A Note on the Efficient Operation of Quantum Radar and the Fair Classical Comparison

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***Abstract*—At the current state of the scientific discourse on quantum radar, the best understood and experimentally feasible types of implementation are based on two-mode-squeezed-vacuum (TMSV) photon states and aimed at the task of target detection. The operating environment, in which an advantage over classical radar may be attainable, is therefore limited to the extreme regimes of very low signal-to-noise ratios (SNRs) and high thermal noise levels as well as confining the required hardware at mK temperatures. In this work, we approach the open question of how to optimally operate a potential quantum radar system. To this end, we define the optimal operation using the detection advantage against classical radar as well as the efficient usage of the resource measurement time. We show that there is a tradeoff between time efficiency and outperformance of classical radar and specify the conditions for such an operation. Building on this aspect, we investigate the concept of the fair classical comparison to facilitate the understanding of its relation to quantum radar.**

***Index Terms*—Detection, efficiency, optimality, quantum radar.**

I. Introduction

The research on quantum radar has brought forth a range of theoretical concepts such as quantum illumination using Gaussian two-mode squeezed-vacuum (TMSV) photon states (theoretical foundations by Lloyd [1]; continuation by Tan et al. [2]), feed-forward sum-frequency generation (Zhuang et al. [3]), the correlation-to-displacement receiver (Shi et al. [4]), the quantum two-mode squeezing radar (Luong et al. [5]), or the quantum-enhanced noise radar (Chang et al. [6]).

The principal measurement procedure of the current state of quantum radar based on TMSV photon states can be summarized as follows [7]: The transmitter generates pairs of entangled microwave photons, of which one partner photon is directed at a target object, and one is kept in a closed loop within the radar. The photons in the signal stream are scattered by the target object and overlaid by thermal noise before reception, whereas the secondary stream of photons is delayed according to the round-trip time of the measurement channel. After having received the signal photons, a correlation operation is applied to both streams to be used later for decision-making (target present or absent). The joint measurement of both signal streams allows us to fully exploit the initial entanglement at transmission. We refer the reader interested in a more general introduction to quantum radar to the publication by Sorelli et al. [7].

The current state of the experimental realization of quantum radar is centered on the use of TMSV photon states. Notable experiments were conducted by Luong et al. [8], [9], Barzanjeh et al. [10], and Assouly et al. [11].

The potential of quantum radar being able to outperform classical radar in detection applications is typically analyzed using the Chernoff bound and a comparison of the error exponents of the two radar schemes. Tan et al. [2] calculated the maximum possible quantum advantage to be 4 on a linear scale, with experimentally viable concepts being limited at present to a theoretical advantage of at most 2. In the experiment by Assouly et al. [11], an advantage of 0.79 dB was achieved. Note that for the quantum two-mode squeezing [5] and the quantum-enhanced noise radar [6], despite the generation of entangled microwave photons, a receiver has been implemented that leads to the circumstance that a classical radar can be devised that outperforms these proposals [7].

Within the scope of this work, we analyze an explicit implementation of quantum radar used for detection purposes, which relies on the TMSV photon state as the transmit signal and the implementation of a joint measurement operation at the radar receiver. This is, to our knowledge, the most prevalent and well-studied concept in the quantum radar discourse. The particular measurement operator we are referring to is in the following denoted $\hat{c}$ and defined as

$$\hat{c} = \hat{x}_R \hat{x}_I - \hat{p}_R \hat{p}_I. \tag{1}$$

Herein, $\hat{x}_R$ and $\hat{p}_R$ are the quadrature operators of the returned measurement mode and $\hat{x}_I$ and $\hat{p}_I$ are the corresponding quadrature operators of the retained idler mode. The structure of (1) thus resembles a cross correlation operation between the quadrature amplitudes of the measurement and idler modes. Note that (1) was proposed by Las Heras et al. [12] and that their devised concept for possible implementation in the experiment shares similarities with the OPA and PC receivers proposed by Guha and Erkmen in [13]. The specific quantum radar concept analyzed by us is thus fulfilling the two important prerequisites for being categorized under the term quantum illumination and enabling a potential advantage over classical radar [5], [7]: A TMSV photon state is generated at the transmitter, and a joint measurement of the returned measurement and retained idler modes is being performed.

