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APPLICATION OF MULTI OBJECTIVE CONSTRAINED OPTIMIZATION IN AERODYNAMIC HIGH LIFT DESING

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Abstract. *This lecture deals with the application of numerical optimization for aerodynamic design of high-lift systems, which is a multi-objective constraint design problem. The applied mathematical fundamentals of numerical optimization are briefly outlined. A description of the design targets and constraints for high-lift wings is given, followed by a detailed analysis of the properties of the flow calculation for the use within optimization and the suitability of optimization algorithms for this type of design problem. Another focus is set on the practical issues resulting from a long time experience in applying numerical optimization to aerodynamic design. of high-lift systems for transport aircraft.*

1 PREFACE

This presentation is an excerpt of the lecture titled "Multi objective constrained optimization and high-lift design applications" held at the VKI lecture series on Optimization Methods & Tools for Multicriteria/Multidisciplinary Design in Aeronautics and Turbomachinery. The full length written contribution can be found at [24].

2 INTRODUCTION

Computational Fluid Dynamics (CFD) has begun to play an important role in aircraft design. Widely known is the class of inverse design methods, which calculate the geometry for a given pressure distribution. The drawback of these methods is that the designer must know about good pressure distributions, which is not so easy to be defined in case of multi-element airfoils.

Another way to do aerodynamic design is direct optimization. Recently, a large number of aerodynamic optimization applications have been reported for transonic airfoil and wing design with a wide range of applied optimization methods, using either potential, Euler or Navier-Stokes methods for the simulation of the flow.

The design of high-lift devices is one of the challenging items in aerodynamic research. While looking at the procedures how high-lift devices are currently designed within industry, it is found, that main aspects of the work are still carried out by wind tunnel tests. Only the preliminary design of the device shapes is assisted by lower order CFD tools like panel-boundary layer methods. The reason is, that in the past the development and application of higher order methods like RANS solvers were mainly targeted to transonic airfoil and wing design. First attempts for a validation and application of these solvers for high-lift configurations at low Mach numbers just started less than 15 years ago [15].

In the past few years there have been two European EC-funded research projects addressing this topic. To mention is EUROLIFT [8], a project dealing with the validation of RANS solvers for high-lift configurations, and its follow up EUROLIFT II, for which another STS is held within this conference.

It has also already been shown, that RANS-simulation is able to capture sensitivities of the flow on geometric changesup to a validation of an optimization procedure for high-lift systems [23]. But up to date, only few attempts to use CFD methods for the design of high-lift configurations were madeand the most efforts were located at research institutes and universities. A summary of this work was collected by van Dam [19]. But in these days CFD is taking the step to become usable for practical design purposes. Although the reached level of accuracy of RANS-simulation has been demonstrated, this type of simulation has not yet found its application in aerodynamic design for high lift systems.

This lecture breifly outlines the mathematical fundamentals of the applied numerical optimization. Afterwards the targeted application to the aerodynamic design of high-lift systems is described. In the following a special section concerns about selecting the appropriate strategy and validating the optimization system. In a last section practical issues that are encountered within the aerodynamic design by numerical optimization are mentioned.

3 CLASSES OF NUMERICAL OPTIMIZATION METHODS

The intention of this section is to give an overview over the mathematical fundamentals of solving an optimization problem with numerical methods. The goal is to clarify the different items and idioms one faces when dealing with numerical optimization. It is intended to be as complete as possible, but also to not exceed the scope of this lecture. Further details can be found for example at [7, 12]

The mathematical formulation of an optimization problem is rather easy

$$F_{obj}\left(\vec{x}\right) \rightarrow min. \tag{1}$$

where F_{obj} is a scalar function, the so called objective function, depending on the vector of design variables $\vec{x}$. Note that the common formulation is to minimize the objective function. If the maximum of F_{obj} is requested, it is possible to use the negative $\tilde{F}_{obj} = -F_{obj}$ or the inverse $\tilde{F}_{obj} = \frac{1}{F_{obj}}$ of the original function, where for the latter it is essential that the sign does not change within the design space.

The optimality of a set of design variables $\vec{x}_{opt}$ is given if the gradient $\nabla F_{obj}\left(\vec{x}\right)$ is zero and the Hessian of the objective function $\boldsymbol{\nabla}^{\mathbf{2}} F_{obj}\left(\vec{x}\right)$ is positive definite, meaning that all eigenvalues are positive. This formulation of course is only the weak optimality condition, since it is only local to the surrounding of $\vec{x}$.

The method of solving an optimization problem is a straight forward loop. With the mathematical formulation of the optimization problem and the rapid advances in computer technology several hundreds of optimization methods and algorithms have been developed. It would be far beyond the scope of the lecture to give a complete overview. But there are some characteristics that help to distinguish between the different types of algorithms. One major criterion is if the algorithm selects new sets of design variables strictly based on the sets already evaluated, as done in the so called deterministic methods, or if the algorithm disturbs the design variables randomly. For the deterministic methods a further classification is possible based on the use of the derivatives of the objective function.

Non-derivative deterministic methods select a new set of design variables based on the objective function values of previously evaluated sets without using any derivatives. The simplex method [14] is a such an algorithm that does not use derivatives. The main idea is to build up a regular body with $n+1$ points in the design parameter space. The algorithm performs inversion, stretching and shrinking while moving through the domain. In the end the simplex collapses into one single point.

