THE TRANSPORT OF ENERGETIC PARTICLES THROUGH THE EARTH'S MAGNETOSPHERE AND ATMOSPHERE

International Space Weather Camp 2024

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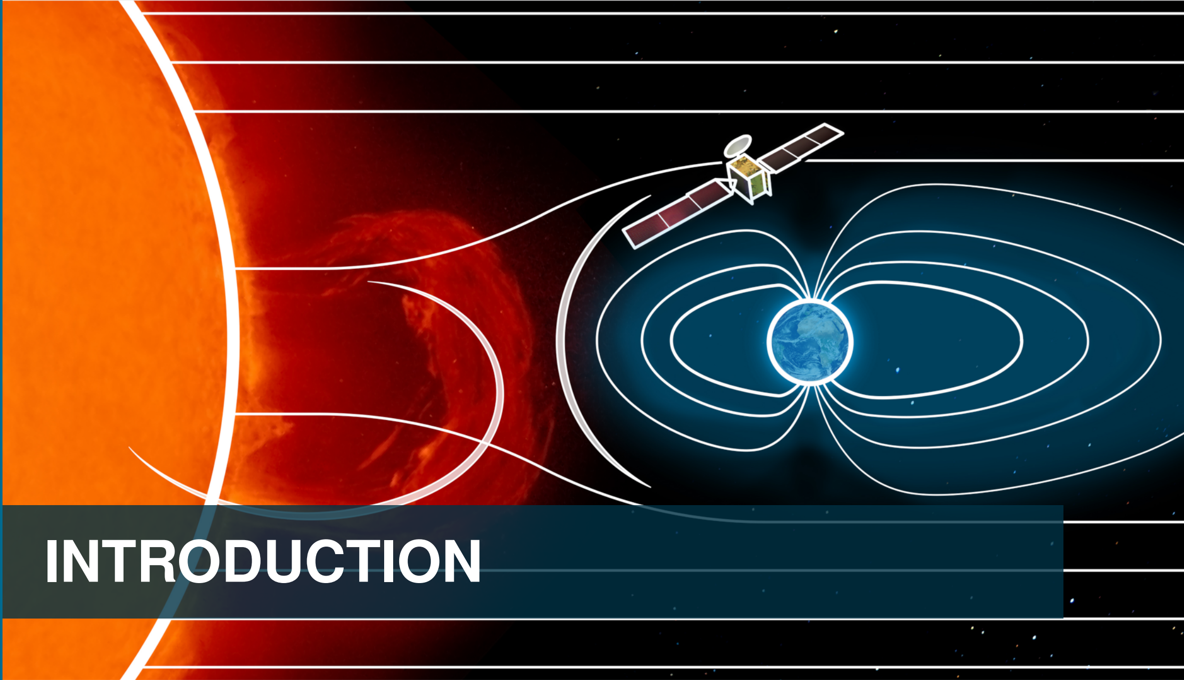
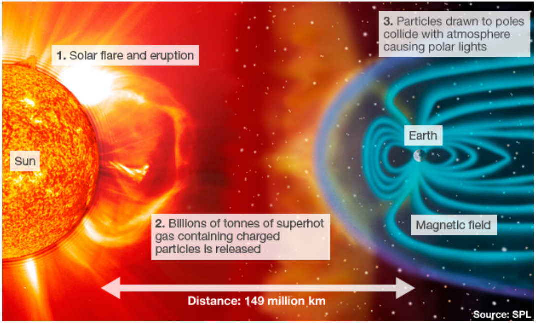
Outline



INTRODUCTION

Particles in the environment of Earth



How solar activity affects the Earth

sun	activity cycle (sunspots: footpoints of magnetic loops)
loops	may change structure and accelerate particles (flare, CME)
solar wind	as background particle population + B-field (→ talks by Borries, Iochem, July 8th)
particles	move along parker spiral, may be accelerated at shock fronts (→ talks by Scherer, Thursday; Shprits, tomorrow; Martens, July 9th)
GCRs	extend the energetic spectrum (→ talk by Heber, yesterday)

Everything except us is plasma

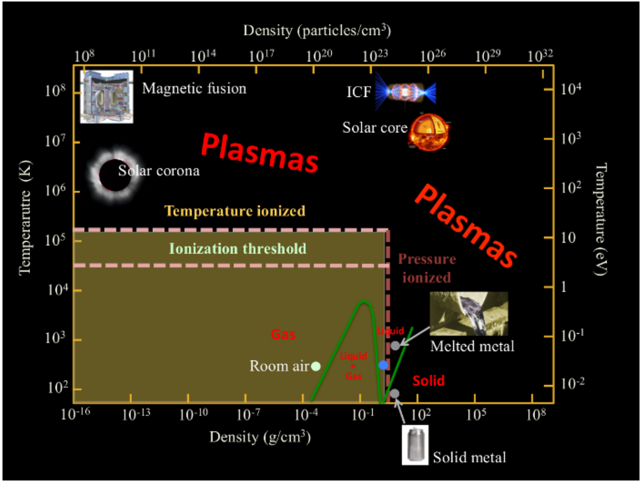
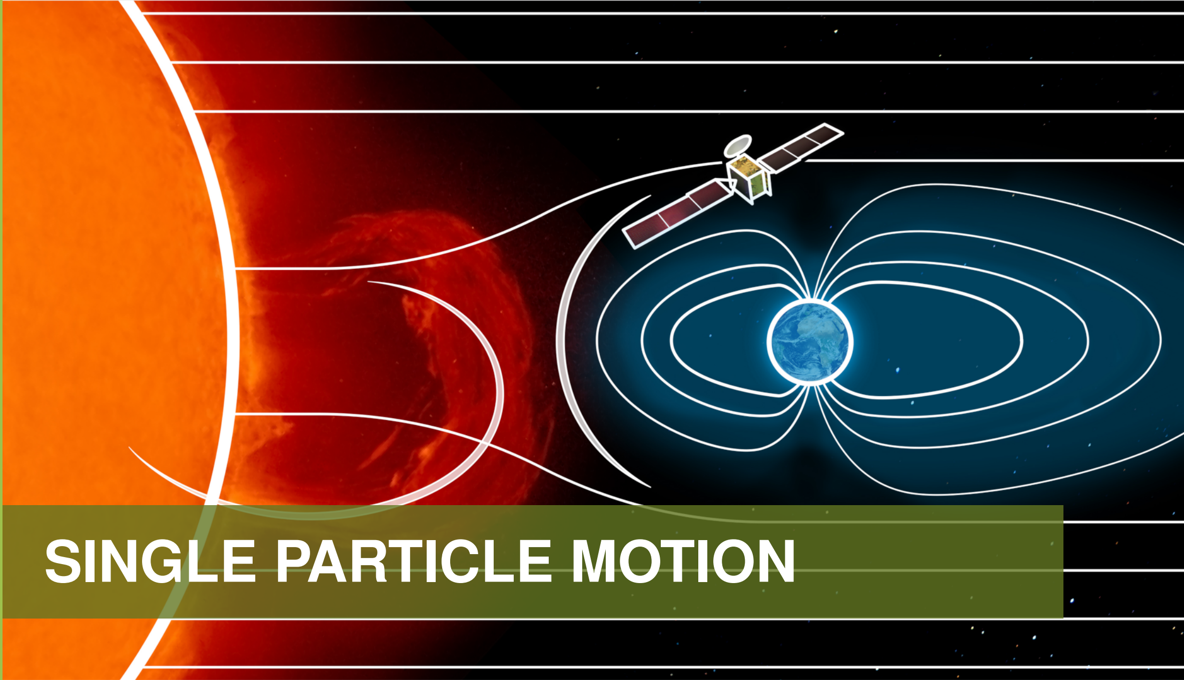


image: Los Alamos National Lab

Plasma: gas of charged particles consisting of free + and - charge carriers

- quasi-neutral
- charges are free: $E_{kin} \gg$ influence by direct neighbours. Thus plasma typically is hot (>some eV).
- abundance
  - on earth: flames, lightning, ionosphere, magnetosphere
  - space: 99%



