Lunar Laser Ranging for Testing Relativity and Studying the Earth-Moon System

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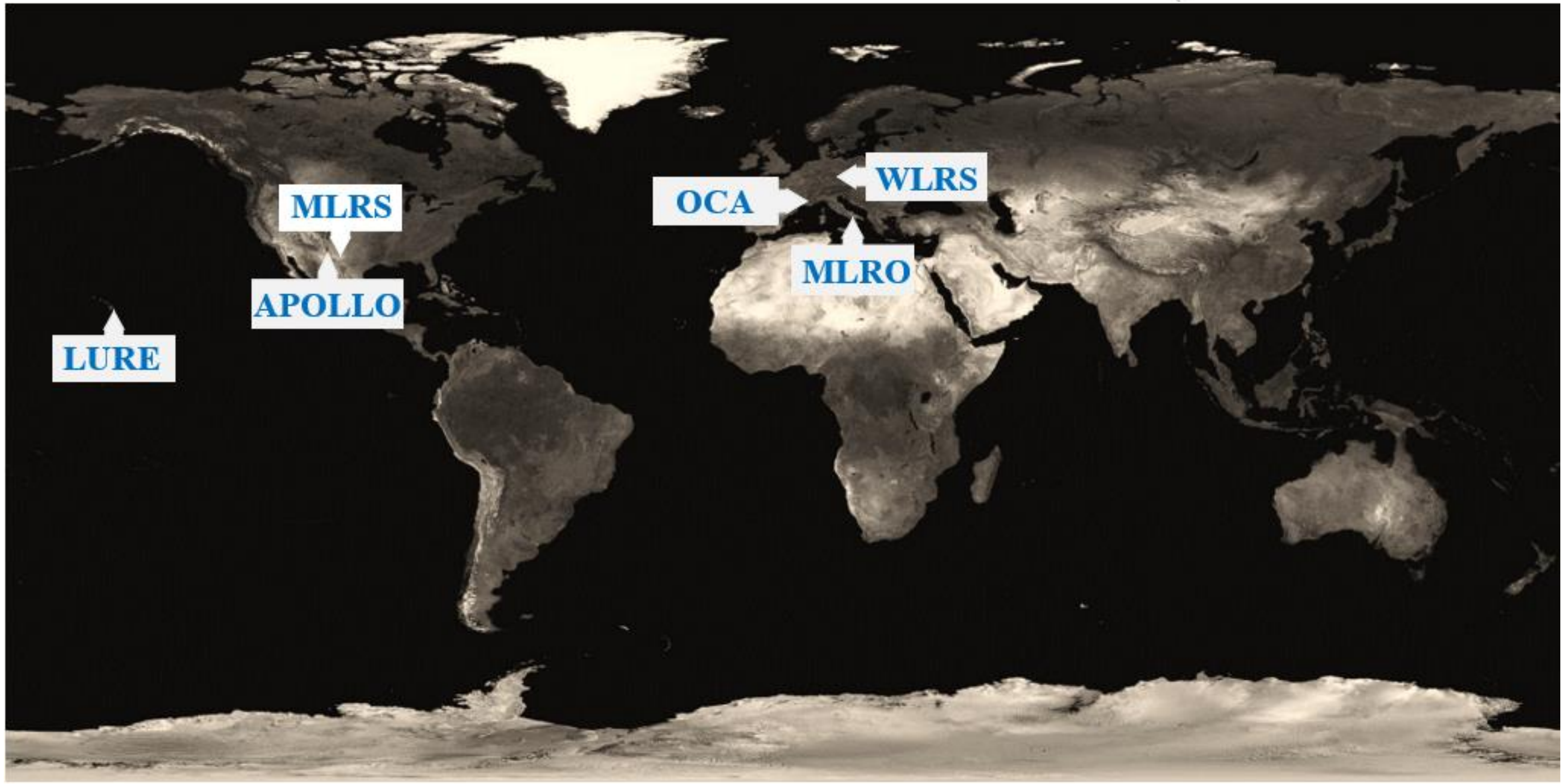
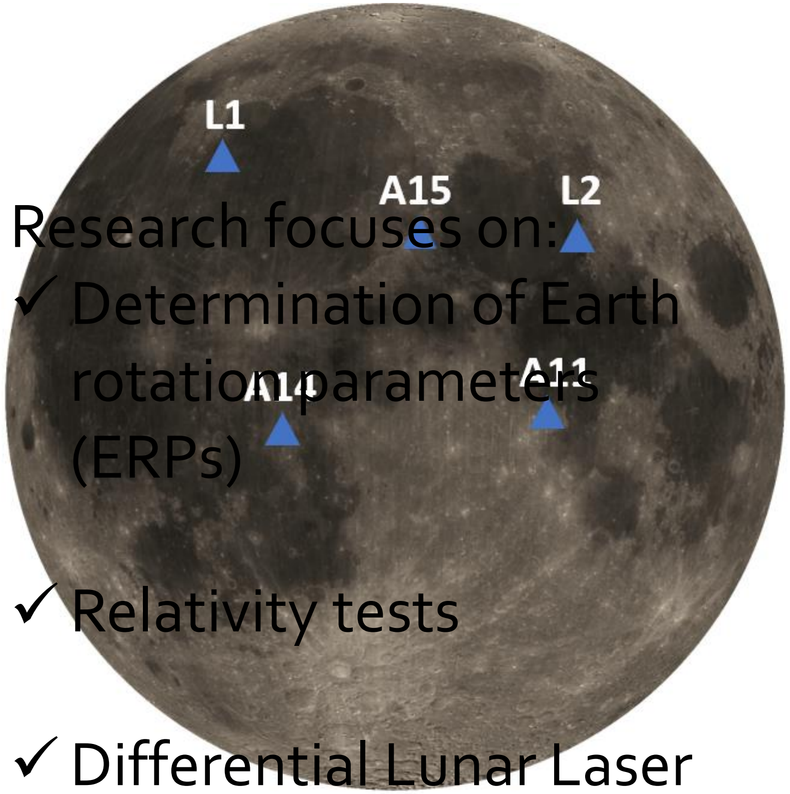
Principle of Lunar Laser Ranging (LLR)



