TOWARDS A PARAMETRIC RELATIONSHIP OF CRACK FRONT TWISTING ON FATIGUE CRACK GROWTH BEHAVIOR

Mixed Mode AG 2025

Jesco Talies, Eric Breitbarth, David Melching, Vanessa Schöne, Florian Paysan

SYSTEMATISCHE IDENTIFIKATION UND PARAMETRISCHE BESCHREIBUNG VON MECHANISMEN DER RISSFRONTVERDREHUNG BEI ZYKLISCH BELASTETEN, DÜNNWANDIGEN ALUMINIUMBLECHEN

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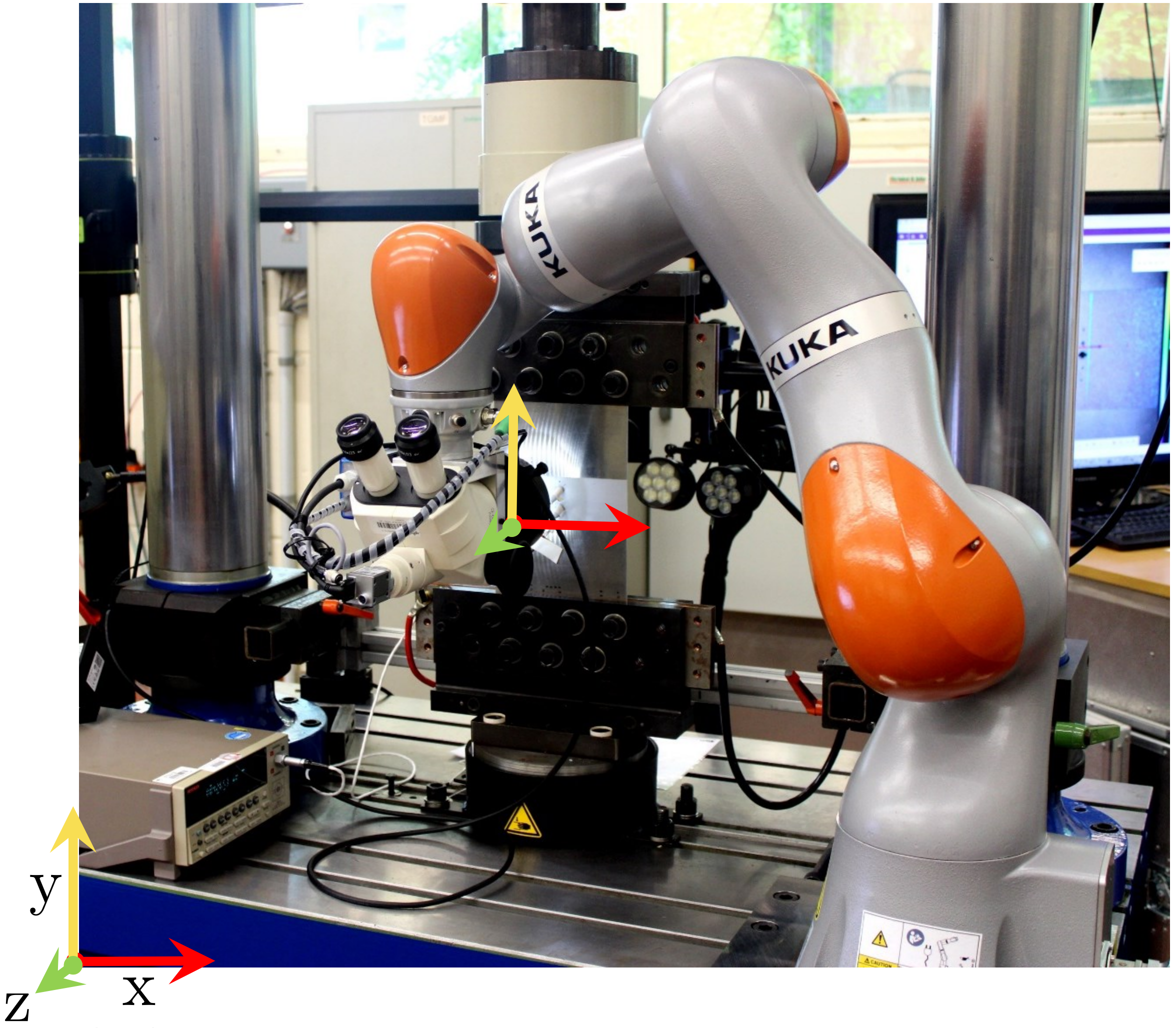
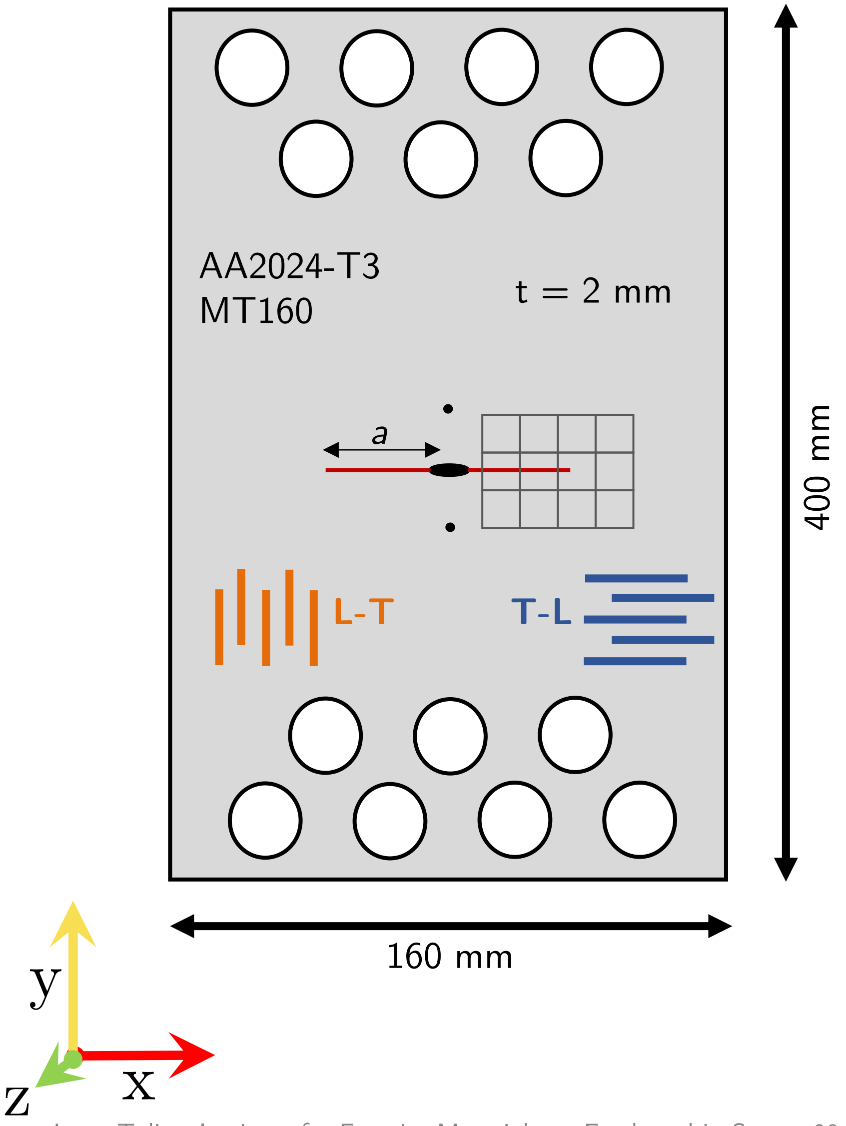
ChatGPT generated image

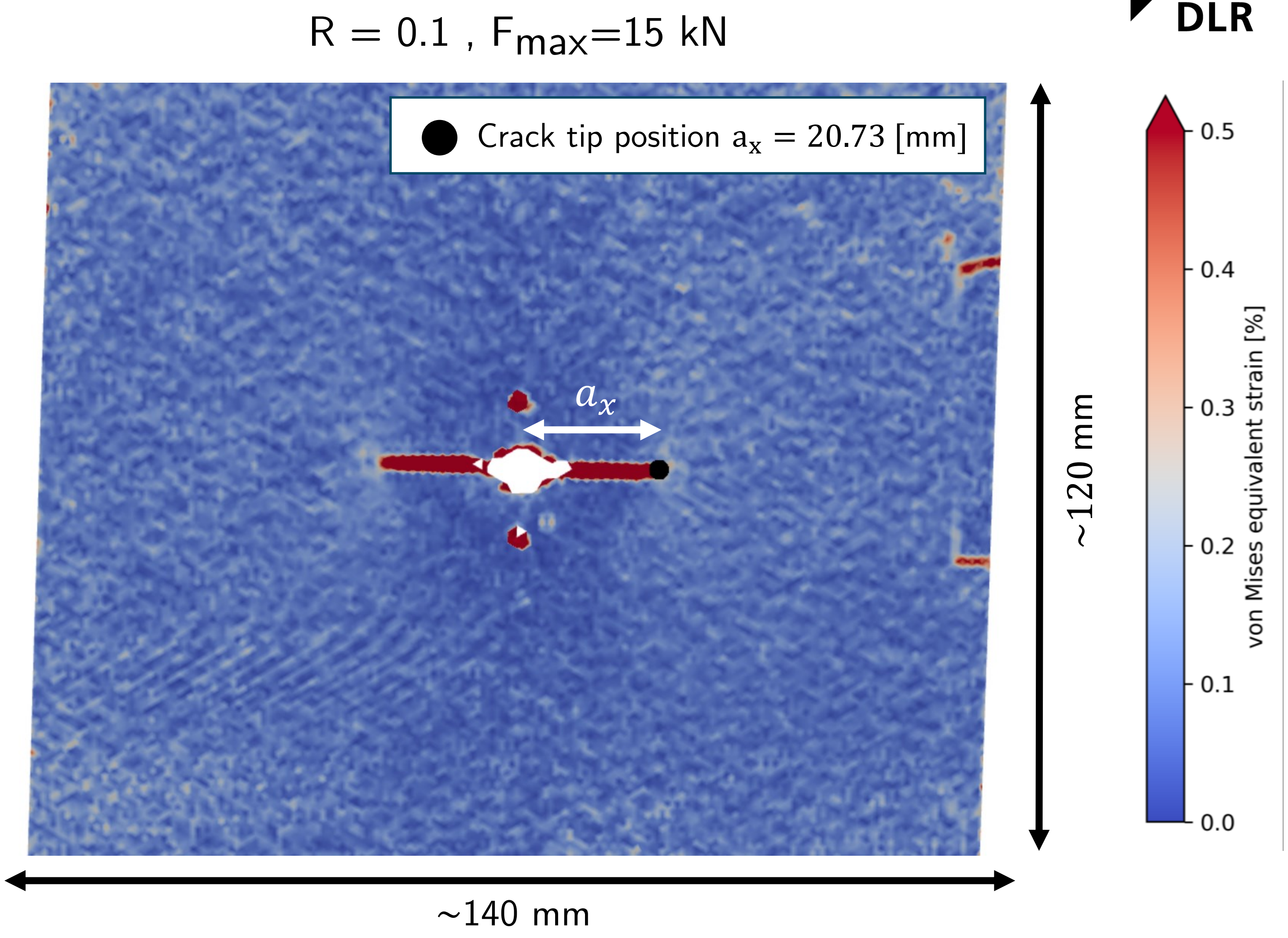
Motivation

Uniaxial fatigue tests



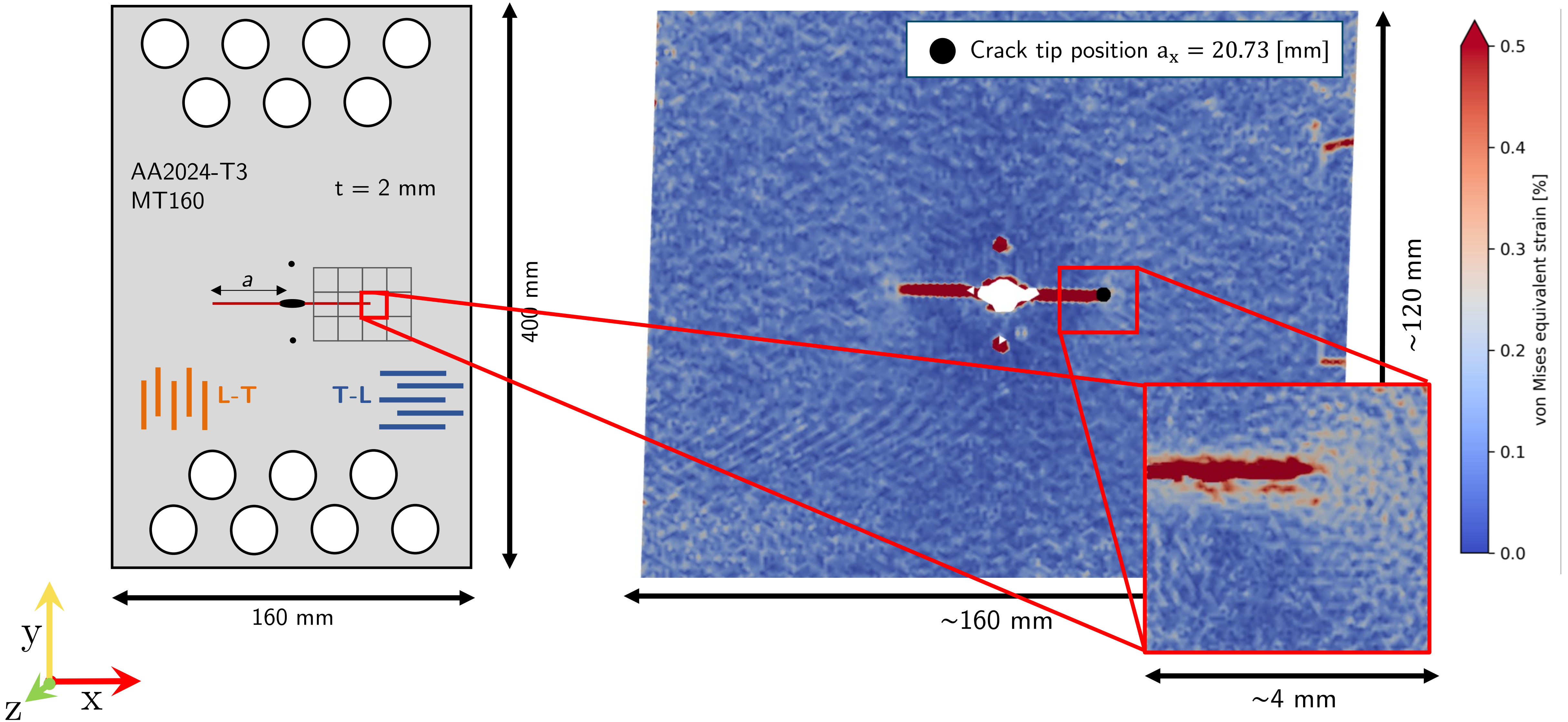
Experimental data – 3D digital image correlation