SINGLE PARTICLE MOTION

Charged Particles in a B-Field

Lorentz-Force:

$$\vec{F}_L = q\,\vec{E} + q\,\vec{v} \times \vec{B}$$

Charged Particles in a B-Field

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as differential equation:

$$\stackrel{\wedge}{=} m\,\ddot{\vec{x}} = q\,\vec{E} + q\,\dot{\vec{x}} \times \vec{B}$$

Charged Particles in a B-Field

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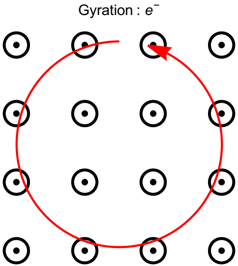
Assuming: energy density particles $\ll$ energy density Field:

$\rightarrow$ single particle movement, no impact on field

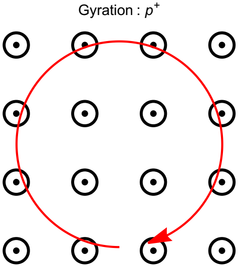
Gyration

Simple case: homogenous B-field, no electric field → Gyration

$$\vec{F}_L = \cancel{q\vec{E}} + \qquad q\,\vec{v} \times \vec{B}$$



Left-Hand rule

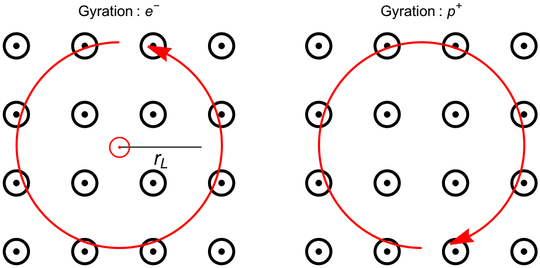


Right-Hand rule

Gyration

Simple case: homogenous B-field, no electric field → Gyration

$$\vec{F}_L = \cancel{q\vec{E}} + \underbrace{q\,\vec{v} \times \vec{B}}_{=\text{centripetal force } F_c = \frac{m\,v^2}{r}} \quad \rightarrow \quad \text{Larmor-Radius: } r_L = \frac{m\,v_\perp}{|q|\,B}$$



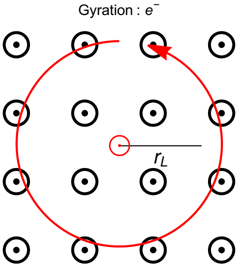
Left-Hand rule

Right-Hand rule

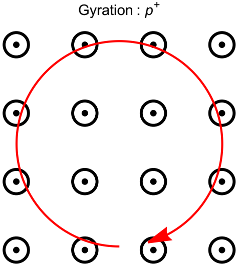
Gyration

Simple case: homogenous B-field, no electric field → Gyration

$$\vec{F}_L = \cancel{q\vec{E}}$$



Left-Hand rule



Right-Hand rule

integral of motion dissapears:

$$\begin{aligned} m\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} &= q\,\vec{v}\times\vec{B} \\ \vec{v}\cdot m\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} &= \vec{v}\cdot q\,(\vec{v}\times\vec{B}) \\ \frac{1}{2}m\frac{\mathrm{d}v^2}{\mathrm{d}t} = \frac{\mathrm{d}W_{kin}}{\mathrm{d}t} &= q\,(\vec{v}\times\vec{B})\cdot\vec{v} = 0 \end{aligned}$$

The energy during gyration is constant.

Cyclotron frequency

equation of motion in pure B-field: $m\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = q\,\vec{v} \times \vec{B}$

component-wise (assuming $\vec{B} = B_z$):

$$m\dot{v}_x = q\,B\,v_y \qquad m\dot{v}_y = -q\,B\,v_x \qquad m\dot{v}_z = 0$$

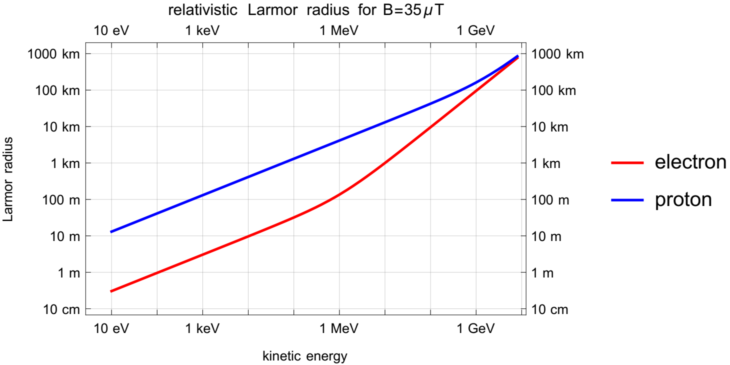
$$\ddot{v}_x = \frac{q\,B}{m}\dot{v}_y = \left(\frac{q\,B}{m}\right)^2 v_y \qquad \ddot{v}_y = -\frac{q\,B}{m}\dot{v}_x = -\left(\frac{q\,B}{m}\right)^2 v_x$$

→ harmonic oscillator: solved by $v_i = v_{0,i}e^{i\omega t}$

cyclotron frequency: $\omega_c = \frac{|q|B}{m}$

Larmor-radius: $r_L = \frac{m\,v_\perp}{|q|\,B} = \frac{v_\perp}{\omega_c}$

Relativistic Larmorradius



Space Weather deals with high velocities. Relativistic effects need to be considered.

Larmor radius (non-relativistic):

$$r_L = \frac{m\ v_\perp}{|q|\ B}$$

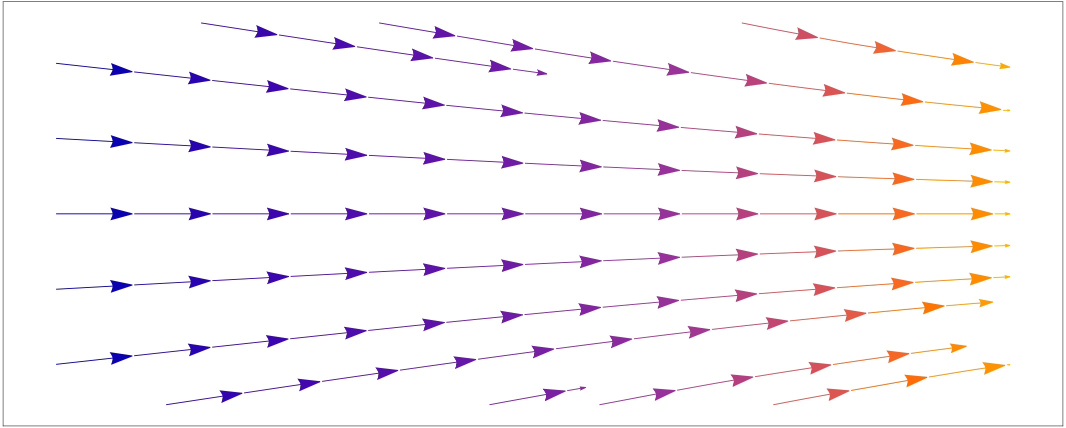
relativistic:

$$r_{L,rel} = \frac{\gamma\ m_0\ v_\perp}{|q|\ B}$$

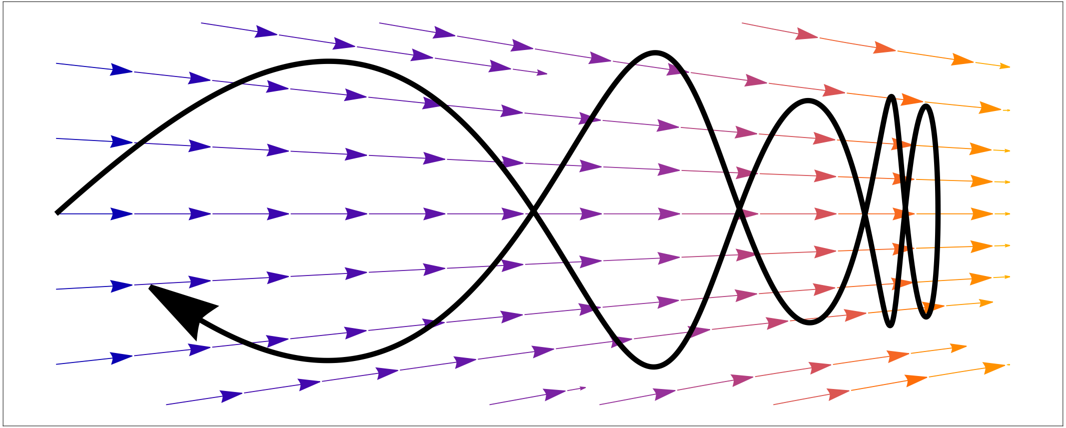
$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

$$E_{kin,rel} = (\gamma - 1) m_0\ c^2$$

Magnetic mirror



Magnetic mirror



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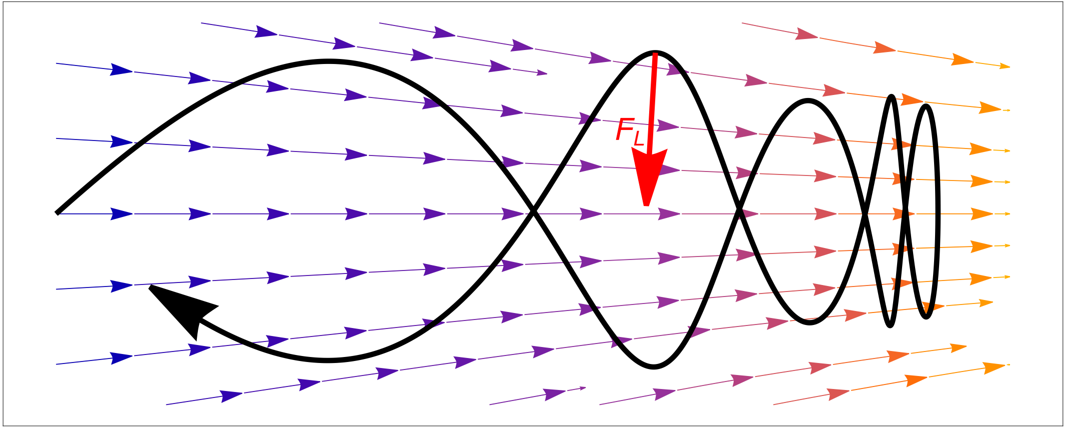
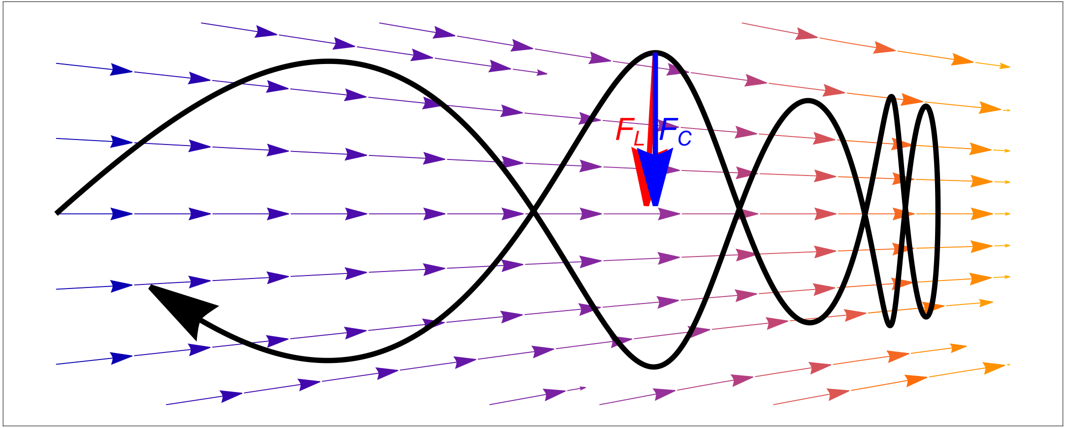
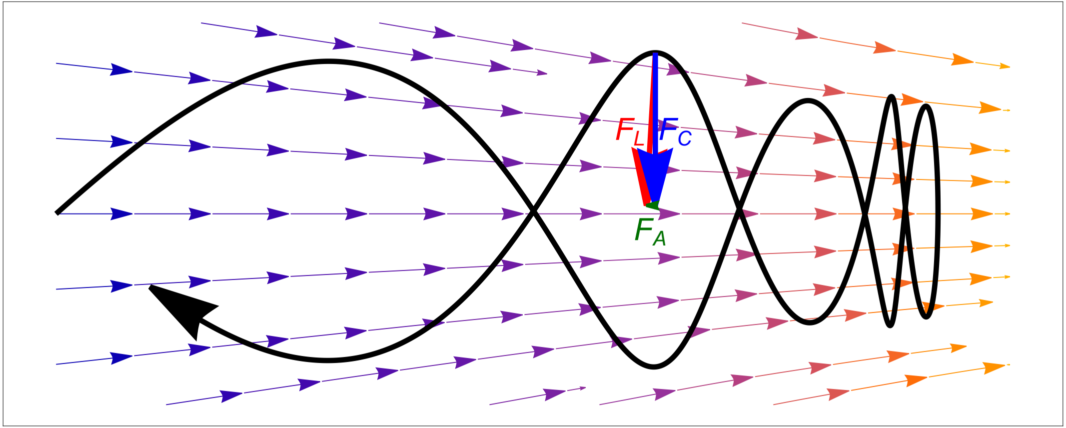