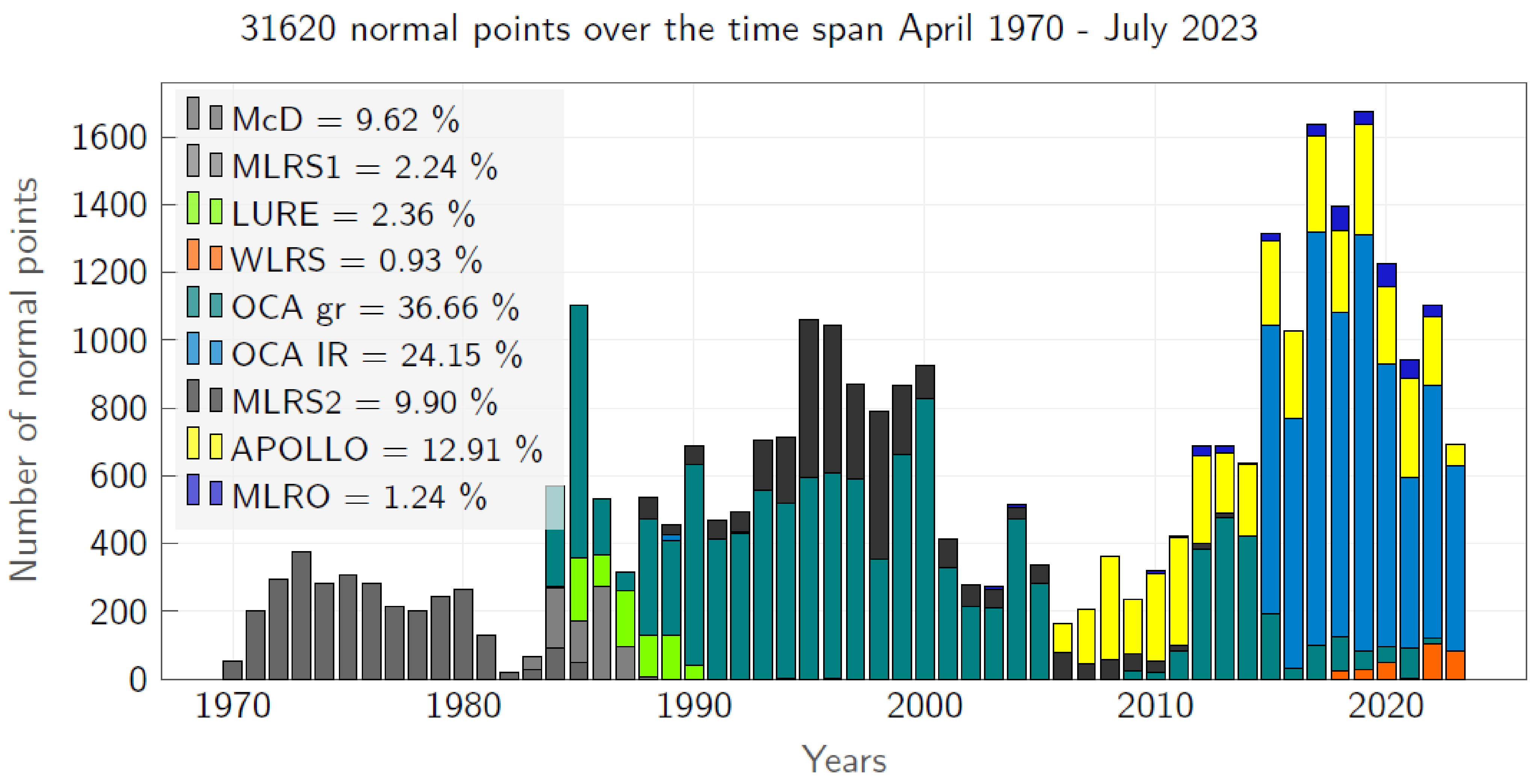


Research focuses on:

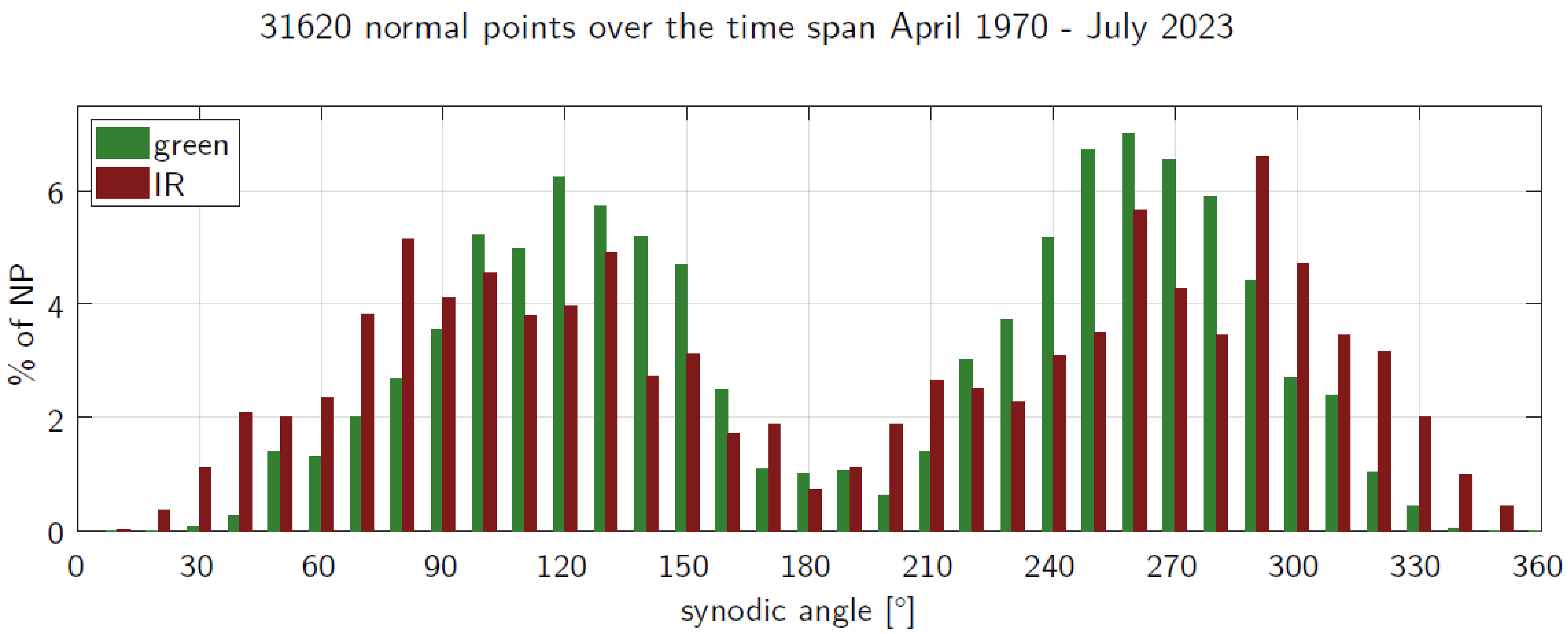
- ✓ Determination of Earth rotation parameters (ERPs)
- ✓ Relativity tests
- ✓ Differential Lunar Laser Ranging (DLLR)