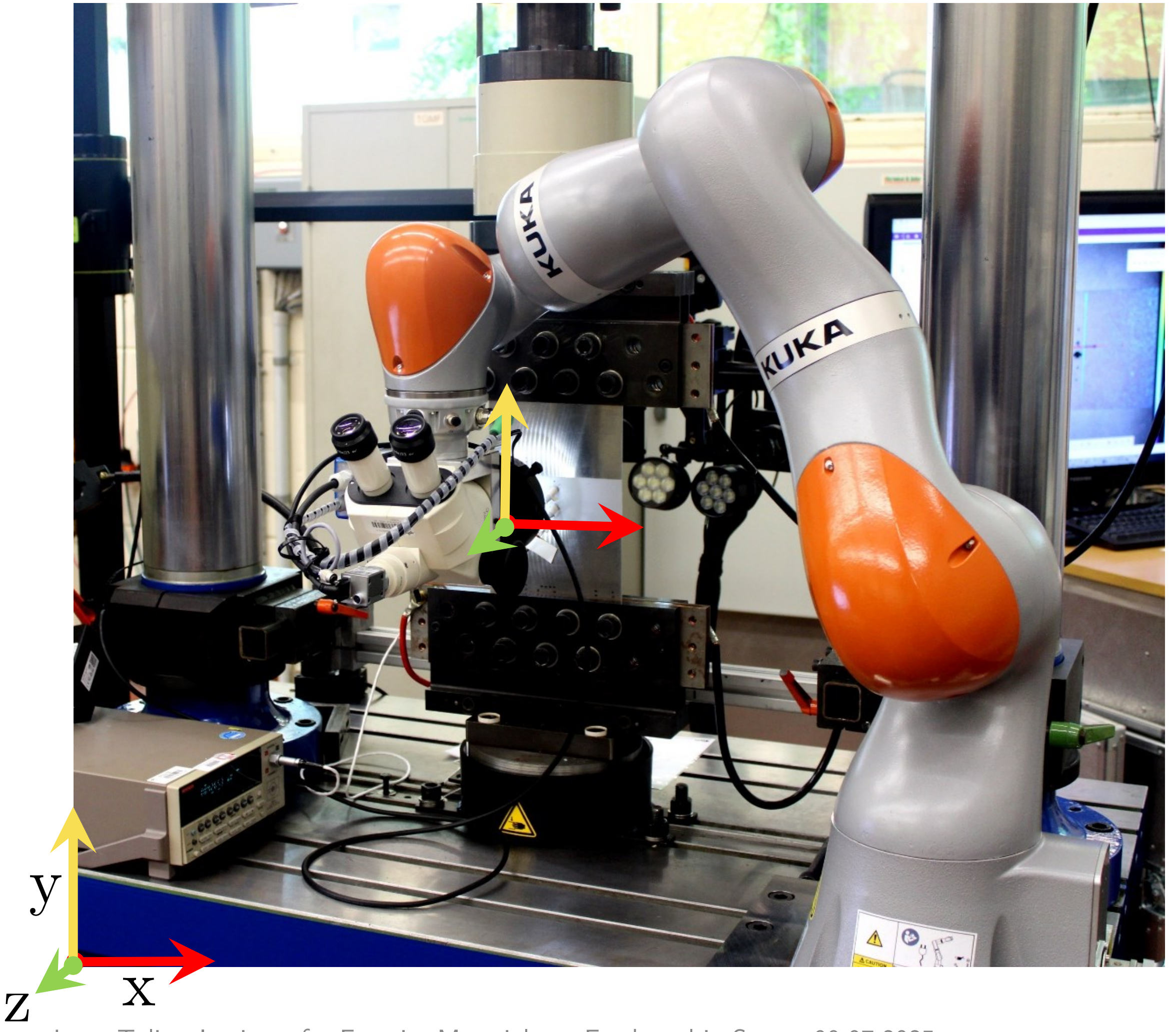
Experimental data – high res. digital image correlation

