

Vibrational similarities in jamming-unjamming of polycrystalline and disordered granular packings

Juan C. Petit^{1,*}, Saswati Ganguly,¹ and Matthias Sperl^{1,2}

¹*Institute of Materials Physics in Space, German Aerospace Center (DLR), 51170 Köln, Germany*

²*Institut für Theoretische Physik, Universität zu Köln, 50937 Köln, Germany*



(Received 5 November 2024; accepted 18 June 2025; published 8 July 2025)

We investigate the vibrational properties of polycrystalline monodisperse and disordered bidisperse granular packings during jamming and unjamming using discrete element method simulations. Both systems deviate from Debye scaling at low frequencies (ω), but only bidisperse packings exhibit a low- ω plateau. The low ω exponent (α) in bidisperse packings evolves smoothly from zero (plateau) to near one (Debye scaling) with increasing packing fraction, whereas in polycrystalline packings, it changes discontinuously near jamming/unjamming, due to the nature of the contact network rearrangements. Despite structural modifications during the compression-decompression cycle, the exponent remains unchanged at the same distance from jamming density, regardless of the history. Nonaffine displacements and contact orientational order further confirm that structural features that impact low- ω vibrational states and, hence, mechanical properties are largely restored upon decompression, reinforcing vibrational similarities between jamming and unjamming states.

DOI: [10.1103/yk2h-zr63](https://doi.org/10.1103/yk2h-zr63)

I. INTRODUCTION

The phenomenon of jamming in granular media is critical for understanding the transition between fluidlike and solidlike states in granular packings [1–11]. This transition is not merely a theoretical concern; it has significant implications for practical applications across various fields, including material science, geophysics, industrial processes, and cell biology [12–19]. As granular systems undergo jamming, they acquire rigidity, a property traditionally associated with crystalline solids, which exhibit long-range order [20,21]. However, disordered solids, which lack this long-range order, also resist external stresses, challenging our conventional understanding of rigidity and mechanical stability [22].

Previous theoretical and experimental studies have explored the mechanical transition of monodisperse crystals to disordered solids [9,11,23,24]. In three-dimensional systems, a key feature of this transition is the emergence of a coexistence region in the mechanical coordination number (Z), appearing as a plateau from $\phi_J \approx 0.64$ to $\phi \approx 0.68$ at $Z \approx 6$, marking the isostaticity of the disordered structure before it transitioned to an ordered state [23]. Supporting this, studies have identified intermediate polycrystalline states with disorderedlike mechanical properties near jamming, which evolve into crystalline behavior under high pressure [9]. This coexistence region between ordered and disordered packings has

been confirmed experimentally and validated through simulations, providing strong evidence of a structural transition [24]. Furthermore, simulations in both two and three dimensions reveal an order-disorder transition displaying an intermediate polycrystalline phase with distinct mechanical properties [11]. Beyond mechanical behavior, the vibrational density of states (DOS) also reflects these structural changes. In a 2D triangular crystal at a fixed packing fraction ϕ , the DOS is highly sensitive to increasing disorder. As disorder grows, it transitions from a crystalline profile (low disorder) to a polycrystalline form (intermediate disorder) and eventually to a fully disordered structure (high disorder). These findings demonstrate that packing structure is directly mirrored in the DOS, particularly at low frequencies, where soft modes, including the boson peak, become dominant.

Although the effect of disorder on the mechanical properties of both polycrystalline and disordered packings has been explored [3,9–11], the change in vibrational behavior during jamming, where particle contacts quickly increase at a certain density, and during unjamming, where particle contacts suddenly decrease at a similar density, has not been fully studied, especially when comparing these two states in a compression-decompression cycle. A compression-decompression protocol can significantly alter the packing structure, thereby impacting the mechanical properties of the system [5,24,25]. These structural changes are expected to be directly reflected in the DOS, capturing the influence of packing modifications on vibrational modes and mechanical behavior. Examination of the difference in vibrational behavior of the packing near jamming and unjamming states, with similar packing fractions but different structural arrangements, can offer important conceptual insights. Most research focuses on either jamming or unjamming, often overlooking the interplay between the two. Comparing polycrystalline monodisperse and disordered

*Contact author: Juan.Petit@dlr.de

bidisperse structures helps clarify whether the soft vibrational modes arise solely from disorder or if they also emerge in systems with partial order (polycrystals). This distinction offers deeper insights into the structural features that govern mechanical behavior.

Our objective is to investigate how the vibrational behavior of two-dimensional (2D) polycrystalline monodisperse and disordered bidisperse packings evolve along the jamming and unjamming path, with a focus on how structural changes affect the corresponding DOS. 2D systems are easier to replicate in experimental setups, including microgravity, facilitating various empirical and fundamental studies that validate our theoretical perspective [26,27]. Insights from simulations of compression and decompression during the jamming-unjamming transition in 2D granular solids can deepen our understanding of seismic wave propagation [26], acoustic wave behavior [28,29], soil compaction, and granular flow in industrial applications [30,31].

II. SIMULATION PROCEDURE

Simulations using the 2D discrete element method are performed with the software MercuryDPM [32–34]. Monodisperse and bidisperse packings are formed by $N = 3000$ particles. The bidisperse packing has a number of large, N_L , and small, N_S , particles with radius r_L and r_S . The size ratio is $\delta = r_S/r_L = 0.73$ with a concentration of small particles of $X_S = N_S\delta^2/(N_L + N_S\delta^2) = 0.5$. Therefore, $\delta = 0$ represents the monodisperse packing. $\delta = 0.73$ and $X_S = 0.5$ are chosen to prevent long-range order [1,2,35]. Simulations are performed using an isotropic compression-decompression protocol with a linear, frictionless contact model. Periodic boundary motion is applied smoothly and symmetrically in all directions using a cosine-based strain profile, ensuring continuous deformation and minimal inertial effects (see Sec. I of the Supplemental Material [36] for details). The system is first compressed from an initial packing fraction $\phi_0 = 0.80$ up to a maximum value $\phi_{\max} = 0.95$, then decompressed back to ϕ_0 at the same rate. The mean volumetric strain rate during the compression-decompression protocol, $\langle \dot{\epsilon}_v \rangle \sim 10^{-4}$, is approximately 100 times higher than in previous work [5]. Despite this, the protocol consistently yields polycrystalline monodisperse and disordered bidisperse packings. Mechanical equilibrium is maintained throughout, with a stable kinetic-to-potential energy ratio of $\sim 10^{-4}$ near the jamming transitions. This process allows for the extraction and analysis of structural variables to assess how monodisperse and bidisperse packings respond to compression and decompression, and their impact on vibrational behavior during the jamming-unjamming transition.

