



Wake vortex lidar measurement processing with large-eddy simulations and machine learning

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Abstract: Aircraft generated wake vortices may pose a hazard to following aircraft during final approach. Light detection and ranging (lidar) instruments and appropriate processing algorithms are employed for characterizing the position and strength of wake vortices. Unavailable ground-truths in field measurements limit obtainable accuracies of processing algorithms. In this paper the employment of lidar simulations with known ground-truths from large-eddy simulations enhanced by a machine learning based lidar processing algorithm is proposed. This processing algorithm generalization offers wake vortex detection rates above 90% and practical characterization accuracies for comparable simulation and field measurement datasets, decreasing the need for the latter.

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1. Introduction

The study of wake vortices, rotations of air directly produced as a consequence of aircraft lift generation, constitutes an unavoidable component for enhancing runway throughput at airports whilst ensuring safe operations [1]. Field measurements, computational fluid dynamics, as well as deterministic and probabilistic fast-time prediction models are tools to reach the unified goal of determining and/or adjusting the life, strength, and path of the wake vortices. Factors influencing the characteristics of wake vortices spread from the aircraft geometry to the prevailing atmospheric conditions and ground topography prevailing at the landing site.

A wake vortex encounter occurs when a follower aircraft flies into a wake vortex produced by a generator aircraft flying ahead. Encounters are most common and dangerous during the final approach and landing phase of an aircraft, as aircraft share similar glide paths and their low altitude does not allow for major piloting maneuvers [2]. Also emerging air vehicles, such as air taxis, may experience wake vortex encounters when a vertiport is installed in runway vicinity. For studying or operationally monitoring wake vortex evolutions at airports light detection and ranging (lidar) instruments, in recent years particularly pulsed coherent Doppler lidars (PCDLs), are employed next to the glide path of approaching aircraft. In particular, PCDLs (used interchangeable with lidar in this paper) with a laser wavelength of $\lambda \approx 1.5 \mu\text{m}$ are most modern, delivering high spatial resolution and highly accurate line of sight (LOS) velocity measurements. Most wake vortex related undertakings involve lidar measurements at some stage during their development (for example the plate line concept in Ref. [3]) or during the approval process by authorities. Lidar measurements consist of LOS velocities to and from the instrument by evaluating the frequency shift between the emitted laser beam and the echo signal backscattered by aerosols. A range height indicator (RHI) scan geometry is sketched in Fig. 1, delivering a two-dimensional velocity field (LOS velocities, V_r) perpendicular to the ground, characterized by the azimuth angle perpendicular to the runway, elevation angle φ , and radial distance r , where

discrete distances are given by R . The primary counter-rotating wake vortex pair generated by the overflying aircraft is given by the starboard (Str) and port (Prt) vortices.

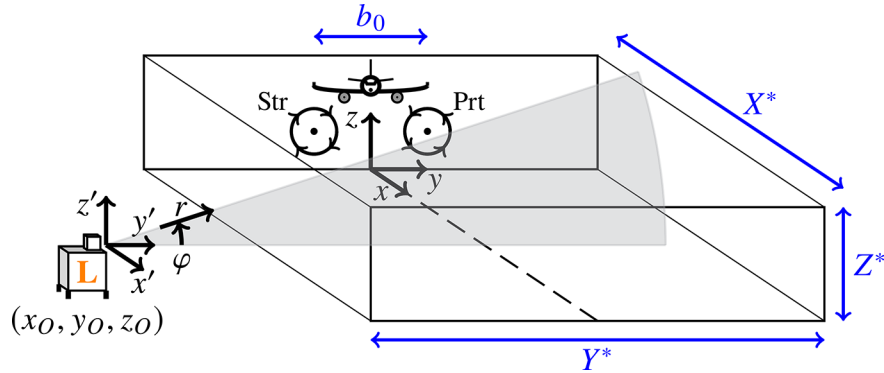


Fig. 1. Lidar at (x_O, y_O, z_O) in the coordinate system $[x, y, z]$ measuring a RHI lidar scan (gray) with starboard (Str) and port (Prt) vortices. The Cartesian RHI scan coordinate system $[x', y', z']$ and the polar coordinate system $[r, \varphi]$ are also given.

The hazard for a following aircraft is assessed based on the position and strength of a wake vortex. Lidar measurements only contain a single velocity component, therefore the vortex centroid cannot be computed straightforwardly using the vorticity field. The strength of a wake vortex is defined by the circulation $\Gamma = \int_A \omega_x dA$, where ω_x is the axial component of the vorticity and A is a perpendicular two-dimensional disc area [1]. Also the circulation cannot be computed by straightforward means from the LOS velocity field measured by a lidar. Note that for quantifying the circulation in measurements $\Gamma_{5-15} = 0.1 \int_{5\text{ m}}^{15\text{ m}} \Gamma(r) dr$ is often employed, representing the averaged circulation over discs of radii from 5 m to 15 m [1].

Therefore, processing of wake vortex characteristics from lidar measurements necessitates algorithms specifically tailored for this task. Numerous processing algorithms have been developed in the past, these can be categorized into physical and data-driven types, or a combination of both [4]. The state-of-the-art physical processing algorithm for $\lambda \approx 1.5 \mu\text{m}$ PCDLs is the radial velocity (RV) method [5]. Its practical accuracy has recently been quantified in turbulent atmosphere and for wake vortices close to the ground [6], revealing the inherent error carried by the method. Nonetheless, physical processing algorithms offer great characterization accuracy, however they often lack automation or fast-time capabilities. During the last decade, machine learning (ML) has been instrumental in increasing the processing speeds (efficiency) and accuracies (effectiveness) in optical measurements [7]. Unlike ML-based methods, traditional physics-based methods often cannot handle vast data amounts efficiently, in particular in combination with graphics processing units (GPUs). Applications of ML in optical measurements includes many ML types [8]: Support vector machines, random forests, artificial neural networks (ANNs), and reinforcement learning. An example ML application for atmospheric measurements using lidars is the error reduction for estimating turbulence, where it was concluded that the dataset composition is highly influential on the effectiveness of the algorithm - there is a lack of generalization [9]. Numerous data-driven processing algorithms have also emerged for processing wake vortices in lidar scans, with the perspective of processing large data amounts quickly and in fast-time. The capabilities of data-driven algorithms range from wake vortex pair identification in lidar scans [10–12], to a more refined location determination of the individual vortices constructing a pair [13–15], to combined location and strength characterization methods [16–18]. To date, the most versatile data-driven processing algorithm is the ML pipeline from Ref. [4], where the deep vision tool you only look once (YOLO) is employed for the rapid identification of the individual

vortices. Based on cutouts defined by YOLO, a regression convolutional neural network (CNN), a type of ANN, architecture [16] identifies the position and strength of the individual vortices. The processing algorithm's accuracy has not yet been investigated, however certainly the accuracy of the RV method cannot be reached, since field measurements lack ground-truth labels. So far, most ML processing of wake vortex field measurements has been performed using ML models also trained on field measurements labeled by physical processing algorithms i.e., training, validation, and test subsets were all based on the same measurement campaign, lidar setup, and atmospheric conditions.

From the above the main research question arises: can data-driven processing algorithms surpass the accuracy of state-of-the-art physical algorithms by employing datasets with ground-truth labels? Furthermore, the question how well a ML model is generalizable, i.e., how well it can be transferred to other locations, varying setups, and/or measurement conditions, remains open. Success is questionable for deviating measurement geometries and varying meteorological conditions occurring at different airports. The collection and labeling of new measurements for each use-case is not effective. These drawbacks may be overcome by employing synthetic lidar measurement data based on dedicated computational fluid dynamics simulations. Recently Ref. [19] studied the evolution of wake vortices in lidar field measurements employing ML models trained on lidar simulations. While this study used idealized wake vortex modeling and focused on low instrument noise measurements, it suggests the practicability of employing lidar simulations for processing field measurements.

In this paper we present a generalization approach for data-driven lidar processing algorithms using the ML pipeline from Ref. [4] as an example. Synthetic lidar scans at realistic detail are simulated under varying atmospheric conditions within high-fidelity hybrid Reynolds-averaged Navier-Stokes (RANS) and large-eddy simulations (LES) from aircraft in their final approach (also referred to as hybrid simulation or simply LES). Throughout the position and strength of the wake vortices is known at ground-truth accuracy level, enabling data-driven processing algorithms to surpass the accuracies of physical processing algorithms. LES resolve the larger turbulent eddies, while the non-resolved flow is modeled by subgrid-scale closures. This method has proven itself for the simulation of wake vortex evolution in ground proximity, where secondary vortices are induced by the diverging primary vortices along the ground surface that subsequently detach from the ground, whereupon the interaction of the primary and secondary vortices causes vortex decay [20–24]. The hybrid LES method applied here constitutes a most realistic approximation of vortex initialization, as the wake vortices are generated by a realistic aircraft geometry in high-lift configuration that conducts a final approach including landing, where also end effects triggered during touchdown contribute to vortex decay [25,26]. Therefore, lidar simulators as part of LES are also referred to as LES lidar simulators (LLS) in this paper. Lidar simulators have previously been implemented in LES [25,27] and studies made use of the known position and strength of wake vortices within these LES [6,28–30]. By employing LES, the expected atmospheric conditions at the location of interest and the campaign setup can be represented. To the authors' knowledge, for the first time LLS scans are employed to train ML models for the complete characterization of wake vortices in lidar field measurements.

Both the detection skill and characterization accuracy of wake vortices employing the proposed generalization of the data-driven processing algorithm are investigated employing two types of LLS, without/with modeled instrument noise, in comparison to the ML pipeline based on field measurements (as in Ref. [4]) and the RV method. The representativeness of the simulated dataset for processing field measurements is discussed as well.

