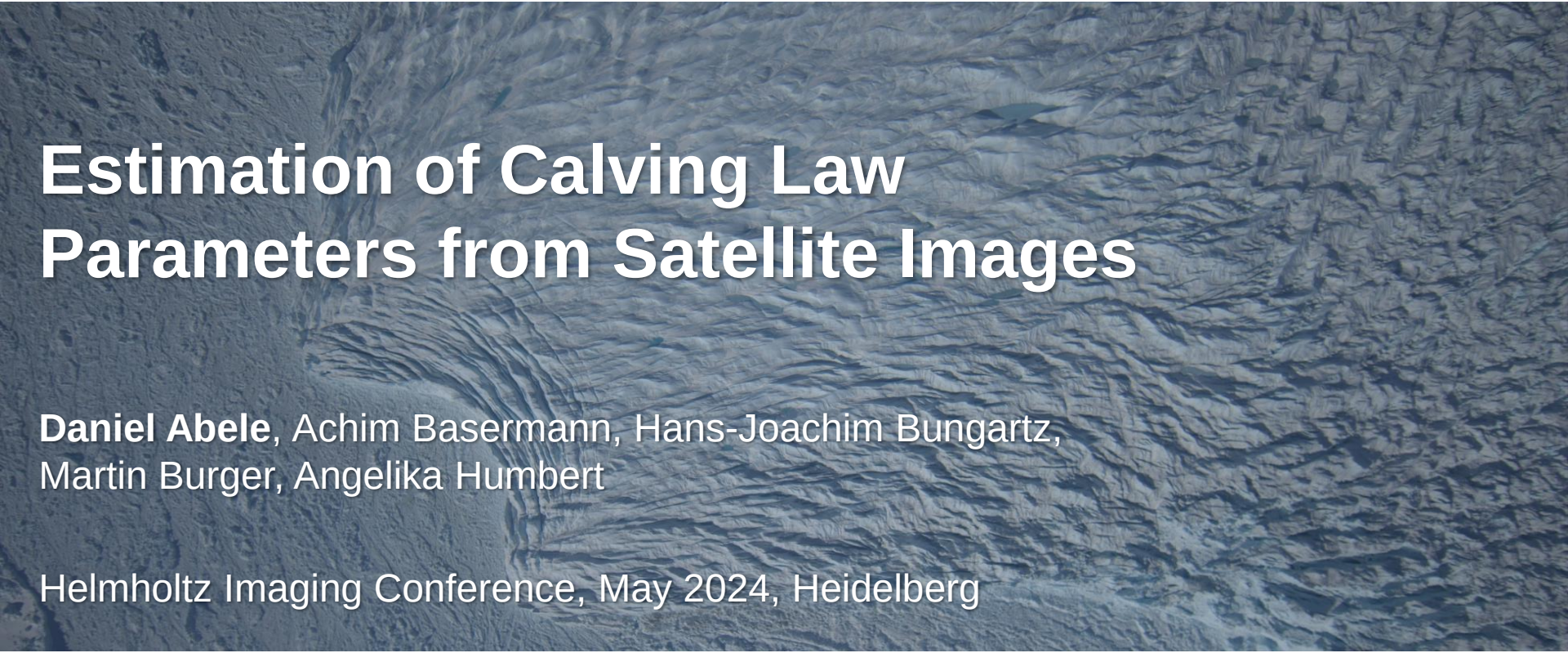
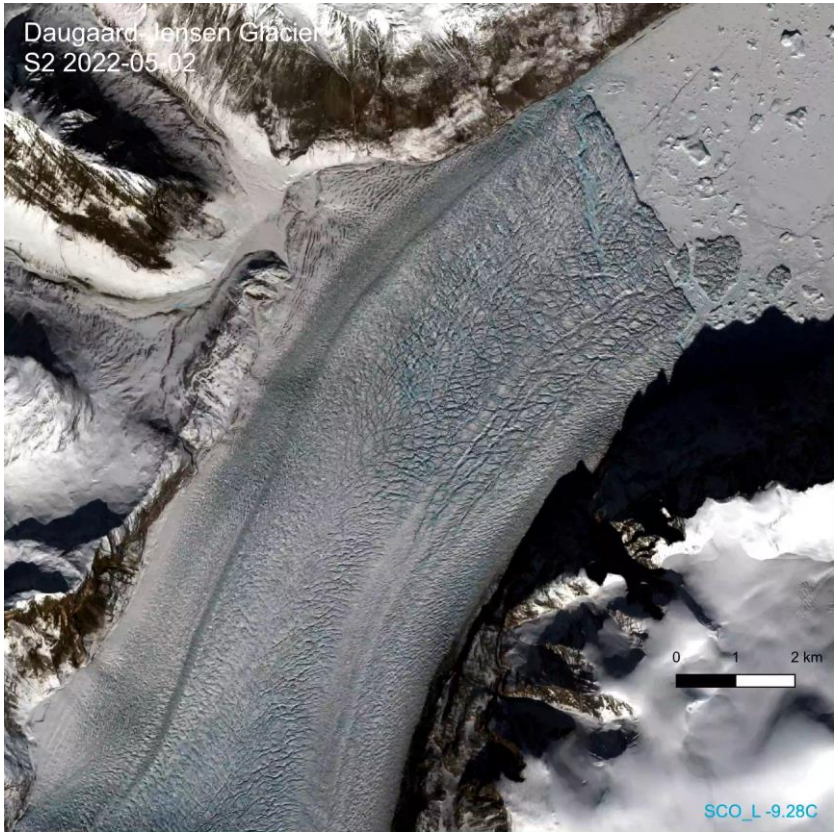


