

Collaborative Code-Based Differential GNSS for Multi-User Positioning

Helena Calatrava¹, Daniel Medina² and Pau Closas¹

Abstract—This paper introduces a collaborative differential GNSS (C-DGNSS) method designed to enhance positioning accuracy in scenarios with limited satellite visibility, relying solely on GNSS pseudorange measurements and requiring no inter-ranging information. C-DGNSS exploits the correlation among single difference observations from multiple users to a common base station, which are not leveraged by conventional DGNSS. The estimation problem is formulated as an iterative weighted least squares (WLS) estimator, known to be optimal under the Gaussian assumption, and its variance lower bound is derived from the Cramér-Rao bound (CRB). C-DGNSS is compared against conventional DGNSS and the ideal DGNSS bound, the latter assuming noiseless pseudorange measurements for the base station. Simulation results demonstrate that the proposed scheme outperforms standard DGNSS even with a single aiding user, approaching the ideal bound as the number of aiding users grows and their geometry improves. Accuracy gains of up to 28.5% are achieved.

Index Terms—Differential GNSS, Collaborative Positioning, Cramér-Rao Bound

I. INTRODUCTION

The demand for precise and reliable positioning information continues to grow, as it is critical for emerging applications in transportation (vehicle-to-everything (V2X), traffic management, search and rescue), healthcare (patient monitoring, asset tracking, telemedicine), and automation (smart factories, industrial robots, autonomous vehicles) [1]. Global navigation satellite systems (GNSS) serves as the backbone for outdoor localization due to its integrity, reliability, high coverage, and wide deployment [2]. Nevertheless, GNSS positioning methods such as single point positioning (SPP) are vulnerable to degradation from several factors, including receiver quality, atmospheric effects, satellite availability, multipath effects, and interference [3].

While precise point positioning (PPP) improves accuracy and provides better atmospheric corrections compared to SPP, it suffers from longer convergence times [4]. Additionally, PPP is less effective at mitigating spatially correlated errors, such as atmospheric variations, when compared to augmentation systems like differential GNSS (DGNSS). Position, velocity, and time (PVT) estimation in GNSS systems primarily relies on pseudorange (or code) and carrier-phase measurements,

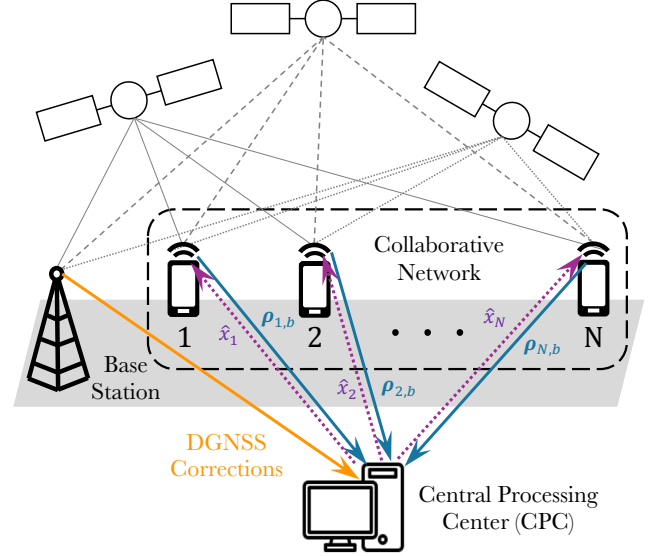


Fig. 1: Overview of the proposed C-DGNSS scheme. A set of $r \in \{1, \dots, N\}$ nearby users transmit their code observables to the central processing center (CPC), which processes these observables alongside DGNSS corrections from the base station. By leveraging the cross-correlation between the received measurements, C-DGNSS achieves position estimates with higher accuracy compared to conventional DGNSS.

both of which can be used within the DGNSS framework. In particular, real-time kinematic (RTK) positioning [5] resolves carrier-phase ambiguities to enable high-precision position estimates [6, Ch. 26]. Although carrier-phase measurements are more precise and typically necessary to meet the requirements of some V2X applications, their quality and reliability often degrade in challenging environments, making it difficult to consistently resolve an RTK fix. This paper focuses on code-based DGNSS.

To improve positioning accuracy, DGNSS uses corrections from a base station to mitigate common errors between the station and the receiver, including atmospheric effects [7]. For code-based DGNSS, when corrections are derived from a single station, the technique is referred to as local DGNSS. This requires the receiver to stay within 10 km of the base station to ensure sufficient common satellites in view and similar atmospheric delays, a condition known as the *short baseline*. In contrast, wide-area differential GNSS (WADGNSS) makes use of a network of reference stations providing corrections valid over a larger region [8]. When integrating code and carrier-phase measurements, WADGNSS becomes conceptually equivalent to PPP [6]. The positioning accuracy of DGNSS can be further enhanced by incorporating corrections

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¹Helena Calatrava and Pau Closas are with Dept. of Electrical and Computer Eng., Northeastern University, Boston, MA (USA). E-mail: {calatrava.h, closas}@northeastern.edu

²Daniel Medina is with the Institute of Communications and Navigation, German Aerospace Center (DLR), Neustrelitz, Germany. E-mail: daniel.ariasmedina@dlr.de

from ionospheric models, such as Klobuchar or NeQuick, and tropospheric models, like the Saastamoinen model [9]. Despite these advances, such methods still face significant challenges in mitigating severe multipath and non-line-of-sight (NLOS) errors, particularly in complex environments like urban canyons and tunnels [10].

