p-anisotropic h-isotropic adaptive Discontinuous Galerkin methods for turbulent flows

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DLR

Outline

- Introduction to modal anisotropic DG
- Mesh refinement by subdivision
- Introduction of refinement indicators
- Description of adaptation strategies
- Numerical experiments
- Conclusion and outlook

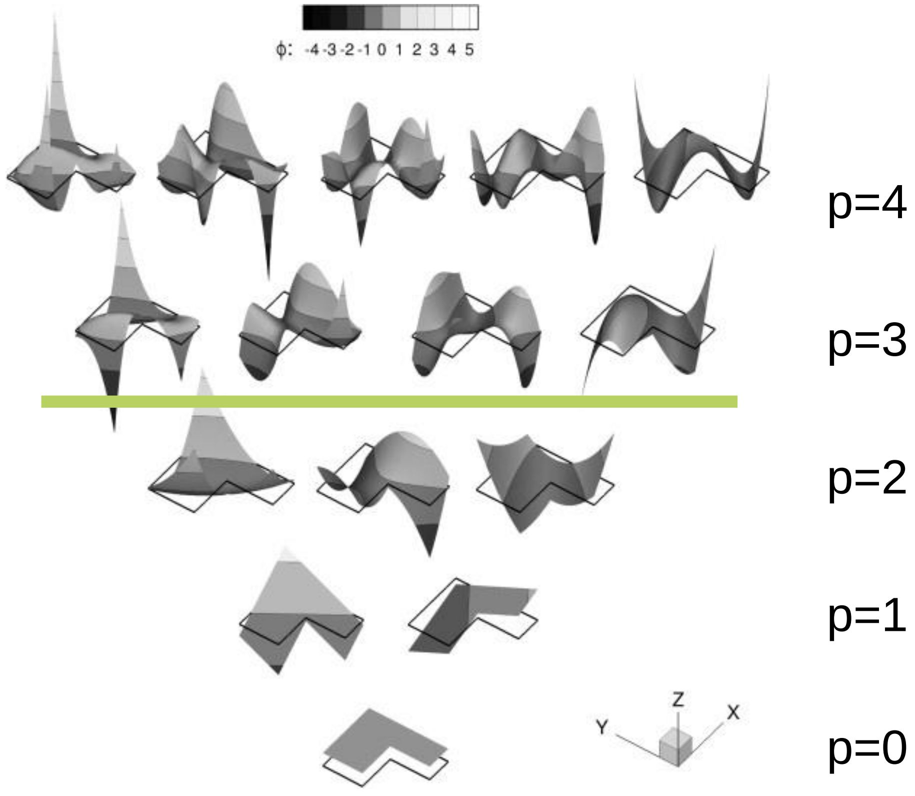
- This work has been conducted in the CFD software by ONERA, DLR and Airbus (CODA)

Isotropic adaptive modal Discontinuous Galerkin in CODA

- Orthonormal hierarchical modal basis in physical space

$$\left\langle \phi_i, \phi_j \right\rangle_\kappa = \delta_{i,j}$$

- Full basis is constructed up to a certain maximum degree
- This maximum degree can be chosen individually for each element



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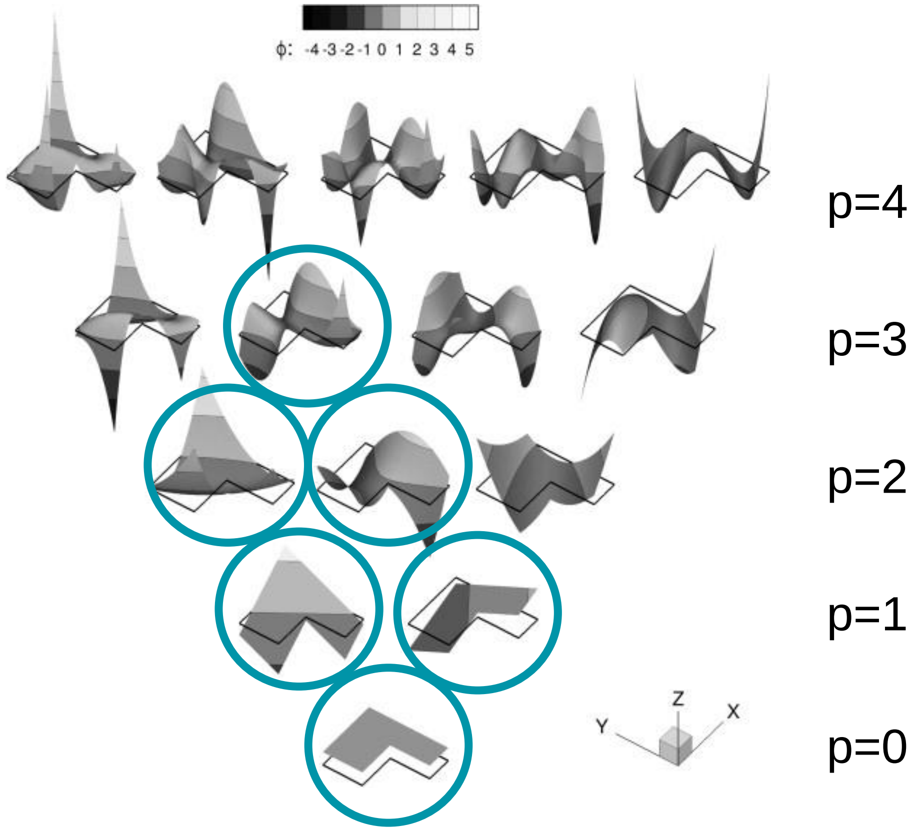
Anisotropic adaptive modal Discontinuous Galerkin in CODA

- Orthonormal hierarchical modal basis in physical space
$$\langle \phi_i, \phi_j \rangle_\kappa = \delta_{i,j}$$
- Full basis is constructed up to a certain maximum degree
- Any subset of basis functions is allowed to be chosen in each element
  - The constant basis function is always included
  - The linear basis functions are also included as we start adapting from second order



