Mitteilung

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Robustness, accuracy and efficiency of the discontinuous Galerkin spectral element method under local mesh refinement in an industrial CFD solver

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The aerospace industry is interested in scale-resolving simulations (SRS) with shortened time to solution. Work is underway at the DLR to develop a “rapid SRS” process chain, which combines: 1) automatic mesh generation, 2) immersed imposition of boundary conditions, 3) use of wall models, and 4) the discontinuous Galerkin spectral element method (DGSEM).

We consider the split-form DGSEM with Legendre-Gauss-Lobatto collocation points on tensor-product elements. This method is promising for large eddy simulation (LES) as it combines:

1) robustness, both in theory (in the sense of entropy stability) and in practice,
2) high resolving efficiency[1] for smooth solutions,
3) high throughput that reduces only moderately (linearly) with increasing design order.

In the present work, we want to investigate to what extent each of these three properties is retained when the DGSEM is applied to geometrically nonconforming meshes.

Our interest in nonconforming DGSEM is two-fold. Firstly, so that we can apply it to automatically generated Cartesian meshes obtained by recursive refinement of cells near solid boundaries in an octree fashion. Secondly, because we want to employ DGSEM also on locally refined curvilinear hexahedral grids in order to efficiently resolve localized solution features such as vortex structures, shear layers or shock waves.

We have extended the DGSEM already available in the CFD software by ONERA, DLR and Airbus [4] (CODA[2]) to support nonconforming pairs of elements by means of the mortar method [3]. We currently have two variants of this approach implemented, depending on whether the least-squares projections to/from mortars are approximated using the collocated quadrature rule of the DGSEM basis itself (we refer to this variant as *Lobatto mortar*), or using the more accurate Legendre-Gauss rule (*Gauss mortar*). A third variant, based on [1], is currently in development and we plan to include it in the comparisons in the final article.

For a first assessment of robustness, we simulate a uniform flow. The domain is the $[-5,5]^3$ cube, with periodic boundary conditions on all 6 sides. The initial condition is given by the primitive state vector (density, velocity vector, pressure) $= (0.7, 0.2, 0.3, -0.4, 1.0)$. We solve the Euler equations (i.e. there is no physical dissipation) using the 7-stage, 3rd order explicit Runge-Kutta time scheme from [5, section 3.3.3] and a time step size of 0.02. We employ the quadratic nonconforming mesh in Figure 2, obtained by applying the following mapping before local refinement:

$$\tilde{x} = a\cos(\omega x)\cos(3\omega y)\sin(4\omega z),$$
$$\tilde{y} = a\sin(4\omega x)\cos(\omega y)\cos(3\omega z),$$

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[1] *In the sense that it can accurately resolve arbitrarily high wavenumbers for a fixed problem size.*

[2] *CODA is the computational fluid dynamics (CFD) software being developed as part of a collaboration between the French Aerospace Lab ONERA, the German Aerospace Center (DLR), Airbus, and their European research partners. CODA is jointly owned by ONERA, DLR and Airbus.*

$$\tilde{z} = a\cos(3\omega x)\sin(4\omega y)\cos(\omega z),$$

where $\omega = \pi/10$ and $a = 10/15$.

We compare two numerical surface fluxes: the central flux of Chandrashekar and the upwind flux of Roe with no entropy fix. The discretization is DGSEM of 5th order. We employ the Chandrashekar flux as numerical volume flux in both cases. Figure 1 shows the results obtained with CODA, and compares them with those obtained using exactly the same numerical methods in the open-source research solver FLUXO [2]. For both mortar method variants combined with a diffusive surface flux, the solution remains equal to the initial one for all times. For the nondissipative central surface flux, neither of the mortar method variants is stable. For the Gauss mortar variant, however, the error growth is slower and divergence occurs later than for the Lobatto one. This behavior is also observed with the FLUXO code.

In the full paper, our intention is to use the inviscid Taylor-Green vortex test case [2] to assess robustness and accuracy in a more challenging problem.

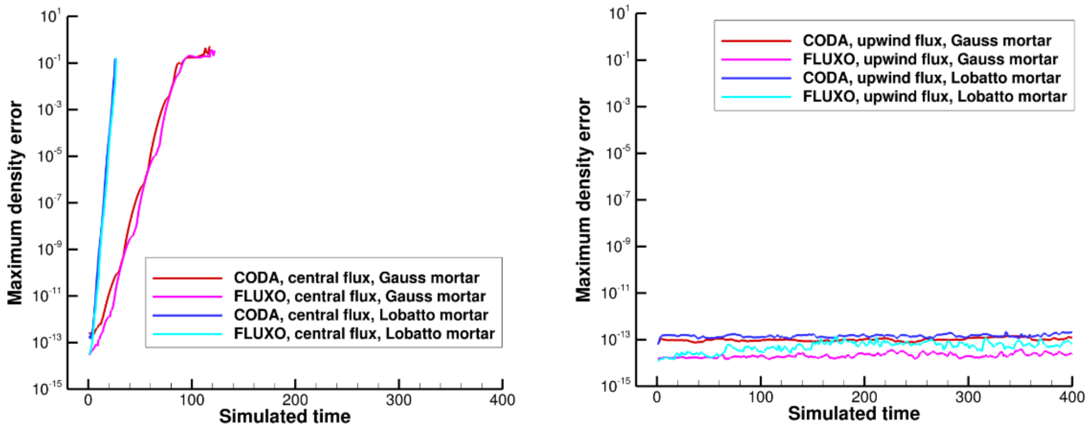

Figure 1: Largest difference in density with respect to the initial condition over all collocation points in the mesh over time. Left: Chandrashekar surface flux. Right: Roe surface flux.

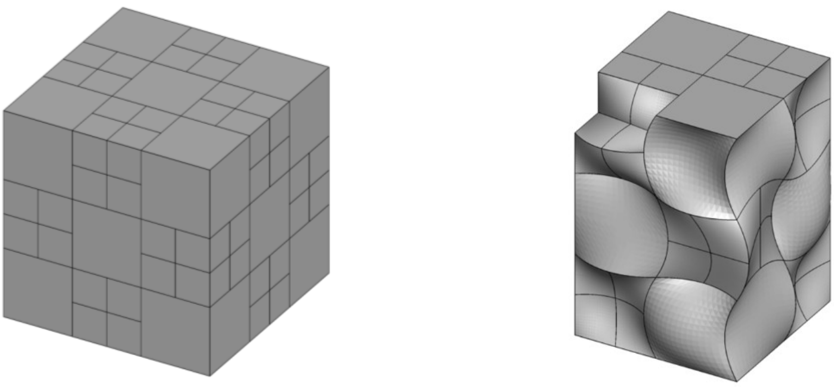

Figure 2: The mesh used to obtain the results in Figure 1. Left: full view. Right: cutaway view.

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