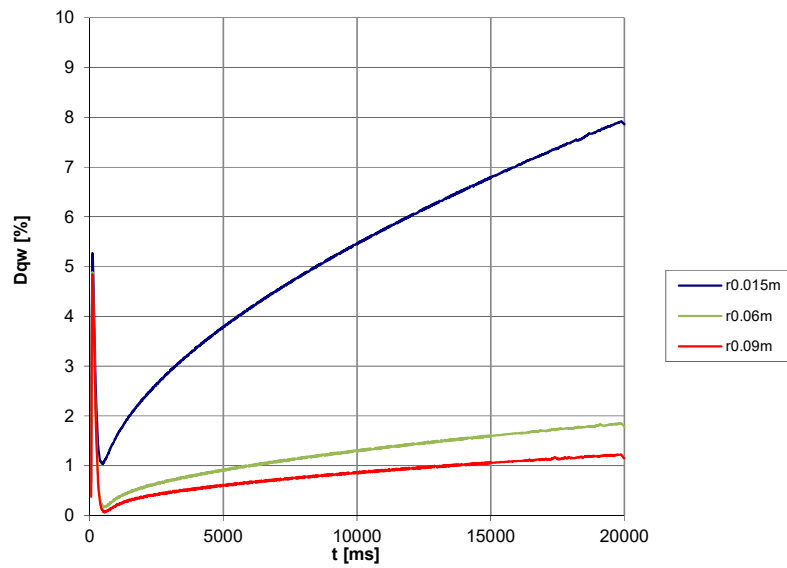
After a short initial time of below approx. 1 second the cylindrical value agrees to the input within $< 0.05\%$. The impact of using a non- instead of an axisymmetric formulation can be seen by evaluating the difference in figure 7.71a with the label “1D”. As expected, there is already a difference from beginning on and it increases with time to approx. 2% after the simulated 15 seconds. The reason for this can be found in the heat penetration into the material. Due to the cylindrical formulation, the material size decreases with increasing depth whereas a one-dimensional representation is constant in size. Therefore, also the heat capacity is different between both representations and a larger amount of heat can penetrate into the material in case of the one-dimensional formulation. Therefore, also a larger surface heat flux is necessary to result in the same surface temperature.

This difference by mis-using a non-cylindrical formulation on a cylindrical (or in our case conical) body is even increasing with decreasing radius r . Transferred to our case of a cone a decreasing radius means moving toward the nose. To visualize this sensitivity, in Figure 7.71b three different radii corresponding to three different axial positions are shown with $x_a \approx 0.325$ m ($r = 0.015$ m), $x_a \approx 0.5$ m ($r = 0.06$ m) and $x_a \approx 0.733$ m ($r = 0.09$ m) for an extended simulation with $t = 20$ s.

At 20 seconds the largest difference is visible with approx. 8% at the closest position to the nose with the smallest radius of $r = 0.015$ m, as expected. With increasing cone length and therefore increasing radius, the axisymmetric formulation (see equation (3.19)) tends to the one-dimensional formulation. Already at the aft end of the model with $r = 0.09$ m the difference drops to approx. 1% after 20 seconds.



(a)



(b)

Figure 7.71: pyHeatflow cylindrical validation and sensitivity with a) heat flux against time at $x_a \approx 0.5$ m ($r = 0.06$ m) and b) heat flux difference between non-cylindrical assumption and input at $x_a \approx 0.325$ m ($r = 0.015$ m), $x_a \approx 0.5$ m ($r = 0.06$ m) and $x_a \approx 0.733$ m ($r = 0.09$ m).

7.6.5 Heat Flux Normalization Study

A general definition of the Stanton number can be given with

$$St = \frac{\dot{q}_c}{\rho_{St} u_{St} (h_{St} - h_w)} = \frac{\dot{q}_c}{\rho_{St} u_{St} c_p (T_{St} - T_w)} \quad (7.29)$$

where ρ_{St} and u_{St} are meant here as reference density and velocity, respectively and h_{St} and h_w are a reference and wall enthalpy, respectively. With the assumption of a perfect gas the second part of the equation can be used with the constant heat capacity of the fluid c_p , a reference T_{St} and wall temperature T_w , respectively.

To normalize the the convective heat flux, proper values need to be defined to capture the mass flow rate and the driving temperature difference [40]. In the framework of experimental investigations the reference state for the density and velocity are most often chosen as the inflow condition ($\rho_{St} = \rho_\infty$ and $u_{St} = u_\infty$) and the reference temperature as the reservoir temperature $T_{St} = T_0$, since all parameters are either measured directly or can be derived by gas dynamic equations. In the framework of boundary layer analysis typically the (local) edge conditions of density ($\rho_{St} = \rho_e$) and velocity ($u_{St} = u_e$) are used together with the (local) adiabatic wall temperature T_{aw} representing the local convective enthalpy potential which drives the heat transport. The adiabatic wall temperature can seldom be measured directly since long testing times would be necessary. In the H2K facility the typical run times of below 30 s are not sufficient to reach a (radiative) equilibrium and therefore in our case the adiabatic wall temperature was estimated via the recovery temperature $T_{St} = T_r$, according to eq. (3.8). The corresponding edge conditions were derived from the inflow based on the Taylor-Maccoll equations (section 3.7), neglecting viscous interactions along the cone.

If not otherwise noted the Stanton number is typically defined with $\rho_{St} = \rho_e$, $u_{St} = u_e$, $T_{St} = T_r$ according to eq. (3.8) and denoted St_e . This decision is based on a parameter study reported in the following.

To choose which version of the Stanton number is more suitable to normalize the heat flux along the sharp cone, several computations with the boundary layer code (see section 3.8) and with TAU (see section 3.9) were performed. In the following the results from the BLcode are used to visualize the outcome. The TAU data showed the same trends. The computations were performed for three different inflow conditions with $p_0 = 18.9$ bar, $T_0 = 475$ K, $p_0 = 19.7$ bar, $T_0 = 490$ K and $p_0 = 20.0$ bar, $T_0 = 500$ K, all at $Ma = 6$, according to performed tests. The computational grid was chosen according to grid convergence index (GCI) analysis (described in section 7.7.2). Fully turbulent (eddy viscosity turbulence model, see section 3.8) and transitional computations were performed, where for the latter a transition location of $x = 0.23$ m and extent of $\Delta x \approx 0.12$ m were prescribed in the code, mimicking approximate locations for our experimental conditions (see e.g. figure 4.42). The difference in heat flux which should be normalized by the corresponding Stanton numbers is visualized in figure 7.72.

The comparison includes two different options for the reference density and velocity with either the inflow (subscript inf) or BL edge condition (subscript e) and three different options for the reference temperature T_{St} . For the latter either the analytical equation (3.8) was used

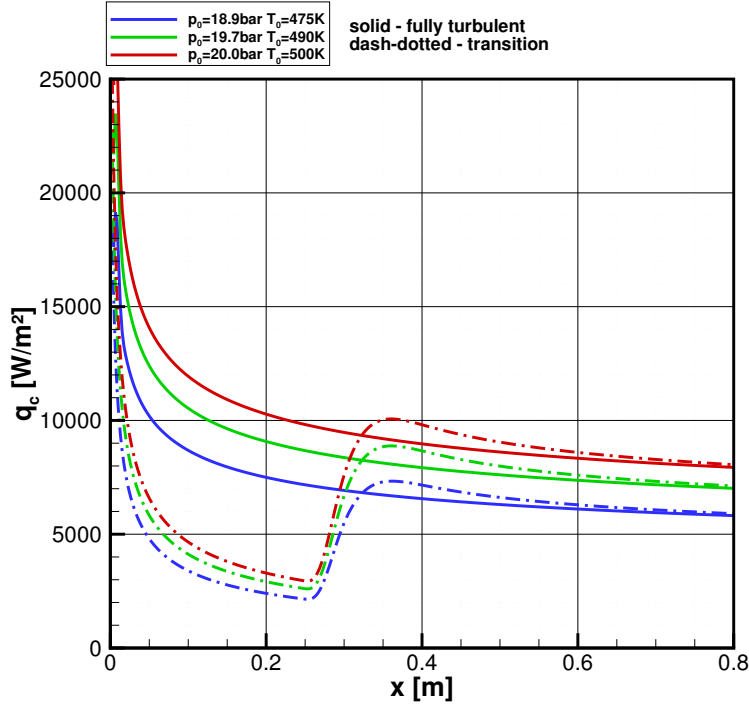


Figure 7.72: Heat flux for Stanton number sensitivity study based on boundary layer code simulations along the 7° cone at three different inflow conditions and $T_w = 340$ K if not otherwise indicated: solid lines for fully turbulent and dash-dotted lines for transitional computations

(subscript T_{aw}), or the wall temperature derived from BLcode computations applying adiabatic wall assumption (subscript $T_{aw} = f(x)$) or the reservoir temperature (subscript T_0). The computed adiabatic wall temperatures are plotted in figure 7.73a along the cone surface x with the analytical result (dashed) and the BLcode derived results for fully turbulent (solid) and transitional computations (dash-dotted). It is visible that in contrast to the constant analytical value (since only depending on the constant cone edge temperature) the results computed by the BLcode are not constant along the x -axis and systematically lower. The impact of the different adiabatic wall temperature estimations is visible in the subplots 7.73b to 7.73f, showing the different definitions of the Stanton numbers. It is visible that the definition with inflow conditions and reservoir temperature St_{inf,T_0} , typically used in experimental frameworks, normalizes the heat flux worst of all shown options. Nevertheless, also with this option a residual of only $\approx 12\%$ persists. The best option would be to normalize via edge conditions and computed adiabatic wall temperature $St_{e,T_{aw}=f(x)}$ which collapses the curves of the three inflow conditions best for both fully turbulent or transitional computations (subplot 7.73c). The next best option is the definition with edge conditions and analytically derived adiabatic or recovery temperature $St_{e,T_{aw}}$ (subplot 7.73e). This is a viable option for a post-processing of the experimental data in this work, since it only necessitates the approximation of the edge conditions via Taylor-Maccoll equations (see section 3.7) and equation (3.8).

The above computations were performed with wall isothermal temperatures. In experiments, the wall temperature varies along the cone due to varying heat loads. This influence is assumed to be negligible due to the limited temperature range which occurs during testing.

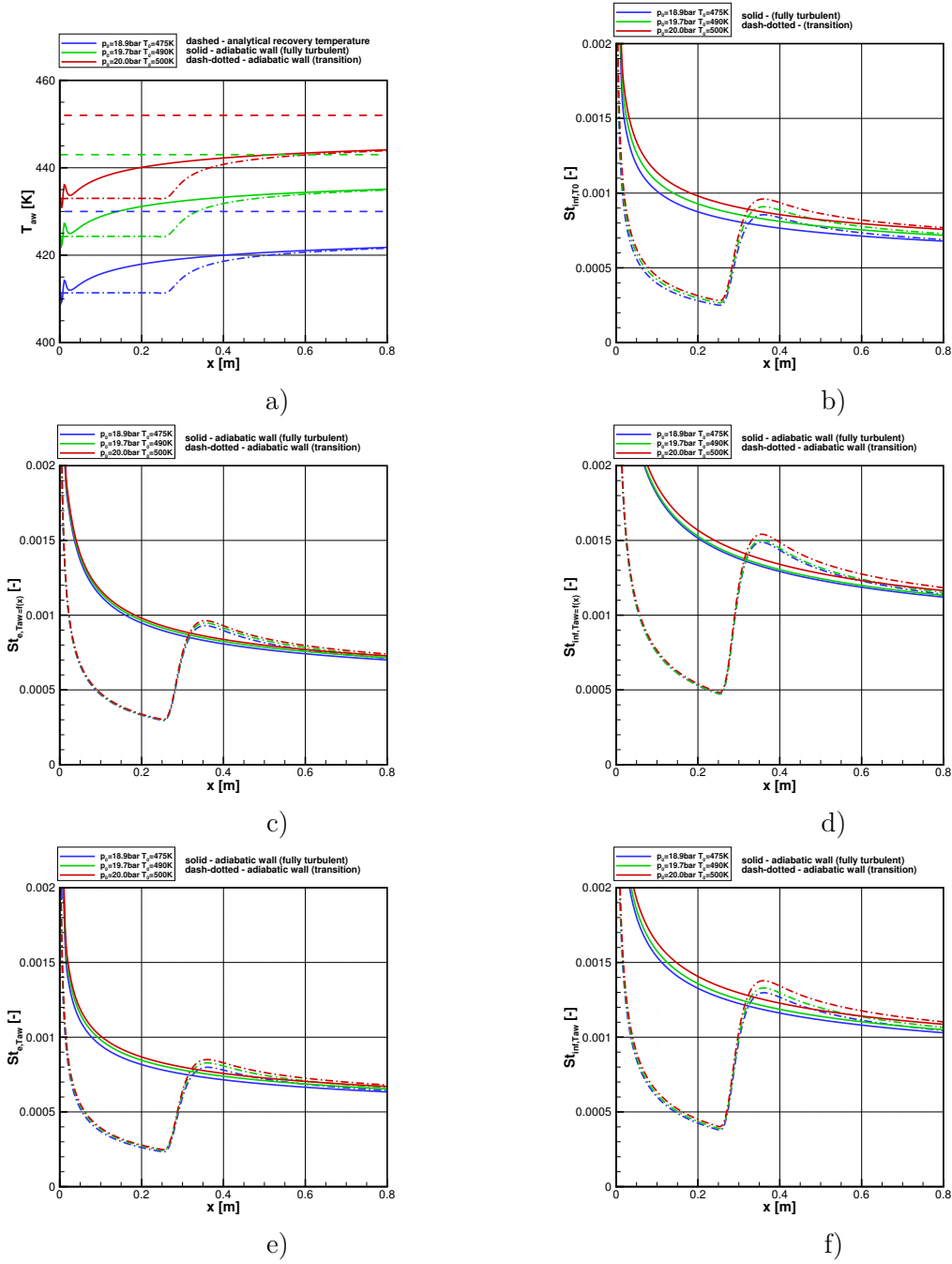


Figure 7.73: Stanton number definition sensitivity study with a) the adiabatic wall temperature along x -axis via analytical equation ((3.8), dashed), fully turbulent computation with adiabatic wall assumption (solid) and transitional computation with adiabatic wall assumption (dash-dotted); b)-f) Stanton numbers with b) $\rho_{St} = \rho_\infty$, $u_{St} = u_\infty$, $T_{St} = T_0$, c) $\rho_{St} = \rho_e$, $u_{St} = u_e$, $T_{St} = T_{aw}$, d) $\rho_{St} = \rho_\infty$, $u_{St} = u_\infty$, $T_{St} = T_{aw}$, e) $\rho_{St} = \rho_e$, $u_{St} = u_e$, $T_{St} = T_r$, f) $\rho_{St} = \rho_\infty$, $u_{St} = u_\infty$, $T_{St} = T_r$. For b) to e): solid lines for fully turbulent and dash-dotted lines for transitional computations.

