

# Considering Uncertainty in Energy System Optimisation

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## Abstract

In energy system optimisation, there is a tendency towards working with models that feature perfect foresight [1, 2]. On the other hand, devices in today's energy systems have been operational for a long time, e.g. one third of home heaters in Germany are older than 20 years [3], some German coal power plants from the 1950s are still in operation [4], and – more anecdotally – according to the company that built and installed it in the Zurich town hall, the world's oldest water-water heat pump from 1936 is still operational. Thus, it is rather optimistic that a prognosis over the complete operational period can be made with good accuracy. In fact, in research it is often correctly stated that scenarios are created rather than prognoses are made. However, if decisions are to be taken, a prognosis is needed, and the further that prognosis looks into the future, the more uncertainty it has.

The issue can be addressed by considering multiple scenarios for the future, instead of just a single one. Options include a sensitivity analysis to test the dependency of the result on the input parameters [5, 6, 7], sometimes in the flavour of Monte-Carlo simulation [8]. Stochastic programming [9] and robust optimisation [10] are two commonly used ways [11, 12] to do so already in the optimisation process.

This contribution gives examples on how to implement these types of multi-scenario approaches into energy system optimisation that is based on linear programming. It also comments on the generation and selection of scenarios, that can then be used for these methods. Besides manually picking optimistic and pessimistic extremes, we discuss the generation of self-consistent time-series using Markov chains [13]. While the methods can be applied in multiple fields, we use charging of battery electric vehicles (EV) as an example.

## 1 Scenario-aware optimisation (EV charging)

Scenario-aware methods can be used to plan energy systems in a manner which accounts for uncertainties within considered scenarios. These methods aim to identify different scenarios that may be encountered in the future, and maximises the system performance individually for each of these scenarios [14]. Our example shows a way, how to include stochastic programming [15] into a linear

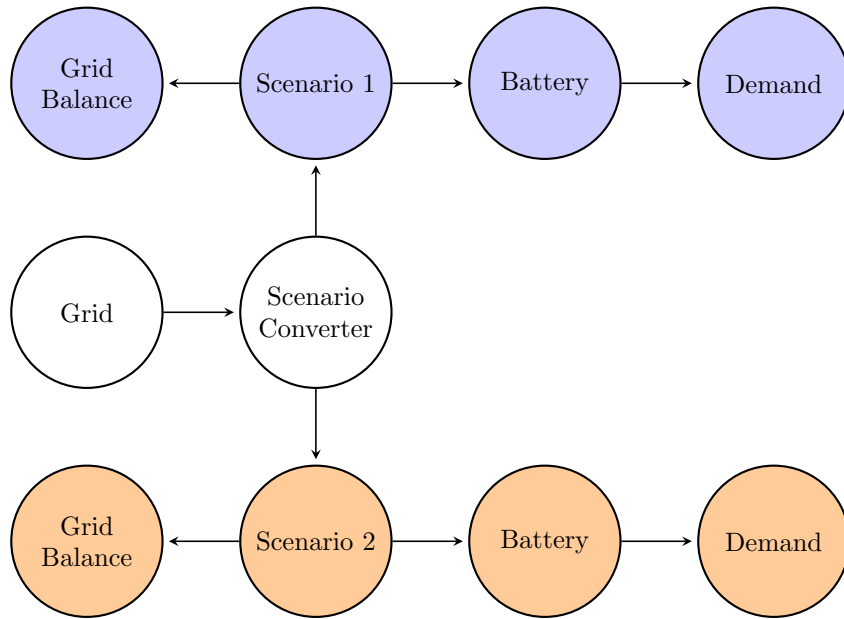


Figure 1: Energy flow diagram with two scenarios for the trips. Grid source is connected to both scenarios via scenario converter. Both scenarios include grid balance, battery and demand.

optimisation framework that does not provide explicit support for stochastic programming.

The energy system model that we used includes the electricity grid for energy supply, EV batteries, and charging stations. To account for the uncertainty, we created two different scenarios to be optimised: one base case scenario (Scenario 1) and an alternative scenario, where the driver sets off two hours earlier (Scenario 2). The electricity price is changing throughout the day, but there is just one common price curve. In particular, to enforce differences in this didactic example, we set very low prices for the periods of time when the car is on the road in the morning in either of the scenarios.

