LEARNING THE SHAPE OF DEMAND: A GEOMETRIC FRAMEWORK FOR REAL-TIME SHARED MOBILITY

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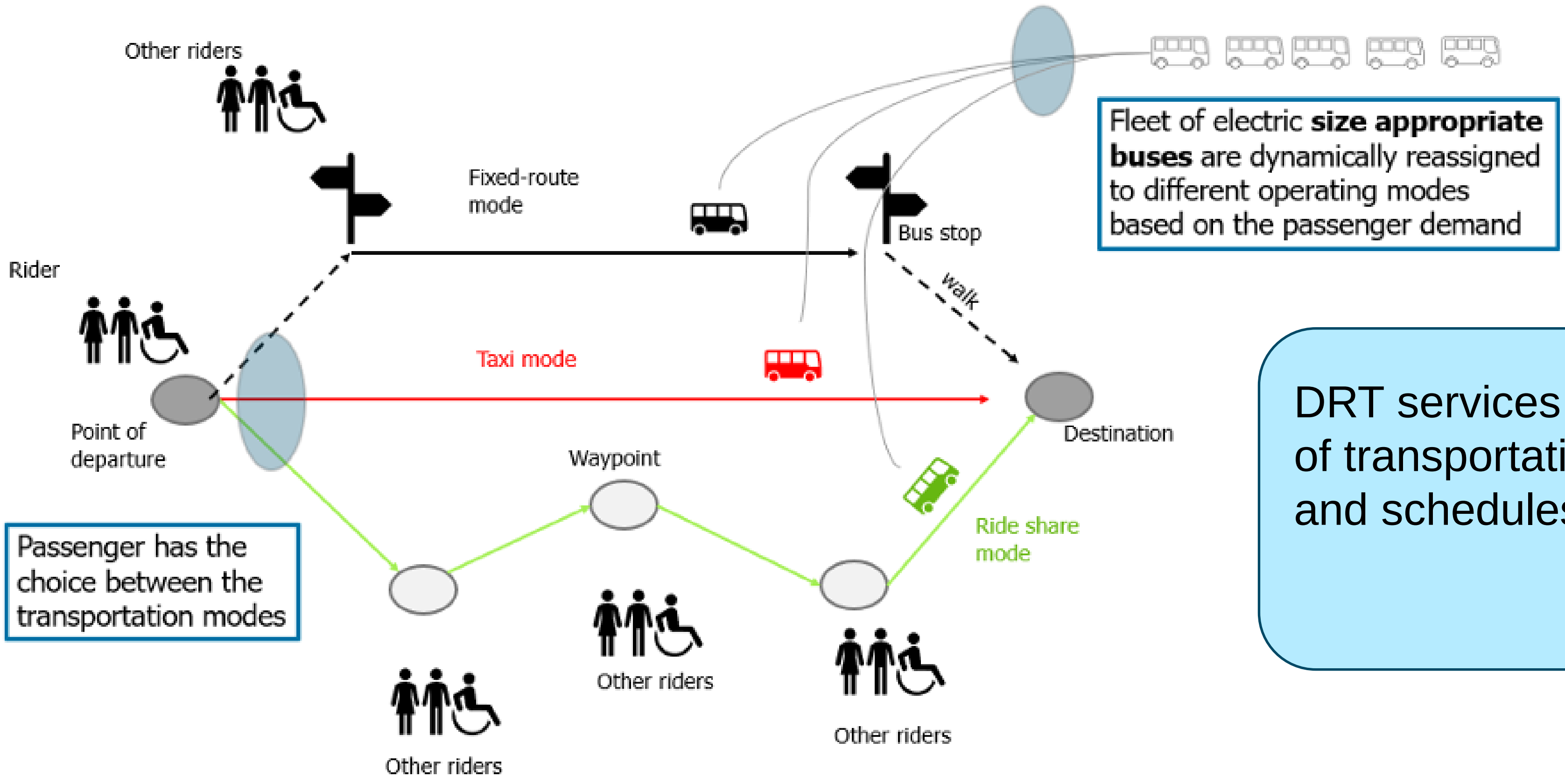
Agenda

- Motivation and Relevance of the Problem
- Related Works
- Identified Gaps and Research Questions
- Mathematical Formulation
- Preliminary Results
- Next Steps