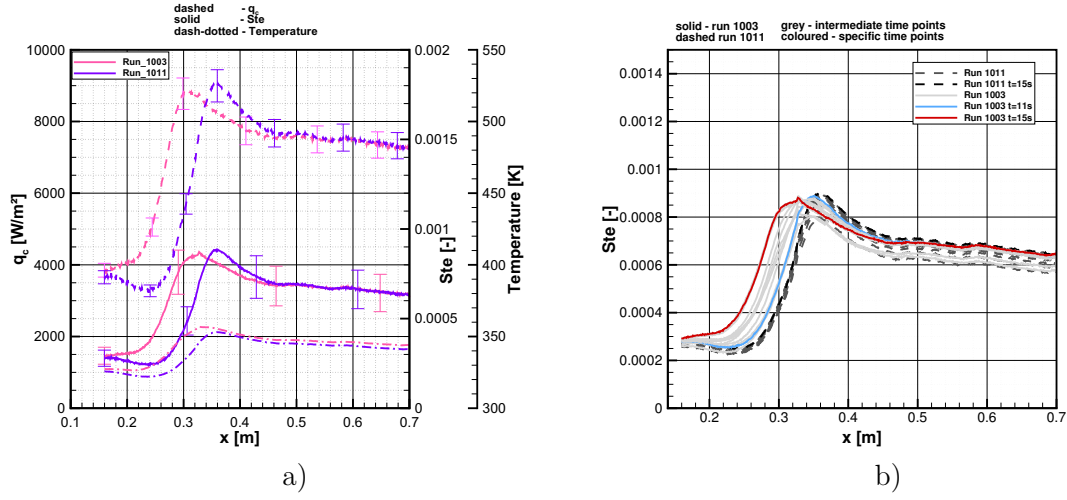


Figure 7.74: Influence of inflow on transition location with a) Convective heat flux, wall temperature and edge based Stanton number seeding influence comparison for Run1003 and Run1011 at $t = 15$ s and b) Stanton numbers for the same runs at different time points, highlighting the influence of seeding on the transition front.

7.6.6 Heat Flux Sensitivity Study

Natural transition location sensitivity:

To show the sensitivity of transition location on the smooth wall heat flux in more detail, Figure 7.74a includes the convective heat flux, wall temperature and edge based Stanton number at $t = 15$ s for Run1003 and Run1011 with and without PIV seeding, respectively. It is visible that the transition front is shifted upstream in case of seeding on. Furthermore, if the temporal evolution of the transition front is investigated in figure 7.74b, it is visible that the front of Run1003 moves towards the position of Run1011 (without seeding) for intermediate time points ($t = 11$ s). This can be attributed to the seeding apparatus which generated non-constant seeding densities for Run1003 (detectable via corresponding PIV raw images, not shown here). For later time points the seeding was re-established so that the transition front moves toward the final position $t = 15$ s. Despite this impact on the transition location, the downstream turbulent Stanton number level is not influenced by this shift, as visible for the values approx. larger than $x > 0.45$ m. Therefore, the analysis of turbulent augmentation due to roughness should not be influenced by this.

Temporal evolution of Stanton number:

Figure 7.75 shows heat flux distributions based on experimental data of the second PIV campaign at FC1 with smooth wall Run3014, 3016 (left column, blue and red symbols, respectively) and rough wall Run3006, 3007 (right column, blue and red symbols, respectively) at $x = 0.5$ m. The two plots in the middle row show the Stanton number St_∞ defined with T_{St} from analytical equation (3.8) and u_{St} , ρ_{St} from inflow conditions. The two plots in the lower row show the Stanton number St_e defined with T_{St} from analytical equation (3.8) and u_{St} , ρ_{St} from edge

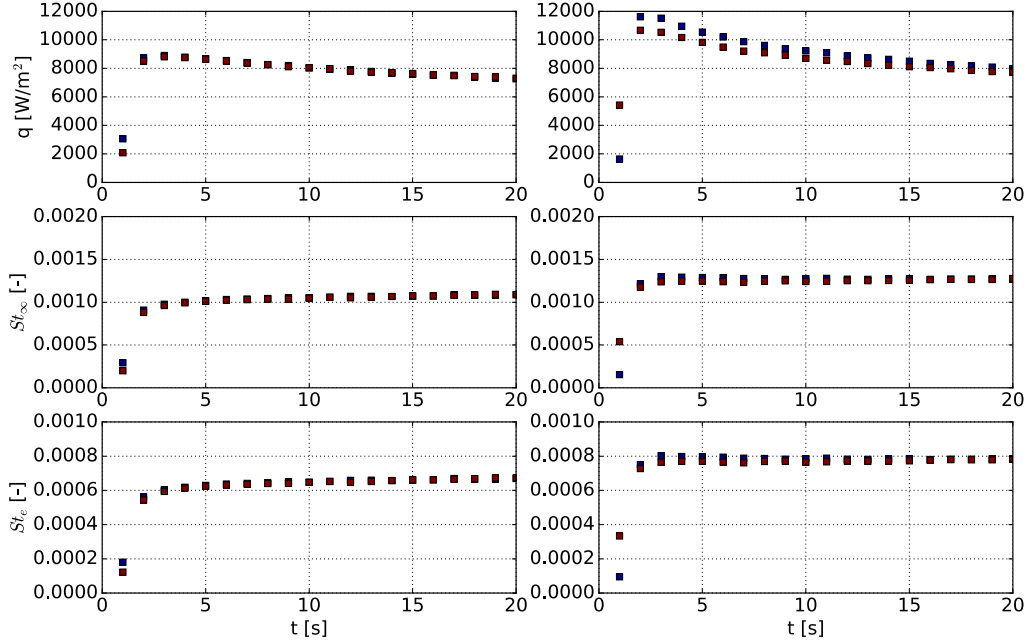


Figure 7.75: Temporal heat flux distributions based on experimental data of second PIV campaign at FC1 with smooth wall Run3014, 3016 (left column, blue and red symbols, respectively) and rough wall Run3006, 3007 (right column, blue and red symbols, respectively) at $x = 0.5$ m.

conditions. It is visible that both St_∞ and St_e are able to normalize the decreasing convective heat flux q (two plots in the upper row) with increasing wall temperatures acceptably to a nearly constant value with differences below $\approx 4\%$ for $t > 10$ s. This translates into an augmentation ratio change of $St_{e,r}/St_{e,s}$ below $\approx 1.5\%$ for $t > 10$ s, visible in figure 7.76.

IR heat flux augmentation:

In the following additional data is given with respect to IR heat flux augmentation.

The sensitivity of the augmentation levels, as shown in figure 4.48 and 4.49, from chosen time interval were found to be negligible. Table 7.28 shows the impact of different spatial and temporal averaging on the heat flux augmentation from IR data. The results are based on different smooth runs (Run3014, 3016, 3019) and rough runs (3006, 3007, 3008). In contrast to augmentations based on the absolute heat flux, the Stanton number augmentations are only marginally affected by the averaging time interval.

Local sensor heat flux augmentation sensitivity:

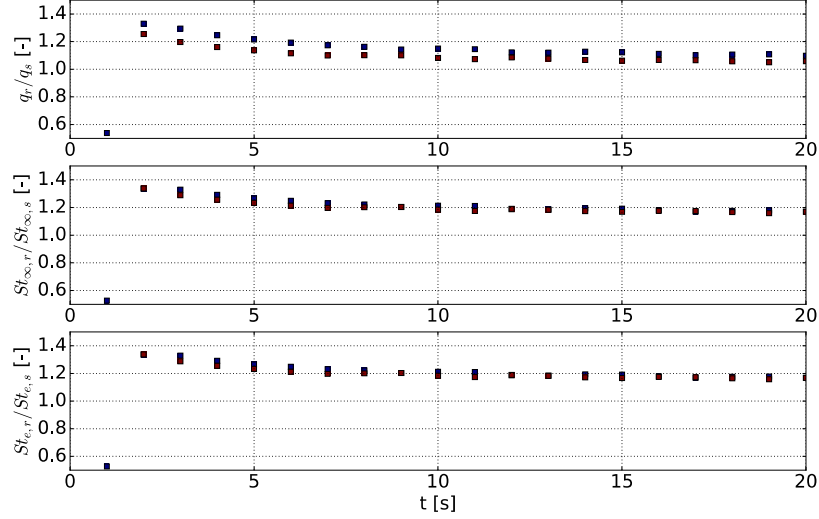


Figure 7.76: Temporal heat flux augmentation distributions based on experimental data of second PIV campaign at FC1 with rough to smooth wall Run3006 to Run3014 and Run3007 to 3016 (blue and red symbols, respectively) at $x = 0.5$ m.

As outlined in section 4.4.3, averaging of local sensor data was necessary due to varying augmentation levels over testing time. Different averaging time periods were checked and found to be negligible. Final results of the HFM and CT augmentations are summarized in table 7.29, 7.30 and 7.31.

Table 7.32 contains statistics of the roughened CT2 (TE63-3) in the Carborundum roughness model heat flux augmentation within $t=2.0 - 4.0$ s

Boundary condition impact on BLcode results:

Due to the different possible formulations of relevant parameters for the modified Krogstad roughness model in the BLcode (see section 3.8), the impact is analyzed for the flow condition FC1. Figure 7.77 shows the impact of using either y^+ and k_s^+ or y_c^+ and $k_{c,s}^+$ (see eq. (7.37) and (7.40)) on calculated roughness Reynolds number k_s^+ and Stanton numbers in figure 7.77a and figure 7.77b, respectively, along the rough cone at EQSR values within $k_s/k = 0.125 - 2.0$. It is visible that the formulations of parameters have large impact on the resulting roughness Reynolds numbers, especially in a region close to the nose and in case of larger EQSR values. This is driven by the combination of larger roughness heights in combination with corresponding smaller boundary layers close to the nose. Since in case of y_c^+ and $k_{c,s}^+$ for of e.g. viscosity the local or corresponding values at the roughness height are used, it has a larger impact than for larger boundary layers downstream the cone. In contrast to the Reynolds numbers, the corresponding Stanton numbers in figure 7.77b show only negligible differences in case of small EQSR values

Table 7.28: IR roughness heat flux augmentation for different spatial and temporal averaging.

Run	Sensor	x [m]	T [K]	q [W/m ²]	St_e [-]	$T_{w,r}/T_{w,s}$ [-]	$\dot{q}_r/\dot{q}_s$ [-]	$St_{e,r}/St_{e,s}$ [-]
10s < t < 14s								
avg.	s	0.45	345	8170	0.0007	1.00	1.00	1.00
avg.	r	0.45	350	9121	0.0008	1.02	1.12	1.21
avg.	s	0.55	343	7887	0.0007	1.00	1.00	1.00
avg.	r	0.55	348	8833	0.0008	1.01	1.12	1.20
avg.	s	0.50	344	8028	0.0007	1.00	1.00	1.00
avg.	r	0.50	349	8976	0.0008	1.01	1.12	1.20
15s < t < 19s								
avg.	s	0.45	353	7779	0.0007	1.00	1.00	1.00
avg.	r	0.45	358	8507	0.0008	1.01	1.09	1.19
avg.	s	0.55	351	7503	0.0007	1.00	1.00	1.00
avg.	r	0.55	356	8256	0.0008	1.01	1.10	1.19
avg.	s	0.50	352	7640	0.0007	1.00	1.00	1.00
avg.	r	0.50	357	8381	0.0008	1.01	1.10	1.19
10s < t < 19s								
avg.	s	0.45	349	7974	0.0007	1.00	1.00	1.00
avg.	r	0.45	354	8814	0.0008	1.02	1.11	1.20
avg.	s	0.55	347	7695	0.0007	1.00	1.00	1.00
avg.	r	0.55	352	8545	0.0008	1.01	1.11	1.19
avg.	s	0.50	348	7834	0.0007	1.00	1.00	1.00
avg.	r	0.50	353	8679	0.0008	1.01	1.11	1.20

and small differences in case of larger EQSR values.

FC impact on heating augmentation:

Figure 7.78 shows comparison between FC1 and FC2 with the rough wall heat flux augmentations for EQSR $k_s/k = 0.125 - 2.0$ computed with the BLcode. Augmentation levels agree quite closely between both inflow conditions FC1 (solid lines) and FC2 (dashed lines) especially for smaller EQSR values encountered in this thesis.