The energy system model optimises the charging of the EV batteries while managing the energy from the electricity grid. In each scenario, energy flows are managed independently using separate batteries, minimising total costs while meeting the demands. In the scenario integrated optimisation model, separate energy flows are provided for both scenarios.

We simulated energy flows over one day in 10-minute time slots using the `oemof.solph` [16] library in the modelling process. It tries to find one strategy to purchase electricity from the grid that is optimal considering multiple possible scenarios. Thus, the grid source is the same across scenarios. For our present example, we created separate energy flows for two scenarios, which are connected using a `Converter`, that makes sure purchased energy is identical in all scenarios. However, as a battery cannot be charged if the vehicle is not present, there is a (virtual) Sink called ‘Grid Balance’. Pushing energy to this outlet means that purchased energy cannot be used. We added a small penalty to make sure not even free electricity will flow from the grid if there is no way to consume it in

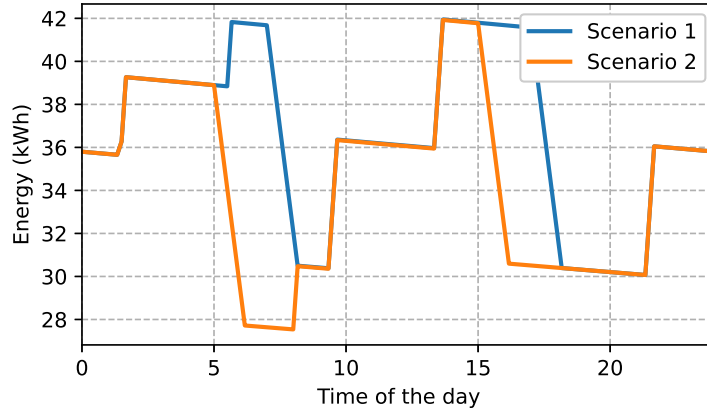


Figure 2: Battery SOC for a day with two scenarios for the trips. An increasing SOC means charging, a slowly decreasing SOC is due to self-discharging. When the vehicle is driving, the SOC decreases. It can be seen that the SOC is different due to the different points in time the vehicle is moving.

any of the scenarios. The energy system graph shown in Fig. 1 visualises the energy flows of the two journey scenarios.

The optimisation results show the most cost efficient energy flows and storage operations under the specified scenarios. This is displayed in Fig. 2, which graphs the energy state of an EV battery for the two scenarios. The method makes sure that the vehicle can only be charged if it is present, preferably scheduling the charging if it is there in all scenarios. However, high fluctuations in the price can justify buying energy even if it is not used in all of the scenarios. In the present example, this can be seen in the morning hours: In the first scenario, the battery of the electric vehicle is charged before travelling, while it is on the road at this time in scenario 2. Vice versa, the battery in scenario 2 is charged while the car is still moving in scenario 1. Except for differences due to the different driving times, a common State of Charge (SOC) is observed between the scenarios. Note that for a different example, even mostly parallel charging might not always result in the same SOC in multiple scenarios: In particular, a longer time span would allow for almost parallel movement, if there is a later opportunity to realign.

The optimisation method we presented individually optimises the energy flows for multiple different EV and grid scenarios, and couples these to a scenario-independent operational strategy for the grid connection point. These scenarios can be real world scenarios based off gathered trip data, but can also be applied to theoretical datasets to account for the numerous uncertainties in data collection. This optimisation code was demonstrated on a two scenario dataset, with the optimisations showing notable differences. In the first scenario, energy management and cost-effectiveness are optimised as the battery of the electric vehicle can be charged by receiving energy from the grid at a time when the early departure scenario cannot.

## 2 Scenario generation (price development)

Considering multiple scenarios instead of just a few calls for a method to automatically generate realistic scenarios. To do so, we blend a diffusion model with a-priori expectations for the state. Diffusion models are often used to model market prices [17], but they can be applied for many cases. While they are simple to apply, they typically assume a constant volatility or diffusion rate [13]. This might be a valid assumption for very short periods of time but it is not expected over several decades, especially, during the transition from fossil fuels to renewable energies. And it is also not modelling periodical changes as they are implied, for example by daily variations in the energy demand. Secondly, diffusion models are typically unbound. In the energy market, however, prices depend on the availability of supply options, that each have a more or less constant price. This is why we add an a-priori probability for the state.