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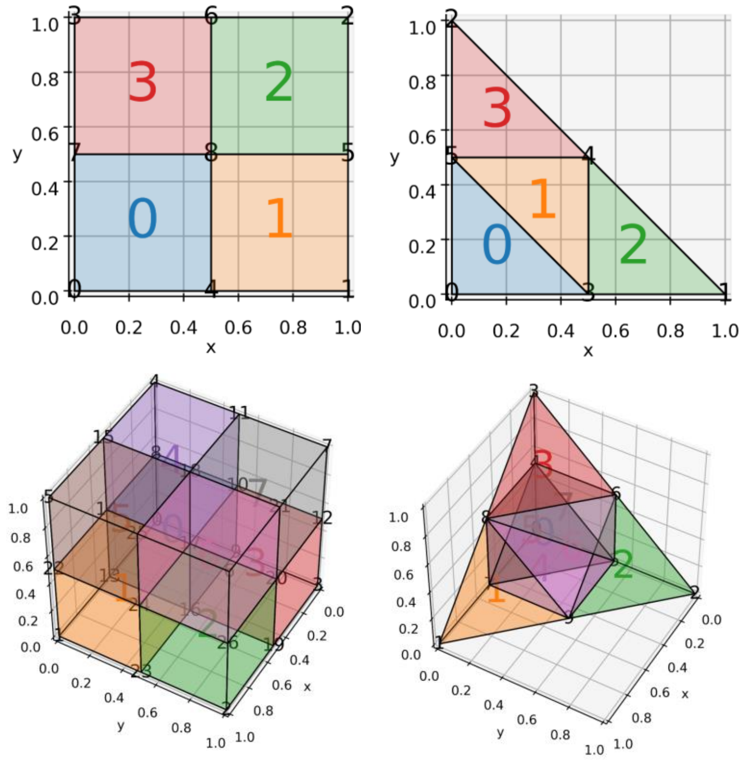
Adaptive mesh refinement using subdivision

- When compared to other techniques (remeshing, …)
  - \+ Possible for every element type (→hybrid meshes) except for polyhedrons
  - \+ Fewer elements are affected
  - \+ Pre-existing expert knowledge for meshing is reused
  - – Spatial orientation of elements is fixed
  - – There are non-conforming element interfaces
  - – Pyramids are problematic



Residual-based error Indicator

- Introduced by Hartmann, Houston and Leicht in [1, 2]
- Measure for the discretization error in each element

$$\eta_\kappa = h_\kappa \, \|\mathbf{R}_\kappa(\mathbf{u}_h)\|_{L^2} + h_\kappa^{\frac{1}{2}} \|\mathbf{r}_{\partial\kappa}(\mathbf{u}_h)\|_{L^2}$$

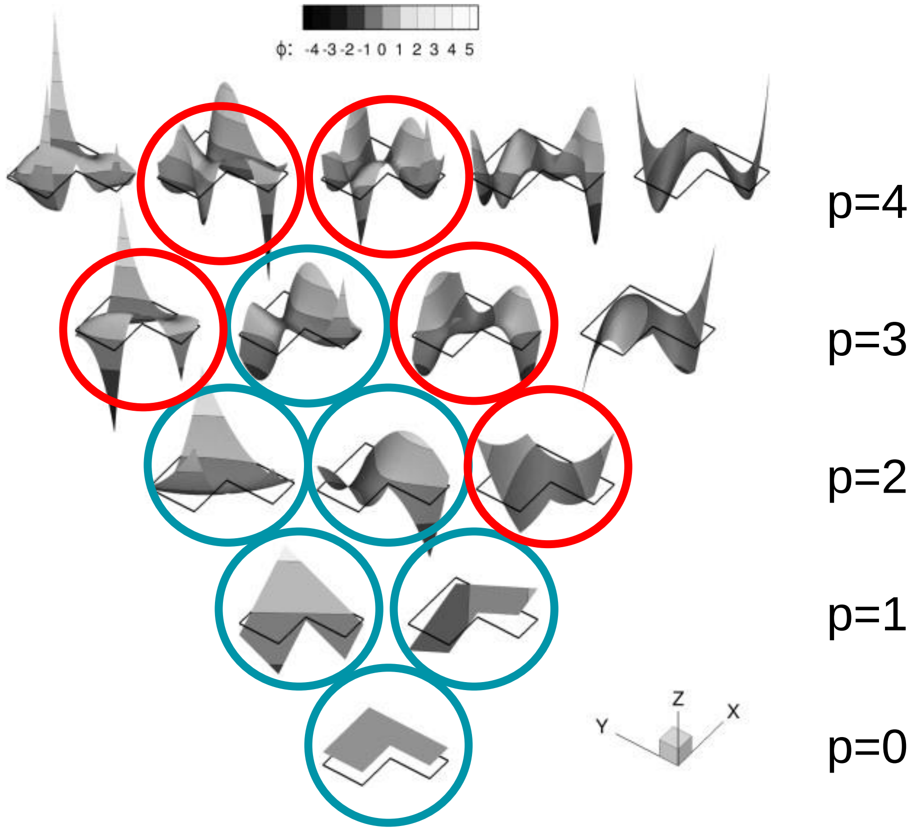
- $\mathbf{R}_\kappa(\mathbf{u}_h)$ is the strong element residual
- $\mathbf{r}_{\partial\kappa}(\mathbf{u}_h)$ is the difference between the numerical and exact fluxes
- Implementation details for the indicator with lifting operators can be found in [Wegener, Hartmann, JCP, submitted]
- Not oriented at a specific goal (e.g. force coefficient) but aims at resolving **all** flow features

[1] Hartmann, R., & Houston, P. (2006). Symmetric Interior Penalty DG Methods for the Compressible Navier–Stokes Equations II: Goal–Oriented A Posteriori Error Estimation. *International Journal of Numerical Analysis & Modeling*, *3*(2), 141–162.

[2] Leicht, T., & Hartmann, R. (2010). Error Estimation and Anisotropic Mesh Refinement for 3d Laminar Aerodynamic Flow Simulations. *J. Comput. Phys.*, *229*(19), 7344–7360.

Least-Squares reconstruction (LSQ)

- Find a higher order reconstructed solution, which is indistinguishable from the numerical solution on the nearest neighbor stencil S_κ in a weak sense
- As the reconstruction is over-constrained, it is solved in a least-squares sense
- $\langle \boldsymbol{u}_\kappa + \boldsymbol{u}_R, \boldsymbol{\phi}_i \rangle_{\kappa'} = \langle \boldsymbol{u}_h, \boldsymbol{\phi}_i \rangle_{\kappa'}$
  $\forall \boldsymbol{\phi}_i \in \boldsymbol{U}^p_{\kappa'}, \kappa' \in S_\kappa$
- As the basis is orthonormal, the coefficients of $\boldsymbol{u}_R$ are the energy contained in the **candidate functions**