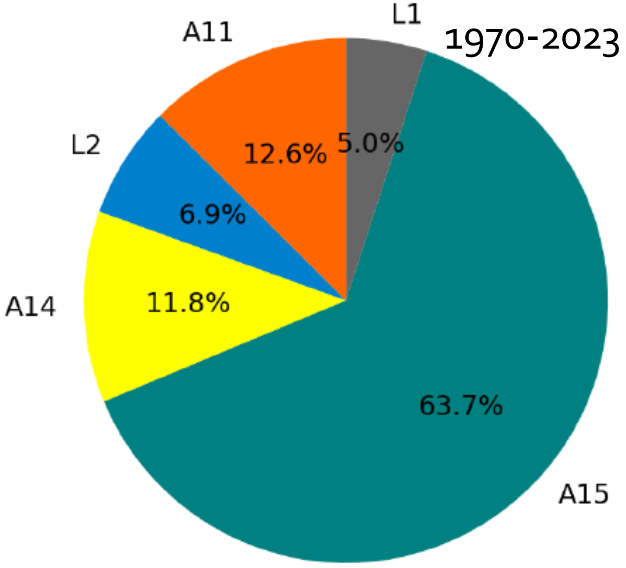
Earth Rotation Parameters

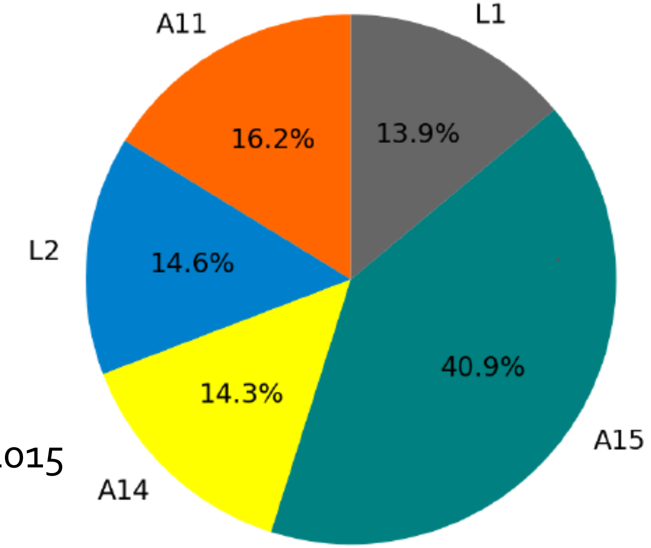
Normal Points (NPs) over 53 Years



Better NP Distribution with IR NPs







after 2015

- ✓ better distribution over lunar orbit
- ✓ better distribution over reflectors
- ✓ More NPs per night

Earth Rotation Parameters (ERPs) Estimation

✓ Connection between ERPs and LLR

$$\mathbf{r}_{\mathrm{GCRS}} = \mathbf{Q}(\mathrm{t})\,\boxed{\mathbf{R}(\mathrm{t})}\,\boxed{\mathbf{W}(\mathrm{t})}\,\mathbf{r}_{\mathrm{ITRS}}$$

$\Delta UT1$ (x_p, y_p)