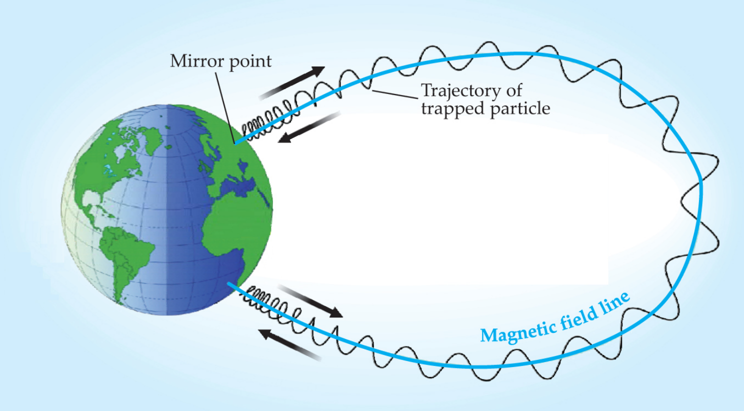
Magnetic mirror



Magnetic mirror



Magnetic mirror - Applied to the Magnetosphere

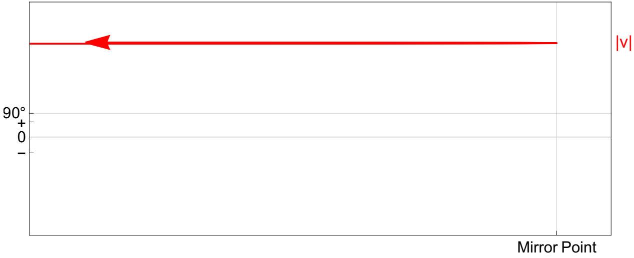


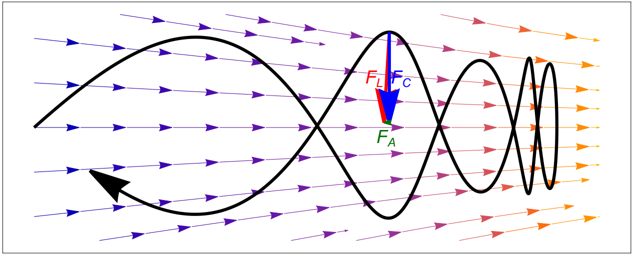
Converging field on one side: magnetic mirror → direction turn

Converging field on both sides: magnetic bottle → bounce motion

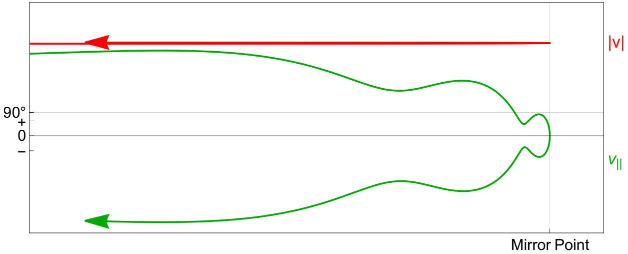
image apapted from Day, Physics Today, 61 (21) 2008

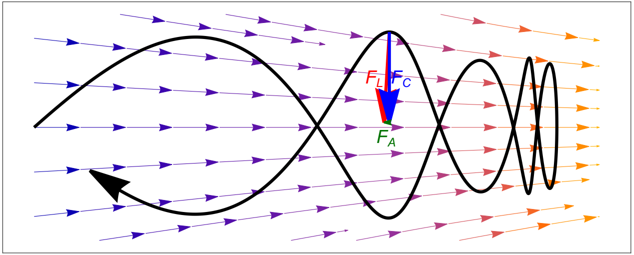
Magnetic mirror



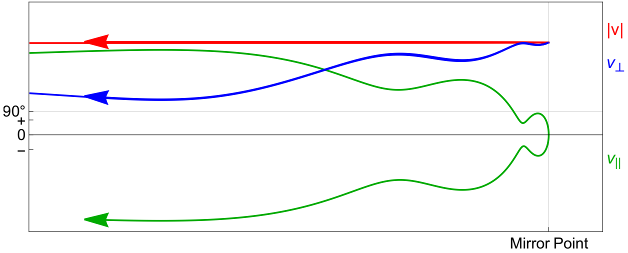


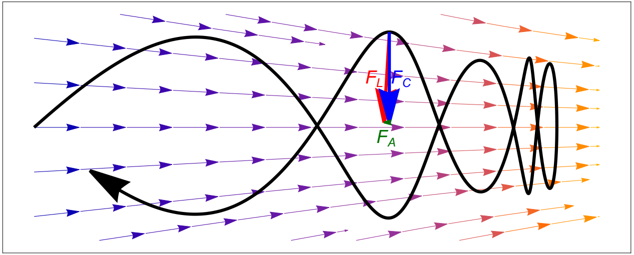
Magnetic mirror





Magnetic mirror



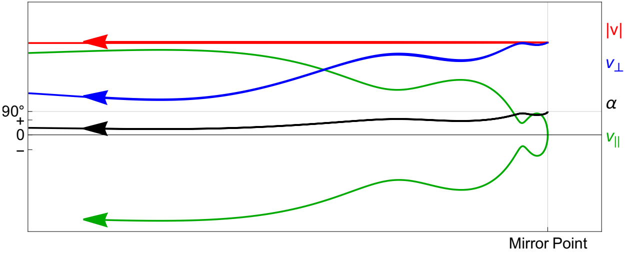


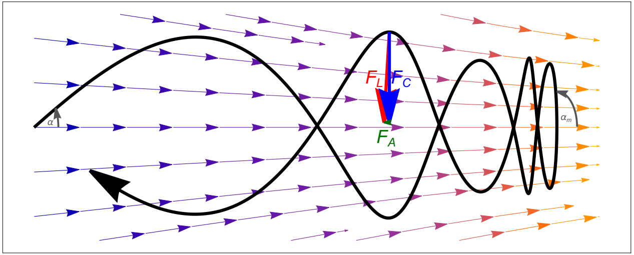
Magnetic mirror

Pitch angle:

$$\alpha = \arctan\left(\frac{v_\perp}{v_{||}}\right)$$

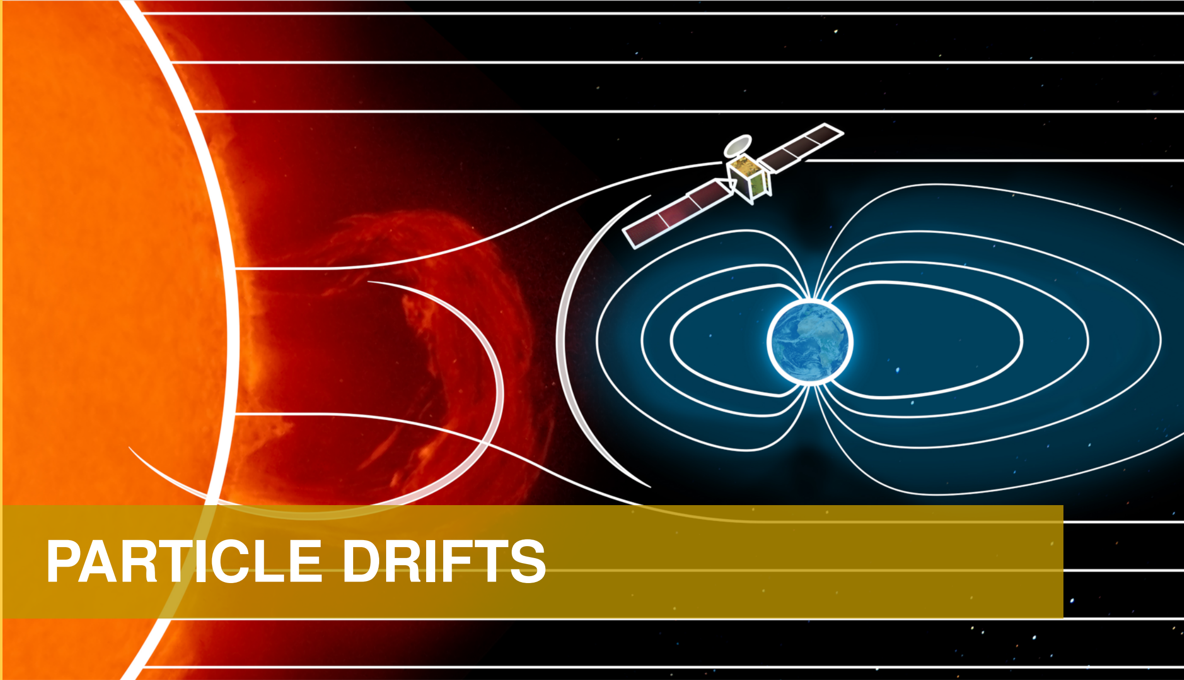
$$\sin^2(\alpha_{eq}) = \frac{B_{eq}}{B_m}$$