1. order derivative deterministic methods are based on the Taylor series expansion of the objective function. In order to reduce the objective function value, the dot product of the gradient $\nabla F_{obj}\left(\vec{x}\right)$ and the perturbation vector $d\vec{x}$ has to be negative. This leads

to methods, that define a search direction that holds negativity and perform a one dimensional search in this direction. This so called line search can again be looked at as a one-dimensional optimization problem. The steepest descent method is the simplest gradient method, since it directly uses the negative gradient for the direction of search. Nevertheless if the design space has a curved shape the search direction is only the best in the near region of the starting point, while further away a search direction other than the negative gradient will yield better results. The appropriate method for this is the use of the so called conjugate gradient method. The big advantage of the conjugate gradient method is that it accounts for the quadratic approximation of the objective function without a need to compute and store the Hessian matrix.

2. order derivative deterministic methods use the second order Taylor series expansion of an arbitrary function. From the optimality condition it follows that the next local extremum is where the gradient vanishes. So if the Hessian is known a system of linear equations has to be solved. Since F_{obj} is normally not of the quadratic form this procedure has to be iterated in order to obtain the minimum, leading to Newton's method. In typical applications the Hessian is unknown and only computable with high effort. Another solution is the use of the variable metric method, also called quasi-Newton method. Similar to the conjugate gradient method, the variable metric method is used to iteratively build up an approximation to the inverse Hessian. The most common way for this is the Broyden-Fletcher-Goldfarb-Shanno (BFGS) update.

There is also quite a large range of optimization algorithms that uses random numbers in one or the other way. Many random based optimization algorithms try to model either physical, chemical or biological mechanisms that have some kind of uncertainty incorporated. Most opular algorithms of this kind are simuated annealingand evolutionary algorithms.

4 HANDLING CONSTRAINTS

The optimization problem may be formulated with regard to some side constraints where there is a distinction between the equality constraints $g(\vec{x}) = 0$ and the inequality constraints $h(\vec{x}) \leq 0$. Sets of the design variables that violate even one constraint are called infeasible and should not be taken into account within the optimization. In practice there is nearly no unconstrained optimization, since in most cases at least the range of the design variables will be limited. The number of all equality constraints and all active inequality constraints reduce the dimension of the optimization problem if they are linearily independent. Thus, if there are more active and independent constraints than design parameters, the problem is ill-posed.

For most optimization problems it is sufficient to ensure that a point that is infeasible is excluded from the further optimization process. For non-derivative based methods the simplest and very effective way is the worsening of the objective function value, i.e. adding a penalty to the objective. But due to the addition of objective function and the penalty function it is possible that for a constraint violation the objective still decreases faster than the penalty rises. For this reason penalty functions are known to softly violate the constraints, i.e. hard limits cannot be guaranteed through the use of penalty functions. On the other hand this behavior the formulation of conditional objective functions, which are parts of the objective that are only used under circumstances.

For gradient based methods the use of penalty functions is only the second choice. The introduction of the penalty function.introduces a singularity to the derivative, and in all cases the assumptions of higher order methods that the function can be approximated by a quadratic form is strictly violated. Additionally the use of penalty functions often leads to a search direction pointing in a direction violating a constraint. The proper method here is to look for a search direction that does not violate the constraint and guarantees a decrease of the objective function value. There are two common methods. The first by Vanderplaats [20] is called the feasible direction method. Here the non-linear constraints are already activated a short distance before they are active in order to smoothly approach the constraint. For the same reason when a constraint is exactly met, a slight push back from the constraint is added to the search direction to guarantee feasibility for non-linear constraints in at least some distance from the starting point. The reduced gradient method of Gill, Murray and Wright [7] proposes to allow some infeasibility within the line search and followed by an additional step along the gradient of the constraint back onto the constraint, which is similar to projecting the end point of the line search on the hyperplane orthogonal to the gradients of the constraints.

5 DEALING WITH MULTIPLE OBJECTIVES

The common method of dealing with multiple objectives with standard optimization algorithms is to combine the single objectives into one major objective function, where the easiest and most frequently used formulation is the weighted sum of the single objectives. For the case that all single objectives are positive the product is also valid, with the advantage that no weighting factors have to be specified. But it has to be remembered that when changing a maximization to a minimization positivity has to be retained.

It is quite obvious that for the weighted linear combination of single objectives the result of the optimization strongly depends on the choice of the weighting factors. This is especially the case when the reduction of one single objective leads to an increase in one or more of the others. The weighting factors determine how much the decrease of some objective is worth the increase of the others. Therefore the choice of the weightings is never straight forward and requires the knowledge of the behavior of the single objectives.

In practical multi-objective optimization the occurrence of concurring single objectives is very probable. But there are two types of sets of design variables. For the first type there exists another set of parameters that improves all single objectives. For the second type there is at least one single objective that is increased. This subset is called the Pareto front. Once on the Pareto front it is not possible to improve one of the single objective without worsening another. There are special methods that do a determination of the Pareto front instead of a true optimization. They are mainly based on evolutionary algorithms, where all members of the final population are part of the Pareto front. It is up to the user to then select the appropriate set of design vectors for the design application.

In spite of using some float numbers, fuzzy logic only differentiates between three different states of single objectives: good, acceptable and bad. The major objective is that in the optimum case all single objectives are at least in an acceptable state, if not good. Handling constraints here is a straight forward formulation as additional objectives that only have the states acceptable or bad. It is clear that deterministic methods are not appropriate for the fuzzy logic implementation, since the main assumption for these algorithms, that the design space is differentiable, is not fulfilled.

