EXPRESSIVE LIMITS
OF QUANTUM RESERVOIR COMPUTING

Photonics West

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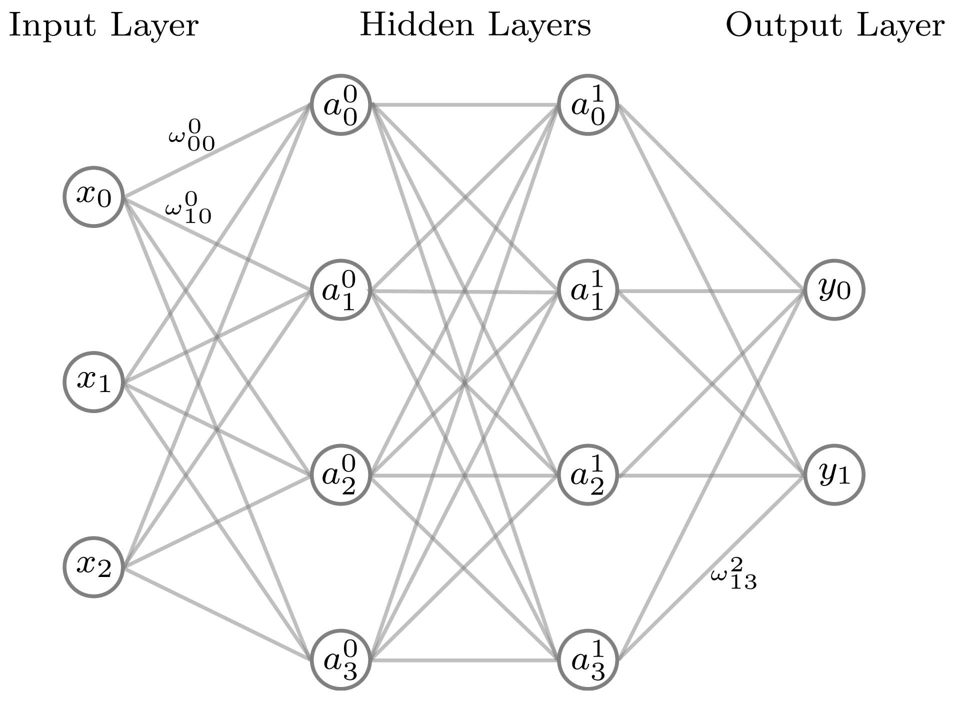
Nils-Erik Schütte, Niclas Götting, Hauke Müntinga, Meike List, and Christopher Gies

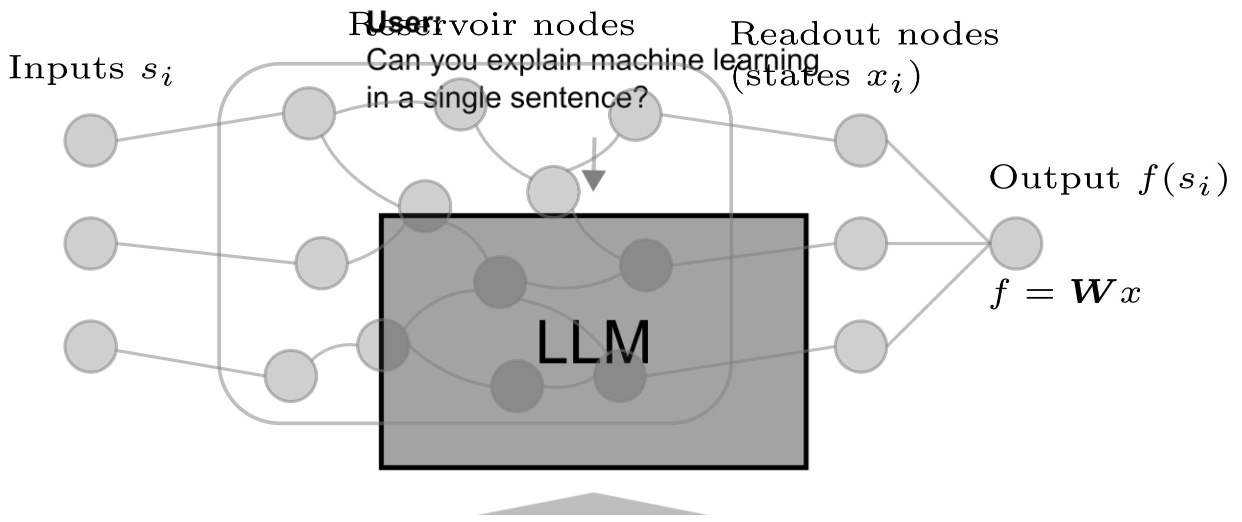
Carl von Ossietzky Universität Oldenburg

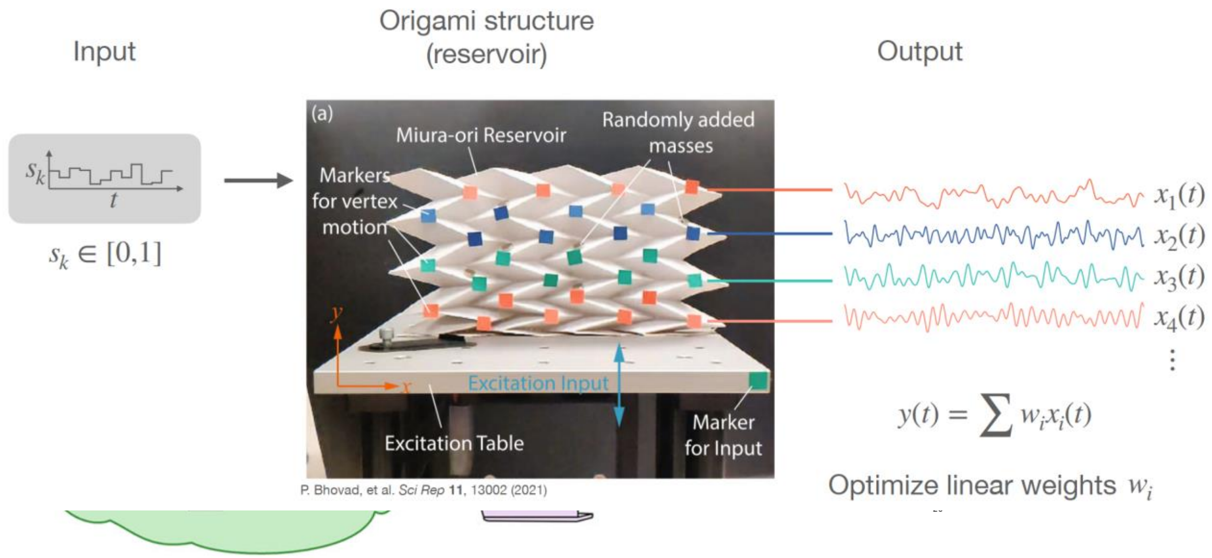
QCi

DLR

Machine learning and reservoir computing



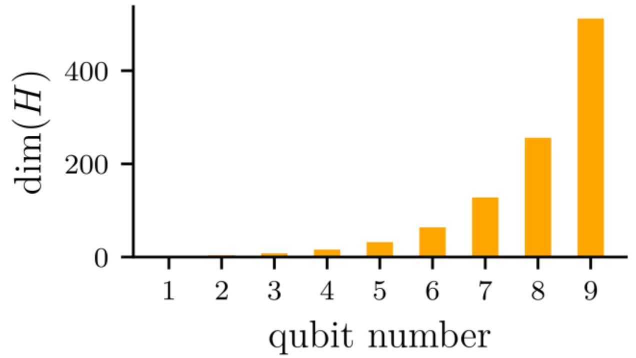




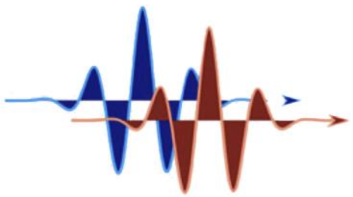
- high energy consumption
- tedious to train

Going quantum…
…quantum reservoir computing (QRC)

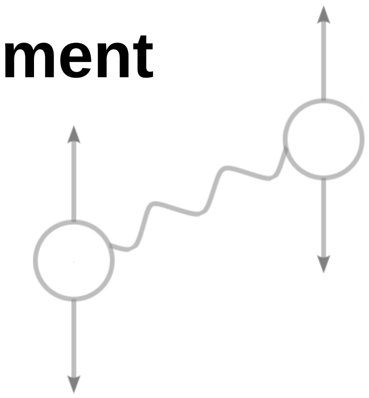
Possible advantages and functionalities of a **quantum** reservoir computer:



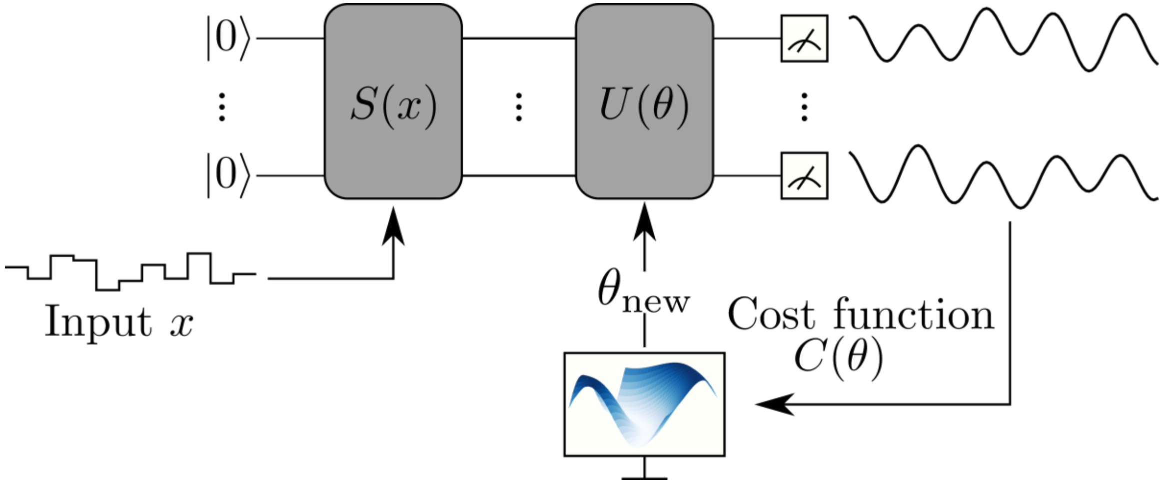
- feature maps into **exponentially large** phase spaces
- native processing of **quantum input**

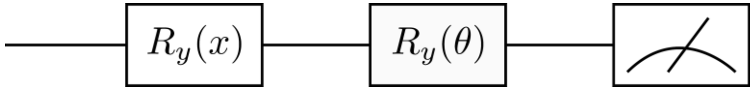


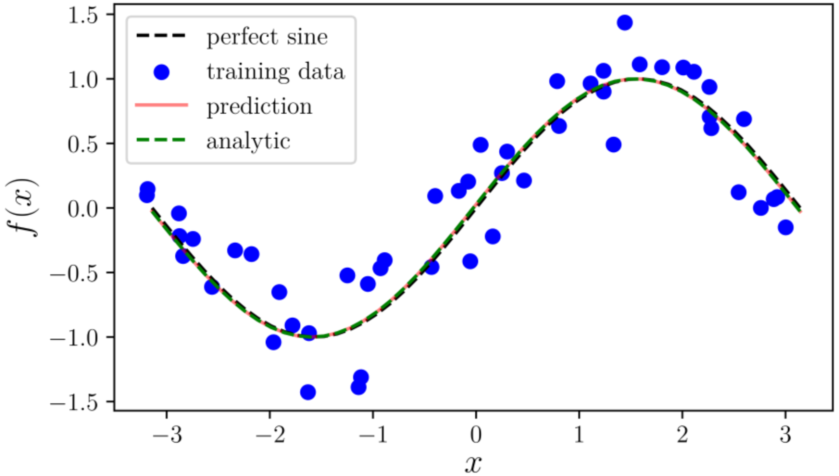
- non-classical correlations via **entanglement**



Gate-based quantum machine learning



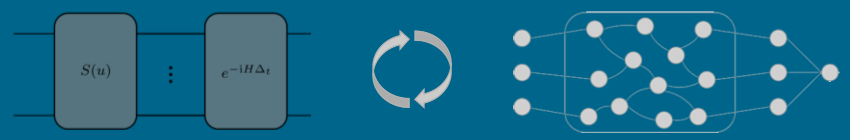




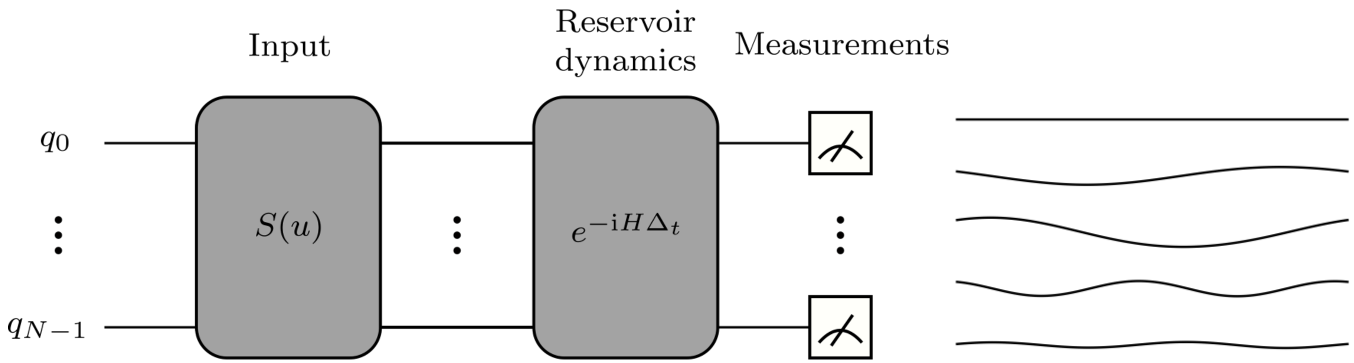
Hybrid approach combining classical optimization with a quantum circuit evaluation:

- parameterized quantum circuits (PQCs)
- training is done classically by minimizing a cost function
- small circuit depth: possible application of NISQ devices

Goals

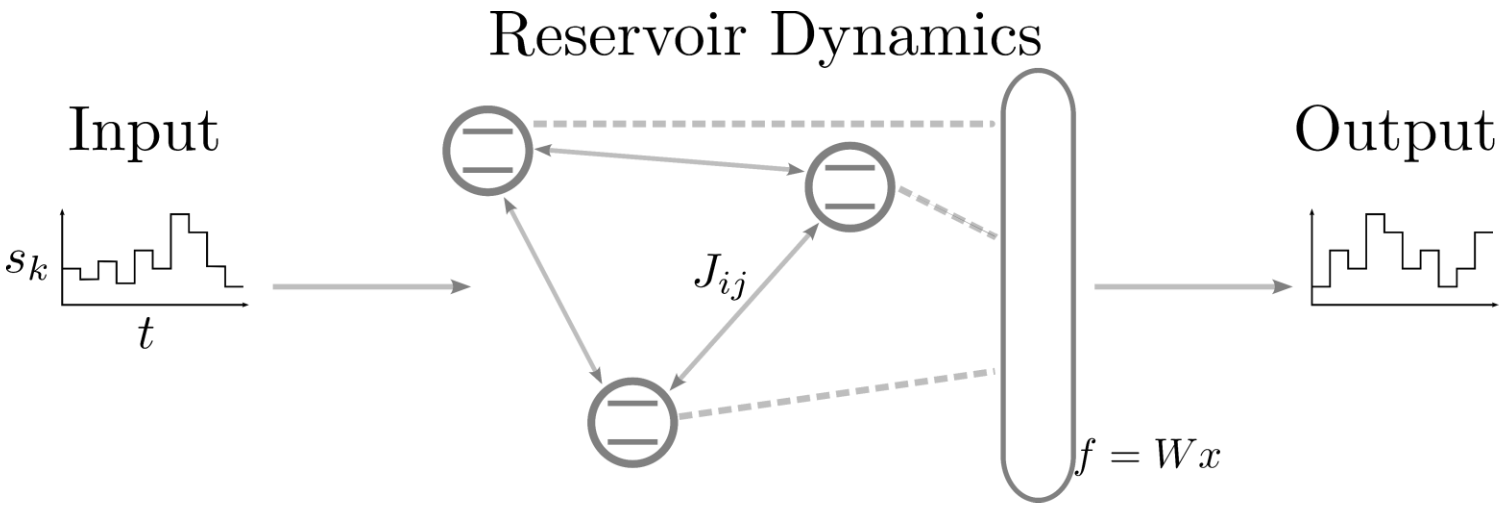


Map both QML paradigms onto each other



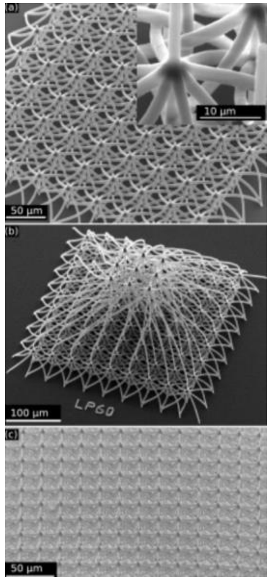
Quantify the expressivity of QRC

Transverse-field Ising model (TFIM)

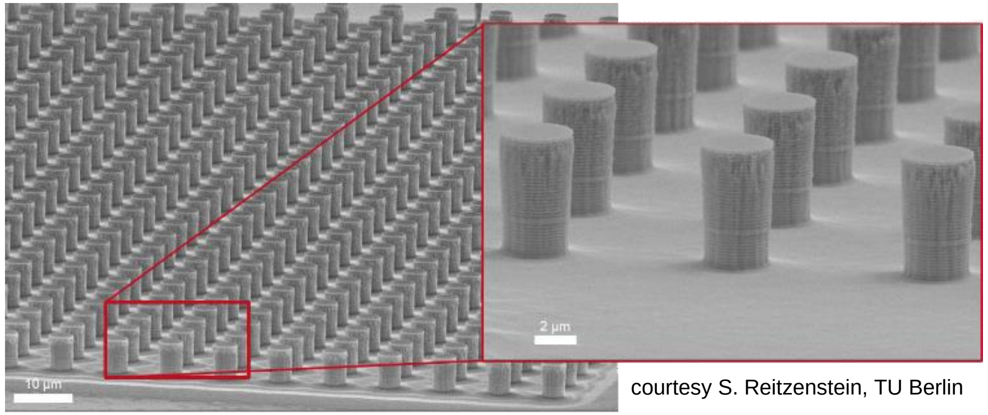


$$H = h \sum_{i=1}^{N} \sigma_z^{(i)} + \sum_{i<j} J_{ij} \sigma_x^{(i)} \sigma_x^{(j)}$$

$$U_{\Delta_t} = \mathrm{e}^{-\mathrm{i} H \Delta_t}$$



Optica **7**, 640 (2020)

