Accounting for Co-Registration Errors in the Interferometric Coherence Model of Semi-Transparent Media

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***Abstract*—Across-track synthetic aperture radar (SAR) interferometry is a well-established technique to obtain the topographic information by using a pair of SAR images, whose coherence influences the topographic height accuracy. Volume decorrelation occurs in presence of semi-transparent media, such as forests and vegetation, and is usually modelled using the vertical scatting profile, thus providing a link between the coherence and the physical parameters of the medium. This has been successfully exploited in polarimetric SAR interferometry and SAR tomography to retrieve three-dimensional structure information of the scene. Future SAR systems are expected to be equipped with larger bandwidths, which also allow for interferometric acquisitions with larger baselines and enhanced height accuracies. In these cases, a significant amount of co-registration decorrelation may occur within the volume, which should be considered in the model of volume decorrelation. This letter presents a novel volume decorrelation model, which accounts for co-registration effects and accurately predicts the coherence for interferometric acquisitions with large bandwidths and/or baselines. Simulation examples are shown for typical scenarios to prove the validity of the model, which is capable of predicting the phase of the complex coherence of a semi-transparent medium with sub-degree accuracy. The proposed model will be a crucial tool to analyze semi-transparent media in future spaceborne SAR missions and drone-borne SAR systems.**

***Index Terms*—Synthetic aperture radar (SAR), SAR interferometry (InSAR), SAR tomography, drones, unmanned aerial vehicle (UAV), volumetric decorrelation, image co-registration, coherence, remote sensing of vegetation**

I. INTRODUCTION

ACROSS-TRACK synthetic aperture radar (SAR) interferometry is a remote sensing technique widely used to obtain the topographic information using a pair of SAR images, acquired with slightly different incidence angles [1]. Interferometric SAR (InSAR) data can also provide volumetric information for semi-transparent media, such as forests and vegetation, as successfully demonstrated by polarimetric SAR interferometry and SAR tomography [2], [3].

The interferometric coherence is defined as the complex correlation between a pair of SAR images and influences the accuracy of the height estimate, i.e., high coherence leads to a better height accuracy [1]. Co-registration consists of shifting the pair of images to a common grid with a sub-resolution accuracy and is an essential step of the InSAR processing chain [4]. An inaccurate co-registration leads to a coherence loss (decorrelation due to co-registration) and the traditional geometric co-registration procedures are designed to work with a single scatterer (or surface scattering).

Although some methods consider a height-dependent co-registration [5], current procedures are not intended to work with volumetric scattering. While for surface scattering it is possible to minimize the decorrelation term due to co-registration by means of coherence maximization techniques [6], this is not always the case for semi-transparent media, where volume decorrelation occurs and can be modelled using vertical scattering profiles [1], [7]. The relationship between the volume decorrelation and the vertical scattering profile is widely used for obtaining three-dimensional structure information of the scene, in spite of the uncertainty in the estimation of the interferometric phase resulting from this decorrelation term.

Future interferometric SAR missions will exploit larger bandwidths and baselines to achieve better height accuracies. Krieger et al. mentioned the possibility of operating TanDEM X with a baseline up to 5 km for the generation of very accurate digital elevation models [8] and bandwidths of up to 1.2 GHz are expected for the next generation of German X band SAR missions, as well as for other commercial systems [9], [10]. Furthermore, drone-based SAR systems, usually equipped with radars with very large bandwidths and very large baselines compared to the platform height, are also being considered for local and frequent InSAR data acquisition [11], [12], [13]. With the increase of bandwidths and baselines, increased accuracy of the shift associated with the co-registration process is required. If semi-transparent media are present and their vertical extent is large, co-registration error may occur within the volume. Therefore, a novel model is required to accurately predict the volume decorrelation, which takes these effects into account.

This letter proposes a novel model, which considers the co-registration error within the volume to accurately predict the volume decorrelation. We analyze the scenarios where the co-registration within the volume is not negligible and provide the conditions where the new model is necessary.

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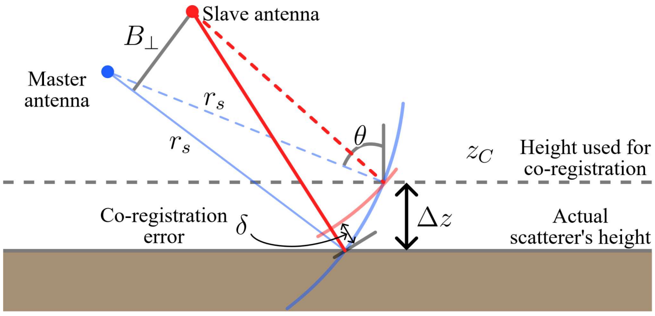


Fig. 1. Geometry of the InSAR data acquisition, where the scatterer's height, shown as a brown area, is vertically displaced from the height of co-registration z_C by Δz.