R = 0.1 , F_{max}=15 kN

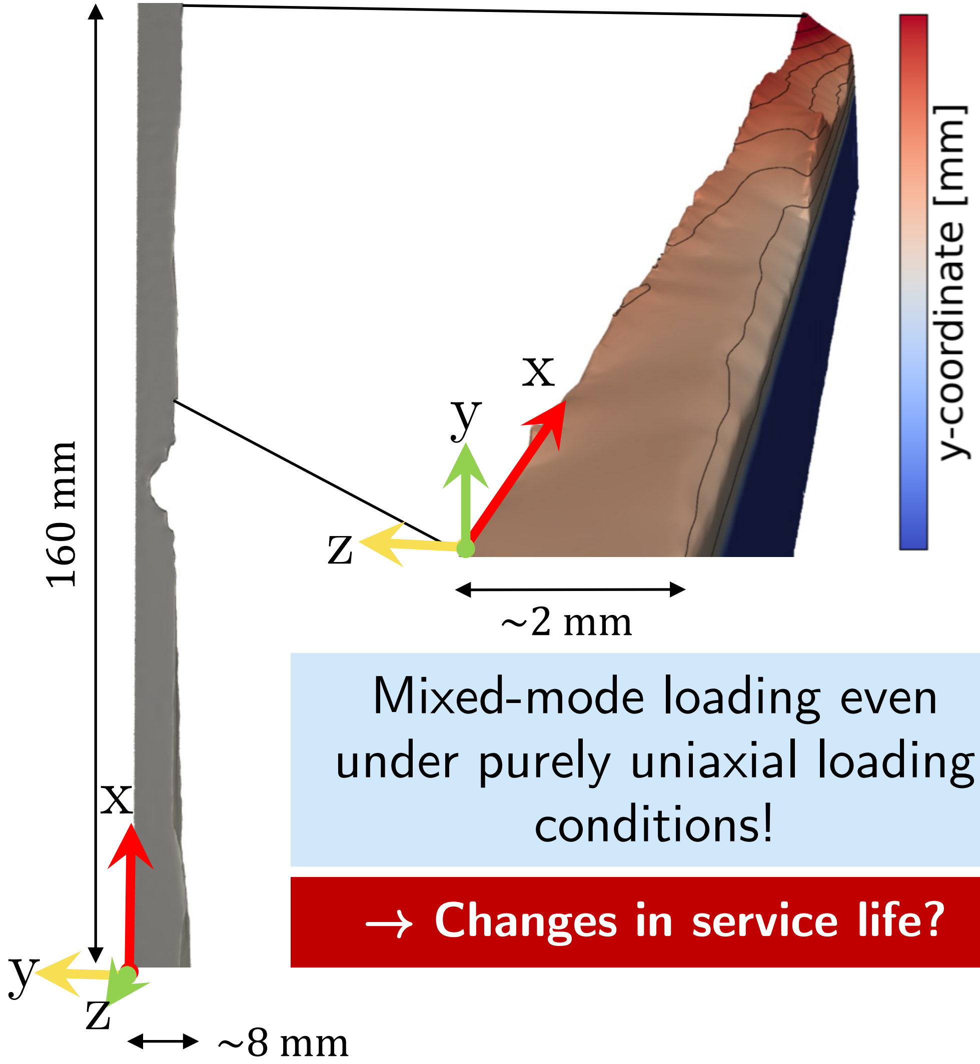


Motivation

Uniaxial fatigue tests



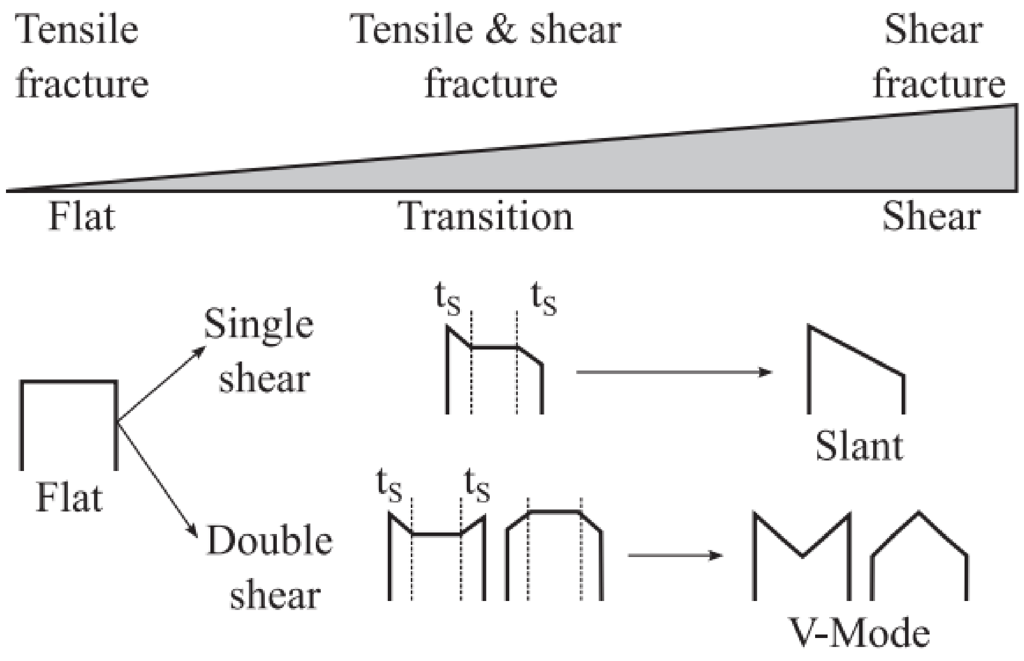
Post-mortem analysis



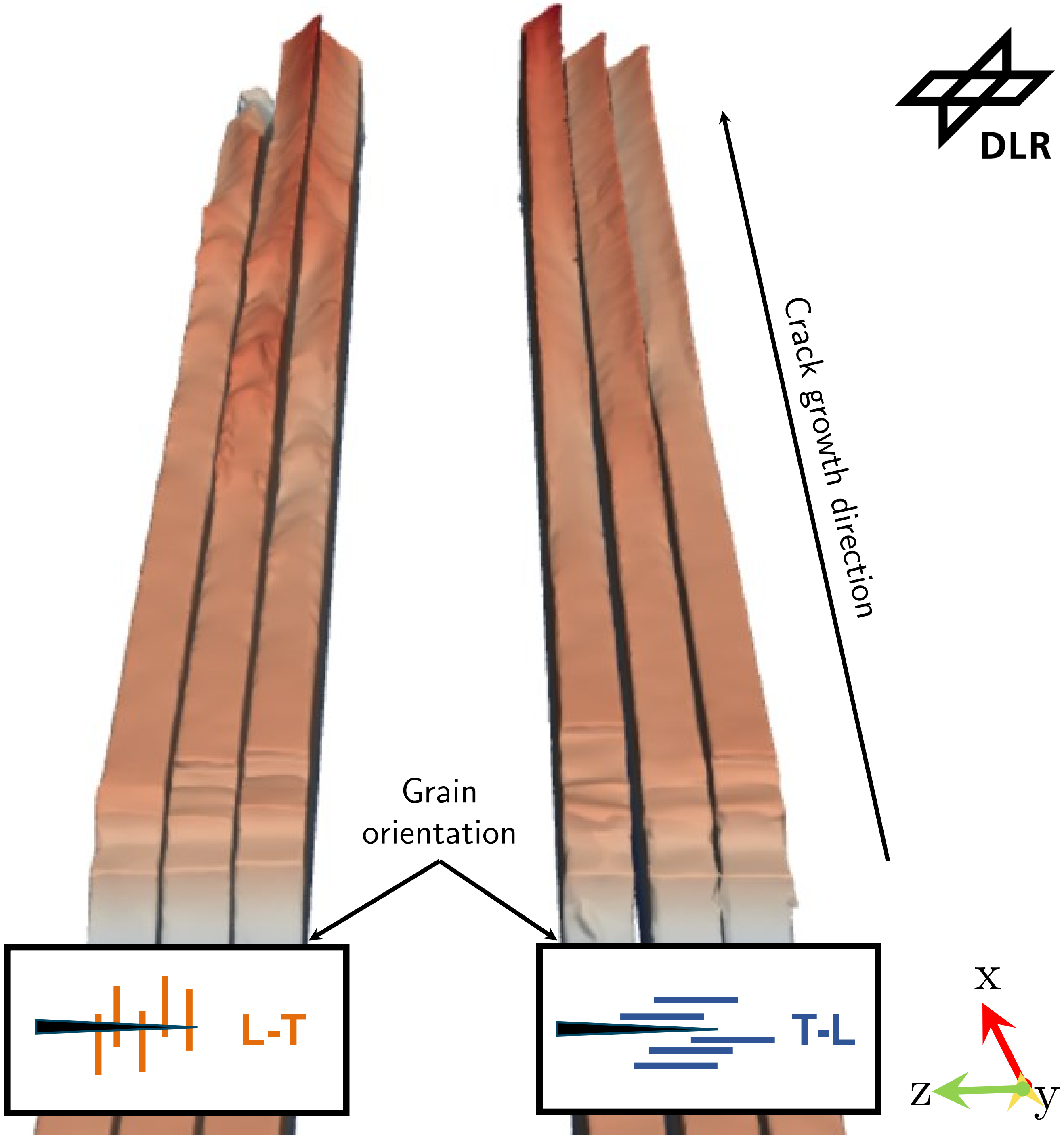
Mixed-mode loading even under purely uniaxial loading conditions!

→ Changes in service life?

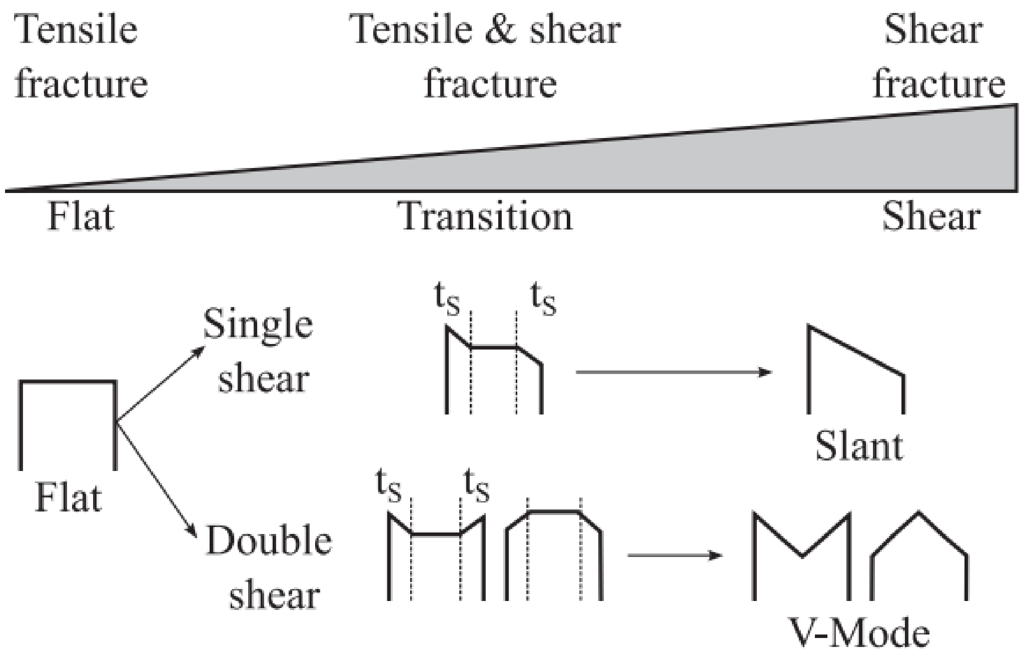
Previous research



Schöne, Vanessa und Paysan, Florian und Breitbarth, Eric
Engineering Fracture Mechanics 313 (2025) 110664



Previous research



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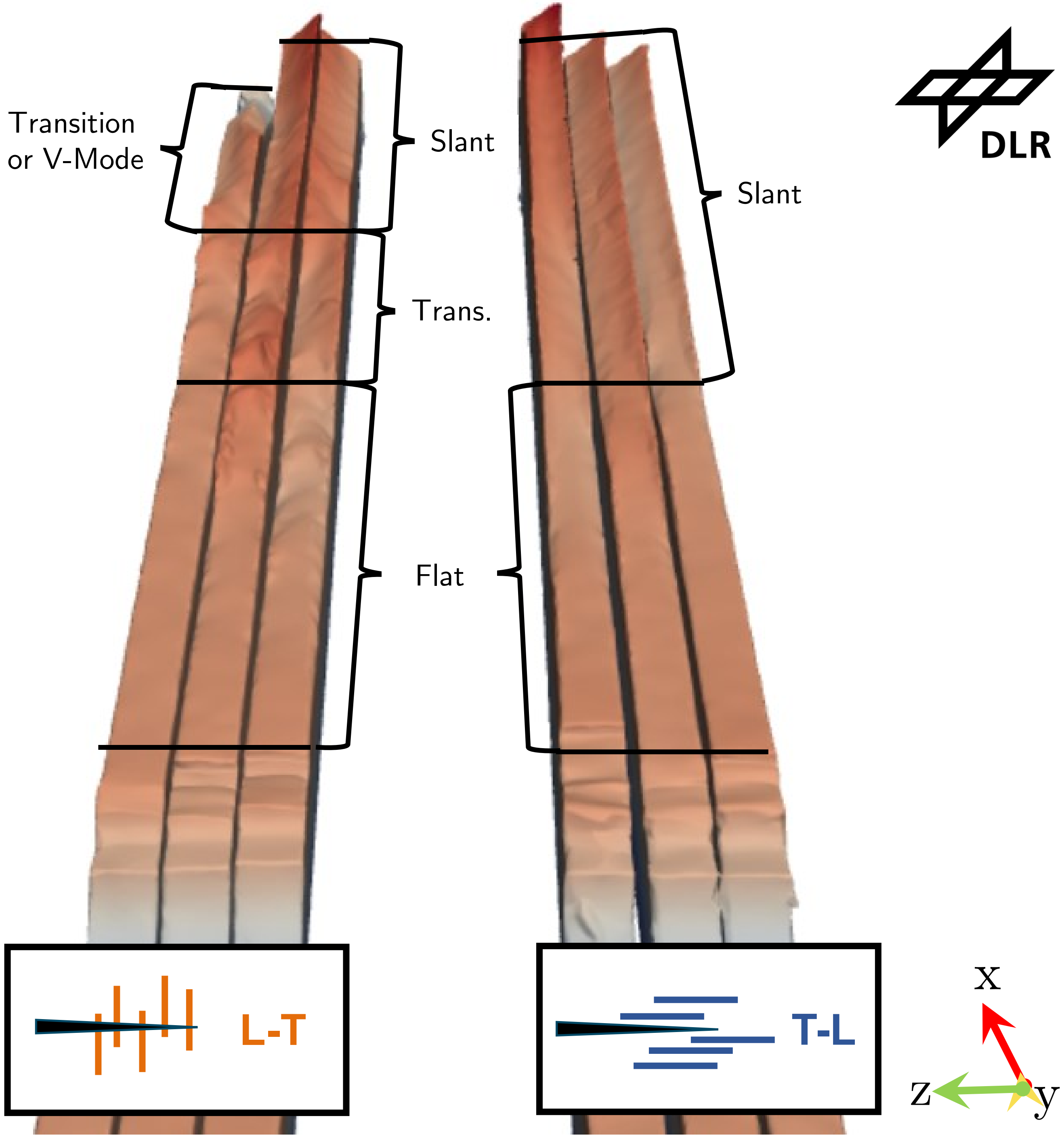
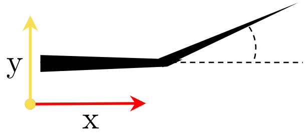


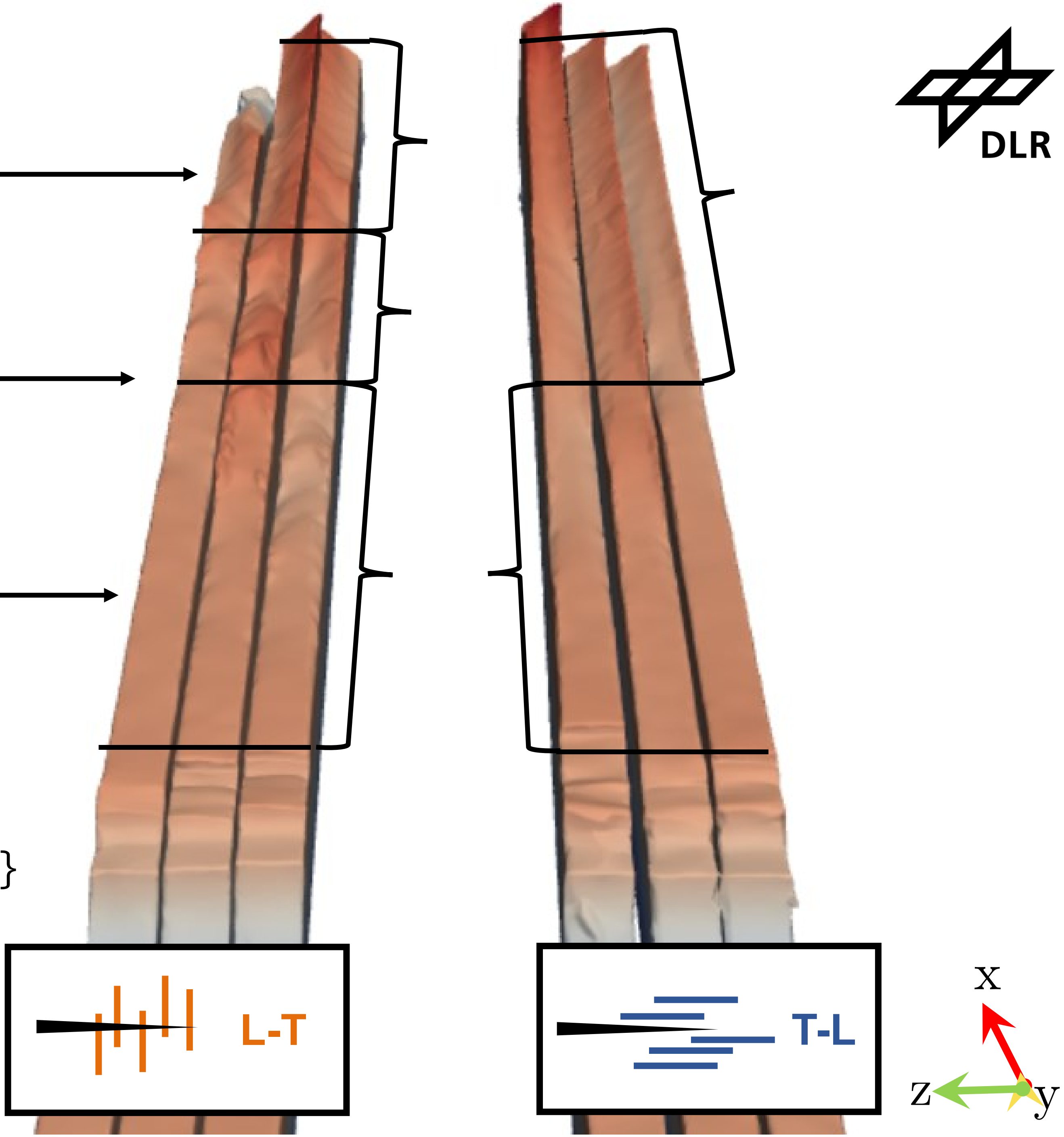
Research question

Material anisotropy $\mathbb{C}$?

Kink angle θ?



Crack length a_x ?
Stress intensity K_i ?



How can we predict mode and angle of the crack front?