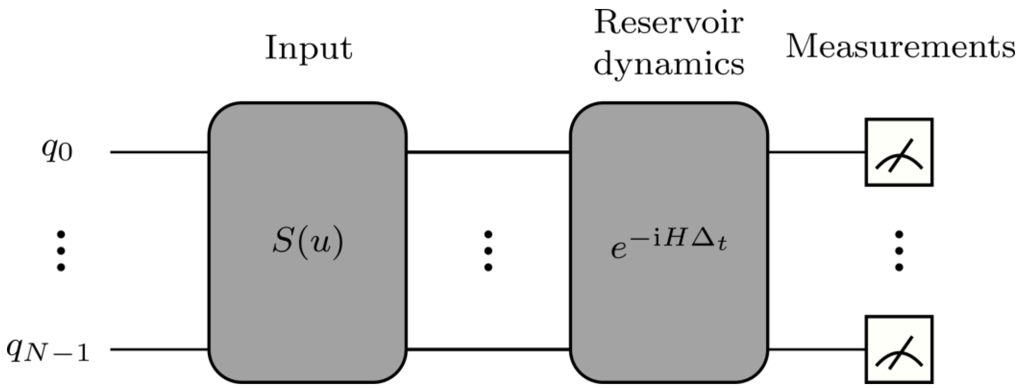


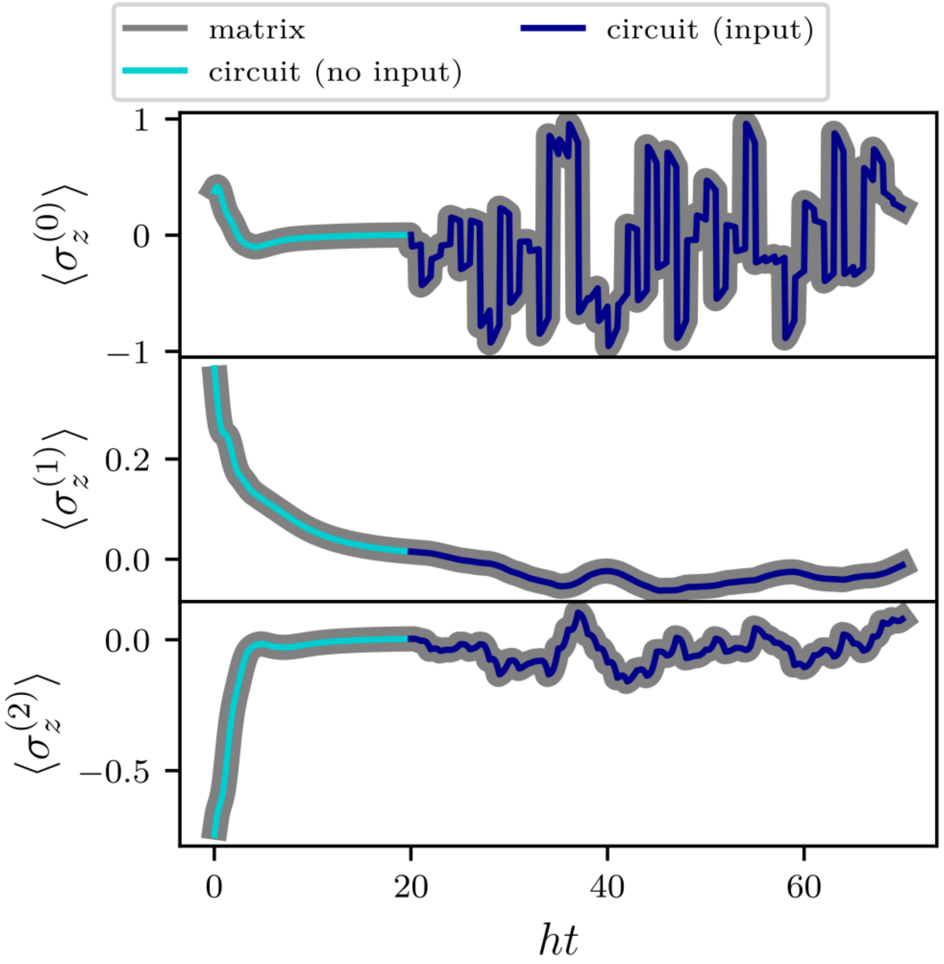
courtesy S. Reitzenstein, TU Berlin

Connection between QRC and gate-based QCs

$$U_{\Delta_t} = \mathrm{e}^{-\mathrm{i}H\Delta_t}$$

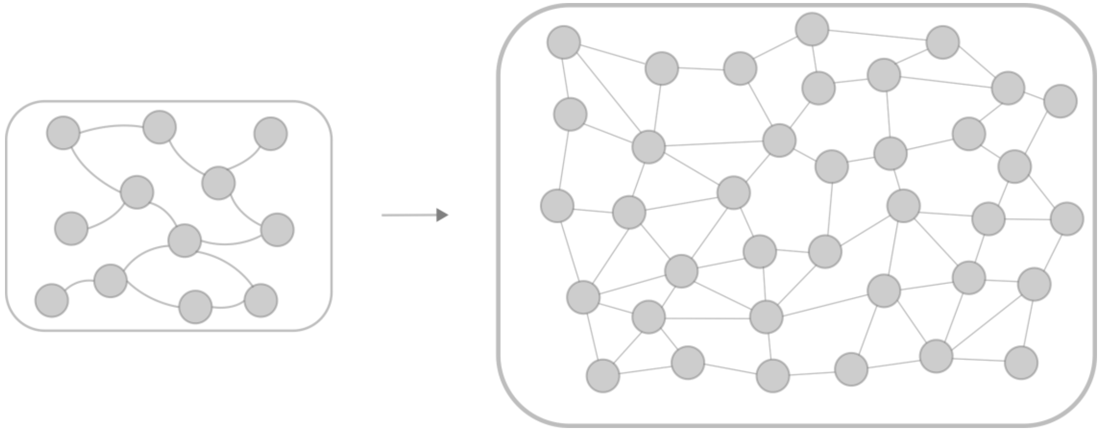
$$\rho \mapsto U_{\Delta_t} \rho U_{\Delta_t}^{\dagger}$$



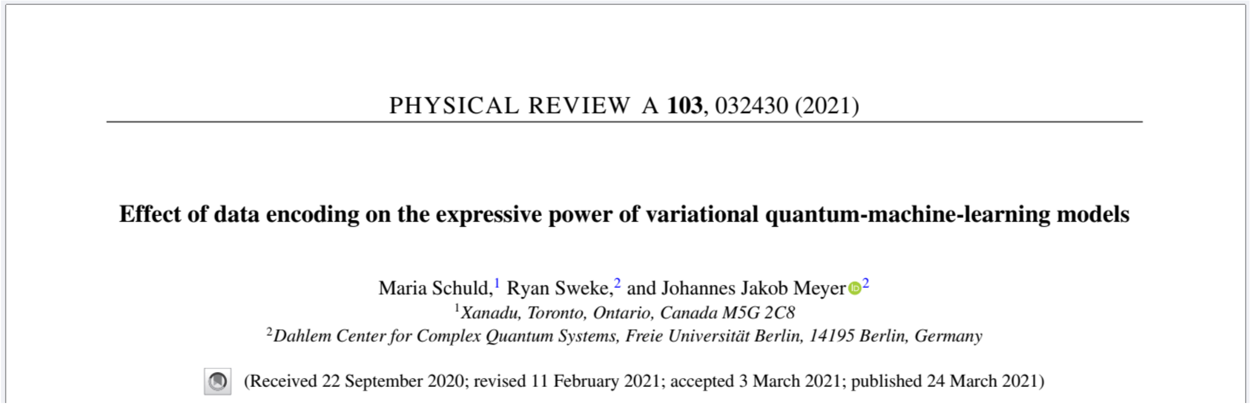


Both approaches are equivalent and can be mapped onto each other

Expressivity



Classical reservoir:
increasing the size of the reservoir increases the available output function space



PHYSICAL REVIEW A **103**, 032430 (2021)

Effect of data encoding on the expressive power of variational quantum-machine-learning models

Maria Schuld,[1] Ryan Sweke,[2] and Johannes Jakob Meyer[2]
[1]*Xanadu, Toronto, Ontario, Canada M5G 2C8*
[2]*Dahlem Center for Complex Quantum Systems, Freie Universität Berlin, 14195 Berlin, Germany*

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Quantum machine learning models are truncated Fourier series

$$f(u) = \sum_{\omega \in \Omega} c_\omega \mathrm{e}^{\mathrm{i}\omega u}$$

The available frequencies are determined by the input encoding gates

What is the expressivity of a QRC?

What do we find for QRC?

- expressivity is limited by the number of input encoding gates
- larger reservoirs do not necessarily increase the available function space
- there are better ansatz circuits in QML as provided by the TFIM

How can we quantify the expressive power in QML?