Basic charged particle movements:

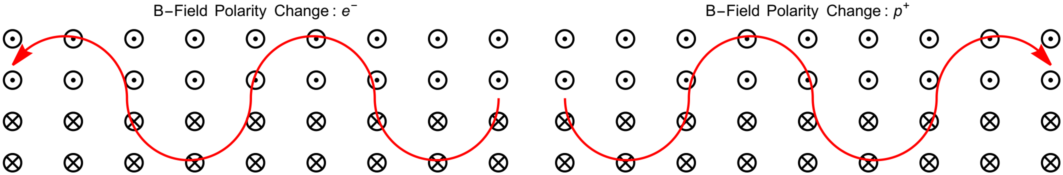
- ✓ Gyration
- ✓ Bounce
- ? Drift



PARTICLE DRIFTS

Opposite B-Fields: Neutral Sheet Current

opposite B-fields, no electric field

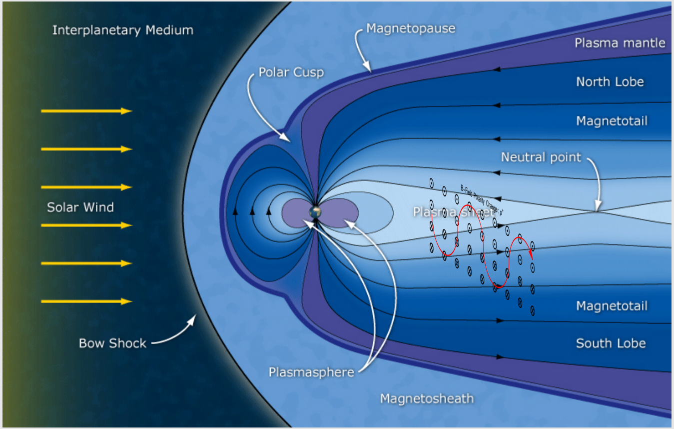


→ drift with charge separation

→ Neutral Sheet Current

Opposite B-Fields: Neutral Sheet Current

Applied to the Magnetosphere



adapted from T. Russel, ESA.

Drift theory

separate velocity components of guiding center and gyration

$$m\,\dot{v}_\parallel = F_\parallel \qquad\qquad m\,\dot{v}_\perp = F_\perp + q\,v_\perp \times \vec{B}$$

$$\begin{aligned}
v_\perp &= v_D + v_G && (1)\\
m\,\dot{v}_\perp &= m\,\dot{v}_D + m\,\dot{v}_G && (2)\\
&= F_\perp + q\,(v_D + v_G) \times \vec{B} && (3)\\
m\,\dot{v}_D + \cancel{m\,\dot{v}_G} &= F_\perp + q\,v_D \times \vec{B} + \cancel{q\,v_G \times \vec{B}} && (4)\\
m\,\dot{v}_D &= F_\perp + q\,v_D \times \vec{B} && (5)
\end{aligned}$$

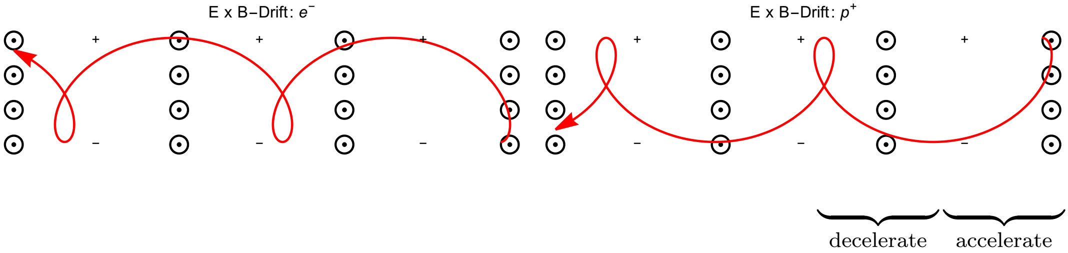
Assuming time-invariant velocity:

$$\begin{aligned}
F_\perp &= -q\,v_D \times \vec{B} && (6)\\
F_\perp \times \vec{B} &= -q\,v_D \times \vec{B} \times \vec{B} = q\,v_D\,B^2 && (7)\\
\frac{F_\perp \times \vec{B}}{q\,B^2} &= v_D \qquad \rightarrow \text{generalized drift velocity} && (8)
\end{aligned}$$

$E \times B$-Drift

homogenous B-field, orthogonal electric field

$$\vec{F}_L = q\,\vec{E} + q\,\vec{v} \times \vec{B}$$



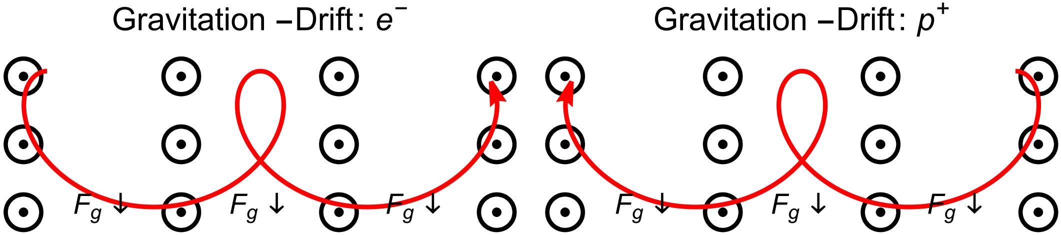
using generalized drift eq: $\vec{v}_{E\times B} = \frac{\vec{F}\times\vec{B}}{q\,B^2} = \frac{\cancel{q}\vec{E}\times\vec{B}}{\cancel{q}\,B^2} = \frac{\vec{E}\times\vec{B}}{B^2}$ (charge independent)

$\rightarrow$ drift to same direction, same speed $\rightarrow$ no resulting current

Gravitation-Drift

homogenous B-field, no electric field, Gravitation

$$\vec{F} = \cancel{q\vec{E}} + q\,\vec{v} \times \vec{B} + m\,\vec{g}$$

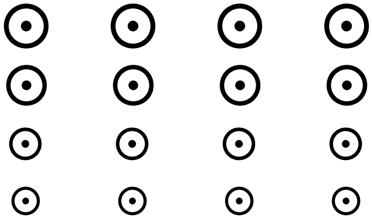


using generalized drift eq: $\vec{v}_g = \frac{\vec{F} \times \vec{B}}{q\,B^2} = \frac{m}{q}\frac{\vec{g} \times \vec{B}}{B^2}$ (charge dependent, mass dependent)

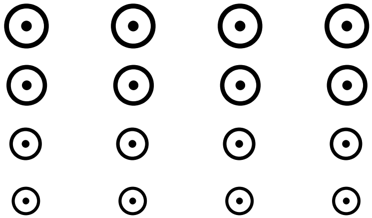
→ drift with charge separation,
→ note that in most cases the gravitation drift is neglegible compared to other drifts

Gradient-Drift

Gradient in the B-field, no electric field



∇B–Drift: e^-



∇B–Drift: p^+

Question: The ∇B-drift is directed to?