The theoretical research regarding the performance of quantum radar thus relies on the foundations of detection theory and hypothesis testing and entails works by Luong et al. [9], [14], Torromé and Barzanjeh [15], Karsa et al. [16], and Russer et al. [17], for instance. In particular, Luong et al. [18], [19], [20] investigated the dependence of the detection performance on the integration time and the correlation coefficient for a general class of noise-type radars, to which quantum radar belongs to. The authors have shown that the measurement time can be decreased by using quantum radar.

In our previous publication *Engineering Constraints and Application Regimes of Quantum Radar* [21], we provide a detailed mapping of the exact operating regime (measurement time, signal-to-noise ratio (SNR), environmental noise level, target object size, and distance) of the currently experimentally feasible type of quantum correlation radar using TMSV photon states and the measurement operator as defined in (1). A particular aspect of our prior publication relevant to the scope of this work is that we recognized that there exists a unique value for the environmental noise power level, at which the SNR can be chosen maximal to achieve a specific quantum advantage: having too little or too much noise, and this quantum advantage can only be made possible at the cost of reducing the SNR. In [21], we did not discuss this curiosity further, but selected for our analysis the maximally possible SNR value, motivated by the rationale that a higher SNR is always desirable, as it leads to a

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possible reduction of the measurement time while achieving the same detection performance.

In this work, we build upon this theoretical foundation and investigate these aspects in further detail. To this end, we establish the notion of an efficient quantum radar, as one, that is implemented and operated such that for any given quantum advantage, the maximal signal power resources are exploited to operate with the least required amount of signal samples. Thus, we propose to introduce the following attributes to the discourse on quantum radar:

1) *Efficient:* For a desired quantum advantage, a quantum radar achieves any detection error probability in a given operating regime using the least amount of the resource signal samples, measurement time, or repetitions of the experiment.
2) *Superior:* A quantum radar outperforms a classical radar in terms of the achievable detection error probability.
3) *Maximally Superior:* A quantum radar saturates the maximally possible quantum advantage in a given operating regime.
4) *Optimal:* A quantum radar is efficient and maximally superior.

We derive the exact operating regimes in which a quantum radar may be assigned one or more of the proposed attributes. In addition, we relate our theoretical analysis to the experiment by Assouly et al. [11].

Building on the quantum radar aspects of this work, we aim at facilitating the understanding of the concept of the classical comparison radar (see Section IV). To this end, we compare two different points of view on the classical system model and the subsequent derivation of different performance metrics: the unmodulated continuous-wave (U-CW) radar, which is described by classical signal theory, as well as the coherent-state (COH) approach, which is based on quantum mechanical formulations (we refer the reader to our publication [21] for a detailed treatment of these two modeling approaches).

It is known that the quantum mechanically formulated COH state resembles on average a classical electromagnetic field [7]. In regions where the signal power vastly exceeds the unavoidable inherent quantum noise of the coherent state, it becomes possible to instead use a classical U-CW signal model. It is thus said that the coherent-state model is valid at all signal power levels, whereas the U-CW model is only applicable at very high signal power levels. This common knowledge, however, leaves an open question: "Where exactly is the transition between the two valid approaches to formulating a physical model of classical radar?" Furthermore, the question of how such a hypothetical transition border relates to the regime in which quantum radar can outperform classical radar arises.

We describe the exact transition between the classical models and their predictions for the detection performances of classical radar depending on the power level. Furthermore, we can relate our result to the regimes in which a quantum advantage is attainable and show that an efficient quantum radar with a quantum advantage of 3 dB can be compared to a classical radar modeled using the U-CW perspective.

We begin by briefly reviewing the key performance metrics used throughout this work in Section II. In Section III, we present a study on the optimal operation of quantum radar with respect to resource efficiency and outperformance of classical radar. In Section IV, the comparison of the classical modeling approaches is discussed. We conclude and summarize our results in Section V.