PHYSICAL REVIEW X **13,** 041020 (2023)

Tackling Sampling Noise in Physical Systems for Machine Learning Applications: Fundamental Limits and Eigentasks

Fangjun Hu[1,*,†] Gerasimos Angelatos[1,2,†] Saeed A. Khan[1,†] Marti Vives[1,3] Esin Türeci[4] Leon Bello[1]
Graham E. Rowlands[2] Guilhem J. Ribeill[2] and Hakan E. Türeci[1]
[1]*Department of Electrical and Computer Engineering, Princeton University, Princeton, New Jersey 08544, USA*
[2]*Raytheon BBN, Cambridge, Massachusetts 02138, USA*
[3]*Q-CTRL, Santa Monica, California 90401, USA*
[4]*Department of Computer Science, Princeton University, Princeton, New Jersey 08544, USA*

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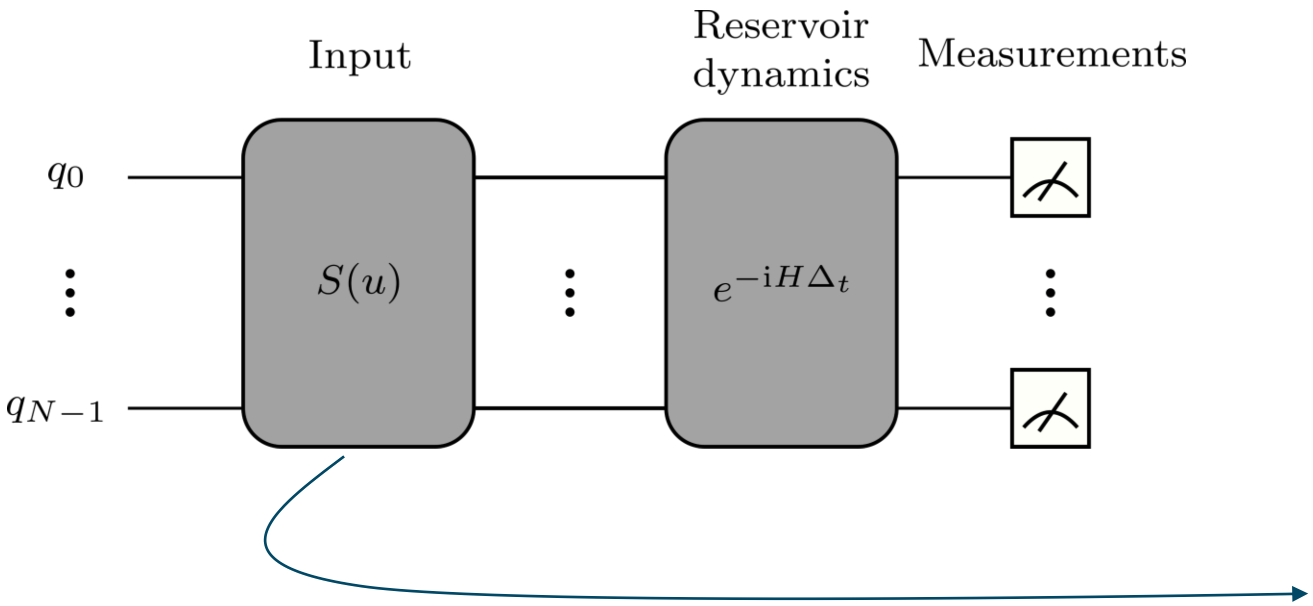
Resolvable expressive capacity (REC):
REC quantifies how many linearly independent functions a system can express

Eigentasks:
Orthogonal functions that can be approximated by the system with minimal error

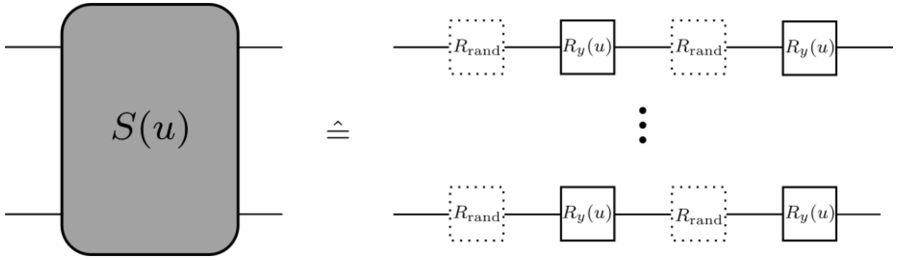
Upper bounds for the expressivity

REC limited by the number of measured degrees of freedom [1]

REC in dependence on the input encoding:

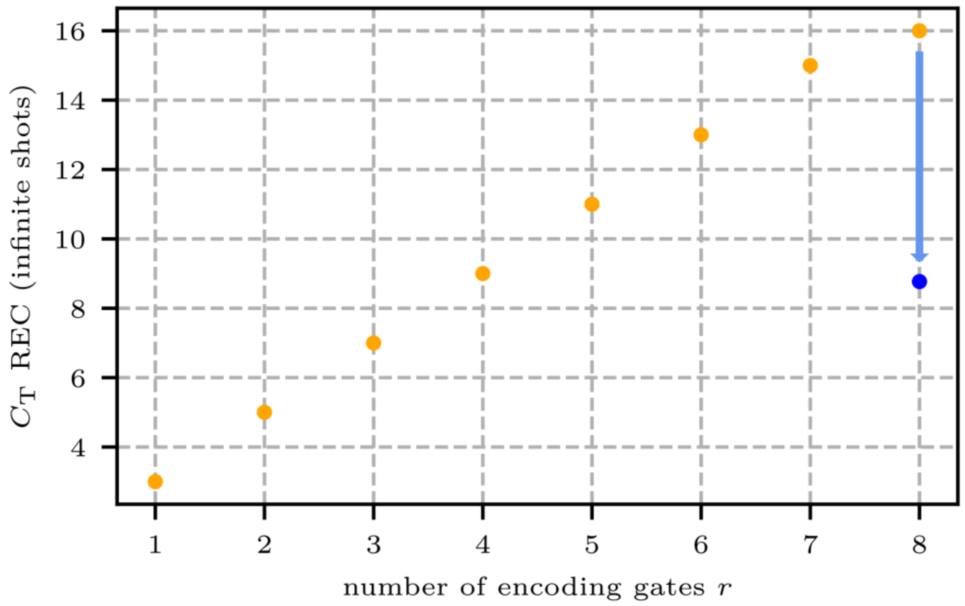


… using single qubit Pauli rotations as input encoding gates

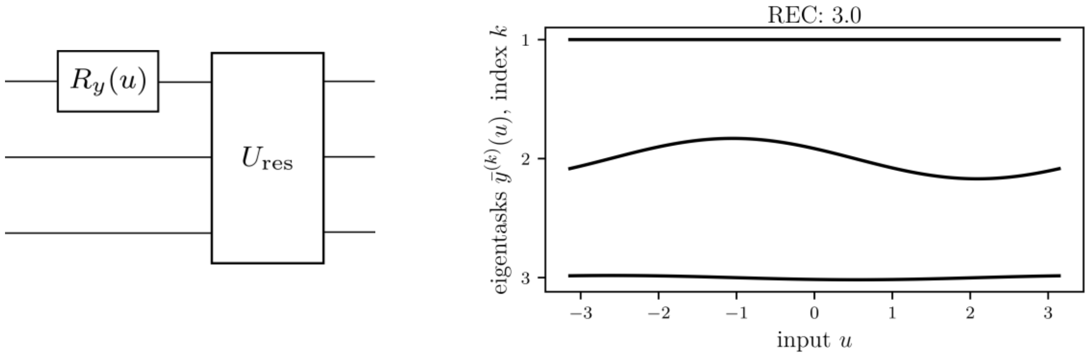


[1] Hu *et al.*, Phys. Rev. X 13, 041020 (2023)

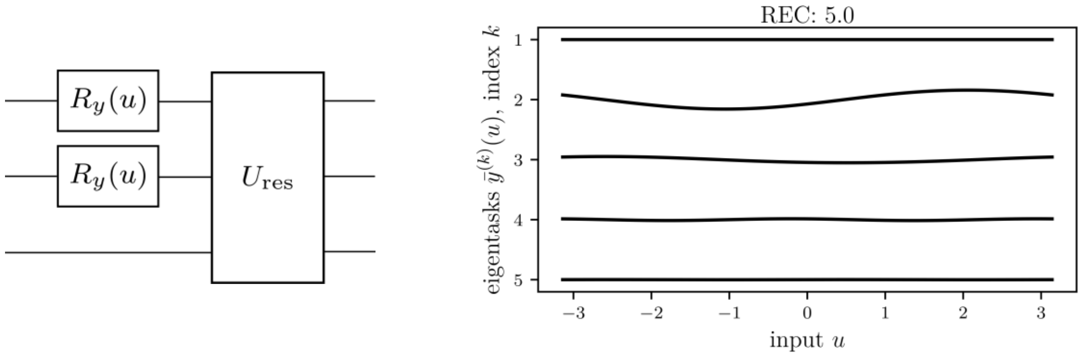
Upper bounds for the expressivity



Infinite shots: $$C_T = 2r + 1$$



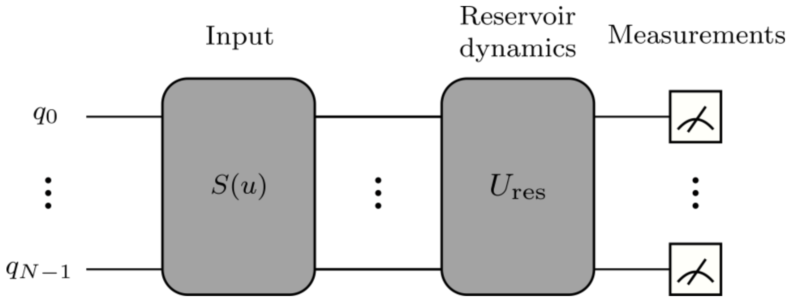
$$f(u) = \sum_{\omega \in \Omega} c_\omega e^{i\omega u}$$ $$|\Omega| = 2r + 1$$