Blockage ratio impact on BLcode friction augmentation:

Figure 7.79 shows the impact of blockage ratio on friction augmentation evolutions with roughness Reynolds numbers k_s^+ , as calculated with the BLcode along a rough wall cone. There are different options to increase the friction Reynolds number to collect this evolution (see formulation in eq. (2.6)). One option is to keep the flow conditions the same (ρ , μ at FC1) and successively increase

Table 7.29: Statistics of filtered HFM roughness heat flux augmentation within $t = 3 - 20$ s of rough wall Run1015 to smooth wall Run1011.

	q		St_e		$\dot{q}_r/\dot{q}_s$		$St_{e,r}/St_{e,s}$	
	[W/m ²]	[W/m ²]	[-]	[-]	[-]	[-]	[-]	[-]
	HFM2	HFM3	HFM2	HFM3	HFM2	HFM3	HFM2	HFM3
1011								
mean	21443	18941	0.0014	0.0012	-	-	-	-
min	20278	17751	0.0013	0.0011	-	-	-	-
max	21820	19299	0.0015	0.0013	-	-	-	-
std	394	389	0.00004	0.00004	-	-	-	-
1015								
mean	24148	19519	0.0016	0.0013	1.13	1.04	1.13	1.05
min	21115	18206	0.0013	0.0012	1.05	1.02	1.05	1.04
max	24688	19835	0.0016	0.0013	1.16	1.06	1.16	1.07
std	730	367	0.00007	0.00004	0.02	0.01	0.02	0.01

k_s at the position corresponding to MS1, which is shown as black line/symbols. It is visible that in this case, the augmentation tapers off at approx. 1.9. The corresponding blockage ratios k_s/δ are plotted in figure 7.79 by the colors of the symbols. It is visible that the tapering-off correlates with approaching a value of $k_s/\delta = 1$ (values above 1 are masked since the range is restricted to 0 to 1). Due to the compressible formulation (see eq. (7.37) and (7.39)) and since a boundary layer code was used, at blockage ratios close or even above 1 relevant parameters are fixed to edge conditions, since no flow field above the BL is calculated. In contrast, due to the conical flow field, the parameters above the BL would change with wall-distance so that the calculated values in this blockage ratio regime can be questioned.

Instead of successively increasing k_s , high Reynolds numbers can also be derived if the density is increased (simulated here with increasing the artificial reservoir condition to 200 bar). With this approach, the same roughness Reynolds number regime is reached at lower k_s values. At the same time values were extracted at an increased running length (6 m) resulting in lower blockage ratios. In this case (green lines/symbols) the augmentation tapers off not before approx. 2.5, reaching much higher values than for the previous case.

A further check was made with another option to increase k_s^+ via artificially increasing the cone length to 50 m and keeping the flow conditions the same (FC1). This is shown as red lines/symbols and the same tapering-off behavior is visible than the case before. In both latter cases the blockage ratio is still below approx. 0.5 so that the previously described bias due to using a BLcode is prevented. The third case (increased running length at FC1) was used in the corresponding plots in section 4 to compare to experimental and literature data.

Prandtl number model impact on BLcode heating augmentation:

Three different variable turbulent Prandtl number models were compared to the nominal constant Prandtl number approach in the BLcode. First, a model according to Rotta [207] was used, which

was already implemented in the BLcode. Second, a model according to Aupoix [17] was used, which was already successfully applied on Holden's high-enthalpy data [176]. Third, compressible smooth wall DNS data by Zhang *et al.* [296], which were derived for flow conditions close to data in this thesis, was fed to the BLcode as tabulated data. Although along a smooth wall, it generally resembled outer layer incompressible data along rough walls as well (see e.g. [55]).

Figure 7.80a shows in calculated turbulent Prandtl number profiles Pr_t at MS1. Only minor variations are visible with the model according to Aupoix [17], whereas the largest are derived by the one according to Rotta [207]. The $Pr_t = 0.9$ constant assumption is visible to not being accurate directly at the wall but represents a rough average of the near wall behavior if compared to data by Zhang *et al.* [296].

The corresponding Stanton numbers are shown in figure 7.80b, where smooth wall results are shown with black lines and rough wall with colored lines, all for a fixed $k_s = 0.5\text{mm}$. The different models show different degrees of changes compared to the nominal $Pr_t = 0.9$ (const.) curve. Most curves show higher Stanton numbers. Only the one according to Zhang *et al.* [296] shows slightly lower values but only close to the end of the cone. Therefore, it is expected that these Prandtl number models do not improve the agreement to the measurement, for which a clear lowering of the Stanton numbers would be necessary.

The above described approaches do not consider near wall behaviors of the turbulent Prandtl numbers. Although the rough wall DNS by Doosttalab *et al.* [55] agrees closely to the applied smooth wall data by Zhang *et al.* [296] in the outer layer, very close to the wall ($y^+ < 20$) Prandtl numbers vary more drastically above the roughness topology (see e.g. figure 8b) in Doosttalab *et al.* [55]). This is well within the roughness sublayer so this behavior is assumed to be highly dependent on the actual roughness topology. This might already be the case for smooth wall data but which is rarely shown in the publications. These sensitivities may have an impact on the computations but were out of the scope of this thesis.

Table 7.30: CT2 (TE63-3) roughness heat flux augmentation within t=2.0 - 4.0s

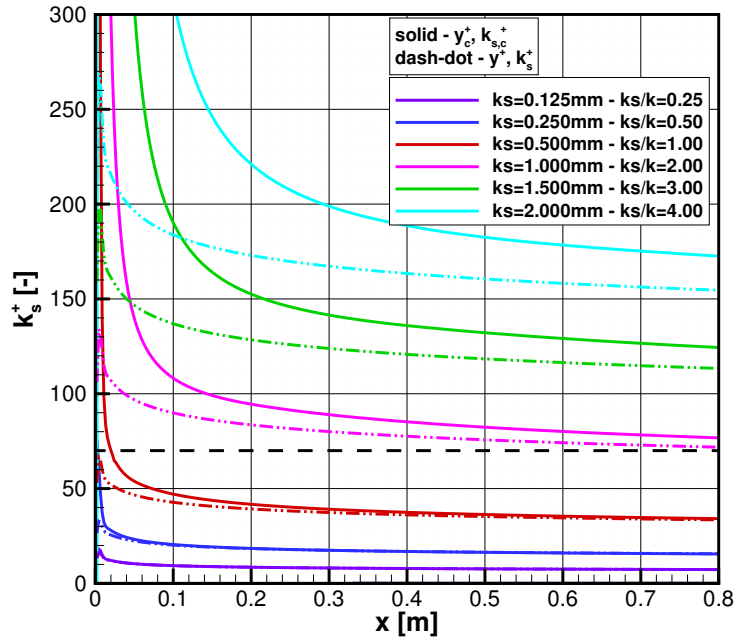
Run	wall	value	$\dot{q}$ [W/m ²] CT2	St_e [-] CT2	$\dot{q}/\dot{q}_{s,avg.}$ [-] CT2	$St_e/St_{e,s,avg.}$ [-] CT2
2023	s	mean	13669	0.00106	1.01	1.03
		min	13393	0.00103	1.01	1.03
		max	13947	0.00108	1.02	1.04
		std	155	0.00001	0.002	0.002
2036	s	mean	13562	0.00106	0.99	1.01
		min	13357	0.00104	0.98	1.00
		max	13709	0.00108	1.00	1.02
		std	116	0.00001	0.005	0.005
2046	s	mean	13605	0.00102	1.01	0.98
		min	13300	0.00099	1.00	0.98
		max	13834	0.00104	1.01	0.99
		std	181	0.00002	0.004	0.003
2051	s	mean	13336	0.00102	0.98	0.97
		min	13089	0.00100	0.98	0.96
		max	13589	0.00105	0.99	0.97
		std	152	0.00001	0.002	0.002
avg.	s	mean	13503	0.00104	1.00	1.00
		min	13245	0.00102	1.00	1.00
		max	13737	0.00106	1.00	1.00
		std	139	0.00001	0.000	0.000
2059	r	mean	14920	0.00113	1.10	1.09
		min	14547	0.00110	1.09	1.08
		max	15367	0.00117	1.12	1.10
		std	251	0.00002	0.008	0.008
2064	r	mean	14267	0.00111	1.06	1.09
		min	13880	0.00108	1.05	1.08
		max	14725	0.00116	1.07	1.10
		std	251	0.00002	0.008	0.008
2071	r	mean	14535	0.00112	1.07	1.07
		min	14220	0.00109	1.06	1.06
		max	14903	0.00115	1.08	1.08
		std	192	0.00002	0.005	0.005
2083	r	mean	14829	0.00112	1.10	1.08
		min	14461	0.00109	1.09	1.07
		max	15348	0.00117	1.12	1.10
		std	256	0.00002	0.008	0.008

Table 7.31: CT3 (TE101-4) roughness heat flux augmentation within t=2.0 - 4.0s

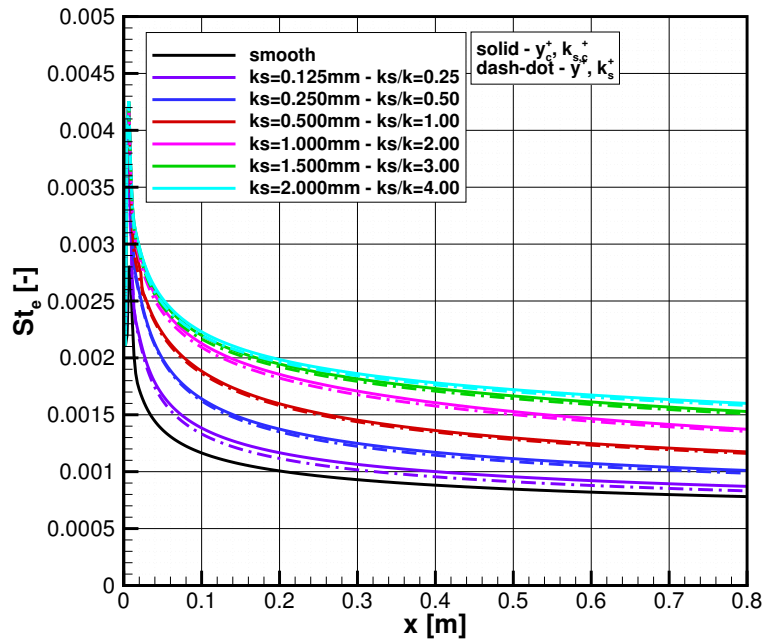
Run	wall	value	$\dot{q}$ [W/m ²] CT3	St_e [-] CT3	$\dot{q}/\dot{q}_{s,avg.}$ [-] CT3	$St_e/St_{e,s,avg.}$ [-] CT3
2023	s	mean	21495	0.00168	1.03	1.05
		min	19459	0.00151	1.02	1.04
		max	23190	0.00183	1.04	1.06
		std	1077	0.00009	0.006	0.006
2036	s	mean	21444	0.00169	0.96	0.98
		min	19475	0.00152	0.95	0.96
		max	23058	0.00183	0.97	0.99
		std	1039	0.00009	0.008	0.008
2046	s	mean	21563	0.00163	1.04	1.01
		min	19599	0.00147	1.02	1.00
		max	23164	0.00177	1.06	1.03
		std	1034	0.00009	0.010	0.010
2051	s	mean	21378	0.00167	0.99	0.98
		min	19383	0.00150	0.99	0.98
		max	22938	0.00181	0.99	0.98
		std	1022	0.00009	0.001	0.001
avg.	s	mean	20926	0.00163	1.00	1.00
		min	18673	0.00144	1.00	1.00
		max	22702	0.00178	1.00	1.00
		std	1155	0.00010	0.000	0.000
2059	r	mean	22929	0.00176	1.10	1.08
		min	20513	0.00155	1.09	1.07
		max	24856	0.00192	1.11	1.09
		std	1189	0.00010	0.007	0.007
2064	r	mean	21651	0.00171	1.05	1.08
		min	19293	0.00151	1.04	1.07
		max	23530	0.00188	1.06	1.09
		std	1236	0.00011	0.003	0.003
2071	r	mean	22361	0.00173	1.04	1.03
		min	19924	0.00153	1.02	1.01
		max	24378	0.00190	1.05	1.04
		std	1259	0.00011	0.008	0.009
2083	r	mean	22197	0.00172	1.06	1.05
		min	19676	0.00151	1.06	1.05
		max	24242	0.00189	1.07	1.06
		std	1335	0.00011	0.004	0.004

Table 7.32: Roughened CT2 (TE63-3) roughness heat flux augmentation within t=2.0 - 4.0s

Run	wall	value	$\dot{q}$ [W/m ²] CT2	St_e [-] CT2	$\dot{q}/\dot{q}_{s,avg.}$ [-] CT2	$St_e/St_{e,s,avg.}$ [-] CT2
2110	s	mean	8550	0.00066	1.00	1.01
		min	8030	0.00062	0.99	1.00
		max	9069	0.00070	1.01	1.01
		std	287	0.00002	0.005	0.005
2111	s	mean	8415	0.00064	1.00	0.99
		min	8058	0.00061	0.99	0.99
		max	8911	0.00068	1.01	1.00
		std	242	0.00002	0.005	0.005
avg.	s	mean	8393	0.00062	1.00	1.00
		min	8054	0.00060	1.00	1.00
		max	8845	0.00066	1.00	1.00
		std	237	0.00002	0.000	0.000
2139	r	mean	9194	0.00072	1.10	1.16
		min	8931	0.00070	1.07	1.13
		max	9552	0.00075	1.12	1.18
		std	193	0.00001	0.014	0.015
2140	r	mean	9209	0.00069	1.10	1.11
		min	8919	0.00067	1.07	1.08
		max	9507	0.00071	1.11	1.12
		std	175	0.00001	0.013	0.013
2141	r	mean	9030	0.00070	1.11	1.15
		min	8597	0.00067	1.06	1.11
		max	9411	0.00073	1.14	1.18
		std	258	0.00002	0.020	0.021
2142	r	mean	8779	0.00070	1.04	1.10
		min	8401	0.00067	1.02	1.07
		max	9076	0.00072	1.08	1.14
		std	212	0.00002	0.019	0.020
2143	r	mean	8824	0.00070	1.05	1.11
		min	8389	0.00067	1.02	1.09
		max	9107	0.00072	1.06	1.13
		std	197	0.00001	0.011	0.012



(a)



(b)

Figure 7.77: Impact of boundary conditions on a) calculated roughness Reynolds number k_s^+ and b) Stanton number along rough cone from BLcode at different EQSR values.