Twist mode

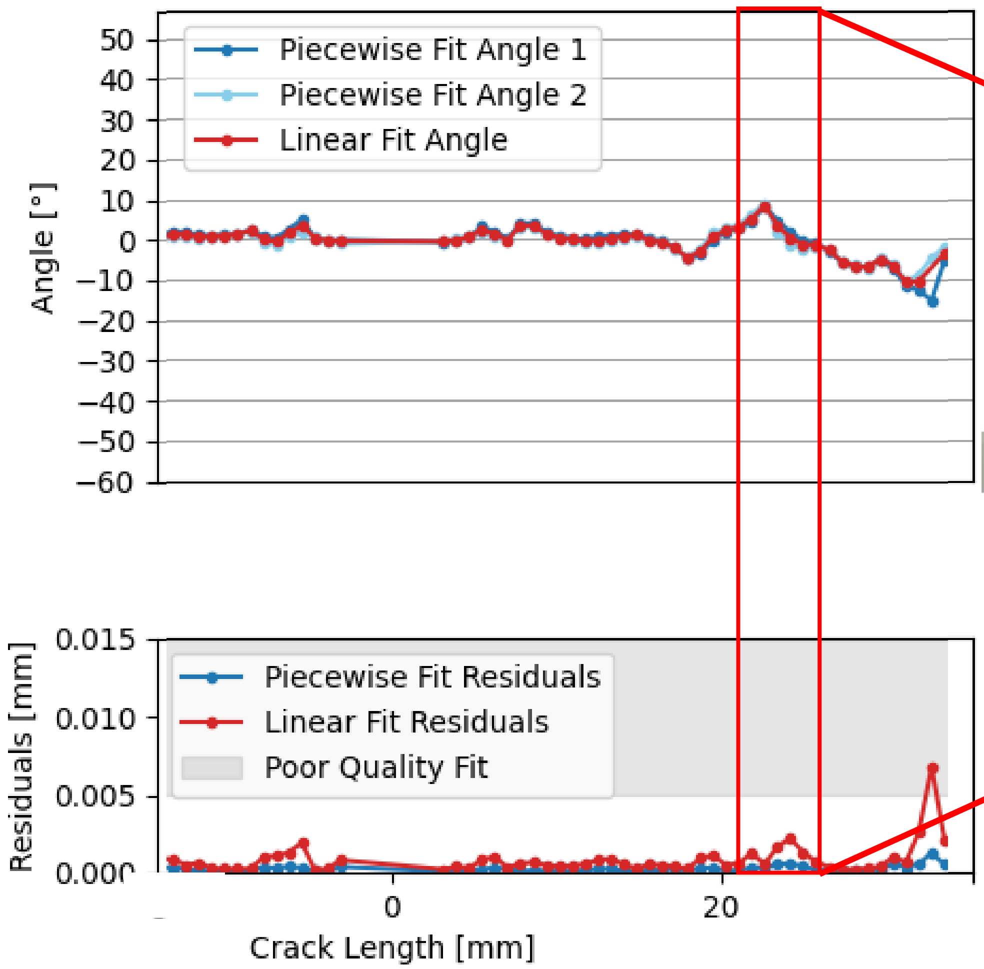
$\mathrm{M}(a_x, t, \mathbb{C}, K_i, \sigma, R, \eta, \theta, \ldots) \in$ {Flat, V−Mode, Trans}

Angle

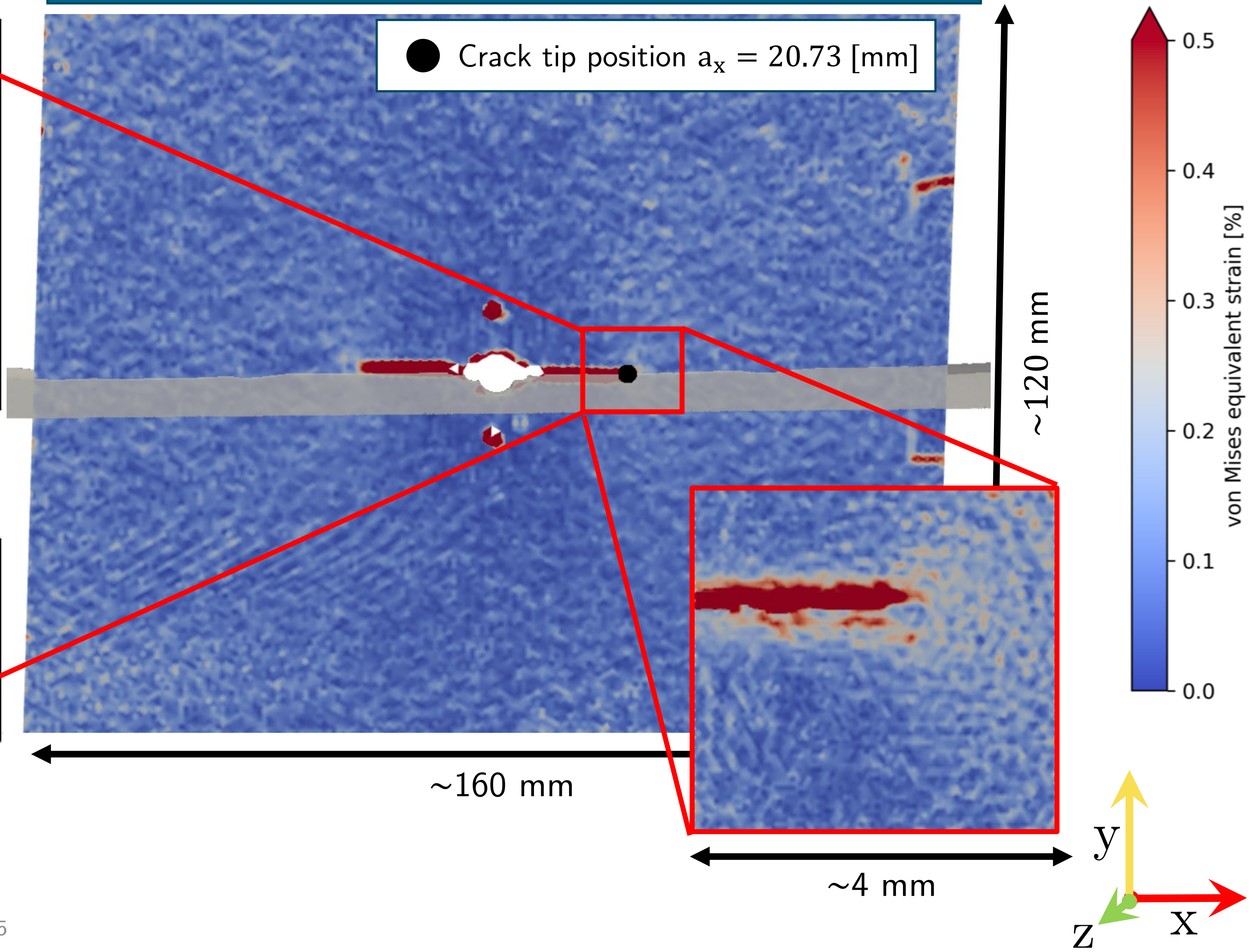
$\varphi(a_x, t, \mathbb{C}, K_i, \sigma, R, \eta, \theta, \ldots) \in [-\pi, +\pi]$

Experimental data – fracture surface angle

3D Fracture surface angle data

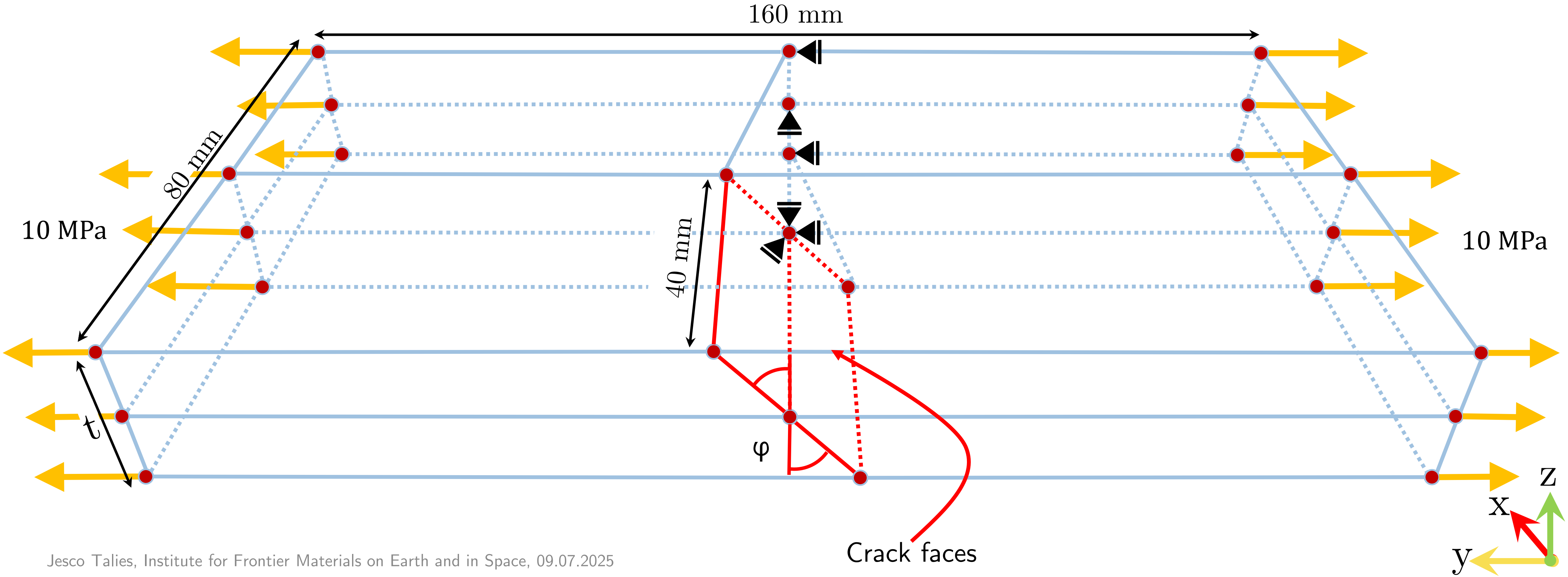


Optical high-resolution crack tip data



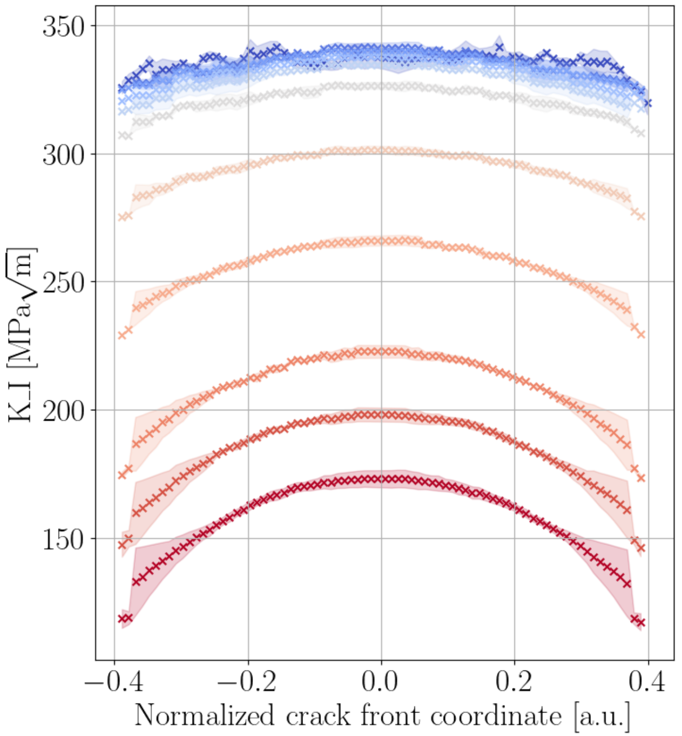
Numerical model - Setup

Ansys mechanical model			
Linear elastic	E = 74.1 GPa	ν = 0.33	a/w = 0.5
Ele. Size = 2 mm	Ele. tip = 0.125 mm	t ϵ [2, …, 10] mm	φ ϵ [0, …, 55] °