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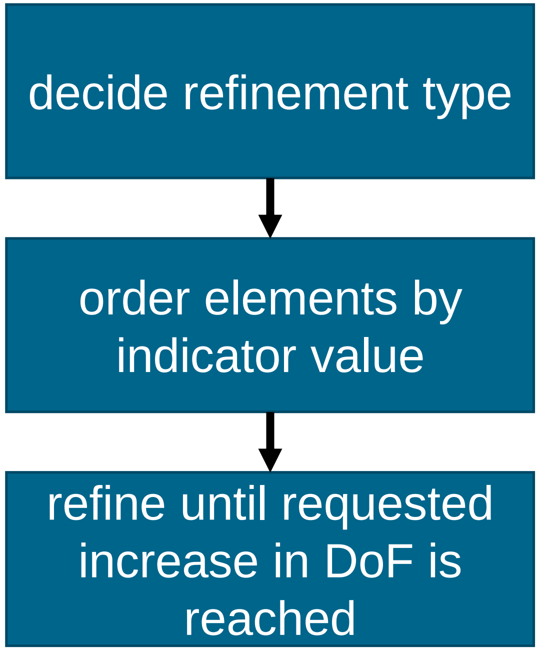
Adaptation strategies

p-isotropic h-isotropic

- Each element is either flagged for p-refinement or h-refinement according to the **exponential decay indicator**
- Elements are sorted by their indicator value
- Elements are refined until the requested increase in DoF is reached



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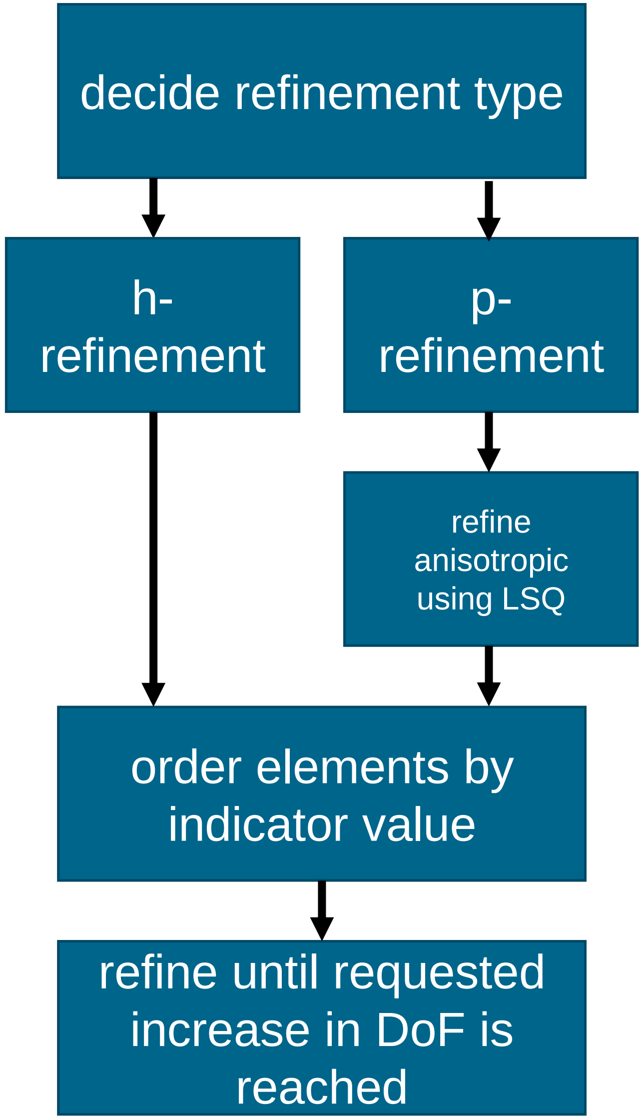
Adaptation strategies

p-anisotropic h-isotropic split

- Each element is either flagged for p-refinement or h-refinement according to the **exponential decay indicator**
- p-refinement is chosen according to the LSQ indicator
- Elements are refined until the requested increase in DoF is reached