GNSS collaborative positioning (CP) [11] and sensor data fusion [12] strategies have been developed to overcome these limitations and meet the need for high accuracy and continuous state estimates [13]. By enabling differencing operations between cooperative receivers, correlated errors such as atmospheric delays can be mitigated, and NLOS errors can also be reduced [10], [14]. Several works in the GNSS CP literature assume the availability of additional technologies providing range information to enhance positioning accuracy. This is particularly useful when a group of users has constrained satellite visibility. For instance, the work in [15] applies multidimensional scaling to a matrix of inter-receiver distances to position users in a local frame, which is then aligned with the real reference frame using the SPP estimates of a subset of receivers with good satellite visibility. In [16], the collaborative dilution of precision (CDOP) is introduced, providing a theoretical analysis of GNSS CP that examines the effects of errors in inter-user ranging information. Notably, it has been shown that CDOP values decrease as the number of users in the collaborative network increases [17]. Both [16] and [17] assume that the observation model contains pseudorange and relative distance (or ranging) measurements, which contributes to solving the problem without requiring a base station. The theoretical limits of GNSS terrestrial localization in collaborative scenarios are investigated in [18], which evaluates deterministic and Bayesian estimation for both distributed and centralized architectures. This aligns with the increasing research interest in GNSS distributed architectures [19], though these schemes fall outside the scope of this work. Remarkably, GNSS CP has recently been explored in the context of privacy-preserving localization [20].

While some works propose hybrid collaborative approaches [18] or focus on GNSS-denied environments [21], GNSS-only CP strategies remain an active area of research, having been shown to improve positioning accuracy by solely leveraging GNSS measurements from multiple users when visibility allows [15]–[17]. For instance, the work in [22] demonstrates how auxiliary inter-vehicular range information can be derived through the exchange of pseudoranges and state estimates within a collaborative network. Pseudorange double differencing for relative positioning in vehicles is explored in [23], where the challenges posed by multipath effects are also addressed. In [24], a base station-less method for massive differencing of GNSS pseudorange measurements is proposed, showing better performance than traditional DGNSS as the number of users in the network increases. This method eliminates the need for a reference station [25], enhancing scalability and flexibility. Urban environments present significant challenges for GNSS-based positioning due to signal occlusion and multipath. A collaborative RTK (C-RTK) approach tailored

for vehicle swarms operating in such settings was proposed in [26]. This method enhances the precision of RTK by allowing users with strong satellite visibility to assist nearby users in more constrained conditions. However, a detailed analysis of how the number of collaborative users and the satellite visibility of open-sky users affect C-RTK performance remains unexplored. Additionally, the applicability of this framework to code-based DGNSS has not been investigated. While other works have improved collaborative GNSS localization in urban areas by leveraging signal visibility and reflection data derived from 3D building models [27], such map-aided techniques are not considered in this paper.

Following the line of research in [26], we propose a collaborative code-based DGNSS method, referred to as collaborative DGNSS (C-DGNSS). This scheme is designed to enhance positioning performance in urban scenarios by leveraging pseudorange measurements from multiple receivers in a centralized network. Our analysis provides valuable insights into the asymptotic performance as a function of the number of collaborative users, satellites in view, and measurement precision. Additionally, we evaluate the impact of different geometric dilution of precision (GDOP) values for the users with constrained visibility. We compare the proposed C-DGNSS estimator against conventional code-based DGNSS and the ideal DGNSS bound, the latter assuming noise-free base station measurements. As a theoretical benchmark, we derive the lower bound on the variance of the proposed C-DGNSS estimator from the Cramér–Rao bound (CRB). The robustness of C-DGNSS against outliers, such as those caused by multipath propagation, is further analyzed in [28].

The remainder of the paper is organized as follows. Section II introduces the observation model for conventional code-based DGNSS. Section III presents the proposed C-DGNSS approach, detailing the observation model, the estimation problem of interest, and the derivation of the CRB as a theoretical benchmark. Additionally, this section explains how users within the network can be grouped into clusters based on their distinct geometric conditions. The simulation setup and results are discussed in Section IV, and the paper concludes with Section V.

II. DGNSS OBSERVATION MODEL

This section presents a detailed derivation of the observation model for conventional code-based DGNSS. Let us assume that K satellites are observed on a single frequency by a receiver r and a base station. At a particular time instant, the pseudorange (or code) measurement at receiver r with respect to satellite s can be expressed as

$$\rho_r^s = \varrho_r^s + I^s + T^s + c(\delta t_r - \delta t^s) + \epsilon_r^s, \quad (1)$$

where $\varrho_r^s = \|\mathbf{p}^s - \mathbf{p}_r\|$ is the true range, being $\mathbf{p}^s$ and $\mathbf{p}_r$ the positions of the satellite and receiver, c is the speed of light, δt^s and δt_r are the clock offsets for the satellite and receiver, ϵ_r^s captures unmodeled errors like multipath and thermal noise, and I^s and T^s denote the ionospheric and tropospheric delays.