Estimation of Calving Law Parameters from Satellite Images

Daniel Abele, Achim Basermann, Hans-Joachim Bungartz, Martin Burger, Angelika Humbert

Helmholtz Imaging Conference, May 2024, Heidelberg

Calving Front



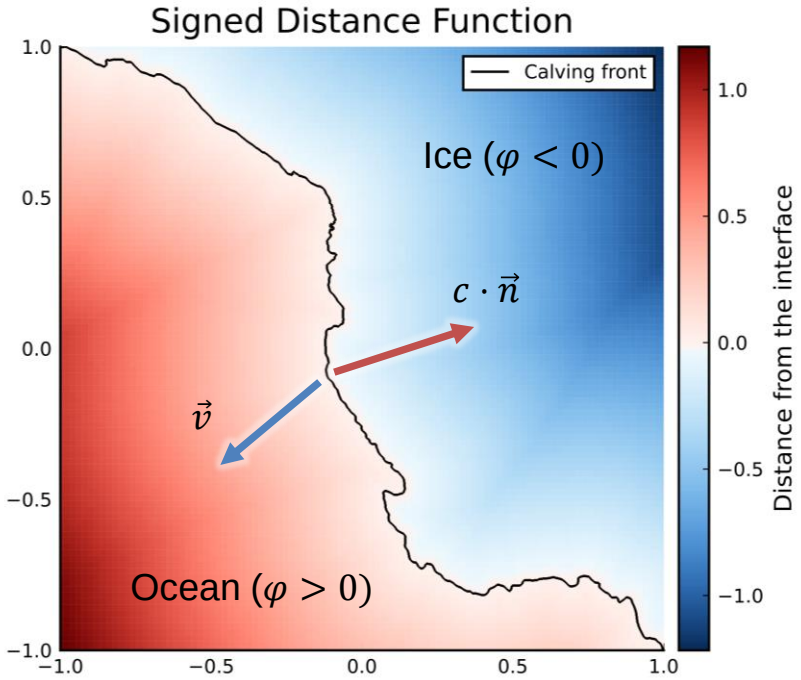
Calving Front Capturing

Level-set Method

$$\frac{\partial \varphi}{\partial t} + \vec{v}\ \nabla\varphi = c\ \|\nabla\varphi\|$$