This paper first details the RANS-LES for the landing aircraft and the associated lidar simulators employed for the generalization in Section 2. Thereafter, Section 3 introduces the lidar processing algorithms for wake vortices relevant in this paper, with a focus on the herein generalized ML pipeline. The obtained and employed datasets from both field measurements and simulations are

described and compared in Section 4. The results follow in Section 5, first providing an accuracy comparison of the two investigated processing algorithms, after which the generalization of the ML pipeline is investigated for detecting and characterizing wake vortices. Finally, the potential of datasets labeled by ground-truths is discussed. Conclusions are provided in Section 6.

2. Lidar simulations within LES of landing aircraft

Two lidar simulators are considered in this paper. On the one hand, lidar simulations with instrument noise and realistic signal-to-noise ratios (SNRs). On the other hand, lidar simulations neglecting noise are considered, allowing the impact of optical measurement errors to be studied. The retrieval of the simulated lidar scans is split into two parts. First, raw LOS velocities are obtained from the RANS-LES from landing aircraft. Then, the velocity fields are processed for the two lidar simulator types. LES and lidar coordinate systems are sketched in Fig. 1.

2.1. RANS-LES from landing aircraft

Herein performed RANS-LES runs are Airbus A340 aircraft landings for four atmospheric scenarios (turbulent crosswinds). In ground vicinity crosswinds may cause wake vortices to remain in the glide path for a considerable amount of time, posing a high-risk scenario for aircraft [31]. High-fidelity simulations of the landing aircraft are computed using a hybrid simulation system, which has previously been described in Ref. [6], below key aspects are recapitulated. A RANS simulation from an Airbus A340 (in high-lift configuration) is performed; obtained solution frames are swept through an LES domain (see Fig. 1), such that the wake vortex evolution can be simulated from its roll-up to its decay [32] using the incompressible Navier-Stokes solver MGLET [33]. The flight speed U_∞ is met with the LES timestep [34],

$$dt = \frac{0.5dx}{U_\infty} \sqrt{1 + \frac{1}{16^2}}, \quad (1)$$

in quiescent atmosphere, resulting from the glide path angle γ necessitating 16 horizontal grid points for every vertical grid point in an equidistant mesh ($dx = dy = dz$). Under crosswind conditions dt is adjusted accordingly. Fundamental wake vortex parameters associated to the considered aircraft type (assuming an elliptic lift distribution) include [1]:

- Root circulation: $\Gamma_0 = (2C_L U_\infty B)/(\pi \Lambda)$.
- Initial separation of the counter-rotating wake vortex pair: $b_0 = \pi B/4$.
- Initial vortex descend speed: $w_0 = \Gamma_0/(2\pi b_0)$.
- Characteristic time: $t_0 = b_0/w_0$.
- Vorticity unit: $\omega_0 = 1/t_0$.

These quantities are employed for normalizing circulations, dimensions, velocities, times, and vorticities in this paper, respectively (marked with an asterisk, *). Parameters utilized in the above equations are defined in Table 1. Times given are initialized when the aircraft flies through the recording lidar measurement plane, and can thus also be called vortex age. Within the LES wake vortices are tracked and characterized via the vorticity component in flight direction and the pressure field - an appropriate approach for horizontal vortex filaments.

The generously sized and highly resolved LES domain allow tracking the full life of wake vortices under crosswind conditions and delay unphysical boundary effects. The side, top, and bottom LES boundaries are conditioned as periodic, free-slip, and no-slip, respectively. Besides the theoretical quiescent atmosphere case shown in Fig. 2, with the near field phase (A), extended near field (B), and vortex phase (C), three crosswind magnitudes are simulated in this paper:

Table 1. Settings of the RANS-LES Airbus A340 landing simulations from Ref. [6]

Parameter	Symbol	Unit	Value
A340 wingspan	B	m	60.4
A340 wing aspect ratio	Λ	-	9.3
True air speed	U_∞	m s^{-1}	75
Lift coefficient	C_L	-	1.4
Glide path angle	γ	deg	3.6
A340 pitch angle	θ	deg	5.5
Initial/root circulation	Γ_0	$\text{m}^2 \text{s}^{-1}$	433.9
Initial vortex spacing	b_0	m	47.4
Initial vortex descend speed	w_0	m s^{-1}	1.5
Characteristic time	t_0	s	32.6
Vorticity unit	ω_0	s^{-1}	0.03
LES domain dimensions	X^*, Y^*, Z^*	-	23.3, 11.7, 2.9
LES grid spacing	dx^*, dy^*, dz^*	-	0.011
Crosswind strengths	χ	-	0, 0.5, 1, 2

- $0.5 w_0$: matches the vortex-induced lateral propagation speed [31].
- $1.0 w_0$: operationally relevant [35].
- $2.0 w_0$: enhanced turbulence case.

Turbulent crosswinds are implemented via a constant pressure gradient along the y -axis such that χw_0 is set at a height of b_0 above the ground, where χ represents a factor. The initialization is performed using a logarithmic boundary layer disturbed by random noise.

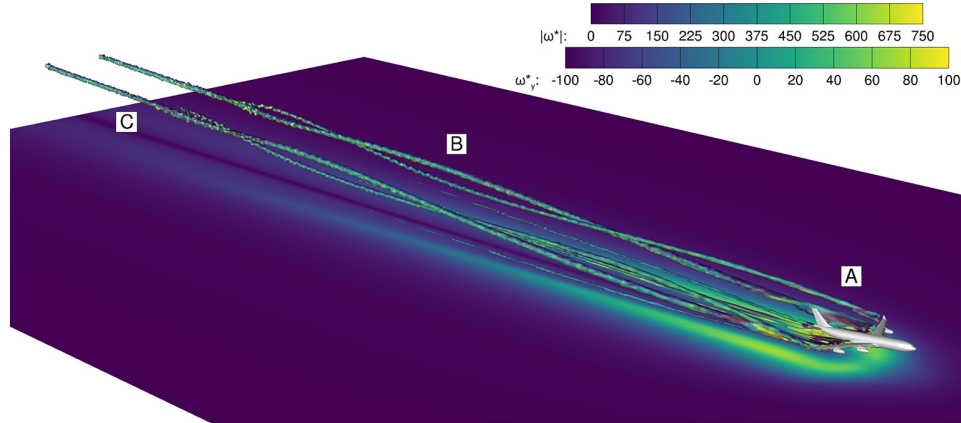


Fig. 2. Airbus A340 landing in quiescent atmosphere. The ground is colored by the overall vorticity magnitude $|\omega^*|$. The shown iso-surface is $|\omega^*| = 300$ while being colored by the spanwise vorticity ω_y^* [6].

2.2. Line of sight (LOS) velocity retrieval

LOS velocities are obtained simultaneously during the numerical simulations from landing aircraft, retrieving raw LOS velocities according to the selected scanning rate ω_s of the lidar

with a time and elevation angle resolution of dt' and $d\varphi$, respectively [6]. Therefore, discrete LOS elevation angles φ_k are considered, along which velocities are retrieved instantaneously at ranges R_j . Since $dt \ll dt'$, temporal nearest-neighbor interpolation is used [6]. A RHI lidar scan is schematically plotted in Fig. 3. Spatial bilinear interpolation is used to obtain the flow velocity vector $\mathbf{V} = [u, v, w]$ from the LES grid. Herein, all lidars are oriented perpendicular to the x -axis of the LES. At (R_j, φ_k) the LOS velocity is computed via the individual LES velocity components $[v, w]$ and the LOS unit direction vector $\mathbf{e}$,

$$V_r = \mathbf{e} \cdot \mathbf{V} = v \cos \varphi + w \sin \varphi. \quad (2)$$

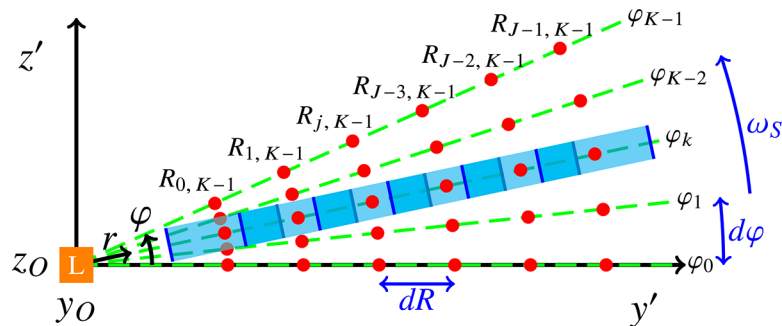


Fig. 3. Schematic RHI lidar scan. Red dots represent locations for which the LOS velocity is estimated along the LOS (green dashed lines). Overlapping range gate windows are drawn for φ_k in blue color (the window width is exaggerated for visibility).

The second part of the lidar simulator can be considered post-processing of the raw RHI lidar scans, delivering the spatial low-pass filter characteristics inherent to lidars as they cannot perform point measurements - reducing velocity magnitudes and gradients. Following parameter definitions can be found in Ref. [36, chap. 1-2]. The following lidar simulators are considered:

- LLS_n: practical type for measurements with realistic SNR (based on Ref. [37]).
- LLS: idealized type for measurements assuming an SNR $\gg 1$ (based on Ref. [38]).

2.3. LLS_n (with instrument noise)

Instead of point measurements, lidars observe regions called sensing volumes of length $\Delta z = (cT_W/2)/\text{erf}[T_W/(2\sigma_P)]$, with the temporal range gate T_W (spatial: $\Delta R = cT_W/2$), the speed of light $c = 0.3 \text{ m ns}^{-1}$, the standard error function $\text{erf}(x) = (2/\sqrt{\pi}) \int_0^x \exp(-\xi^2) d\xi$, and the temporal pulse width σ_P , computed where the pulse power drops to the e^{-1} level from the peak (spatial: $\Delta p = c\sigma_P/2$). The temporal full width at half maximum (FWHM) of the pulse τ_P (spatial: $\Delta r = c\tau_P/2$) is given in Table 2, where $2\sigma_P = \tau_P/\sqrt{\ln 2}$.