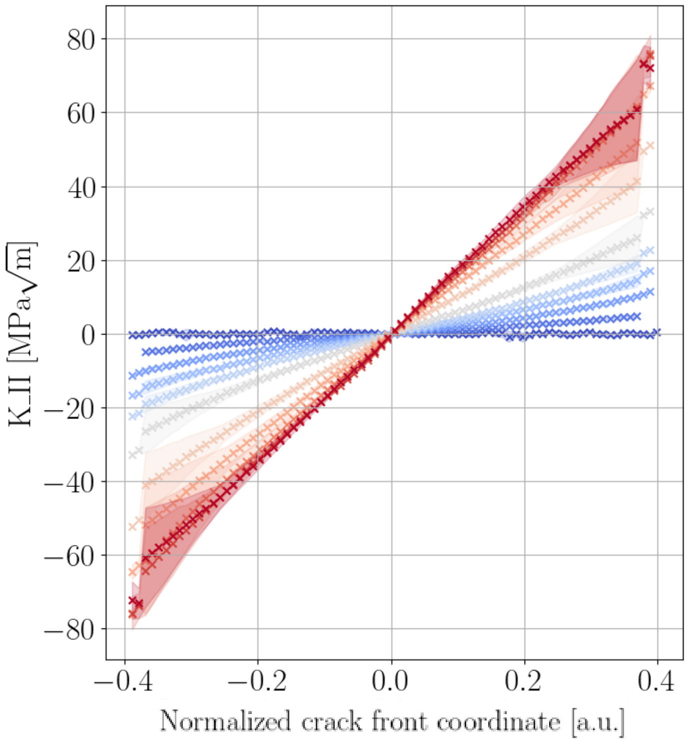


Numerical model – stress intensity factors

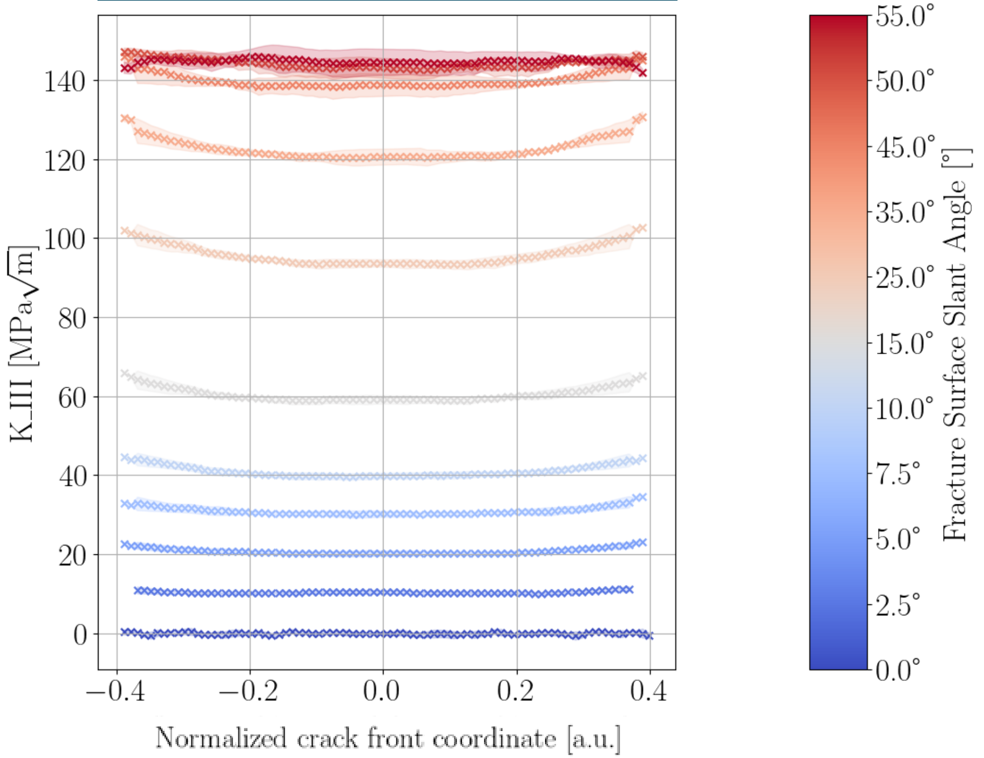
K_I



K_{II}



K_{III}



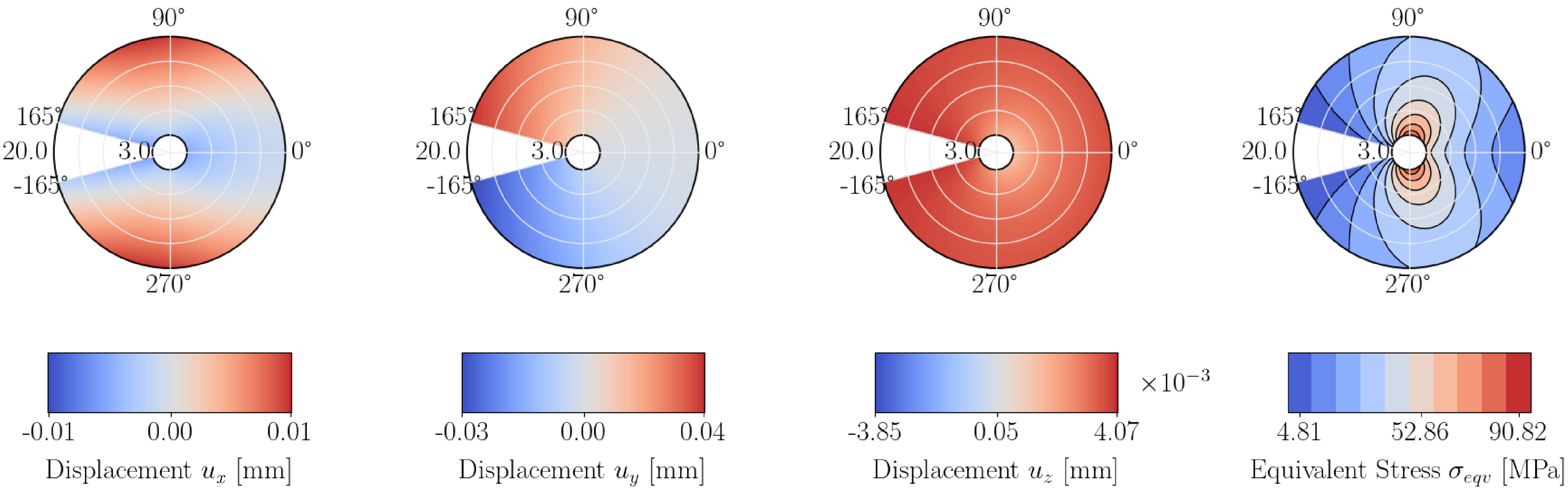
Twisted crack fronts lead to complex stress at the crack front even under purely uniaxial loading conditions

+ switch meshing to hexahedral elements

Numerical model – crack tip field

φ ϵ 0 °

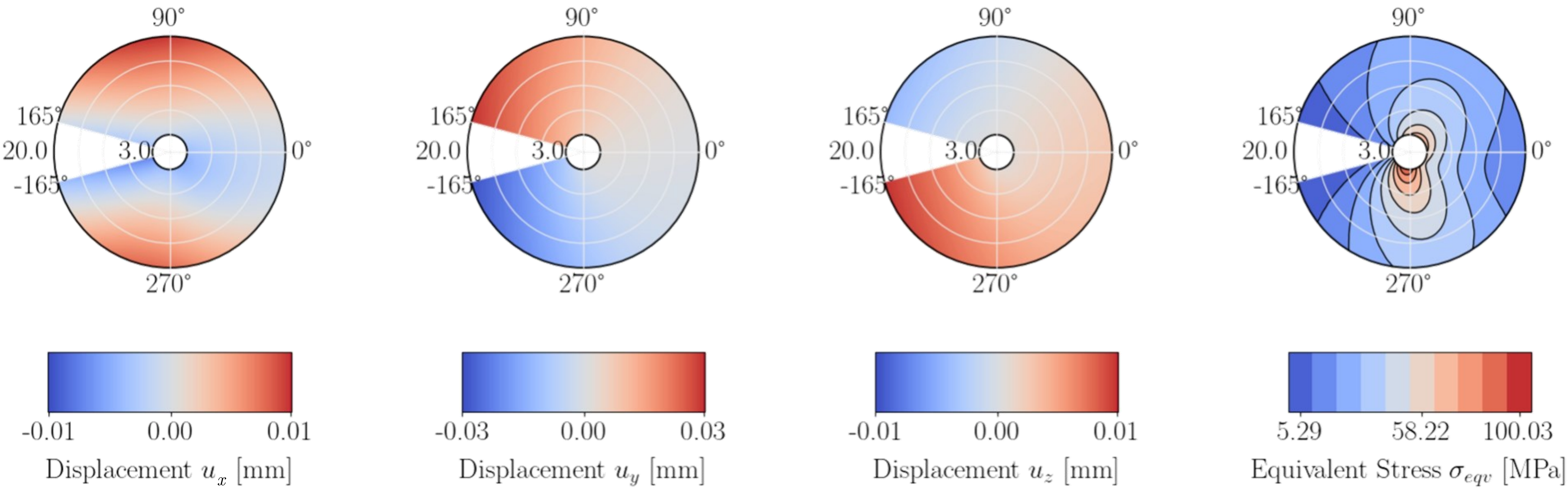
t = 10 mm



Numerical model – crack tip field

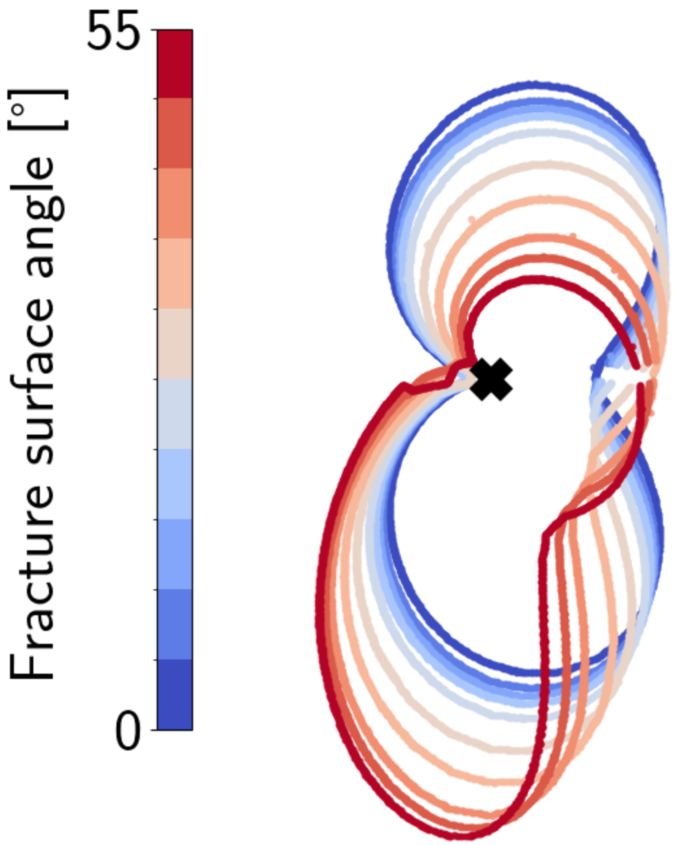
φ ϵ 35 °

t = 10 mm



Crack tip field characterization

Contours for σ_{eqv} = const.



Quantitative characterization

Williams series expansion

Stress

$$\sigma_{ij}(r,\varphi) = \sum_{n=1}^{\infty} r^{\frac{n}{2}-1}\left[\underbrace{a_n\widetilde{M}_{ij}^{(n)}(\varphi)}_{\text{Mode I}} + \underbrace{b_n\widetilde{N}_{ij}^{(n)}(\varphi)}_{\text{Mode II}} + \underbrace{c_n\widetilde{L}_{ij}^{(n)}(\varphi)}_{\text{Mode III}}\right]$$

Displacement

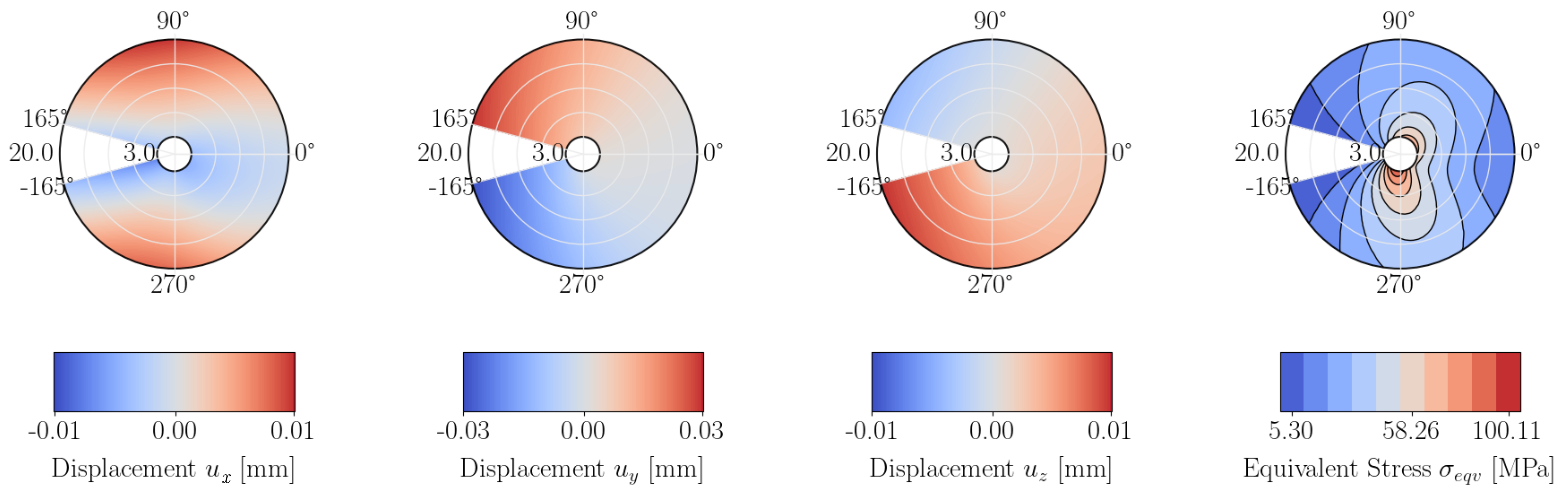
$$u_i(r,\varphi) = \frac{1}{2\mu}\sum_{n=1}^{\infty}\left[a_n\widetilde{F}_i^{(n)}(\varphi) + b_n\widetilde{G}_i^{(n)}(\varphi) + c_n\widetilde{H}_i^{(n)}(\varphi)\right]$$

$$\mu = \frac{E}{2\cdot(1+\nu)}$$

Melching, D., & Breitbarth, E. (2023) *International Journal of Fatigue*
Kuna, Meinhard (2013) "*Finite elements in fracture mechanics*"