Motivation and Relevance of the Problem



SUSTAINABLE DEVELOPMENT GOALS

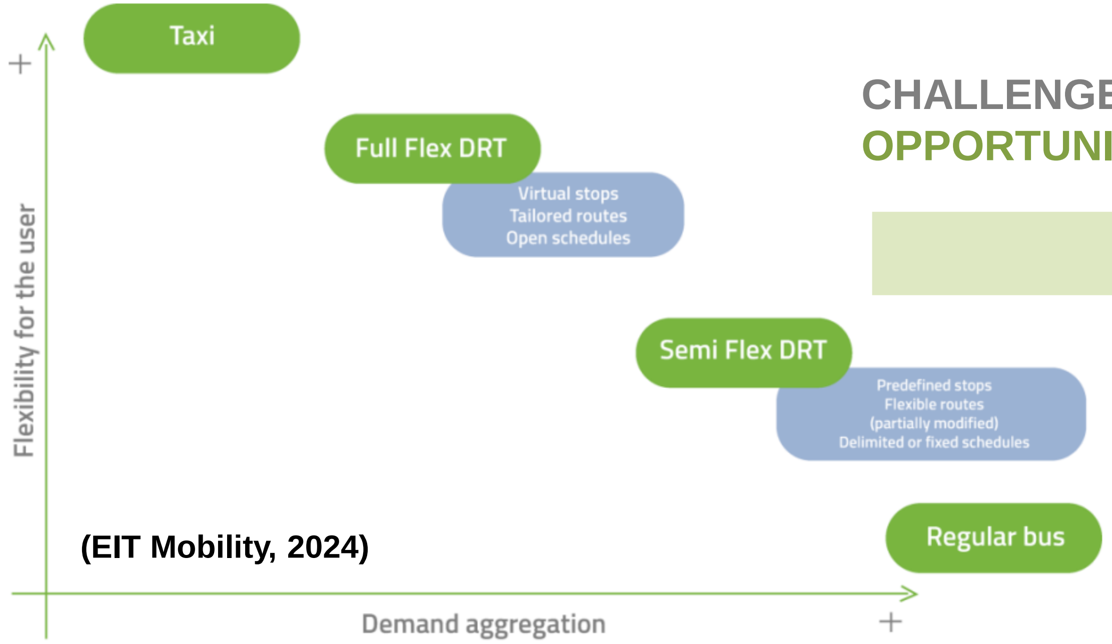
DRT services should be **adaptable** mode of transportation which can **adjust** routes and schedules based on **user** requests.

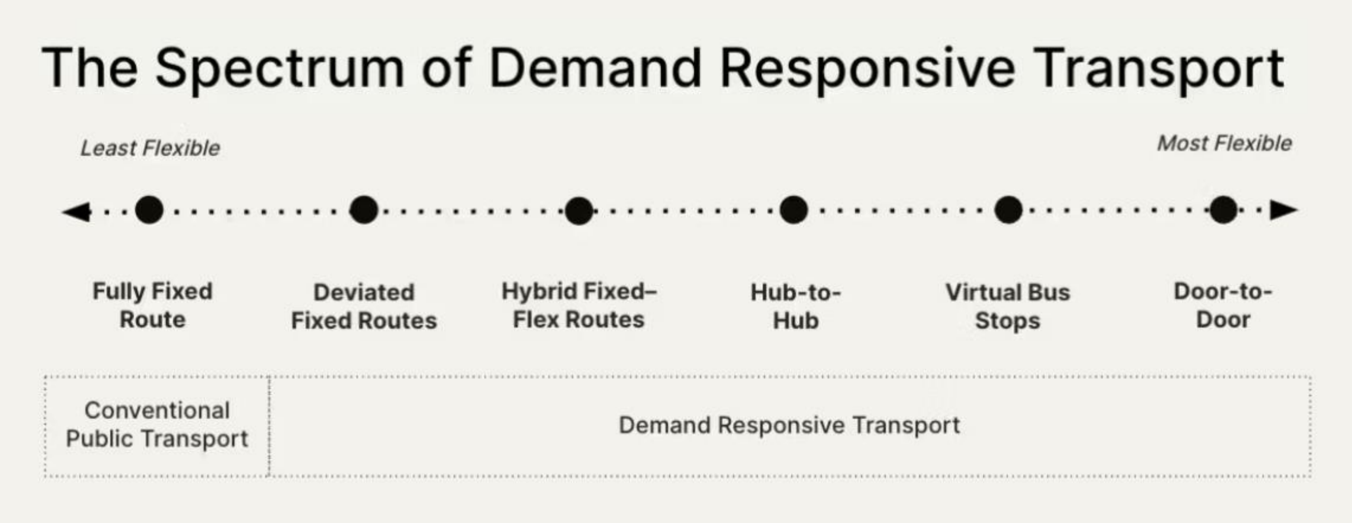
(Krell & Hunkin, 2024)

Demand Responsive Transport (DRT) in a nutshell

- **Personalized** and **low cost**
- Enabler of **multi-modal travel**
- Support **economic development** and **social incusion** (especially VRUs)

