

An analysis of inter-satellite link topologies in future GNSS constellations: operational constraints and figures of merit.

Giulia Schievano, Gabriele Giorgi, Grzegorz Michalak

Institute of Communications and Navigation German Aerospace Center (DLR)

BIOGRAPHY

Giulia Schievano holds an M.Sc. in Aerospace Engineering from the University of Padova, Italy. Following her master's degree, she worked as a post-graduate researcher at CISAS (Centro di Ateneo di Studi e Attività Spaziali "Giuseppe Colombo") in Padova, focusing on multi-GNSS positioning. She joined the System Evolution group of the German Aerospace Center (DLR) in 2022, where her current field of research centers on orbit determination with inter-satellite and ground-to-satellite links. Concurrently, she is pursuing her PhD at the Technical University of Munich (TUM), under the Satellite Geodesy Chair.

Gabriele Giorgi received a M.Sc. in Space Engineering from the University of Rome "Sapienza" (Italy, 2006), and a doctorate in Aerospace engineering from the Delft University of Technology (the Netherlands, 2011). He is now a senior researcher at the German Aerospace Center (DLR) in Weßling-Oberpfaffenhofen (Germany), where he is involved in the development of technologies and architectures for next generation navigation satellite systems

Grzegorz Michalak received his Ph.D. in Physical Sciences (Astronomy) in 2000 from Wroclaw University, Poland. In 2001 he joined the orbit determination group at the German Research Centre for Geosciences (GFZ) in Potsdam, Germany, where his main activities focused on precise orbit determination of GNSS and LEO satellites using real and simulated data. In 2021 he joined the German Aerospace Center (DLR), where he works on Galileo evolution concepts exploiting inter-satellite links.

ABSTRACT

Future Global Navigation Satellite Systems (GNSSs) will integrate advanced technologies and methodologies to enhance the accuracy, robustness and resilience of navigation services. A key and promising innovation in this context is the integration of Inter-Satellite Links (ISLs), which facilitate low-latency, high-rate data transmission and accurate time and range measurements between satellites. One of the crucial aspects in the integration of ISLs is the determination of their connectivity strategies. In this study, we focus on the scheduling and planning of ISLs within a constellation modeled after the Galileo system, consisting of 24 Medium-Earth-Orbit (MEO) satellites, equipped with 2 terminals each. Our approach investigates the conditions under which a required topology (or contact plan) can be achieved, given a number of physical and operational constraints. To delineate the requirements for establishing an effective contact plan, we first define the limitations derived from mechanical and physical characteristics of the ISL terminals used. These constraints include the terminals' field-of-regard (FOR), their transmission power levels limiting the maximum ISL range attainable, and potentially additional restrictions on the physical layer. The core of the scheduler consists in creating a graph which encompasses all the feasible links under the constraints outlined above. With two terminals per satellite, it becomes theoretically possible to establish a path connecting all satellites in the constellation in a closed chain. The close path topology allows the possibility to relay data with low latency enabling distributed clock solutions, tight system synchronization, high density of ISL measurements, and the implementation of closed-loop checks for time dissemination. In this paper, we present a scheduling strategy that ensures a systematic and full exploration of the initial graph and its potential topologies. The combination of constraints under which suitable closed topologies are found has been thoroughly analysed and the operational boundaries (in particular in terms of physical limitations on terminals) is outlined and discussed in detail.

I. INTRODUCTION

Global Navigation Satellite Systems (GNSS) are supported by a global network of sensor stations, communication stations and data processing facilities. Sensor stations collect GNSS measurements, specifically L-band pseudo-ranges (code) and carrier phase data. These measurements are then gathered by the processing facilities to perform Orbit Determination and Time synchronization (ODTS). The ODTS estimates a large number of parameters, including satellite state vectors, parameters of dynamical models (e.g. solar radiation pressure), troposphere delays, station coordinates, satellite and station clocks, phase ambiguities etc. These computations cannot be performed autonomously onboard in current GNSS architectures, due to limited computational power and a lack of low-latency communication channel linking ground and space segments. Hence, the ground

network is necessary to provide this service and to support the constellation. However, it is expensive and challenging to maintain a global coverage. Inter-Satellite Links (ISLs) in GNSS will unlock a manifold of new possibilities and opportunities to improve the performance of modern navigation systems. When deployed, ISLs will provide a way to achieve autonomous and efficient data dissemination in space, with the support of a reduced ground segment. Furthermore, a new type of measurements such as inter-satellite (pseudo)ranges, (pseudo)range rates, and clock offsets will be available to be integrated in the ODDS process (see Fernández (2011)). This would enhance the precision and accuracy of orbit and clock determination, as shown in Michalak et al. (2021) and Schlicht et al. (2020). Another key application lies on two-way time transfer (TWTT) enabled by ISLs: TWTT makes possible to retrieve clock offsets with sub-ns accuracy, which can be used for a synchronization algorithm based, for example, on a distributed clock ensemble approach (see Trainotti et al. (2022)).

Given their potentials and applications, ISLs have been extensively studied since the 1990s, when, Ultra-High-Frequency (UHF) cross-link measurement and data communication system were introduced for GPS Block IIR satellites to implement the AUTONAV function. These satellites were able to provide navigation function for 180 days without a ground contact, thanks to long term prediction ephemeris and the integration of inter-satellite (pseudo)range measurements to aid the satellites' autonomous orbit prediction (see Menn and Bernstein (1994)). Now that we are facing the rise of a new generation of modern navigation systems, ISLs have been again considered, planned or applied to reach better performances levels in terms of accuracy, robustness, autonomy and integrity (for example for Galileo 2nd Generation, BeiDou and other commercial communication constellations such as Starlink and IRIS²).