II. Performance Metrics

The analysis of the potential of quantum radar to outperform classical radar is typically approached by means of the error exponent ξ and the Chernoff bound with respect to the detection error probability P_E in the special case of binary hypothesis testing [7]: The Chernoff bound is given as

$$P_E(M) \leq \frac{1}{2}\exp\{-M\xi\} \tag{2}$$

with M being the number of integrated samples obtained from successive and identical repetitions of the experiment. This bound is asymptotically tight, meaning that

$$\xi = -\lim_{M\to\infty}\frac{\log P_E(M)}{M}. \tag{3}$$

The quantity referred to as quantum advantage is the ratio of the error exponents of the quantum and classical radar schemes, typically given in dB, providing insights into the potential of quantum radar to outperform classical radar.

The realizable detection performance of any radar, quantum or classical, is fully described by the receiver operating characteristic (ROC), linking the probability of a false alarm event P_{FA} to the probability of detection P_D [22]. In the special cases discussed herein, it can be shown that the relevant ROCs are fully parameterized by the minimal error probability $P_{E,\min}$ [21]. This quantity provides the combined probability of making an erroneous decision, that is, deciding on the presence of a target when there is none (false alarm) or missing a target. The ROCs yield in this case

$$P_D\big(P_{\mathrm{FA}}; P_{E,\min}\big) = \Phi\big(\Phi^{-1}(P_{\mathrm{FA}}) - 2\Phi^{-1}\big(P_{E,\min}\big)\big), \tag{4}$$

with $\Phi(\bullet)$ being the complementary cumulative distribution function of a standard normal distribution and $\Phi^{-1}(\bullet)$ being the respective inverse. Hence, as we have shown in [21], the minimal error probability $P_{E,\min}$ of a scenario with prior probabilities equal to 50% for the presence and absence of a target can be used as the single parameter defining the ROC of the detection problem at hand, even when the prior probabilities are unknown or different from the 50% case. A smaller value $P_{E,\min}$ is always desirable and leads to improved ROCs. We thus proceed in this work to refer to the minimal error probability as the figure of merit for our analyses.

Following the results shown in [21], we express the achievable minimal error probability as

$$P_{E,\min} = \Phi\left(\sqrt{\frac{Mr/2}{p_2}}\right). \tag{5}$$

Herein, r is the SNR observable at the input of the radar receiver without further analog or digital signal processing. The parameter p_2 is introduced to obtain a compact description, applicable to the radar schemes considered in this context (quantum radar using the TMSV photon state for transmission and a receiver performing a correlation operation; classical radar modeled using the coherent photon state; and classical radar modeled using the classical U-CW perspective). This quantity comprises parameters such as the noise photon rate N_{th}, the SNR, and the power loss in the measurement channel κ. Hence, we have that $p_2 = p_2(r, N_{\mathrm{th}}, \kappa)$. We refer the reader to [21, Table I] for further details.

Note that κ comprises all quantities relevant in radar engineering, such as the power level of the transmitter, the gain of the antenna, or the target range, for instance, and can be derived from the radar range equation. In [21], we have focused our analysis on this set of engineering parameters and determined the limits of what can be achieved by quantum radar in terms of the detection range and object size. Within the scope of this work, we consider κ as a parameter but do not determine it any further as the main conclusions that can be drawn from the presented analyses are independent of the exact values of these quantities.

The quantum advantage Q_A is in the following, by referring to our previous publication [21], expressed as

$$Q_A = \frac{p_2^{\mathrm{COH}}}{p_2^{\mathrm{TMSV}}}, \tag{6}$$

with p_2^{TMSV} and p_2^{COH} corresponding to the parameter p_2 computed for the TMSV quantum radar and the classical COH radar, respectively (see [21, eq. (27)]). We kindly refer the reader to [21, eq. (28)] showing the full equation for computing the quantum advantage.

Note that the formula for computing the quantum advantage Q_A is independent of the number of samples. By contrast, the minimal error probability (5) depends explicitly on M. The quantum advantage is, therefore, a measure of the potential ability of quantum radar

to outperform classical radar, while the minimum error probability provides a means of evaluating the specific performance of both quantum and classical radar. We will use these circumstances in the following to provide a quantitative, precise, and comprehensible analysis of the optimal and efficient operation of quantum radar.