Numerical simulations show that this model better predicts the InSAR coherence for acquisitions with large bandwidths and baselines.

II. PROBLEM FORMULATION AND MODEL DERIVATION

Fig. 1 shows an InSAR acquisition geometry in the zero-Doppler plane, where the ground, assumed to be the source of surface scattering, is shown as a brown area. In the co-registration process, the shift associated with a certain range is calculated using the acquisition geometry and the a-priori knowledge of the scatterer's height [4]. If the scatterer's height is exactly known, the shift will be equal to the difference between the ranges of the master and the slave to the actual scatterer's position, represented in Fig. 1 by the blue solid line and the red solid line, respectively.

If a height z_C, different than the actual scatterer's height, is used in the shift estimation, a co-registration error occurs. In this case, the resulting shift is given by the difference of the ranges from the master and the slave to the fictive scatterer's position at the wrong height, represented in Fig. 1 by the blue dashed line and the red dashed line, respectively. As a result, the co-registration error δ will correspond to the difference between the red solid line and the red dashed line in Fig. 1.

Assuming that the difference between the height used for co-registration and the actual scatterer's height is Δz, the co-registration error δ can be approximated as follows [4]:

$$\delta = \frac{B_\perp}{r_s \sin\theta} \Delta z, \tag{1}$$

where $B_\perp$, θ, and r_s denote the perpendicular baseline, the incidence angle, and the slant range, respectively. The multiplicative contribution of co-registration to the coherence γ_C caused by this co-registration error is given by [14], [15]:

$$\gamma_C = \operatorname{sinc}\left[\frac{pB_r}{c}\gamma_s^2 \delta\right], \tag{2}$$

where B_r, c, and γ_s denote the range bandwidth, the speed of light, and the coherence contribution due to the spectral or baseline decorrelation, respectively. The constant p equals 1 for bistatic, single-pass acquisitions and 2 for repeat-pass acquisitions. In (2), the function sinc is defined as $\operatorname{sinc}(x) = \sin(\pi x)/(\pi x)$, and γ_s is included to account for the loss of effective range resolution due to the baseline decorrelation [15]. According to (2), there is no decorrelation due to co-registration in absence of the co-registration error and the decorrelation due to co-registration is indeed negligible for surface scattering, where techniques are used to find the scatterer's height and therefore the shift, which maximize the coherence magnitude.

If a semi-transparent medium is considered, however, multiple scattering occurs at different heights for a given slant range. Since current co-registration techniques assume a single scattering height, it is not possible to correctly co-register all scatterers within the volume, located at different heights, using a single co-registration shift.

For some cases, namely if the volume height (or the equivalent one, also considering a possible extinction through the medium) is significantly smaller than the height of ambiguity and the geometrical co-registration is performed using a height within the extent of the volume, the co-registration errors within the volume are negligible and the coherence contribution γ_V due to the volume decorrelation is modelled using the following well-known expression [1], [2]:

$$\gamma_V = \frac{\displaystyle\int_{z_0}^{z_0+h_v} g_v(z)\exp\left[-jk_z z\right] dz}{\displaystyle\int_{z_0}^{z_0+h_v} g_v(z)\, dz}, \tag{3}$$

where j, z_0, h_v, and z denote the imaginary unit, the height of the ground, the height of the volume, and the vertical axis, respectively. The vertical scattering profile $g_v(z)$ represents the amount of backscatter occurring at different heights. The vertical wavenumber k_z is the phase-to-height conversion factor associated with the acquisition geometry, which is expressed as follows [1]:

$$k_z = \frac{2p\pi B_\perp f_c}{c r_s \sin\theta}, \tag{4}$$

where f_c denotes the central frequency of the transmitted signal.