III. RESULTS

A. Jamming and unjamming transitions

Figures 1(a)–1(d) illustrate the fraction of particles in contact, $n_q = N_q^c/N_q$, and the mean contact number, $\langle Z_q \rangle = \sum_{i=1}^{N_q} Z_i^q/N_q$, as a function of relative packing fraction, $\hat{\phi} = \phi - \phi_{\max}$. Here, $q \in [L, S]$, Z_i^q is the number of contacts for the i th large (L) or small (S) particle, and N_q^c is the total

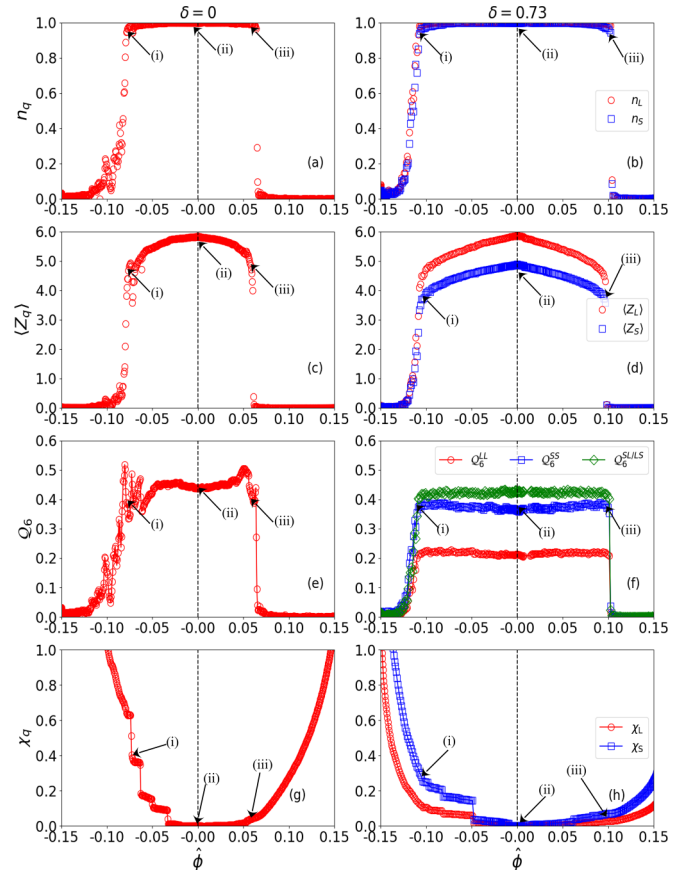


FIG. 1. (a), (b) Fraction of particles in contact, n_q ; (c), (d) mean contact number, $\langle Z_q \rangle$; (e), (f) contact orientational order, Q_6 ; and (g), (h) the cumulative nonaffine parameter, X_q , as a function of $\hat{\phi}$ for monodisperse (left panel) and bidisperse packings (right panel) as the systems are compressed to ϕ_{\max} and decompressed. For the bidisperse packing, the $\delta = 0.73$ and $X_S = 0.5$. Here, $q \in [L, S]$, except in the monodisperse case where $n_L = n_S$ by definition. X_q is intentionally scaled by 1000 to enhance the readability of the y axis. The arrows indicate (i) near jamming, (ii) ϕ_{\max} , and (iii) near unjamming. Particle configurations on these states are shown in Fig. 2. The dashed line corresponds to $\hat{\phi} = \phi - \phi_{\max} = 0$ or $\phi = \phi_{\max}$.

contact number for large or small particles. $\hat{\phi}$ distinguishes between compression and decompression branches, with compression defined as $\hat{\phi} = \phi - \phi_{\max} \leq 0$ and decompression as $\hat{\phi} = \phi_{\max} - \phi > 0$. For monodisperse and bidisperse packings, n_q and $\langle Z_q \rangle$ exhibit a sharp increase upon compression, with $n_q \approx 1$ and $\langle Z_q \rangle \sim 4 - 5$, at a specific $\hat{\phi}$ consistent with the jamming density. As $\hat{\phi} \rightarrow 0$, n_q and Z_q approaches their respective maximum values of 1 and 6. During decompression, n_q and Z_q drop discontinuously at a different $\hat{\phi}$ than near jamming, indicating the transition to an unjammed structure.

The jamming (ϕ_J) and unjamming (ϕ_{uJ}) densities are determined using the five-point finite difference method [$\sim O(\Delta\phi^4)$] to compute the derivative of the particle fraction, n_q , with respect to ϕ . The method provides a pronounced peak at ϕ_J/ϕ_{uJ} , indicating a rapid shift in particle arrangement that marks the onset of either a jammed or unjammed structure, see Sec. II in [36]. The extracted jamming and