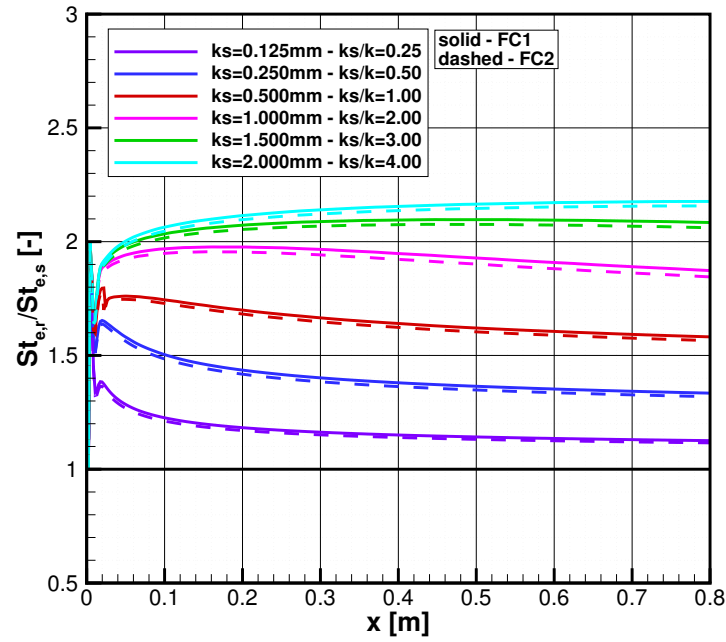


Figure 7.78: Rough-to-smooth wall convective heat flux edge based Stanton number from BLcode for different EQSR values at FC1 (solid lines) and FC2 (dashed lines).

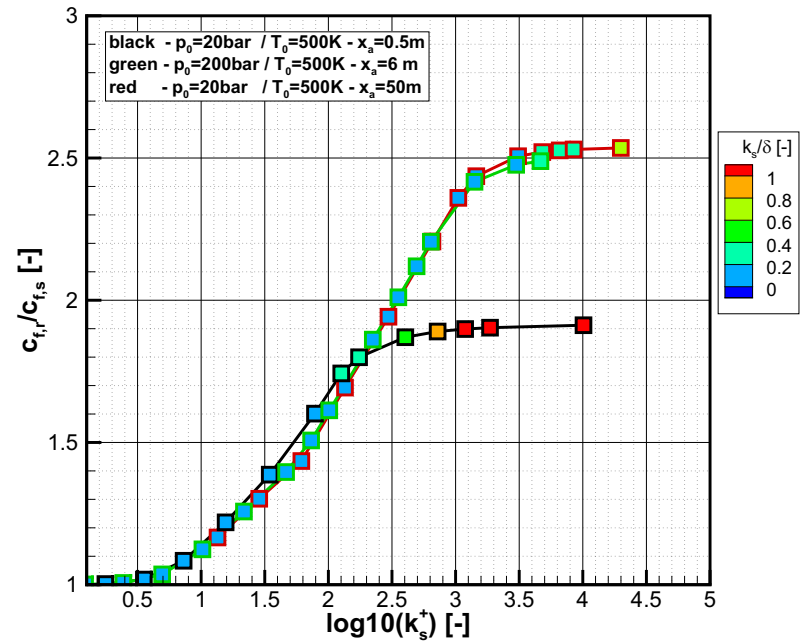


Figure 7.79: Calculated skin friction augmentation against roughness Reynolds numbers k_s^+ along rough cone from BLcode at different EQSR values and assumptions.

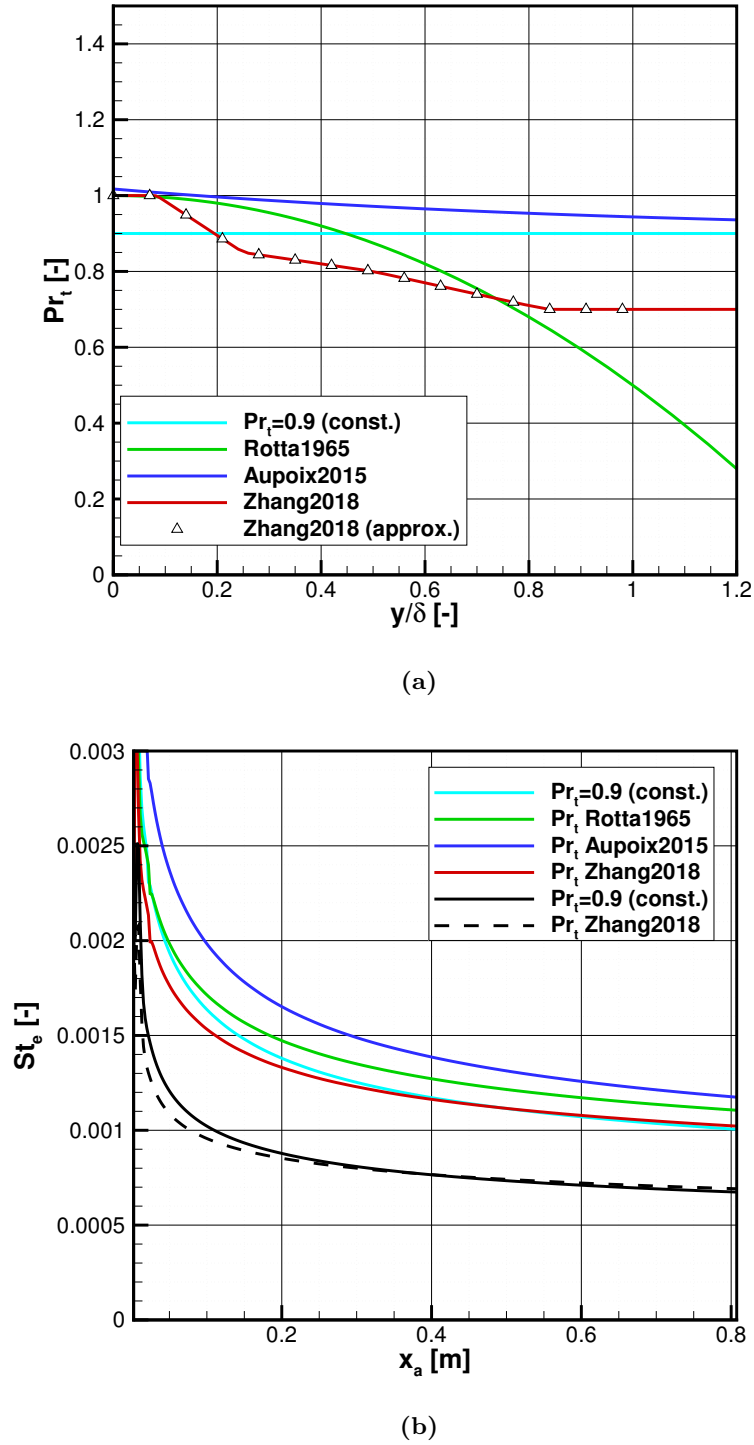


Figure 7.80: Calculated a) turbulent Prandtl number profiles Pr_t at MS1 and b) corresponding Stanton numbers along cone from BLcode with different Prandtl number assumptions. Smooth wall results with black lines and rough wall with colored lines, all with $k_s = 0.5\text{mm}$.

Table 7.33: ANSYS CT in PEEK simulation cases

label	h [W/(m ² · °C)]	T_{aw} [K]	T_w	contact CT-PEEK
Run1011 Tw	-	-	IR	full contact
Run1011 Tw, noContact	-	-	IR	no contact
conv., h(x=0.65), TAUmin	77.4	438	-	full contact
conv., h(x=0.65), TAUmin, noContact	77.4	438	-	no contact
conv., h(x=0.65), TAUmax	92.5	438	-	full contact
conv., h(x=0.65), TAUmax, noContact	92.5	438	-	no contact

7.6.7 ANSYS CT Analysis

To support the analysis of CT heat flux signals (see e.g. figure 4.45) ANSYS simulations were performed. To this end, the CT inside the surrounding PEEK material is approximated as axisymmetric configuration with the corresponding axis of rotation at the mid-point of the CT. See figure 7.81 for the setup with the axis at $x = 0$ m (most left vertical line of CT) and the y direction perpendicular to the wall. The light brown parts correspond to the CT whereas the green and brown parts to the PEEK material. The green part represents a PEEK disc which was necessary to insert the CT (light brown) into the main PEEK body (brown) with later flush mounting it to the surface. The PEEK-PEEK interfaces were always assumed to establish perfect contact in the ANSYS computations. A structured mesh was used and a refinement resulted in no substantial differences of the results (see figure 7.82a). The material properties of PEEK and CT (the latter assumed purely from Chromel) were the same as used in pyHeatflow (see section 7.6.1). The contact surfaces between CT and PEEK were either switched on (perfect contact) or off (adiabatic) to check the influence of lateral heat flux between materials (see sections between green and light brown surfaces in figure 7.81). Two different kinds of boundary conditions were used at the upper surface with either convective heating or pre-scribed surface temperatures. For the first, heat transfer coefficients h and adiabatic wall temperatures T_{aw} (as bulk temperature) were taken from turbulent TAU computations. Since different turbulence models give different transfer coefficients, a minimum and maximum value was chosen representing a possible range. For the second boundary condition, wall temperatures T_w were taken from CT signals and IR data on PEEK surfaces near the sensor TE83-3 position $x = 0.65$ m. All other surfaces were assumed adiabatic.

This is a simplified version of the real conditions due to several reasons. E.g the conical shape of the model is ignored since only a region of maximal 60 mm was computed with the focus on the CT-PEEK intersection. This intersection is assumed either perfect or non-existing, although the real situation is most likely somewhere in between. The CT has an internal structure with the two thermocouple materials but here it was modeled by only one solid part made of the one material. Apart from these simplifications, the simulations were deemed helpful for the understanding of the experimental measurements. Table 7.33 summarizes the labels and most important parameters of the simulated cases.

First, resulting heat flux q and surface temperatures T on PEEK parts can be used as sanity check, since no lateral and contact influences are expected. If not otherwise noticed, the heat flux

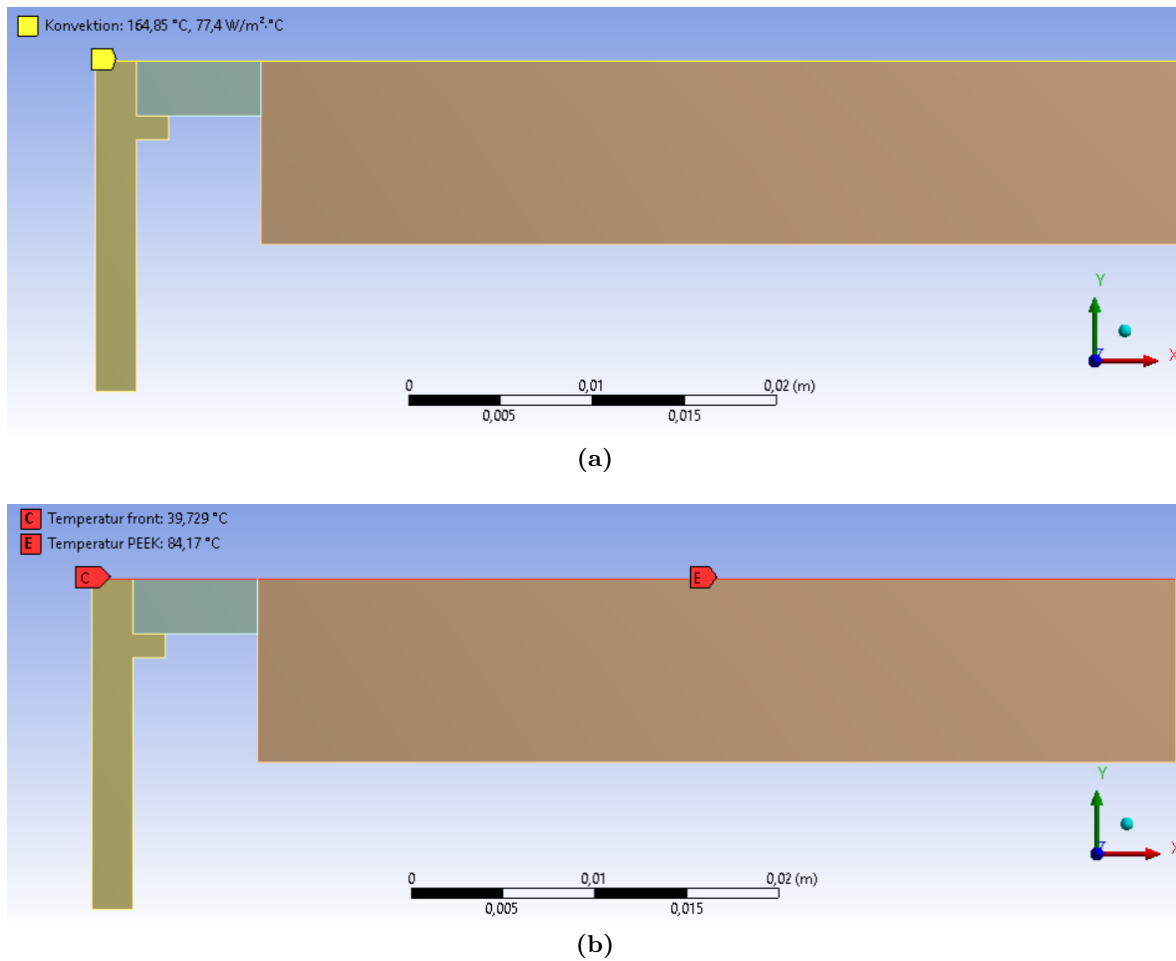


Figure 7.81: ANSYS simulation of CT in PEEK setup in a) for convective boundary conditions and in b) for pre-scribed surface temperature conditions.