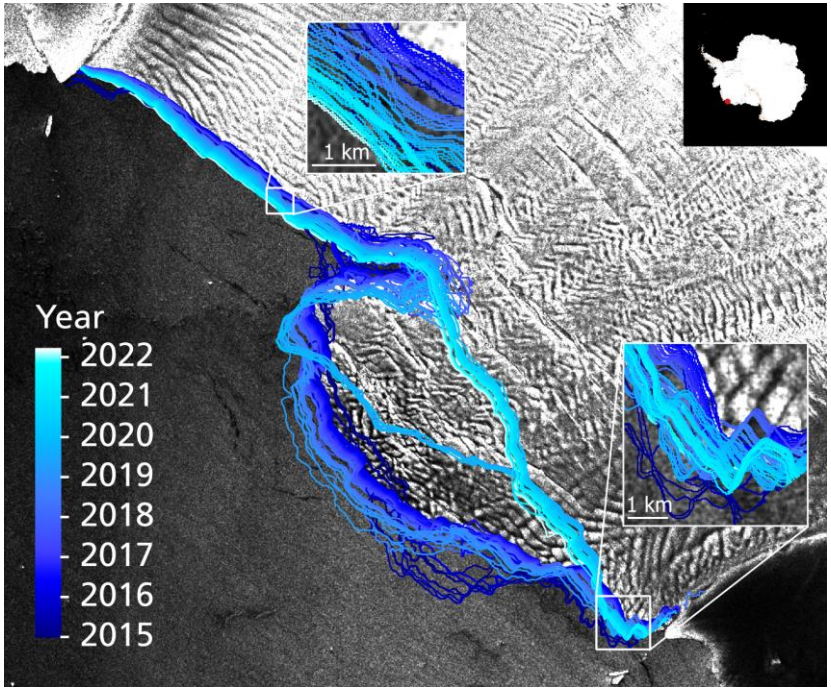
Von Mises Calving Law

$$c = \|\vec{v}\| \frac{\sigma_{VM}}{\sigma_{max}}$$



Observation Data

- Time series of ice fronts
- one per month, with gaps
- extracted from satellite images using ML



Baumhoer, Celia et.al. (2023) IceLines – A new data set of Antarctic ice shelf front positions

Parameter Estimation

Simple Optimization Problem with PDE Constraint?

Complications:

- Velocity $\vec{v}$ and stresses σ_{VM} from ice sheet model
  => Precompute using observations
- Reinitialization to signed distance function
  => Small enough time spans
- Restrict objective function to area around front
  => Add weights

Optimization Problem

$$\min_{\sigma_{max}} \frac{1}{2} \sum_i \|(\varphi(t_i) - \varphi_i^*) w_i\|^2$$

$$\text{s.t. } \forall i$$

$$\frac{\partial \varphi}{\partial t} + \vec{v} \cdot \nabla \varphi = c(\sigma_{max}) \quad \text{in} \quad [t_{i-1}, t_i]$$

$$\varphi(t_{i-1}) = \varphi_{i-1}^*$$

Observations φ_i^*, Gaussian weights $w_i = \exp(-\frac{|\varphi_i^*|}{\alpha})$

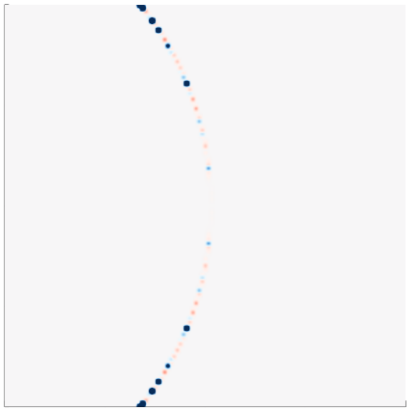
Adjoint system

$$\frac{\partial \lambda}{\partial t} + \vec{v} \ \nabla \lambda = 0$$

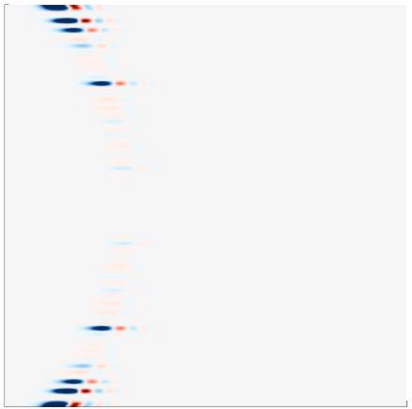
$$\lambda(t_i) = (\varphi(t_i) - \varphi_i^*) w_i^2$$