Williams fitting

Finite element simulation crack tip field data



Truncated Williams series

$$u_i(r,\varphi) \approx \frac{1}{2\mu} \sum_{n=-M}^{N} \Big[\underbrace{a_n \tilde{F}_i^{(n)}(\varphi)}_{\text{Mode I}} + \underbrace{b_n \tilde{G}_i^{(n)}(\varphi)}_{\text{Mode II}} + \underbrace{c_n \tilde{H}_i^{(n)}(\varphi)}_{\text{Mode III}} \Big]$$

Least squares fit

$$\underset{a_i,b_i,c_i}{\arg\min} \sum_i^{x,y,z} \left[w_i \left(u_{y,data} - u_{y,williams} \right) \right]^2$$

Using weights w_i to balance contributions

Quantitative descriptors

a_i, b_i, c_i for i $\in [-M,..,N]$

Relate to fracture mechanics

$$K_I = 2\pi\, a_1$$
$$K_{II} = -2\pi\, b_1$$
$$K_{III} = 2\pi\, c_1$$

Melching, D., & Breitbarth, E. (2023) *International Journal of Fatigue*
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Williams fitting - example

$\varphi \in 35\,°$, t=10 mm

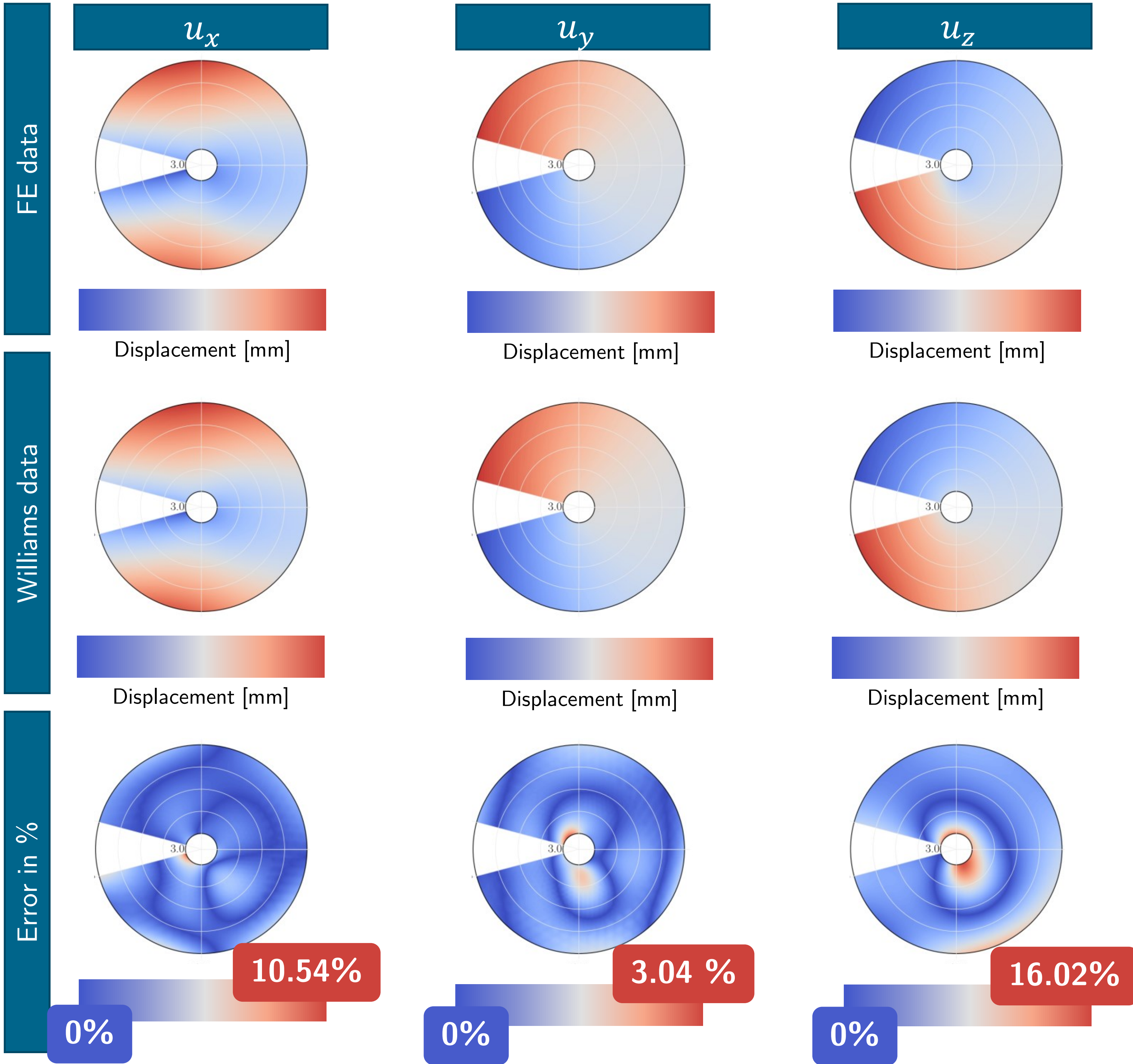
$r_{min} = 3$ mm , $r_{max} = 20$ mm

$R_r = 17 \, \frac{\text{points}}{\text{mm}}$, $R_r = 1 \, \frac{\text{points}}{\text{deg}}$

Terms: $i \in [-1, 0, 1, \ldots, 9]$

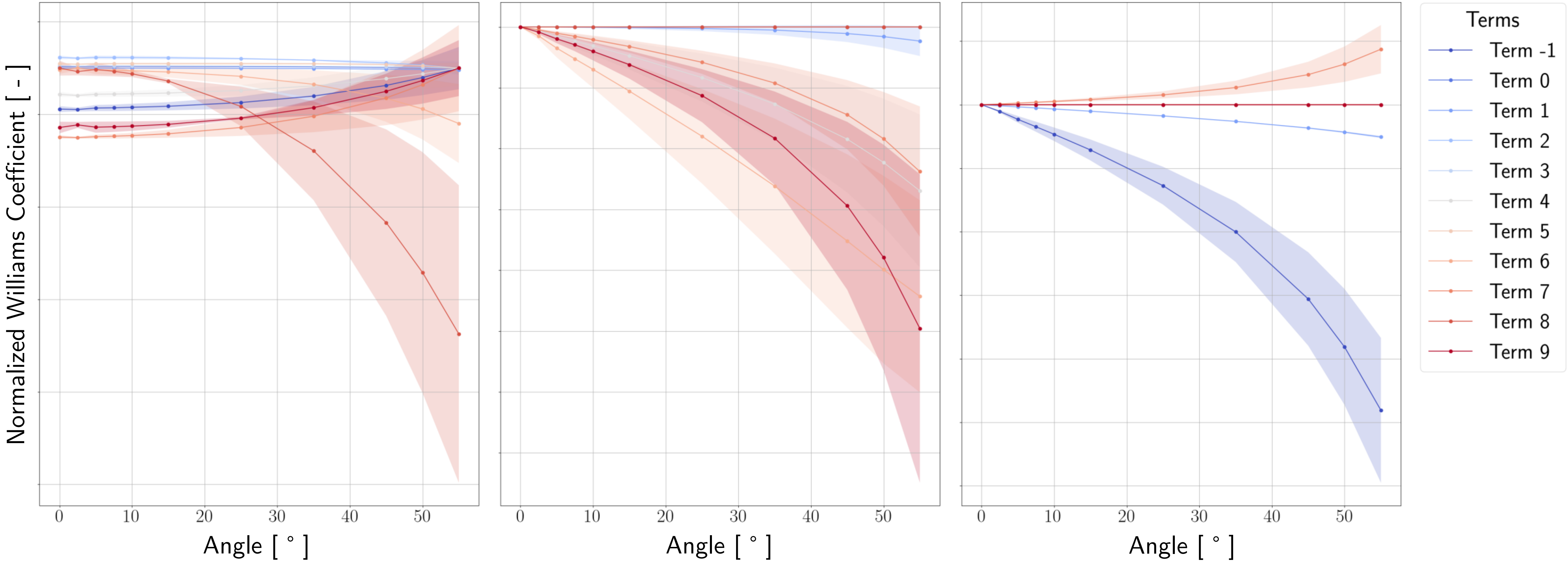
$w_i = 1$ for $i \in [x, y, z]$

Preliminary parameters. Convergence studies for optimal fitting parameters are in preparation.



Williams fitting - summary

Williams coefficients a_n | Williams coefficients b_n | Williams coefficients c_n



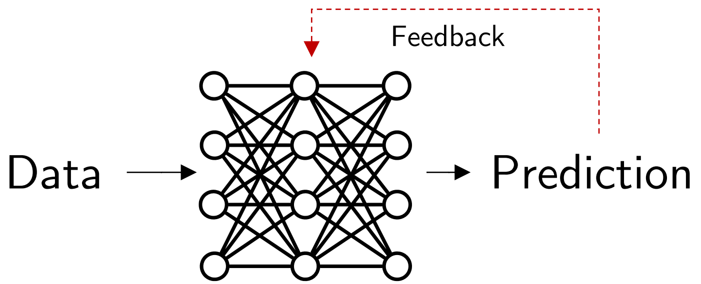
Intermediate research question: How does the Williams field relate to the crack front angle?

$$\cancel{\varphi(a_x, t, \mathbb{C}, K_i, \sigma, R, \eta, \theta, \ldots)} \in [-\pi, +\pi] \quad \Rightarrow \quad \varphi(a_n, b_n, c_n) \in [-\pi, +\pi]$$

Symbolic regression - introduction

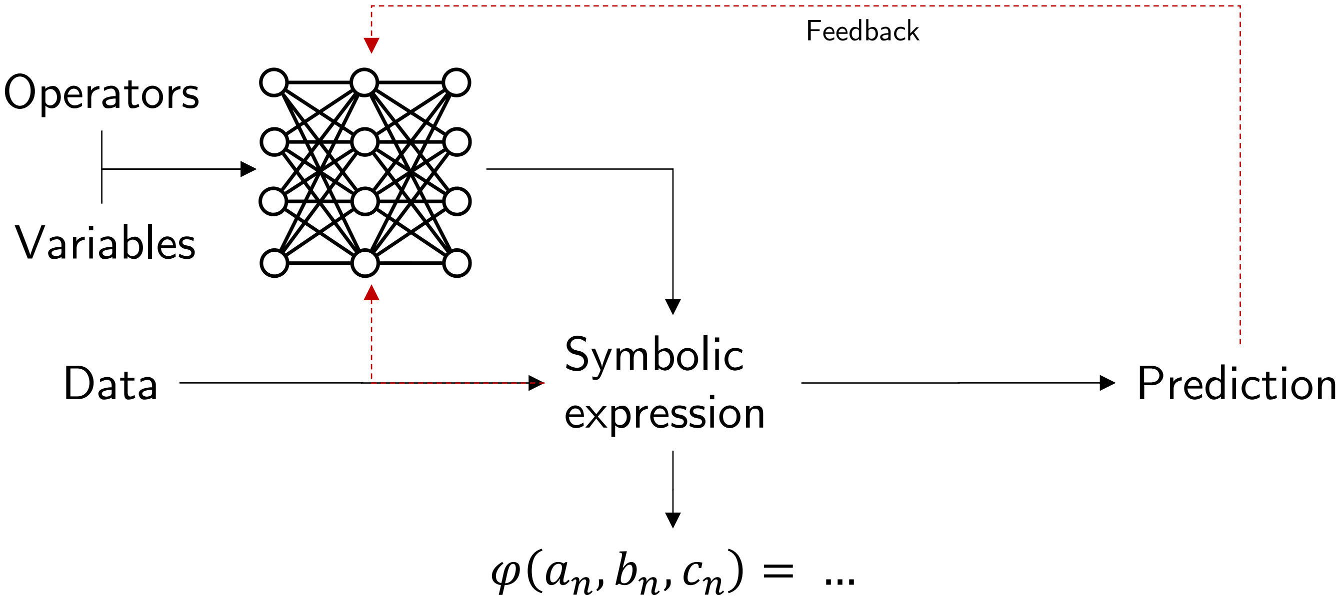
Machine learning	Symbolic regression (part of ML)

Train a model f to predict values from data



- Usually opaque black-box
- Not interpretable
- Rarely transferable to new domains
- Prone to overfitting / poor generalization