A) both drifts to left

B) both drifts to right

C) e-drift left, p-drift to right

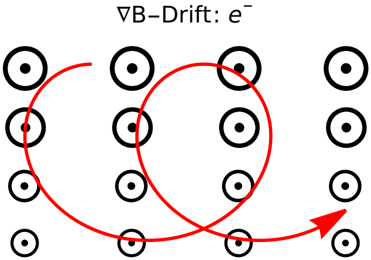
D) e-drift right, p-drift to left

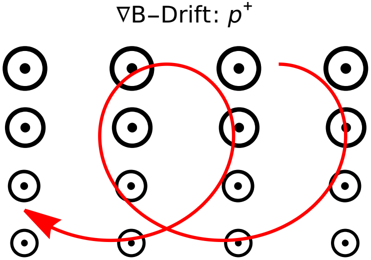
E) e-drift up, p-drift to down

F) both drifts up

Gradient-Drift

Gradient in the B-field, no electric field



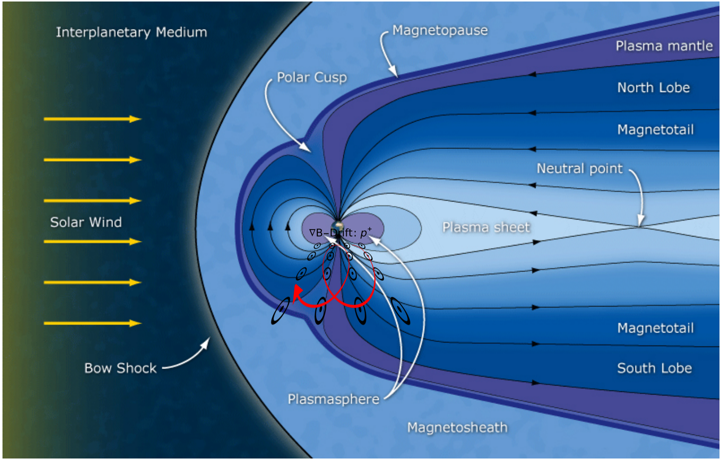


Correct answer is:

D) e-drift right, p-drift to left

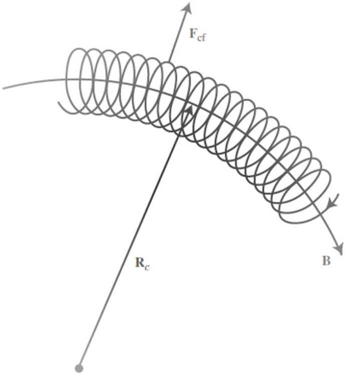
$\vec{v}_{\nabla B} = \frac{\mu}{q\,B^2}\vec{B}\times\nabla B = \frac{W_{\mathrm{kin}_\perp}}{q\,B}\frac{\vec{B}\times\nabla B}{B^2}$ (charge dependent, energy dependent)

Applying Drifts



adapted from T. Russel, ESA.

Curvature drift



Simplified approach:

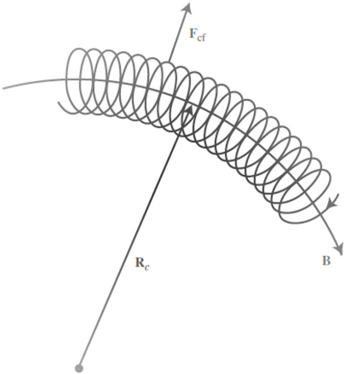
a particle on bound field lines experiences

centrifugal force: $F_{cf} = \frac{mv_{\parallel}^2}{r_c^2} \vec{r}_c$

generalized drift velocity: $\vec{v}_{cf} = \frac{\vec{F}_{cf} \times \vec{B}}{q\,B^2} = \frac{mv_{\parallel}^2 \vec{r}_c \times \vec{B}}{r_c^2\,q\,B^2}$

image: Inan & Golkowski, Page 34

Curvature drift



Simplified approach:

a particle on bound field lines experiences centrifugal force: $F_{cf} = \frac{mv_\parallel^2}{r_c^2}\vec{r}_c$

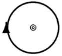
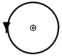
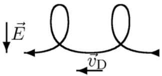
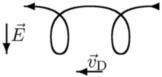
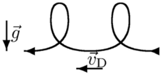
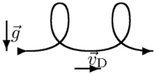
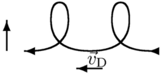
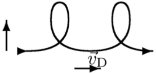
generalized drift velocity: $\vec{v}_{cf} = \frac{\vec{F}_{cf} \times \vec{B}}{q\,B^2} = \frac{mv_\parallel^2 \vec{r}_c \times \vec{B}}{r_c^2\,q\,B^2}$

However, assuming a pure bend magnetic field in vacuum, according to the Ampere's law ($\nabla \times \vec{B} = \mu_0 \vec{j} + \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$), there has to be a gradient.

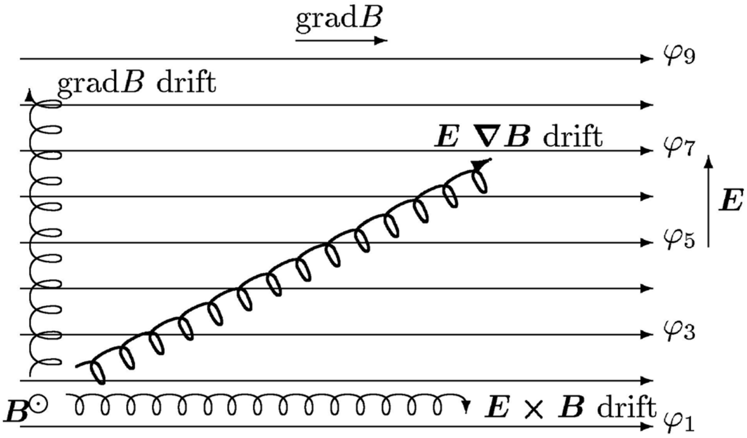
We skip this for today.

image: Inan & Golkowski, Page 34

Summary drifts

$\vec{B}$ upward through the paper	Charge	
	positive	negative
B homogenous		
E homogenous		
Gravitation		
Inhomogeneous B ↑grad $\lvert\vec{B}\rvert$		