The received time signal by a lidar is called the backscatter signal. By computing the Doppler frequency shift between a set reference frequency f_{ref} of the lidar with the backscatter signal, the averaged LOS velocity $\bar{V}_r$ for (R_j, φ_k) can be obtained. Multiple samples in time are required for transferring the backscatter to the frequency domain via the fast Fourier transform. The set bandwidth B_0 uses L spectral channels, dictating the need for $I = 2L$ samples in time, represented by the red circles. The sampling interval is $\delta t = 1/B_0$ in time and $\delta R = c\delta t/2$ in space. Within each sensing volume a range gate window ΔR is used for sampling the backscatter, enclosing $R_{j,k}$ (filled red dots separated by dR) drawn as blue regions in Fig. 3.

We further focus on a single range gate window at range R_j (overall case $R_{j,k}$) from the lidar in Fig. 4. A single laser pulse P_n moves through the sensing volume, modeled of Gaussian shape,

$$g(i') = \exp \{ - [(i' - I/2) \delta R / \Delta p]^2 / 2 \}. \quad (3)$$

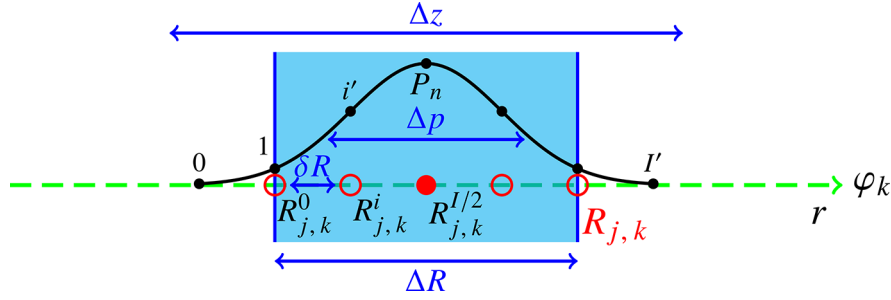


Fig. 4. Blue range gate window for estimating the LOS velocity at $R_{j,k}$ along LOS φ_k . Red circles are sampling points. A Gaussian laser pulse moves through the sensing volume. Black dots along the pulse represent individual aerosol sub-signal weightings.

The laser pulse moving through the sensing volume interacts with $I' = I$ aerosols, these reflect light back to the lidar receiver. Thus, the LOS aerosol spacing $h = \delta R$. However, noise light sources are backscatter towards the lidar too. Thus, according to Ref. [37] the backscatter received by a lidar at a point in time t_i for a single range gate window and a single laser pulse can be described by,

$$Z(t_i; n) = \mathcal{R}\{Z_S(t_i; n)\} + Z'_N(t_i; n), \quad (4)$$

where,

$$Z_S(t_i; n) = \sqrt{\text{SNR} \frac{2B_f}{B_0} \frac{\delta R}{\sqrt{\pi} \Delta p}} \sum_{i'=0}^{I-1} \zeta(i + i'; n) g(i') \exp \{ 2\pi j t_i [f_{\text{ref}} + (2/\lambda) V_r(R_{j+i'}, \varphi_k)] \}, \quad (5)$$

corresponds to the echo signal of interest and Z'_N represents background noise modeled by $\varepsilon_1(i)$, a pseudo-random parameter realized by a Gaussian probability density function with a mean of zero and unit variance (distinct for each sample). The square root is associated with the pulse energy E_p and utilizes $B_f = 50$ MHz, defined by the SNR computation (see below). The aerosol noise modeled by $\zeta(i + i'; n) = \varkappa [\varepsilon_2(i + i'; n) + j\varepsilon_3(i + i'; n)]$, with $j = \sqrt{-1}$, is adapted from Ref. [37]. The origin of this aerosol noise is the inhomogeneous spread of aerosols in the atmosphere, unpredictable refractions, and differing aerosol shapes/sizes. Both ε_2 and ε_3 represent pseudo-random parameters given by a Gaussian probability density function with a mean of one and unit variance (distinct for each sample and aerosol). This factor is directly related to the new power scaling parameter $\varkappa$. The introduction of empirically determined $\varkappa = 0.2124/\sqrt{2}$ allows the input SNR to be retrieved when practically evaluating the backscatter spectrum in the frequency domain using $\text{SNR}(R_j, \varphi_k) = \sum_{f_-}^{f_+} S_D(f_i; R_j, \varphi_k) / \sum_{f_-}^{f_+} S_N(f_i; \varphi_k)$, where $f_{\pm} = f_{\text{ref}} \pm B_f/2$, S_D and S_N are defined below [39]. The initialized SNR corresponds to the mean SNR field across lidar RHI scans from the considered field measurements with an additional pseudo-random factor (mean of unity and a variance of 0.25). The complex aerosol noise and weighting of the backscatter from an individual aerosol is multiplied by a complex sub-signal created by the aerosol LOS velocity found at $R(j + i') = R_j + (i' - I/2) \delta R$.

The full echo signal of interest for t_i is a sum of all I' aerosol sub-signals, with further signal power components multiplied. For implementation purposes, Eq. (5) differs from Ref. [37], assuming a perfectly centered laser pulse within the range gate window at every sampling point.

Given that every LOS is resolved for one instantaneous point in time and that a multitude of pulses are employed for averaging the backscatter signal (see below), on average thus centered, the suggested simplification is considered appropriate. The above process, Eq. (5), is repeated for all I sample points such that the solution can be inserted into

$$S(f_i; R_j, \varphi_k) = \frac{1}{N_a} \sum_{n=1}^{N_a} \left| \sum_{i=0}^{I-1} W(i\delta t) Z(i\delta t - T_W/2) \exp\left(-2\pi j \frac{li}{I}\right) \right|^2, \quad (6)$$

where each time signal component is multiplied with,

$$W(t) = \begin{cases} 1, & t \leq T_W, \\ 0, & t > T_W, \end{cases} \quad (7)$$

giving the Fourier transform of the temporal backscatter to the lidar. This process is repeated for N_a pulses, defined by the pulse repetition frequency f_P and lidar scanning rate, such that most noise components introduced above cancel when averaging the spectra, with variations of the set SNR. We further remove a pure noise signal S_N [obtained from $Z(t_i) = \varepsilon_4$, defined analogous to ε_1 , and Eq. (6)] from the averaged spectrum S ,

$$S_D(f_i; R_j, \varphi_k) = S(f_i; R_j, \varphi_k) - S_N(f_i; \varphi_k), \quad (8)$$

giving the Doppler spectrum. Within the Doppler spectrum the peak and its related frequency f_{peak} determine the Doppler frequency shift (with respect to f_{ref}), and thus the lidar LOS velocity,

$$\bar{V}_r = \frac{\lambda(f_{\text{peak}} - f_{\text{ref}})}{2}. \quad (9)$$

2.4. LLS (without instrument noise)

When no instrument noise is assumed a single laser pulse suffices for computing $\bar{V}_r$. This enables obtaining the Doppler frequency shift using the backscatter spectrum center of gravity (as well as f_{ref}), rather than the peak. Instrument noise causes non-linear low-frequency spatial filtering, leading to different center of gravity and peak frequencies [39, chap. 2]. As a result, the averaged LOS velocity for (R_j, φ_k) can be estimated directly from the individual aerosol LOS velocities as well as a normalized weighting for each aerosol velocity [38],

$$\bar{V}_r(R) = \int_{-\infty}^{+\infty} Q_S(s) V_r(R + s) ds \approx \sum_j \frac{h_j Q_S(s_j)}{\sum_i h_i Q_S(s_i)} V_r(R + s_j), \quad (10)$$

with $h = 1$ m and the range gate weighting function (RWF) given by the convolution of the Gaussian pulse and the spatial indicator function of the range gate $W(r)$ [from Eq. (7)],

$$Q_S(s) = \int_{-\infty}^{+\infty} g(s - r) W(r) dr = \frac{1}{cT_W} \left[\text{erf}\left(\frac{2s/c + T_W/2}{\sigma_P}\right) - \text{erf}\left(\frac{2s/c - T_W/2}{\sigma_P}\right) \right], \quad (11)$$

with s as the coordinate along the LOS with respect to R . See Ref. [38] for a minor difference in the normalization for the Gaussian pulse $g(s)$.

3. Lidar processing algorithms for characterizing wake vortices

Throughout this paper two lidar processing algorithms for wake vortex characterization are employed, the state of the art physical algorithm (RV method) and the state of the art data-driven algorithm (the ML pipeline). The former is employed for providing the labels of the field

measurement dataset investigated, the latter is developed further through the generalization proposed. Both processing algorithms have previously been published, below recapitulates the key aspects of the two processing algorithms from Ref. [5] and Ref. [4], respectively. This section concludes with specifying metrics used for assessing the wake detection and characterization capabilities of the processing algorithms.

3.1. Radial velocity (RV) method

First mention of the RV method was in Ref. [5], with this paper focusing on this original version, given that it processed the field measurements analyzed in this paper. Newer versions of the RV method include superior accuracy in ground vicinity [40]. The RV method first determines the position of a vortex via the polar coordinate system $[r, \varphi]$. The maximum of the functional,

$$D(R_j) = \sum_{k=0}^K [\bar{V}_r(R_j, \varphi_k)]^2, \quad (12)$$

determines the distance from the lidar to the vortex center R_C using averaged LOS velocities $\bar{V}_r$ along all LOSs. The corresponding elevation angle φ_C is estimated via the mean of the elevation angles associated with the highest positive and negative $\bar{V}_r$ at R_C .