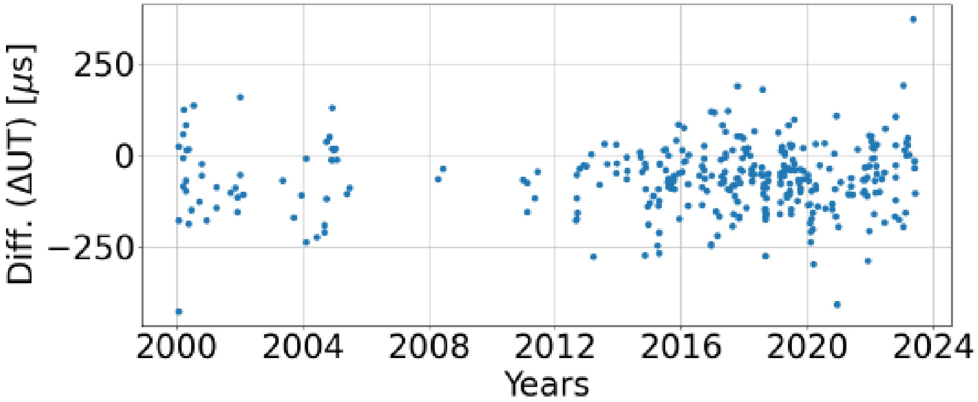
✓ LLR NPs selection

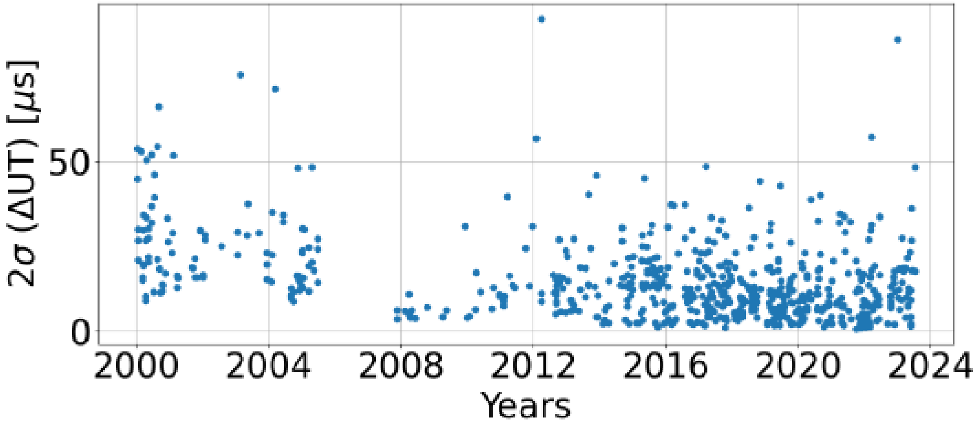
- all observatories
- min. 15 NPs per night, 519 nights
- 04.1984 - 05.2023

✓ LLR analysis

- until 1983: Kalman Earth Orientation Filter (KEOF) series COMB2019;
  from 1983: IERS C04 EOP series
- ERPs fixed for nights not considered in the fit
- standard parameters of Earth-Moon system with either $\Delta UT1$, x_p or y_p

ΔUT1 differences to the a-priori IERS C04 EOP series

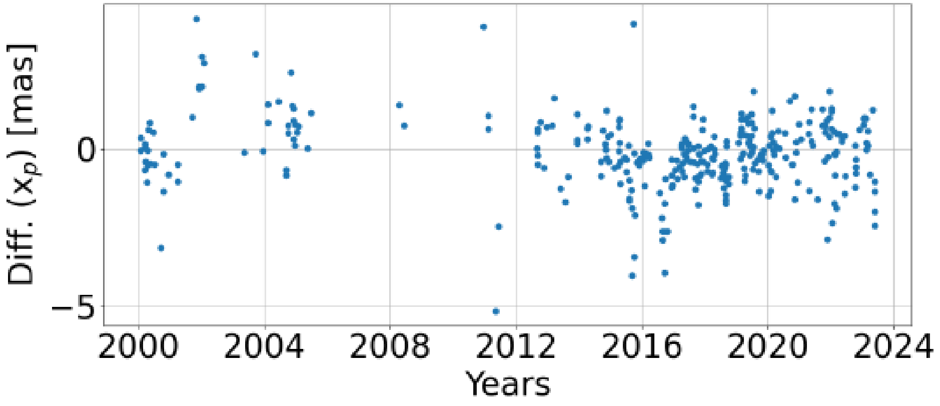




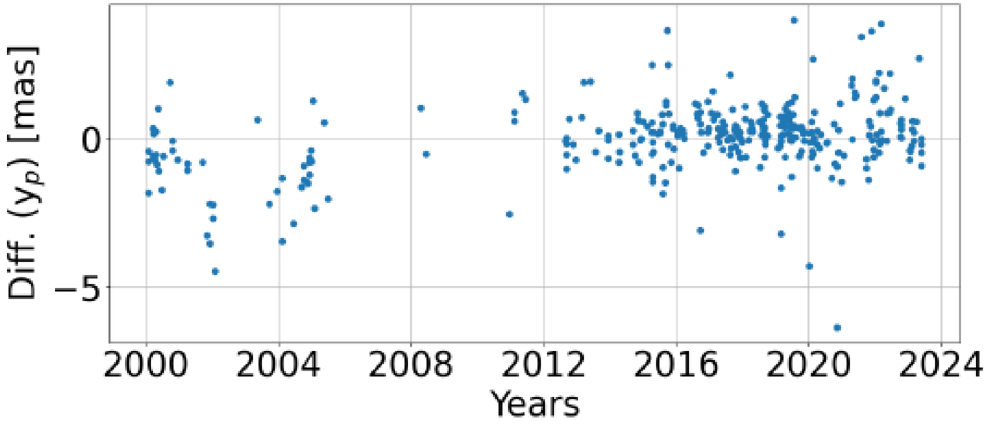
2×ΔUT1 uncertainties with wrms of 13.67 μs