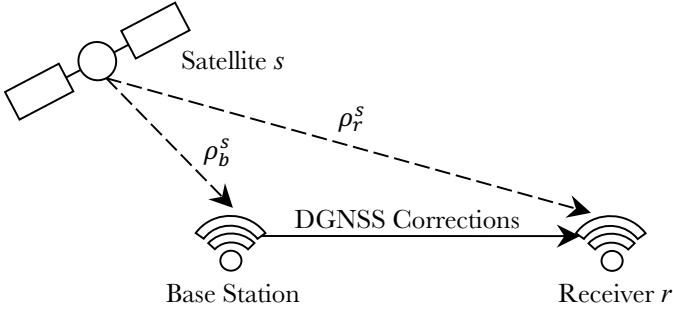


Fig. 2: Illustration of satellite s , base station b and receiver r involved in the single difference operation in (2) for conventional DGNSS.

With conventional DGNSS, atmospheric errors and satellite clock offsets are mitigated using corrections from a base station at a known location that tracks all satellites in view [6]. This can be modeled as the single difference between the pseudorange measurements at receiver r and the base station b . For satellite s , it can be expressed as

$$\rho_{r,b}^s = \rho_r^s - \rho_b^s = \varrho_{r,b}^s + c\delta t_{r,b}^s + \epsilon_{r,b}^s, \quad (2)$$

where atmospheric errors and satellite clock biases are effectively canceled under the assumption of well-synchronized measurements [29]. For ease of notation, we define $h^s(\mathbf{x}_r) = \varrho_r^s + c\delta t_r$, where $\mathbf{x}_r = [\mathbf{p}_r, \delta t_r]^T$ is the unknown parameter vector for receiver r . Considering all satellites in view, (2) is extended to vector form as

$$\boldsymbol{\rho}_{r,b} = [\rho_{r,b}^1, \dots, \rho_{r,b}^K]^T = \mathbf{h}(\mathbf{x}_r) - \mathbf{h}(\mathbf{x}_b) + \boldsymbol{\epsilon}_{r,b}. \quad (3)$$

Given the non-linearity of the range term, (3) is linearized as $\boldsymbol{\rho}_{r,b} \approx \mathbf{h}(\mathbf{x}_{r,0}) + \mathbf{H}_r \boldsymbol{\delta}_r - \mathbf{h}(\mathbf{x}_b) + \boldsymbol{\epsilon}_{r,b}$. Here, $\boldsymbol{\delta}_r = \mathbf{x}_r - \mathbf{x}_{r,0}$ is the deviation of the unknown parameter vector from the linearization point $\mathbf{x}_{r,0}$, and $\mathbf{H}_r \in \mathbb{R}^{K,4}$ is the geometry matrix containing the satellite-to-receiver line-of-sight (LOS) vectors. For simplicity, we do not explicitly indicate the dependence of this matrix on $\mathbf{x}_{r,0}$. The observed-minus-computed (O-C) residual is defined as $\mathbf{y}_{r,b} = \boldsymbol{\rho}_{r,b} - (\mathbf{h}(\mathbf{x}_{r,0}) - \mathbf{h}(\mathbf{x}_b)) \approx \mathbf{H}_r \boldsymbol{\delta}_r + \boldsymbol{\epsilon}_{r,b}$, and can be modeled as follows

$$\mathbf{y}_{r,b} | \boldsymbol{\delta}_r \sim \mathcal{N}(\mathbf{H}_r \boldsymbol{\delta}_r, \boldsymbol{\Sigma}_{r,b}), \text{ with } \boldsymbol{\delta}_r \in \mathbb{R}^4. \quad (4)$$

The covariance matrix accounts for the contributions of both the receiver and the base station as

$$\boldsymbol{\Sigma}_{r,b} = \mathbb{E}[\boldsymbol{\epsilon}_{r,b} \boldsymbol{\epsilon}_{r,b}^T] = \boldsymbol{\Sigma}_r + \boldsymbol{\Sigma}_b = 2\boldsymbol{\Sigma}, \quad (5)$$

where we use $\boldsymbol{\Sigma} = \sigma_\rho^2 \mathbf{W}^{-1}$ for simplicity. The weighting matrix $\mathbf{W} \in \mathbb{R}^{K,K}$ typically depends on the satellite elevation, and σ_ρ is the zenith-referenced standard deviation of the pseudorange measurements. Similar to conventional DGNSS, the base station coordinates are assumed to be known with high accuracy. This makes $\mathbf{x}_b$ a known quantity and leaves $\mathbf{x}_r$ as the only unknown in this model.

Remark 1. If we assume the receiver and base station to have the same stochastic model, then the third equality in (5) applies.

III. PROPOSED COLLABORATIVE METHOD

Building on the conventional DGNSS observation model, C-DGNSS combines pseudorange measurements across multiple users to achieve enhanced positioning performance, particularly in scenarios with limited satellite visibility. Let us assume that K satellites are observed on a single frequency by N receivers and a base station.