Wake vortex circulations are estimated by minimizing the functional

$$G(\Gamma_n) = \sum_{k=0}^K \left[\bar{V}_r(R_{C,n}, \varphi_k) - \tilde{V}_r(R_{C,n}, \varphi_k; \Gamma_{n=1}, \Gamma_{n=2}) \right]^2, \quad (13)$$

where analytical Burnham-Hallock vortices [41] model each vortex (count n) in a velocity field $\bar{V}_r$. The employed RV method implementation allows removing a so-called background scan (superscript B) from the wake vortex scan (superscript V) via

$$\bar{V}_r(R_j, \varphi_k) = \bar{V}_r^V(R_j, \varphi_k) - \langle \bar{V}_r^B(\varphi_k) \rangle. \quad (14)$$

3.2. Machine learning (ML) pipeline

This data-driven processing algorithm includes three fundamental steps. First, pre-processing modifies the lidar scans to focus on the wake vortices present within. Second, the widely recognized one-stage object-detection model YOLO is employed for detecting individual wake vortices within the lidar scans. Finally, regression CNNs are trained on the individual wake vortices allowing the complete position and strength characterization of wake vortices.

3.2.1. Data pre-processing

Both the lidar scans as well as the target labels require pre-processing. Methods employed for the scans include: removing LOS with low elevation angles (those prone to measure the boundary layer or objects near the ground), mitigating the effect of background winds by subtracting an appropriate background lidar RHI scan, and a SNR filter with a threshold of $T_{\text{SNR}} = 0.02$. Finally, the RHI scans are converted to a rectangular image by projecting the scan onto a Cartesian grid. Prior to feeding these pre-processed scans into YOLO, their LOS velocities are converted into pixel intensities using min-max normalization and a scaling where the maximum intensity is $2^{16} - 1$ (16-bit), given that image files are required as input.

The target labels required by YOLO are rectangular boxes, named bounding boxes (BBs), containing the object of concern. The known vortex center is converted to a bounding box with width $BB_{\text{width}} = b_0$ and height $BB_{\text{height}} = b_0$ (associated to the vortex generating aircraft).

3.2.2. Vortex detection

Lidar scans may contain none, one, two or even multiple wake vortices, when a first long-lived vortex pair is still visible while another pair is generated by a following aircraft. Object-detection ML models can flexibly detect any number of vortices using bounding boxes. One-stage detection models, YOLO being among the most popular, allow the detection to be efficient, and therefore quick. The ML pipeline employs YOLOv4 [42], constructed by three building blocks: One, a COCO dataset [43] pre-trained convolutional-based collection of layers named CSPDarknet-53 [44]. Two, a spatial pyramid pooling [45] and path-aggregation network [46]. Three, YOLOv3's [47] so-called detection head, which segments the input into Cartesian grids of different scales, with each cell predicting three bounding boxes, allowing vortices of different sizes and shapes to be identified. Bounding boxes are thereafter classified and a confidence is computed for judging the bounding box quality. The confidence contains the intersection over union (IoU) of the estimated (est) and labeled (lbl) bounding boxes, $IoU = |BB_{est} \cap BB_{lbl}| / |BB_{est} \cup BB_{lbl}|$, the probability of the bounding box containing any vortex, and the probability of the bounding box containing a specific class of object i.e., a starboard or port vortex. The first two items of the previous listing give the so-called objectness. Non-maximum suppression (NMS) of type greedy NMS is employed to avoid multiple bounding boxes identifying the same vortex [42,48].

The loss function of YOLOv4 is three-fold. The bounding box loss is complete IoU [49] where the spatial separation between the centers, overlap area, and aspect ratio are evaluated. Both objectness and classification are binary-cross entropy losses. The remainder of this paper refers to YOLOv4 by simply YOLO. An example of a processed lidar scan with the output by YOLO, rectangular bounding boxes associated with a class and confidence score, is presented in Fig. 5 with associated target labels given by circular markers. YOLO is trained for 235 epochs (iterations of the training subset).

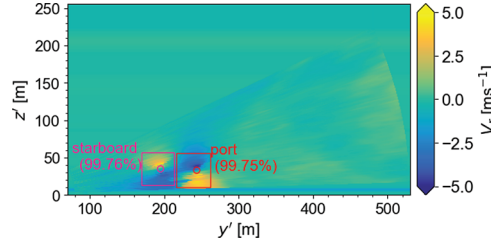


Fig. 5. Example of YOLO-estimated bounding boxes with confidence and class based on a field measurement scan. Circles represent the labels.

3.2.3. Vortex characterization

The circulation and fine-tuned position of the vortex center of each wake vortex is obtained via the regression CNN [16] shown in Fig. 6. Each wake vortex is thus individually characterized, where the YOLO-estimated bounding box is projected onto a 64×64 pixel grid (padding is applied for small bounding boxes). Afterwards the velocity fields are normalized such that the training subset has a mean LOS velocity of zero and a variance of unity. The circulation and localization tasks are performed individually, thus the output layer has an output dimension of 1 or 2, respectively. Labels are the Cartesian vortex center coordinate within the 64×64 grid coordinate system $[yy, zz]$ and the Γ_{5-15} value. The architecture employs convolutional-pooling (ConvPool) blocks containing convolutional layers with 32 kernels of size 3×3 for the first ConvPool block, each subsequent ConvPool block doubles the amount of kernels. Max-pooling layers follow with a kernel of size 2×2 and a stride of 2. The regression task is completed with dense layers. Throughout the CNN the employed activation function is of rectified linear unit

type [50], the optimizer is Adam [51], and the loss function is of mean squared error (MSE) type. Overfitting is prevented via early training stopping if no validation subset loss reduction is noted for 30 epochs. Wake vortex characterizations for the lidar scan from Fig. 5 are given in Fig. 7 with crosses representing vortex center estimations, circles giving the target labels, and the legend printing the respective circulations.

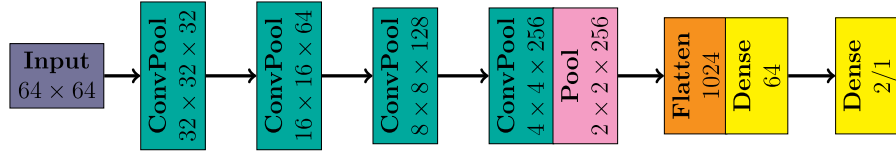


Fig. 6. Regression CNN architecture for estimating the circulation of individual wake vortices. Output dimensions of the hidden layers are also given [4].

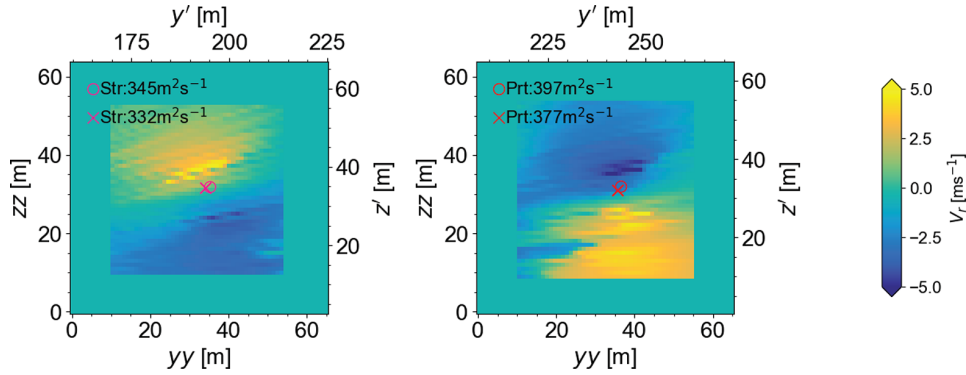


Fig. 7. Example of the regression CNN characterizing the starboard (Str, left) and port (Prt, right) wake vortices. Circles represent the labels and crosses are CNN estimates. The legend gives the estimated and target label circulations.

3.3. Wake vortex detection and characterization metrics

The wake vortex detection is evaluated using the number of true positives (TPs), false positives (FPs), and false negatives (FNs) with respect to the label. In the case of the ML pipeline, for a YOLO-detected vortex to be considered a true positive the confidence must be at least 20 %, the Euclidean distance between the centers of the estimated and labeled bounding boxes is less than or equal to 42 m, corresponding to an offset of half the diagonal of a large $60 \times 60 \text{ m}^2$ bounding box, and the estimated class must match that of the label. Furthermore, each available label may only match one estimated bounding box. When multiple estimates fulfill the aforementioned conditions, the bounding box with the smallest Euclidean distance between the centers of the estimated and labeled bounding boxes is chosen, with the other estimations consequently considered false positives. The RV method does not separately detect and characterize wake vortices. For a test subset the rate of detected vortices out of all labeled vortices and the rate of how many detected vortices also correspond to a label are respectively given by

$$\text{recall} = \frac{\text{TP}}{\text{TP} + \text{FN}}, \quad \text{precision} = \frac{\text{TP}}{\text{TP} + \text{FP}}. \quad (15)$$

The wake vortex characterization is evaluated with the mean error (ME) or root MSE (RMSE). The circulation error is $E_{\Gamma_{5-15}} = \Gamma_{5-15}^{\text{est}} - \Gamma_{5-15}^{\text{lbl}}$ and for the positional error the Euclidean distance

between the estimated and labeled vortex centers (y_C , z_C) is employed,

$$D_C = \sqrt{(y_C^{\text{est}} - y_C^{\text{lbl}})^2 + (z_C^{\text{est}} - z_C^{\text{lbl}})^2}. \quad (16)$$

4. Dataset families

For evaluating the suggested generalization approach of employing simulated lidar scans for processing field measurements (FMs), this paper considers the lidar setup from Fig. 1, where the investigated lidar is the Vaisala WINDCUBE 200s with the settings given in Table 2. These settings were used at the herein investigated field measurements campaign, and therefore also for the lidar simulators. Note that wake vortex parameters employed for normalization in this paper (Γ_0 , b_0 , w_0 , t_0 , ω_0) differ between the simulated and measured aircraft types.