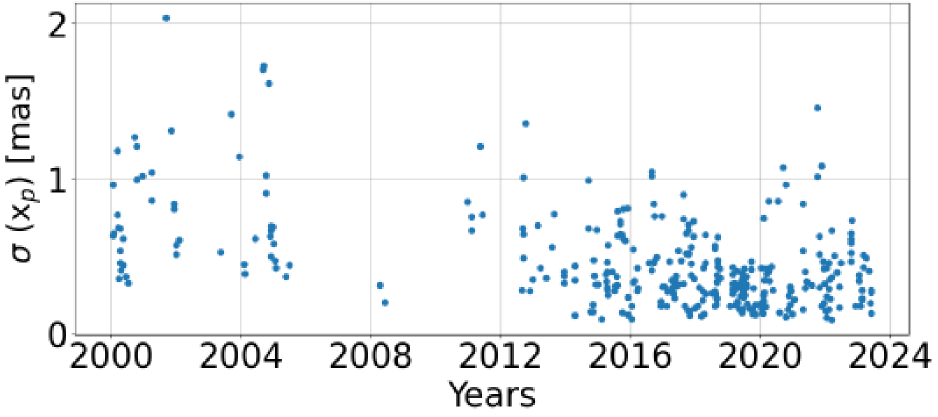
Earth Rotation Parameters (ERPs) Estimation

x_p differences to the a-priori IERS C04 EOP series

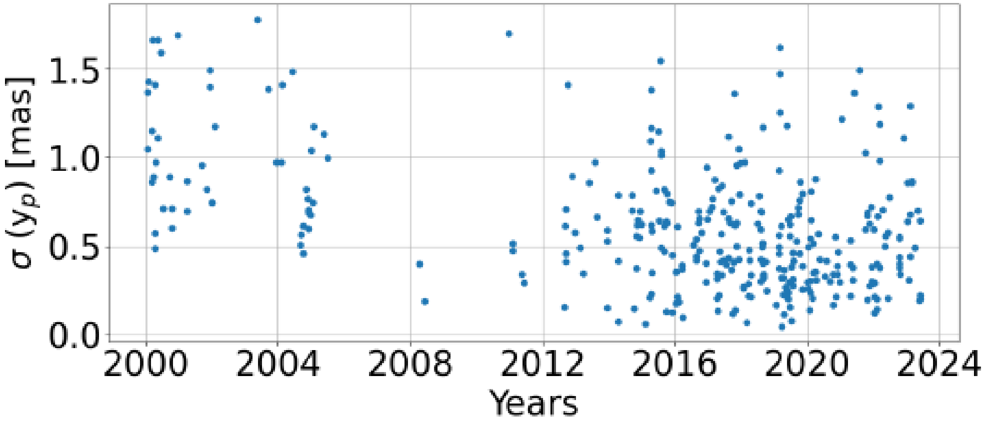


y_p differences to the a-priori IERS C04 EOP series





x_p uncertainties with wrms of 0.52 mas



y_p uncertainties with wrms of 0.66 mas

Relativity Tests

Relativity Tests

- ✓ Equivalence of inertial and gravitational mass
  - Earth and Moon as test bodies
  - in the gravitational field of the Sun

    $\Delta\left(\frac{m_g}{m_i}\right)_{EM}$: difference between the ratio $\frac{m_g}{m_i}$ for the Earth and Moon
  - in the gravitational field of galactic dark matter

    Δa_{EM}: additional acceleration of the Moon relative to the Earth towards the galactic center

- ✓ Equivalence of active and passive gravitational mass

  $\Delta\left(\frac{m_a}{m_p}\right)_{Al,Fe}$: difference between the ratio $\frac{m_a}{m_p}$ for Al and Fe of the Moon