6 HIGH-LIFT SYSTEM DESIGN – A MULTI-OBJECTIVE MULTI-DISCIPLINARY CONSTRAINED OPTIMIZATION PROBLEM

Before dealing with the aims of high-lift design, the question of the aerodynamic necessity of high-lift devices should be answered, as this directly clarifies the design targets. First let us concentrate on a typical transonic transport aircraft. Such an aircraft typically ends its cruise flight at an altitude of approximately 11km flying at $C_L \approx 0.5$ and $M_\infty \approx 0.8$, or $V_{EOC} \approx 460$kts respectively. Since this is a condition of equilibrium flight, the equation

$$L = m \cdot g = C_L \cdot \frac{\rho}{2} \cdot V^2 \cdot A \tag{2}$$

holds, and gives the relation between the necessary lift coefficients for decelerated flight. A typical flight speed at final approach is $V_A = 135$kts , resulting in a velocity ratio $\frac{V_{EOC}}{V_A} \approx 3.4$. Together with the density ratio $\frac{\rho_{11km}}{\rho_{0km}} = 0.2971$ the necessary lift coefficient can be calculated to $C_{L_A} = 1.72$. Remembering that even a single element airfoil can only reach maximum lift coefficients up to $C_{L_{max}} \approx 2.0$, it is obvious that simple wings are not able to get the performance needed. So for take-off and landing the wing has to be adapted by deploying high-lift devices.

The design targets of high-lift devices can be divided into two classes: regulatory needs and performance targets. The first class concerns primarily the regulations of the Federal Aviation Administration FAA contained in the Federal Aviation Regulations (FAR) part 25 [4]. One of the major aspects is the definition of the different flight segments for takeoff and landing, where each has its own regulations concerning speeds and climb performance. For each segment FAR part 25 specifies a minimum speed ratio related to stall speed V_S and a minimum climb gradient either with all engines operating or with one engine inoperative.

In addition to these performance issues are an increasingly important point of interest, especially for the take-off. The first item here is a fast climb to cruise flight level. A second one, which has gained in significance in recent years, is the noise impact on the environment. The growing socio-political demands for noise reduction in the vicinity of airports has led to more and more stringent regulations, which can be found in FAR part 36 [5]. These regulations define procedures to measure noise (FAR B36.3) as well as the maximum noise levels permissable (FAR B36.5). Flyover noise is directly correlated to the aerodynamic performance, as the sound intensity on the ground varies roughly as the inverse square of the distance form the source. Thus an increased take-off performance leads to a greater altitude over the measurement point and thereby to a reduced noise level.

The requirements on a high-lift system of a transport aircraft can be reduced to two primary aerodynamic parameters: a) the maximum lift coefficient $C_{L_{max}}$ and b) the lift to drag ratio L/D. The first one is the primary driver for the landing case and the second one for the take-off case.

7 HIGH-LIFT DEVICES FOR TRANSPORT AIRCRAFT

The development of different classes of high-lift devices is an ongoing process. A nearly complete summary of devices used today is given by Rudolph [16]. This lecture focuses on passive devices, since active devices like blown flaps are not commonly used on transport aircraft. They have their application mainly in military aircraft.

For the leading edge there are mainly three types of devices: a) the so called slat, which is deflected by a translational and rotational movement normal to the wing front spar, resulting in a gap between the slat and the so called fixed leading edge (FLE); b) the droop nose device, which is deflected only in a rotational movement, so that no gap is formed; c) the Krueger flap, which is a thin plate that is deflected from the lower side of the wing leading edge.

Modern trailing edge devices all work on the principle of Fowler [6]. The device is moved completely towards the trailing edge and is also rotated. The big benefit of the Fowler-flap is the increase of wing area. For a conventional transport aircraft the wing area is increased roughly 25% by only deploying the flap.

The parameterization of a high-lift wing can be divided into three classes: a) outline parameters, b) positioning parameters and c) shape parameters. The first class of parameters includes all parameters that can be observed in the plan view of the wing. This class covers the spanwise extent of the slats and flaps and the local chord length of the devices and the fixed main wing. These parameters are usually not constant along the span, so the minimum set is chosen by specifying these parameters at the three wing sections root, kink and tip. The second set of parameters covers the deflection of the deployed devices, the so called setting. The third class of parameters covers the shape of the devices. In general not the complete shape is free for design. In contrast, the shape of the clean airfoil that is designed for cruise flight, cannot be changed by the high-lift design. The shape that only may be modified for high-lift design are the portions that divide the devices from the fixed wing.

A number of constraints limiting the design envelope of the high-lift system resulting from multidisciplinary topics. The first important class are the requirements from structural design. The demands here are directly correlated to deformation aspects. In order to limit the deformations minimum thicknesses must be given. Since the positions of the front and rear spars forming the wing box are designed for the wing load under cruise conditions, they are fixed. The flaps and slats must remain outside the wing box along with the required actuators and kinematics. A second class of constraints is derived from the kinematics used for deploying the high-lift devices. The kinematics are targeted to be as simple as possible to reduce system complexity in terms of actuators, moving parts as well as system weight. For the high-lift design the kinematics constrain the dependencies between the settings for the different flight segments. The established procedure currently is to design for the landing segment, defining the maximum deflections. The other settings are then derived by use of the kinematical laws.

8 SELECTING AN APPROPRIATE OPTIMIZATION ALGORITHM

Over the last decades many different optimization algorithms have been developed, each with their own strengths and weaknesses. Therefore it is hard to find one optimizer ideally suited to all kinds of problems. In fact the right choice of the optimization algorithm depends heavily on the method used to evaluate the objective function. For that reason the question of which optimization strategy should be used for our purpose can be answered not before knowing the numerical method to evaluate the objective function, respectively the flow behavior.