III. Efficient Quantum Radar Operation

We begin our analysis of optimality, efficiency, and superiority of quantum radar with a derivation of the efficient operating regime based on our results presented in [21]. To this end, we determine a problem formulation that allows us to obtain the optimal operating environment expressed by the environmental noise photon rate N_{th}, which allows the quantum radar to be operated with the minimally possible error probability and the minimal amount of signal samples required for achieving this. This is possible since we can formulate in the following an expression for the minimal error probability $P_{E,\min}^{\text{TMSV}}$, which depends explicitly on the quantum advantage Q_A, the noise photon rate N_{th}, the total power loss κ, and the number of signal samples M. Thus, it is possible to define a desired quantum advantage for any given power loss in a radar measurement channel and compute the achievable detection performance as a consequence of M and N_{th}.

With the known dependence of the noise photon rate on the temperature T of the measurement environment and the signal frequency f of the radar, given in [7] as

$$\frac{T}{f} = \frac{h}{k_B}\left(\log\left(1+\frac{1}{N_{\text{th}}}\right)\right)^{-1}, \tag{7}$$

it is then possible to select the quantum radar operating parameters using our results. In (7), $h/k_B \approx 0.048$ K/GHz is the ratio of Planck's constant and Boltzmann's constant, respectively.

It is important to note the following restrictions, which apply in the following: the power loss is limited, in a linear scale, to $0 < \kappa < 1$. For the discussed type of quantum correlation radar using TMSV transmit photon states, the quantum advantage is limited to the suboptimal range of $1 < Q_A < 2$. The noise photon rate is lower bounded according to

$$N_{\text{th}} > \frac{Q_A - 1}{2 - Q_A}, \tag{8}$$

which follows from the derivations leading to [21, eq. (34)]. To ensure the applicability of the central limit theorem and the validity of the Gaussian assumption in the derivations presented in [21], it is required that $M \gg 1$. This last requirement is easily adhered to, since the number of signal samples has to vastly exceed 10^6 in moderately demanding measurement situations (−50 dB power loss in the measurement channel) and is chosen far greater than 10^9 in more practically relevant scenarios with −80 dB power loss in the measurement channel (see [21, Fig. 5]).

A. Formulation of an Optimization Problem

With (6) and the result for the parameter p_2^{COH} from our publication [21], the parameter p_2^{TMSV} can be expressed as

$$p_2^{\text{TMSV}}(N_{\text{th}}, Q_A) = \left(1+\frac{1}{2N_{\text{th}}}\right)Q_A^{-1}. \tag{9}$$

By referring to [21, eq. (33)], we have that $r = r(N_{\text{th}}, Q_A, \kappa)$. This relation provides a method for determining the SNR required for achieving a desired quantum advantage Q_A depending on the noise environment quantified by the noise photon rate N_{th} and the total power loss κ in the radar measurement channel. The explicit definition of the function $r(\bullet)$ is not shown here, as it is a rather large and laborious formula. We kindly refer the reader to our prior publication [21] for all details.

By combining (5), (9), and $r(\bullet)$ from (33) in [21], we obtain the minimal error probability $P_{E,\min}^{\text{TMSV}}$ in case of the TMSV quantum radar as

$$P_{E,\min}^{\text{TMSV}}(N_{\text{th}}, Q_A, \kappa, M) = \Phi\left(\sqrt{M\widetilde{r}(N_{\text{th}}, Q_A, \kappa)}\right), \text{ with} \tag{10}$$

$$\widetilde{r}(N_{\text{th}}, Q_A, \kappa) = \frac{r(N_{\text{th}}, Q_A, \kappa)Q_A}{2\left(1+\frac{1}{2N_{\text{th}}}\right)}. \tag{11}$$

Herein, we introduce the parameter $\widetilde{r}$ as the "effective" SNR.

The complementary cumulative distribution function of a standard normal distribution $\Phi(\bullet)$ in (10) is a strictly monotonically decreasing function. Therefore, it is desirable to increase the product $M\widetilde{r}$ to decrease $P_{E,\min}^{\text{TMSV}}$, and that, for any M, it is always advantageous to increase $\widetilde{r}$. Furthermore, it is known from our analysis in [21] that the SNR r can be chosen maximal only for a specific value N_{th}, while, at the same time, achieving a certain quantum advantage on a specific radar measurement channel. In [21], we chose for our initial analysis directly this maximum. However, as it follows from (10), the relevant quantity of interest that has to be maximized is the effective SNR as defined in (11).