Motivation and Relevance of the Problem





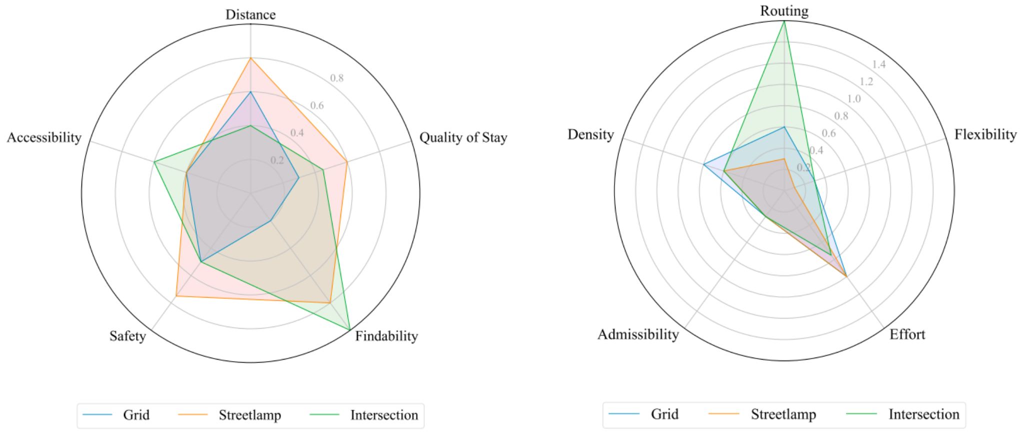
CHALLENGES/
OPPORTUNITIES

- **Door-to-door services**: Highly flexible, providing direct point-to-point transportation anywhere within a designated zone, allowing passengers to be picked up and dropped off at their preferred locations.
- **Virtual bus stops**: Predefined pick-up and drop-off points, usually within a short walking distance, offering less flexibility compared to door-to-door services.
- **Hub-to-hub services**: Operate between fixed points of interest (e.g., transport stations or city centers), with potential flexibility for point-to-hub or hub-to-point routes in specific areas.
- **Hybrid fixed-flex routes**: Combine fixed routes with set schedules during peak hours and flexible on-demand service during off-peak times, providing a balance between fixed and flexible transportation.
- **Deviated fixed routes**: Follow a set path but allow small deviations for picking up or dropping off passengers beyond the usual bus stops, offering limited flexibility.

Related Works: Harmann et al. (2022, 2023)

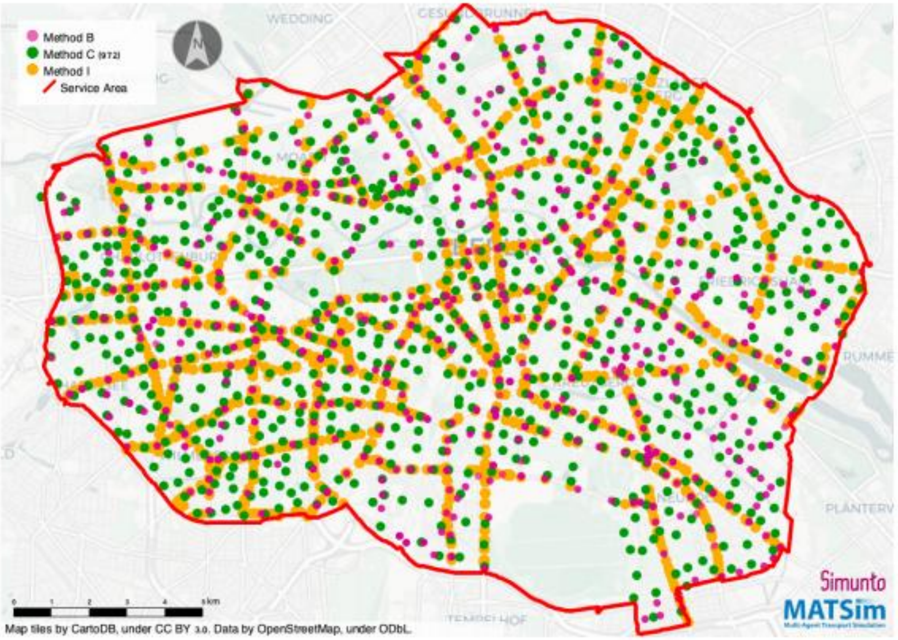
(a) Grid with a distance of $100m$.   (b) Streetlamps.   (c) Intersection.