The best way to make a decision on the computational method is first to look at the flow features that have to be simulated. Fig. 1 shows the two-dimensional flow field around the wing section of a 3-element airfoil called NHLP90 L1T2 [13]. The shaded regions are those where viscous effects are dominant. Along with transition and the typical boundary layer regions near the walls the flow around multi-element wings has two additional features: a) re-circulation areas in the cut-outs, b) mixing of boundary layers and wakes of preceding elements, also known as confluent shear layers. Additionally, since maximum lift conditions are of interest, the occurrence of flow separation must be included in the flow model. Although the flow velocity is subsonic ($M_\infty \approx 0.2$) the acceleration on the upper side of the slat can be so strong that the flow can locally become supersonic. For the choice of the flow modeling method this implies that compressible solvers are strongly recommended.

For all these reasons, tools based on the Reynolds-averaged Navier-Stokes equations (RANS) are the methods of choice. Nevertheless other methods have already been applied to high-lift flows. There are two summaries available for the requirements of calculating high-lift flows [11] and for the state of the art of RANS computations [17].

RANS solvers require the discretization of the volume by a computational grid. For the purpose of optimization it is evident that the quality of the grid in terms of resolution of the critical flow features is kept constant for all the different configurations. At DLR the grid generator MegaCads has been developed, which is able to fulfill this requirement (Brodersen et al. [2]). The key features that make the grid generator applicable for optimization, are: a) the parametric concept, allowing the generation of grids of constant quality around changing geometries; b) restart functionality instead of grid movement;

c) non-interactive mode that allows to run the grid generation in a batch job on high performance computers.

Within this lecture the flow simulation software used is the DLR FLOWer code [10], which is a structured finite-volume RANS solver. It solves the compressible unsteady RANS equations. For steady flow the equations are integrated using a time-marching 5-stage Runge-Kutta scheme until a steady state solution is reached. Typical convergence acceleration techniques, like multigrid, implicit residual smoothing and local time stepping are applied. For the discretization of the governing equations a central scheme of second order accuracy in time and space is used. For turbulence modelling FLOWer offers algebraic models as well as transport equation models, beneath them the algebraic model of Baldwin and Lomax [1] (BL), the one equation model of Spalart and Allmaras with the Edwards modification [3, 18] (SAE) and the two equation k-ω–model of Wilcox [21].

When talking about the results of CFD calculations two questions are always asked: a) which turbulence model was used and was it suitable for the calculated flow? and b) which grid with which resolution has been used and is the solution grid converged? While for aerodynamic performance analysis the focus is on the accuracy of the values of the aerodynamic coefficients, in aerodynamic design optimization the focus is on whether the sensitivities on geometrical changes are correct. It is definitely not necessary to achieve a high accuracy as long as the influence of the geometry on the flow is captured. In the case that the error made in the flow calculation is a systematic one, being the same for each evaluated configuration, there is no effect on the optimum set of design parameters.

In order to answer these questions an investigation was made using a simpler test case. The four-digit NACA airfoil series,was used for this purpose at a flow condition typical for high-lift flows, given by $M_\infty = 0.2$, $Re_\infty = 3 \cdot 10^6$ and $\alpha = 10°$. The two design variables used are the maximum camber f and the chordwise position of maximum camber x_f. The thickness is held constant at $\frac{d}{c} = 12\%$.

In the following the complete design spaces resulting form step-by-step calculations are compared for three turbulence models each computed on two different grids. The turbulence models used are the Baldwin-Lomax–model, the SAE–model and the k-ω–model. The grids have 320×64 and 480×96 points, so the finer grid has a 50% higher density. The corresponding design spaces are shown in fig. 2, where the objective function values are plotted as iso-line depending on the two parameters. First discussing the grid resolution, it can be seen for all three turbulence models, the optimum parameter combination is not affected by the size of the grid. Only the k-ω–model shows slight differences. It has to be pointed out, that the absolute values of the objective functions for the two grids differ, since the dashed and solid lines do not match exactly. Of course this statement only holds to a certain limit. Using excessively coarse grids will also have an impact on the optimum. Experience shows that using grids typical for engineering purposes is sufficient.

The other important aspect is the choice of the turbulence model. The comparison of the different design spaces shows a strong dependency of the optimal parameter combination on the model used. Thus the right choice of model is evidently necessary for successful optimization and a wrong model can lead to completely false solutions. The question which turbulence model is the right one for the targeted application, can only be answered by comparing to experiments. Detailed validation investigations have shown that the SAE–model seems to be well suited for high-lift flows [8, 23].

Before selecting an optimization algorithm, it is necessary to determine the proper criteria for the selection. These are mainly driven by the characteristics of the flow calculation as these are: a) time-consuming calculations and b) truncation errors of the iterative solution procedure, resulting in a noisy objective function. For these reasons the optimization algorithm must be highly efficient and robust concerning noisy functions. Since the computational effort will be high, it is also an advantage, if the optimization algorithm is able to detect the global optimum, and not merely the closest local optimum.

Some effort has been undertaken to analyze one representative of each class of algorithms for its suitability to aerodynamic optimization by the author [22]. A summary of the obtained results is sketched in fig. 3. The principle of testing is a statistical evaluation over a hundred optimization runs on an analytic test function with a global optimum and a local optimum nearby, each run starting at a different randomly chosen starting point.

The efficiency is evaluated by the number of function evaluations necessary to get a converged optimization solution. Globality here is the ration of detecting the true global optimum related to the number of runs. Finally the robustness is examined by running different tests with different levels of random noise ϵ_F added to the analytical function. There is some general behavior of the optimization strategies to be noted here, that is typical for the class the chosen algorithms belong to. Firstly, the random based optimizers are the only ones that are truly global. But they are also highly inefficient. In contrast, all higher-order deterministic methods are really local. For the outlined case, two optima were present and a ratio of 50% for the smooth function simply denotes that every time the next optimum is hit. But the higher-order methods, especially the gradient based algorithms are highly efficient on smooth functions. The obvious drawback of the Newton algorithm results from the possibility that this type of method may not be able to converge. For the gradient based algorithms it can be seen that the high efficiency is lost, if the function is noisy. This results from errors occurring in the gradient evaluation by finite differences on non-smooth functions, which might be different when using adjoint methods [9]. In summary the best compromise between the three criteria is found to be a Simplex-type algorithm for the targeted application.