in the following plots corresponds to wall-normal component. Figure 7.83a shows different heat flux results extracted at a surface point at mid x distance of the main PEEK part together with the values from 1D pyHeatflow method (see section 3.4.2.1) including estimated uncertainties (red line). ANSYS convective cases show higher values after wind tunnel start but converge to the same slope after a few seconds and the TAUmin case agrees closely to the result from pyHeatflow. The case with prescribed Run 1011 wall temperature (“Run1011 Tw”) agrees very closely to pyHeatflow, since no lateral effects are expected. This is also visible for the corresponding case without contact between CT and PEEK (although hardly visible since cases “Run1011 Tw” and “Run1011 Tw, no contact” nearly collapse). Small differences are most likely due to differences in the mesh. The corresponding computed PEEK wall temperatures are shown in figure 7.83b. Also here, the TAUmin case agrees best with data from IR (red line) and no effect of the contact situation is visible. No data for Run1011 Tw cases are shown in this plot, since the IR temperatures were used as input.

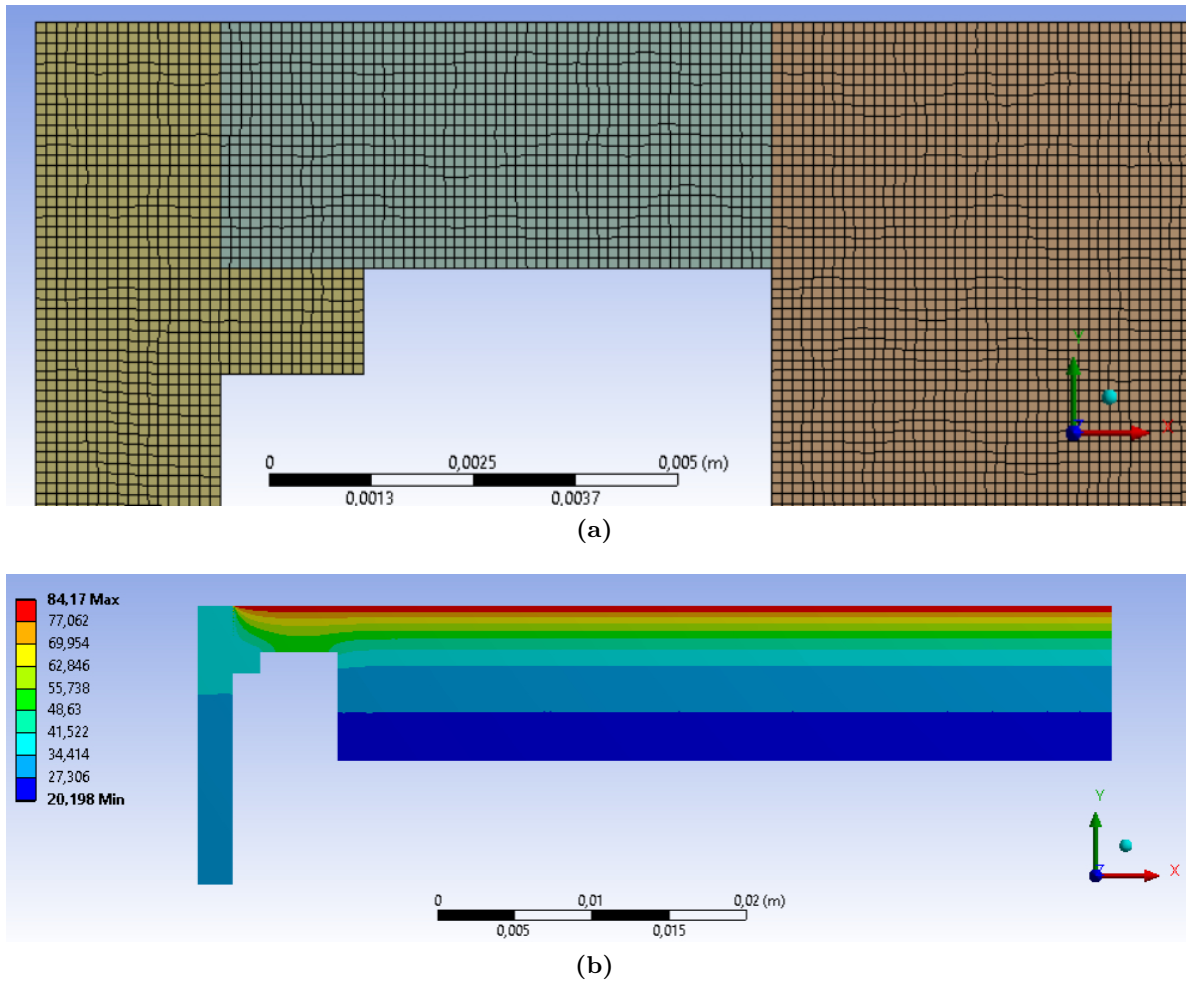
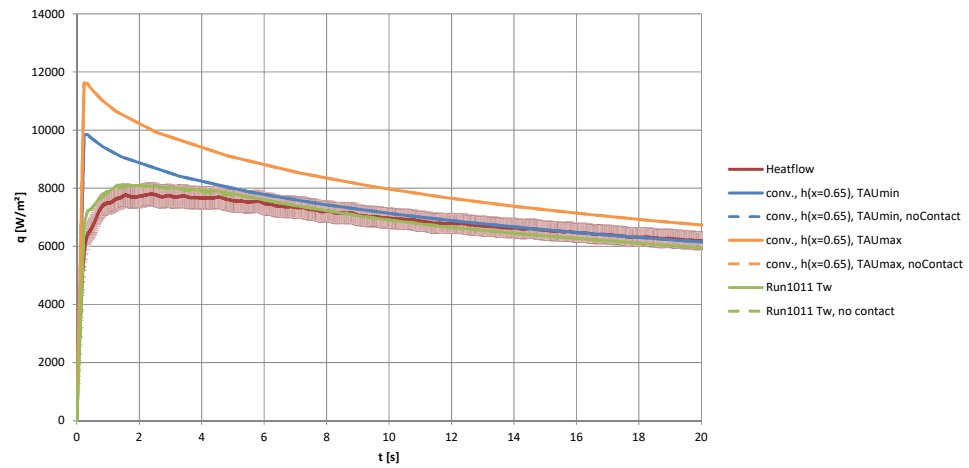


Figure 7.82: ANSYS simulation of CT in PEEK with grid in a) and temperature distribution at $t = 20\text{s}$ in b).

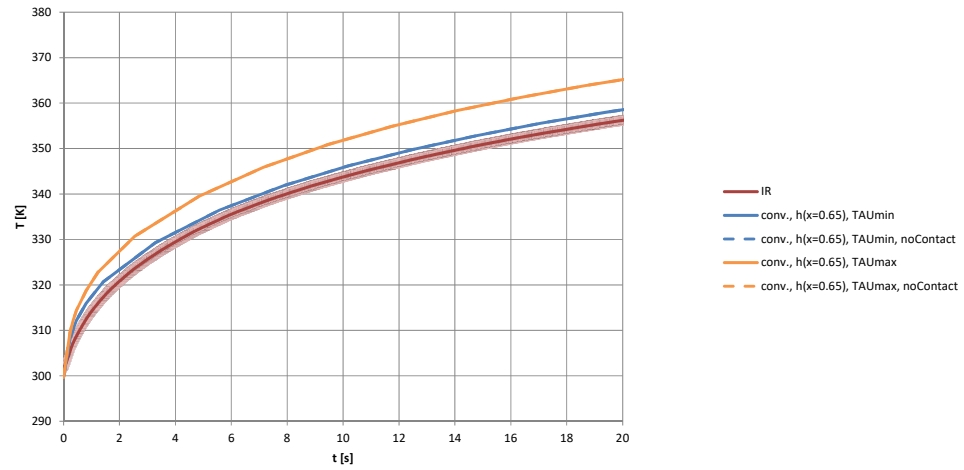
In contrast, the heat flux and surface temperatures on the CT part are of interest with respect to lateral and contact influences. In figure 7.84 the wall-normal heat flux at the axis point on the surface of the CT is shown with the red line indicating the (post-processed) temperature signal from TE83-3 transformed into heat flux via pyHeatflow. It is visible that after a first overshoot, there is a minimum after around 2 – 4 seconds with a following increase until the end of run. The corresponding values from ANSYS convective cases show heat flux values close but slightly lower compared to the Heatflow minimum. In contrast to the experimental data, the ANSYS convective cases show a monotonic decrease until the end of run. The cases without contact between CT and PEEK show a slightly lower decrease but no increase. This decrease with increasing surface temperatures is an expected trend for a constant convective heating. A sanity check of the Heatflow results was made with ANSYS by applying the CT temperature signal on the CT surface and switching off the CT-PEEK contact. It is visible that with this case (“Run1011 Tw noContact”) the experimental trend can be rebuild.

In figure 7.85a the corresponding surface temperatures and in figure 7.85b the backside temperatures on the CT axis are shown. It is visible in a) that the measured CT temperatures (red line) are only reached via the convective cases when the CT-PEEK contact surface is switched on (solid lines) whereas no-contact cases are colder (dashed lines). This is mainly due to a lateral heat flux from the hotter PEEK surfaces to the colder metallic material of CT, which is also visualized in figure 7.86 with the lateral heat flux q_l along the vertical CT-PEEK junction. In this plot, the negative heat flux values correspond to a flow of energy from PEEK to CT. PEEK has a lower heat conductivity which leads to higher surface temperatures at the same convective heating (see figure 7.82b). This leads to an additional lateral heating from the hotter PEEK to the colder CT which also result in higher CT surface temperatures than from the convective heating alone. If this increased CT temperature information is used as input to a method only considering 1D heating (either Heatflow or the “Run 1011 Tw, noContact” case), an erroneous higher wall-normal heat flux is calculated which would be necessary to result in this increased surface temperatures.

Due to this information, it is assumed, that the experimental CT 1D Heatflow values are biased by lateral heat flux and it was decided that the most reliable value of the Heatflow results is the one where the first minimum is reached, before lateral heating effects increase over time. This value is also taken for run-to-run and smooth-to-rough comparisons in section 4.4.



(a)



(b)

Figure 7.83: CT in PEEK ANSYS simulation results of a) heat flux and b) wall temperatures on PEEK.