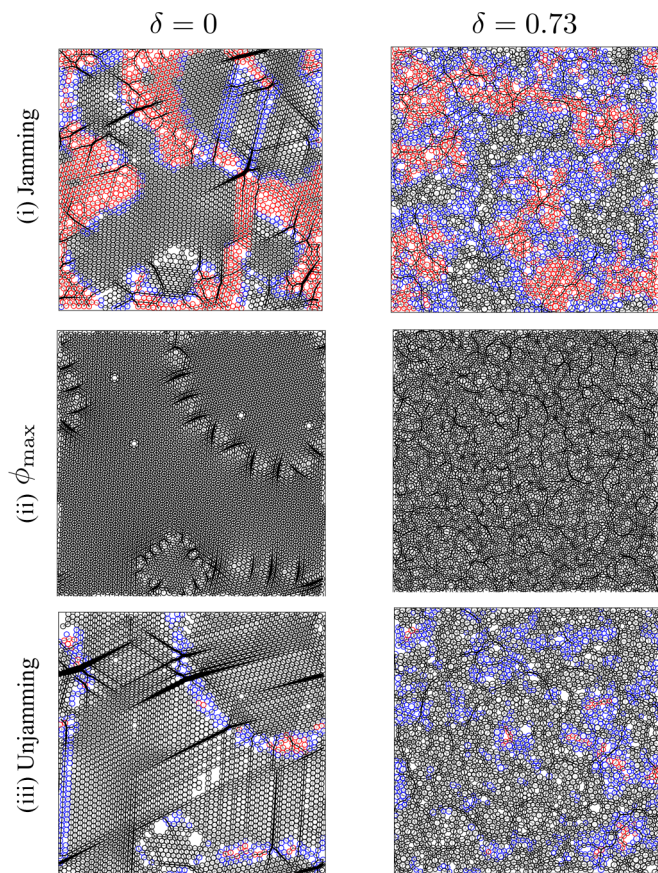


FIG. 2. Particle configurations and force chains for monodisperse ($\delta = 0$) and bidisperse ($\delta = 0.73, X_S = 0.5$) packings are shown near jamming (i), at ϕ_{\max} (ii), and near unjamming (iii), consistent with Fig. 1. The width of the force chain lines corresponds to normal forces greater than $2\langle F_n \rangle$, ensuring the visibility of the particle colors. Black, blue, and red circles represent the nonaffine motion of the i th particle, corresponding to low ($\chi_i < 1$), intermediate ($1 \leq \chi_i < 5$), and high ($\chi_i \geq 5$) values, respectively.

unjamming densities for monodisperse packing are $\phi_J \approx 0.87$ and $\phi_{uJ} \approx 0.88$. These values are consistent with the formation of some crystallization inside the packing but far from the triangular lattice structure of ~ 0.90 observed in Ref. [3]. Polycrystals are already formed near the jamming density (i), remain intact at ϕ_{\max} (ii) after particle rearrangement during further compression, and persist even near the unjamming state (iii). Figure 2 reveals polycrystalline structures with inhomogeneous force chains, where regions of high nonaffine particle motion (colored red, $\chi_i \geq 5$) are primarily localized near grain boundaries, delineating distinct local structural domains and highlighting deviations from affine deformation. For the bidisperse packing, the unjamming density is $\phi_{uJ} \approx 0.84$, which is 0.5% higher than ϕ_J . This value agrees with that reported for amorphous bidisperse mixtures in Refs. [3,6]. The structure near jamming in bidisperse packings appears disordered, as indicated by the more uniform distribution of force chains. This disordered character becomes more evident at the maximum packing fraction, ϕ_{\max} , where regions of distinct degrees of nonaffine motion are more broadly distributed throughout the system; see the right panel

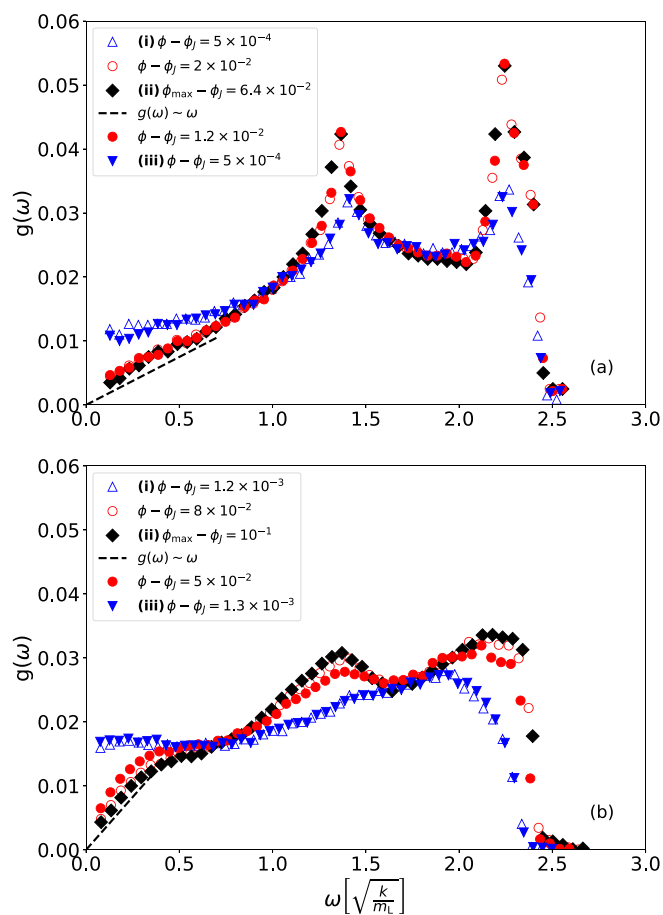


FIG. 3. The DOS, $g(\omega)$ versus ω , for (a) monodisperse and (b) bidisperse packings along the compression-decompression process. Open red and blue (Δ , \circ) symbols represent the system during the compression, whereas solid red and blue (∇ , \circ) represent the decompression at different distances from jamming/unjamming. $\phi_{\max} = 0.95$ is in black solid (\diamond). The Debye scaling law, $g(\omega) \sim \omega$, for 2D solids is represented in black dashed lines. The jamming and unjamming densities for monodisperse packing are $\phi_J \approx 0.87$ and $\phi_{uJ} \approx 0.88$, while for bidisperse packing, $\phi_J \approx 0.83$ and $\phi_{uJ} \approx 0.84$, respectively.

of Fig. 2. Such a structure evolves with smooth incremental changes along compression-decompression paths. Subtle structural changes in both packings during this process are also supported by the structure factors presented in Sec. III of the Supplemental Material [36].