A. Collaborative DGNSS Observation Model

The proposed C-DGNSS method operates under the assumption that all users, along with the base station, send their pseudorange measurements to a CPC responsible for estimating the positioning solution for the entire user network. C-DGNSS integrates single differences between multiple users $r \in \{1, \dots, N\}$ and the base station as follows

$$\tilde{\boldsymbol{\rho}} = [\boldsymbol{\rho}_{1,b}^T, \dots, \boldsymbol{\rho}_{N,b}^T]^T = \mathbf{h}(\tilde{\mathbf{x}}) - \mathbf{h}(\tilde{\mathbf{x}}_b) + \tilde{\boldsymbol{\epsilon}}, \quad (6)$$

where $\tilde{\cdot}$ denotes the vertical stacking operation applied across multiple users, being $\tilde{\mathbf{x}}_b = \mathbf{1}_{N,1} \otimes \mathbf{x}_b$, $\tilde{\mathbf{x}} = [\mathbf{x}_1^T, \dots, \mathbf{x}_N^T]^T$, $\otimes$ is the Kronecker product, and $\tilde{\boldsymbol{\epsilon}} = [\boldsymbol{\epsilon}_{1,b}^T, \dots, \boldsymbol{\epsilon}_{N,b}^T]^T$. The O-C residual is given by $\tilde{\mathbf{y}} = \tilde{\boldsymbol{\rho}} - (\mathbf{h}(\tilde{\mathbf{x}}_0) - \mathbf{h}(\tilde{\mathbf{x}}_b)) \approx \tilde{\mathbf{H}}\tilde{\boldsymbol{\delta}} + \tilde{\boldsymbol{\epsilon}}$, which can be modeled as

$$\tilde{\mathbf{y}} | \tilde{\boldsymbol{\delta}} \sim \mathcal{N}(\tilde{\mathbf{H}}\tilde{\boldsymbol{\delta}}, \tilde{\boldsymbol{\Sigma}}), \text{ with } \tilde{\boldsymbol{\delta}} \in \mathbb{R}^{4N}. \quad (7)$$

In this expression, $\tilde{\boldsymbol{\delta}}$ contains the error vector for the N users as $\tilde{\boldsymbol{\delta}} = [\boldsymbol{\delta}_1^T, \dots, \boldsymbol{\delta}_N^T]^T$. The observation matrix is augmented as $\tilde{\mathbf{H}} = \text{diag}(\mathbf{H}_1, \dots, \mathbf{H}_N)$. If all the users observe the same satellites, we have $\tilde{\mathbf{H}} = \mathbf{I}_N \otimes \mathbf{H}$, where $\mathbf{I}_p$ is the $p \times p$ identity matrix. Given the similar satellite geometry among users, it is typically assumed that $\mathbf{H}_i = \mathbf{H}$, $\forall i$. The covariance matrix takes the form

$$\tilde{\boldsymbol{\Sigma}} = \mathbb{E}[\tilde{\boldsymbol{\epsilon}} \tilde{\boldsymbol{\epsilon}}^T] = \begin{bmatrix} \boldsymbol{\Sigma}_{1,b} & \boldsymbol{\Sigma}_b & \cdots & \boldsymbol{\Sigma}_b \\ \boldsymbol{\Sigma}_b & \boldsymbol{\Sigma}_{2,b} & \cdots & \boldsymbol{\Sigma}_b \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{\Sigma}_b & \boldsymbol{\Sigma}_b & \cdots & \boldsymbol{\Sigma}_{N,b} \end{bmatrix} = \tilde{\mathbf{C}}_N \otimes \boldsymbol{\Sigma}, \quad (8)$$

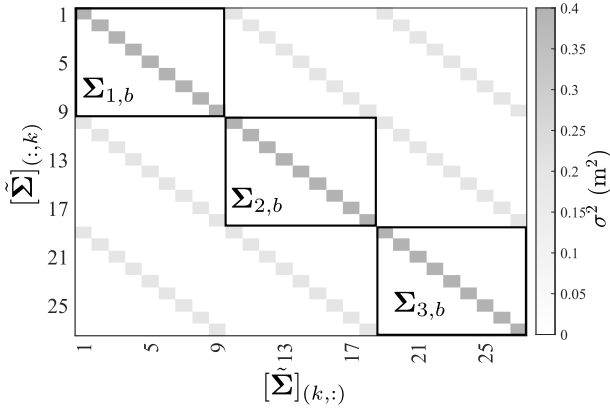
where the off-diagonal elements capture the contribution from the base station measurement covariance (see Figure 3a).

Remark 2. If we assume common weighting matrices and identical satellites in view between users and the base station, then the second equality in (8) applies. The collaborative (or mixing) matrix is denoted as $\tilde{\mathbf{C}}_N = [\mathbf{I}_N + \mathbf{1}_{N,N}]$, and relates the base and user measurements [26]. Here, $\mathbf{1}_{p,q}$ represents an $p \times q$ matrix of ones.

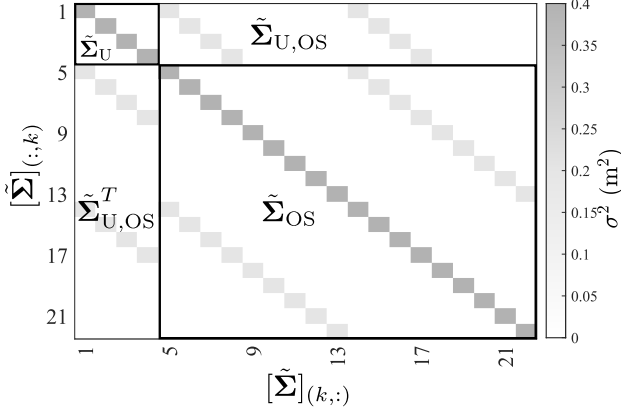
B. Collaborative DGNSS Estimator

A sufficient condition for the least squares solution is that the total number of independent measurements must be greater than or equal to the number of unknowns in the system. For the C-DGNSS estimator, this corresponds to [16]

$$\sum_{n=1}^N \text{Card}(\mathcal{K}_n) \geq 4N, \quad (9)$$



(a) Matrix in (8) for $K = 9$, $N = 3$, $\sigma_\rho^2 = 20 \text{ cm}^2$, and $\mathbf{W} = \mathbf{I}$. The diagonal blocks correspond to the $K \times K$ covariance matrix in (5), used in conventional DGNSS.