Perspective	Weighting	Criteria	Description	Rating-Examples (1-5)
User	25%	Safety	Safety during pick-up and drop-off.	1: not safe 5: very safe
	20%	Accessibility	Accessibility by mobility-impaired people and for every user in general.	1: badly reachable 5: well reachable
	20%	Distance	Average walking distance to virtual stop.	1: large distance 5: close distance
	20%	Findability	Identification of virtual stop.	1: difficult to find 5: easy to find
	15%	Quality of stay	Environmental elements that make the stay pleasant and comfortable.	1: uncomfortable location 5: very comfortable
Provider	30%	Routing	Routing options for ensuring trip efficiency.	1: long detours 5: detour-minimal
	25%	Effort	Applying of virtual stop locations in network.	1: high effort 5: easy
	20%	Density	Ratio between stops and service area.	1: very low stop-density 5: high network-coverage
	15%	Admissibility	Restrictions to be considered.	1: not allowed 5: legal location
	10%	Flexibility	The adaptability of virtual stop locations.	1: not possible 5: very flexible



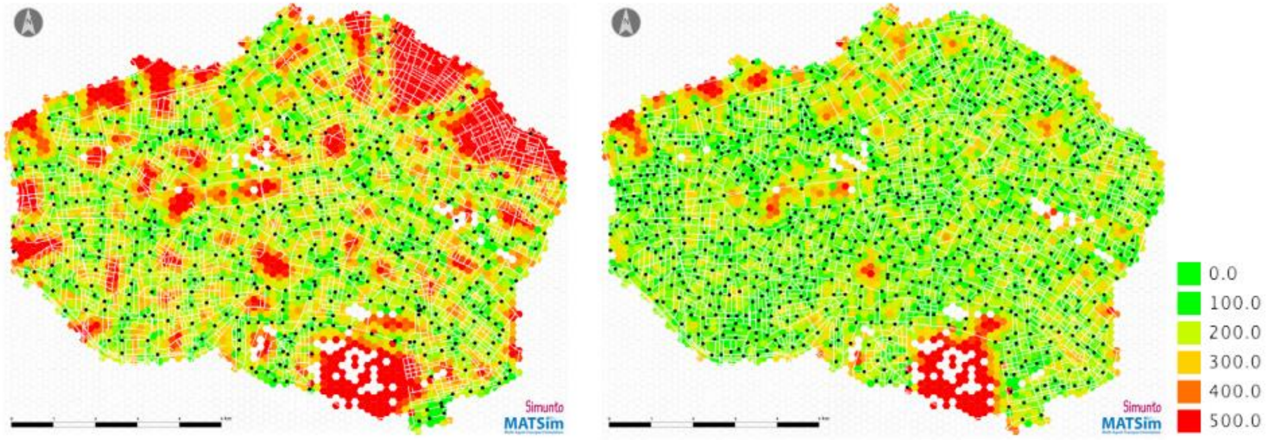
(a) Users.   (b) Providers.

Related Works: Tamleh et al. (2024)



(a) Resulting stop locations in service area

Virtual stops	Base case (18734)	Method B 972	Method C 972	Method I 2147	Method C 2147
DRT rides	43768	43070	43335	43492	43637
Pooling ratio	1.21	1.32	1.25	**1,41**	1.23
Fleet mileage [km]	150,993	139,375	147,075	**124,341**	147,845
eVMT fraction [%]	5	**3**	4	**3**	5
Ø total travel time [s]	976	1078	1017	1030	**990**
Ø waiting time [s]	149	133	136	**132**	144
Ø in vehicle travel time [s]	762	754	766	**708**	761
Ø walk distance [m]	88	256	153	254	**114**
Ø detour ratio	1.087	1.095	**1.088**	1.106	1.090

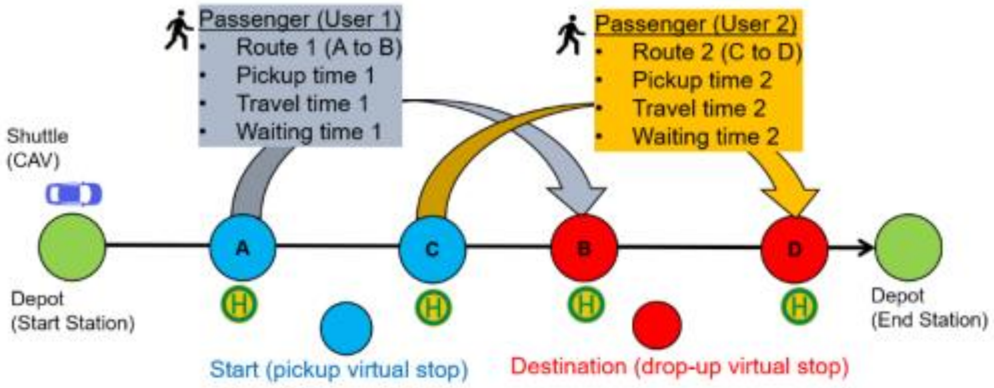


(a) Method B

(b) Method C (972)