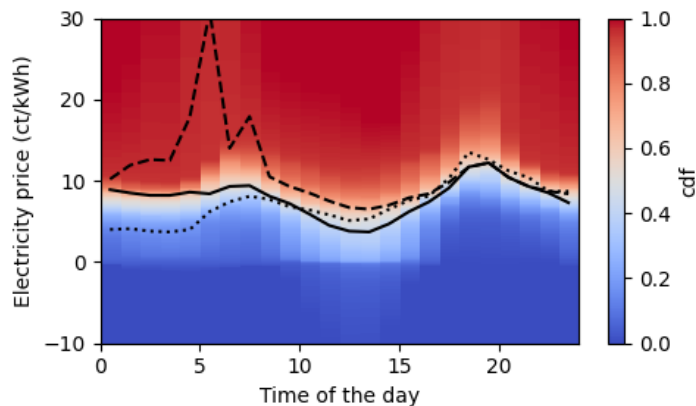


Figure 3: Cumulated distribution function of the electricity spot market price over the course of a random day between December 2023 and November 2024 (data from [18]). Estimating probabilities from a low number of events marks a challenge by itself. Thus, the probability of extreme prices has a significant uncertainty. The black lines show random price development scenarios generated using the presented method.

Applied to the problem of electricity price variations during the day, the state is the current electricity price  $c_t$ , and the diffusion is the price change  $\delta_t$ . To model the fact that price variations happen in a certain band, we introduce an a-priori probability density function (pdf)  $p(c, t)$  as shown in Fig. 3. It can be seen, that it is uncertain what the price will be, but certain changes are expected, e.g. prices tend to be higher in the morning and in the evening. As in a discrete time grid with time resolution  $\Delta t$

$$c_t = c_{t-\Delta t} + \delta_t, \quad (1)$$

the probability density for the price change  $p(\delta, t)$  does not guarantee that  $c_t$  stays inside the allowed range. For example, if the price is at the lowest possible value for  $c_t$  and  $p(c_t, t + \Delta t) = 0$ , the price change  $\delta_{t+\Delta t}$  needs to be sufficiently high in that time interval. To solve this, a-priori probability  $p(c, t)$  might

be convolved with a probability resulting from the last state and the change probability  $p(\delta, t)$

$$p'(c, t + \Delta t) = p(c, t + \Delta t) * (c_t + p(\delta, t)), \quad (2)$$

where ‘\*’ marks the convolution of the two functions. However, this approach does not guarantee that the scenarios resemble the a-priori probability. Thus, we rather want to find a pdf  $p'(\delta, t)$  that is compatible with the pdf  $p(c, t)$ , in particular it should not introduce a bias.

As a solution, we model the change as a Gaussian walk in quantile space. This is equivalent to Brownian motion in  $[0:1]$  for a continuous time scale. The result is a time series  $0 \leq P(t) \leq 1$ . The values of this time series are then mapped to the costs  $P(t) \rightarrow c(t)$  using the cdf  $P(c, t)$  of the a-priori values

$$P(c, t) = \int_{-\infty}^c p(c', t) dc'. \quad (3)$$

This way, the resulting total distances are automatically self-consistent with the chosen a-priori pdf  $p(c, t)$ .

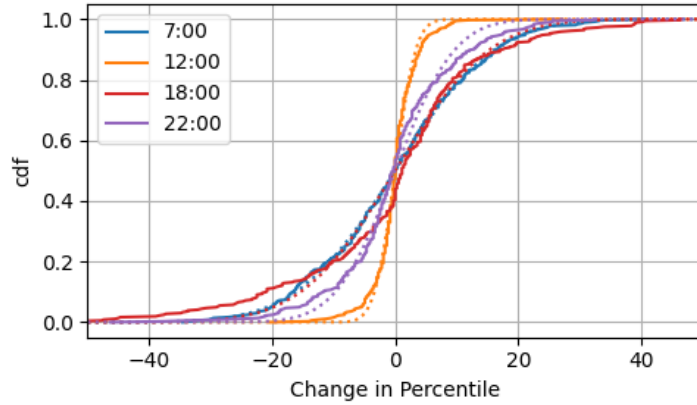


Figure 4: Cumulated distribution functions of the change in percentile of the spot price for example hours of a random day between December 2023 and November 2024. Actual quantiles are shown in solid lines, dashed lines show the approximation using normal distributions.