Table 2. RHI scan settings of the Vaisala WINDCUBE 200S lidar

Parameter	Symbol	Unit	Value
Laser wavelength	λ	μm	1.543
Pulse energy	E_P	μJ	50
FWHM pulse width	τ_P	ns	200
Pulse repetition frequency	f_P	kHz	20
Pulses for accumulation	N_a	-	1000
Reference beam frequency	f_{ref}	MHz	69.3
Bandwidth	B_0	MHz	250
Temporal range gate	T_W	ns	140
Sensing volume length	Δz	m	35.6
Spectral channels count for 0 – 125 MHz	L	-	32
Elevation angle resolution	$d\varphi$	deg	0.2
Scanning rate	ω_S	deg s^{-1}	4
Range gate spacing	dR	m	3

4.1. Field measurements (FMs)

In 2019 a comprehensive measurement campaign was undertaken at Vienna Airport (for a more detailed overview of the campaign see Ref. [3]). A total of three lidars were positioned at five positions starboard to the glide path of aircraft landing on runway 16. The lidar setups relevant for this work are given in Table 3.

Table 3. Lidar setups at Vienna Airport. Lidar positions in flight direction are given with respect to the runway threshold assumed at aircraft altitude of 50 ft [3].

Parameter	Symbol	Unit	Value
Number of lidars	-	-	5
Elevation angle limits	$[\varphi_-, \varphi_+]$	deg	[0, 25], [0, 20], [0, 18], [1, 28], [0, 29]
Range limits	$[R_-, R_+]$	m	[80, 530]
x-position lidar	x_O	m	(230, 140, -30, -200, -370)
y-position lidar	y_O	m	-(222.5, 279.0, 343.3, 245.6, 272.5)
z-position lidar	z_O	m	(1.0, 0.0, 1.0, -4.0, -6.0)

Lidar measurements from the campaign have so-far been only partially processed with the RV method [5], providing the position and circulation labels. The dataset family contains 17 003

RHI scans of wake vortices generated by aircraft of the RECAT-EU aircraft categories A to D by EUROCONTROL and the European Union Aviation Safety Agency [52]. The majority of aircraft are Airbus A320 (CAT-D), only few CAT-A (Airbus A388) are represented in the dataset. Root circulation values (Γ_0) for the field measurements are derived from Mode S data as described in detail in the appendix of Ref. [53], the remaining wake vortex parameters (b_0 , w_0 , t_0 , ω_0) can be computed for each overflight using the equations noted in Section 2.1. For ML the dataset family is divided into subsets of 80 : 10 : 10, for training, validation, and testing, respectively (13618, 1757, 1628). Note that RHI scans pertaining to one vortex evolution are kept together. The vortex distributions with respect to vortex strength as well as age are shown in Fig. 8. Most vortices in the training subset are young (they have recently been generated) i.e., $t^* \leq 4$ resulting in a fewer weak vortices than strong vortices.

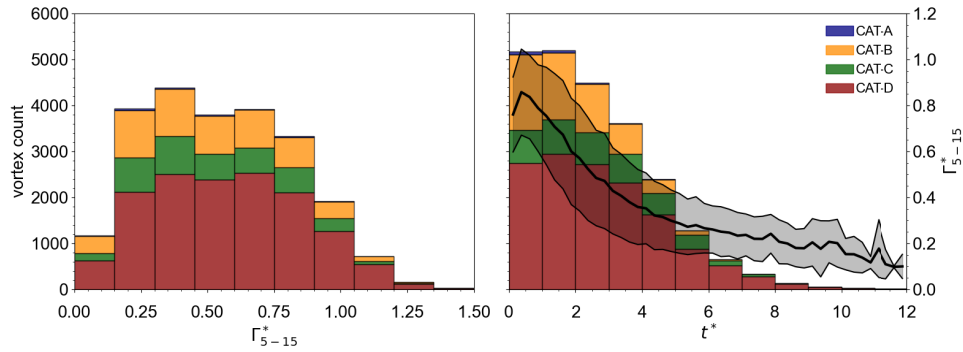


Fig. 8. Distributions of normalized vortex circulation Γ_{5-15}^* (left) and vortex age t^* (right) of the field measurement training subset. The temporal evolution of Γ_{5-15}^* is plotted with standard deviations on the right. Aircraft categories are given by color.

4.2. Simulated data (LLS and LLS_n)

In an effort to replicate the lidar setup from the field measurements at Vienna Airport, Table 4 summarizes the amount and positions considered by both LES lidar simulators. Tabulated values in front the semi-colon represent lidar configurations corresponding to the field measurement data, additional values represent further lidar configurations used for generalization purposes.

Table 4. LLS parameters [6]. There are 26 equidistant x_O^* positions, each with all possible y_O^* and the associated elevation angle spectrum.

Parameter	Symbol	Unit	Idealized Vaisala WindCube 200S
Number of lidars	-	-	312
Elevation angle limits	$[\varphi_-, \varphi_+]$	deg	[0, 20], [0, 25], [0, 18], [0, 28], [0, 29]; [0, 20]
Range limits	$[R_-, R_+]$	m	[80, 530]; [210, 660]
x^* -position lidar	x_O^*	-	[0.0, 17.1] with a spacing of 0.67
y^* -position lidar	y_O^*	-	$\pm(5.9, 4.7, 7.2, 5.2, 5.7; 10.0)$
z^* -position lidar	z_O^*	-	0

In order to establish the difference in complexity and optical realism between LLS and LLS_n scans, Fig. 9 compares a vortex pair. A wake vortex appears as a dipole (a patch of positive velocities and a patch of negative velocities) in a lidar scan. The LLS scan (left) captures the expected low-pass filter characteristic, however the LLS_n scan (right) also includes background noise, evident from the LOS pattern common in field measurements. Also aerosol noise, sudden

and single LOS velocity outliers, is present in Fig. 9 right. The effect of pulse accumulation is shown in Fig. 10, where N_a is increased from 1 to 1000. While one pulse ($N_a = 1$) results in a nearly pure noise scan, an increase in pulses reveals the atmospheric field with the wake vortices.

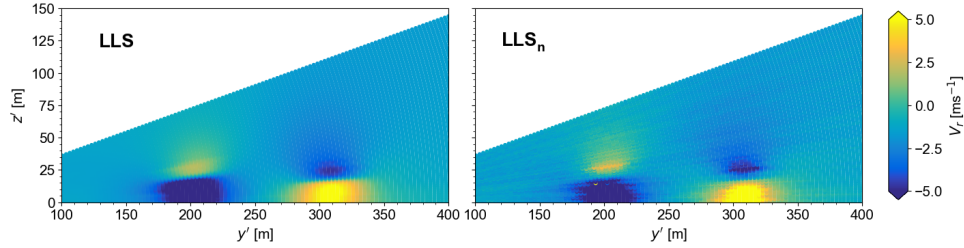


Fig. 9. Zoomed in example LLS (left) and LLS_n (right) wake vortex RHI lidar scans.

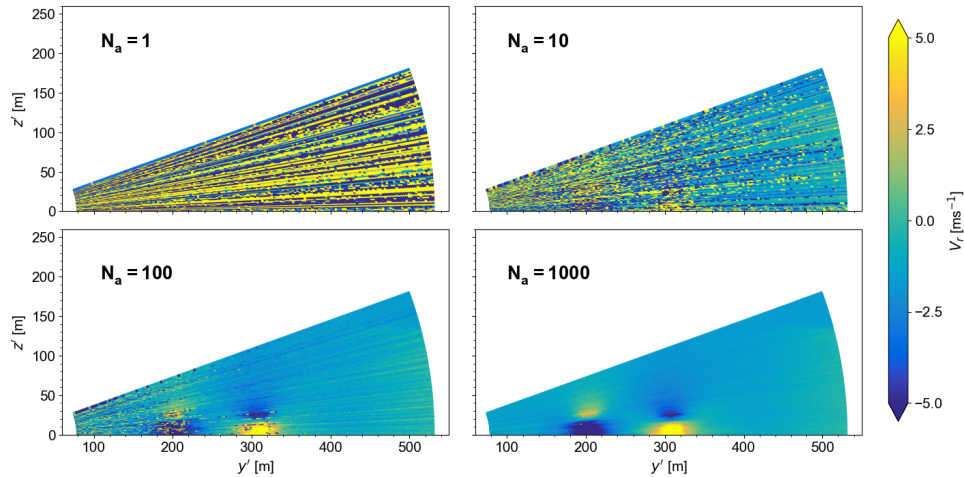


Fig. 10. Example LLS_n RHI lidar scans for varying pulse accumulation count N_a .

Both dataset families, LLS and LLS_n , contain 17 679 simulated RHI scans each, split into 80 : 10 : 10, for training, validation, and test subsets, respectively (14250, 1859, 1570), where RHI scans pertaining to one measurement plane along the LES x -axis are kept together. The position and circulation labels emerge directly from the LES. Figure 11 shows also most simulated vortices to exist for $t^* \leq 4$, however featuring less old and weak wake vortices than the field measurements. Wake vortices are on average generated at lower heights in the simulations, significantly accelerating vortex decay as also observed in field measurements at Munich airport [54]. During the Vienna measurement campaign the flight heights in the lidar planes varied between 40 m and 74 m [53], whereas the LES domain enables heights from 47 m down to the ground. In comparison to the field measurements, the reduced vortex strength during roll-up is on average not as pronounced and throughout the entire lifetime the vortex strength is higher. While it appears difficult to conclude whether the simulations or the field measurements provide more reliable vortex strength evolutions, the highlighted differences must be considered during the ML results analysis.

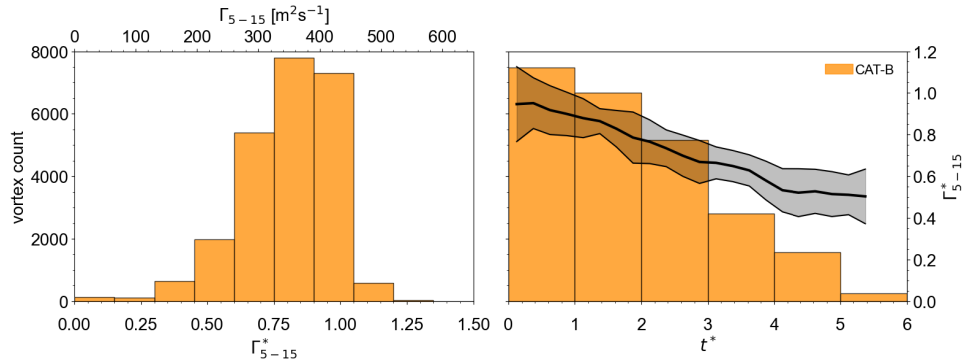


Fig. 11. Distributions of (normalized) vortex circulation Γ_{5-15}^* and Γ_{5-15} (left) and the vortex age t^* (right) of the LLS and LLS_n training subsets. The temporal evolution of Γ_{5-15}^* is plotted on the right with the respective standard deviations in black.