9 VALIDATING AN OPTIMIZATION ENVIRONMENT

Before starting to optimize some arbitrary configurations, a verification and validation is needed, in order to ensure reliable answers from the aerodynamic optimization procedure. For this purpose a comparison is made to experimental data. Woodward and Lean [25] published experimentally determined dependencies of the maximum lift coefficient of the NHLP90 L1T2 configuration on the positioning of the slat at different deflection angles. In fig. 4 a comparison is shown between the measured values of maximum lift coefficient (iso-line) and the search path of two optimizations of the slat for maximum lift coefficient, where the second was performed using a two times coarser grid. It is observed that for the normal grid with a medium resolution corresponding to best engineering practice, the optimum is located exactly. The optimization using a coarser grid is not far off in its result, since the measured values for the two detected configurations only vary by 1%. Three topics previously discussed are confirmed by this result: a) numerical optimization is able to detect an optimal aerodynamic configuration; b) grid sensitivity and transition play a minor role for this type of optimization; c) the chosen turbulence model is suitable for high-lift design.

10 APPLYING NUMERICAL OPTIMIZATION - PRACTICAL ISSUES

Basically devising a method for numeric optimization sounds quite simple: a) select an optimization tool; b) select a flow simulation tool; c) couple them for data exchange. But those assuming that running this code will produce optimum results for his type of application will almost certainly be disappointed. One sentence is evidently true for practical optimization: You will exactly get what you have asked for, and nothing else. Mistakes or uncertainties in the definition of the design problem will be reflected by unusable designs. This section will attempt to give some advice based on some extensive experience in applying numerical optimization to aerospace design.

10.1 Noisy functions

Assuming that an iterative solver or some other kind of numerical method including round-off errors is used for the evaluation of a configuration it is clear that a certain level of random noise of the objective function must be expected. The flow solver used by the author for example uses the maximum deviation in the lift coefficient over the last couple of iterations as a criterion to stop the calculation. For the optimization it may be assumed that this is directly the noise level of the objective function. Looking at a zoomed image of the convergence history (fig. 5) of the initial NACA 4412 airfoil shows the problem. It is not a priori sure that the remaining oscillations are completely captured in the interval used for testing the convergence. Especially low frequency oscillations have the potential for a large uncertainty in the calculated coefficients. In conclusion it is obvious to have a good insight on the convergence behavior in order to estimate the noise level of the objective function.

10.2 Multi-objective formulation

It can be stated that every aerodynamic design is at least a multi-objective optimization. The common understanding of improving aerodynamic performance is to increase lift while reducing drag (nevertheless there are cases where it might be desirable to increase the drag, e.g. spoiler design). These are the first two objectives. From an optimizers point of view the formulation would be

$$F_{obj}\left(\vec{x}\right) = -w_1 C_L + w_2 C_D, \tag{3}$$

while the aerodynamicist would use of course the lift to drag ratio

$$F_{obj}\left(\vec{x}\right) = -\frac{C_L}{C_D}. \tag{4}$$

What is the advantage of using formulation (4) over (3)? First of all the second one does not imply any arbitrary weighting coefficients. A look at the gradients for both formulations

$$\delta F_{obj}\left(\vec{x}\right) = -w_1 \nabla C_L \cdot \delta\vec{x} + w_2 \nabla C_D \cdot \delta\vec{x} \tag{5}$$

and

$$\delta F_{obj}\left(\vec{x}\right) = -\left(\frac{1}{C_D}\nabla C_L \cdot \delta\vec{x} - \frac{C_L}{C_D^2}\nabla C_D \cdot \delta\vec{x}\right) \tag{6}$$

respectively, show that the optimum for both formulations is only the same, if the gradients of the single objectives ∇C_L and ∇C_D approach zero independently. In this special case the single objectives are independent and can also be optimized independently. But this is not the usual case. For most multi-objective problems, the optimum is found as a compromise between the single objectives, where none is at an optimal point with respect to one single objective alone. Now, even if the weighting factors in (3) are set accordingly, so that the gradient is exactly the same for the initial configuration

$$w_1 = \frac{1}{C_{D_{ini}}}, w_2 = \frac{C_{L_{ini}}}{C_{D_{ini}}^2}, \tag{7}$$

this will not result in the same optimum, since C_L and C_D at the optimum will definitely differ from $C_{L_{ini}}$ and $C_{D_{ini}}$.

The optimum of a multi-objective optimization is usually a compromise between single objectives. If constraints apply this extends to the most feasible compromise. It is understandable that the complexity of the design space increases with each additional single objective or constraint. Introducing a new constraint can never improve an existing optimal compromise. In contrast in the best case, the compromise is not worsened, which is only the case if the new objective is independent of the other ones. An additional constraint always bears the risk that a feasible solution cannot be found anymore. For these reasons it is advantageous to reduce the dimension and complexity of an optimization problem to the lowest possible level. Trying to do an all-in-one optimization from scratch without a deep understanding of the design space bears more risk of failure than it allows for a successful design to be achieved. It is better at first to slowly increase the complexity of a design case to study the influence of new objectives and constraints.