B. Mechanical reponse during compression-decompression

The mechanical response of the packings under compression and decompression can be characterized by the density of states (DOS), which quantifies the distribution of vibrational modes as a function of frequency. It is defined as $g(\omega) = \frac{1}{N} \sum_{i=1}^N \delta(\omega - \omega_i)$, where ω_i is the frequency of the i th vibrational mode, typically obtained by diagonalizing the dynamical matrix \mathbf{D} , see Sec. IV of Ref. [36]. The DOS offers insight into the evolution of vibrational modes with changing packing density. Figure 3 shows the DOS for monodisperse and bidisperse packings at three key states: near jamming

(i), at $\phi_{\max} = 0.95$ (ii), and near unjamming (iii). Several intermediate cases are also included for comparison. During compression, typical van Hove singularities emerge at high frequencies near the jamming density in the monodisperse packing, as indicated by the open blue Δ in Fig. 3(a). This observation aligns with the polycrystalline structure depicted in Fig. 2(i) (left panel). In the bidisperse system, prominent van Hove singularities are not observed due to the intrinsic disorder introduced by the different particle sizes, see Fig. 3(b) and its corresponding configuration in Fig. 2(i) (right panel). With further compression, the van-Hove singularities in the monodisperse case become more pronounced and broadened. Whereas in the bidisperse packing, two similar peaks appear at high ω . The more pronounced case can be seen at ϕ_{\max} in Fig. 3. These features indicate increased order after compression, as shown in Fig. 2(ii) and Fig. 1. A key feature of the DOS near jamming is the plateau at low ω for bidisperse packing. This is known as the boson peak, characterized by an excess of vibrational modes [2,7–11]. The DOS for the polycrystalline packing does not display a low- ω plateau. However, it still deviates from the expected Debye scaling. This deviation suggests that the vibrational properties of the polycrystalline packing more closely resemble those of a disordered packing rather than a single crystal structure. As the system undergoes further compression, the DOS evolves (open red \circ) and, at ϕ_{\max} (solid \diamond), follows the well-known Debye law at low ω (black dashed line). This behavior suggests that polycrystalline monodisperse and disordered bidisperse packings behave as continuum elastic mediums in certain limits. Therefore, the disordered bidisperse structure shifts from behaving like disordered solids at low compression to resembling crystalline solid behavior at high compression, see Fig. 3. This transformation is particularly pronounced at ϕ_{\max} , where the emergence of peaks at specified values of ω (see Fig. 3), similar to those in the monodisperse case, suggests the development of some degree of local order. This observation is consistent with findings in 3D bidisperse packings, as noted in [24,25,37,38], where local order appears at the jamming density and becomes increasingly prominent with further compression.

During decompression, the DOS for both systems evolves (solid \circ), exhibiting behavior similar to that observed during compression near unjamming. Interestingly, despite the distinct local structural features characterized by nonaffine displacements [39] (see Fig. 2) at ϕ_J and ϕ_{uJ} values observed for monodisperse and bidisperse packings near the jamming and unjamming states, their corresponding density of states remains strikingly similar at these points (see Fig. 3). Even the intermediate DOS remains similar at comparable distances from the jamming/unjamming densities. A difference in the DOS is expected as the packing structure evolves during a complete cycle of compression and decompression. This structural change has been shown to affect the jamming density in previous work [5].

To better understand the evolution of the DOS during compression decompression, particularly at low frequencies, we fitted the low- ω region in Fig. 3 with a power law, $D(\omega) \sim \omega^\alpha$. The extracted exponent is shown in Fig. 4. For monodisperse packings, α remains low but nonzero near the jamming/unjamming density, see Fig. 4(a). As $\Delta\phi$