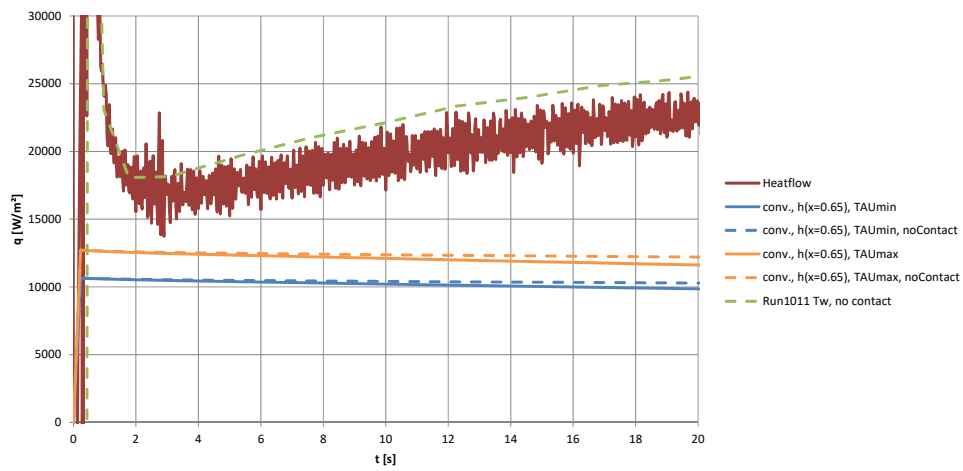
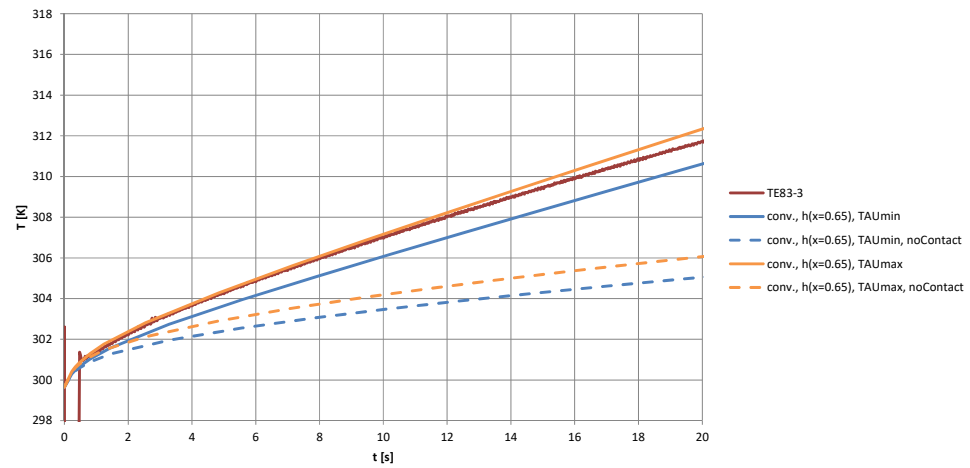
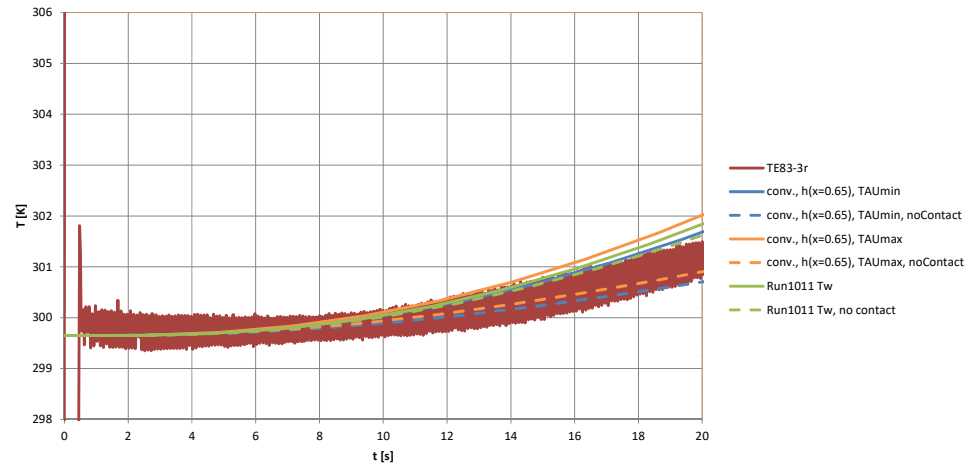


Figure 7.84: CT in PEEK ANSYS simulation results of heat flux on CT.



(a)



(b)

Figure 7.85: CT in PEEK ANSYS simulation results of a) wall temperatures and b) backside temperatures on CT.

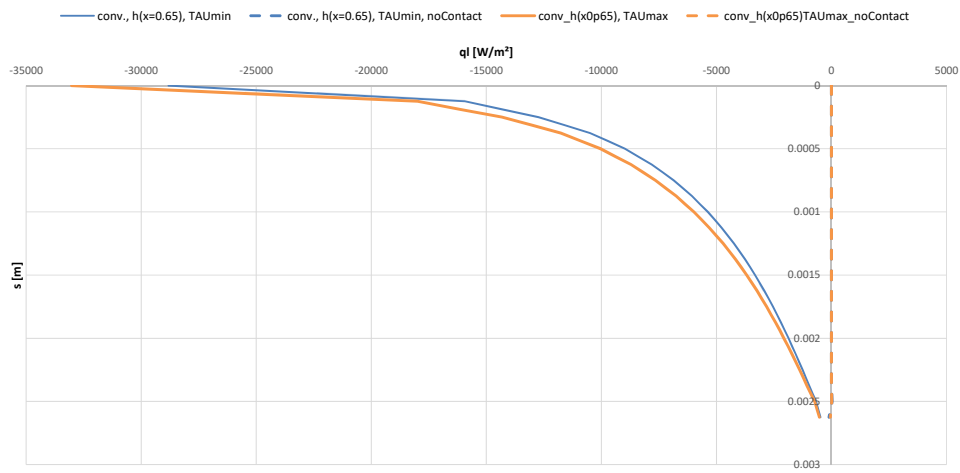


Figure 7.86: CT in PEEK ANSYS simulation results of lateral heat flux along the vertical CT-PEEK junction.

7.7 Additional Numerical Investigations

7.7.1 Modified Krogstad Roughness Model Validation

Within the two-layer eddy viscosity model, in the framework of this thesis an extension was implemented to account for flow over rough walls. This model is based on work by Fluid Gravity Engineering within the ExoMars project to include rough wall fluxes along the Mars capsule in the aerothermodynamic database (see [161]). This was achieved by the implementation of an equivalent sand grain based model in a Cebeci-Smith turbulence model, based on the modification of the Van Driest damping function which damps out eddy viscosity near the wall in the inner of the two-layers of the algebraic model. A detailed description of this modified compressible Krogstad model can be found in [161], [136]. This implementation is a modification of a model, originally proposed by [136] and with ideas from [260]. The implemented modifications to the boundary layer code are summarized in the following.

The original (smooth wall) formulation of the eddy viscosity μ_t in the inner and outer layer of the boundary layer code is

$$\left(\frac{\mu_t}{\mu}\right)_i = \frac{\rho}{\mu} (k_1 y D) \left| \frac{\partial U}{\partial y} \right| \quad \text{for} \quad (0 \leq y \leq y_m) \quad (7.30)$$

$$\left(\frac{\mu_t}{\mu}\right)_o = \frac{\rho}{\mu} k_2 U_e \delta_k^* \bar{\gamma} \quad \text{for} \quad (y_m < y) \quad (7.31)$$

with the Van Driest damping function D

$$D = 1 - e^{\left(\frac{-y_c^+}{A^+}\right)} \quad (7.32)$$

including the constant $A^+ = 26$ and the kinematic displacement thickness δ_k^* , in which the density throughout the BL is set constant, according to

$$\delta_k^* = \int_0^\infty \left(1 - \frac{U}{U_e}\right) dy \quad (7.33)$$

and the mean intermittency factor $\bar{\gamma}$ with

$$\bar{\gamma} = \frac{1 - \text{erf} \left[k_3 \left(\frac{y}{\delta} - k_4 \right) \right]}{2} \quad (7.34)$$

The above equations include model constants k_1 to k_4 according to 0.4, 0.0168, 5.0 and 0.78, respectively. The location of the boundary between the inner and outer layer y_m is defined where eq. 7.30 is equal to 7.31.

Along rough walls, the damping function in eq. (7.30) was re-formulated to effectively reduce the amount of damping close to the rough wall according to

$$D = D_s + D_r \quad (7.35)$$

in which the smooth wall damping D_s is set equivalently to the Van Driest function (eq. (7.32)) (see [260]) with

$$D_s = 1 - e^{\left(\frac{-y_c^+}{A^+}\right)} \quad (7.36)$$

but, in the framework of the current modifications, with a re-formulated inner scaled wall distance according to

$$y_c^+ = y^+ \sqrt{\frac{\rho}{\rho_w} \frac{\mu_w}{\mu}} = y \sqrt{\frac{\tau_w}{\rho} \frac{\rho}{\mu}} \quad (7.37)$$

The second version in eq. 7.37 was added to clarify that the modification is effectively a scaling later suggested by Trettel and Larsson [256]. Trettel and Larsson derived a smooth wall-normal distance scaling for TBL's with the objective to include effects of compressibility and wall heat flux by combining the idea of a scaling in the logarithmic region and a stress balance. The rough wall additive term to the damping function in eq. (7.35) is either used according to Van Driest

$$D_{r,VD} = e^{\left(\frac{-y_c^+ R_{VD}^+}{A^+ k_{s,c}^+}\right)} \quad (7.38)$$

with $R_{VD}^+ = 60$ or according to a formulation after Krogstad

$$D_{r,K} = e^{\left(\frac{-y_c^+}{A^+} \left(\frac{R_K^+}{k_{s,c}^+}\right)^{\frac{3}{2}}\right)} \sqrt{1 + e^{\frac{-R_K^+}{k_{s,c}^+}}} \quad (7.39)$$

with $R_K^+ = 70$. In the above equations, also the (equivalent) sand grain roughness Reynolds number was re-formulated according to

$$k_{s,c}^+ = k_s^+ \sqrt{\frac{\rho k_s}{\rho_w} \frac{\mu_w}{\mu k_s}} = k_s \sqrt{\frac{\tau_w}{\rho k_s} \frac{\rho k_s}{\mu k_s}} \quad (7.40)$$

This re-formulation of the Reynolds number $k_{s,c}^+$ instead of using k_s^+ was necessary due to observed excessive heat flux augmentations for large roughness values which were deemed unphysical (see also [161]). It is interesting to note that, although this re-formulation was based on engineering judgment, it is equivalent to a re-scaling according to [256] (again more clearly visible with the second formulation in eq. 7.40). This scaling was later also applied to the roughness Reynolds number by Modesti *et al.* [162] to study the compressibility effects on supersonic DNS channel flow along cubic roughness elements. Modesti *et al.* was only able to scale his roughness function properly when not only ΔU^+ but also the roughness Reynolds number is transformed via e.g.

Trettel's scaling. Although he achieved the best results by applying a slightly different scaling via $k_{s,c}^+ = k_s^+(\rho_{k_s}/\rho_w)(\mu_w/\mu_{k_s})$.

For the implementation of the above equations, in case of $k_{s,c}^+ < 65$ the formulation of Van Driest (eq. 7.38) is more accurate to reproduce corresponding experimental small roughness data (see figure 1 in [136]). For larger roughness the Krogstad model is more accurate so that eq. 7.39 is used if $k_{s,c}^+ \geq 65$.

Figure 7.87a and figure 7.87b show inner-scaled velocities and resulting roughness function ΔU^+ , respectively, against EQSR Reynolds number k_s^+ from integration of $U^+ = \int_0^{y^+} [2dy^+ / (1 + \sqrt{1 + (2l^+ D)^2})]$ with corresponding terms in eq. 7.35 to 7.39 for different formulations according to [260] and [136]. The dimensionless mixing length was used here with $l^+ = \kappa y^+$. Also in this plot, it is visible that a combination of the two models is closer to the data by Nikuradse [173].

This model, within the TINA CFD code of Fluid Gravity Engineering, was validated within the ExoMars project against data of the Passive Nostip Technology (PANT) program with very good agreement (see [161]). The model produced conservative predictions in comparison to experimental heat flux augmentations along flat plate configurations within the ExoMars project (see [167]). This flat plate data was also used to validate the modified Krogstad model formulation in the current boundary layer code implementation (see figure 7.88). Additionally, it was checked favorably against rebuildings from [176] of data by [104], [105] on a rough wall 6° cone.

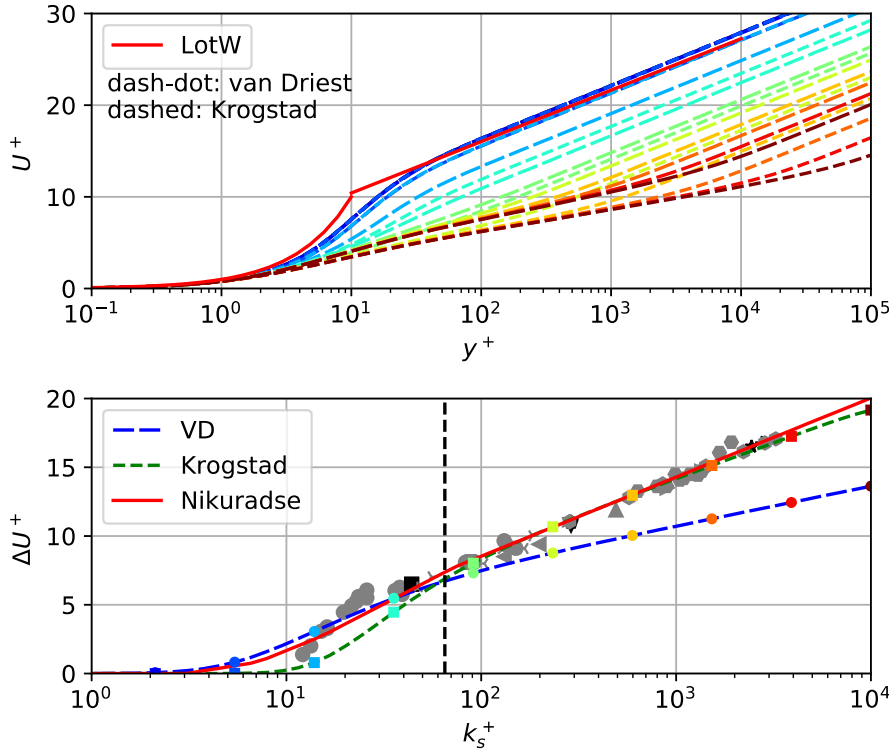


Figure 7.87: a) Inner scaled velocities and b) roughness function ΔU^+ against EQSR Reynolds number k_s^+ from integration of corresponding terms in eq. 7.35 to 7.39 for different formulations according to [260] and [136] and data points corresponding to “circle” Bandyopadhyay1987 $\lambda/k=3.8$, “square” Berg1977(Ma6.0) [24] $\lambda/k=4.$, “delta” Betterman1965 [25] $\lambda/k=4.2$, “gradient” Latin2000(Ma2.9) [139] $\lambda/k=3.9$, “left triangle” Liu1966 [152] $\lambda/k=4.$, “right triangle” Perry1963 [185] $\lambda/k=4.$, “pentagon” Bowersox2001(Ma2.7) [33] $\lambda/k=3.9$, “star” Pritchett2001(Ma0.22) [195] $\lambda/k=3.9$, “plus” Pritchett2001(Ma0.56) [195] $\lambda/k=3.9$, “cross” Leonardi2003 [148] (DNS) $\lambda/k=4.$, “hexagon corner up” Moore1951 [185] $\lambda/k=3.9$, “hexagon” Perry1963 [185] $\lambda/k=4.$, “1” Perry1969 [187] $\lambda/k=3.6$, “triangle up” Lee2012(DNS) [143] $\lambda/k=4.$, “triangle left” Liu1966 [152] $\lambda/k=4.$, “triangle right” Leonardi2007(DNS) [150] $\lambda/k=4.$; black symbols and gray symbols for compressible and incompressible data, respectively.