Related Works: Tcheumadjeu & Rummel (2024)

(a) VS as bus stop (b) VS as on-street parking (c) VS as parking bay



No	Criteria	Description
1	Form of stop	Autonomous driving Manual driving
2	Type of stop	Bus stop, Shuttle stop Parking bay On-street Parking
3	Dimension of the stop	Total length and width Corner point of the stop Geometry as shape
4	Availability of communication infrastructures on the stop	To guarantee the correct management of the stop E.g. Car2Infrastructure (C2I)
5	Location/position	Geo-coordinate
6	Location type	on-street virtual stop
		off-street virtual stop
7	Parking Capacity	In term of number of vehicles
8	Street name	The name or address of the street
9	Operating time	Information about the opening days and hours

Related Works

Study	Methodology	Findings
Tamleh et al. (2024)	Agent-based simulation using MATSim in Berlin; tested grid, intersection, and clustering virtual stop designs; analyzed system performance metrics like vehicle kilometers traveled (VKT), pooling ratio, walking distance.	Intersection-based stops reduced VKT by 18%, improved pooling; increased walking distances to ~250m; virtual stops enable better efficiency than door-to-door service.
Harmann et al. (2022, 2023)	Utility-based evaluation of stop placement strategies (grid, streetlamp, intersection); considered user comfort and provider efficiency in a mid-sized German city (Braunschweig).	Streetlamp stops best for user safety and visibility; intersection stops best for routing; grid method least effective overall; stop strategy must balance user and operator needs.
Tcheumadjeu & Rummel (2024)	Criteria-based design framework developed; defined legal, technical, and user-centered requirements for placing and selecting virtual stops;	Various roadside elements (e.g., parking bays, intersections) can act as virtual stops if criteria are met; legal and accessibility criteria significantly affect stop usability; framework enables context-aware deployment.

Identified Gaps and Research Questions

- All previous works treat virtual stops as **preselected, fixed spatial candidates**. These are optimized or filtered using performance metrics, but the stop locations themselves are not dynamically generated.
- Most prior studies use graph-based models, which only capture **pairwise relationships**, such as rider-to-stop or vehicle-to-rider. This limits the system's ability to represent shared rides, group flows, or pooling dynamics.
- Routing metrics are **aggregate**: vehicle kilometers, detours, occupancy. They don't explain why inefficiencies occur or how flows interact structurally in the network.
- Once virtual stops are generated or selected, they remain **fixed or slowly adaptive**. There is no support for continuous, demand-responsive reconfiguration of the stop network.

How can virtual stops be dynamically placed and coordinated in real-time, using the underlying structure of mobility demand?

To what extent emergent virtual stops adapt to real-time demand changes and what is their impact on operational performance?

Which placement criteria support equitable access for vulnerable and underserved populations to these locations?

Mathematical Formulation: Simplicial Complex

1. We model the city as a **directed, weighted multigraph** $G = (V, E, w)$, where $w: E \to \mathbb{R}_{>0}$

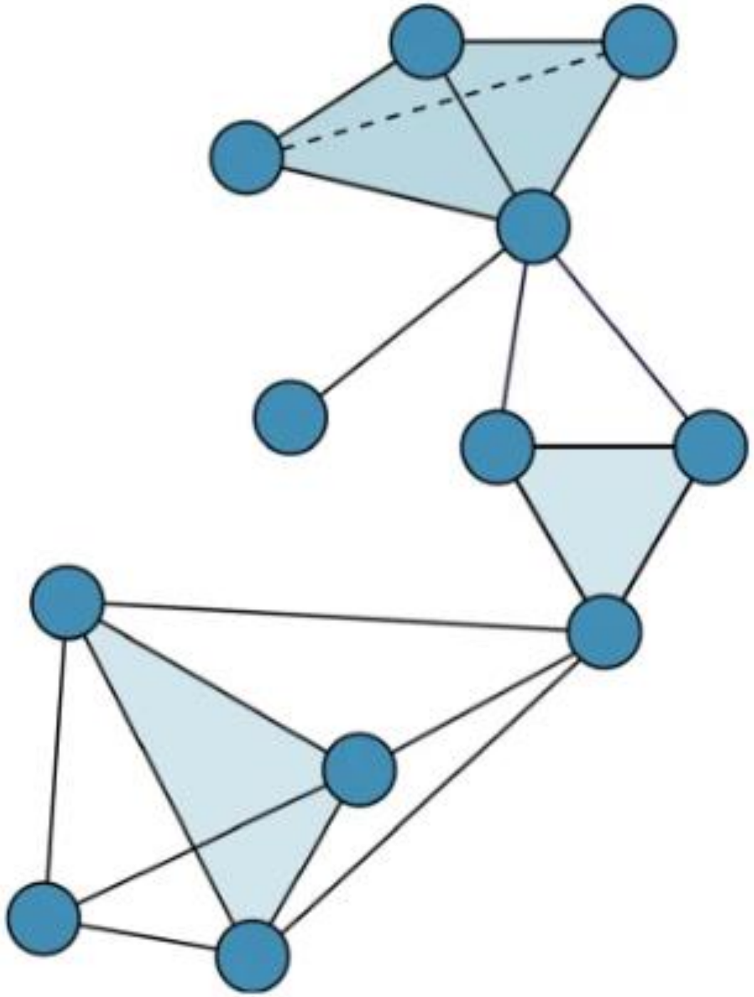
- Nodes V: street intersections or geospatial anchors;
- Edges E: directed roads, possibly multiple per node-pair
- Weights $w(e)$: road lengths or travel times.