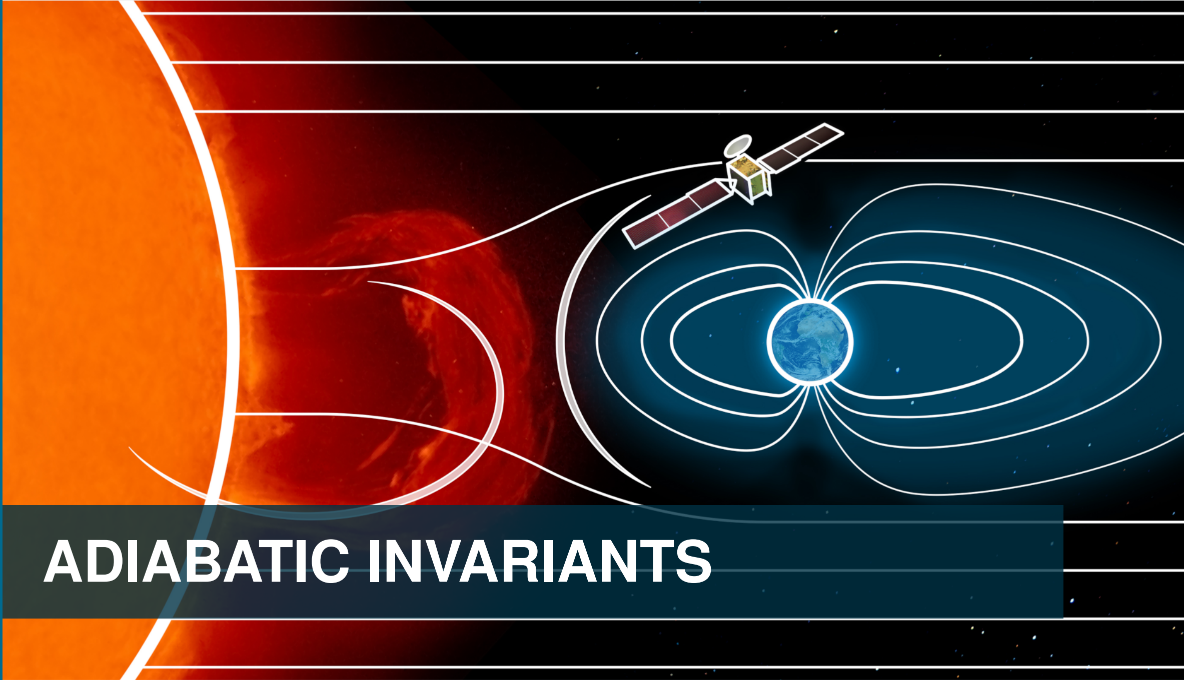
Drift with energy change - $E\nabla B$-drift



∇B-drift moves "upward" but no energy change as there is no E-Field

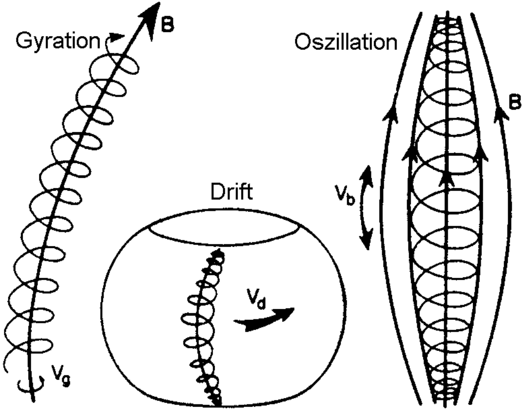
$E \times B$-drift moves along same potential → no energy change

combined:
$E\nabla B$-drift changes energy

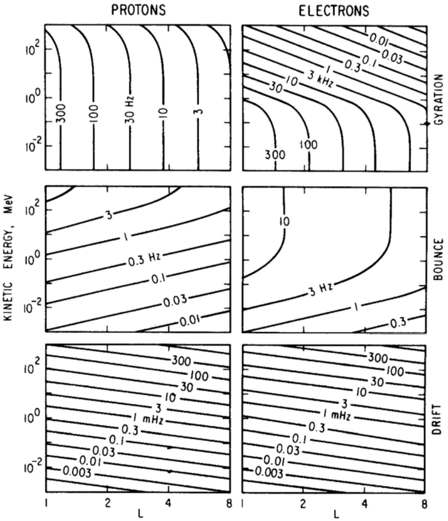


ADIABATIC INVARIANTS

Main particle motions



Main particle motions - timescales



Time scales:

gyration: kHz

longitudinal motion: seconds

drift: 15 min (1000 s)

Schulz and Lanzerotti (1974)

Adiabatic invariants

precondition: spatial and temporal B-field changes are small compared to motion:

- gyration time: $\frac{1}{B}\frac{\partial B}{\partial t} \ll \frac{\omega_c}{2\pi}$
- field-parallel motion: $\frac{\nabla B_\parallel}{B} \ll \frac{\omega_c}{2\pi v_\parallel}$
- gyration orbit: $\frac{\nabla B_\perp}{B} \ll \frac{\omega_c}{2\pi v_\perp}$ or $\frac{\nabla B_\perp}{B} \ll \frac{\omega_c}{2\pi v_D}$

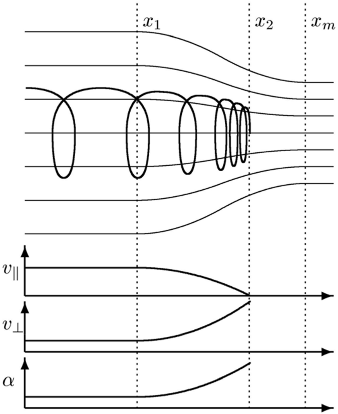
In a cyclic motion the action integral of the momentum over a full circle is:
$J_i = \oint p_i \,\mathrm{d}q_i \approx const$

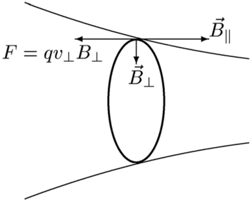
magnetic moment invariant: $J_1 = \oint m\, v_\perp\, r_L \mathrm{d}\psi = 2\pi\, m\, v_\perp\, r_L = 4\pi \frac{m}{|q|}\mu = const$

longitudinal invariant: $J_2 = \int_{s_1}^{s_2} m\, v_\parallel \mathrm{d}s = \int_{s_2}^{s_2} m\sqrt{v^2 - \frac{2\mu\, B}{m}}\mathrm{d}s = const$

flux invariant: $J_3 = \oint m\, v_D r\, \mathrm{d}\psi = \frac{4\pi m}{|q|} \cdot M = const$

Magnetic mirror - 1st adiabatic invariant





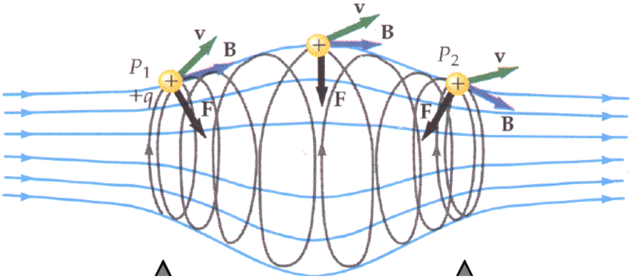
$$\frac{2\mu}{mv^2} = \frac{\sin^2\alpha_1}{B_1} = \frac{\sin^2\alpha_2}{B_2}$$

magnetic moment is constant:

$$\begin{aligned}
\mu = \frac{W_{kin,\perp}}{B} &= \frac{m\,v_\perp^2}{2\,B} \\
= \frac{m\,v^2\ \sin^2\alpha}{2\,B} &= const \\
\frac{m\,v^2\ \sin^2\alpha_1}{2\,B_1} &= \frac{m\,v^2\ \sin^2\alpha_m}{2\,B_m} \\
\frac{m\,v^2\ \sin^2\alpha_1}{2\,B_1} &= \frac{m\,v^2}{2\,B_m} \\
\frac{\sin^2\alpha_1}{B_1} &= \frac{1}{B_m}
\end{aligned}$$

→ estimate B_m at mirror point

Acceleration - 2nd adiabatic invariant



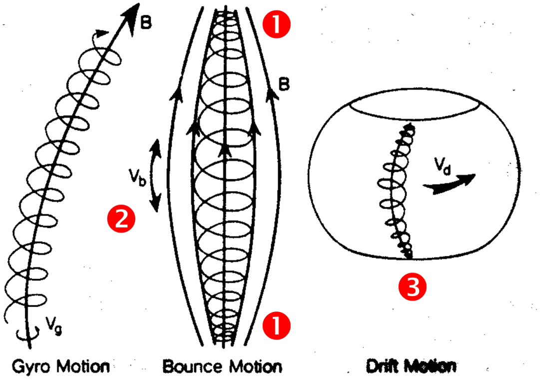
Longitudinal invariant (2nd adiabatic invariant):