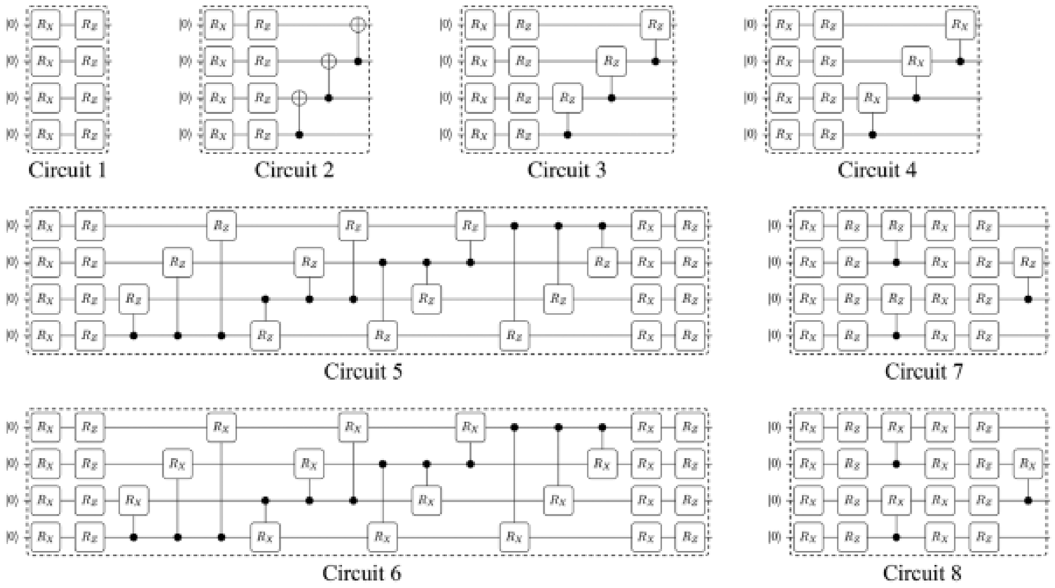


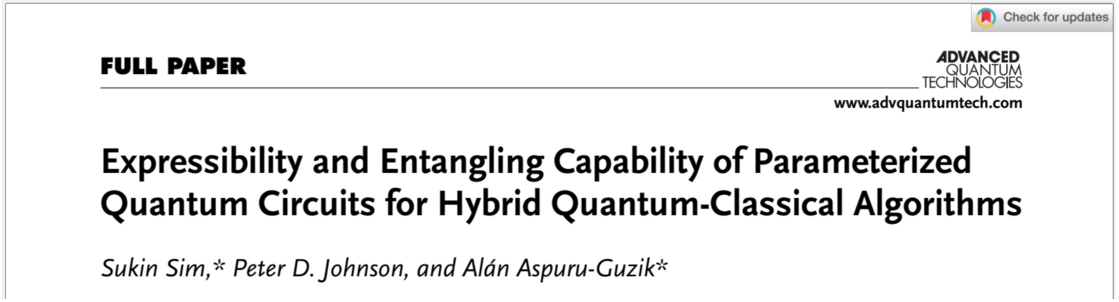
Finite shots (8 encodings): $$C_T \approx 8.77$$

REC of gate-based ansatz circuits

Idea: consider ansatz circuits as reservoirs in QRC



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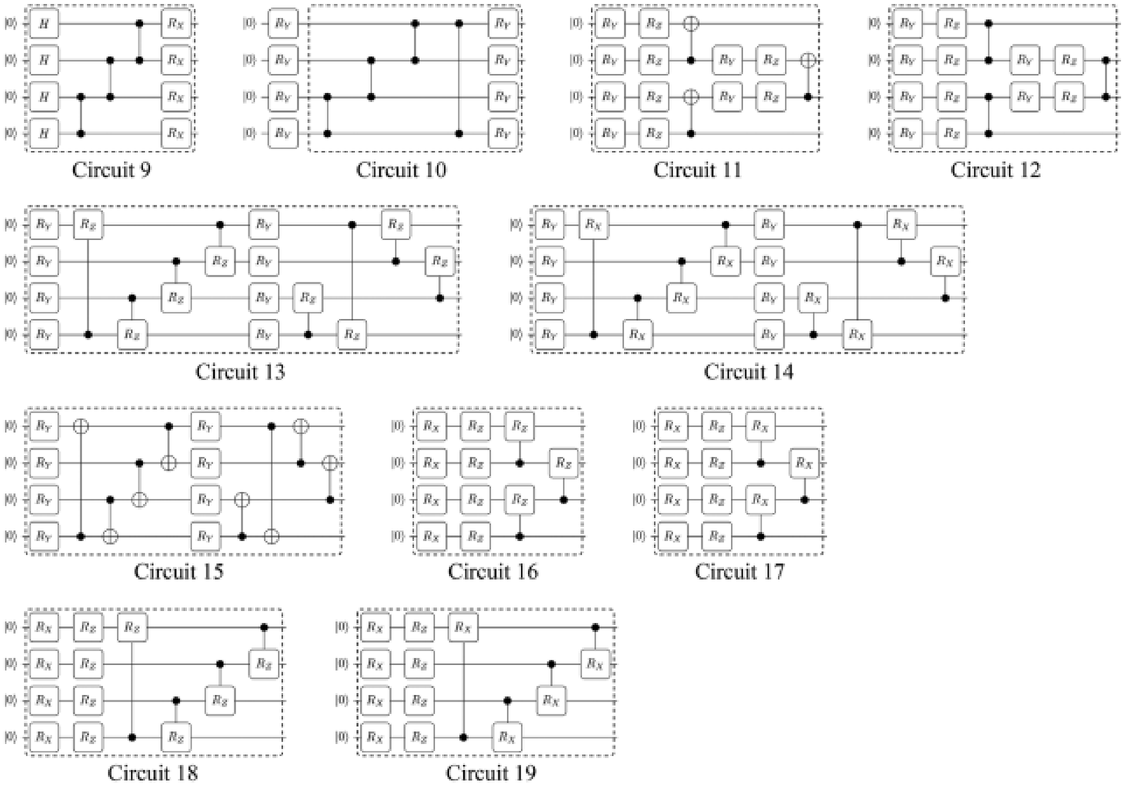
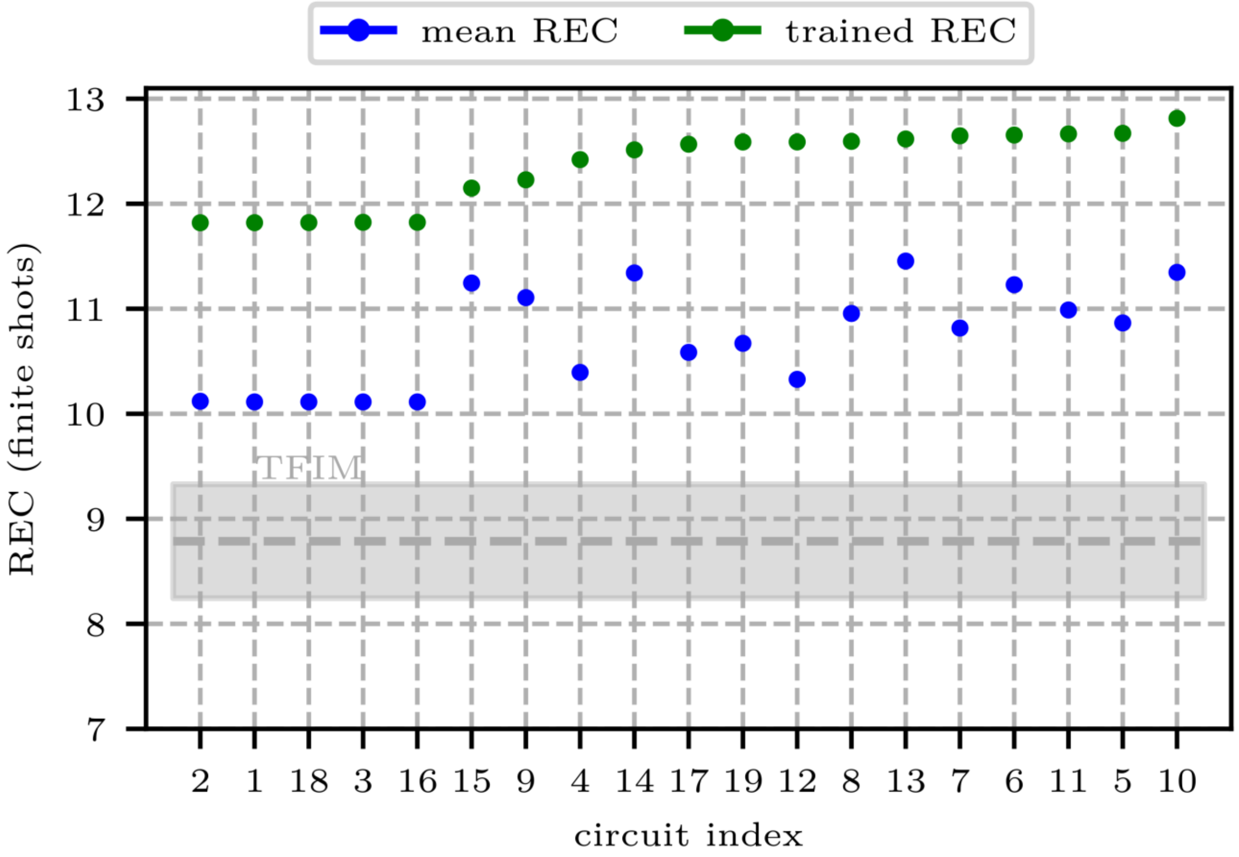
Expressibility and Entangling Capability of Parameterized Quantum Circuits for Hybrid Quantum-Classical Algorithms