In this paper, we analyse the problem of contact plan design of inter-satellite links, i.e., how to organise ISL connectivity in time and space considering the terminals' hardware limitations and the physical constraints. Specifically, we characterize the terminal field-of-regards by maximum elevation and azimuth angles, defined with respect to the satellite's body reference frame, which describe the terminal's angular operational envelope. In this way, the constraints are applicable both to optical and radio frequency (RF) terminals. We consider also maximum ISL range attainable which reflects transmission power limits. Another desirable option is the implementation of simultaneous two-way links, which offer advantages especially in terms of time synchronization (enabling simultaneous two-way time transfer). To establish such links with optical terminals, it is necessary to utilize hardware capable of emitting polarized signals or operating at distinct wavelengths. However, this approach restricts communication to complementary terminals pairs that transmit and receive signals with matching polarization or wavelength on the transmit and receive lines, respectively. The impact of this constraint on the construction of ISL topologies is also analysed. Finally, we also integrate in the ISL scheduling a minimum required duration for the persistence of the link. This accounts for the time needed for the ISL acquisition and tracking, before useful data can be exchanged, ensuring also that the link can be maintained for a sufficient duration to support stable communication and measurement generation. The shape of the resulting topology is influenced by these constraints and, in the first place, by the number of terminals on board the satellite. In this study, it is considered to embark two identical terminals per satellite. The presence of a second terminal permits the formation of closed topology chains (as in Figures 1a and 1b), defined as a closed paths that pass through all satellites once, starting and ending at the same node (i.e., Hamiltonian cycle). Maintaining a closed topology introduces the need of a precise contact plans

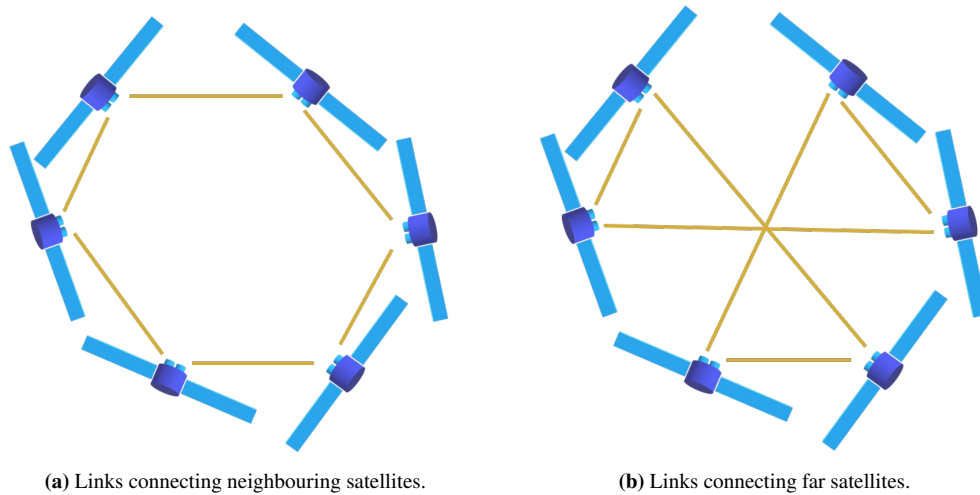


Figure 1: Two example of Hamiltonian cycles.

to program satellites' operations and organize the topologies' switches according to the variability of the visibility constraints. This study aims at investigating the contact plan problem starting from a graph theory perspective, considering various

constraint scenarios. The primary focus is determining the lower bound of the operational envelope for which closed topologies can be found.

This paper is organized into the following sections: Section II introduces the essential notions and terminology of graph theory, establishing a common ground for the study; Section III details the simulation set-up, with focusing on the definition of the critical constraints; in Section IV, we provide a detailed description of the scheduling algorithm's steps, and, finally, in Section V, the results obtained are presented, providing a comprehensive overview of the study's findings.

II. GRAPH THEORY AS A FRAMEWORK

Graph theory provides the underlying methodology for studying the contact plan design problem. It offers a robust and formally established framework for addressing a variety of complex problems, including those related to networks. Specifically, graphs facilitate the analysis of connectivity, pathfinding, distances, and other network-related properties. In this context, a graph is mathematically defined by two essential components: nodes and edges. Nodes represent distinct entities or objects and are typically illustrated as points, while edges symbolize the relationships or interactions between the nodes in form of lines or curves.

Graphs can be classified based on several attributes, as in Diestel and Diestel (2017):

- *directed or undirected*: depending on whether or not the edges have an associated direction;
- *weighted and unweighted*: whether edges have a numerical weight, which quantifies the strength or significance of the connection between vertices;
- *sparse or dense*: refers to the relative number of edges compared to the number of nodes. A sparse graph has relatively fewer edges, while a dense graph has many.

In the context of satellite constellations, the computed graph encompasses all feasible connections (edges) between the satellites (nodes). This graph can be classified as undirected and unweighted, while its density is contingent upon the specific constraints applied and the scenario analysed. From this initial graph, the desired topology, i.e., the actual links to be established, is extracted thanks to the search function of the algorithm. This solution or topology is a so-called Hamiltonian path, i.e. a path that visits all satellites once; Hamiltonian paths can be open chains or close (cycles), which means that the chain starts and ends at the same satellite.

Determining the existence of closed paths is classified as an NP-complete problem, i.e., non-deterministic polynomial time. For this class of problems there is no known efficient way to find a solution in polynomial time, if no other assumptions or heuristics are applied. While numerous algorithms exist to deal with NP-complete problems and path search, we select our solution applying a systematic and exhaustive scan of the graph. This exhaustive scan verifies the existence of at least one topology, even in scenarios where the constraints are particularly stringent. Since a complete exhaustive search of the full graph is numerically unattainable in most cases, we have introduced an user-defined timeout to halt the search, as we will discuss in Section IV. Therefore, our primary objective is not to identify the topology that minimizes a specific figure of merit, (focus of previous studies as e.g. Nardin et al. (2021)), rather, we aim to investigate the conditions under which closed topologies can be established, thereby providing a fundamental understanding of the problem of the ISL scheduling problem in a GNSS constellation of satellites each equipped with two ISL terminals.

III. SIMULATION SET-UP

Satellites' positions and orientations are required for all analysed epochs to design the constellation's contact plan. The required inputs are obtained through simulation: a constellation modeled similarly to Galileo, in terms of orbital slots and attitude, is created via GROOPS (Gravity Recovery Object Oriented Programming System, a geodetic toolkit for gravity field recovery and GNSS processing, see Mayer-Guerr et al. (2021)). Orbit, attitude and constellation parameters used in the simulation are given in Table 1. The orbits are simulated for a period of one day.