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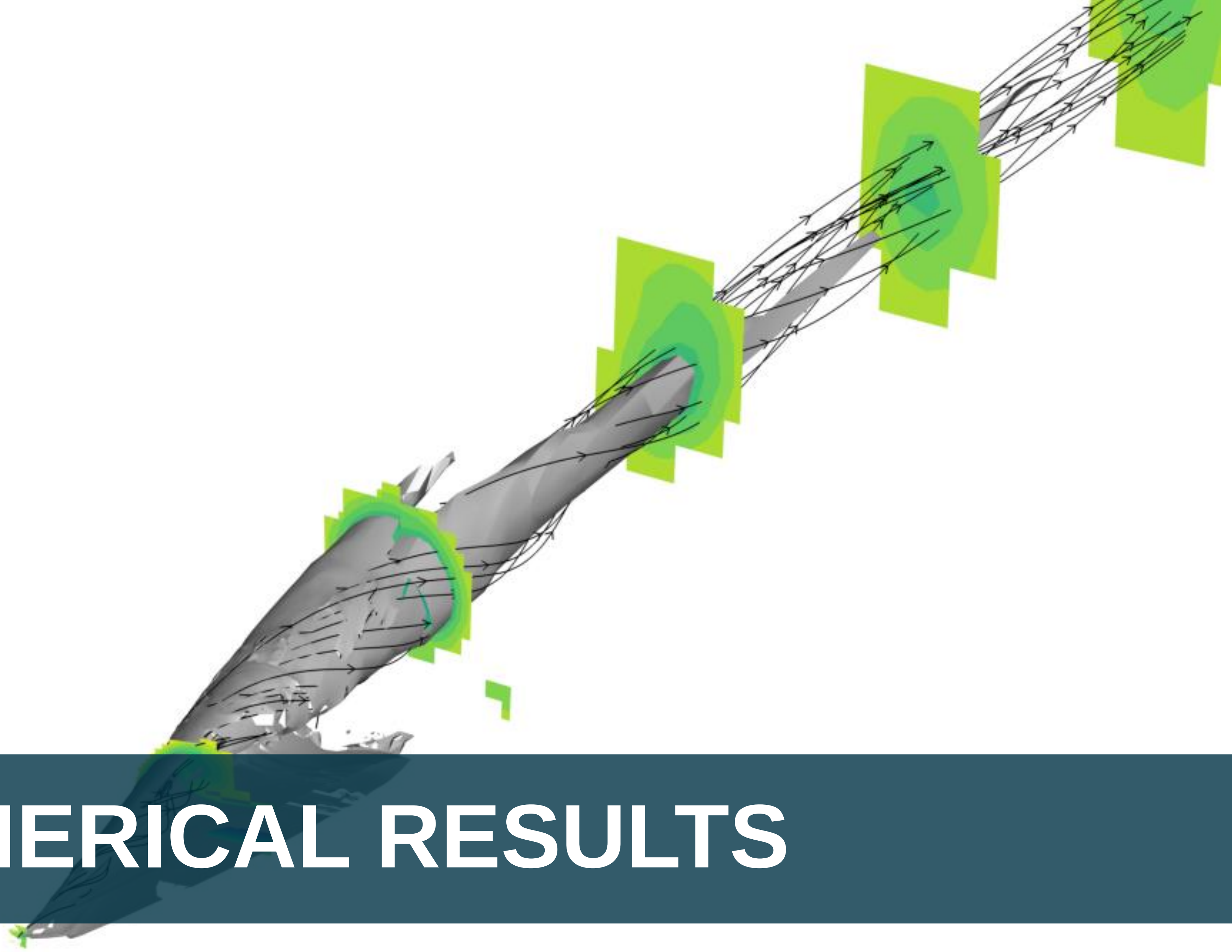
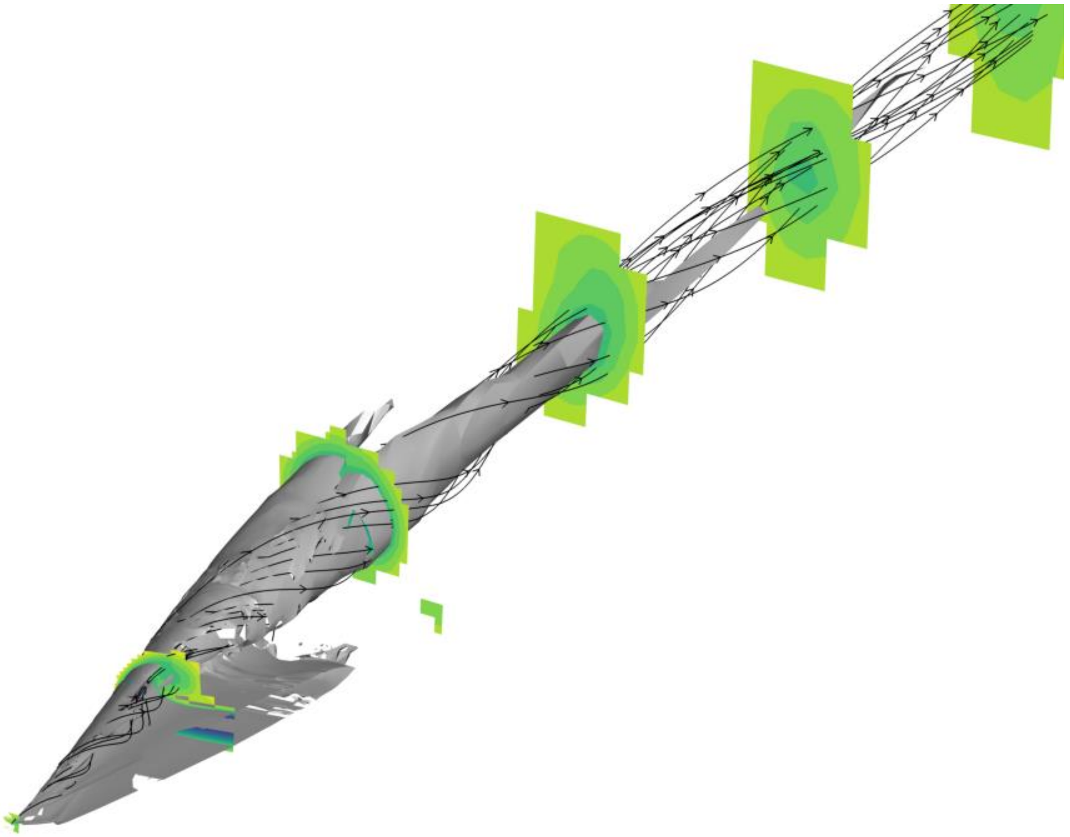