In other cases, the decorrelation due to co-registration within a volume may not be negligible. Let us assume that the co-registration is performed using the height of the ground z_0 which is perfectly known. Considering the signal returns from the top of the volume, there is a difference between the height used for co-registration $z_C = z_0$ and the actual scatterer's height $z_0 + h_v$. In this scenario, the height difference equals the volume height h_v, and the coherence contribution due to co-registration can be obtained using (1) and (2) by substituting Δz with h_v:

$$\begin{aligned} \gamma_C &= \operatorname{sinc}\left[\frac{pB_r}{c}\gamma_s^2 \frac{B_\perp}{r_s \sin\theta} h_v\right] \\ &= \operatorname{sinc}\left[\gamma_s^2 \frac{B_r}{f_c}\frac{h_v}{h_{\text{amb}}}\right], \end{aligned} \tag{5}$$

where $h_{\text{amb}} = 2\pi/k_z$ denotes the height of ambiguity, which can be expressed as a function of the vertical wavenumber.

In (5), the factor B_r/f_c is the fractional bandwidth, whose theoretical maximum value is 2. While the fractional bandwidth is below 0.1 for most spaceborne SAR systems, it can be larger than 1 for more recent drone-borne SAR systems

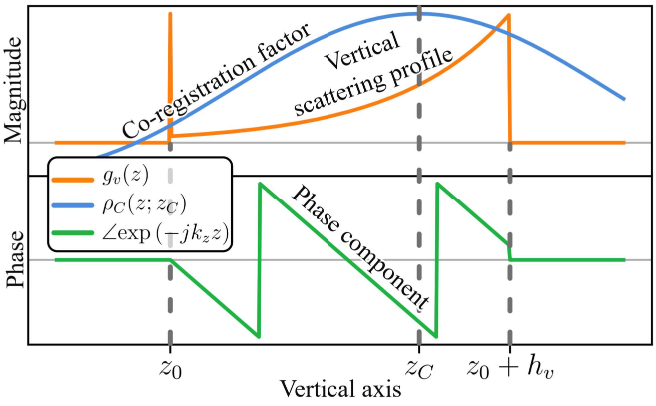


Fig. 2. Graphical description of the three functions which are used to model the volume decorrelation considering the decorrelation due to co-registration. The orange, blue, and green lines depict the vertical scattering profile g_v, co-registration factor ρ_C, and the phase of the complex exponential $\exp(-jk_z z)$, respectively.

[13]. The factor h_v/h_{amb} becomes large for acquisitions with very large ratio between the baseline and the sensor height (as h_{amb} is inversely proportional to this ratio), which explains why the drone-borne case is very relevant too. The condition for a negligible co-registration error can therefore be derived from (5) by imposing a minimum acceptable value of γ_C, namely γ_C^*. By inverting (5) for $\gamma_C = \gamma_C^*$, a condition of the ratio between the volume height and the mainlobe width h_C of the sinc-function in (5) can be obtained:

$$\frac{h_v}{h_C} < \alpha, \tag{6}$$

where the main-lobe width h_{amb} can be expressed as:

$$h_C = \frac{1}{\gamma_s^2} \frac{1}{B_r/f_c} h_{\text{amb}}. \tag{7}$$

The actual value of the parameter α will be discussed later on in Section III.

In case the condition in (6) is not fulfilled, the following more general expression of the coherence contribution due to volume should be considered which accounts for a considerable amount of decorrelation due to co-registration within the volume. This can be obtained by substituting the numerator of (3) with a weighted integral over the vertical profile:

$$\gamma_{V,C} = \frac{\displaystyle\int_{z_0}^{z_0+h_v} g_v(z)\rho_C(z; z_C) \exp\left[-jk_z z\right] dz}{\displaystyle\int_{z_0}^{z_0+h_v} g_v(z)\, dz}, \tag{8}$$

where $\rho_C(z; z_C)$ is a real-valued function, referred to as the co-registration factor, representing the coherence contribution due to incorrect co-registration expected from a vertical point z assuming that a height z_C is used within the co-registration process. An analytical form of ρ_C can be obtained observing that the portion of volume with a large co-registration error contributes less to the overall volume decorrelation and has a similar form as (5):

$$\rho_C(z; z_C) = \operatorname{sinc}\left[\frac{z - z_C}{h_C}\right]. \tag{9}$$