5. Results

With the aim of allowing simulated lidar scans to process field measurements using the ML pipeline, the remainder of this paper compares several combinations of training, validation, and test subsets, with the former two strictly belonging to the same dataset family. To denote which dataset has been used for training and which for testing, we introduce the notation *trained on (evaluated on)*, e.g., *FM (LLS)* refers to the performance of the ML pipeline, which is trained on field measurements and tested on LLS data.

First, this section compares the characterization accuracy of the ML pipeline and the RV method, highlighting the potential of the ML pipeline to outperform the state-of-the-art RV method. Then, the generalization of the ML pipeline is analyzed using the simulated dataset families. Finally, the impact of employing ground-truth labels on the wake vortex characterization accuracy of the ML pipeline is assessed.

5.1. Characterization accuracy of lidar processing algorithms

The practical characterization accuracy of the RV method has been described in Ref. [6] using LLS data. The ML pipeline has not previously been assessed. By using the LLS dataset family across all subsets for the ML pipeline, Table 5 reveals that the ML pipeline is theoretically superior to the RV method in turbulent situations and ground vicinity. The RMSE provides the expected error magnitude for a single vortex characterization. The mean error reveals the systematic overestimation of the vortex circulation, with the standard deviation σ giving the error spread. The ML pipeline detects over 99 % of present wake vortices. The key advantages of the ML pipeline compared to the RV method are:

- An accuracy increase of at least 33 % for both vortex position and circulation.
- Vortices close to walls are handled superiorly, resulting in a smaller systematic overestimation of wake vortex strength.
- Each RHI lidar scan is processed automatically within approximately 0.1 s employing an NVIDIA Tesla V100 GPU, two orders of magnitude faster than the RV method.

5.2. Generalizing the ML pipeline using LES lidar simulations

Having established that the characterization accuracy of the ML pipeline can surpass the RV method, we aim to achieve the same on field measurements. Since the ground-truth of field

Table 5. Performance of the ML pipeline for $LLS(LLS)$, in comparison to the state-of-the-art RV method [5] evaluated in Ref. [6]

Parameter	Metric	$LLS(LLS)$	$RV(LLS)$
D_C [m]	RMSE	3.1	4.6
	ME	2.4	3.7
	σ	1.9	2.7
Γ_{5-15} [m ² s ⁻¹]	RMSE	23.9	38.7
	ME	1.2	19.9
	σ	23.8	33.1

measurements is not available, all metrics are computed with respect to the RV-labels. The proposed generalization of the ML pipeline is studied employing three ML pipeline cases:

- $FM(FM)$: reference case from Ref. [4], all subsets share one dataset family.
- $LLS_n(FM)$: generalization by training with lidar simulations and realistic SNR values.
- $LLS(FM)$: generalization by training with idealized lidar simulations and $SNR \gg 1$.

5.2.1. Detection of wake vortices

The recall and precision rates for bins of normalized vortex strength are plotted for the three cases in Fig. 12. Bootstrap resampling, random selection of vortices with return (a vortex may be present multiple times in the test subset), provides the uncertainty boundaries. As expected, $FM(FM)$ delivers the highest recall rates across all vortex strength bins, as the training and test subsets differ only in considered overflights, while the setup, data distribution, and scan quality are identical. Only extremely weak wake vortices are occasionally missed, on the one hand due to the low number of weak vortices in the dataset (see Fig. 8), on the other hand weak vortices are rarely coherent. While strong and coherent vortices feature a similar structure, weak and incoherent vortices are diverse. The high number of strong and coherent vortices allows the ML pipeline to learn their pattern, unlike the low number of weak diverse vortices.

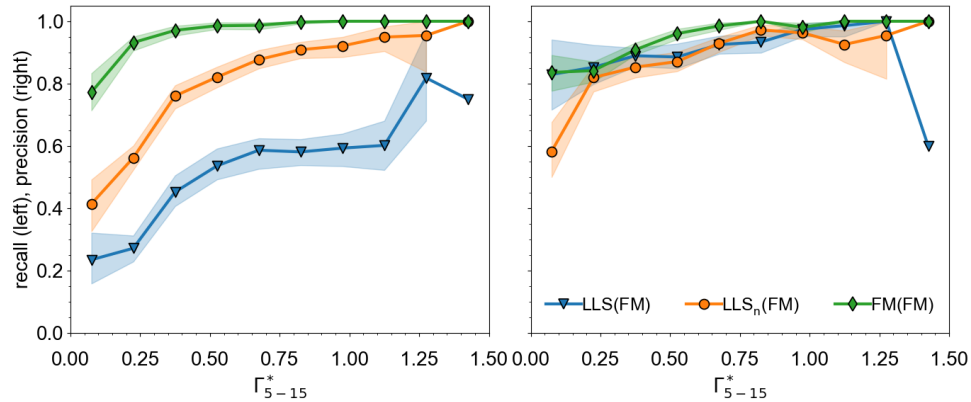


Fig. 12. Recall (left) and precision (right) with respect to the normalized vortex circulation for three ML pipeline cases applied to field measurements. Bootstrap resampling gives the uncertainty.

High recall rates should not correlate with low precision rates, it would indicate that wake vortices are not detected by YOLO but simply assumed everywhere in lidar scans i.e., there would

be many false positives, while all labeled vortices would be found too (true positives). Figure 12 right demonstrates the opposite for all investigated ML pipeline cases, stronger vortex bins feature a higher precision, weaker vortex bins feature a lower precision. Thus, the presence of wake vortices in lidar scans is properly learned by YOLO. Note that the circulation bin representing the strongest vortices is sparse and should be considered with care.

The generalization cases i.e., the ML pipelines trained by the simulation datasets, obtain lower recall rates than training with field measurements i.e., $FM(FM)$. It is expected that ML pipelines using training and test subsets not from the same dataset family obtain lower wake vortex detections. The $FM(FM)$ recall rates can therefore be considered an upper limit. By comparing the vortex distributions of the lidar simulations (Fig. 11) and the field measurements (Fig. 8), it is evident that the former includes far less weak vortices, explaining the weak circulation bins to feature the highest difference in recall rates compared to the $FM(FM)$ case. The impact of instrument noise on the recall rates is studied by $LLS(FM)$ and $LLS_n(FM)$. Particularly for vortices of medium and strong strength the LLS_n -trained ML pipeline obtains recall rates up to 35 % higher than the LLS -trained ML pipeline. Figure 12 left thus highlights the importance of modeling aerosol and background noise i.e., optical measurement errors, when simulating practical lidar measurements (as would be expected from field measurements). The major difference in the recall rate indicates that the two noise sources alter the lidar scan pattern strongly, hindering less realistic LLS scans to train the ML pipeline sufficiently.

5.2.2. Detecting wake vortices generated by different aircraft categories

It was established in the introduction that the composition of the dataset is essential for successful data-driven lidar processing. Field measurements and lidar simulations employed in this paper differ in the vortex generating RECAT-EU aircraft categories:

- FM dataset family: CAT-A, CAT-B, CAT-C, CAT-D (see distribution Fig. 8)
- LLS_n dataset family: CAT-B (see distribution in Fig. 11)

This section compares training the ML pipeline with different aircraft categories (representative of the field measurements) in comparison to one aircraft category. Figure 13 splits recall curves into aircraft categories for the ML pipeline cases $FM(FM)$ and $LLS_n(FM)$, with the former corresponding to training with different aircraft categories and the latter corresponding to training with a single aircraft category. The ML pipeline case $FM(FM)$ unveils that including all expected aircraft categories in the training subset results in wake vortices generated by all aircraft categories to be detected similarly well. A correlation between recall rate and size of the wake vortex generating aircraft category is partly visible. Only the largest aircraft category (CAT-A) deviates for weak vortices from this correlation due to the limited amount of data.

The ML pipeline case $LLS_n(FM)$ directly shows that CAT-B wake vortices, which the LLS_n dataset family consists of, are detected superiorly to other aircraft categories across nearly all circulation bins. Unsurprisingly, CAT-D (representing aircraft sized most different to the Airbus A340) offers the lowest recall rates. When focusing on processing CAT-B scans, satisfactory detection performance is achieved. Indeed, in both ML pipeline cases recall rates above 90 % are achieved when a respectable amount of wake vortices are binned:

- $FM(FM)$: 90 % recall at $\Gamma_{5-15}^* \geq 0.175$ (c.f. Figure 8)
- $LLS_n(FM)$: 90 % recall at $\Gamma_{5-15}^* \geq 0.375$ (c.f. Figure 11)

Therefore, the suggested generalization offers comparable wake vortex detection rates when the evaluation scenario is represented well by the simulations used for training. Here, simulated medium to strong vortices of CAT-B aircraft are recalled well, whereas not-simulated weaker vortices and those generated by different aircraft categories are struggled with. The normalized

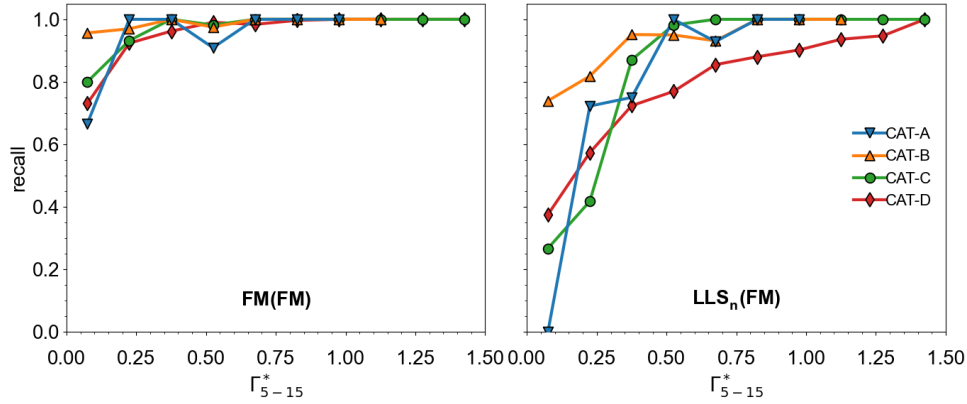


Fig. 13. Recall with respect to the normalized vortex circulation for two ML pipeline cases split into aircraft categories.