For the step in quantile space, we use a time-dependent normal distribution. The fact that the distribution is symmetric is important, so that the step does not introduce a bias relative to the a-priori pdf. Figure 4 displays that the approximation of the step using a normal distribution is plausible. Random price scenarios generated the presented method have already been displayed in Fig. 3. Scenarios like this can now be used for stochastic analysis as discussed in Sec. 1.

### 3 Scenarios as constraints (flexibility market)

Sometimes, it is not possible to explicitly optimise for scenarios. The decision if and when to place electric vehicles in a market for flexible load is an example

for this. Modelling these cases is particularly difficult as perfect foresight and flexibility are contradictory when using standard optimisation techniques. An optimisation model that “knows in advance” if the flexibility is actually used, contradicts the nature of a flexibility. We solve this issue by forcing the optimiser to find a solution that works in both extreme cases: If vehicle is used as flexible load the full time and if is not used at all. This view implements a minimalistic version of robust stochastic optimisation [19] for a dispatch problem.

We model that market in a way that is aligned with current evaluations of the transmission grid operators [20]: It might be possible to get cheap electricity during a period of time, but as a precondition the charging capacity needs to be reserved for that specific period of time. Using this market design, re-dispatch will not cost any money (no compensation is offered for the capacity), but even a (small) revenue is generated when electricity can be sold instead of curtailed.

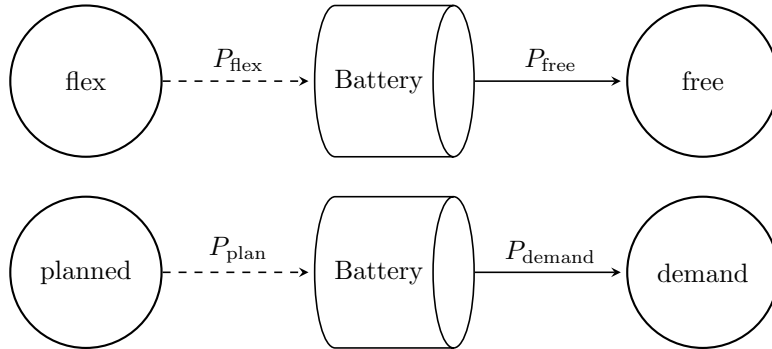


Figure 5: Model for the usage of BEV batteries to provide flexibility. Both usage models, planned and flexible, share the same battery but can only use it one at a time.

We implement the energy system graph displayed in Fig. 5, again using `oemof.solph` [16]. To make sure, only one of the two options, flexibility or planned charging, is used at a time, the respective power  $P_x$  is constrained using binary status variables  $Y_x$  with

$$P_x \leq Y_x \times P_{x,\text{cap}} \quad x \in \{\text{flex}, \text{plan}\} \quad (4a)$$

with

$$Y_{\text{flex}} + Y_{\text{plan}} \leq 1. \quad (4b)$$

To make sure, the battery has sufficient capacity to flexibly take energy if it is offered, the total capacity of the battery is shared between planned and flexible use

$$E_{\text{flex}} + E_{\text{plan}} \leq E_{\text{battery},\text{cap}}, \quad (5)$$

even though  $E_{\text{flex}}$  cannot be used to fulfil a demand, so it is effectively forced empty. This reserved capacity can only be freed if there is demand

$$P_{\text{free}} \leq P_{\text{demand}}. \quad (6)$$

If we did not set this limit, it was possible to offer more flexibility than there is demand.

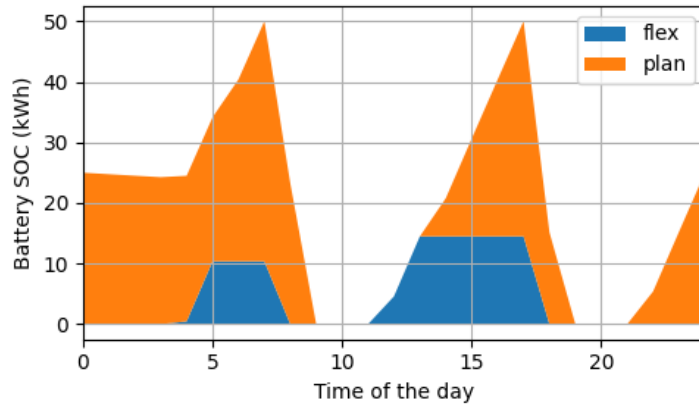


Figure 6: Battery SOC (flex and planned) of an electric vehicle with two trips a day. The SOC at midnight is set to 50 %. It can be seen that remaining capacity is used to offer flexibility.