NUMERICAL RESULTS

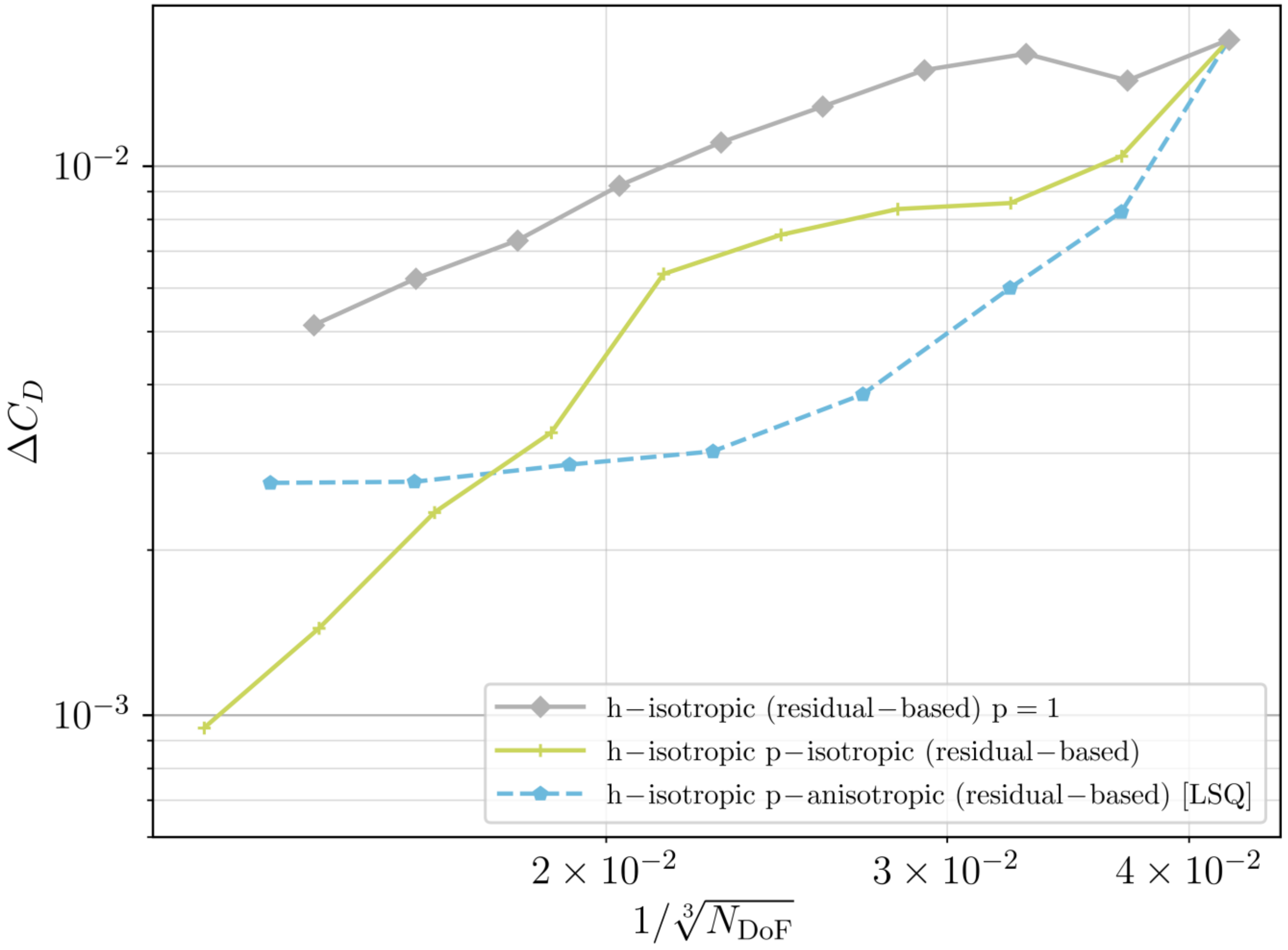
Laminar delta wing

- Laminar flow at
  - Ma: 0.3
  - Re: 4000
  - α: 12.5°
- Structured hexahedral grid with degenerate hexahedrons subdivided into regular element types
- Has already been part of the European ADIGMA project, thus reliable reference results are available



Laminar delta wing

- hp-adaptation outperforms pure h-adaptation
- In early stages p-anisotropic outperforms p-isotropic
- p-anisotropic adaptation stalls



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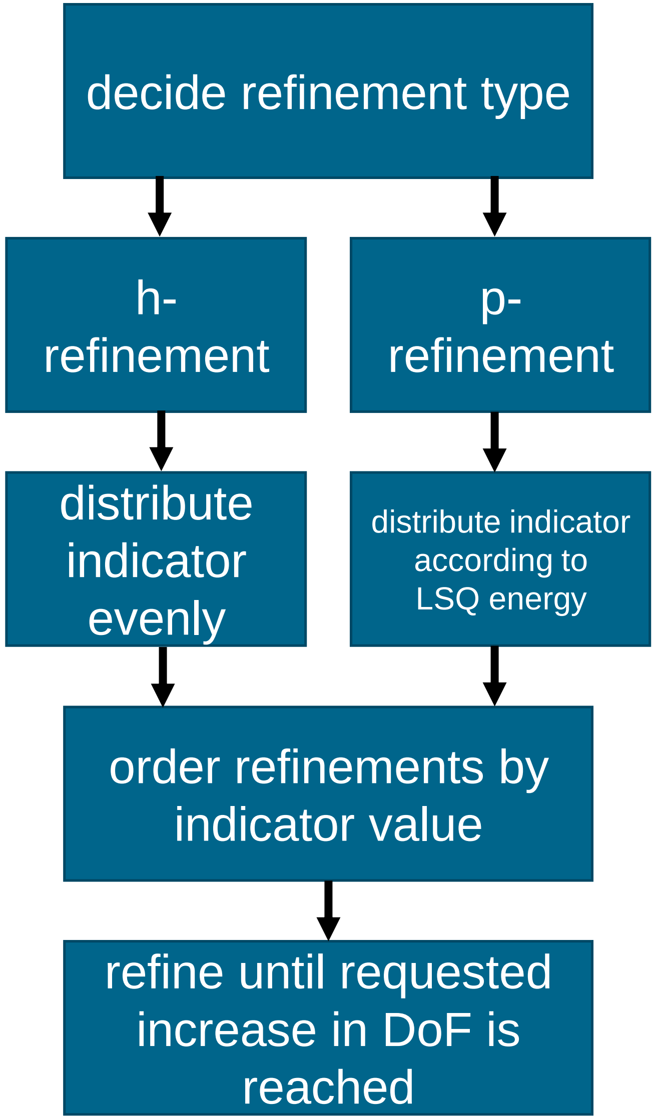
Adaptation strategies

p-anisotropic h-isotropic combined

- Each element is either flagged for p-refinement or h-refinement according to the **exponential decay indicator**
- Indicator is split between different candidate functions according to their energy
- Elements are refined until the requested increase in DoF is reached