Sukin Sim, Peter D. Johnson, and Alán Aspuru-Guzik**



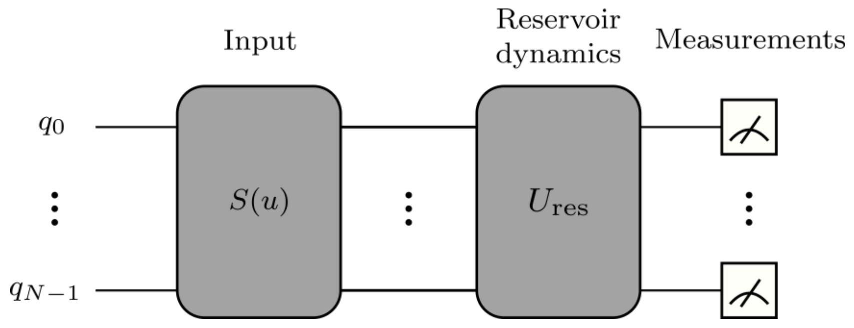
REC of gate-based ansatz circuits

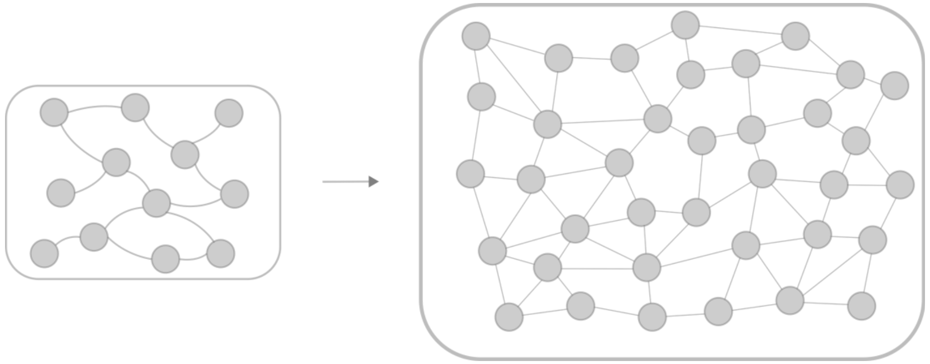


Conclusion and Outlook

Expressivity in QRC is limited by the number of input encoding gates!

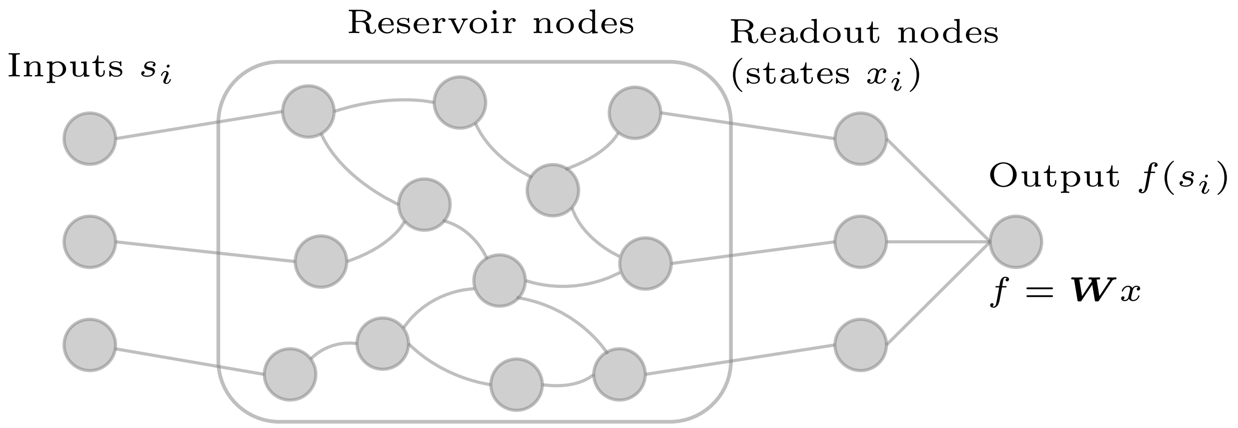
Increasing the size or complexity of the reservoir does not increase the available function space!

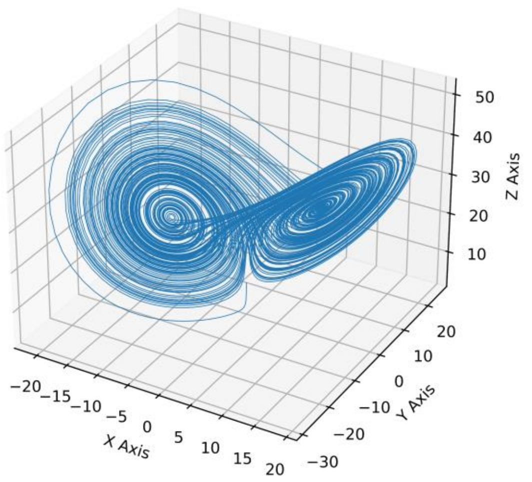




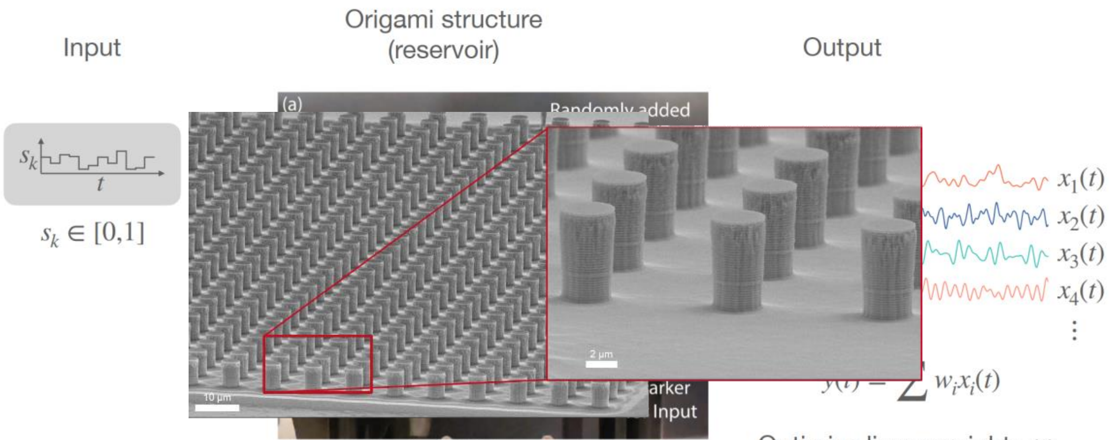
Thank you for your attention!