10.3 Understanding optimality

Along with the direct impact of using optimization methods for aerodynamic design to get improved configurations, another big advantage of applying this method is the ability to learn about optimality of aerodynamic configurations. The normal case when running an aerodynamic optimization is that the result may be unexpected. The optimizer is able to change so many parameters simultaneously that the effects improving the aerodynamic behavior cannot be directly assigned to any of them. An inspection of the results and a detailed comparison of initial and optimized configurations helps to understand the reasons for optimality of parameter combinations. It also allows figuring out which parameters always tend towards their bounds and so can be fixed in future optimizations. But there are also some lessons that can be learned when analyzing the way the optimizer took to get to the optimum. In the following two short examples will be given for this purpose.

The first example is a multi-point design of a landing configuration

$$\begin{aligned} F_{obj}\left(\vec{x}\right) &= -0.5\left(C_L\left(\alpha = 0°\right) + C_{L_{max}}\right) \\ \vec{x}^T &= \left(x_F, y_F\right). \end{aligned} \tag{8}$$

In a previous section it was shown, that plotting the objective function as an iso-contour depending on the two parameters is a good way to inspect a design space. In the present example the design space is not evaluated in a step-by-step calculation, but by an interpolation of the data from the optimizers search path (fig. 6). In this way not only the shape of the design space can be observed after optimization but also the contributing design points can be observed. Im the present example it can be seen, that the design points are competing, and the resulting multi-point optimum is a compromise between both.

A second example deals with unexpected results. The task is a simple drag minimization by vertical slat positioning for a take-off configuration

$$\begin{aligned} F_{obj}\left(\vec{x}\right) &= C_D \\ \vec{x}^T &= \left(y_S\right). \end{aligned} \tag{9}$$

The corresponding dependency of the drag coefficient on the vertical displacement, as it results from the optimization, shows a sharp optimum with a 19% reduction of drag compared to the initial position (fig. 7). Now looking at a velocity profile (given in terms of total pressure) of the boundary layer shows big differences that are the reason for the obtained drag reduction. While for the initial configuration the slat wake and the boundary layer have just started to mix, the optimum configuration only shows one shear layer. Additionally the shape of the profile looks similar to a laminar boundary layer, although the flow is fully turbulent. This was only possible by moving the slat to a position at which the shear layers of wing and slat mix in exactly this way. Slight changes would already result in a significant increase of drag.

11 FIGURES

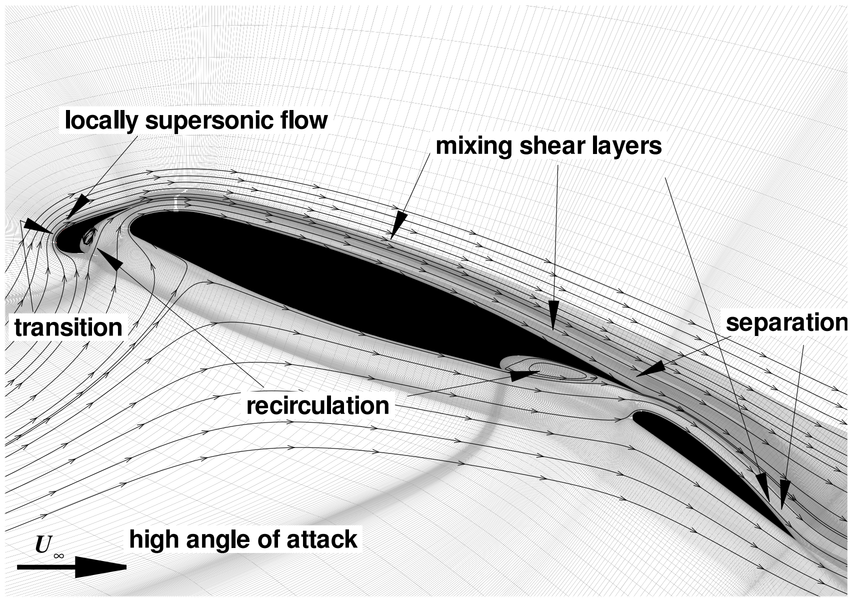

Figure 1: Flow field around the wing section of a 3-element wing

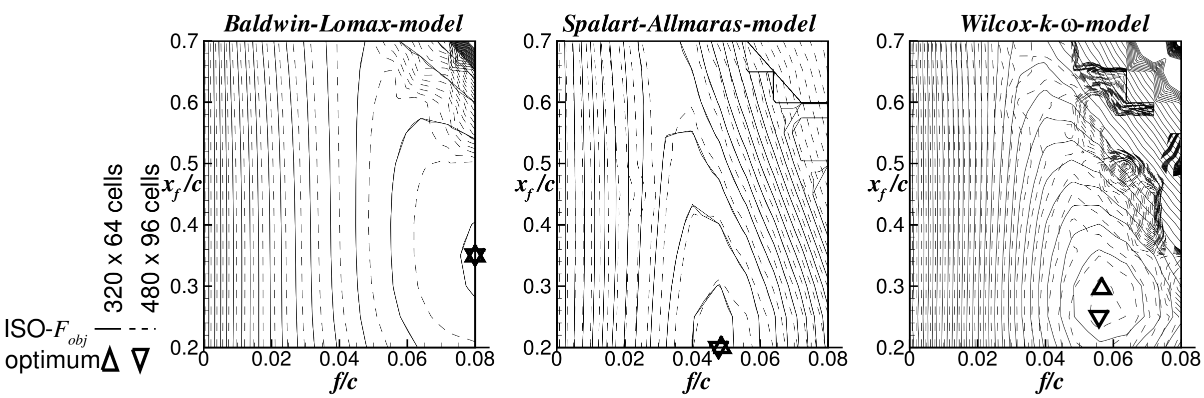

Figure 2: Calculated design spaces of the NACA-XX12 airfoil series for different turbulence models and grid resolutions

12 CONCLUSIONS

The lecture presented the successful application of numerical optimization methods to the design of high-lift devices. Based on a detailed analysis of the design problem and the flow simulation method, the appropriate optimization method was selected in order to achieve a validated and reliable optimization tool. Beneath this, the critical issues for the formulation of multi-objective constraint optimization problems has been mentioned.