Solver diffusion at low order/resolution

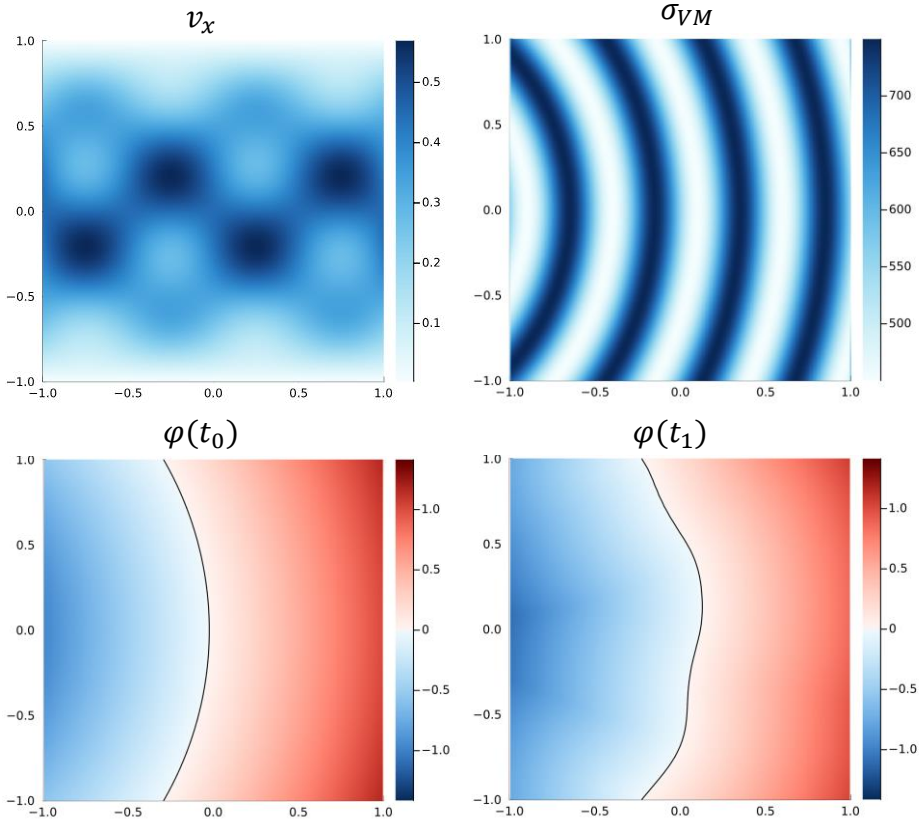
$\lambda(t_i)$



$\lambda(t_{i-1})$



Experiment with Synthetic Observations

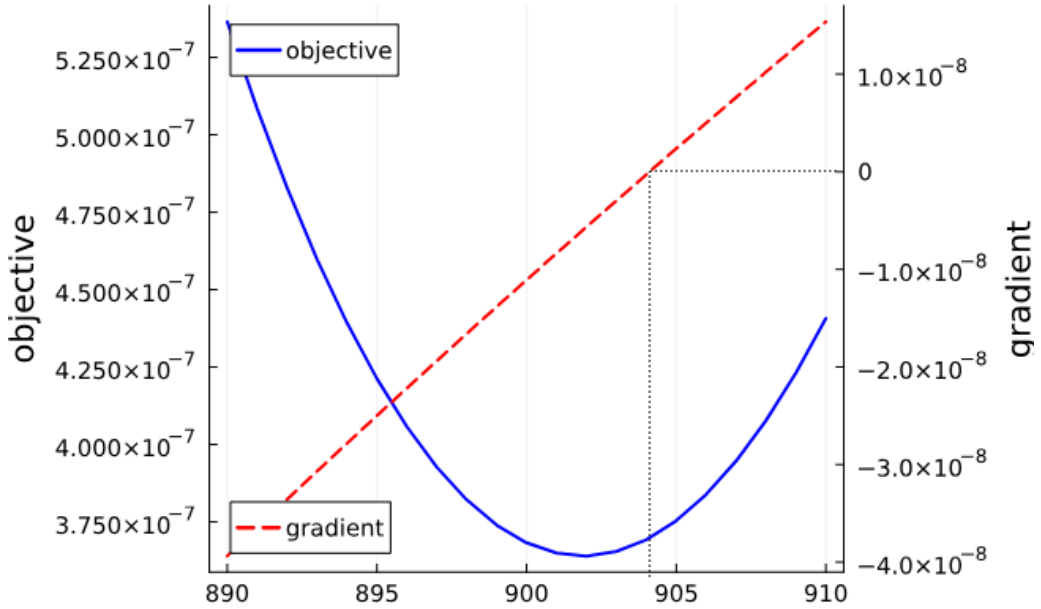


Weight parameter α	Minimizer $\sigma_{\max}$ (target 900)
1 dx	895.5
1/2 dx	899.6
1/4 dx	900.4
1/8 dx	900.2
1/16 dx	900.3
1/32 dx	900.5

Solver: Trixi.jl
https://trixi-framework.github.io/Trixi.jl

Adding Noise

Distorted synthetic data with random velocity field



Next Steps

- Testing
  - More realistic synthetic data from ice sheet model
  - Real observations
- Investigate errors with noisy data
- Tuning of α/weights, resolution, solver order
- Scale up to cover continental ice sheets

Thank you! Questions?

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Ice Sheet Model