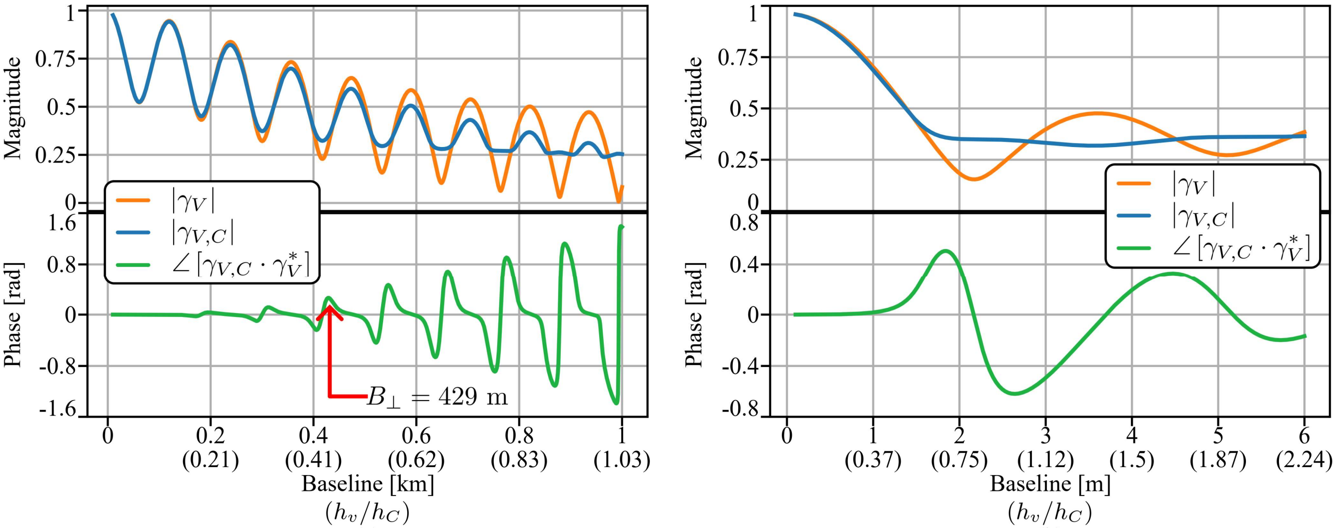
TABLE I
SIMULATION PARAMETERS

SAR scenario	X-band low-earth orbit	Drone-borne with wide-fractional bandwidth
Center frequency	9.8 GHz	2.5 GHz
Bandwidth	1.2 GHz	3 GHz
Incidence angle	3.6°	60°
Altitude	514 km	100 m
Slant range	635 km	200 m
Volume scenario	Random-volume-over-ground	Random-volume-over-ground
Extinction coefficient	0.6 dB/m	0.3 dB/m
Ground-to-volume-ratio	0.3	0.6

A graphical interpretation of the expression in (8) is shown in Fig. 2, where the vertical scattering profile g_v, the co-registration factor ρ_C, and the phase component of the complex exponential $\exp(-jk_z z)$ are depicted in orange, blue and green, respectively, as a function of the vertical coordinate z. The difference between the conventional model in (3) is indeed the co-registration factor, which is centered around z_C with a maximum value of 1. The conventional model is a special case of (8), where the co-registration factor is constant, which is the case for a large value of the sinc main-lobe h_C compared to the volume height.

As the co-registration factor ρ_C in (8) is a function of height used for the co-registration z_C, the coherence contribution due to volume decorrelation $\gamma_{V,C}$ will also be a function of z_C. Therefore, different coherence values are expected for different values of z_C. Similarly, as for surface scattering, the coherence contribution due to volume decorrelation and accounting for co-registration effects provided in (8) can be evaluated for multiple co-registration heights z_C and the value with the maximum magnitude could be selected. Evaluating the volume decorrelation with different co-registration heights is equivalent to shifting the co-registration factor over the vertical axis.

Depending on the vertical scattering profile, assumptions can be made on the co-registration height which maximizes the coherence contribution due to volume decorrelation. For a vertical scattering profile of vegetation with a very large extinction through medium [7], assuming that no ground contribution is present, most of the backscatter contribution originates from the canopy. Therefore, a co-registration factor $\rho_C(z; z_C)$ centered near the canopy height (e.g., $z_C \simeq z_0 + h_v$) would lead to the maximum coherence value. For single-layered vertical scattering profiles with most of the backscatter concentrated in a small vertical extent, the co-registration factor centered near the region with high backscattered power would yield maximum coherence magnitude. In these cases, the backscattered power originating from heights other than z_C would be small. Therefore, the consideration of the co-registration factor ρ_C does not impact the output of (8), i.e., the model in (3) would be sufficient to model the volume decorrelation. For a general multi-layered vertical scattering profile with the backscatter distributed within the volume, an approximation of the co-registration height for maximum coherence is difficult to provide. Moreover, the backscatter of a point away from