Machine learning and reservoir computing



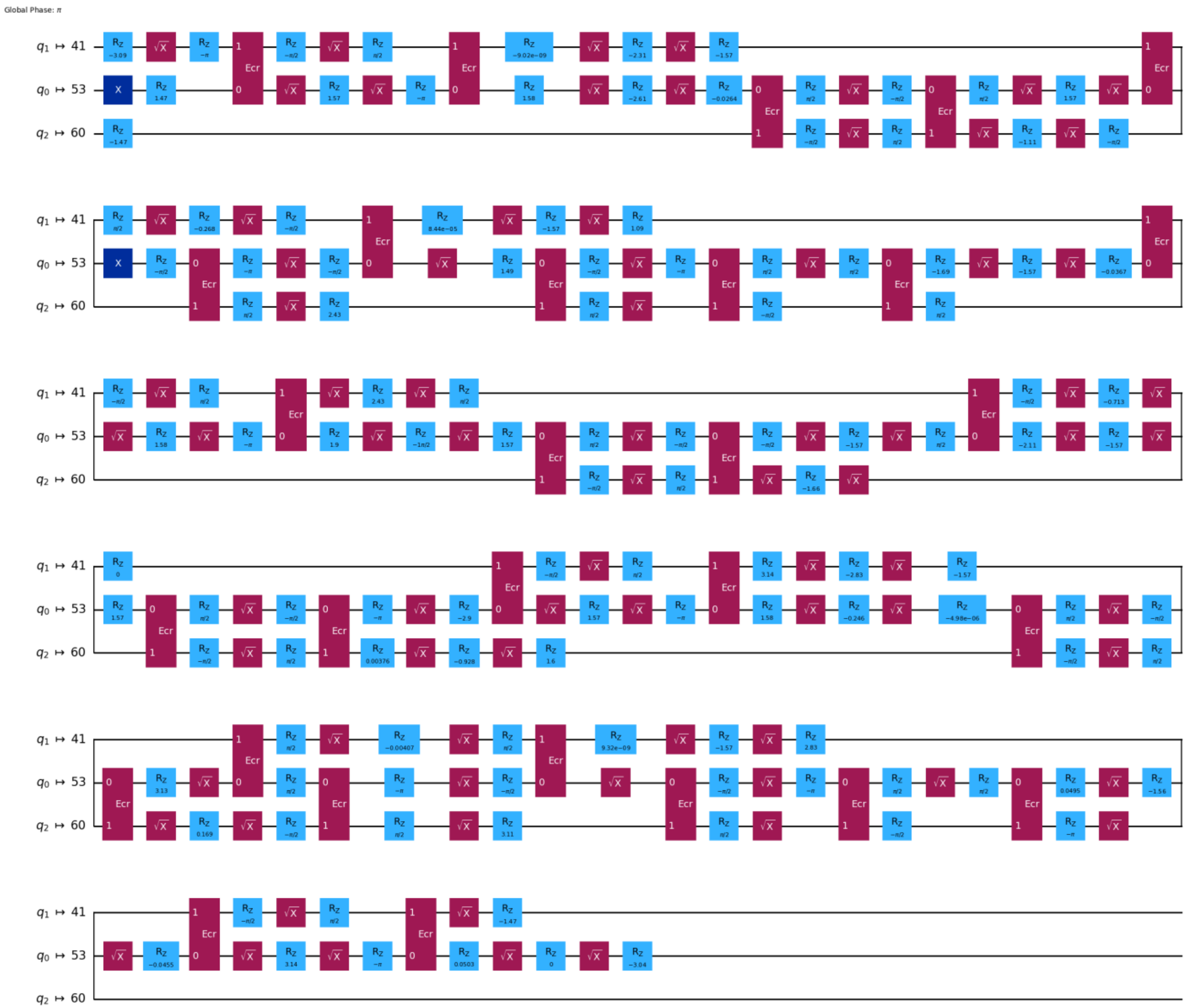


- fixed internal weights
- only readout weights are trained
- often used for time series processing



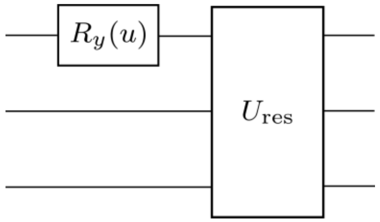
P. Bhovad, et al. *Sci Rep* **11**, 13002 (2021)

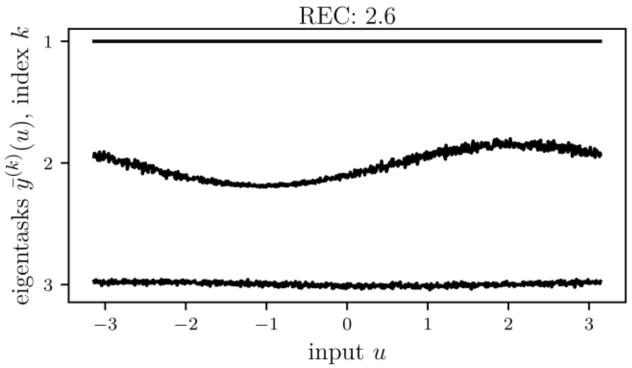
Transpilation of the time-evolution of the TFIM



Influence of shot noise

Measurement statistics increase the noise-to-signal ratios (NSRs) of the eigentasks reducing the REC

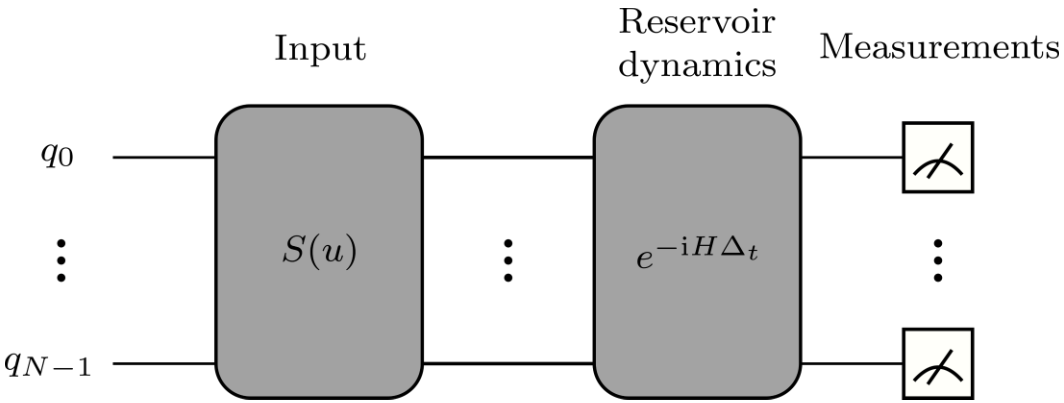


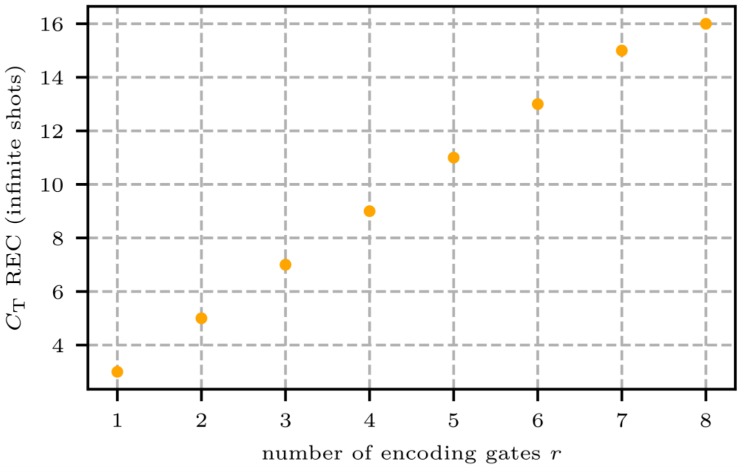


REC of the TFIM for finite shots:

$$C_{\mathrm{T}} \approx 8.77$$

Resolvable expressive capacity of the TFIM



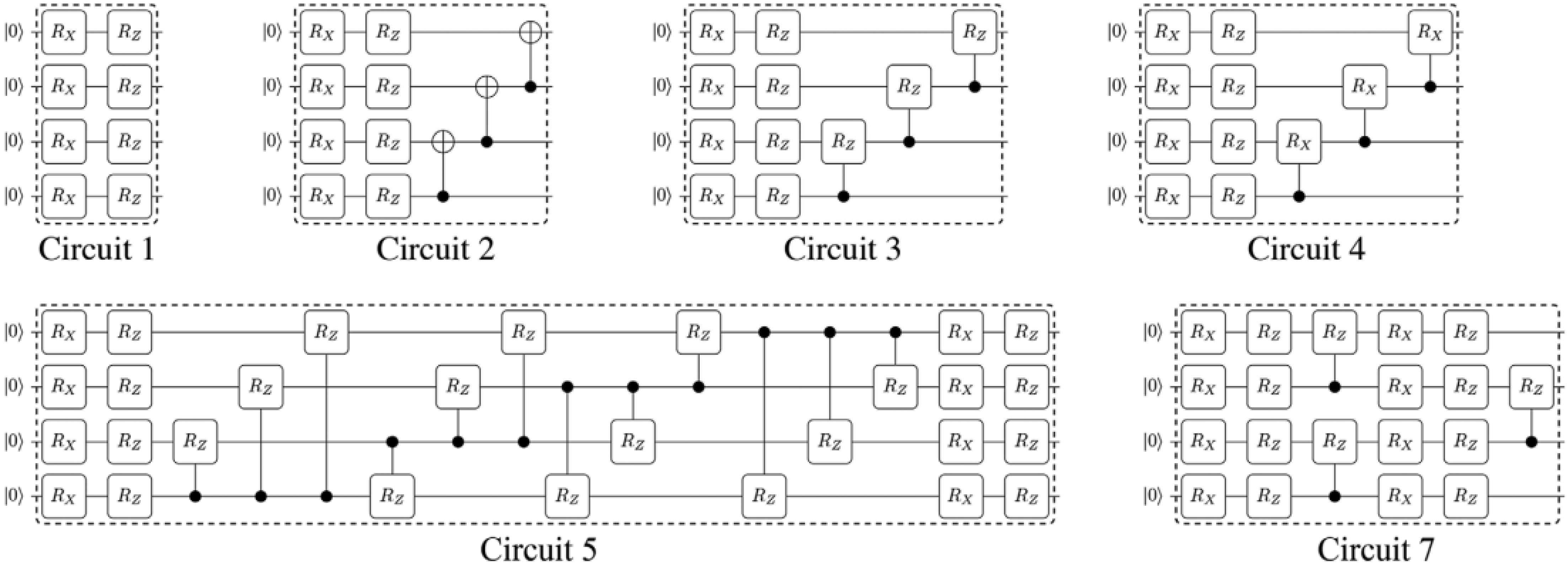


The time-evolution of the TFIM can produce the maximal number of orthogonal functions