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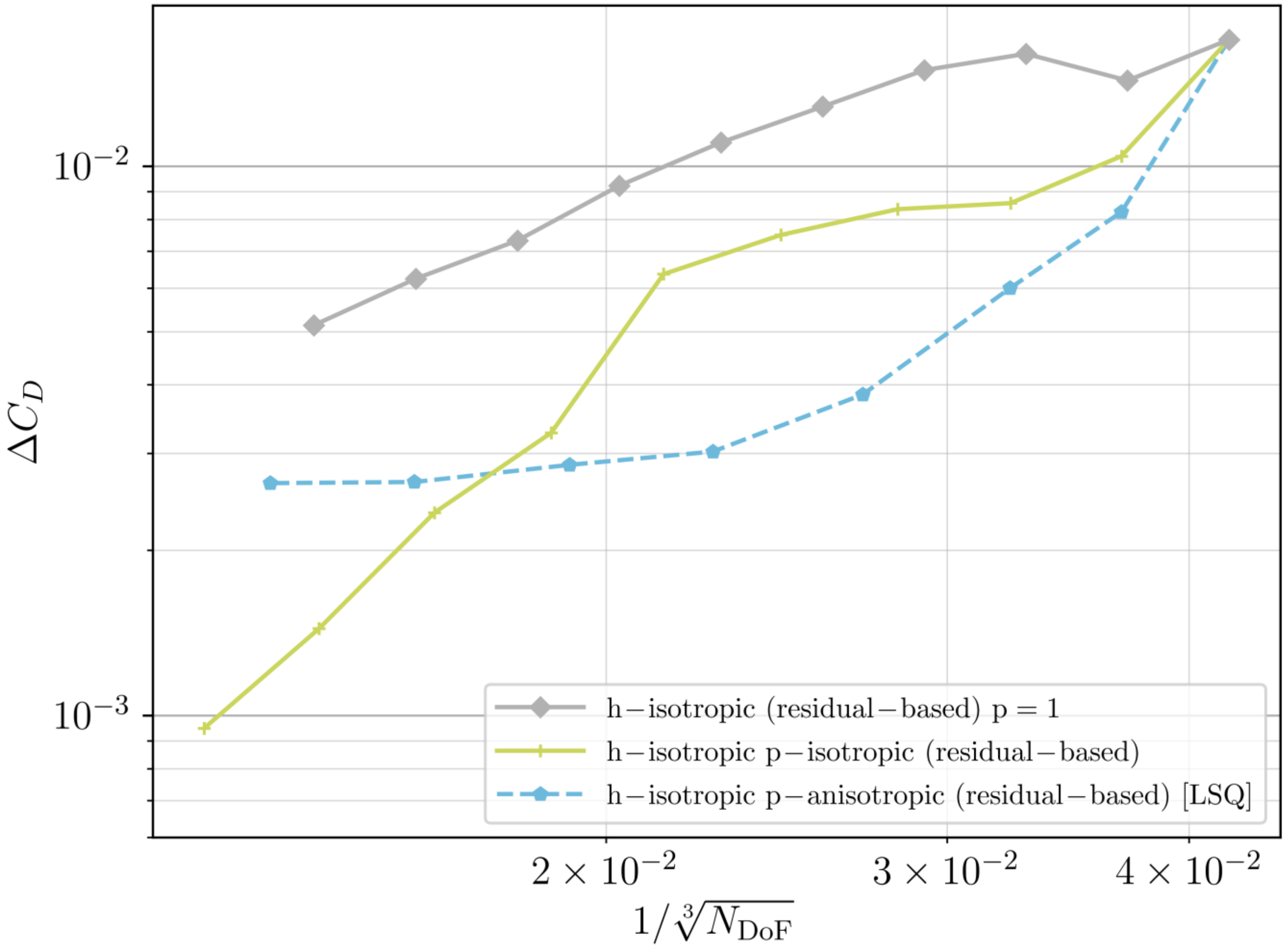
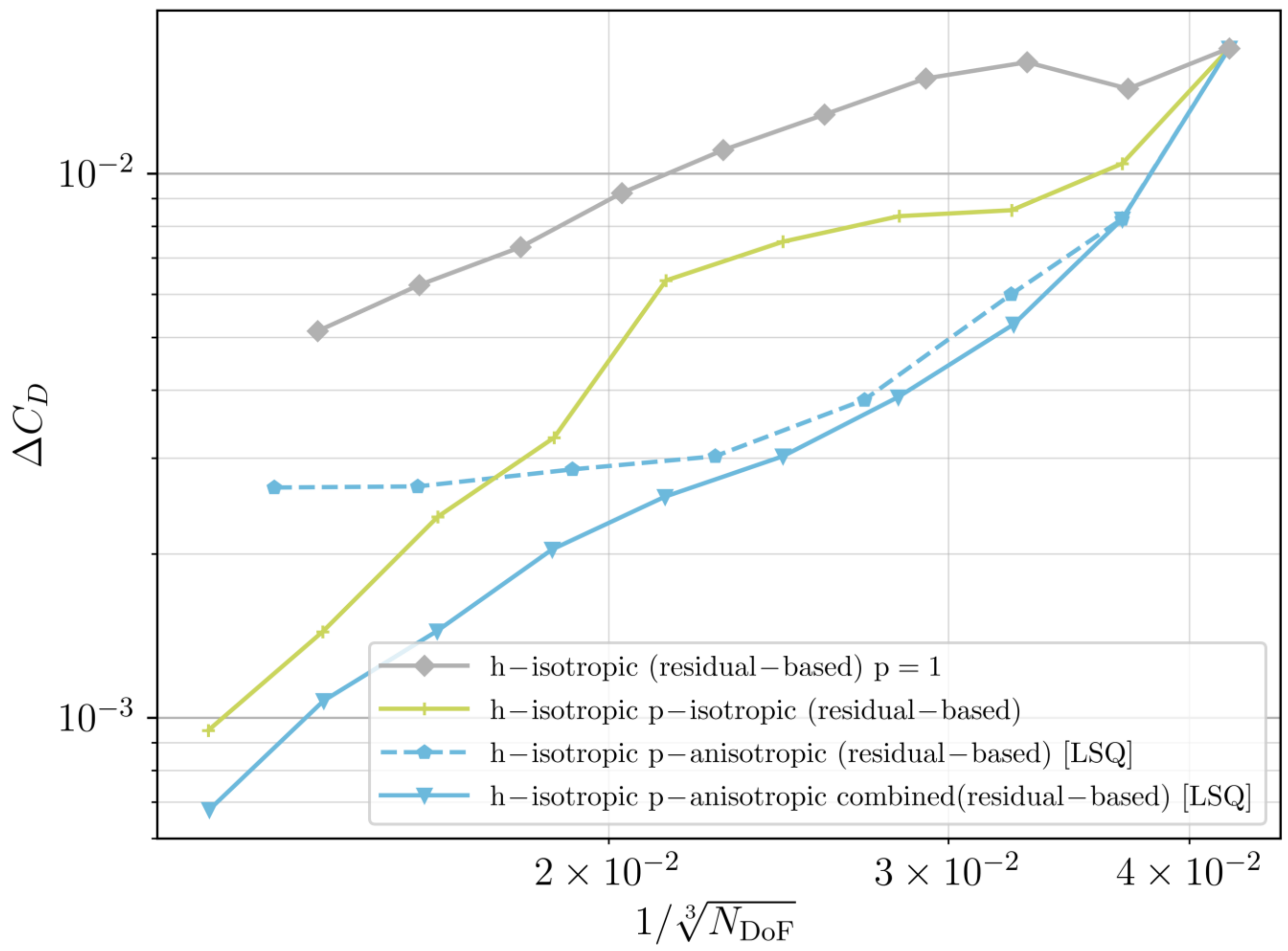
Laminar delta wing



