Mitteilung

Fachgruppe: Turbulenz und Transition

Quantitative comparison of results from DNS and nonlinear parabolized stability equations for the subharmonic transition process

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Background and Motivation

The laminar-turbulent transition process in a flat plate boundary layer can be viewed as a sequence of instabilities, beginning with the spatial growth of two-dimensional (2D) Tollmien-Schlichting (TS) waves. As these waves amplify to certain finite amplitudes, they evolve into a three-dimensional (3D) vortical structure, forming distinct Λ-shaped vortices. These three-dimensional disturbances rapidly grow to large amplitudes, finally causing the breakdown to turbulence. Among the possible transition mechanisms, the experiments conducted by Kachanov and Levchenko [1] provided a detailed understanding of the subharmonic resonance process, also known as the H-type transition. In this scenario, disturbances arise with twice the period and twice the wavelength of the fundamental 2D TS waves, leading to a staggered pattern of Λ-vortices (see Figure 1). The H-type transition has already been extensively studied numerically, with both direct numerical simulations (DNS) (e.g., [2]) and the more computationally efficient nonlinear parabolized stability equations (NPSE) (e.g., [3]) yielding closely aligned results in the weakly nonlinear regime. However, a detailed quantitative comparison between these two methods in the more strongly nonlinear stage of the transition process has not yet been thoroughly presented. This contribution aims to address this gap, with a particular focus on identifying the limitations of NPSE—specifically, investigating the location prior to transition where NPSE ceases to deliver accurate results.

Methodology and Results

Direct simulations were carried out using the incompressible spectral element method (SEM) code Nek5000 [4]. The SEM decomposes the physical domain into spectral elements, where the flow field solution is represented by a sum of Lagrange interpolants defined by an orthogonal basis of Legendre polynomials within each element. All parameters were chosen to match the conditions of the wind-tunnel experiments conducted by Kachanov and Levchenko [1]. The disturbances were introduced via a blowing and suction strip where the wall-normal velocity was prescribed to model the effect of vibrating ribbon-induced disturbances. As in the experiments [1], the transition scenario is referred to as 'controlled,' since the initial disturbance wavelengths and frequencies are fixed such that only the fundamental 2D TS (reduced frequency $\mathrm{F} = 124 \times 10^{-6}$) and the oblique subharmonic waves (reduced frequency $\mathrm{F} = 62 \times 10^{-6}$) are introduced without any random components. The amplitudes of these two harmonics were chosen to achieve subharmonic resonance between 2D and 3D disturbances, closely following the experimental setup [1]. Figure 1 shows an instantaneous snapshot of the vortical structures developing inside the boundary layer for the H-type transition, where the characteristic staggered arrangements of the Λ-vortices are visible. This subharmonic breakdown was also investigated using compressible NPSE, which enables the study of the nonlinear evolution and interaction of 2D TS waves and 3D oblique waves, potentially up to the breakdown stage where significant changes in skin friction occur. The NPSE results were obtained with the NOLOT code (see [3]). The spatial development of selected time- and spanwise-Fourier components of the disturbances from the DNS simulation was extracted to provide a benchmark for direct comparison with NPSE. Figure 2 shows an excellent agreement between DNS and NPSE results for both the initialised and some selected nonlinearly generated harmonics, up to a streamwise location where nonlinear effects remain weak. Note that deviations observed further upstream are due to differences in mode initialization between the two approaches, leading to varying transients. However, after these initial transients have decayed, the agreement between DNS and NPSE becomes excellent. This agreement, facilitated by using a common base flow in both DNS and NPSE—thereby minimizing inaccuracies due to base flow differences rather than methodological discrepancies—provides a strong foundation for further analysis. Specifically, the final contribution will focus on identifying the point at which NPSE begins to lose accuracy in the highly nonlinear regime closer to the transition point, where deviations from DNS become significant and NPSE may ultimately fail to converge.