The final topology is highly dependent on the constraints considered, which also affect the success of the searching process (i.e., the process of scanning the initial graph to find a close topology). The constraints considered are defined by a series of parameters summarised in Table 2. For each of the parameters listed, a concise description is provided, along with the corresponding values used in the simulation environment.

Field-of-regard: maximum azimuth and maximum elevation We assume to embark two identical terminals onboard each satellite. The line-of-sight (LOS) between the two satellites is specified through elevation and azimuth angles, characterized with respect to satellites' body frame (i.e., the frame defined by the three orthogonal axes x , y and z integral with the satellite body, with z pointing to the nadir direction). The elevation is the angle between the LOS and its projection on x - y plane, while the azimuth is the angle between the x axes and the LOS projection. To define the field-of-regard, we set the maximum azimuth

Table 1: Keplerian orbital parameters, constellation definition and satellite’s attitude law.

Parameter	Symbol [unit]	Values
Semi major axis	a [km]	26,560
Eccentricity	e [-]	0
Inclination	i [deg]	55
Right ascension of ascending node	Ω [deg]	0, 120, 240 (for different orbital planes)
Argument of perigee	ω [deg]	0
Mean anomaly	M [deg]	0, 45, ... 360
Constellation definition	Walker δ 24/3/1	
Attitude law	Smoothed yaw steering	

Table 2: Factored operational constraints used in the analyses. Besides some preliminary analyses, the link preference has been set always to the longest link.

Parameters	Abbreviation [unit]	Values
Maximum azimuth	$MaxAz$ [deg]	90, 135, 180, 225, 270, 315, 360
Maximum elevation	$MaxEl$ [deg]	30, 45, 60, 75, 90
Maximum range	R_{max} [km]	25,000, 30,000, 60,000
Minimum persistence	t_{min} [min]	1, 10, 20, 30, 40, 50, 60
Complementary flag	fl_{compl}	activated, deactivated
Link preference	$link\ guide$	shortest, longest

$MaxAz$ and the maximum elevation $MaxEl$ to delimit the solid angle which the terminals can sweep. It should be noticed that in this paper we do not consider limitations due to obstruction of solar panels or other satellites’ elements.

The visibility constraints are specified in a general manner, applicable to both optical and RF terminals. In the case of optical terminals, which produce narrow beams, these angles represent the capability of a mechanical gimbal platform to scan a three-dimensional space, offering two degrees of freedom. For RF systems, which typically generate broader beams, the directional and shape characteristics of the antenna pattern are often achieved using an antenna array (with or without mechanical rotations). This antenna array consists in multiple spatially arranged elements and it produces a focused radiation pattern using phase control (see Ehyae (2011)). The values of $MaxAz$ and $MaxEl$ considered in the simulations are specified in Table 2. We decided not to analyse elevation angles < 30 deg and azimuths < 90 deg, because of the results obtained by Deprez and Giorgi (2021), where it was accessed that below that threshold the co-visibility was not sufficient to find close topologies.

Maximum range In order to successfully establish a link, the satellites must be within the maximum range R_{max} attainable for a given transmit power. In our simulation, R_{max} varies between 25,000 km and 60,000 km. The initial value of 25,000 km is chosen to be slightly larger than the distance between two adjacent inter-plane satellites (22,500 km), and the larger distance referring to the upper bound of the range between any two satellites of the Galileo MEO constellation.

Minimum link persistence We also integrate in the ISL scheduling a minimum required duration for the persistence of the link. This accounts for the time needed for the ISL acquisition and tracking, before useful data can be exchanged, ensuring also that the link can be maintained for a sufficient duration to support stable communication and data exchange. After the t_{min} interval, the link can be extended in time, provided that the visibility requirements are met. We assess the impact of t_{min} ranging from 1 to 60 minutes.

Complementary flag The implementation of simultaneous two-way links with optical terminals may require separating transmit and receive paths in wavelength and/or polarization. To establish such links, it is necessary to utilize terminals capable of emitting polarized signals and/or operating at distinct wavelengths. However, this approach restricts communication to complementary terminals pairs that transmit and receive signals with matching polarization or wavelength on the transmit and receive lines, respectively. This statement is true if the lines cannot be swapped due to technical design limitations. The impact of this constraint on the construction of close ISL topologies is also analysed. We set a flag fl_{comp} that permits to switch from any-to-any connections to complementary-only connections, and both scenarios are analysed.

Link length priority This parameter guides the search process by prioritizing longer or shorter links. When the flag is set to “longest link”, the satellite prioritizes connections to the farthest visible satellite, tending to facilitate inter-orbital plane connections. Conversely, when the “shortest link” option is selected, intra-orbital plane connections are generally selected,

with the exceptions of a few inter-plane links selected to enable a fully connected topology. The first option is selected for the majority of the analyses, as it produces the topology with the most diverse geometry.

IV. ALGORITHM DESCRIPTION

In this section a detailed description of the algorithm steps is provided; the key points of the process are the graph creation, the graph pruning, the path search and, when it occurs, the backtrack process. We also provide a motivation for the timeout parameter. Each of these steps is explained in the following paragraphs.

Graph creation In this step, the initial graph is generated with all satellites as nodes and all possible links as edges. The constraints that are applied and checked at this stage are:

1. **Earth masking:** the link between two satellites must not be obstructed by the Earth and the atmosphere to avoid signal distortion and attenuation. To satisfy this condition, the nearest point of the link's segment must be at least 1,000 km above the Earth's surface.
2. **Field-of-regard:** satellites must fall in mutual terminal field-of-regards, ensuring that the link can be established.
3. **Range limitations:** satellites must be close enough to establish a connection with reasonable transmit power, which translates into a maximum allowable range between the two satellites.