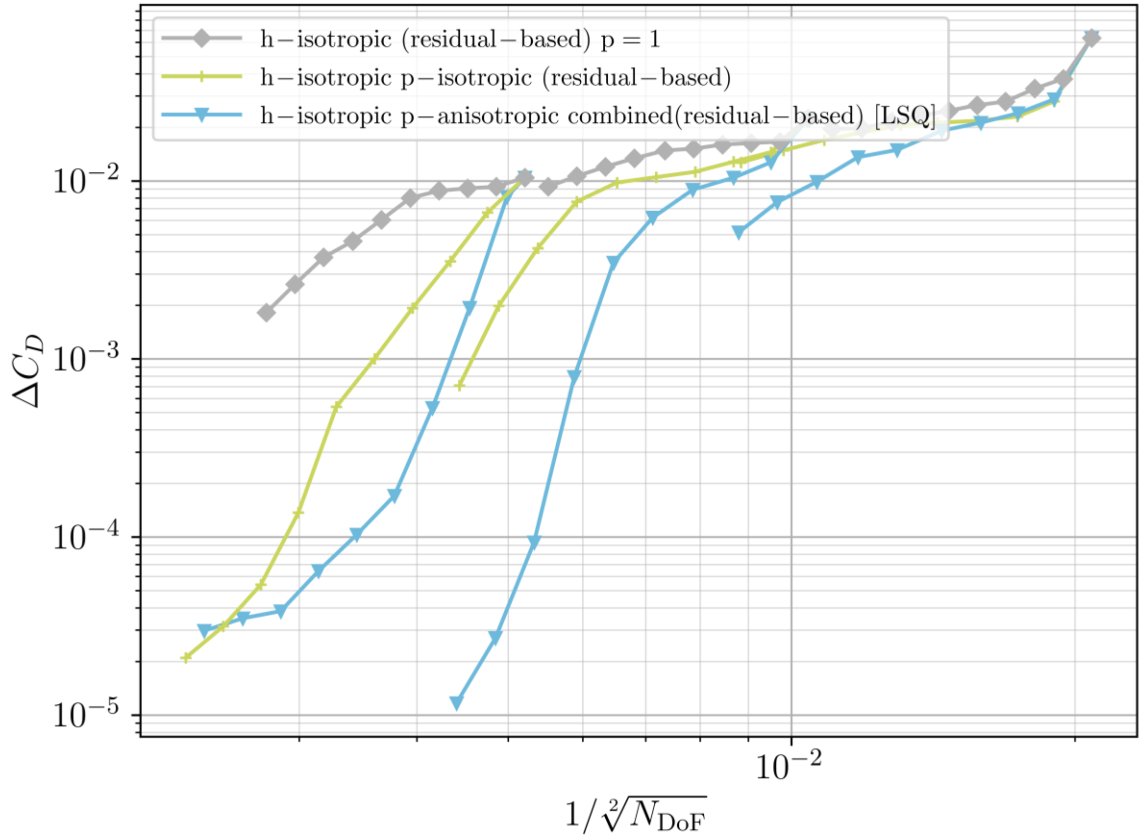
Laminar delta wing

- New refinement strategy keeps benefits from early stages
- Can keep the advantage and converges to the correct value



Juskowski airfoil

- Part of HiFiCFD 2024
- RANS-SA negative model with the QCR2000 modification
  - Ma: 0.15
  - Re: 6,000,000
  - α: 0°
- Comparison to the reference drag from the SANS solver
- hp-adaptive simulations outperform h-adaptive simulations in terms of accuracy per DoF
- p-anisotropic adaptation outperforms p-isotropic adaptation



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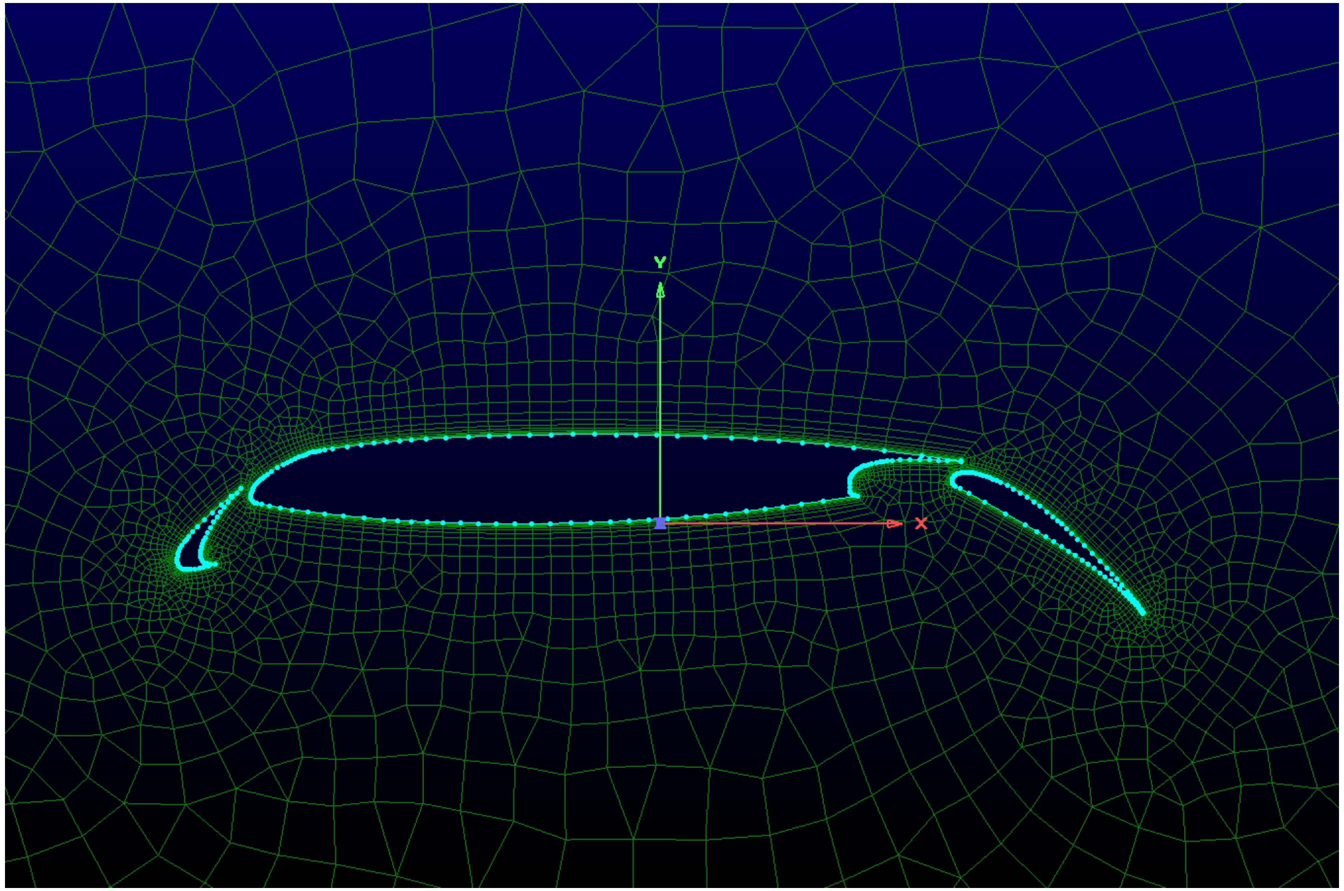
2D Multi-Element airfoil

- HLPW4, Case 1
  - 3 element high-lift airfoil
- RANS-SA negative model
  - Ma: 0.2
  - Re: 5,000,000
  - α: 16°
- Initial mesh has 6645 elements
- Each iteration increases DoF by a factor of 1.2 each iteration