It is now possible to formulate an optimization problem that results in the optimal value $N_{\text{th,eff}}$ for the noise environment, which allows to use the minimal number of samples for achieving a desired quantum advantage given a specific radar measurement channel. To this end, (10) is reformulated as

$$M_{\text{req}}\left(P_{E,\min}^{\text{TMSV}}, N_{\text{th}}, Q_A, \kappa\right) = \frac{\left(\Phi^{-1}\left(P_{E,\min}^{\text{TMSV}}\right)\right)^2}{\widetilde{r}(N_{\text{th}}, Q_A, \kappa)} \tag{12}$$

with M_{req} being the required number of samples. The optimization problem is defined as

$$N_{\text{th,eff}} = \arg\min_{N_{\text{th}}} M_{\text{req}}\left(P_{E,\min}^{\text{TMSV}}, N_{\text{th}}, Q_A, \kappa\right). \tag{13}$$

The above equation can be further simplified by recognizing that $\Phi^{-1}(\bullet)$ in the numerator of (12) does not depend on N_{th} and can thus be ignored. Furthermore, the search for the minimum of the reciprocal of a function is, in this case, identical to the search for the maximum of the function (note that $\widetilde{r}(\bullet) \geq 0$). Thus, it follows that

$$N_{\text{th,eff}} = \arg\max_{N_{\text{th}}} \widetilde{r}(N_{\text{th}}, Q_A, \kappa). \tag{14}$$

Note that (14) is independent of the number of samples M.

The optimizer $N_{\text{th,eff}}$ of (13) results as well in the minimal error probability, since we have that

$$\arg\min_{N_{\text{th}}} P_{E,\min}^{\text{TMSV}}(N_{\text{th}}, Q_A, \kappa, M) \tag{15}$$

$$= \arg\max_{N_{\text{th}}} \widetilde{r}(N_{\text{th}}, Q_A, \kappa), \tag{16}$$

which is identical to (14) and thus (13). To arrive at (16) from (15), we used (10) and the mentioned circumstance that $\Phi(\bullet)$ is a strictly monotonically decreasing function. Hence, the minimizer of $\Phi(\bullet)$ is the maximizer of the respective argument, of which only $\widetilde{r}(\bullet)$ depends on N_{th}.

B. Solution to the Optimization Problem

The solution to (14) subject to the restriction (8) is given by (17), as shown at the bottom of the page. In addition to this general result, we introduce two hypothetical cases, being a total blockage of the measurement channel ($\kappa \to 0$) and a loss-less channel ($\kappa \to 1$), to be used later as limiting scenarios. We get

$$N_{\text{th,eff}}^{\kappa\to 0}(Q_A) = \frac{2}{2-Q_A} - 3/2 \tag{18}$$

$$N_{\text{th,eff}}(Q_A, \kappa) = \left(2 - Q_A\left(4 + 3\sqrt{1 + Q_A(\kappa-2) - Q_A^2(\kappa-1) + 2\kappa}\right)\right)\left(2\left(Q_A^2 - Q_A - 2\right)\right)^{-1} \tag{17}$$

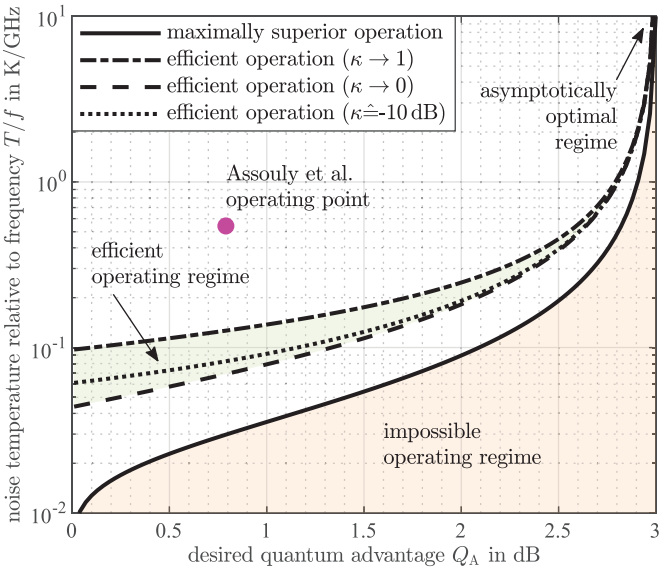

Fig. 1. Mapping of the efficient, optimal, and impossible (see [21]) quantum radar operating regimes.

and

$$N_{\text{th,eff}}^{\kappa\to 1}(Q_A) = \frac{2 - 4Q_A - 3Q_A\sqrt{3 - Q_A}}{2Q_A^2 - 2Q_A - 4} \tag{19}$$

for the two required noise photon rates, respectively.