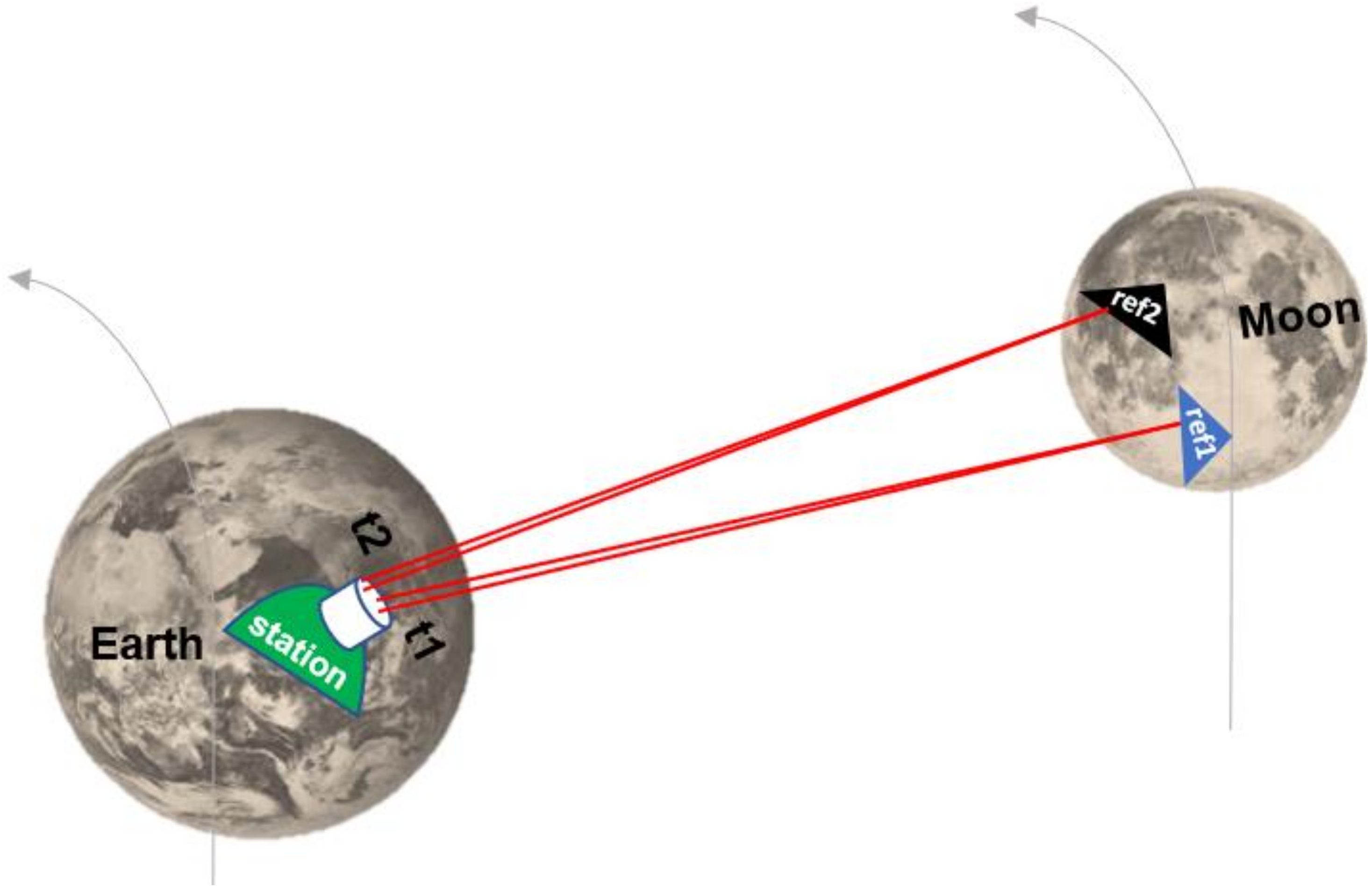
- ✓ Temporal variation of the gravitational constant G
- ✓ PPN parameters γ (space-curvature) and β (non-linearity of gravity)

Estimation Results for Relativity Tests

Parameter	Estimation result	Data time span	Data number	Source
$\frac{\dot{G}}{G}$	$(-5.0 \pm 9.6) \cdot 10^{-15}\ yr^{-1}$	04.1970-04.2020	27485	Biskupek et al. (2021)
$\gamma - 1$	$(1.7 \pm 1.6) \cdot 10^{-4}$			
$\beta - 1$	$(6.2 \pm 7.2) \cdot 10^{-5}$			
$\Delta\left(\frac{m_g}{m_i}\right)_{EM}$	$(-0.6 \pm 2.1) \cdot 10^{-14}$	04.1970-03.2021	28093	Zhang et al. (2022)
Δa_{EM}	$(2.3 \pm 4.1) \cdot 10^{-17}\ \mathrm{m}.s^{-2}$			
$\Delta\left(\frac{m_a}{m_p}\right)_{Al,Fe}$	$\pm 3.9 \cdot 10^{-14}$	04.1970-04.2022	30172	Singh et al. (2023)

Differential Lunar Laser Ranging

Principle of Differential Lunar Laser Ranging (DLLR)



- ✓ one station – two reflectors
- ✓ short switching interval
- ✓ range difference
- ✓ expected accuracy ~30 µm
- ✓ atmospheric error, significantly reduced

Characteristics of Differential Lunar Laser Ranging

LLR and DLLR data:

- ✓ same number (28093)
- ✓ same time span (04.1970-03.2021)
- ✓ same observatories and reflectors
- ✓ same data distribution
- ✓ DLLR data accuracy

  increased 200 times compared to real LLR
- ✓ DLLR switching interval (1.5 min)

Parameter	LLR	DLLR
θ_m [rad]	4.4×10^{-9}	1.4×10^{-11}
ω_{mz0} [rad/s]	2.1×10^{-16}	0.6×10^{-18}
kvc [rad/d]	1.1×10^{-10}	0.8×10^{-12}
f_c	1.6×10^{-6}	0.6×10^{-8}
X_{m0} [km]	8.5×10^{-5}	2.3×10^{-5}
Y_{m0}	3.0×10^{-5}	1.9×10^{-5}
Z_{m0}	2.9×10^{-5}	1.1×10^{-5}

Estimation of lunar orientation, rotation and interior largely improved

- ✓ inclination between XY-plane of inertial frame and lunar mantle equator θ_m
- ✓ initial z-component of angular velocity of lunar mantle ω_{mz0}
- ✓ friction coefficient between the core and mantle kvc
- ✓ oblateness of the lunar core f_c
- ✓ Initial position of lunar mass center (X_{m0}, Y_{m0}, Z_{m0})

LLR and DLLR Combination

Ratio of estimation accuracy of LLR as well as DLLR&LLR for parameters related to lunar orientation, rotation and interior

Case	Data timespan	Reflector/Reflector baseline (RB)	Parameter accuracy ratio Acc_LLR/Acc_DLLR&LLR			
			θ_m	ω_{mz0}	kvc	f_c
LLR	04.1970-03.2021 (real LLR)+ 04.2021-03.2026 (simulated LLR)	A11, A14, A15, L1, L2	8.9	48.3	58.6	107.0
LLR&DLLR	LLR+04.2021-03.2026 (simulated DLLR)	LLR: 5 reflectors DLLR: RBs from random combination of 5 reflectors				

Only 5-year DLLR data added, parameter estimation related to lunar orientation, rotation and interior improved a lot, especially the oblateness of lunar core f_c improved by 107-fold

Conclusions and Outlook

- ✓ ERPs benefit greatly from improved LLR data, especially from IR NPs with high number of NPs per night and better distribution

- ✓ LLR can conduct many different relativity tests
  - ➢ Equivalence Principle of inertial and gravitational mass
  - ➢ Equivalence Principle of active and passive gravitational mass
  - ➢ Temporal variation of the gravitational constant
  - ➢ PPN parameters

- ✓ DLLR would significantly improve parameter estimation related to the lunar orientation, rotation and interior, if really achieving the expected high accuracy

- ✓ Combination of LLR and DLLR is more practical to benefit from the DLLR advantages when DLLR data time span is short

- ✓ Combination of LLR and DLLR for the investigations of ERPs and relativity tests

References and Acknowledgements

Biskupek, L., Müller, J., & Torre, J. M. (2021). Benefit of new high-precision LLR data for the determination of relativistic parameters. Universe, 7(2), 34. https://doi.org/10.3390/universe7020034

Biskupek, L., Singh, V.V., Müller, J., Zhang, M. (2024). Potential of Lunar Laser Ranging for the Determination of Earth Orientation Parameters. In: Freymueller, J.T., Sánchez, L. (eds) Gravity, Positioning and Reference Frames. REFAG 2022. International Association of Geodesy Symposia, vol 156. Springer, Cham. https://doi.org/10.1007/1345_2024_238

Singh, V. V., Müller, J., Biskupek, L., Hackmann, E., & Lämmerzahl, C. (2023). Equivalence of Active and Passive Gravitational Mass Tested with Lunar Laser Ranging. Physical review letters, 131(2), 021401. https://doi.org/10.48550/ARXIV.2212.09407.

Zhang, M., Müller, J., Biskupek, L., & Singh, V. V. (2022). Characteristics of differential lunar laser ranging. Astronomy & Astrophysics, 659, A148. https://doi.org/10.1051/0004-6361/202142841

Zhang, M. (2023). Characteristics and Benefits of Differential Lunar Laser Ranging (Doctoral dissertation, Fachrichtung Geodäsie und Geoinformatik der Leibniz Universität Hannover).

Zhang, M., Müller, J., & Biskupek, L. (2024). Advantages of combining Lunar Laser Ranging and Differential Lunar Laser Ranging. Astronomy & Astrophysics, 681, A5. https://doi.org/10.1051/0004-6361/202347643

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Thank you

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