(b) Matrix in (14) for $N_U = 1$, $K_U = 4$, $N_{OS} = 2$, $K_{OS} = 9$, $\sigma_\rho^2 = 20 \text{ cm}^2$, and $\mathbf{W} = \mathbf{I}$.

Fig. 3: Heatmaps of the C-DGNSS model covariance matrix. (Top): All users share the same satellites in view. (Bottom): A cluster of urban users experiences constrained visibility. Unlike conventional DGNSS, C-DGNSS provides access to the off-diagonal blocks, offering additional information that enhances positioning accuracy.

which is guaranteed to be satisfied if every user in the scenario has visibility to at least 4 satellites. Here, $\mathcal{K}_n$ represents the set of satellites visible to the n -th user. If the condition in (9) is met, the system in (6) can be formulated as a weighted least squares (WLS) problem as

$$\hat{\tilde{\boldsymbol{\delta}}} = \arg \min_{\tilde{\boldsymbol{\delta}} \in \mathbb{R}^{4N}} \left\{ (\tilde{\mathbf{y}} - \tilde{\mathbf{H}}\tilde{\boldsymbol{\delta}})^\top \tilde{\mathbf{W}} (\tilde{\mathbf{y}} - \tilde{\mathbf{H}}\tilde{\boldsymbol{\delta}}) \right\}, \quad (10)$$

where $\tilde{\mathbf{W}} = \tilde{\boldsymbol{\Sigma}}^{-1}$ and $\hat{\tilde{\boldsymbol{\delta}}} = [\hat{\boldsymbol{\delta}}_1^\top, \dots, \hat{\boldsymbol{\delta}}_N^\top]^\top$. This can be solved iteratively, where at the k -th iteration one has

$$\hat{\tilde{\boldsymbol{\delta}}}^{k+1} = \hat{\tilde{\boldsymbol{\delta}}}^k + (\tilde{\mathbf{H}}^{k\top} \tilde{\mathbf{W}} \tilde{\mathbf{H}}^k)^{-1} \tilde{\mathbf{H}}^{k\top} \tilde{\mathbf{W}} \tilde{\mathbf{y}}^k. \quad (11)$$

The geometry matrix $\tilde{\mathbf{H}}^k$ and the O-C $\tilde{\mathbf{y}}^k$ depend on the positioning solution at every iteration. Given the Gaussian assumption of the stochastic model in (7), the C-DGNSS estimator is optimal and equivalent to the maximum likelihood estimator (MLE).

It is important to understand what leads to the improved performance of the estimator in (10) compared to conventional DGNSS. The cross-correlations in (8) provide essential

information, drawing a parallel to their role in GNSS joint position and attitude (JPA) estimation. In JPA, multiple sets of measurements from different antennas for a single user are leveraged, with the correlations between these measurements offering valuable information for improved positioning [30]. One might assume that this effect does not extend to the C-DGNSS method, where only one set of measurements is available per user and the unknown parameters are uncorrelated among them. However, we hypothesize that this is not the case, as significant improvements have been observed in the closely related C-RTK method presented in [26], particularly in scenarios where some users experience limited satellite visibility. In such cases, the measurements that contribute additional information to the joint position estimates are those from satellites observed exclusively by users with advantageous geometries. This additional information leads to an improvement in the network CDOP.

C. Performance Benchmark

We derive the CRB as a theoretical performance benchmark for the variance of the C-DGNSS estimator. For the i -th parameter, the CRB provides a lower bound on the variance of its estimate by

$$\text{Var}([\hat{\tilde{\boldsymbol{\delta}}}]_i) \geq \text{CRB}([\hat{\tilde{\boldsymbol{\delta}}}]_i) = [\mathbf{J}(\tilde{\boldsymbol{\delta}})]_{ii}^{-1}, \quad (12)$$

where the Fisher information matrix (FIM) is defined as

$$[\mathbf{J}(\tilde{\boldsymbol{\delta}})]_{i,j} = -\mathbb{E} \left[\frac{\partial^2 \ln p(\tilde{\mathbf{y}}; \tilde{\boldsymbol{\delta}})}{\partial \tilde{\delta}_i \partial \tilde{\delta}_j} \right] = [\tilde{\mathbf{H}}^\top \tilde{\mathbf{W}} \tilde{\mathbf{H}}]_{i,j}. \quad (13)$$

To evaluate the CRB for a specific user r , we extract the corresponding 4×4 submatrix from the FIM using the operator $\text{block}_r(\mathbf{X}) = \mathbf{X}[4(r-1)+1 : 4r, 4(r-1)+1 : 4r]$, where the indices follow a one-based indexing convention, and $r \in \{1, 2, \dots, N\}$. This results in $\text{CRB}(\hat{\tilde{\boldsymbol{\delta}}}_r) = \text{block}_r(\mathbf{J}^{-1}(\tilde{\boldsymbol{\delta}}))$. To assess the 3D positioning performance, we use the trace of the CRB submatrix as a scalar metric, given by $\text{Tr}([\text{CRB}(\hat{\tilde{\boldsymbol{\delta}}}_r)]_{1:3,1:3})$. The performance of the C-DGNSS estimator is compared against both conventional DGNSS and what we refer to as the ideal DGNSS bound. The latter corresponds to an ideal DGNSS scenario where the zenith-referenced code variance for the base station is zero.