Train a model f to predict functions of varying complexity that fit the data



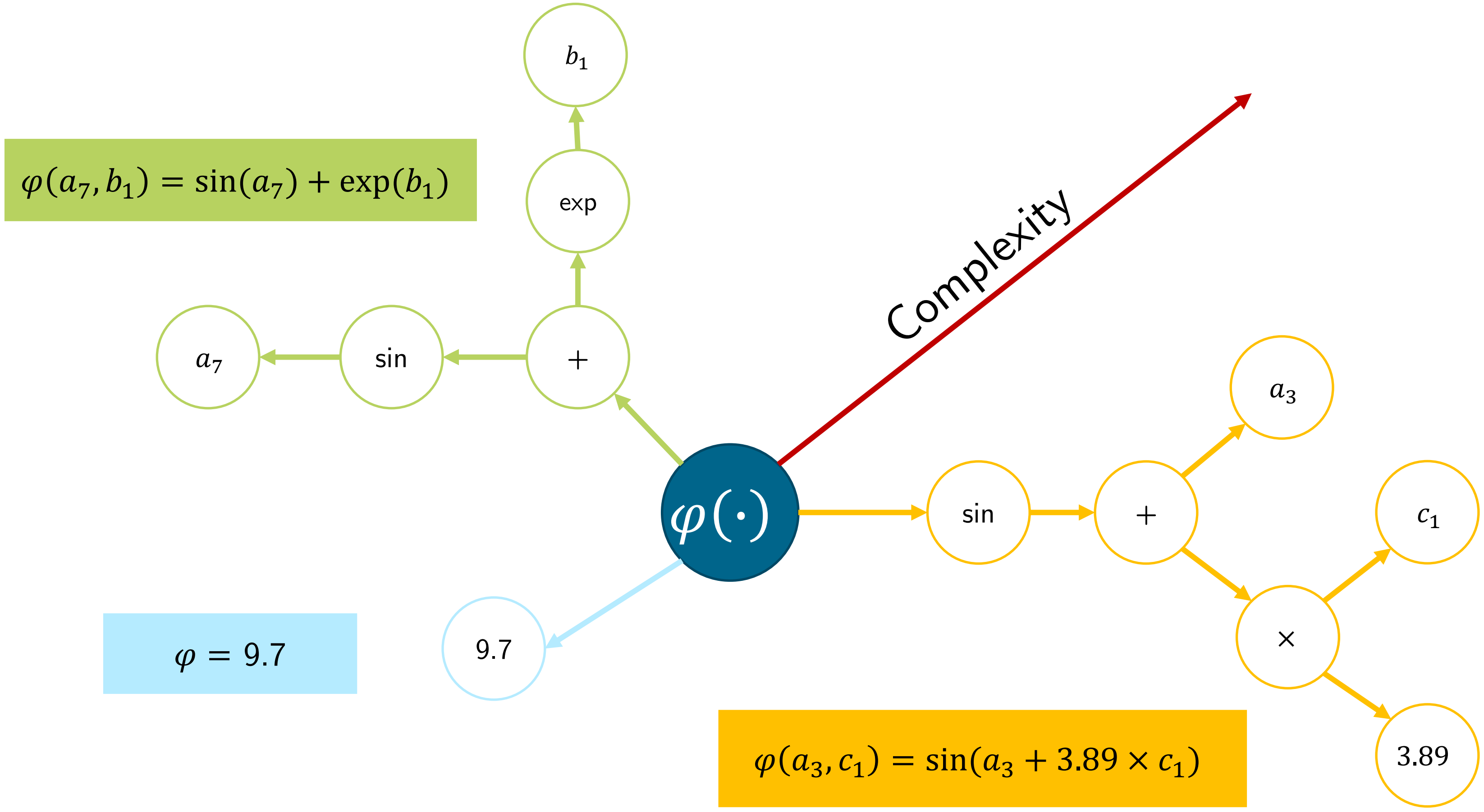
- Interpretable
- Potentially analytically derivable from known expressions
- Potentially general (within assumptions)

Symbolic regression - introduction

Operators
$\sin(\cdot)$
$\cos(\cdot)$
$\exp(\cdot)$
$\log(\cdot)$
$\cdot + \cdot$
$\cdot - \cdot$
const.
...

Variables
a_n
b_n
c_n

Expression Tree(s)



Symbolic regression – setup

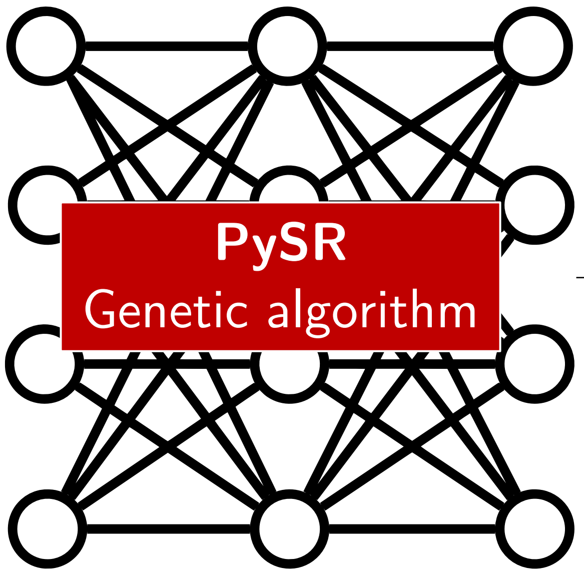
Operators

$\sin(\cdot)$, $\cos(\cdot)$, $\exp(\cdot)$, $\log(\cdot)$, $\mathrm{sqrt}(\cdot)$

$\cdot + \cdot$, $\cdot - \cdot$, $\cdot \times \cdot$, $\cdot \div \cdot$, const.

Variables

a_{-1}	b_{-1}	b_{-1}
a_0	b_0	c_0
a_1	b_1	c_1
…	…	…



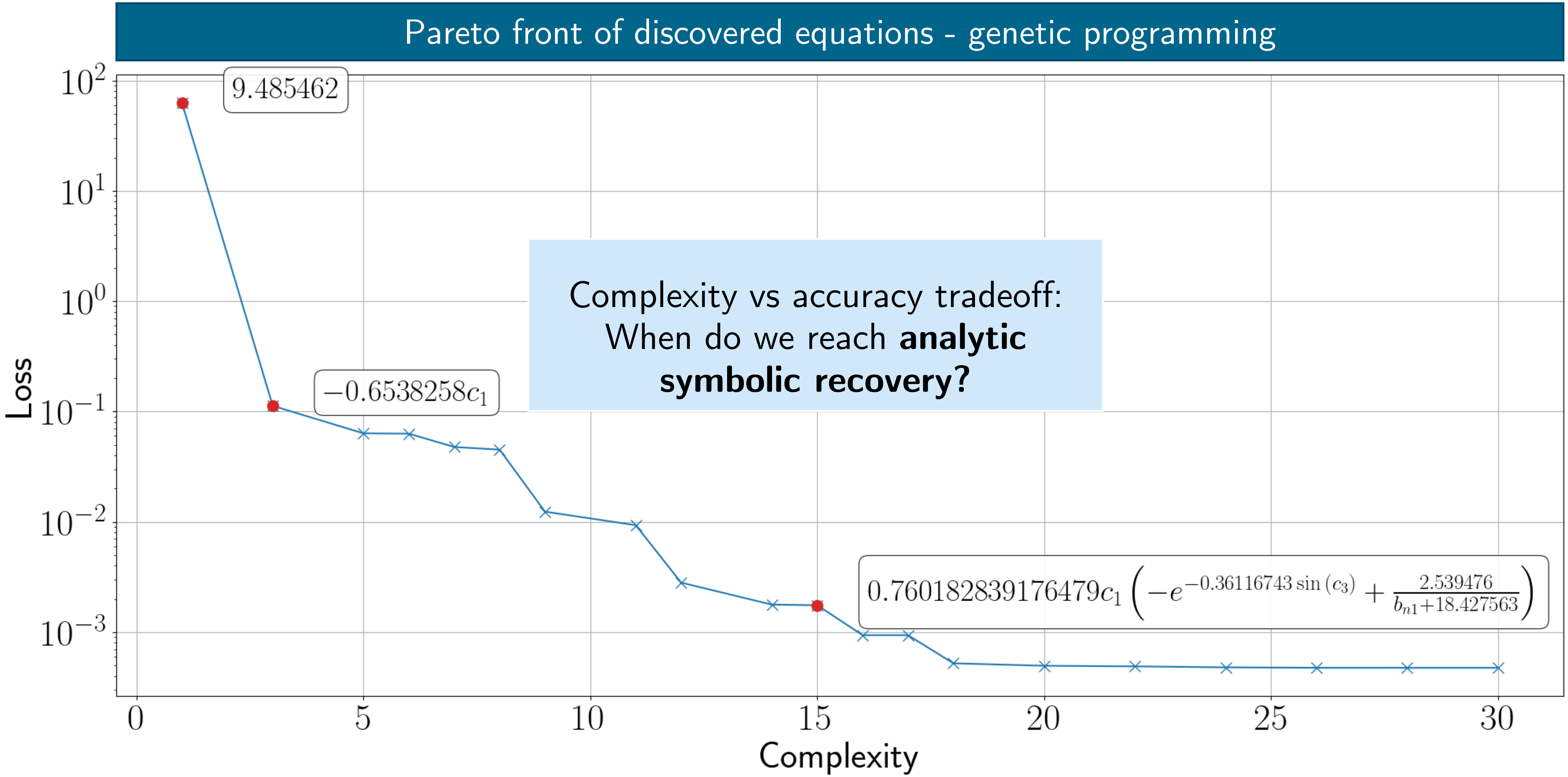
$$L_2(\hat{\varphi}, \varphi) = \sum_{i=1}^{N} (\varphi_i - \hat{\varphi}_i)^2$$

$$\varphi(a_n, b_n, c_n) = \ldots$$

Dataset

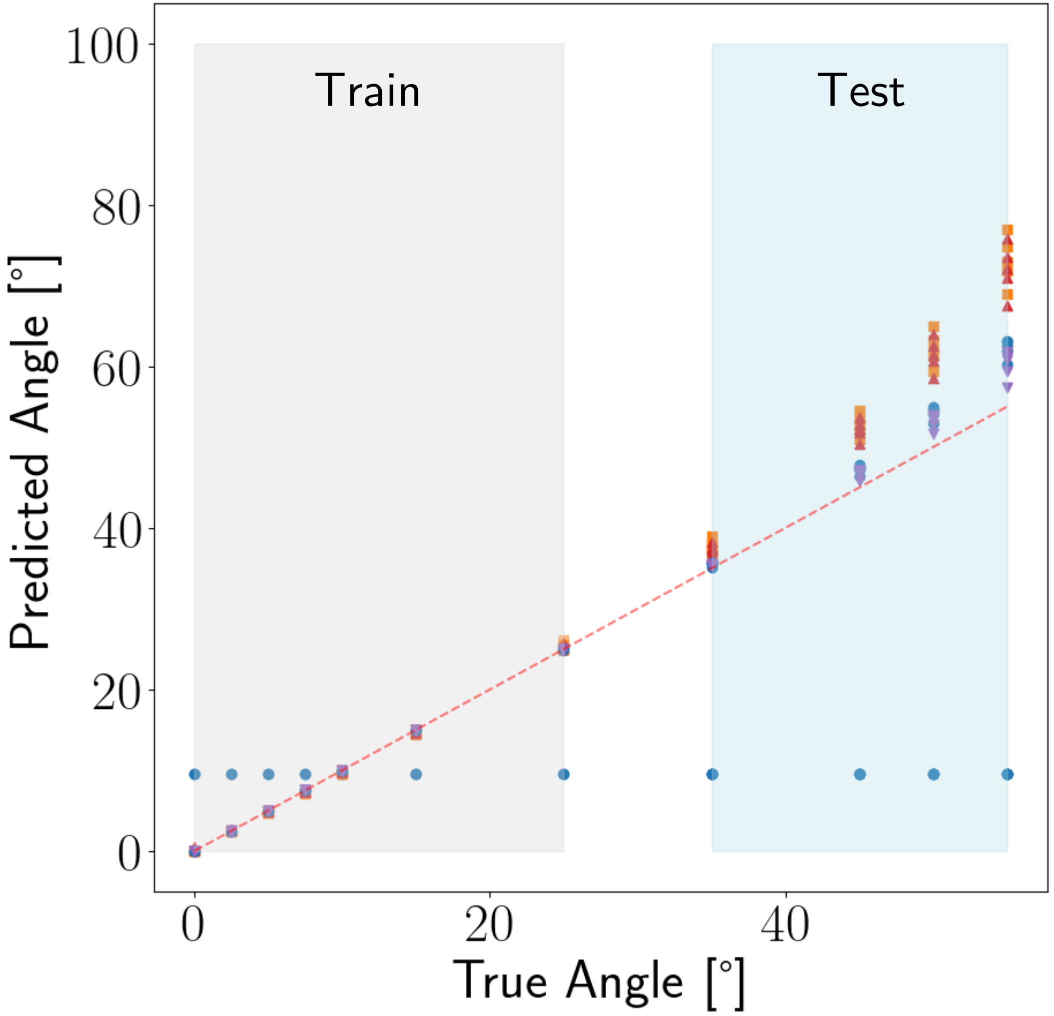
$\hat{\varphi}$	a_{-1}	…	a_9	b_{-1}	…	b_9	c_{-1}	…	c_9
0	3.64	…	4e-9	-0.01	…	-1e-9	0.11	…	2e-9
0	1.88	…	-3e-7	4e-4	…	2e-9	-0.02	…	1e-9
…	…	…	…	…	…	…	…	…	…
35	7.18	…	-2e-7	35.72	…	-3e-7	-83.7	…	3e-7