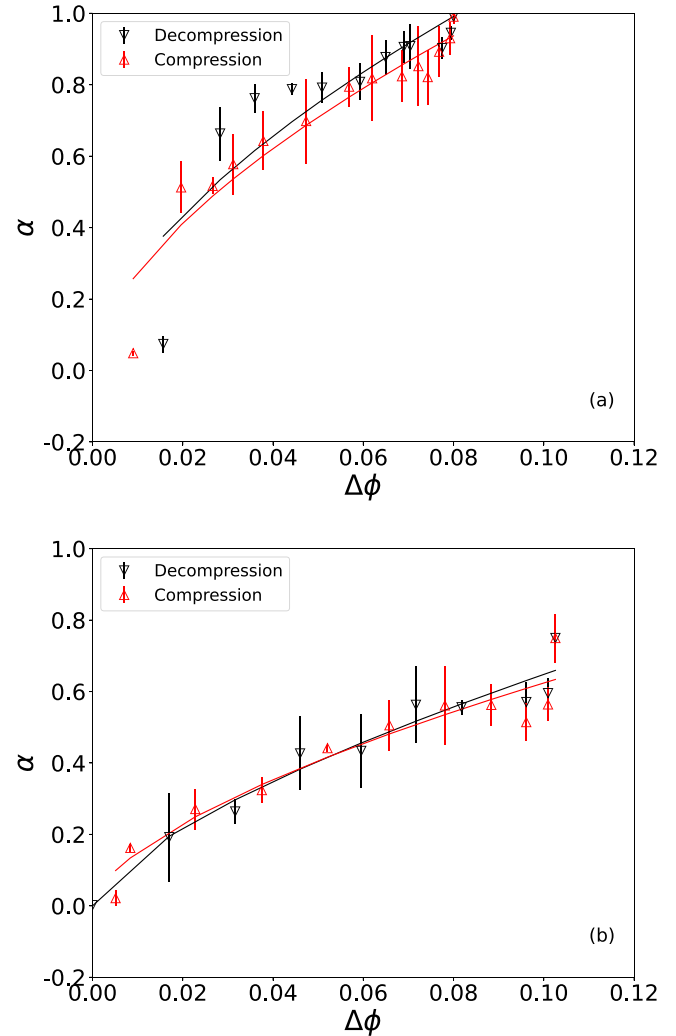


FIG. 4. Low- ω exponent α as a function of $\Delta\phi = \phi - \phi_{J,uJ}$. The exponent α is shown during compression and decompression for (a) $\delta = 0$ and (b) $\delta = 0.73$, extracted by fitting the low- ω region of the DOS in Fig. 3 to $D(\omega) \sim \omega^\alpha$. Continuous lines represent power-law fits $\alpha \approx \alpha_0 + A\Delta\phi^{\nu_{\text{mono}}}$ for monodisperse and $\alpha \approx \alpha_0 \Delta\phi^{\nu_{\text{bi}}}$ for bidisperse ones, with extracted exponents: $\nu_{\text{mono}} \approx 0.60$, $\nu_{\text{bi}} \approx 0.62$ during compression and $\nu_{\text{mono}} \approx 0.59$, $\nu_{\text{bi}} \approx 0.67$ during decompression. Error bars represent the variance from the mean across three independent realizations, quantifying statistical uncertainty.

increases, α undergoes a discontinuous change from the jamming/unjamming state, then rises continuously until reaching $\alpha = 1$ at high $\Delta\phi$, consistent with Debye scaling. In contrast, for bidisperse packings, α transitions smoothly from zero—indicating a plateau—near the jamming/unjamming density to $\alpha \approx 0.8$ at high $\Delta\phi$, approaching Debye scaling, see Fig. 4(b). The relationship between the low-frequency exponent α and the distance from the jamming or unjamming point, $\Delta\phi = \phi - \phi_{J,uJ}$, follows a power-law trend: $\alpha \approx \alpha_0 + A\Delta\phi^{\nu_{\text{mono}}}$ for monodisperse packings and $\alpha \approx \alpha_0 \Delta\phi^{\nu_{\text{bi}}}$ for bidisperse ones. The fitted exponents are $\nu_{\text{mono}} \approx 0.60$ and $\nu_{\text{bi}} \approx 0.62$ during compression, and $\nu_{\text{mono}} \approx 0.59$, $\nu_{\text{bi}} \approx 0.67$ during decompression.

The nonzero α near the jamming/unjamming transition in polycrystalline packings confirms the absence of a low- ω

plateau in the DOS while still deviating from Debye scaling. The discontinuity in α may be related to rearrangements near jamming/unjamming, as indicated by the change in mean contact number (from 0 to greater than 5) shown in Fig. 1(c) and localized nonaffine displacements of the particles around the grain boundaries (left panel of Fig. 2). In contrast, for bidisperse disordered packings, the smooth evolution of the DOS and α is accompanied by apparent differences in mean contact numbers between large and small particles, which arise due to variations in the rattler fraction rather than distinctions in the contact network. The absence of the plateau and the discontinuity in polycrystalline packings may also stem from the strain rate used ($\dot{\epsilon} \sim 10^{-4}$), far higher than the slower process performed in [9], where a plateau is observed for polycrystals. Therefore, this proves to be a more nuanced problem that requires additional simulations at varying strain rates to capture subtle changes in DOS in the low ω regime as the system undergoes fast or quasistatic compression-decompression cycles.

C. Structural response during compression-decompression

The evolution in the vibrational density of states suggests an underlying connection between structural rearrangements and the evolution of vibrational modes. To further investigate this relationship, we analyze the changes in the packing structure by computing the contact orientational order (COR), Q_6 , which quantifies the formation of potential sixfold lattice structures (see its definition in Sec. V in [36]). This variable avoids the ambiguities of the bond orientational order (BOR) and is more sensitive to sudden changes in contact numbers or other structural features compared to the structure factor, see Ref. [37]. Figures 1(e) and 1(f) display the evolution of Q_6 for monodisperse and bidisperse packings. Monodisperse packings exhibit variations in Q_6 after jamming due to particle rearrangement, stabilizing under further compression, see Fig. 1(e). Notably, Q_6 at ϕ_{\max} remains below 1, indicating that the system is not fully ordered and deviates significantly from the ideal value of $Q_6 = 1$ expected for perfect hexagonal packing [11]. During decompression, Q_6 slightly increases, suggesting some organization due to stored energy before dropping near unjamming (see Fig. 2). For bidisperse packings, we quantify local Q_6 values between large-large (LL), small-small (SS), and small-large (SL) particles, each defined in [36]. Figure 1(f) illustrates that SL contacts predominantly govern the evolving structure, while LL and SS contacts are slightly reduced near $\hat{\phi} = 0$. This reduction occurs because particles of the same size experience slight overlap at ϕ_{\max} , leading to distortions that introduce disorder into the packing. Interestingly, all partial Q_6 values near jamming (i) and unjamming (iii) are identical, see the arrows in Fig. 1(f). This suggests that while the bidisperse packing structure evolves during compressions, configurations are restored upon decompression, such that jamming and unjamming states exhibit similar values for the structural quantifiers like Q_6 . Identifying jamming and unjamming configurations in monodisperse packings in Fig. 1(c) is more challenging. Variations in Q_6 near jamming/unjamming further support the idea that rearrangements may be responsible for the absence of the plateau and the α discontinuity in polycrystalline systems. The arrows

in panels (i)–(iii) of Fig. 1(c) highlight similar local Q_6 values, consistent with the structure factor in [36]. Therefore, the preserved packing structure in both monodisperse and bidisperse systems near jamming/unjamming likely results in comparable vibrational and mechanical behavior. This is confirmed in Fig. 3 and reinforced by the low- ω DOS exponent in Fig. 4, which remains similar near jamming and unjamming, indicating a consistent structural influence on vibrational properties.