But: For now, we neglected the measurement statistics

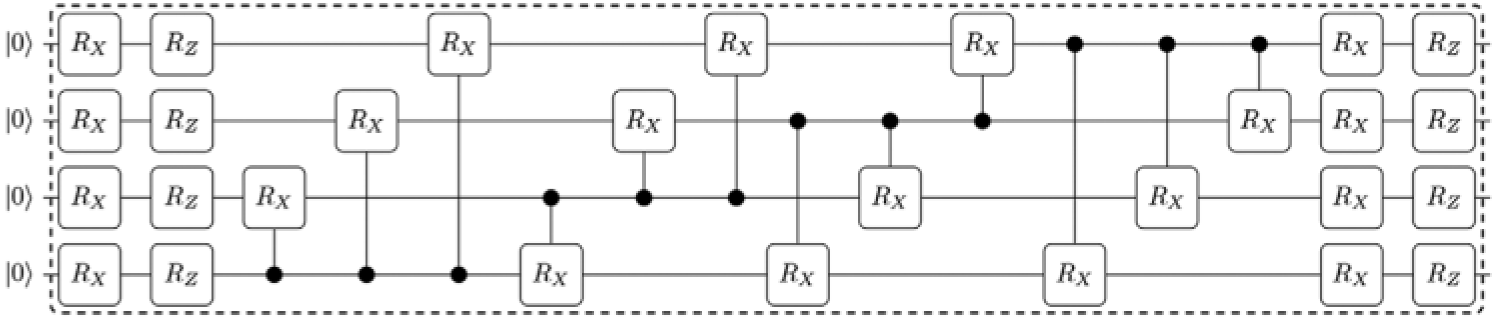
REC of the TFIM for finite shots: $C_\mathrm{T} \approx 8.77$

Ansatz circuits

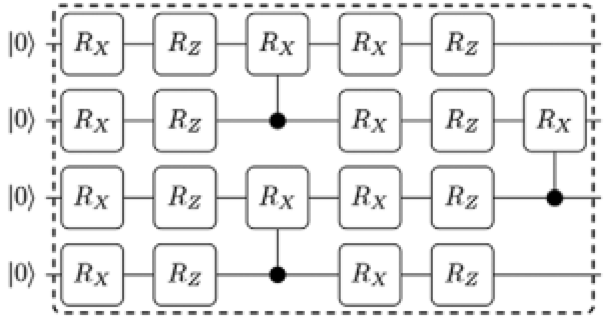


Sim *et al*., Adv. Quantum Technol., 2 **12** 1900070 (2019)

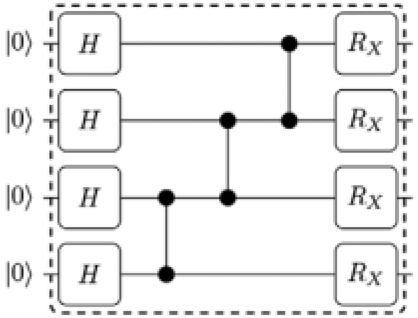
Ansatz circuits



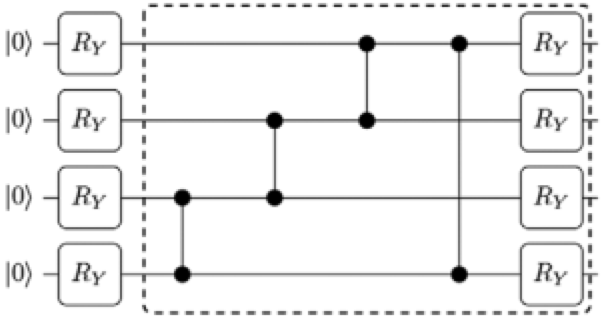
Circuit 6



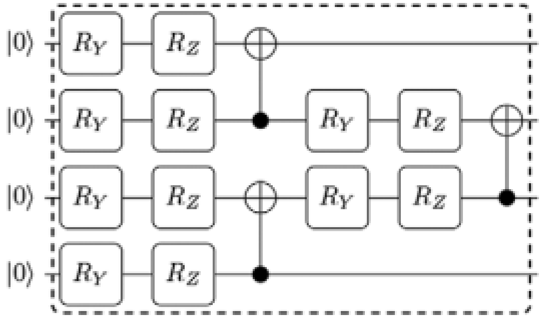
Circuit 8



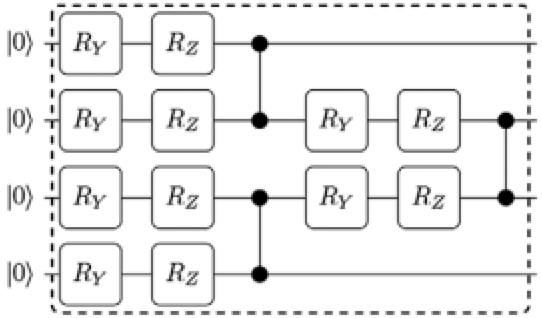
Circuit 9



Circuit 10



Circuit 11



Circuit 12

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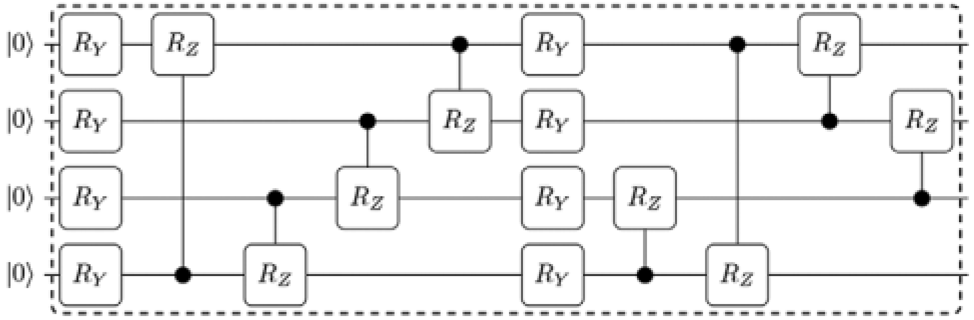
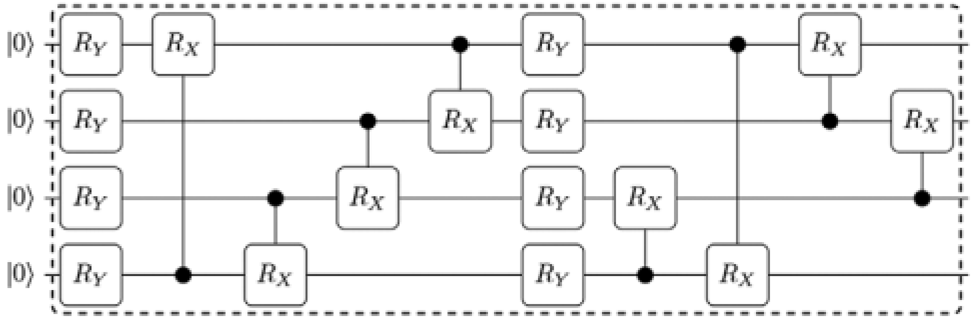
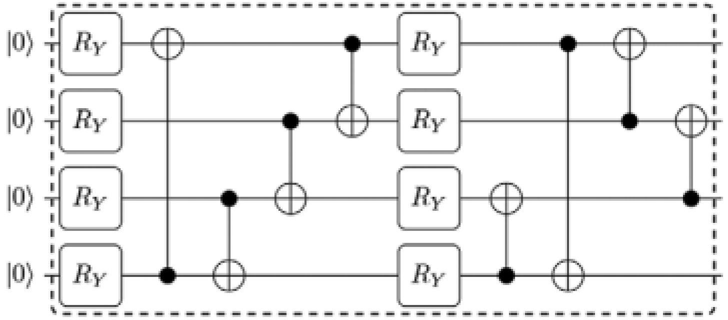
Ansatz circuits



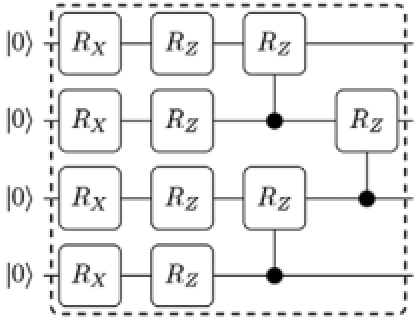
Circuit 13



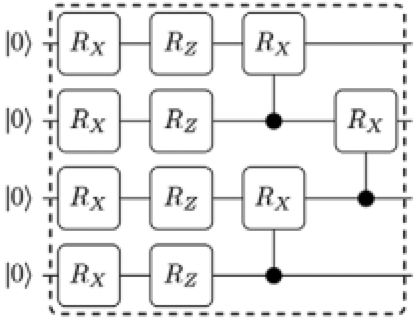
Circuit 14



Circuit 15



Circuit 16



Circuit 17

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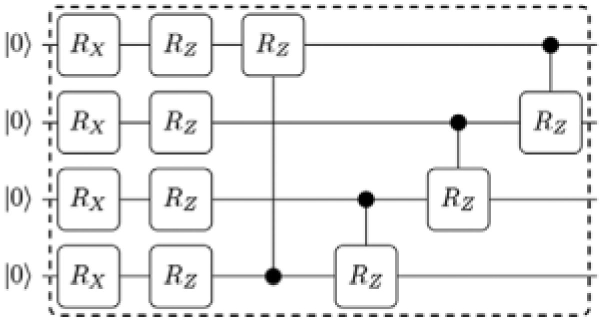
Ansatz circuits



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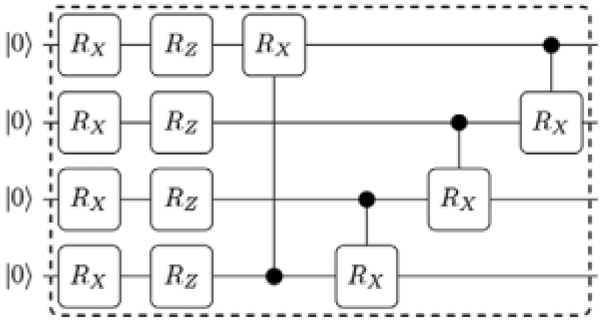

Circuit 19

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