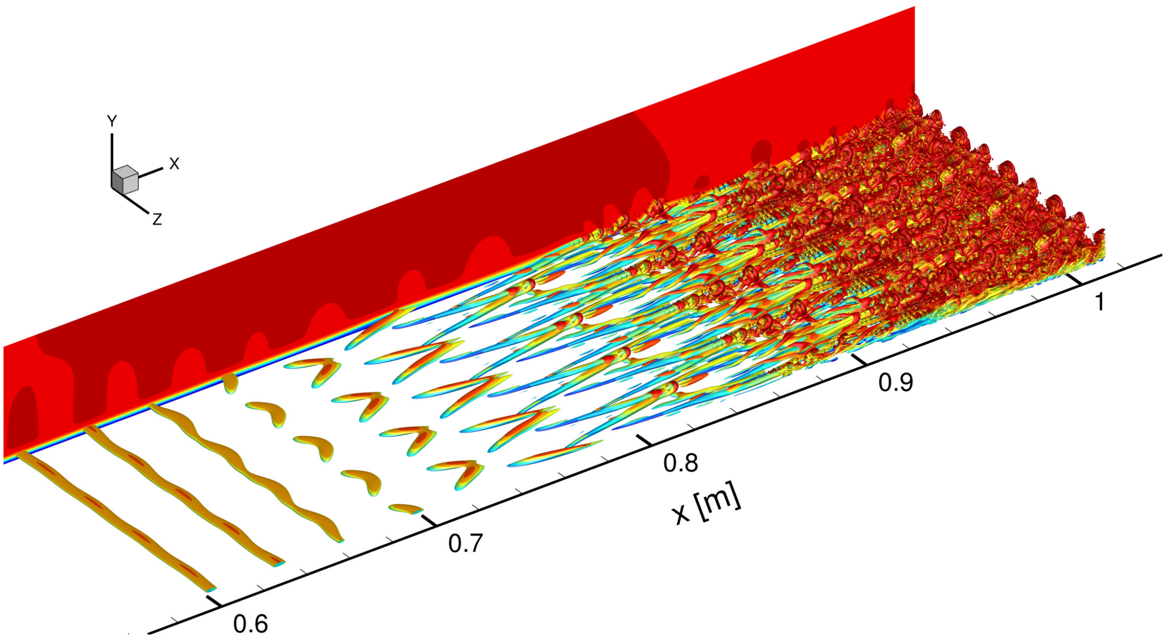

Figure 1: Instantaneus isosurfaces of the Q-criterion coloured by the streamwise velocity, showing the staggered Λ-vortices.

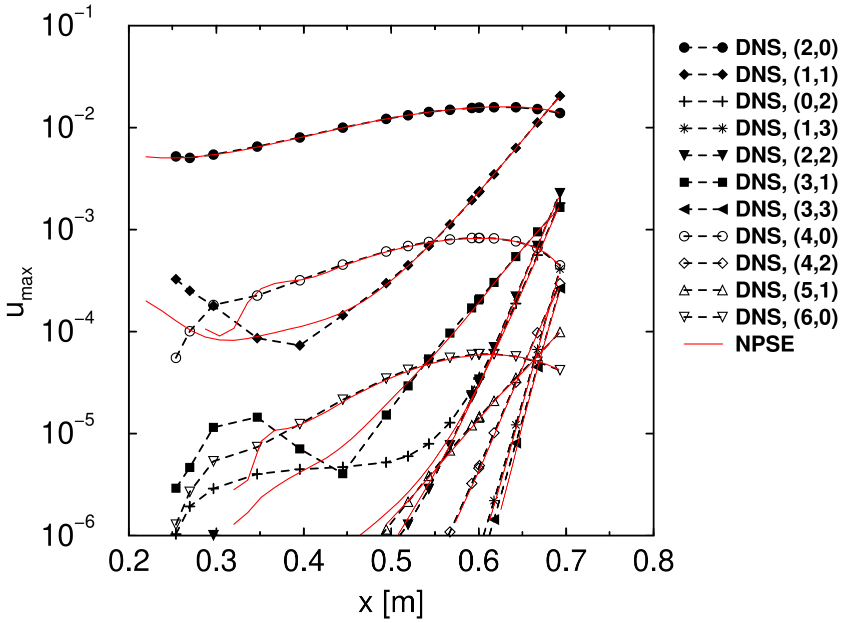

Figure 2: Comparison of amplification curves for the streamwise velocity component between DNS (symbols) and NPSE (solid red lines) across various harmonics commonly considered in the literature. The initialised 2D and 3D TS waves are denoted by $(2,0)$ and $(1,1)$, respectively, with the nonlinearly generated harmonics labeled as (m,n), where m and n represent multiples of the frequency and the spanwise wavenumber of the oblique TS mode $(1,1)$, respectively.

References

[1] Y. S. Kachanov and V. Y. Levchenko. The Resonant Interaction of Disturbances at Laminar-Turbulent Transition in a Boundary Layer. *Journal of Fluid Mechanics* 138 (1984), pp. 209–247.

[2] H. F. Fasel, U. Rist, and U. Konzelmann. Numerical Investigation of the Three-Dimensional Development in Boundary-Layer Transition. *AIAA Journal* 28.1 (1990), pp. 29–37.

[3] S. Hein. "Nonlinear Nonlocal Transition Analysis". DLR Forschungsbericht 2005-10. PhD thesis. University of Stuttgart, 2005.

[4] P. F. Fischer, J. W. Lottes, and S. G. Kerkemeier, 2008 *Nek5000 Web page*. http://nek5000.mcs.anl.gov