2. Then, **user demand** is observed over time: $G(t) = (V(t), E(t))$, with $V(t) = \{v \in V | \text{ user active at } t\}$.

3. From $G(t)$, we construct a **simplicial complex** $\mathcal{K}_t$, where:

- 0-simplices $\sigma^0(t) = V(t)$
- 1-simplices $\sigma^1(t) = E(t)$
- 2-simplices $\sigma^2(t) =$ directed triangles $(i \to j \to k \to i)$

This complex captures **connectivity**, **flows**, and **local cyclicity: How?**



Mathematical Formulation: Forman-Ricci Curvature

DLR

4. For each edge $e \in \sigma^1$, we define **Forman-Ricci curvature** (Sreejith et al., 2016; Samal et al.,2018) as:

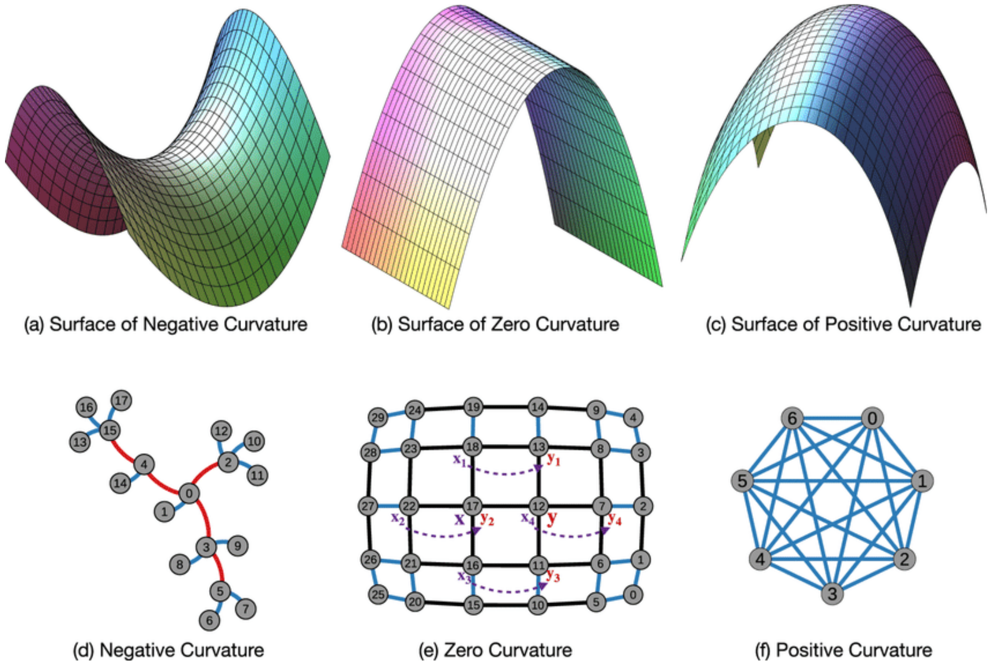
$$\mathrm{Ric}_F(e) = w(e)\left[\left(\frac{w(u)}{w(e)} + \frac{w(v)}{w(e)}\right) - \sum_{e' \to u} \frac{w(u)}{\sqrt{w(e)w(e')}} - \sum_{v \to e''} \frac{w(v)}{\sqrt{w(e)w(e')}}\right]$$

Where:

- $w(u) = \deg^{in}(u) + \deg^{out}(u)$

The sums run over **incoming edges to** u and **outgoing edges from** v. This captures **asymmetry and cost** in routing flows.

Edges with **high positive curvature** may signal **clustering of demand or routing bottlenecks**, useful for **virtual stop detection**.