(a) (b)

Fig. 3. (a) Values of coherence model assuming a X-band satellite-borne SAR system is evaluated for baselines ranging from 0.1 km to 1 km. (b) Values of coherence model assuming a drone-borne SAR system is evaluated for baselines ranging from 0.1 m to 6 m. The orange and blue lines depict the magnitude of the coherence where the co-registration decorrelation is not considered $|\gamma_V|$ and considered $|\gamma_{V,C}|$, respectively. The green lines depict the phase differences between the coherence obtained from the two models $\angle\gamma_{V,C} \cdot \gamma_V$.

the co-registration height z_C could have a significant impact when modelling the coherence using (3). As a conclusion, for some volumes, i.e., forests with random-volume-over-ground (RVoG) characteristics [2], the use of the refined expression for the coherence contribution in (8) is necessary to properly model the volume decorrelation.

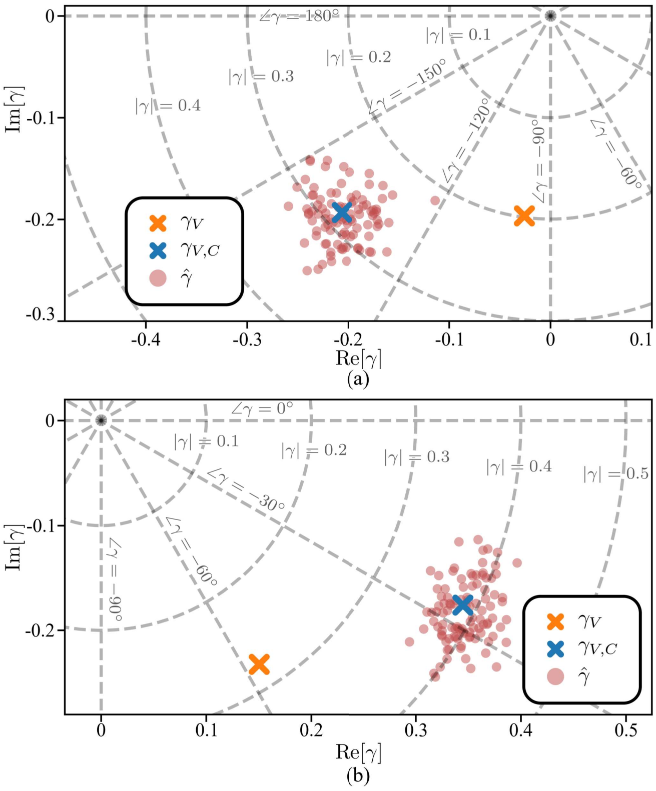
III. SIMULATION RESULTS

Simulations were performed for scenarios represented by the parameters in Table I to investigate how decorrelation due to co-registration impacts the volume decorrelation. Fig. 3 (a) depicts the coherence values for an X-band SAR system flying in a low-earth orbit with a ultrawide bandwidth of 1.2 GHz [9], [10]. The volume decorrelation is evaluated for values of the perpendicular baseline varying from 100 m to 1 km and assuming a repeat-pass acquisition. The ratios between the volume height h_v and the scaled heights of ambiguity h_C in (7) are evaluated for different baselines and are shown in the horizontal axis between brackets. The orange and blue lines depict the magnitude of the coherence contribution due to volume decorrelation obtained with (7) and (8), respectively. The green line depicts the difference between phases of the coherence in the two models. When the effects of co-registration are considered, the value with the maximum magnitude was selected from the multiple coherences obtained from (8) for different co-registration heights z_C.

As expected from the analysis of the previous section, a considerable difference between the two models is observed from Fig. 3 (a) when the condition in (6) is not fulfilled, i.e., when the ratio h_v/h_C exceeds a given threshold. In this specific case, this happens for baselines larger than 386 m (baseline-height-ratio=0.0008). Note, that the baseline equivalent to this value in the bi-static single-pass acquisition is two time larger (772 m). The maximum difference in coherence magnitude and phase for baselines smaller than 386 m were 0.05 and 0.12 rad ($6.6°$), respectively. For a baseline of 429 m, the ratio h_v/h_C is slightly larger than 0.4 (marked with a red arrow in Fig. 3 (a)) and a significant amount of phase difference (0.26 rad, $15°$). While the difference between the two models occurs only for baselines with relatively small coherence magnitudes, low coherences are indeed expected from volume scattering, for which the new model is required.