D. Network Clustering

Users in the collaborative network are grouped into different clusters based on their satellite visibility conditions. In this work, we distinguish between an urban cluster and an open-sky cluster, similar to [26]. The urban cluster consists of N_U users operating in constrained-visibility conditions and observing only K_U satellites. The open-sky cluster comprises the remaining N_{OS} users, who benefit from favorable satellite visibility and access a set of $K_{OS} \geq K_U$ satellites. The total number of users in the network is given by $N = N_U + N_{OS}$.

To model the distinct geometry between the two clusters, the error covariance matrix in (8) is expressed as

$$\tilde{\Sigma} = \begin{bmatrix} \tilde{\Sigma}_U & \tilde{\Sigma}_{U,OS} \\ \tilde{\Sigma}_{U,OS}^T & \tilde{\Sigma}_{OS} \end{bmatrix} = \begin{bmatrix} 2\Sigma & \mathbf{D}_{N_{OS},K_U,K_{OS}} \\ (\mathbf{D}_{N_{OS},K_U,K_{OS}})^T & \tilde{\mathbf{C}}_{N_{OS}} \otimes \Sigma \end{bmatrix}, \quad (14)$$

being $\mathbf{D}_{N_{OS},K_U,K_{OS}} = \mathbf{1}_{1,N_{OS}} \otimes [\Sigma \ \mathbf{0}_{K_U,K_{OS}}]$ and $\Sigma = \sigma_\rho^2 \mathbf{W}^{-1}$. Here, the second equality incorporates the assumptions outlined in Remarks 1 and 2, as well as the specific scenario $N_U = 1$. This simplified scenario allows us to understand the changes in the correlation structure of the error covariance matrix caused by the limited visibility of the users in the urban cluster. Specifically, for the urban user, the only measurements that exhibit correlation with others in the network are the pseudoranges corresponding to satellites shared between the two clusters, as reflected in matrix $\mathbf{D}$. This is illustrated in Figure 3b and contrasts with Figure 3a, which represents a scenario where all users share the same satellite geometry. Nevertheless, the key contribution to improving the CDOP comes from additional satellites visible to open-sky but not urban users, which enhance the overall network geometry.

IV. RESULTS

This section presents an evaluation of the proposed C-DGNSS method through a series of experiments. We first describe the experimental setup, followed by an analysis of the impact of collaborative users, measurement precision, and urban user geometry on positioning performance.

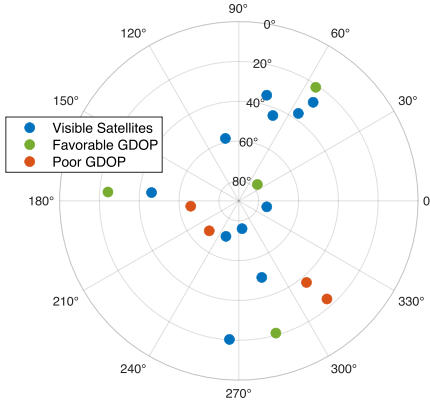


Fig. 4: Skyplot of satellites tracked by the base station, with those contributing to favorable and poor GDOPs for urban users colored in green and red.

A. Experimental Setup

An elevation mask of 20° is applied, with the base station tracking a total of $K = 19$ satellites. The satellite geometry is depicted in Figure 4. The urban cluster comprises $N_U = 2$ users, each observing $K_U = 4$ satellites with a GDOP denoted as GDOP_U . We evaluate the impact of the number of open-sky users (N_{OS}), the number of satellites visible to these users (K_{OS}), and the zenith-referenced pseudorange variance (σ_ρ^2) on the 3D position root mean square error (RMSE). The effect of favorable and poor GDOP conditions on the urban user cluster is also explored. A summary of the findings can be found

in Table I. The weighting matrix is set to $\mathbf{W} = \mathbf{I}$, which corresponds to the assumption that noise across satellites is identically distributed. Additionally, assumptions in Remarks 1 and 2 are adopted. To obtain statistically meaningful results, 10^3 Monte Carlo runs are conducted for each variation of parameter value and experiment. Finally, the results presented focus on the first user in the urban cluster, but this is without loss of generality because the performance is consistent across all urban users due to the symmetry in the model.

B. Impact of Collaborative Users

The impact of the number of collaborative open-sky users in the scenario is illustrated in Figure 5. The effect of K_{OS} is also analyzed, with results for the C-DGNSS estimator presented for $K_{OS} = \{8, 14, 19\}$, respectively. In this simulation, we fix $\sigma_\rho^2 = 20 \text{ cm}^2$ and $\text{GDOP}_U = 2.5$. The improvement achieved by the proposed C-DGNSS estimator compared to conventional DGNSS is evident even with the addition of a single collaborative user, highlighting the efficiency of the proposed approach. Specifically, for $N_{OS} = 1$, C-DGNSS achieves a precision gain in positioning RMSE of 5.2%, 8.1%, and 9.3% for $K_{OS} = \{8, 14, 19\}$, respectively, compared to the initial error of 1.46 m. C-DGNSS demonstrates a decreasing positioning RMSE as the number of open-sky users increases. However, beyond approximately $N_{OS} = 30$, the rate of improvement slows considerably. This indicates that once a substantial number of aiding users are present in the network, the benefit of adding additional collaborative users diminishes. Remarkably, C-DGNSS requires aiding users to have better satellite geometry than the aided users to provide performance improvements. Additionally, an increasing number of aided users does not negatively affect C-DGNSS performance, underscoring its scalability.