In the example study, an electric vehicle with a battery size of 50 kWh, a maximum charging power of  $P_{\text{flex, cap}} = P_{\text{plan, cap}} = 11$  kWh, and 90 % charging efficiency is used. The vehicle does one trip between 7am and 9am, and another trip between 5pm and 7pm. Not to distract from the flexibility market use, we assume electricity prices to be constant with time. We further assume that flexible electricity is cheaper than planned electricity. We express that as a slightly negative cost at  $P_{\text{flex}}$ . The results are shown in Fig. 6. It can be seen that the full capacity is used, the SOC reaches both, 0 % and 100 %. As the optimiser has perfect foresight, this is expected and acceptable. Remaining capacity that is not needed for the rides, is used to offer flexibility. However, in reality, the full capacity of the battery will not be used: In the real world,  $E_{\text{flex}}$  can serve the demand, thus charging the battery using  $P_{\text{flex}}$  would replace charging it using  $P_{\text{plan}}$ .

## 4 Summary and Outlook

In the present work, we discussed methods that allow for considering uncertainty in linear optimisation models. We did so following examples connected to the charging of electric vehicles. The concepts, however, can be applied in other contexts.

Section 1 introduced the alternative use of a `Converter` in `oemof.solph` to do stochastic optimisation. While stochastic optimisation itself is well established, the representation of the scenarios as part of the energy system graph is at least uncommon. We believe that this flexible approach can open the door to more sophisticated models even in the abundance of frameworks that are explicitly designed for stochastic optimisation.

To generate scenarios, we presented a method in Sec. 2. It uses a Markov chain that models a Gaussian walk in quantile space to guarantee plausible and self-consistent time series. As the method is based on a Markov chain, long-ranging interdependencies that go beyond a limited step-size cannot be

modelled. The strength of this method is that it can universally applied and produces results that resemble the a-priori pdf and feature a realistic change of the quantity between the time-steps. This way, effects like collective drifts are considered automatically if they are part of the input distribution. In the future, we plan to apply the method for the development of long-term future scenarios, that are based on prognosis. For example, expected long-term trends for electricity prices due to increased availability of renewable energies could be combined with price fluctuations on a shorter time scale.

Finally, in Sec. 3, we presented a method using scenarios only in constraints, not in the optimisation goals. An approach like this can be advised if the uncertainty of the future itself (not only the choice between multiple scenarios) is to be modelled. This approach might allow to quantify flexibility in the operation of an energy system while still following an optimisation approach.

It is noteworthy, that a combination of the methods is perfectly possible: Randomly created scenarios can be used in stochastic optimisation, and the highest SOC within these scenarios can be used to calculate the amount of flexibility that can be offered throughout all scenarios.

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## References

- [1] F. Plazas-Niño, N. Ortiz-Pimiento, and E. Montes-Páez, “National energy system optimization modelling for decarbonization pathways analysis: A systematic literature review,” *Renewable and Sustainable Energy Reviews*, vol. 162, p. 112406, 2022. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S1364032122003148>
- [2] M. Sporleder, M. Rath, and M. Ragwitz, “Design optimization of district heating systems: A review,” *Frontiers in Energy Research*, vol. 10, 2022. [Online]. Available: <https://www.frontiersin.org/journals/energy-research/articles/10.3389/fenrg.2022.971912>
- [3] BDEW, “Studie: Wie heizt Deutschland 2023?” 2024. [Online]. Available: <https://www.bdew.de/energie/studie-wie-heizt-deutschland/>
- [4] Bundesnetzagentur, “Kraftwerksliste,” 2024. [Online]. Available: <https://www.bundesnetzagentur.de/DE/Sachgebiete/ElektrizitaetundGas/Unternehmen-Institutionen/Versorgungssicherheit/Erzeugungskapazitaeten/Kraftwerksliste/kraftwerksliste-node.html>
- [5] A. Gimelli, M. Muccillo, and R. Sannino, “Optimal design of modular cogeneration plants for hospital facilities and robustness evaluation of the results,” *Energy Conversion and Management*, vol. 134, pp. 20–31, 2017. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S019689041631113X>
- [6] L. Romero Rodríguez, J. M. Salmerón Lissén, J. Sánchez Ramos, E. Ángel Rodríguez Jara, and S. Álvarez Domínguez, “Analysis of the economic feasibility and reduction of a building’s energy consumption and emissions when integrating hybrid solar thermal/pv/micro-chp systems,” *Applied Energy*, vol. 165, pp. 828–838, 2016. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S0306261915016530>
- [7] J. M. Salmerón Lissén, L. Romero Rodríguez, F. Durán Parejo, and F. J. Sánchez de la Flor, “An economic, energy, and environmental analysis of pv/micro-chp hybrid systems: A case study of a tertiary building,” *Sustainability*, vol. 10, no. 11, 2018. [Online]. Available: <https://www.mdpi.com/2071-1050/10/11/4082>