Fig. 3 (b) depicts the difference between the two volume decorrelation models for a drone-borne SAR system, whose parameters are provided in the right column of Table I [13]. Perpendicular baselines ranging from 0.1 m to 6 m were used to evaluate the coherence model assuming a repeat-pass acquisition. In this case, the wide-fractional bandwidth and the large ratio of the baseline to the sensor height make it harder to achieve the condition in (6). Similar to the space-borne case, differences between the two models are apparent when the ratio h_v/h_C is larger than 0.4, which therefore also represents a reasonable value for the parameter α in (6). The maximum baseline, which fulfills the condition in (6) was 1.1 m (baseline-height-ratio=0.01), for which a phase difference of 0.02 rad ($1.3°$) is obtained. The maximum phase difference of 0.62 rad ($35.4°$) occurred for a baseline of 2.2 m. In real scenarios, the volume height can easily exceed the value of 3.5 m considered in Fig. 3 (b). Therefore, the consideration of co-registration effects in the volume decorrelation becomes fundamental when analyzing InSAR coherences obtained from wideband, drone-borne radars.

Further simulations were performed to assess the performance of the novel model. The SAR signal of a semi-transparent medium was simulated by assuming a large number of point targets randomly distributed within the extent of the volume. The vertical scattering profile was taken into account by selecting the amplitudes of each scatterer according



(a)

(b)

Fig. 4. (a) Coherence estimated from simulated radar data assuming a X-band satellite-borne SAR system with 636 m baseline is shown as red circles on the complex plane. (b) Coherence estimated from simulated radar data assuming a drone-borne SAR system with 1.8 m baseline is shown as red circles. The coherence values obtained from models that take the co-registration decorrelation within the volume into account and those that do not take them into account are depicted as orange and blue crosses, respectively.

to its vertical position. In this simulation, the volume was assumed to exhibit RVoG characteristic [2], [3]. The pair of simulated SAR data were co-registered with referenced to multiple heights and the maximum magnitude was selected after a multi-look by 400 samples.

The red dots in Fig. 4 (a) show 100 coherence estimates $\hat{\gamma}$ assuming an X-band SAR scenario, where the parameters in Table I were used to generate the simulated data. In this simulation, the baseline was 636 m. The values are depicted on a complex plane, where the magnitude and phase of the mean estimated coherence were 0.28 and -2.4 rad ($-136°$), respectively. The volume decorrelation obtained from (3), i.e., without the consideration of co-registration decorrelation, is $0.2 \cdot \exp\left(-j \cdot 97°\right)$ and is marked on the complex plane as an orange cross. The magnitude and phase of the estimated coherence is biased by 0.08 and 0.67 rad ($38.5°$), respectively, using this model. The blue cross in Fig. 4 (a) shows the volume decorrelation value obtained from (8), where the co-registration decorrelation within the volume is considered. Upon coherence maximization, the magnitude and phase of 0.28 and -2.4 rad ($-136.9°$), respectively, are obtained, with a bias in phase of only 0.01 rad ($0.9°$)!

Simulation results considering a drone-borne SAR system are shown in Fig. 4 (b), where the parameters from Table I are used and a baseline of 1.8 m is assumed. In the figure, 100 estimated coherences $\hat{\gamma}$ obtained from a multi-look by 400 samples are depicted as red dots. The magnitude and phase of the mean estimated coherence are 0.39 and -0.08 rad ($-27.6°$), respectively. The value of coherence obtained from (3) and (8) are depicted in orange and blue crosses, respectively, which visually shows that the new model is capable of accurately predicting the volume decorrelation. Significant magnitude and phase biases of 0.12 and 0.51 rad ($29.5°$), respectively, are obtained if the estimated coherences are compared to the model in (3). In contrast, a negligible magnitude and phase bias of 0.005 and 0.01 rad ($0.54°$), respectively, are obtained by using the proposed model, which accounts for co-registration effects.

IV. CONCLUSIONS

In this letter, a novel model for the volume decorrelation occurring in semi-transparent media is presented, where the effects of co-registration are considered to accurately predict the expected coherence contribution. Analyses based on simulations show that the proposed model is able to predict phases with sub-degree accuracy and is therefore fundamental to avoid significant magnitude and phase biases in spaceborne and drone-borne SAR systems with wide bandwidths and large baselines.

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