C-DGNSS consistently enhances accuracy under varying geometries of the open-sky users, with better results observed for higher K_{OS} . For example, for $N_{OS} = 50$, the difference in RMSE between $K_{OS} = 8$ and $K_{OS} = 19$ is of 12.5 cm. This demonstrates the capability of C-DGNSS to leverage pseudorange measurements from multiple users in the network, even under differing visibility conditions. Moreover, it highlights that the benefits for aided users extend beyond the satellites they share in common with the aiding users. At $N_{OS} = 60$, the improvement provided by C-DGNSS with respect to conventional DGNSS is of 15.38 cm, 24.16 cm, and 28.06 cm, for $K_{OS} = \{8, 14, 19\}$, respectively. Notably, achieving the ideal DGNSS bound is equivalent to a scenario where DGNSS operates with a noiseless base station, corresponding to a positioning accuracy of approximately 1 meter in this experiment. We observe that this bound is not reached. However, the behavior of the C-DGNSS estimator suggests that as K_{OS} and N_{OS} increase, it may converge toward it. A thorough theoretical analysis of the estimator asymptotic performance for increasing K_{OS} and N_{OS} is beyond the scope of this paper but lays the foundation for future work.

TABLE I: Summary of the observed influence of key parameters on C-DGNSS performance when compared to conventional DGNSS.

Parameter	Impact Description
N_{OS} : Number of open-sky users	Performance improvement for $N_{OS} \geq 1$ subject to $K_{OS} \geq K_U$ (Figure 5).
K_{OS} : Number of satellites visible to open-sky users	Performance improvement for $K_{OS} \geq K_U$, subject to $N_{OS} \geq 1$ (Figure 5).
σ_ρ^2 : Zenith-referenced pseudorange variance	Performance improvement accentuates for increasing σ_ρ^2 (Figure 6).
GDOP _U : Geometric dilution of precision	Increasing $\mathcal{I}(\%)$ for worse GDOP _U values (Table II).
N_U : Number of urban users	No impact.
K_U : Number of satellites visible to urban users	Similar to GDOP _U .

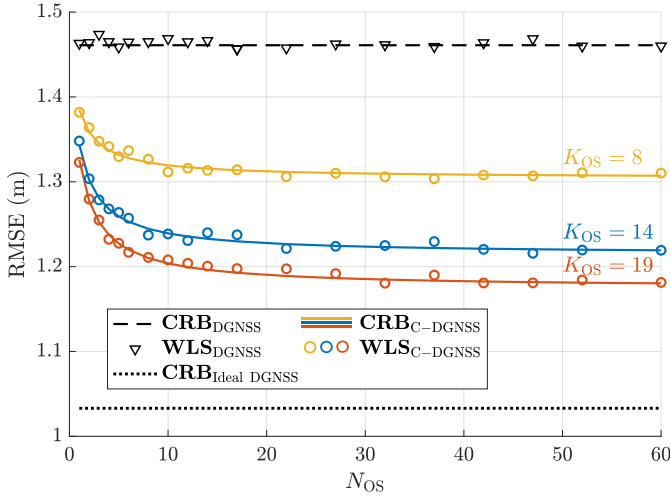


Fig. 5: Impact of N_{OS} on 3D positioning RMSE: positioning RMSE for the DGNSS and C-DGNSS estimators, along with square-root CRBs, as a function of N_{OS} . C-DGNSS results are shown for $K_{OS} = \{8, 14, 19\}$. We fix parameters at $N_U = 2$, $K_U = 4$, $\sigma_\rho^2 = 20 \text{ cm}^2$, and use geometry for favorable GDOP_U in Figure 4.

C. Impact of Measurement Precision

In the second experiment, the zenith-referenced standard deviation for the pseudorange measurements, σ_ρ^2 , spans a low noise level of 0.1 m^2 and a relatively high noise level of 1 m^2 . The results of this analysis, where we have fixed $N_{OS} = 8$ and $\text{GDOP}_U = 2.5$, as presented in Figure 6. As expected, reducing the pseudorange noise leads to a decrease in the positioning RMSE for both DGNSS and C-DGNSS estimators, confirming the general trend that lower measurement noise improves positioning accuracy. C-DGNSS consistently outperforms conventional DGNSS across the entire range of noise levels, with the advantage being particularly pronounced at higher noise levels. This can be explained by the additional information that C-DGNSS has access to, specifically the cross-correlations of errors received from the base station among users. Consequently, as the variance of the base station measurements σ_ρ^2 increases, the challenge for conventional DGNSS becomes more significant, while C-DGNSS is able to better exploit these correlations to improve performance. However, it is important to note that this analysis assumes both the base station and user error variances increase together (see Remarks 1 and 2). Similar to the analysis presented in Figure 5, improved performance is observed as K_{OS} increases.

