Goal-oriented optimal sensor placement for PDE-constrained inverse problems

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This talk presents an application of a linear transient PDE-constrained inverse problem to the example of advection-diffusion problems on a real-world environment $\Omega \subset \mathbb{R}^n$ for $n \in \{2,3\}$. Given is a forward operator $F(m) = \mathcal{B}u$, where $u \in V$ is the solution of the linear partial differential equation $r(m,u) = f$ and $\mathcal{B} : V \mapsto \mathbb{R}^q$ is the observation operator. The objective is to utilize the discrete measurements, represented by the vector $d \in \mathbb{R}^q$, to infer the parameter set m_{op}, denoted by

$$m_{\text{op}} = \underset{m \in \mathcal{M}}{\arg\min} \frac{1}{2} ||\mathcal{F}(m) - d||^2_{\Gamma^{-1}_{\text{noise}}} + \mathcal{R}(m)\,.$$

In this application, the transport of pollutants is simulated using the advection-diffusion equation. The inverse problem is to identify the source of the pollutant and then predict its dispersion. Unfortunately, this leads to a highly underdetermined inverse problem. Therefore, a regularization $\mathcal{R}(m)$ is used to solve this problem. First, a classical L^2-regularization [3] is compared with a sparsity-enforcing regularization [1],

$$\mathcal{R}(m) := \alpha \|m\|_{\mathcal{M}(\Omega)}\,, \text{where } \|m\|_{\mathcal{M}(\Omega)} = \sup\{\langle m, \varphi\rangle : \varphi \in C(\overline{\Omega}), \|\varphi\|_{C(\overline{\Omega})} = 1\}\,.$$

For both methods, a numerical implementation is presented and evaluated for its applicability to critical infrastructure protection.
Moreover, a Bayesian formulation of the inverse Problem allows the quantification of the uncertainty associated with the prediction of the parameter m. In particular, goal-oriented uncertainty estimation represents a promising approach with regard to evacuation scenarios. In this approach, the uncertainty is quantified for a specific quantity of interest [2]. In a crisis management context, this could be a particular evacuation point or even a specific route. Additionally, this talk discusses the application of goal-oriented uncertainty estimation in the context of pollutant transport in a complex environment.

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