Graph pruning The graph is pruned to retain a subset of links meeting specific conditions, ordered to prioritize certain connections. This involves three pruning stages and a final check:

1. **First pruning phase:** it eliminates links leading to dead ends, i.e., edges leading to satellites that have no other links; it also detects and reports the presence of satellites without links or connecting to only one other satellite, throwing a warning and ending the computation.
2. **Second pruning phase:** it computes links duration and retains only those exceeding the minimum persistence threshold t_{min} .
3. **Third pruning phase or Treecheck:** it ensures no isolated satellites remain after the second pruning phase and before the search begins. If an isolated satellite is encountered, the algorithm throws a warning and stops, because the constellation cannot be fully connected.
4. **Dirac Theorem check:** it is possible to apply Dirac's theorem to the undirected, unweighted graph that was created in the previous steps. This theorem states that if a graph has N nodes, each of them must have at least $N/2$ degrees of freedom (i.e., edges), to be guaranteed the existence of a Hamiltonian path. If this condition is not met, the path could still be found, so this theorem provides a sufficient, but not necessary condition to prove the existence of a solution. However, if the condition is not met, the algorithm notifies that the calculation may take longer than expected. This check has no critical role in the algorithm flow, but it gives a preliminary information on the complexity of the scenario analysed and the expected computation time.

Path search The search for a path is performed systematically, examining all potential ISLs until a solution is found, as shown in Fig. 2. Initially, a random satellite is chosen as a first node, followed by subsequent satellites from the list of those visible and available from the terminal (i.e., not already linked to others). This process continues until the search either loops back to the first node, closing the path, or cannot proceed further. Specifically, this means that the selected node has no linked satellites or all potential connections are already occupied. In this case a 'dead end' is encountered and a backtrack process is necessary. This involves 'undoing' the last iteration, deleting the last link (which led to the dead end) from the topology, and restoring the satellites' and terminals' lists.

The search concludes with: a) the closed path is found, or b) the timeout is reached before achieving the desired result.

Backtrack process The backtrack process consists in trying different options and undoing them if they lead to a dead end. When this occurs, the algorithm backtracks to the previous decision point and explores a different path, changing the chosen satellite. The number of backtracks performed can be interpreted as a measure of the complexity of the scenario analysed. Theoretically, since the backtrack process is employed, the search can continue until at least one path is found (if it exists).

Timeout The computational time tends to significantly increase when all the constraints in Table 2 become stringent, since the graph search extends in depth with a growing number of backtracks. For this reason a timeout has been set for each analyses to 2 minutes. Without this timeout, the computation time could reach several days per scenario. When the timeout is reached the search is stopped without having found a closed path. Since path-finding is an NP-problem, there are no theorems providing a necessary and sufficient condition to prove the existence of a solution from the initial graph. Thus, the timeout is necessary,

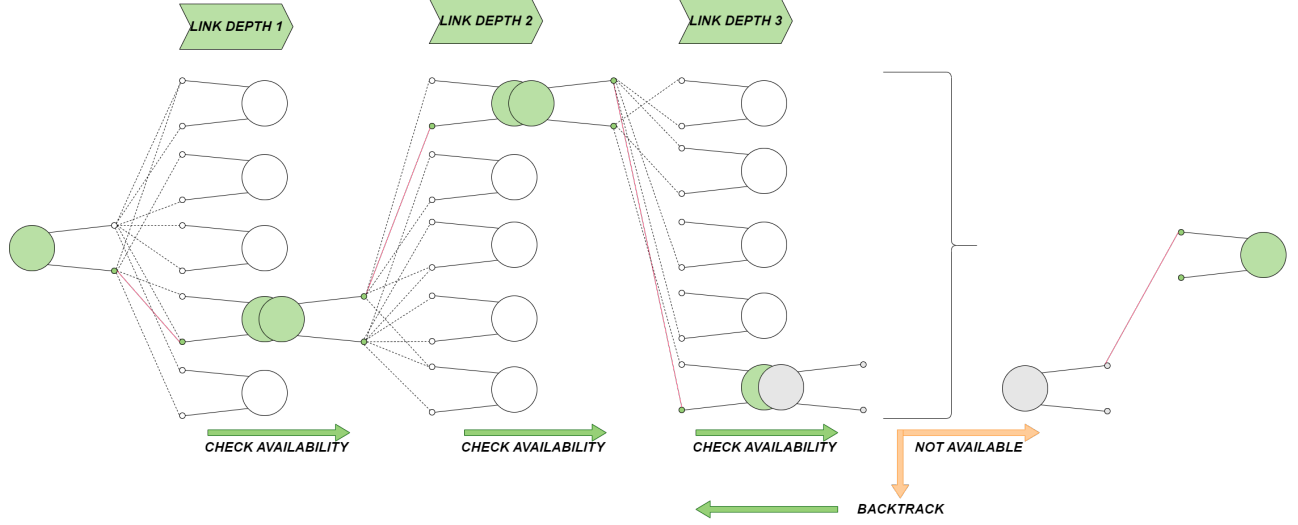


Figure 2: Hamiltonian search path. The first random node connects via first terminal to the first terminal of another node, selected from the list of the visible ones. The actual link established is shown in red. The link depth parameter indicated the number of iteration the algorithm performed, at this point *link depth* = 1. The second node connects to a free terminal of an next free node fulfilling again the link length requirement. If no satellites or terminals are available anymore in the list of the last node, a backtrack is performed, restoring the previous *link depth* and deleting the last edge and the last node from the search tree. The search continues until a solution it is found and the last link created connects the second terminal of the first node selected.

since the number of combinations between links is at most $O(N - 1)! = O(10^{22})$, with $N = 24$ (for a complete graph). The order of magnitude of the number of possible Hamiltonian paths, given the initial graph, is discussed in Section V (Table 4).

V. RESULTS

This Section presents results and some findings on graphs and topology structures: the first two sections provide relevant observations and insights obtained inspecting the graphs and topologies, while in the third we examine the impact of the operational constraints on the resulting topology and on the search output.

Preliminary Graph Results The matrix plots in Figure 3 show the satellite mutual co-visibility for snapshots taken every 2.5 hours, selecting a hemispherical field-of-regard and $R_{max} = 60,000$ km. Many of the possible links (in green) persist in the graph for extended periods, up to 24 hours. Hence, they would not be discarded during the second graph pruning (i.e., in the algorithm phase in which the link persistence is checked). Consequently, selecting those active links, it is possible to find an any-to-any closed topology, that would last up to 24 hours. Examples of such long-lasting topology can be seen in the matrix plot 4a and 4b, the former produced setting the link priority to “longest”, the latter on “shortest” (the Figures 4a and 4b adopt the same color code of 3).