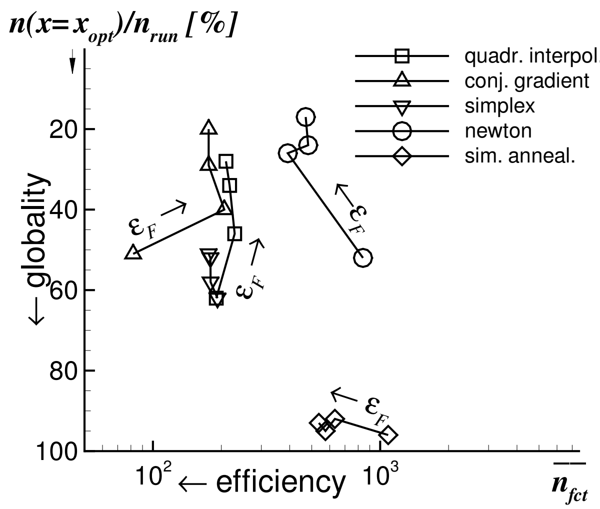

Figure 3: Comparison of robustness, efficiency and globality of different classes of optimization algorithms shown on an analytic test case with a global and a second local minimum

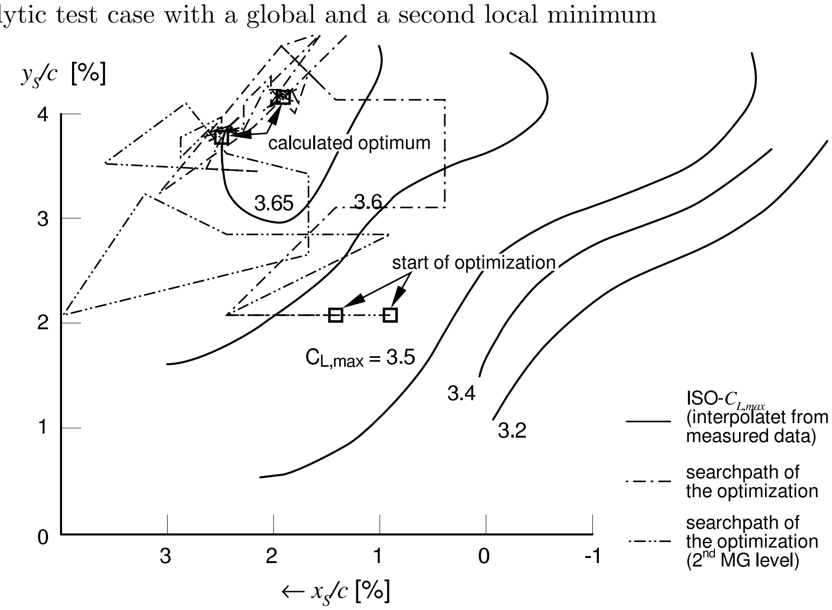

Figure 4: Measured iso-lines of maximum lift coefficient depending on the slat position overlaid with the search path of two optimizations with different grid resolutions

Additionally the potential of using numerical optimization methods to the aerodynamic design has been shown for the example of high-lift system design.

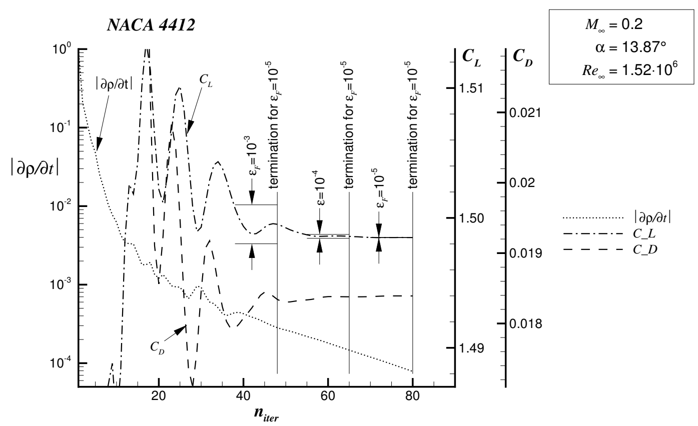

Figure 5: Influence of the termination criterion for the lift coefficient on the remaining error in the aerodynamic coefficients shown for the convergence of the NACA 4412 airfoil

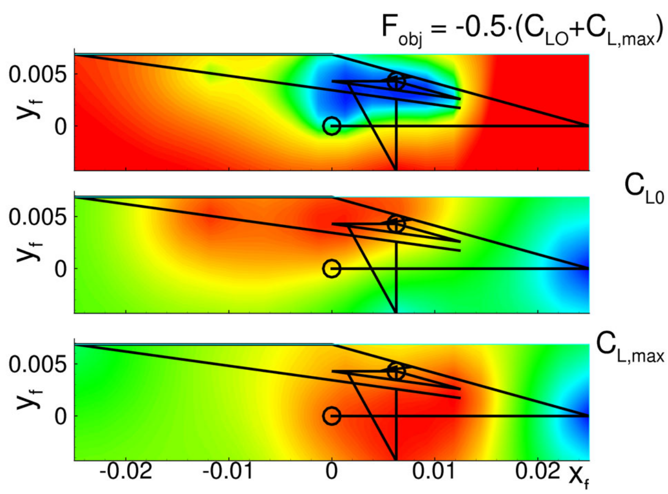

Figure 6: iso-contours of the objective function and the single design points of a multi-point optimization extracted from the search path of the optimization

REFERENCES

[1] B.S. Baldwin and H. Lomax. *Thin layer approximation and algebraic model for seperated turbulent flows.* Paper 78-257, AIAA, 1978.