The local configurations near jamming and unjamming can also be explored by quantifying the nonaffine motion of particles during compression decompression and examining its influence on the low- ω behavior of the DOS in Fig. 3. Figures 1(g) and 1(h) depict the evolution of cumulative nonaffine displacements (\mathcal{X}_q) [39,40] as a function of $\hat{\phi}$ for both monodisperse and bidisperse packings, with $q \in [L,S]$ (see Sec. VI of [36]). \mathcal{X}_q consistently decreases during compression showing a sudden drop near jamming for monodisperse packing, whereas for bidisperse, packing is continuous. The sharp decrease in \mathcal{X}_q near $\hat{\phi} = 0$ in both systems indicates that an increase in density close to ϕ_{\max} significantly restricts the movement of nonaffine particles. Instead, the particles undergo affine displacements, indicating that the systems follow an elastic behavior. During decompression, the nonaffine motions increase again, but are more continuous, showing fewer discontinuous jumps. Since discontinuous changes in \mathcal{X} signal significant particle rearrangements [41], the absence of these discontinuities during decompression suggests that the overall packing structure at ϕ_{\max} is preserved. These findings align with the consistent Q_6 values in Figs. 1(e) and 1(f) across jamming and unjamming states and with the $S(q)$ data presented in [36].

D. Normal modes at jamming-unjamming

Nonaffine displacements are analyzed near jamming, at ϕ_{\max} , and near unjamming to assess their impact on the low- ω vibrational modes in polycrystalline and disordered packings. Eigenvectors for $\omega = 0.05$, which lie within the plateau region or deviate from Debye's law in the DOS, are extracted and shown in Fig. 5. Each eigenvector is color coded based on the magnitude of nonaffine displacement of each particle (χ_i): red for high ($\chi_i > 5$), blue for intermediate ($1 \leq \chi_i < 5$), and black for low ($\chi_i < 1$). At ϕ_{\max} , eigenvectors in polycrystalline monodisperse and disordered bidisperse packings display wavelike behavior with minimal nonaffine motion, reflecting macroscopic solid behavior consistent with Debye's law [see Figs. 5(b) and 5(e)]. In contrast, near jamming and unjamming, several localized modes emerge with varying nonaffine displacements, particularly in regions prone to rearrangements [see Figs. 5(a) and 5(c) for monodisperse and 5(d) and 5(f) for bidisperse packings). While Figs. 5(a), 5(c), 5(d), and 5(f) reveal differences in the local structures in the eigenvectors near jamming and unjamming, the low- ω DOS and the exponent α remain similar. In addition, rattlers (colored gray) appear in distinct clustered structures in monodisperse systems near jamming and unjamming, whereas in bidisperse packings, they are more dispersed and localized, further influencing the vibrational properties of each system in a distinct manner.

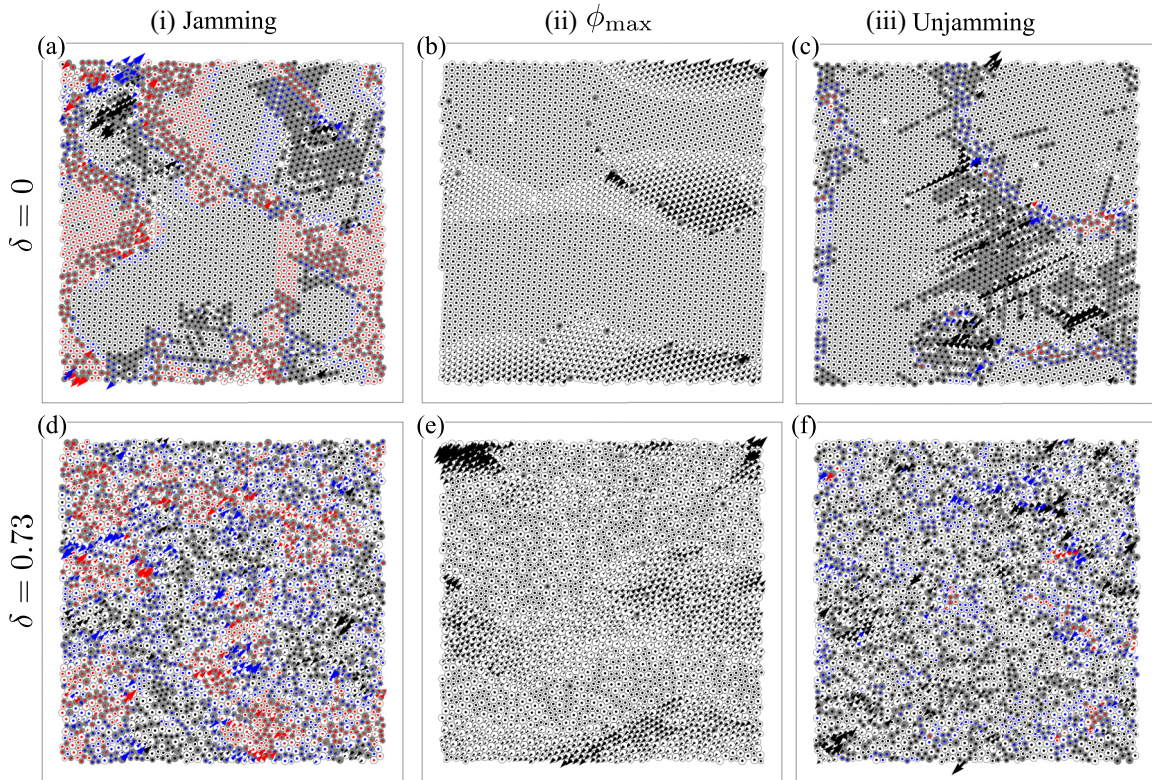


FIG. 5. Eigenvectors for $\omega = 0.05$. (Top panel) Eigenvectors of the monodisperse and (bottom panel) bidisperse systems associated with packing fractions near jamming (i), at ϕ_{\max} (ii), and near unjamming (iii). The displacement fields marked with red, blue, and black arrows indicate large ($\chi_i \geq 5$), intermediate ($1 \leq \chi_i < 5$), and low ($\chi_i < 1$) nonaffine particle values, respectively. Gray disks identify particles with less than four contacts (rattlers). Note the wavelike behavior of the particles at ϕ_{\max} consistent with the elastic nature of the system. Jamming and unjamming give rise to localized soft modes, which drive the deviation from Debye scaling in monodisperse packings and the emergence of a plateau in bidisperse packings, as observed in Fig. 3.