Table 3: Overall number of links present in the graph for epoch 1 for varying field-of-regards and maximum distance set to 60,000 km.

El [deg] Az [deg]	15	30	45	60	75	90
45	0	24	38	56	80	80
90	8	48	86	128	168	168
135	8	80	156	240	312	312
180	12	120	258	408	516	516
225	12	120	258	408	516	516
270	12	120	258	408	516	516
315	12	120	258	408	516	516
360	12	120	258	408	516	516

In Figures 5a, 5b and 5c, graphs with varying field-of-regard are shown. As expected, the narrower the field-of-regard, the

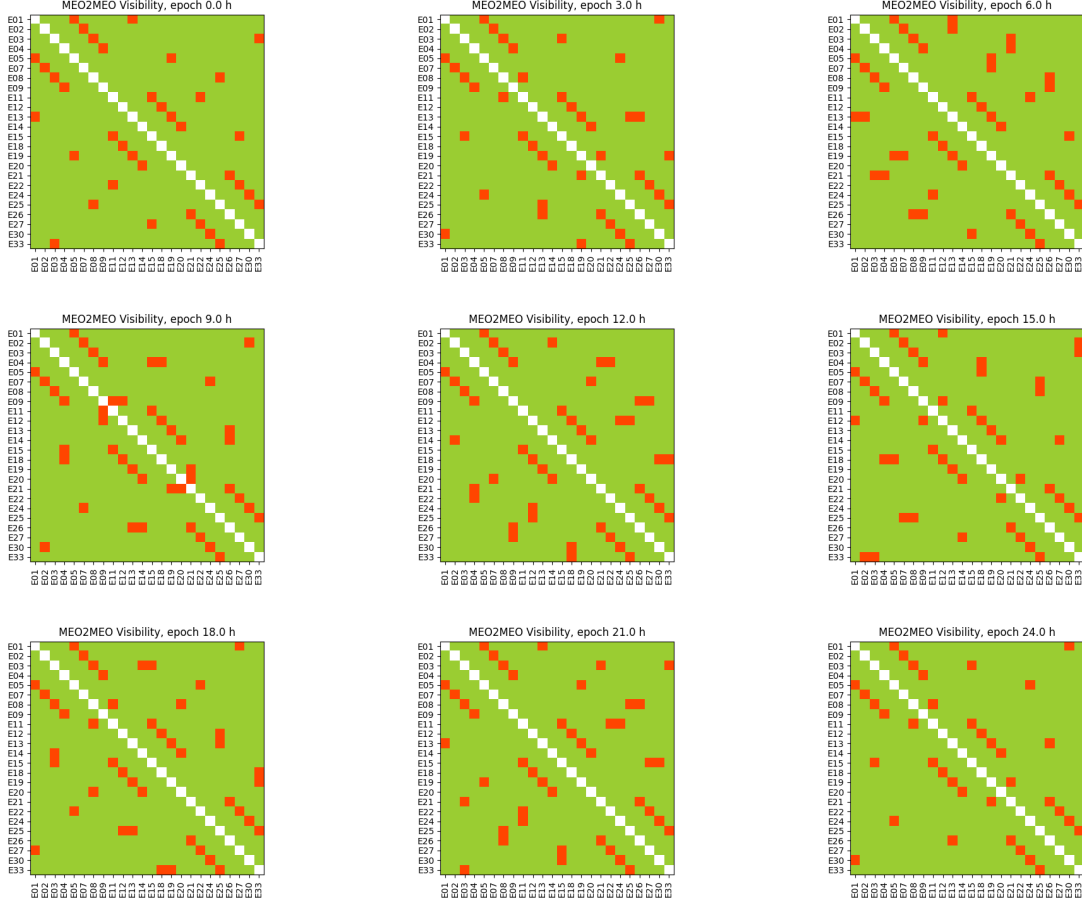


Figure 3: Co-visibility matrices with hemispherical field-of-regard (red not visible, green visible).

sparser the final graph will be (i.e., less number of possible connections). Finding a topology starting from a dense graph (like the one in Figure 5c), it is much easier with respect to the former. Denser connectivity offers more options and possibilities for path exploration, making it easier to find a close topology. The overall number of potential links for varying $MaxAz$ and $MaxEl$ is also shown in Table 3. It can be noticed that the number of links increases until certain field-of-regard amplitudes and then it remains constant.

Table 4: Order of magnitude of possible paths explorations for epoch 1 for varying field-of-regards and maximum distance set to 60,000 km.

El [deg] Az [deg]	15	30	45	60	75	90
45	0	0	0	0	0	0
90	0	0	0	0	0	0
135	0	0	$O(10^9)$	$O(10^{14})$	$O(10^{16})$	$O(10^{16})$
180	0	$O(10^7)$	$O(10^{14})$	$O(10^{19})$	$O(10^{21})$	$O(10^{21})$
225	0	$O(10^7)$	$O(10^{14})$	$O(10^{19})$	$O(10^{21})$	$O(10^{21})$
270	0	$O(10^7)$	$O(10^{14})$	$O(10^{19})$	$O(10^{21})$	$O(10^{21})$
315	0	$O(10^7)$	$O(10^{14})$	$O(10^{19})$	$O(10^{21})$	$O(10^{21})$
360	0	$O(10^7)$	$O(10^{14})$	$O(10^{19})$	$O(10^{21})$	$O(10^{21})$

We also compute the number of possible paths that can be explored. Given a complete graph, the total number of possible paths is $n_{paths} = (N - 1)!$, with N number of nodes/satellites. Since our graphs are not complete, even in the hemispherical field-of-regard case, we have that $n_{paths} = N_{links,sat_1} \cdot N_{links,sat_2} \cdot \dots \cdot N_{links,sat_{24}}$, excluding reciprocal links. The result of this computation can be found in Table 4 (order of magnitude). For such large set of possible paths to be explored no algorithm

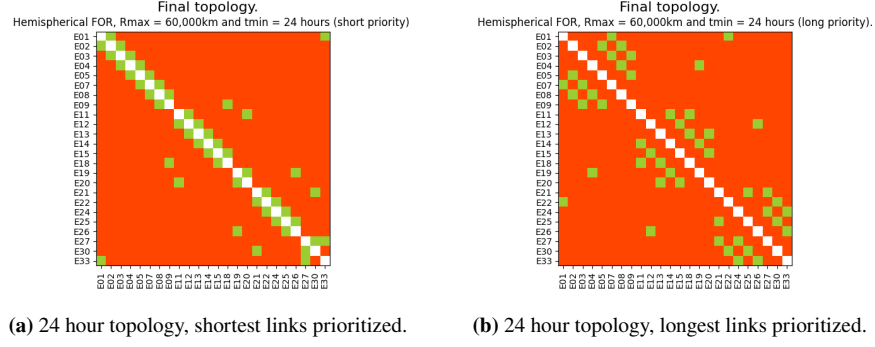


Figure 4: Two example of topologies lasting 24 hours, 4a obtained with short link priority and 4b with long link priority (red not established, green established).