C. Discussion

The circumstance that in (17), the required noise environment allowing for the collection of a minimal amount of signal samples depends only on the desired quantum advantage and a given radar measurement channel, becomes explicit as $N_{\text{th,eff}} = N_{\text{th,eff}}(Q_A, \kappa)$. Operating a quantum radar in any other noise environment, that is, $N_{\text{th}} \neq N_{\text{th,eff}}$, requires the collection of more signal samples than minimally necessary to achieve the same detection performance. Furthermore, it shall be highlighted that the optimizer $N_{\text{th,eff}}$ does not depend on a specific value of the minimal error probability $P_{E,\min}^{\text{TMSV}}$. Thus, any desired detection performance can be achieved in a time-efficient way by operating in the noise environment defined by $N_{\text{th,eff}}$, if enough signal samples have been collected.

We formalize the introduction of the attributes noted in Section I and summarize the previous prosaic statements as follows:

1) *Efficient:* For any $P_{E,\min}^{\text{TMSV}} \in [0, 1]$, $Q_A \in (1, 2)$ and $\kappa \in (0, 1)$, it follows, by referring to (12), that

$$M_{\text{req}}\left(P_{E,\min}^{\text{TMSV}}, N_{\text{th}}, Q_A, \kappa\right) > M_{\text{req}}\left(P_{E,\min}^{\text{TMSV}}, N_{\text{th,eff}}(Q_A, \kappa), Q_A, \kappa\right) \tag{20}$$

if, by referring to [21, eq. (34)] and (8)

$$N_{\text{th,eff}}(Q_A, \kappa) \neq N_{\text{th}} \in \left(\frac{Q_A - 1}{2 - Q_A}, \infty\right). \tag{21}$$

2) *Superior:* A quantum radar is operated, such that it achieves a quantum advantage $Q_A > 1$.
3) *Maximally Superior:* A quantum radar is operated, such that it achieves the maximal quantum advantage $Q_{A,\max}$, by referring to [21, eq. (36)], of

$$Q_{A,\max} = 2 - \frac{1}{1 + N_{\text{th}}}. \tag{22}$$

4) *Optimal:* Quantum radar is efficient and maximally superior. In the following, we show that this can only be achieved asymptotically.

Fig. 1 shows a mapping of the efficient quantum radar operating regime (green-shaded area) depending on the desired quantum advantage and the power regime quantified as the ratio T/f. The figure includes a mapping of the operating regime denoted as "impossible" (orange-shaded area; see [21, eq. (34)]) as well.

The efficient operating regime does not coincide with the maximally attainable quantum advantage. There is a significant gap between the green-marked area and the border to the impossible operating regime. An efficient quantum radar with $Q_A \hat{=} 3$ dB is only possible with $T/f \approx 20$ K/GHz. Thus, we can bound the asymptotically optimal regime with $Q_A \to 2$ to values $T/f > 20$ K/GHz. With room-temperature noise ($T \approx 300$ K), this corresponds to frequencies $f < 15$ GHz.

The two limiting cases $N_{\text{th,eff}}^{\kappa\to 0}$ and $N_{\text{th,eff}}^{\kappa\to 1}$ act as hard boundaries for the efficient operation of a quantum radar. The dotted line in Fig. 1 marks an exemplary operating case with a power loss of $\kappa \hat{=} -10$ dB. This line coincides well with the lower extreme case $N_{\text{th,eff}}^{\kappa\to 0}$ for desirable quantum advantages of $Q_A > 2$ dB. We may, therefore, in good approximation, use this bound as representative of practically relevant scenarios with expected power losses of $\kappa \ll -10$ dB.

Let us discuss the experiment by Assouly et al. [11] indicated in Fig. 1 by the magenta-colored dot. The devised experimental quantum radar is superior compared to a classical radar with a quantum advantage of $Q_A \hat{=} 0.79$ dB [11]. However, in an operating regime of $T/f = 0.542$ K/GHz [11], we see that Assouly et al.'s quantum radar [11] is not maximally superior. Based on our analysis, the quantum radar is not efficient also.