- [8] L. Schmeling, P. Schönfeldt, P. Klement, S. Wehkamp, B. Hanke, and C. Agert, “Development of a decision-making framework for distributed energy systems in a german district,” *Energies*, vol. 13, no. 3, 2020. [Online]. Available: <https://www.mdpi.com/1996-1073/13/3/552>
- [9] G. Mavromatidis, K. Orehounig, and J. Carmeliet, “Design of distributed energy systems under uncertainty: A two-stage stochastic programming approach,” *Applied Energy*, vol. 222, pp. 932–950, 2018. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S0306261918305580>
- [10] P. Gabrielli, F. Fürer, G. Mavromatidis, and M. Mazzotti, “Robust and optimal design of multi-energy systems with seasonal storage through uncertainty analysis,” *Applied Energy*, vol. 238, pp. 1192–1210, 2019. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S0306261919300649>
- [11] A. Alonso-Travesset, D. Coppitters, H. Martin, and J. de la Hoz, “Economic and regulatory uncertainty in renewable energy system design: A review,” *Energies*, vol. 16, no. 2, 2023. [Online]. Available: <https://www.mdpi.com/1996-1073/16/2/882>
- [12] A. Soroudi and T. Amraee, “Decision making under uncertainty in energy systems: State of the art,” *Renewable and Sustainable Energy Reviews*, vol. 28, pp. 376–384, 2013. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S1364032113005790>
- [13] A. Mateo Gonzalez, A. Munoz SanRoque, and J. Garcia-Gonzalez, “Modeling and forecasting electricity prices with input/output hidden markov models,” *IEEE Transactions on Power Systems*, vol. 20, no. 1, pp. 13–24, 2005. [Online]. Available: <https://ieeexplore.ieee.org/abstract/document/1388488>
- [14] C. Li and I. E. Grossmann, “A review of stochastic programming methods for optimization of process systems under uncertainty,” *Frontiers in Chemical Engineering*, vol. 2, 2021. [Online]. Available: <https://doi.org/10.3389/fceng.2020.622241>
- [15] J. R. Birge and F. Louveaux, *Introduction to Stochastic Programming*. Springer, 2011.
- [16] U. Krien, P. Schönfeldt, J. Launer, S. Hilpert, C. Kaldemeyer, and G. Pleßmann, “oemof.solph—a model generator for linear and mixed-integer linear optimisation of energy systems,” *Software Impacts*, vol. 6, p. 100028, 2020. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S2665963820300191>
- [17] R. Norberg, “The markov chain market,” *ASTIN Bulletin*, vol. 33, no. 2, pp. 265–287, 2003. [Online]. Available: <https://doi.org/10.1017/S0515036100013465>
- [18] Netztransparenz.de. [Online]. Available: <https://www.netztransparenz.de/>
- [19] Z. Chen, M. Sim, and P. Xiong, “Robust stochastic optimization made easy with rsome,” *Management Science*, vol. 66, no. 8, pp. 3329–3339, 2020. [Online]. Available: <https://doi.org/10.1287/mnsc.2020.3603>
- [20] 50hertz, amprion, tennet, and T. BW, “Nutzen statt Abregeln” gem. §13k EnWG,” in *Informationsveranstaltung zur Vorstellung der Teilnahmemodalitäten*, 2024.