YOLO-estimated bounding box area $A_{BB}^* = (BB_{height} \times BB_{width}) / b_0^2$ gives additional evidence that simulating the particular evaluation scenario generalizes the ML pipeline further, enabling higher accuracies. Consider Fig. 14, where the normalized YOLO-estimated bounding box area is plotted with respect to the normalized vortex circulation for each aircraft category. If $A_{BB}^* = 1$, YOLO has learned to differentiate wake vortices generated by different aircraft categories. For $FM(FM)$ all aircraft categories are close to $A_{BB}^* = 1$. Bounding boxes estimated for CAT-A generated wake vortices are least reliable, presumably due to the low count of CAT-A data.

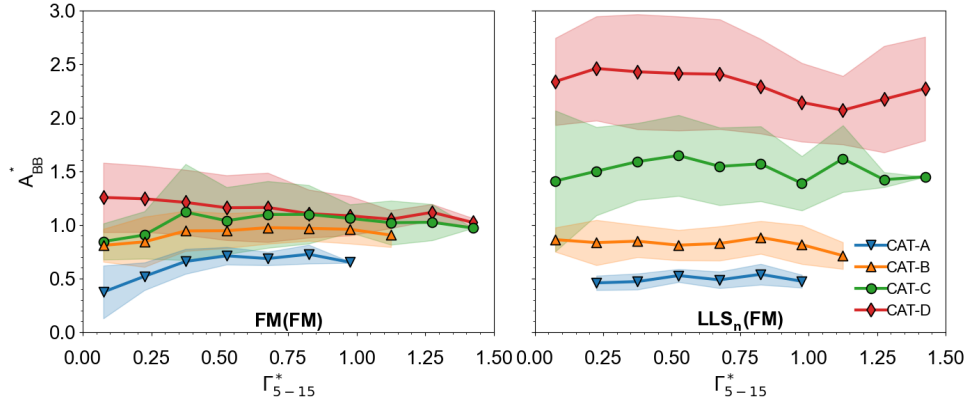


Fig. 14. Mean normalized YOLO-estimated bounding box area (with the standard deviation giving the uncertainty) with respect to the normalized vortex circulation for two ML pipeline cases split into aircraft categories.

Instead, Fig. 14 right immediately reveals that the training subset contains simulations of the Airbus A340 CAT-B aircraft, as $A_{BB}^* \approx 1$ for every circulation bin of that category (orange color, upper triangle). All other aircraft categories feature under-/overestimation of the bounding boxes corresponding to the factor differentiating the various aircraft b_0 (sizes). Example b_0 values are provided in Table 6. Generalized data-driven processing algorithms achieve higher accuracies when considering a wide range of aircraft categories in the lidar simulation dataset.

Table 6. Initial wake vortex pair separations b_0 for example aircraft types assuming an elliptic lift distribution

RECAT-EU aircraft category	Aircraft type	b_0 [m]
CAT-A	Airbus A388	62.7
CAT-B	Airbus A340 (simulated)	47.4
CAT-C	Boeing 763	37.4
CAT-D	Airbus A320	26.8

5.2.3. Characterization of wake vortices

The ML pipeline estimates a wake vortex position and circulation using the regression CNNs, where the YOLO-estimated bounding boxes constitute the input. For the following assessment only vortices detected by both the RV method (the label) and YOLO are considered, i.e., true positives. As above, this section compares the reference case FM (FM) with the herein suggested generalization LLS_n (FM), where ground-truth lidar simulation data serves as ML pipeline input.

In Fig. 15 top the normalized position error is plotted for each vortex with respect to the normalized vortex circulation. Symbols are colored by the YOLO confidence, revealing that weak vortices are not only detected worse, but also YOLO is less certain about its detections. Fortunately, weak vortices are a lower hazard to following aircraft and thus of less concern. The dashed and solid lines represent the localization capabilities of YOLO and the complete ML pipeline with the regression CNN, respectively. Including the regression CNN halves D_C^* across almost the entire vortex strength spectrum for the case FM (FM). This same effect is only present for wake vortices of strength $\Gamma_{5-15}^* > 0.5$ when employing the generalization case LLS_n (FM), in fact YOLO even provides lower position errors for weak vortices. Presumably, the regression CNN requires larger data amounts than YOLO. Correlating the vortex share (Fig. 11 left) and the positional accuracy performance (Fig. 15 top right) hints that approximately 2000 samples within a normalized circulation bin width of 0.15 are sufficient for reasonable vortex localization.

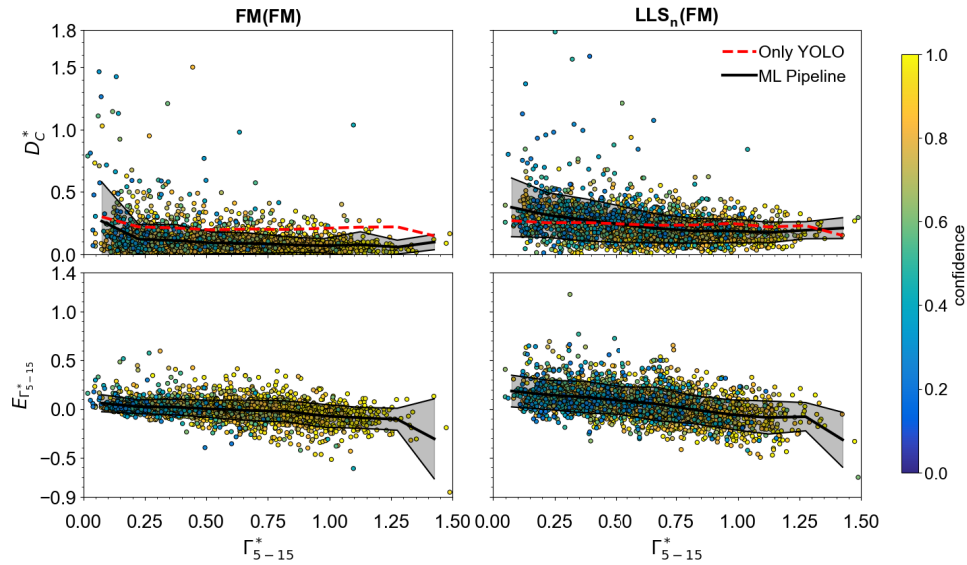


Fig. 15. Normalized vortex localization error (top) and error of the normalized vortex circulation (bottom) with respect to the normalized vortex circulation for vortices colored by the confidence score for two ML pipeline cases FM (FM) (left) and LLS_n (FM) (right). Mean curves for YOLO and the complete ML pipeline (with standard deviation).

The circulation estimation error in Fig. 15 bottom shows both ML pipeline cases to overestimate the strength of weak vortices; the generalization case $LLS_n (FM)$ overestimates the vortex strength across nearly the entire spectrum. Additionally, the uncertainty (standard deviation) in the vortex strength is higher. Table 7 provides the corresponding RMSE values, manifesting nearly twice the positional and circulation inaccuracy for the ML pipeline trained with LLS_n scans. However, it must be remembered that the evaluation is with respect to RV-labels, not the ground-truth.

Table 7. Dimensional and normalized RMSE values of the ML pipeline for four cases

Parameter	Metric	$FM (FM)$	$LLS_n (FM)$	$FM (LLS_n)$	$LLS_n (LLS_n)$
D_C [m]		4.4	7.9	6.3	3.2
D_C^*	RMSE	0.16	0.28	0.13	0.07
Γ_{5-15} [m ² s ⁻¹]		29.5	54.7	48.4	29.0
Γ_{5-15}^*		0.10	0.18	0.11	0.07

5.3. Increasing the characterization accuracy using ground-truth labels

The aim of the herein proposed generalization is to improve lidar processing algorithms by employing lidar simulations with a known ground-truth. As detailed in Section 4.1, the herein considered field measurements are labeled with the RV method. Therefore, the accuracies discussed in Section 5.2.3 do not allow a fair conclusion whether our proposed generalization improves lidar processing algorithms for characterizing wake vortices. Comparison of Figs. 8 and 11 right reveals that the average vortex strength over the vortex lifetime is lower for RV-labels (FMs) than the simulation labels, also called simulation truth (ST). Thus, by plotting the difference between the circulation labels $\Delta\Gamma_{5-15}^* = \Gamma_{5-15}^{*,ST} - \Gamma_{5-15}^{*,FM}$ and $E_{\Gamma_{5-15}^*}$ for the ML pipeline case $LLS_n (FM)$ in Fig. 16, the vortex circulation overestimation from Fig. 15 bottom right can be related to the difference in the dataset labels.

- The roll-up phase, $0 < t^* \lesssim 1$, should be disregarded given that RV-labels for vortices vary strongly in their magnitude (presumably due to the complex vortex structure during roll-up not matching the RV method assumed Burnham-Hallock vortices [41]).
- At $t^* > 1$, the two curves in Fig. 16 correlate well, both simulation labels and ML estimates increase their overestimation throughout the vortex lifetime in comparison to the RV-labels.

Obviously, training with LLS_n scans and evaluating on field measurements inevitably results in a systematic error in the ML pipeline estimates. If ground-truths were available for the field measurements, either prior or post scaling of the circulation values could be undertaken. Therefore, we expect the estimates by the ML pipeline to be of higher precision if the benchmark would rest upon a true basis.