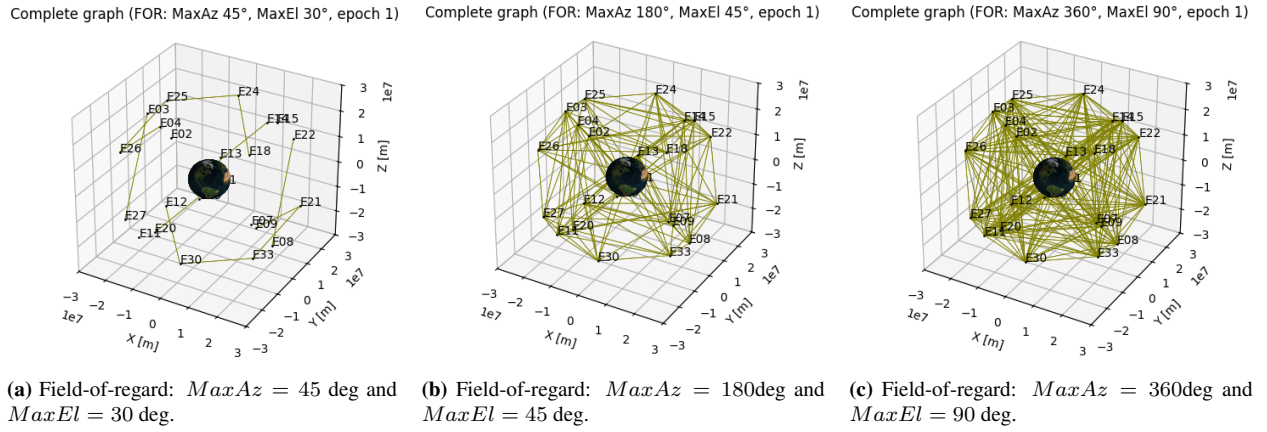


Figure 5: Initial graph with all possible connections with different field-of-regards. Graph obtained for time slot number 1 and $R_{max} = 60,000$ km.

can perform a complete search, therefore, we set the timeout.

Topology shapes and link priority In this section we analyse the outcome of the search when using some particular constraints: specifically, we assess the boundaries of the co-visibility. Additionally, the impact the link guide on the shape of the final topology is analysed.

A preliminary analysis was carried out with a hemispherical field-of-regard, a link duration of 1 minute, a complementary flag deactivated (i.e. all terminals can connect with any-to-any configuration), and a maximum range $R_{max} = 25,000$ km. As the average distance to a neighbour satellite is 22,500 km, we anticipate identifying a topology shaped mainly by links between adjacent co-planar satellites. This resulting topology is illustrated in Figure 6a. Similar consideration can be done when limiting the field-of-regard, especially the $MaxEl$, to angles close to 22.5 deg, which is the angle with which a satellite observes its neighbour on the same orbital plane with respect to its local reference frame. Notably, closed paths can be found easily in these scenarios, highlighting that a limited transmit power can be accepted when the ISL terminals are characterized by an hemispherical field-of-regard.

From the comparison of Figures 6a and 6b, we observe the impact of link priority (i.e., shortest or longest) on the topology configuration. When set to the shortest links, the fostered connections are intra-plane, while the longest links setting enables geometrically diverse measurements. Running several analyses, we assessed that the priority of the links does not influence the outcome of the search, as the topology is either found or not. As a consequence, we have set the priority to longest to favor inter-plane diverse measurements.

Operational constraint and search outputs The analyses described in this paragraph focus on investigating the lower bound in terms of physical constraints for which closed topologies can still be found. This is evaluated by exploring all combinations

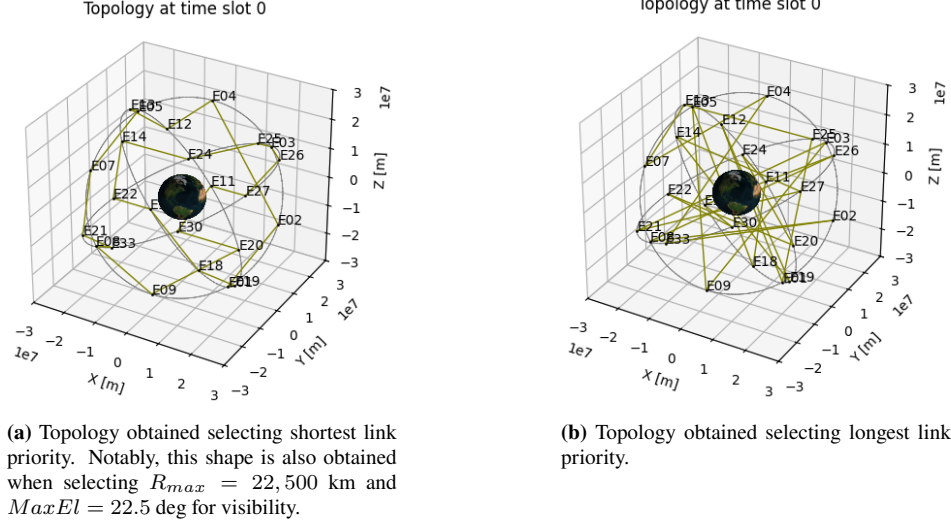


Figure 6: Topology obtained selecting shortest/longest link priority.

of the parameters in Table 2 (except for the link guidance, always set to “longest” for the reasons explained in the previous paragraph). The algorithm is employed for extensive analyses sweeping the multi-dimensional constraint space.