Symbolic regression – pareto optima



Symbolic regression – performance

Symbolic regression fits



Sampled equations

Eq. 1

$$\varphi = \text{const.} = 9.49$$

Eq. 2

$$\varphi = -0.65\, c_1$$

Eq. 3

$$\varphi = -0.65\, c_1 - \sin(c_2)$$

Eq. 10

$$\varphi = -0.76\, c_1 \left(-e^{-0.4\sin(c_3)} + \frac{2.5}{b_{-1} + 18.4}\right)$$

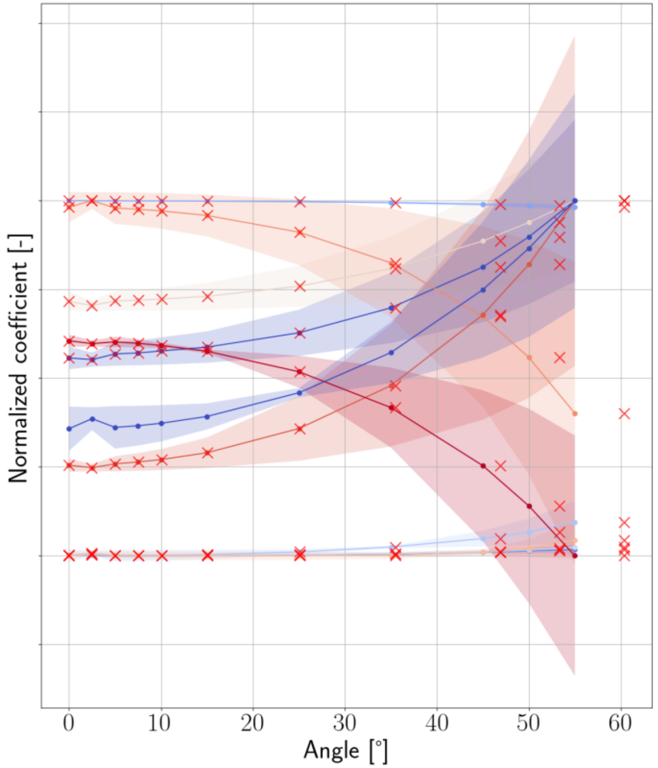
Eq. 19

$$\varphi = -0.47\, b_4 + c_1 \left(0.47 b_4 + 0.24 c_3 - 0.72 + \frac{0.72}{b_{-1} + \sqrt{c_0}}\right) + c_3 - c_4 + \cdots$$

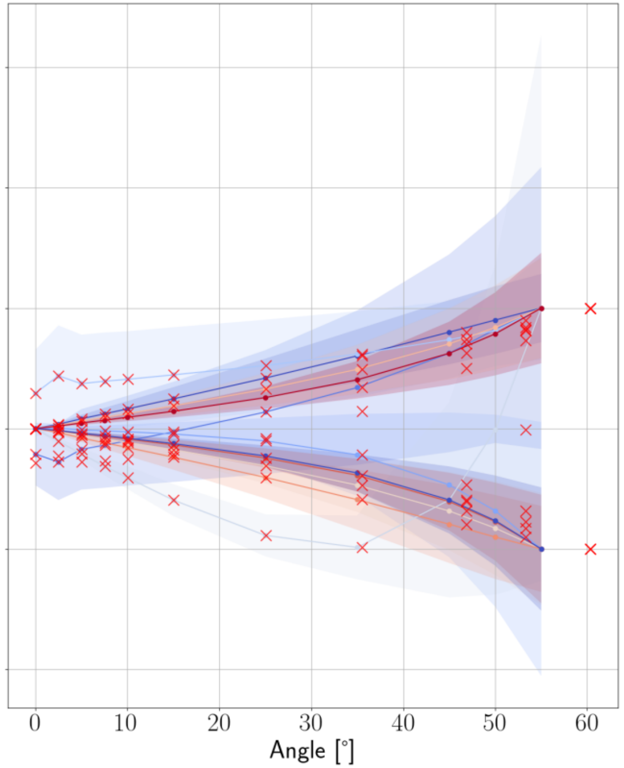
Model appears to distinguish between relevant and irrelevant coefficients and fits nicely even for lower complexity equations

Symbolic predictions – Eq. 19

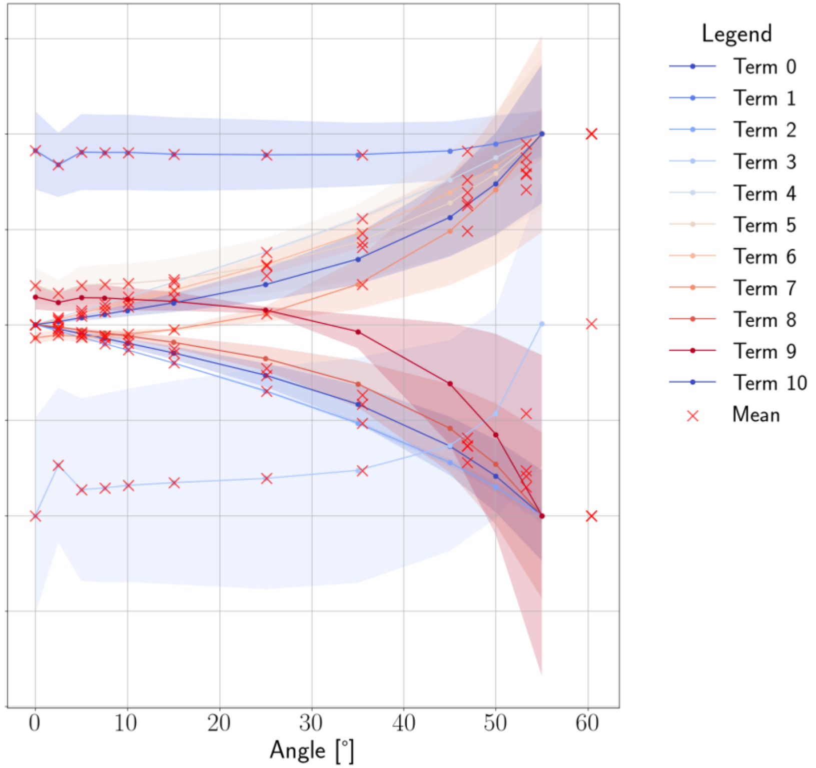
Williams coefficients a_n



Williams coefficients b_n



Williams coefficients c_n



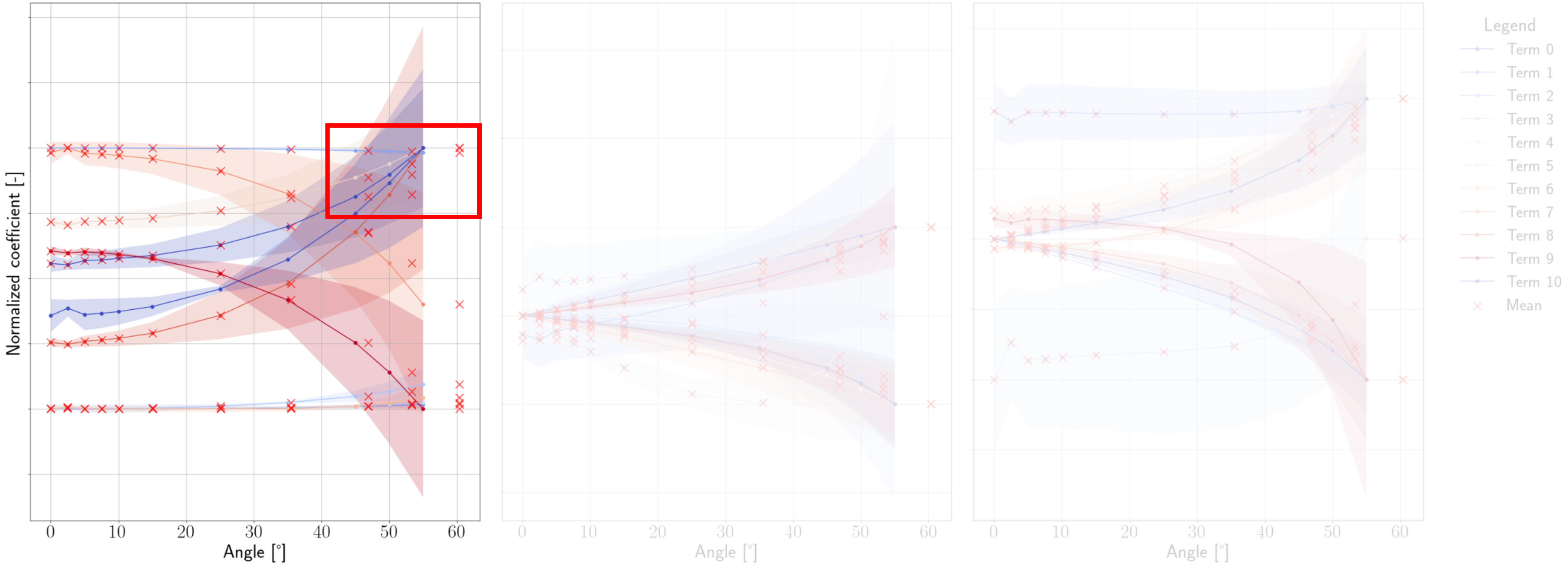
The discovered equation largely is able to predict the correct angles

Symbolic predictions – Eq. 19

Williams coefficients a_n

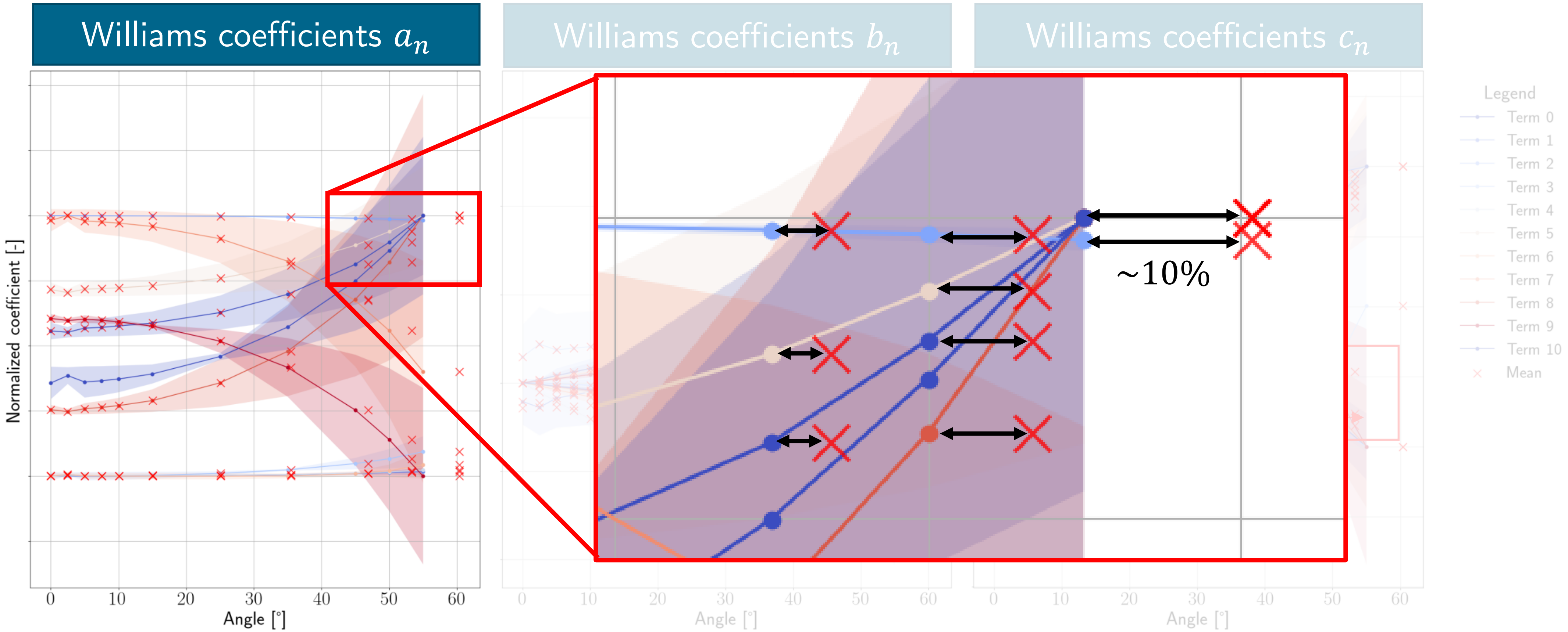
Williams coefficients b_n

Williams coefficients c_n



There is a remaining offset.
likely no full symbolic recovery → Improve symbolic regression approach

Symbolic predictions – Eq. 19



There is a remaining offset.
likely no full symbolic recovery → Improve symbolic regression approach

Next steps

FE Model	Symbolic regression	Experimental data
Hexahedral elements	Improve (Williams) series coefficient fitting	MT160 for varying thickness and varying R ratios
Add kink angle	Explore DL based approaches	AA2024-T3 and AA7075-T651
More sophisticated material model	Add physical priors (e.g. via units)	Validate models on microscope DIC data
	Explore uncertainty and noise	**Determine more material characteristics that can be essential for the symbolic regression task**
	Analytic correctness along pareto front	

Thank you for your attention and feedback!

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