As the field measurements employ error-prone labels, Table 7 provides characterization accuracies also for the cases $FM (LLS_n)$ and $LLS_n (LLS_n)$, which process lidar simulation scans with known ground-truths. The proposed generalization method provides more reliable and accurate wake vortex characterizations. Indeed, the positional RMSE is halved and the circulation RMSE is reduced by 40 %. In addition, the characterization accuracy by the generalized ML pipeline is not largely affected by different atmospheric turbulence levels, as indicated by the separated turbulent crosswind strengths in Table 8 - RMSEs only increase marginally. Undeniably, training and evaluating the ML pipeline on one dataset family (LLS_n) provides an additional advantage to the more accurate labels, field measurements still differ compared to LLS_n data which lack for example large-scale turbulence structures occurring in the atmospheric background. Studies with more accurate field measurement labels provided by another measurement technique like particle image velocimetry or numerical simulations of even more realistic atmospheric fields may help to shed light on this.

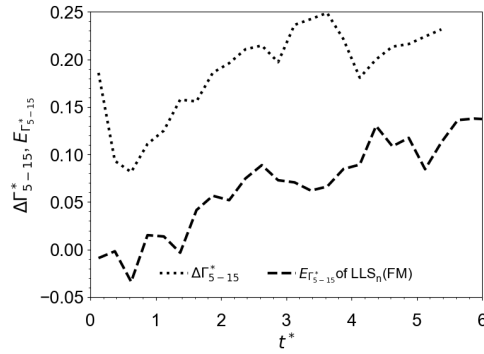


Fig. 16. Comparison of the difference between the normalized field measurement (FM) and lidar simulation truth (ST) circulation labels via $\Delta\Gamma_{5-15}^*$ to the error of the normalized vortex circulation $E_{\Gamma_{5-15}^*}$ of $LLS_n(FM)$, comparing the ML pipeline and the RV-labels, with respect to normalized vortex age.

Table 8. Characterization accuracy of the ML pipeline split by turbulent crosswind strength χ w_0 for the case $LLS_n(LLS_n)$

Parameter	Metric	0.0 w_0	0.5 w_0	1.0 w_0	2.0 w_0
D_C^*	RMSE	0.06	0.06	0.07	0.08
Γ_{5-15}^*		0.06	0.06	0.07	0.08

6. Conclusion

Light detection and ranging (lidar) simulations with known ground-truths from large-eddy simulations (LES), enhanced by a machine learning (ML) based processing algorithm, allow reliable, accurate, automatic, and fast wake vortex characterizations. So far, processing algorithms were limited in speed or by the unknown ground-truths in field measurements. Furthermore, most data-driven processing algorithms employ training and test subsets from one dataset family. To the authors' knowledge the herein generalization of data-driven processing algorithms constitutes the first effort of employing LES data for processing field measurements using ML.

Wake vortices generated during the final approach of an Airbus A340 aircraft have been simulated with hybrid numerical simulations consisting of Reynolds-averaged Navier-Stokes (RANS) simulations, addressing the flow around the aircraft geometry, as well as LES, covering vortex roll-up behind the aircraft geometry until final vortex decay. Within the LES domain we simulated lidars at multiple locations under various turbulent crosswind conditions. Two types of LES lidar simulators (LLS_n) have been implemented, differing in whether instrument noise is considered. When no noise has been simulated, implying that the signal-to-noise ratio (SNR) is assumed extremely high, the simulated lidars are referred to as LLS . Lidar simulations with aerosol and background noise modeled, taking into account realistic SNR values, are referred to as LLS_n . The lidar setup from a campaign at Vienna Airport [3] was modeled.

Wake vortices have been processed employing the state-of-the-art processing algorithm from Ref. [4]. First, vortices are detected using the object-detection ML model YOLO. Second, the individual vortices are characterized using regression Convolutional Neural Networks. Training has been conducted for three different dataset families, the field measurements as a reference, the LLS scans, and the LLS_n scans, where the latter two cases represent the herein proposed generalization of the data-driven processing algorithm. The ML pipeline trained with and applied to LLS data shows that it can outperform the wake vortex processing radial velocity (RV) method [5] by more than 33 % in position and strength characterization accuracies, however so far the missing ground-truth in field measurements hindered achieving such accuracies.

The generalized ML based lidar processing algorithm with LLS_n data (the ground-truth is known) obtains serviceable vortex detection rates and characterization accuracies on field measurements for the simulated aircraft category as well as the covered medium to strong vortices. Vortices of other strengths or generated by alternative aircraft categories are subject to lower detection rates. This demonstrates the requirement for generated lidar simulations to represent the particular expected wake vortex scenario well. In addition, this paper established the importance of modeling lidar scans with realistic SNR, via aerosol and background noise components, for the generalization approach to obtain reliable and accurate wake vortex characterizations. Considering noise increases the probability of detecting strong vortices by 35 %.

The presented generalization approach for lidar processing algorithms aims to increase the overall accuracy of data-driven processing algorithms, in order to even surpass the accuracy of physical lidar processing algorithms. Herein, we showcased that provision of ground-truth labels allows improving the accuracy of the ML pipeline further. This suggests that lidar simulations with a known ground-truth of the expect aircraft categories and atmospheric conditions may set a new standard for lidar processing algorithm accuracies. In particular, the herein studied generalization of the ML pipeline has the potential to replace the RV method, given its superior automatic, rapid, and accurate wake vortex processing.

Nomenclature

Symbols			
$[dx, dy, dz]$	LES domain resolution, m	Λ	Wing aspect ratio
$[r, \varphi]$	Polar coordinates, m, deg	λ	Lidar laser wavelength, m
$[u, v, w]$	Velocity components, $m\ s^{-1}$	ω	Vorticity, s^{-1}
$[X, Y, Z]$	LES domain dimensions, m	$\mathbf{V}$	Velocity vector, $m\ s^{-1}$
$[x, y, z]$	LES coordinates, m	ω_0	Vorticity unit, $1\ s^{-1}$
$[yy, zz]$	Boundary box coordinates, m	ω_S	Scanning rate, $deg\ s^{-1}$
χ	Wind strength factor	σ	Standard deviation
$\Delta\Gamma_{5-15}$	Circulation difference simulation and field measurements	σ_P	Pulse width e^{-1} from peak, s
Δp	Pulse width e^{-1} from peak, m	τ_P	FWHM pulse width, s
ΔR	Range gate width, m	θ	Aircraft pitch angle, deg
Δr	FWHM pulse width, m	ε	Pseudo-random parameter
δR	Sampling interval, m	$\varkappa$	Pulse power scaling parameter
δt	Sampling interval, s	φ	Elevation of lidar beam, deg
Δz	Sensing volume length, m	ζ	Aerosol noise
Γ	Vortex circulation, $m^2\ s^{-1}$	A_{BB}	Bounding box area, m^2
γ	Glide path angle, deg	B	Aircraft wingspan, m
Γ_0	Initial/root circulation, $m^2\ s^{-1}$	B_0	Bandwidth, Hz
Γ_{5-15}	Vortex circulation averaged in a 5-m to 15-m centered disc, $m^2\ s^{-1}$	b_0	Initial vortex spacing, m
		B_f	SNR bandwidth, Hz
		c	Speed of light in air, $m\ s^{-1}$
		C_L	Lift coefficient
		$d\varphi$	Elevation angle resolution, deg

D_C	Euclidean error between vortex center estimations, m	V_r	LOS velocity, m s^{-1}
dR	Range gate spacing, m	Superscripts	
E	Error	,	Lidar coordinates
E_P	Pulse energy, J	*	Normalized by parameters
f	frequency, Hz	Subscripts	
f_P	Pulse repetition frequency, Hz	–, +	Minimum, Maximum
f_{ref}	Reference beam frequency, Hz	0	Initial/Reference
g	Gaussian laser pulse	C	Vortex center
h	Spacing of aerosols sampled along a LOS, m	O	Lidar origin
I	Sample number	Acronyms	
L	Number of spectral channels	BB	Boundary box
N_a	Pulses for accumulation	CNN	Convolutional neural network
P	Laser pulse	DLR	German Aerospace Center
P_P	Pulse power, W	FM	Field measurement
Q_S	Range gate weighting function	FN	False negative (detection)
R	Discrete distance from lidar, m	FP	False positive (detection)
S	Average signal spectrum	FWHM	Full width at half maximum
s	Coordinate along LOS with respect to R	IoU	Intersection over union
S_D	Doppler signal spectrum	LES	Large-eddy simulation
S_N	Pure noise spectrum	Lidar	Light detection and ranging (instrument)
t	Time, s	LLS	LES lidar simulator
t_0	Characteristic time, s	LLS _n	LES lidar simulator with noise
T_{SNR}	SNR threshold	LOS	Line of sight
T_W	Range gate width, s	ME	Mean error
U_∞	Freestream velocity, m s^{-1}	ML	Machine learning
W	Range gate indicator function	MSE	Mean squared error
w_0	Initial vortex descend velocity, m s^{-1}	NMS	Non-maximum suppression
Z	Backscatter signal	PCDL	Pulsed coherent Doppler lidar
Z_N	Noise component of backscatter signal	RANS	Reynolds-averaged Navier-Stokes
Z_S	Relevant component of backscatter signal	RHI	Range height indicator
		RMSE	Root mean squared error
		RV	Radial velocity
		RWF	Range gate weighting function
		SNR	Signal-to-noise ratio
		ST	Simulation truth

TP	True positive (detection)	Est	Estimated
YOLO	You only look once	Lbl	Labeled
Abbreviations		Prt	Port
ConvPool	Convolutional-pooling	Str	Starboard

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Data availability. The generated LES lidar simulator data underlying the results presented in this paper are available in Ref. [55]. The analyzed field measurement data underlying are not publicly available at this time for confidentiality reasons but may be obtained from the authors upon reasonable request.

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