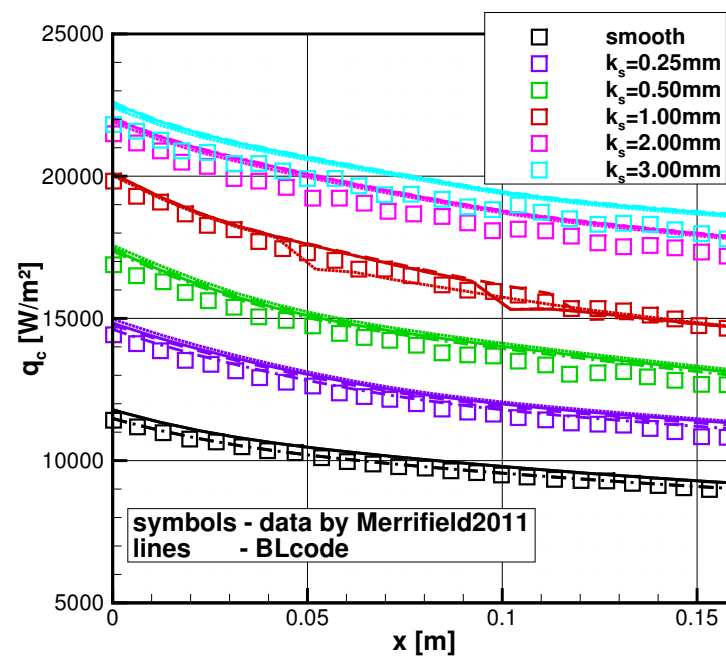


Figure 7.88: Heat flux augmentation calculated by compressible Krogstad model along flat plate at Mach 7 with comparison to data according to [161].

7.7.2 Numerical Sensitivity Studies

This chapter contains sensitivity studies with respect to spatial convergence for the numerical methods for flow simulations, i.e. boundary layer code (see chapter 3.8) and TAU (see chapter 3.9), and for heat flux analyses, i.e. “Heatflow” or “pyHeatflow” (see chapter 3.4.2.1) and ANSYS. Additionally, the numerical methods, turbulence models for both fluid dynamics numerical tools applied in this thesis were analyzed.

GCI for boundary layer code and TAU

For both fluid dynamic methods, the spatial convergence was analyzed via grid convergence index (GCI) according to Slater *et al.* [233] and Roache [206]. It had successfully been applied for hybrid TAU solutions in previous studies [133], [82].

The GCI is based on Richardson extrapolation and approximately relates the difference of solutions obtained by an arbitrary grid refinement value and arbitrary order of method to the difference which would be expected from a grid refinement study of the same case on grids using a grid doubling with a second-order method. It is defined between a finer (index i) and coarser mesh (index j) according to

$$GCI_{ij} = F_s \frac{\epsilon_{ji}}{r_{ji}^p - 1} \quad (7.41)$$

with the safety factor F_s , the relative difference $\epsilon_{ji} = (f_j - f_i)/f_i$ of a chosen solution value f , the refinement ratio r_{ji} and order of convergence p . The safety factor is chosen with $F_s = 1.25$ for a study with three grids ($i = 1$: fine, $i, j = 2$: medium, $i, j = 3$: coarse) [233]. The order of convergence p can be derived for a three grid level analysis according to [297]

$$p = \frac{1}{\log(r_{21})} \left| \log \left| \frac{\epsilon_{32}}{\epsilon_{21}} \right| + q(p) \right| \quad (7.42)$$

with the term $q(p)$

$$q(p) = \log \left(\frac{r_{21}^p - s}{r_{32}^p - s} \right) \quad (7.43)$$

and the parameter s

$$s = \text{sign} \left(\frac{\epsilon_{32}}{\epsilon_{21}} \right) \quad (7.44)$$

If a constant refinement ratio is used throughout the analysis, the term $q(p)$ in equation (7.43) vanishes. In our case the refinement ratios are not exactly equal due to the necessity of an effective refinement, defined later.

The Richardson extrapolated value ($i = 0$) based on solutions of the two finest grid levels ($i = 1, j = 2$) is defined with

Table 7.34: Spatial convergence of boundary layer code grids

grid level i [-]	num. of elements [-]	$\dot{q}$ [W/m ²]	ϵ_{i0} [%]	i,j [-]	r_{ji} [-]	ϵ_{ji} [%]	GCI_{ji} [%]
0	-	9365	0.00				
1	37168	9295	0.74				
2	16415	9205	1.69	1,2	1.50	0.97	0.93
3	7335	9004	3.84	2,3	1.50	2.19	2.16

$$f_0 \cong f_1 + \frac{f_1 - f_2}{r_{21}^p - 1} \quad (7.45)$$

A consistency check can be performed if the grid refinement levels are in the so-called asymptotic range. For that, the parameter ϕ_{GCI} should be close to unity with $\phi_{GCI} = GCI_{23}/(r^p GCI_{12}) \approx 1$, as suggested by Roache [206].

The solution value f chosen in this thesis is the wall heat flux at an axial length of $x_a = 0.5$ m. It represents a value sensitive to grid dependance and of interest for comparison to experimental data. For selected sensitivity analyses, also the skin friction coefficient c_f and for TAU calculations the wall pressure p_w were extracted and compared at the same wall position.

Boundary layer code

First, a sensitivity analysis for the boundary layer code was performed. All grid convergence computations were performed with the eddy viscosity model for fully turbulent flow and the nominal input parameters $Ma_\infty = 6.06$, $T_{0,\infty} = 500$ K, $p_{0,\infty} = 20$ bar, $T_w = 340$ K and $p_e = 0.02422$ bar. In case of the structured grid of the Boundary layer code a constant defined refinement of $r = 1.5$ between all three grid levels was applied in both directions, along and perpendicular to the wall (see table 7.34). The corresponding order of convergence was calculated to $p = 2.03$, which is very close to the expected second order accuracy of the boundary layer code [98]. The ratio $\phi_{GCI} = 1.01$ shows that the grid solutions are in the asymptotic range.

The grid convergence index only quantifies the influence of spatial discretization, not the validity of applied models or schemes. Therefore additional sensitivity calculations with the two-layer eddy-viscosity turbulence model against the mixing-length model was performed with nominal inflow conditions. This resulted in $\Delta\dot{q} \approx 1.9\%$ and $\Delta c_f \approx 2.4\%$ on the finest mesh. If all investigated differences (including the spatial convergence) are added in quadrature a total variation of $\Delta\dot{q} \approx 2.1\%$ and $\Delta c_f \approx 2.4\%$ need to be accepted for boundary layer code results on the finest grid. All results for comparison to experimental data were performed on this finest mesh.

TAU

All TAU code grid convergence computations were performed with the RSM model for fully

Table 7.35: Spatial convergence of TAU grids

grid level i [-]	num. of elements [-]	$\dot{q}$ [W/m ²]	ϵ_{i0} [%]	i,j [-]	r_{ji} [-]	ϵ_{ji} [%]	GCI_{ji} [%]
0	-	8787	0.00				
1	903358	8621	1.89				
				1,2	1.50	2.08	2.42
2	401814	8441	3.94				
				2,3	1.50	4.42	5.13
3	178635	8068	8.18				

turbulent flow and the nominal input parameters $Ma_\infty = 6.06$, $T_{0,\infty} = 500$ K, $p_{0,\infty} = 20$ bar and $T_w = 340$ K. In case of the hybrid TAU grids, an effective refinement ratio $r_{ji,eff} = (N_i/N_j)^{1/D}$ was used with the number of elements N and the dimension of the case D [206]. For the present axisymmetric case the dimension was $D = 2$. A nearly constant refinement of $r \approx 1.5$ was realized by globally changing a pre-defined grid resolution in the complete field downstream the shock region. Additionally, the number of structured sub-grid layers along the wall was refined by an equal ratio (fine: 60 layers, medium: 40 layers, coarse: 27 layers). The corresponding order of convergence was calculated to $p = 1.80$, which is close to the suggested value $p = 1.75$ by Rakowitz [200] for TAU calculations with mixed first and second order methods. The ratio $\phi_{GCI} = 1.02$ shows that the grid solutions are in the asymptotic range.

Besides the spatial discretization different influences were additionally addressed for the TAU calculations by parameter variations. TAU has different heat flux evaluation schemes implemented, evaluating either the temperature gradient via Fourier law of heat conduction or directly via the energy equation. A comparison resulted on the finest mesh in $\Delta\dot{q} \approx 1\%$. The first layer parallel to the wall was nominally kept at $y^+ \approx 0.1$. A sensitivity study with $y^+ \approx 0.01$ resulted in $\Delta\dot{q} \approx 0.1\%$, $\Delta c_f \approx 0.25\%$ and $\Delta p_w \approx 0.006\%$. For higher order accuracy the upwind schemes in TAU have different methods for the reconstruction of gradients. A comparison resulted in $\Delta\dot{q} \approx 0.5\%$, $\Delta c_f \approx 0.37\%$ and $\Delta p_w \approx 0.008\%$. At nominal conditions the use of the Spalart Allmaras turbulence model with Edwards modification (SAE) or $k - \omega$ instead of the RSM turbulence model resulted in $\Delta\dot{q} \approx 10 - 13\%$, $\Delta c_f \approx 8 - 9\%$ and $\Delta p_w \approx 0.5\%$. If all investigated differences (including the spatial convergence) are added in quadrature total variations of $\Delta\dot{q} \approx 13.2\%$, $\Delta c_f \approx 9.2\%$ and $\Delta p_w \approx 0.5\%$ need to be accepted for TAU results on the finest grid. The skin friction velocity resulted in $\Delta u_\tau \approx 4.6\%$. If not otherwise stated, all results for comparison to experimental data were performed on the finest grid.

Heatflow

The grid for the explicit one-dimensional heat flux calculations via “Heatflow” or “pyHeatflow” were checked with two test cases. First, a semi-infinite wall with constant material parameters loaded with a constant conductive heat flux without radiation enables an analytical derivation of the resulting wall temperature increase according to [95]:

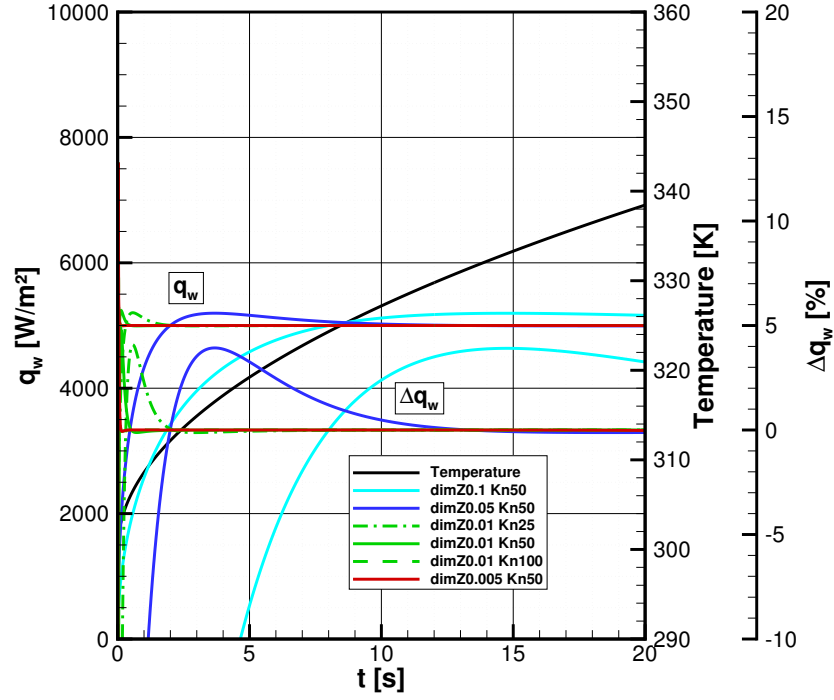


Figure 7.89: pyHeatflow grid sensitivity ($T_{w,analytical}$)

$$T_w(t) = T_w(0) + 2\sqrt{\frac{1}{\pi\lambda c_p \rho}} \dot{q}_w \sqrt{t} \quad (7.46)$$

The resulting temperature can be used as input to “Heatflow” or “pyHeatflow” to calculate the wall heat flux. Figure 7.89 shows the wall temperature increase for an input of $T_w(0) = 300 \text{ K}$ $\dot{q}_w = 5000 \text{ W/m}^2$ and constant material parameters for PEEK ($\lambda = 0.26 \text{ W/(mK)}$, $c_p = 1299 \text{ J/(kgK)}$, $\rho = 1275 \text{ kg/m}^3$). The figure also contains results from “pyHeatflow” with this temperature evolution as input at different grid resolutions, where “dimZ” means the total thickness in meters of the one-dimensional material probe normal to the wall and “Kn” the number of equidistant points within the material. The PEEK segments in H2K were designed to have a minimum thickness of approx. 10 mm, according to the typical heat penetration (see chapter 3.4.2). Therefore, the parameter for the total thickness of material in the heat flux computations was nominally set to “dimZ0.01”. Additionally, computations with larger and smaller thicknesses, between 5 mm and 100 mm and 25, 50 and 100 points were performed (50 being the default “Kn50”). The resulting conductive wall heat flux $\dot{q}_w$ as well as the percentage difference $\Delta\dot{q}_w$ to the input value are shown in figure 7.89. Besides the large thicknesses (“dimZ0.1” and “dimZ0.05”), all results show smaller deviations than 0.1 % above $t \approx 4 \text{ s}$. The nominal case was chosen as “dimZ0.01” and “Kn50”, which is within 0.01% above $t \approx 2 \text{ s}$. The case “dimZ0.005” and “Kn50” shows the beginning influence of the heat penetration, since above $t \approx 15 \text{ s}$ the wall heat flux starts to deviate from the reference. But even in this case the difference is still below 0.05 % at $t = 20 \text{ s}$.