Figures from 7a to 8c depict the constraints space in 3D plots. In Figures 7, the complementary flag is deactivated (any-to-any links possible) while in Figures 8, the complementary flag is activated (restrictions on linking of the terminals). We placed the three physical constraints related to the terminal’s mechanics on the x , y , and z axes, represented by $MaxAz$, $MaxEl$, and R_{max} , respectively. Additionally, we present multiple plots for different values of minimum link duration time t_{min} (i.e., the minimum required duration for the persistence of the link). For the sake of clarity, we have chosen to present only the plots corresponding to $t_{min} = 1, 30, 60$ minutes, although our analyses were performed for t_{min} varying by 10 minutes, as shown in Table 2.

Each point in these plots represents a one-day analysis conducted using a given combination of $MaxAz$, $MaxEl$, R_{max} , and t_{min} . The input files analysed contain data for one day, therefore the data is organized into frames of time interval equal to t_{min} , for a total number of frames $N_{frames} = 1440/t_{min}$ (since t_{min} is expressed in minutes and $1day = 1440$ min). In this way, we can compute the ratio between the number of frames for which a topology has been found and the total number of frames as $N_{frames,found}/N_{frames,total}$. This ratio, labeled as *success ratio*, quantifies the ratio of frames for which a closed topology could be found within the given timeout for that combination of constraints.

A green triangle indicates that at least one closed topology can always be found for all the N_{frames} . On the other hand, when a red cross is shown, a closed topology is never discovered. The intermediate shades in color represent cases where the algorithm successfully identifies the topology for some of the frames (represented with a circle); in other words, the shade and size of the dot represents the success ratio $N_{frames,found}/N_{frames,total}$.

Several key findings can be inferred from the simulations generated under the constraints outlined in Table 2. We organize them according to the use of the complementary flag:

1. **Complementary flag off (any-to-any terminal connection).** Looking at Figures in 7, we can observe the following:
 - a) from Figure 7a (with $t_{min} = 1$ min), increasing the maximum range R_{max} allows for reduction of the maximum azimuth $MaxAz$ from 225 deg to 180 deg; furthermore, there is little impact of the $MaxEl$ - closed topologies can be found even for $MaxEl = 30$ deg.
 - b) increasing link persistence time requires wider field of regards and larger maximum range to find always a closed topology, see Figures 7b and 7c;
 - c) shortening the link persistence reduces the absolute lower limits for which a close topology can always be found; e.g., having $t_{min} = 1$ min, the lower bound is $R_{max} = 25,000$ and $MaxAz = 225$ deg, while for $t_{min} = 60$ min, it is for $R_{max} = 30,000$ and $MaxAz = 315$ deg;
2. **Complementary flag on (connection to compatible terminals only).** From the results presented in Figures 8, we can observe that:

- a) no closed topology is guaranteed to be found for $MaxAz < 360$ deg, within the timeout set; hence, maximum azimuth aperture is necessary for the terminals' field-of-regard;
 - b) for longer t_{min} of 60 min, no closed topology can be assured if $R_{max} < 30,000$ km; thus, longer link persistence times imply larger transmitting power to assure finding of a closed topology, as in Figure 8c.
3. Some other relevant conclusions can be drawn inspecting at **the number of timeouts reached**:
- a) for some of cases with a success ratio of less than 100%, the algorithm reaches the timeout, indicating that the search failed to find the topology within 2 minutes. In other scenarios, the search exits right after the *treecheck* indicating that there are nodes with no connections. In the former case, the algorithm's inability to find the topology might not be due to the absence of a valid topology, but rather to the excessive time required to find it. Therefore, we re-run two of the scenarios with less than a 100% success ratio, reducing the length of the analyses from 1 day to 2 hours, and extending the *timeout* up to 1 hour. We perform this computation for:
 - i. $MaxAz = 135$ deg, $MaxEl = 90$ deg, $R_{max} = 60,000$ km, $t_{min} = 1$ min and fl_{comp} deactivated;
 - ii. $MaxAz = 315$ deg, $MaxEl = 90$ deg, $R_{max} = 30,000$ km, $t_{min} = 30$ min and fl_{comp} activated.

For 3(a)i, in the run with the timeout set at 2 minutes, the *success ratio* was equal to 82,7% (see also 7a), in the run with timeout raised to 1 hour, the *success ratio* increases to 100%. While running 3(a)ii the success ratio increases from 29.2% to 34.2% (see Figure 8a).

The timeout option can then be too restrictive for some scenarios, in which closed topologies can still be found if larger time intervals can be allocated to the graph search.

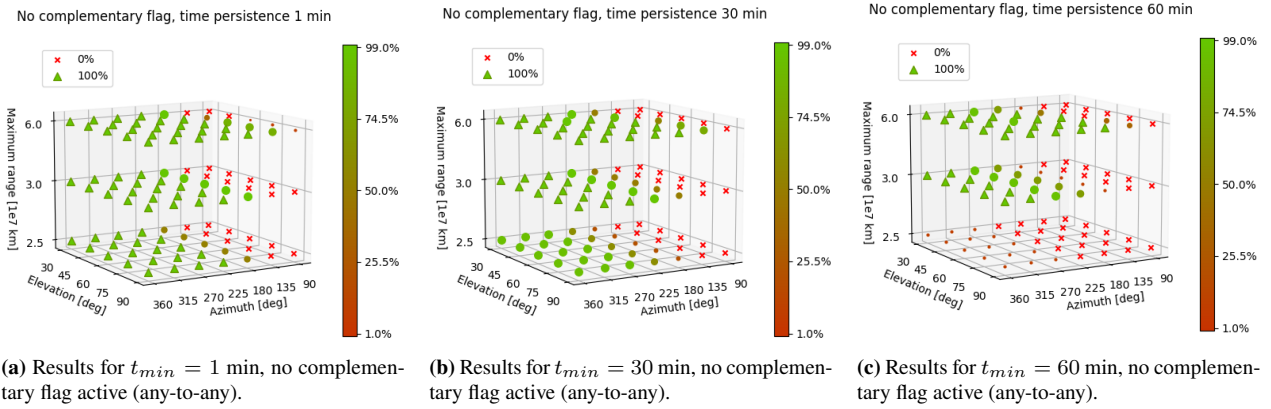


Figure 7: Closed topology search results for different link persistence times in an any-to-any terminal connection scenario (no complementary flag).

