Mitteilung

Fachgruppe: Numerische Aerodynamik

Entropy-stable fluxes for high-order Discontinuous Galerkin simulations of high-enthalpy flows.

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Accurate simulations of high-enthalpy reacting multi-species flows are a crucial component of the research and development cycle for design of new spacecraft [1]. As available computing power increases, so does the range of physical phenomena that can be modelled. At the same time, development of new numerical methods that are able to fully leverage modern computing architectures is a crucial part of CFD development [2]. Discontinuous Galerkin (DG) methods possess many properties that make them the preferred approach for future CDF solver development: 1) higher-order solution representation 2) low numerical dissipation 3) less stringent requirements on grid-shock alignment 4) improved computational efficiency due to data locality.

One of the desired properties of numerical schemes is entropy conservation [3,4], as it helps ensure stability of the resulting scheme. Within the Discontinuous Galerkin Spectral Element Method (DGSEM), this requires entropy-conservative numerical volume and surface flux

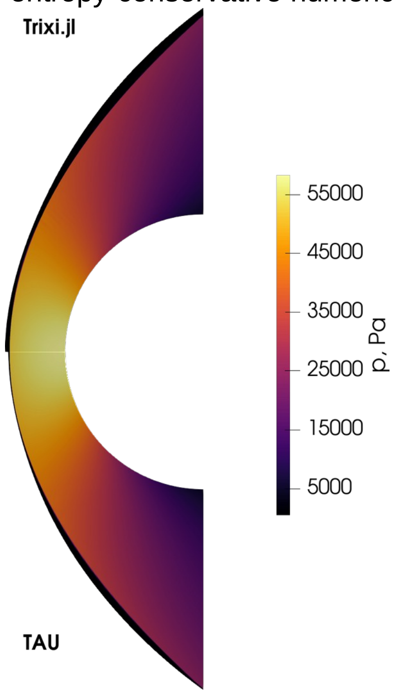

Figure 1: Pressure field for supersonic flow of calorically perfect gas around a cylinder.

functions [5]. Whilst for calorically perfect gases the derivation of such flux functions is relatively straightforward [6], for flows where the internal energies are temperature-dependent, most approaches rely on simplified models for the internal energies and specific heats [7,8], which either reduces the range of applicability of such models, or requires precomputation of curve fits. In [9] a more generic approach to computation of entropy-conservative fluxes for flows with temperature-dependent energies was proposed, which relies on linear reconstruction of energies and specific heats from tabulated values.

In the present work, the approach developed in [9] is leveraged for high-order DG simulations of reacting multi-species flows. Comparisons to the DLR TAU solver [10] are carried out for two-dimensional reacting flows in order to verify the developed approach and assess its performance.

Preliminary results show excellent agreement between solutions obtained with the TAU solver and the developed approach as implemented in the Trixi.jl framework [11] for the case of a calorically perfect gas, as seen on Fig. 1 for the case of an inviscid Mach 10 flow around a cylinder.

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