[2] O. Brodersen, M. Hepperle, A. Ronzheimer, C.-C. Rossow, and B. Schöning. *The parametric grid generation system MegaCads.* In B.K. Soni, J.F. Thompson, J. Häuser, and P. Eisemann, editors, *5th International Conference on Numerical Grid Generation in Computational Field Simulation*, pages 3535–362. National Science Foundation (NSF), 1996. http://www.megacads.dlr.de.

[3] J.R. Edwards and S. Chandra. *Comparison of eddy viscosity-transport turbu-*

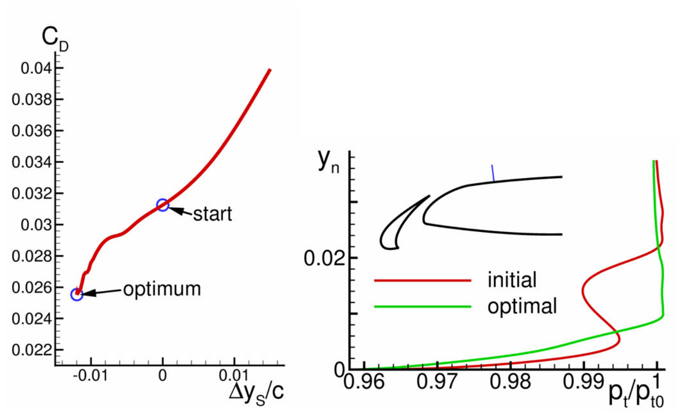

Figure 7: Result of a slat vertical placement optimization on drag

lence models for three-dimensional, shock-separated flowfields. Journal of Aircraft, 34(4):756–763, 1996.

[4] Federal Aviation Administration FAA. *Airworthiness standards: Transport category aircrafts, FAR part 25.* http://www.airweb.faa.gov.

[5] Federal Aviation Administration FAA. *Noise standards: Aircraft type and airworthiness certification, FAR part 36.* http://www.airweb.faa.gov.

[6] H.D. Fowler. *Variable lift. Western Flying*, pages 31–33, Nov. 1931.

[7] P.E. Gill, W. Murray, and M.H. Wright. *Practical Optimization.* Academic Press, London, England, 1993.

[8] H. Hansen, P. Thiede, F. Moens, R. Rudnik, and J. Quest. *Overview about the European high lift research programme EUROLIFT.* Paper 2004-767, AIAA, 2004.

[9] A. Jameson. *Aerodynamic shape optimization using the adjoint method.* In *CFD-Based Aircraft Drag Prediction and Reduction*, VKI-LS 2003-02. VKI, 2003.

[10] N. Kroll, C.-C. Rossow, K. Becker, and F. Thiele. *MEGAFLOW - A numerical flow simulation system. Aerospace Science Technology*, 4:223–237, 2000.

[11] F.T. Lynch, R.C. Potter, and F.W. Spaid. *Requirements for effective high lift CFD.* In *ICAS Proceedings 1996*, pages 1479–1492. ICAS, 1996.

[12] H.D. Mittelmann and P. Spellucci. *Decision tree for optimization software.* http://plato.asu.edu/guide.html.

[13] I.R.M. Moir. *Measurements on a two-dimensional airfoil with high-lift devices.* Number A2 in AGARD-AR-303. AGARD, 1994.

[14] J.A. Nelder and R. Mead. *A simplex method for function minimization. Computer Journal*, 7:308–313, 1965.

[15] S.E. Rogers, N.L. Wiltberger, and D. Kwak. *Efficient simulation of incompressible viscous flow over single and multi-element airfoils.* Paper 92-0405, AIAA, 1992.

[16] P.K.C. Rudolph. *High-lift systems on commercial subsonic airliners.* CR 4746, NASA, 1996.

[17] C.L. Rumsey and S.X. Ying. *Prediction of high lift: Review of present CFD capability. Progress in Aerospace Sciences*, 38:145–180, 2002.

[18] P.R. Spalart and S.R. Allmaras. *One-equation turbulence model for aerodynamic flows.* Paper 92-0439, AIAA, 1992.

[19] C.P. van Dam. *The aerodynamic design of multi-element high-lift systems for transport airplanes. Progress in Aerospace Sciences*, 38:145–180, 2002.

[20] G.N. Vanderplaats. *Numerical Optimization Techniques for Engineering Design.* McGraw-Hill Book Company, New York, USA, 1984.

[21] D.C: Wilcox. *Turbulence Modelling for CFD.* DCS Ind., La Canada, USA, 1994.

[22] J. Wild. *Direct optimization of multi-element airfoils for high-lift using Navier-Stokes equations.* In K.D. Papailiou, D. Tsahalis, J. Périaux, Ch. Hirsch, and M. Pandolfi, editors, *Computational Fluid Dynamics '98 - Proceedings of the 4th European Computational Fluid Dynamics Conference*, pages 383–390, Chichester, UK, 1998. Wiley & sons.

[23] J. Wild. *Validation of numerical otimization of high-lift multi-element airfoils based on Navier-Stokes-equations.* Paper 2002-2939, AIAA, 2002.

[24] J. Wild. *Multi objective constrained optimization and high-lift design applications.* In VKI Lecture Series LS-2004-06, *Optimization Methods & Tools for Multicriteria/Multidisciplinary Design in Aeronautics and Turbomachinery*, VKI (2005)

[25] D.S. Woodward and D.E. Lean. *Where is high-lift today? A review of past UK research programmes.* Number 1 in AGARD-CP-515. AGARD, 1993.