Mathematical Formulation: Hodge Theory

5. We define **boundary operators** on this complex:

- $d_0 : C_0 \to C_1$ (node-to-edge incidence matrix)
- $d_1 : C_1 \to C_2$ (edge-to-face incidence matrix)

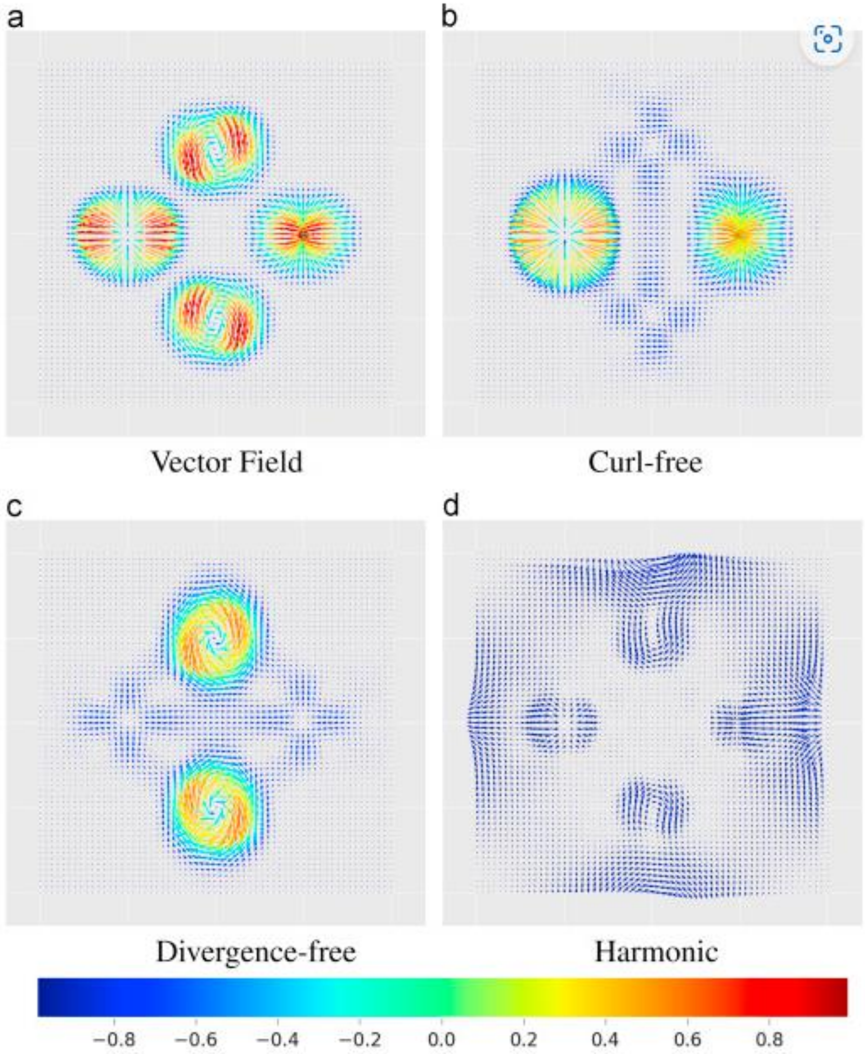
6. Then we build **Laplace-type operators**, letting us measure divergence, curl, and topological holes.

$\Delta_0 = d_0^T d_0$, $\Delta_1 = d_0\, d_0^T + d_1^T d_1$

7. Let $\omega(t) \in C_1$ be an edge-signal (flow induced by user demand), then we can apply the **Hodge Decomposition:**

$$\omega(t) = d_0^T f + d_1\, g + h$$

- $d_0^T f$: gradient field (potential-driven motion)
- $d_1\, g$: curl field (circular flows, loops of inefficiency)
- $h \in \ker d_0 \cap \ker d_1^T$: harmonic (global imbalance)