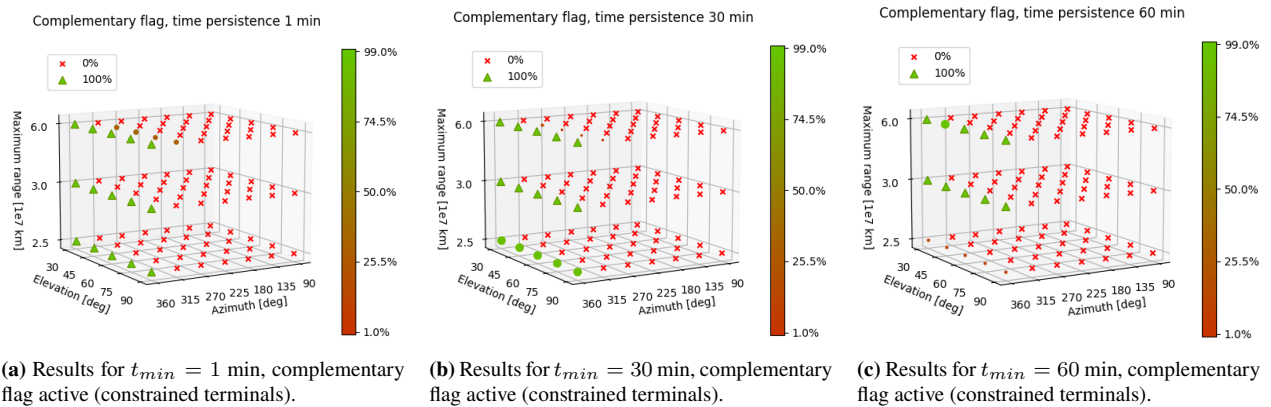


Figure 8: Closed topology search results for different link persistence times in the scenario in which terminals connects only between compatible pairs (complementary flag).

VI. SUMMARY AND CONCLUSIONS

This study addresses the operational envelope of inter-satellite link terminals, when used to enable topologies in the form of Hamiltonian cycles, i.e. closed paths connecting all satellites in the constellation. Different scenarios are investigated by considering two different link guides and examining the resulting topology forms. The topologies generated from long links provide more geometrically diverse measurements. In contrast, the topologies obtained by fostering short links are possible with less transmission power, as they tend to connect neighbour satellites always co-visible.

Systematic analyses are performed throughout the constraints space. The results demonstrated that, without restrictions on linking terminals, closed topologies could be found within the timeout for a significant number of constraint combinations. Very limited dependence on the maximum elevation angle attainable by the terminals is observed. This might be relevant for terminal design and worn-out, since it leads to the execution of smaller pointing maneuvers. When the linking terminals are restricted by an active complementary flag, a maximum azimuth of at least 360 deg is required for the field-of-regard to find of a closed topology. This indicates that when a link channel separation is implemented via frequency and/or polarization separation, large field-of-regards are necessary to enable close link topologies.

Finally, we perform additional analyses with constraints slightly below than the found lower bound (success ration $< 100\%$) with an increased timeout to further explore the search space. We show that increasing the timeout does indeed allow to find closed paths for 100% of the epochs sampled in some scenarios. The timeout to be set is user-dependent, and should vary according to the computational resources available and maximum permissible latency before a solution is output (including a negative response), trading computational time with sought success ratio. As a future development we could set a variable timeout or, in case the number of possibilities allows it (following the values in Table 4), stop when the algorithm has actually exhausted all possible paths.

ACKNOWLEDGEMENTS

This work has been performed in the framework of project Kepler Vision, founded by the Institute of Communications and Navigation of the German Aerospace Center (DLR).

REFERENCES

- Deprez, C. and Giorgi, G. (2021). Operational envelope and link scheduling for inter-satellite links in next-generation gnss. In *2021 IEEE Aerospace Conference (50100)*, pages 1–13. IEEE.
- Diestel, R. and Diestel, R. (2017). Extremal graph theory. *Graph theory*, pages 173–207.
- Ehyaie, D. (2011). *Novel approaches to the design of phased array antennas*. PhD thesis, University of Michigan.
- Fernández, F. A. (2011). Inter-satellite ranging and inter-satellite communication links for enhancing gnss satellite broadcast navigation data. *Advances in Space Research*, 47(5):786–801.
- Mayer-Guerr, T., Behzadpour, S., Eicker, A., Ellmer, M., Koch, B., Krauss, S., Pock, C., Rieser, D., Strasser, S., Suesser-Rechberger, B., Zehentner, N., and Kvas, A. (2021). GROOPS: A software toolkit for gravity field recovery and GNSS processing. *Computers and Geosciences*, page 104864.
- Menn, M. and Bernstein, H. (1994). Ephemeris observability issues in the global positioning system (gps) autonomous navigation (autonav). In *Proceedings of 1994 IEEE Position, Location and Navigation Symposium-PLANS'94*, pages 677–680. IEEE.
- Michalak, G., Glaser, S., Neumayer, K., and König, R. (2021). Precise orbit and earth parameter determination supported by leo satellites, inter-satellite links and synchronized clocks of a future gnss. *Advances in Space Research*, 68(12):4753–4782.
- Nardin, A., Fraire, J. A., and Dovis, F. (2021). Contact plan design for gnss constellations: A case study with optical intersatellite links. *IEEE Transactions on Aerospace and Electronic Systems*, 58(3):1981–1995.
- Schlicht, A., Marz, S., Stetter, M., Hugentobler, U., and Schäfer, W. (2020). Galileo pod using optical inter-satellite links: A simulation study. *Advances in Space Research*, 66(7):1558–1570.
- Trainotti, C., Dassié, M., Giorgi, G., Khodabandeh, A., and Günther, C. (2022). Autonomous satellite system synchronization schemes via optical two-way time transfer and distributed composite clock. In *Proceedings of the 35th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2022)*, pages 3646–3661.