IV. DISCUSSION AND CONCLUSION

The results presented in this study provide insights into the similarities in the vibrational behavior of polycrystalline and disordered granular solids during jamming and unjamming transitions. Using discrete element method simulations, the research investigates the vibrational density of states (DOS) near jamming and unjamming, emphasizing deviations from Debye scaling and the role of nonaffine displacements. A key finding is that while bidisperse packings exhibit a low- ω plateau, polycrystalline packings do not, yet both systems deviate from Debye's law. The DOS for bidisperse packing smoothly evolves from zero (plateau) to near one (Debye scaling) which is captured by the low- ω exponent α . The presence of two particle sizes promotes a more uniform local structure, as reflected in the orientation order parameter Q_6 in Fig. 1(f) and the local nonaffine displacement shown in the right panel of Fig. 2. In contrast, for monodisperse packing, α exponent exhibits a non-smooth behavior near jamming/unjamming. Unlike in the bidisperse system, the polycrystalline structure exhibits nonaffine displacements (left panel of Fig. 2) that are more localized around grain boundaries, along with large rearrangements marked by a sudden change in Q_6 [Fig. 1(e)]. This may explain the absence of the plateau in this system. We found that α (see Fig. 4) remained consistent across

compression and decompression for similar values of packing fractions. Despite structural modifications occurring throughout the process, no significant history dependence is observed in the DOS, suggesting that vibrational properties are preserved across jamming and unjamming transitions. The study also highlights the role of structural features, such as the evolution of the contact network, in shaping the vibrational response. Analysis of nonaffine displacements and contact orientational order reveals that while structural changes occur during compression, features of the configurations contributing to the macroscale quantifiers are largely restored upon decompression, contributing to the vibrational similarities between jamming and unjamming states.

These findings point toward a connection between microstructural features—such as contact network rearrangements, orientational order, and nonaffine displacements—and the vibrational response of granular materials during jamming and unjamming. While the results highlight distinct behaviors between polycrystalline and disordered packings, particularly in the evolution of the low-frequency density of states, further work is needed to generalize these observations. Nonetheless, the trends observed here provide a promising basis for future investigations into the role of microstructure in governing mechanical stability in disordered solids. Future work will focus on extending these findings to a broader range of size ratios

and small particle concentrations, with particular emphasis on the first and second jamming transitions in bidisperse packings. In these systems, a discontinuous transition has been observed, separating jammed states dominated by large particles from those involving both large and small particles [4]. This extension of the current study will deepen our understanding of the complex interplay between particle size distribution and the jamming behavior in granular materials.

ACKNOWLEDGMENTS

We thank Till Kranz and Matthias Schröter, for proofreading, fruitful discussions, and providing constructive criticism

about results and the paper. This work was supported by the Deutscher Akademischer Austauschdienst (DAAD) under Grant No. 57424730. S.G. acknowledges support from Deutsche Forschungsgemeinschaft through Grant No. FU 309/11-1.

DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

-
- [1] T. S. Majmudar, M. Sperl, S. Luding, and R. P. Behringer, Jamming transition in granular systems, *Phys. Rev. Lett.* **98**, 058001 (2007).
- [2] C. S. O'Hern, L. E. Silbert, A. J. Liu, and S. R. Nagel, Jamming at zero temperature and zero applied stress: The epitome of disorder, *Phys. Rev. E* **68**, 011306 (2003).
- [3] A. Donev, S. Torquato, F. H. Stillinger, and R. Connelly, Jamming in hard sphere and disk packings, *J. Appl. Phys.* **95**, 989 (2004).
- [4] J. C. Petit, N. Kumar, S. Luding, and M. Sperl, Additional transition line in jammed asymmetric bidisperse granular packings, *Phys. Rev. Lett.* **125**, 215501 (2020).
- [5] N. Kumar and S. Luding, Memory of jamming–multiscale models for soft and granular matter, *Granul. Matter* **18**, 58 (2016).
- [6] D. Vågberg, P. Olsson, and S. Teitel, Glassiness, rigidity, and jamming of frictionless soft core disks, *Phys. Rev. E* **83**, 031307 (2011).
- [7] M. van Hecke, Jamming of soft particles: geometry, mechanics, scaling and isostaticity, *J. Phys.: Condens. Matter* **22**, 033101 (2010).
- [8] L. E. Silbert, A. J. Liu, and S. R. Nagel, Vibrations and diverging length scales near the unjamming transition, *Phys. Rev. Lett.* **95**, 098301 (2005).
- [9] C. P. Goodrich, A. J. Liu, and S. R. Nagel, Solids between the mechanical extremes of order and disorder, *Nat. Phys.* **10**, 578 (2014).
- [10] L. Zhang, Y. Wang, Y. Chen, J. Shang, A. Sun, X. Sun, S. Yu, J. Zheng, Y. Wang, W. Schirmacher, and J. Zhang, Disorder-induced vibrational anomalies from crystalline to amorphous solids, *Phys. Rev. Res.* **3**, L032067 (2021).
- [11] H. Tong, P. Tan, and N. Xu, From crystals to disordered crystals: A hidden order-disorder transition, *Sci. Rep.* **5**, 15378 (2015).
- [12] J. Amend, N. Cheng, S. Fakhouri, and B. Culley, Soft robotics commercialization: Jamming grippers from research to product, *Soft Robot.* **3**, 213 (2016).
- [13] I. Zuriguel, A. Garcimartín, D. Maza, L. A. Pugnaloni, and J. Pastor, Jamming during the discharge of granular matter from a silo, *Phys. Rev. E* **71**, 051303 (2005).
- [14] A. Jiang, Robotic Granular Jamming, Ph.D. thesis, King's College London, 2014.
- [15] I. R. Peters, J. M. Amundson, R. Cassotto, M. Fahnstock, K. N. Darnell, M. Truffer, and W. W. Zhang, Dynamic jamming of iceberg-choked fjords, *Geophys. Res. Lett.* **42**, 1122 (2015).
- [16] R. Kostynick, H. Matinpour, S. Pradeep, S. Haber, A. Sauret, E. Meiburg, T. Dunne, P. Arratia, and D. Jerolmack, Rheology of debris flow materials is controlled by the distance from jamming, *Proc. Natl. Acad. Sci. USA* **119**, e2209109119 (2022).
- [17] E. Lawson-Keister and M. L. Manning, Jamming and arrest of cell motion in biological tissues, *Cell Biol.* **72**, 146 (2021).
- [18] L. Atia, J. J. Fredberg, N. S. Gov, and A. F. Pegoraro, Are cell jamming and unjamming essential in tissue development? *Cells Dev.* **168**, 203727 (2021).
- [19] L. Oswald, S. Grosser, D. M. Smith, and J. A. Käs, Jamming transitions in cancer, *J. Phys. D: Appl. Phys.* **50**, 483001 (2017).
- [20] D. Forster, *Hydrodynamic Fluctuations, Broken Symmetry, and Correlation Functions* (Benjamin, Reading, Massachusetts, 1975).
- [21] P. C. Martin, O. Parodi, and P. S. Pershan, Unified hydrodynamic theory for crystals, liquid crystals, and normal fluids, *Phys. Rev. A* **6**, 2401 (1972).
- [22] J. P. Sethna, M. K. Bierbaum, K. A. Dahmen, C. P. Goodrich, J. R. Greer, L. X. Hayden, J. P. Kent-Dobias, E. D. Lee, D. B. Liarte, X. Ni, K. N. Quinn, A. Raju, D. Z. Rocklin, A. Shekhawat, and S. Zapperi, Deformation of crystals: Connections with statistical physics, *Annu. Rev. Mater. Res.* **47**, 217 (2017).
- [23] Y. Jin and H. A. Makse, A first-order phase transition defines the random close packing of hard spheres, *Physica A* **389**, 5362 (2010).
- [24] M. Hanifpour, N. Francois, V. Robins, A. Kingston, S. Vaez Allaei, and M. Saadatfar, Structural and mechanical features of the order-disorder transition in experimental hard-sphere packings, *Phys. Rev. E* **91**, 062202 (2015).
- [25] B. A. Klumov, Y. Jin, and H. A. Makse, Structural properties of dense hard sphere packings, *J. Phys. Chem. B* **118**, 10761 (2014).
- [26] D. Fischer, R. Stannarius, K. Tell, P. Yu, and M. Sperl, Force chains in crystalline and frustrated packing visualized by stress-birefringent spheres, *Soft Matter* **17**, 4317 (2021).
- [27] Y. Grasselli, G. Bossis, A. Meunier, and O. Volkova, Dynamics of a 2D vibrated model granular gas in microgravity, *Granul. Materials* **71** (2017).
- [28] K. Tell, Elastic wave propagation in granular packings, Dissertation, Universität zu Köln, 2020.

- [29] K. Tell, C. Dreißigacker, A. C. Tchapnda, P. Yu, and M. Sperl, Acoustic waves in granular packings at low confinement pressure, *Rev. Sci. Instrum.* **91**, 033906 (2020).
- [30] J. Müller, B. Rattenbacher, K. Tell, C. Rösch, T. Welsch, M. Maurer, M. Sperl, and M. Schnellenbach-Held, Space hardware for concrete sample production on ISS “MASON concrete mixer”, *npj Microgravity* **9**, 57 (2023).
- [31] O. D’Angelo, M. Sperl, and W. T. Kranz, Rheological regimes in agitated granular media under shear, *Phys. Rev. Lett.* **134**, 148202 (2025).
- [32] P. A. Cundall and O. D. Strack, A discrete numerical model for granular assemblies, *Géotechnique* **29**, 47 (1979).
- [33] T. Weinhart, L. Orefice, M. Post, M. P. van Schrojenstein Lantman, I. F. Denissen, D. R. Tunuguntla, J. Tsang, H. Cheng, M. Y. Shaheen, H. Shi *et al.*, Fast, flexible particle simulations—an introduction to MercuryDPM, *Comput. Phys. Commun.* **249**, 107129 (2020).
- [34] J. C. Petit, N. Kumar, S. Luding, and M. Sperl, Bulk modulus along jamming transition lines of bidisperse granular packings, *Phys. Rev. E* **106**, 054903 (2022).
- [35] P. K. Morse and E. I. Corwin, Geometric order parameters derived from the Voronoi tessellation show signatures of the jamming transition, *Soft Matter* **12**, 1248 (2016).
- [36] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/yk2h-zr63> for details of the simulation algorithms and definitions.
- [37] J. C. Petit and M. Sperl, Structural transitions in jammed asymmetric bidisperse granular packings, *Granul. Matter* **25**, 43 (2023).
- [38] A. S. Clarke and H. Jónsson, Structural changes accompanying densification of random hard-sphere packings, *Phys. Rev. E* **47**, 3975 (1993).
- [39] S. Ganguly, S. Sengupta, P. Sollich, and M. Rao, Nonaffine displacements in crystalline solids in the harmonic limit, *Phys. Rev. E* **87**, 042801 (2013).
- [40] S. Ganguly, S. Sengupta, and P. Sollich, Statistics of non-affine defect precursors: tailoring defect densities in colloidal crystals using external fields, *Soft Matter* **11**, 4517 (2015).
- [41] S. Ganguly, P. S. Mohanty, P. Schurtenberger, S. Sengupta, and A. Yethiraj, Contrasting the dynamics of elastic and non-elastic deformations across an experimental colloidal Martensitic transition, *Soft Matter* **13**, 4689 (2017).