Mathematical Formulation: Final Steps

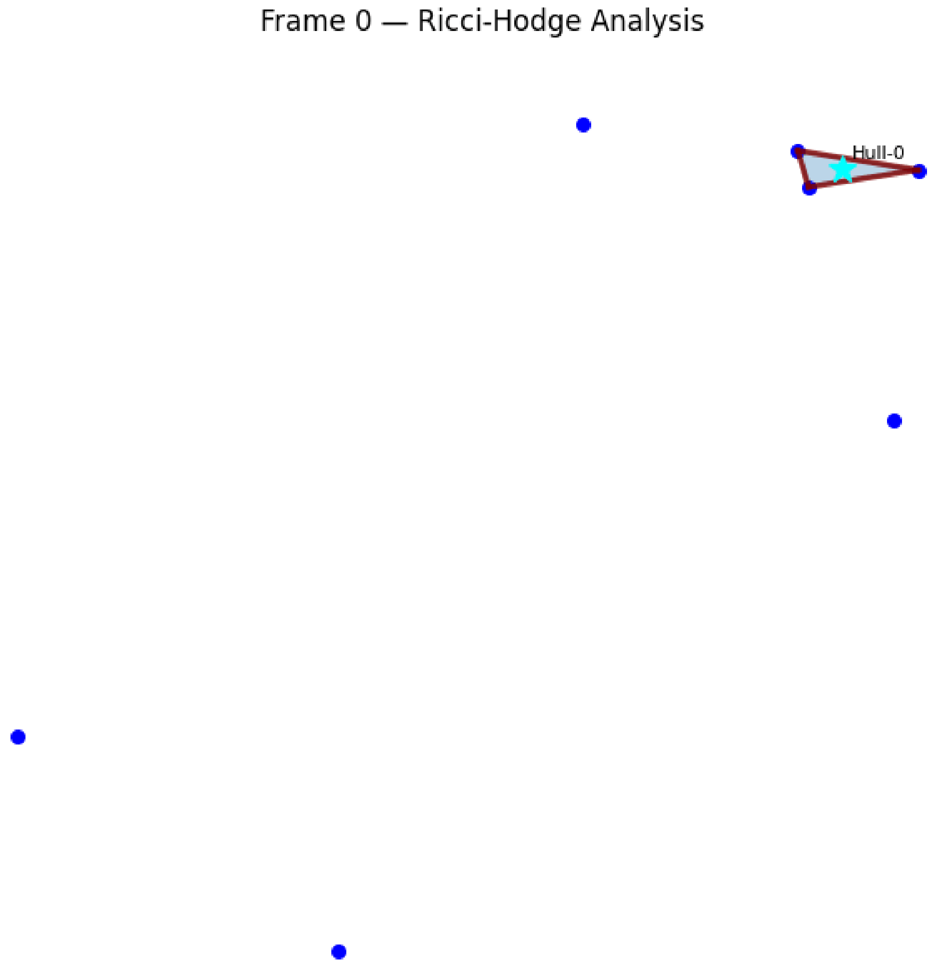
At each time t define:

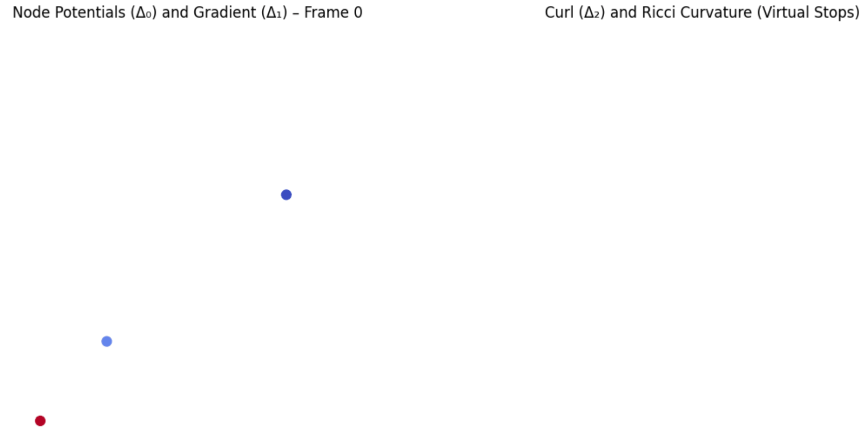
- **High-curvature subgraph** $H_t \subseteq G_t$ as:

$$H_t = \{e \in E : \mathrm{Ric}_F(e) > \theta\}$$

- Then extract **connected components**
- Compute **convex hulls** $\mathcal{C}_t \in \mathbb{R}^2$ of node positions in each component
- Define **virtual stops** as **centroids** of these hulls.

Preliminary Results





Next Steps

- Deploy the framework on real-world datasets such as ride-hailing traces or public transport logs to test its scalability and robustness.
- Validate inferred virtual stops by comparing them to actual boarding/alighting data when available.
- Track the temporal evolution of topological features such as the number of connected components and cycles to assess network dynamics.
- Combine the methodology with predictive transport-demand models (e.g., agent-based simulations) to forecast virtual stop configurations under different urban scenarios.
- Analyze how emergent simplicial complexes evolve in response to simulated changes in user behavior, time-of-day variations, or policy interventions.
- Adapt stop detection logic to prioritize the needs of vulnerable users, such as individuals with reduced mobility or those in underserved neighborhoods.
- Use curvature and density metrics to identify areas of spatial inequality and ensure equitable service coverage.
- Incorporate user-centric thresholds for accessibility, walking distance, and service frequency.
- Optimize virtual stop placement with respect to operational goals like reducing fleet size, minimizing detours, or maximizing coverage. detected virtual stops.

Aknowledgements

Thank you for listening!
Questions?