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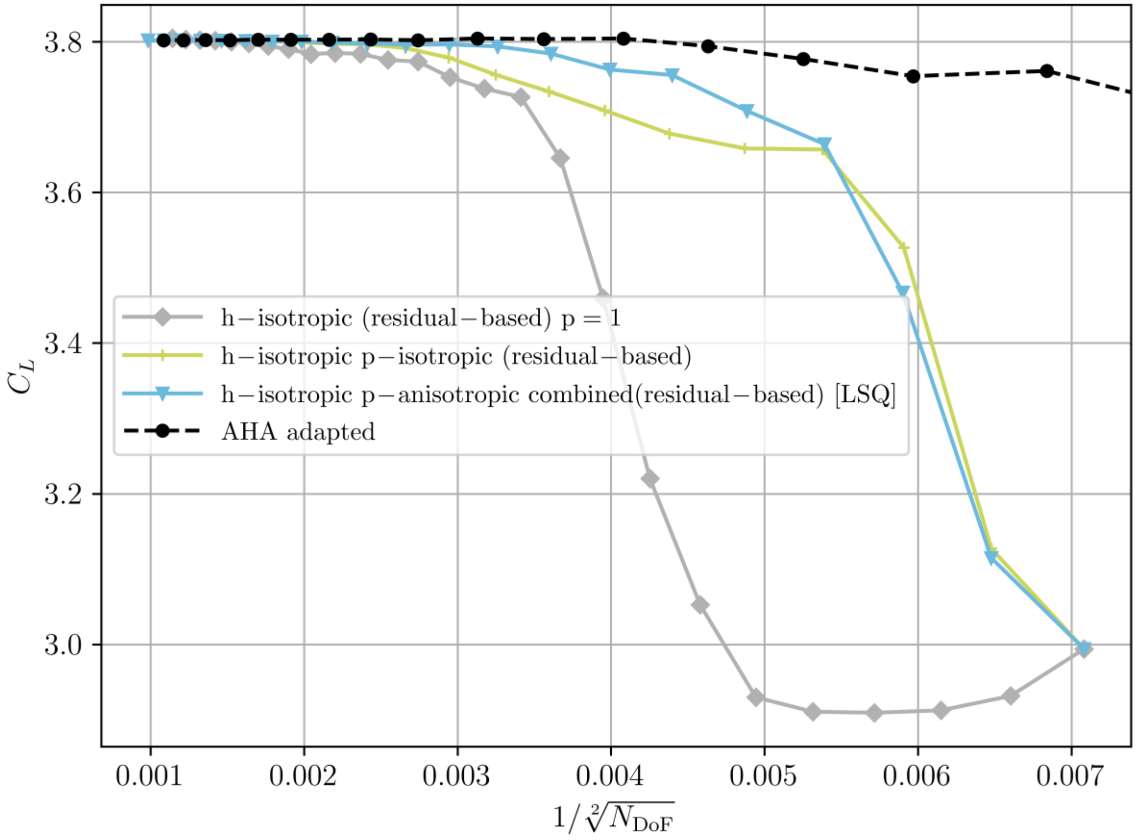
2D Multi-Element airfoil

- hp-adaptive simulations outperform h-adaptive simulations in terms of accuracy per DoF
- p-anisotropic adaptation outperforms p-isotropic adaptation after the 3rd adaptation
- All methods converge to the reference solution



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2D Multi-Element airfoil

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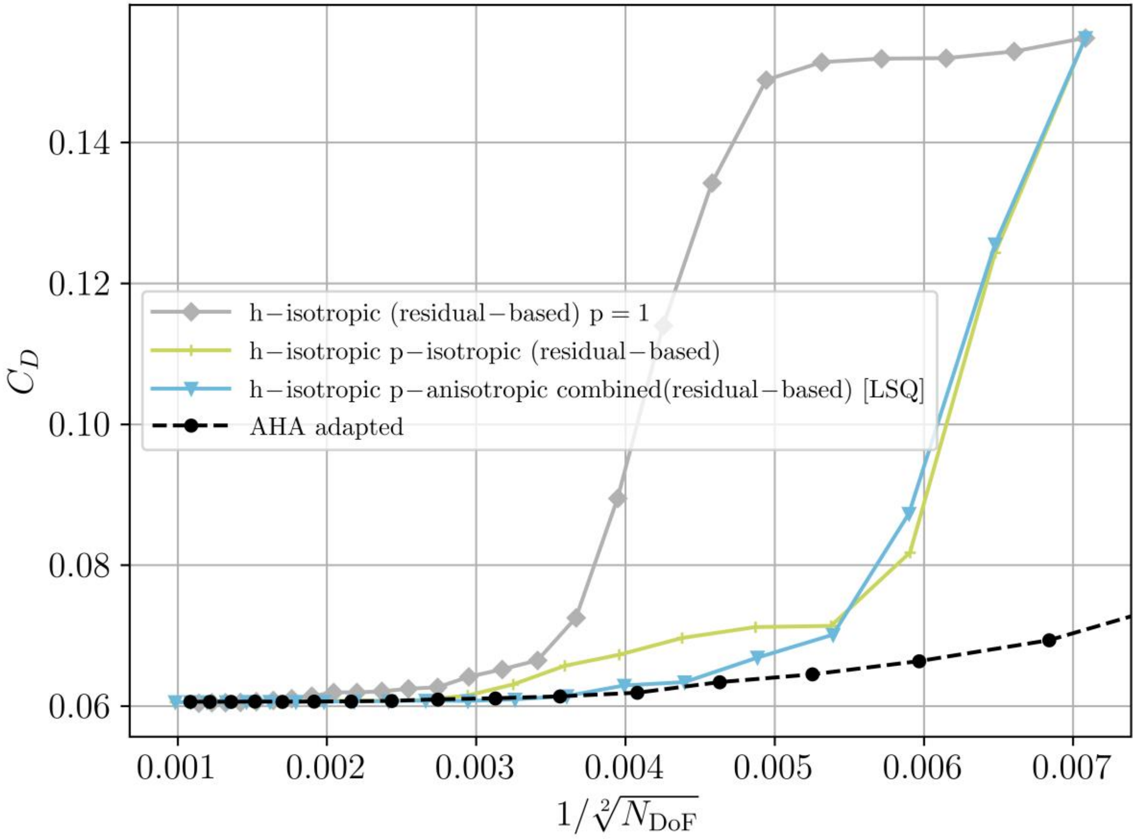

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Conclusion and Outlook

Conclusion

- The combined formulation for refinement is necessary to take advantage of p-anisotropic refinement
- The p-anisotropic refinement strategy outperforms the p-isotropic refinement

Outlook

- Application to a more complex 3D flow
- Compare the effectiveness of a LSQ driven anisotropic adaptation to an adjoint based anisotropic adaptation

Acknowledgements

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