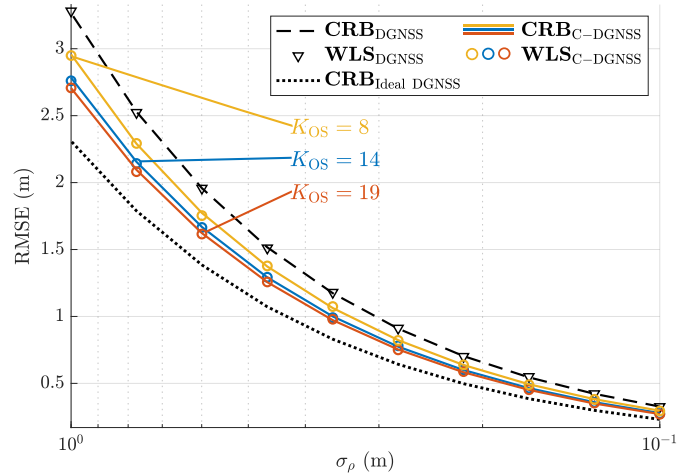


Fig. 6: Impact of σ_ρ^2 on 3D positioning RMSE: positioning RMSE for the DGNSS and C-DGNSS estimators, along with square-root CRBs, as a function of σ_ρ^2 . C-DGNSS results are shown for $K_{OS} = \{8, 14, 19\}$. We fix parameters at $N_U = 2$, $K_U = 4$, $N_{OS} = 8$, and use geometry for favorable GDOP_U in Figure 4.

Particularly, for $\sigma_\rho^2 = 1 \text{ m}^2$, the difference in RMSE between $K_{OS} = 8$ and $K_{OS} = 19$ is of 25 cm. Nevertheless, as already discussed, this difference decreases to 9 cm for instance at $\sigma_\rho^2 = 36 \text{ cm}^2$. Overall, this experiment demonstrates the capability of the collaborative scheme to effectively leverage aiding users, even as the variance of pseudorange measurements from the base station increases.

D. Impact of Aided Cluster Geometry

Results for varying GDOP values in the urban user cluster are presented in Table II. While C-DGNSS provides improvement for both favorable and poor GDOPs, the best absolute results are achieved with better GDOP values, owing to the more favorable conditions of a user performing DGNSS under superior geometry. To quantify the performance enhancement of C-DGNSS compared to conventional DGNSS, the relative improvement metric is calculated as [14]:

$$\mathcal{I}(\%) = \frac{\xi_{\text{DGNSS}} - \xi_{\text{C-DGNSS}}}{\xi_{\text{DGNSS}}}, \quad (15)$$

where ξ_{DGNSS} and $\xi_{\text{C-DGNSS}}$ correspond to the positioning RMSE for the DGNSS and C-DGNSS estimators, respectively. The results show that the relative improvement is significantly higher for poor GDOP_U, at 28.5%, compared to favorable

TABLE II: Impact of favorable and poor GDOP conditions for the urban cluster, with $\text{GDOP}_U = \{2.5, 20\}$, whose corresponding geometries are illustrated in Figure 4. We fix parameters at $N_U = 2$, $K_U = 4$, $N_{OS} = 50$, $K_{OS} = 19$, and $\sigma_\rho^2 = 20 \text{ cm}^2$. Results are presented for the estimator and the bound derived in Section III-C.

Method	$\text{GDOP}_U = 2.5$		$\text{GDOP}_U = 20$	
	Estimator	Bound	Estimator	Bound
DGNSS (m)	1.47	1.46	12.01	12.00
Ideal DGNSS (m)	-	1.03	-	8.49
C-DGNSS (m)	1.18	1.18	8.57	8.59
$\mathcal{I}(\%)$	19.7	19.2	28.6	28.5

GDOP_U , which yields 19.2%. Correspondingly, under poor urban GDOP_U , the proposed C-DGNSS estimator achieves the ideal DGNSS bound. These events occur because a poor GDOP_U amplifies positioning uncertainty due to unfavorable satellite geometry, increasing the influence of base station errors on the overall RMSE. This allows C-DGNSS to effectively leverage the cross-correlations of errors among users, which are inherently shared through the common base station measurements. Overall, the higher relative improvement observed for poor GDOP highlights the robustness of C-DGNSS in addressing challenging geometric conditions.

V. CONCLUSION

This paper presents the statistical modeling and performance analysis of a collaborative, code-based DGNSS method designed to improve position estimation in scenarios with limited satellite visibility. Relying solely on GNSS pseudorange measurements and requiring no inter-ranging information, the proposed approach is evaluated against both conventional and ideal DGNSS, the latter assuming a noiseless base station. Simulations show that C-DGNSS outperforms conventional DGNSS even with a single aiding user, with further gains as more users join. Relative improvements of up to 28.5% are observed under poor geometry conditions, with performance approaching the ideal bound. C-DGNSS relies on aiding users with superior satellite geometry to provide performance gains and maintains consistent performance regardless of the number of aided users, demonstrating its scalability. A similar method incorporating carrier-phase measurements was presented in [26], where RTK-level performance was achieved under constrained visibility conditions by leveraging double difference observations from multiple users. However, the impact of network size and user geometry was not analyzed—an aspect addressed in the present work for the proposed code-based approach. Future work will incorporate carrier-phase measurements into this analysis and analytically characterize the asymptotic C-DGNSS performance as a function of user count and geometry, based on the derived theoretical bounds. Robustness to outliers is further investigated in [28].

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