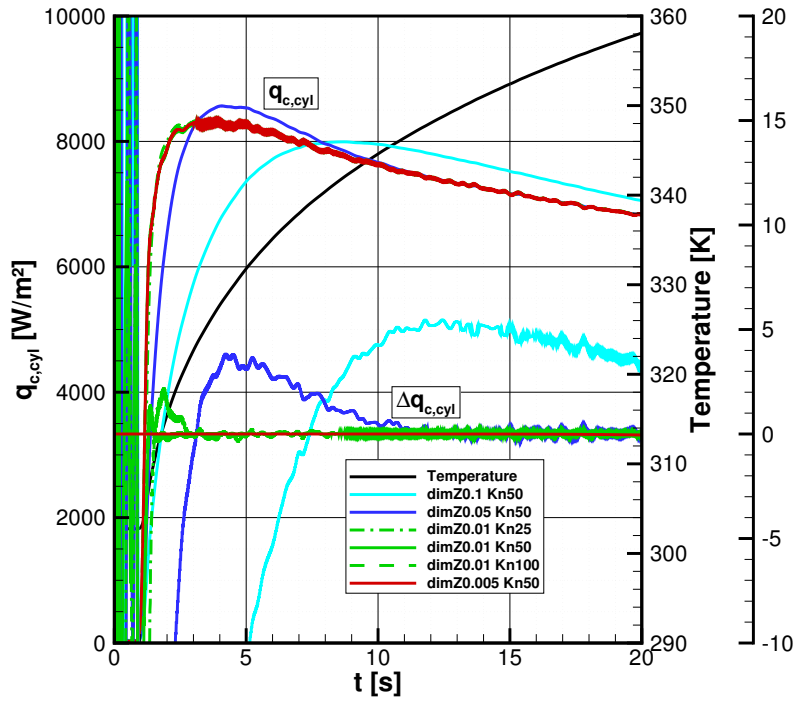


Figure 7.90: pyHeatflow grid sensitivity ($T_{w,experiment}$)

A further grid sensitivity was performed by applying a wall temperature evolution from IR of a selected experimental result with Run1011 (near $x_a = 0.5m$ pixel data (px400 py217)). The resulting convective heat flux $\dot{q}_{c,cyl}$ and the deviations $\Delta\dot{q}_{c,cyl}$ are shown in figure 7.90. The same general behavior can be seen compared to the analytical test case. All grids with “dimZ0.01” or smaller show a deviation $\Delta\dot{q}_{c,cyl} < 0.2\%$ for $t > 3s$. Therefore, the nominal grid with “dimZ0.01” and “Kn50” was chosen.

ANSYS

The grid for the heat flux calculations via the commercial finite-element software ANSYS was also checked with the same analytical semi-infinite wall temperature evolution as for the “pyHeatflow” method was used. A simple 2D block of PEEK was created with either a structured grid or inflation option was used. Again constant material parameters for PEEK ($\lambda = 0.26 \text{ W/(mK)}$, $c_p = 1299 \text{ J/(kgK)}$, $\rho = 1275 \text{ kg/m}^3$) were assumed for the check. The analytical surface temperature was applied as boundary condition on the upper block surface and an adiabatic condition was set for all other surfaces. The test simulation based on the described grid approach resulted in deviations of $\Delta\dot{q} < 0.2\%$ for $t > 0.14s$.

7.7.3 Particle Module Extensions Validation

In the following a short summary of the relevant equations and models is given, which was used within this thesis to extend the Lagrangian model in TAU to estimate motion of particles with mass.

If the following is assumed:

- high particle to fluid density ratio ($\rho_p/\rho_f \gg 1$),
- small, spherical particle dimensions ($d_p \approx 1 \mu\text{m}$),
- predominance of viscous effects,
- negligence of added-mass effects (depending on approx. half of the weight of displaced fluid),
- negligence of pressure-gradient effects (depending on approx. $\mu_f d_p^3$),
- negligence of gravity effects (depending on approx. $\mu_f d_p^3$) and
- negligence of Basset-history effects (integrated effect is small)

the equation of motion for a particle can be given with

$$\frac{d\vec{V}_p}{dt} = \frac{\vec{V} - \vec{V}_p}{\tau_p} = \frac{\vec{V}_{rel}}{\tau_p} \quad (7.47)$$

including the particle response time

$$\tau_p = \frac{4}{3} \frac{\rho_p d_p^2}{c_D Re_p \mu} \quad (7.48)$$

including the drag coefficient c_D and the following definitions

$$Ma_p = \frac{|\vec{V}_{rel}|}{\sqrt{\gamma RT}} \quad (7.49)$$

and

$$Re_p = \frac{\rho |\vec{V}_{rel}| d_p}{\mu} \quad (7.50)$$

There are different drag models suitable for different flow regimes. Two different models were chosen, deemed suitable for the flow regime in this thesis. First, the Henderson model (see [100] and [254]) was implemented, which is suitable for Re_p smaller than the laminar-turbulent transition and $Ma_p < 6$. Second, the Tedeschi model (see [250] and [61]) was implemented, which covers $Re_p \lesssim 200$ and $Ma_p \lesssim 1$. Both are defined in the following.

The Henderson model is split into three regimes according to

$$c_D = \begin{cases} c_{D,H,sub}, & \text{if } Ma_p \leq 1.0 \\ c_{D,H,trans}, & \text{if } 1.0 < Ma_p \leq 1.75 \\ c_{D,H,sup}, & \text{if } Ma_p > 1.75 \end{cases} \quad (7.51)$$

The subsonic part is defined with

$$c_{D,H,sub} = 24 \left(Re_p + S \left[4.33 \left(\frac{3.65 - 1.53 \frac{T_p}{T}}{1 + 0.353 \frac{T_p}{T}} \right) e^{-0.247 \frac{Re_p}{S}} \right] \right)^{-1} \\ + e^{-0.5 \frac{Ma_p}{\sqrt{Re_p}}} \left(\frac{4.5 + 0.38 [0.03 Re_p + 0.48 \sqrt{Re_p}]}{1 + 0.03 Re_p + 0.48 \sqrt{Re_p}} + 0.1 Ma_p^2 + 0.2 Ma_p^8 \right) \\ + 0.6 S \left(1 - e^{-\frac{Ma_p}{Re_p}} \right) \quad (7.52)$$

with the molecular speed ratio

$$S = \frac{U - U_p}{\sqrt{2RT}} = \sqrt{\frac{\gamma}{2}} Ma_p \quad (7.53)$$

The supersonic part is defined with

$$c_{D,H,sup} = \frac{\left(0.9 + \frac{0.34}{Ma_\infty^2} + 1.86 \sqrt{\frac{Ma_\infty}{Re_{\infty,d_p}}} \left(2.0 + \frac{2.0}{S_\infty^2} + \frac{1.058}{S_\infty} \sqrt{\frac{T_p}{T}} - \frac{1}{S_\infty^4} \right) \right)}{\left(1.0 + 1.86 \sqrt{\frac{Ma_\infty}{Re_{\infty,d_p}}} \right)} \quad (7.54)$$

with $Re_{\infty,d_p} = \rho_\infty U_\infty d_p / \mu_\infty$ and the freestream molecular speed ratio

$$S_\infty = \sqrt{\frac{\gamma}{2}} Ma_\infty \quad (7.55)$$

And the transonic part is defined as a linear combination of the above with

$$c_{D,H,trans} = c_{D,H,sub}(Ma_p = 1.0, Re_p) \\ + \frac{4}{3} (Ma_\infty - 1.0) [c_{D,H,sup}(Ma_\infty = 1.75, Re_{\infty,d_p}) - c_{D,H,sub}(Ma_p = 1.0, Re_p)] \quad (7.56)$$

In this work all particles were assumed to be at the surrounding fluid temperature $T_p = T$.

The Tedeschi model is defined with

$$c_{D,T} = \frac{24}{Re_p} K \left(1 + 0.15 (K Re_p)^{0.687} \right) \xi(Kn) C \quad (7.57)$$

where K needs to be found iteratively from

$$g(K) = a_1 K^{1.687} + a_2 K - 1 = 0 \quad (7.58)$$

together with the factors a_1 and a_2

$$a_1 = \frac{9}{4} 0.15 \frac{L}{a} \frac{Kn}{\epsilon'} \left(\frac{2a}{L} \frac{S\sqrt{\pi}}{Kn} \right)^{0.687} \quad (7.59)$$

$$a_2 = 1 + \frac{9}{4} \frac{L}{a} \frac{Kn}{\epsilon'} \quad (7.60)$$

and

$$S' = (1 - K) S \quad (7.61)$$

where the parameter S is defined as in eq. (7.53) of the Henderson model. Additionally,

$$\epsilon' = \frac{3}{8} \frac{\sqrt{\pi}}{S'} \left(1 + S'^2 \right) \operatorname{erf}(S') + \frac{e^{-S'^2}}{4} \quad (7.62)$$

and

$$\xi(Kn) = 1.177 + 0.177 \frac{(0.851Kn^{1.16} - 1.0)}{(0.851Kn^{1.16} + 1.0)} \quad (7.63)$$

Also, in eq. (7.57) either $C = 1$ or if $Ma_p > 0.3$

$$C = 1 + \left(\frac{Re_p^2}{Re_p^2 + 100} \right) e^{\frac{-0.225}{Re_p^{2.5}}} \quad (7.64)$$

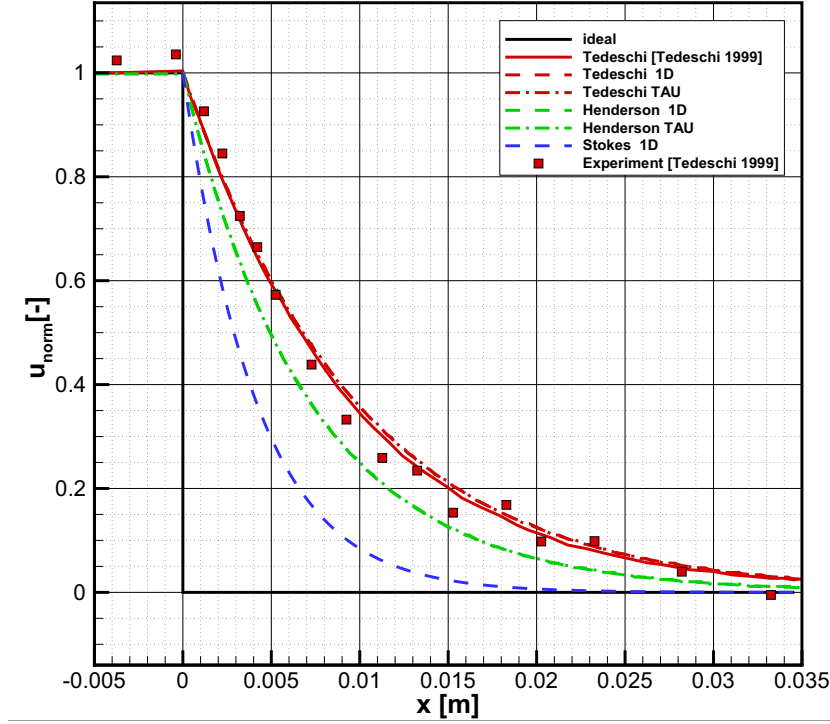


Figure 7.91: TAU particle module validation with data and drag model from Tedeschi *et al.* [250].

For $Re_p \lesssim 1$ or highly rarefied flow $L = d_p$ and $Kn = \lambda/d_p$ whereas for $Re_p > 1$ or near continuum flow $L = \delta = d_p/\sqrt{Re_p}$ and $Kn = (\lambda/d_p)\sqrt{Re_p}$.

A simplified version of the Tedeschi model, valid for $Re_p \ll 1$, $Ma_p \ll 1$, was given in [61] and was also implemented for which in eq. (7.57) $C = 1$ and

$$K = K_{simple} = \frac{1}{(1 + \frac{9}{2}Kn)} \quad (7.65)$$

To validate the extended TAU module, the particle velocity downstream of an oblique shock was calculated. Since the flow conditions downstream of the shock are constant, this scenario can also be simulated with an easier 1D approach, which was implemented in a Python script. The numerical integration in the Python script was performed with the SciPy package “dopri5” solver, applying the same drag models (in the graphs labeled with “1D”). The results of this script and the TAU module are validated with literature data from Tedeschi *et al.* [250] for a 8° wedge flow test case at $Ma = 2.3$ with particles of diameter $d_p = 1.4$ micron. Good agreement between both approaches is visible in figure 7.91 and therefore the particle module was deemed suitable for further analysis.