To operate Assouly et al.'s quantum radar [11] efficiently, one might reduce the noise temperature or increase the signal frequency such that the magenta dot is shifted vertically into the green regime. Another possibility would be to make improvements to the experiment setup, such that a higher quantum advantage is attainable (e.g., by decreasing system imperfections that lead to unwanted power losses). This would translate the dot horizontally to the right.

IV. Fair Classical Comparison

In the following, the question of the discrepancy between the performance predictions derived from the COH and U-CW models is approached from the perspective of hypothesis testing and detection theory. The detection performance is expressed using the ROC, which, in the discussed cases, can be quantified by a single quantity being the minimal error probability $P_{E,\min}$ [21]. The discrepancy between the quantum mechanical COH model and the classical U-CW model can thus be formalized by comparing the two resulting predictions for the minimal error probability.

A. Model Mismatch

We define $\Delta P_{E,\min}$ as the mismatch between the detection performance predictions as

$$\Delta P_{E,\min} = P_{E,\min}^{\text{COH}} - P_{E,\min}^{\text{U-CW}}. \tag{23}$$

Herein, $P_{E,\min}^{\text{U-CW}}$ is the minimal error probability of the classical radar derived using the classical signal theoretical model (U-CW perspective), and $P_{E,\min}^{\text{COH}}$ is the analog quantity derived from the quantum mechanical COH formulation. With the results of our prior work [21], the minimal error probabilities yield

$$P_{E,\min}^{\text{U-CW}} = \Phi\left(\sqrt{Mr/2}\right) \tag{24}$$

and

$$P_{\text{E,min}}^{\text{COH}} = \Phi\left(\sqrt{\frac{Mr/2}{1 + \frac{1}{2N_{\text{th}}}}}\right). \tag{25}$$

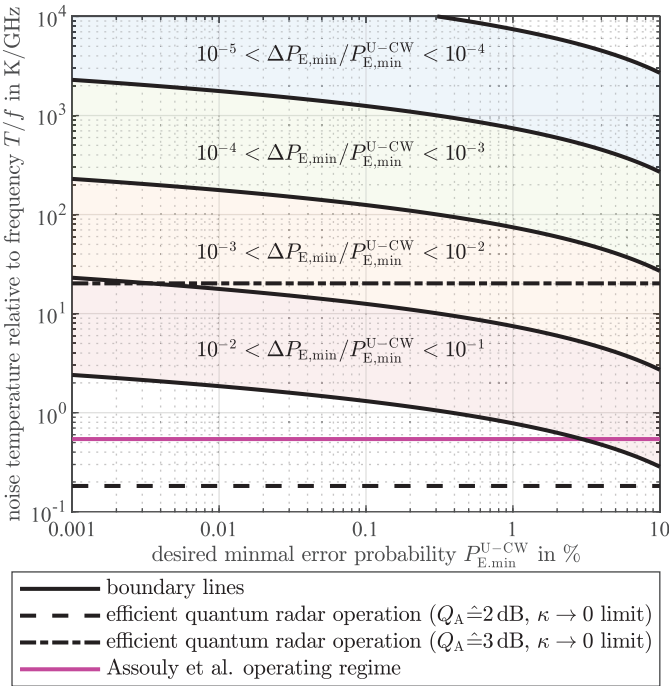

Fig. 2. Mapping of the classical radar operating regimes depending on the discrepancy between the quantum and classical model predictions.

The range of values of the discrepancy $\Delta P_{E,\mathrm{min}}$ is determined by recalling that $N_{\mathrm{th}} > 0$, with which it follows that

$$1 + \frac{1}{2N_{\mathrm{th}}} > 1. \tag{26}$$

By comparing (24) and (25), it follows that

$$P_{E,\mathrm{min}}^{\mathrm{U\text{-}CW}} < P_{E,\mathrm{min}}^{\mathrm{COH}} \quad \Leftrightarrow \quad \Delta P_{E,\mathrm{min}} > 0. \tag{27}$$

The classical prediction $P_{E,\mathrm{min}}^{\mathrm{U\text{-}CW}}$ of the detection performance of a classical radar thus represents a lower bound on the predicted detection performance. Given that the quantum mechanical point of view $P_{E,\mathrm{min}}^{\mathrm{COH}}$ is applicable in every power regime N_{th}, and that, with [21, eq. (12)], $P_{E,\mathrm{min}}^{\mathrm{COH}} \to P_{E,\mathrm{min}}^{\mathrm{U\text{-}CW}}$ for high power levels $N_{\mathrm{th}} \to \infty$, we conclude the following: The detection performance of the classical radar degrades (i.e., the error probability increases) by decreasing the absolute power level of the radar operating point (i.e., reducing N_{th}), while leaving the SNR r constant.

Combining (23)–(25) and solving for N_{th} yields

$$N_{\mathrm{th,req}} = \frac{1}{2}\left(\left(\frac{\Phi^{-1}\left(P_{E,\mathrm{min}}^{\mathrm{U\text{-}CW}}\right)}{\Phi^{-1}\left(P_{E,\mathrm{min}}^{\mathrm{U\text{-}CW}} + \Delta P_{E,\mathrm{min}}\right)}\right)^2 - 1\right)^{-1}. \tag{28}$$

Herein, $N_{\mathrm{th,req}}$ is the required power level, for which the predictions of the detection performances using the classical U-CW model and the quantum mechanical COH model are sufficiently identical up to a discrepancy $\Delta P_{E,\mathrm{min}}$ and an error probability $P_{E,\mathrm{min}}^{\mathrm{U\text{-}CW}}$ predicted by the classical model.

B. Discussion

Fig. 2 shows the required noise level $N_{\mathrm{th,req}}$ (expressed as T/f; see (7)) and the desired detection performance for different relative prediction accuracies $\Delta P_{E,\mathrm{min}}/P_{E,\mathrm{min}}^{\mathrm{U\text{-}CW}}$. The dashed and dashed-dotted black lines indicate the T/f regimes for an efficient quantum radar operation.

An efficiently operated quantum radar with a quantum advantage of $Q_A \hat{=} 2$ dB may, according to Fig. 2, only be compared to a classical radar whose performance prediction is not explainable by a purely classical system model. In the case of an efficient quantum radar with $Q_A \hat{=} 3$ dB, the classical radar model predictions match by two to three orders of magnitude.

With an increasing superiority of the quantum radar, while operating efficiently, the classical radar can be described with increasing precision by a classical signal model. We may, therefore, distinguish between classical in a physical sense, meaning the absence of quantum mechanical phenomena such as entanglement, and classical in an engineering sense, meaning that the radar is sufficiently described by classical physics.

The colored line in Fig. 2 marks the operating regime of the experiment by Assouly et al. [11]. Note that the predictions for the classical radar are deviating by approximately more than one order of magnitude. Consequently, a classical comparison experiment might have to be implemented in the same cryostatic environment as the quantum radar.

V. Conclusion

We have shown that, by referring to Section III, although quantum radar is always superior to classical radar, it cannot be maximally superior and efficient at the same time. Hence, the cost of operating a quantum radar maximally superior is a loss of efficiency in terms of the measurement time or the number of signal samples necessary to achieve a desired detection performance. Maximally outperforming a classical radar may therefore not be desirable in every operational scenario. We may attribute quantum radar as being asymptotically optimal with $Q_A \to 2$ in an operating regime of $T/f > 20$ K/GHz. Note that these considerations are valid for an ideal implementation of quantum radar, whereas a real system might suffer from an additional performance degradation. Our results act therefore as absolute operational limits.

The implications of our results presented in Section IV are twofold. We have shown that the choice to model the classical radar's detection performance using classical signal theory or a quantum mechanical description impacts the predicted detection performance depending on the environmental conditions and therefore the power level in which a potential experiment is set up. This implies that, if a classical radar were to be implemented in a comparison experiment, great care has to be taken to adjust the environmental conditions in conjunction with the quantum radar experiment. If the classical model describes the classical radar sufficiently precise in the quantum radar's operating regime, the classical radar may be set up under relaxed conditions, possibly at room temperature. The second implication refers to the point of view of a classical radar operator: an ideal and optimal quantum radar compares to a sufficiently classically described classical radar, and therefore to a real operational system.

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