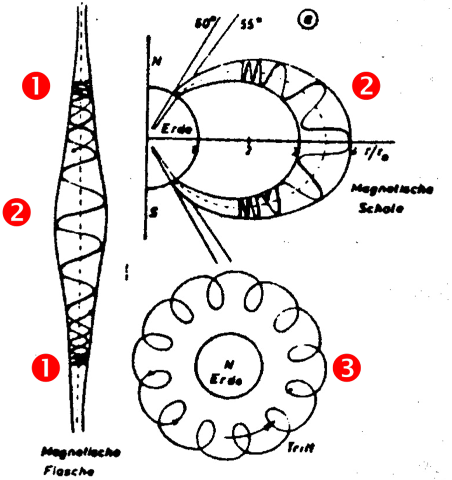
$$J_2 = \int_{s_1}^{s_2} m v_{\parallel} \, \mathrm{d}s = const.$$

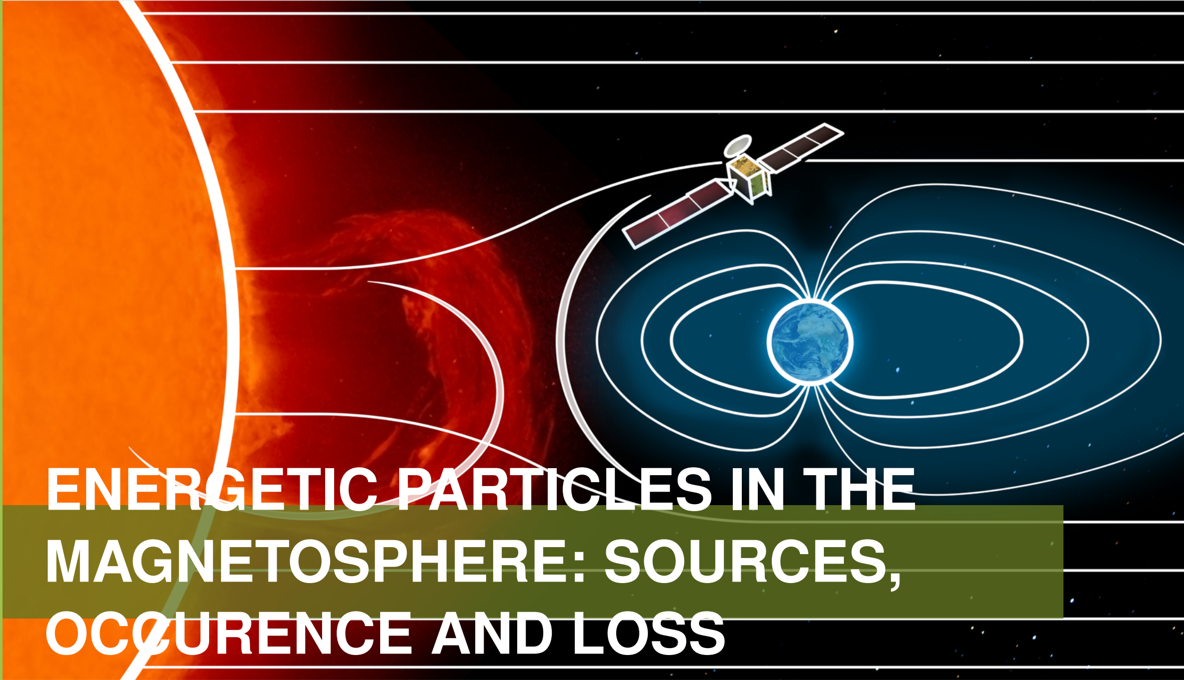
Fermi acceleration type 1: converging mirror points increase $v_{\parallel}$

3rd adiabatic invariant

Flux invariant

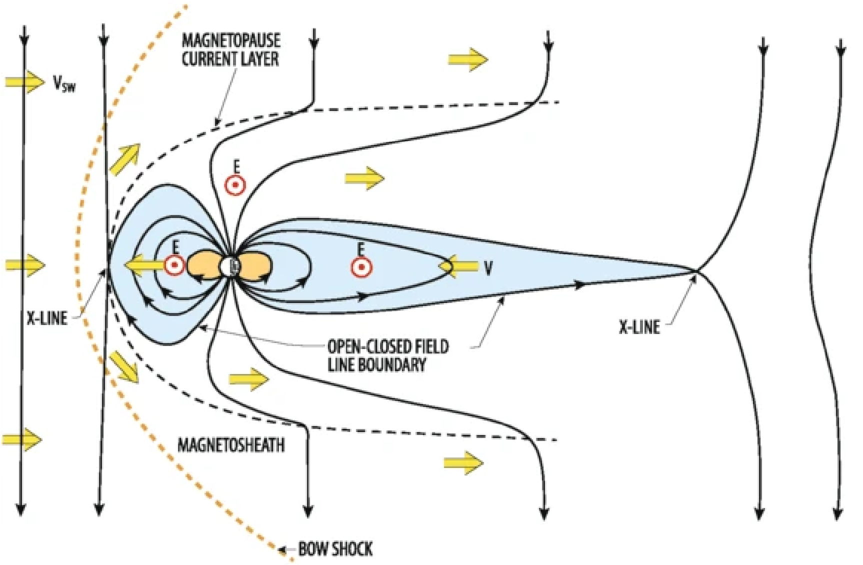






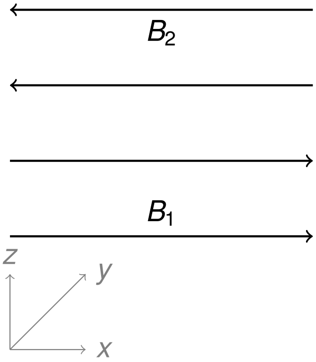
ENERGETIC PARTICLES IN THE MAGNETOSPHERE: SOURCES, OCCURENCE AND LOSS

How/Where do particles enter the magnetosphere?



Lyons 1992

Reconnection



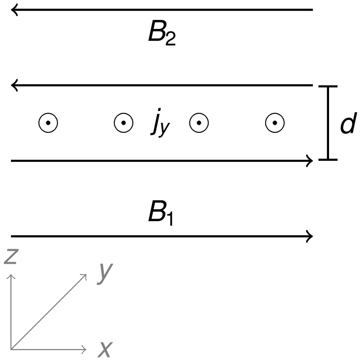
Ampere's Law:

$$\nabla \times \vec{B} = \mu_0 \, \vec{j}$$

$\vec{j}$ current density

$$\begin{pmatrix} \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} \\ \frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x} \\ \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \end{pmatrix} = \mu_0 \, \vec{j}$$

Reconnection



Ampere's Law:

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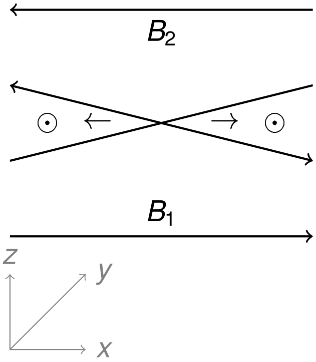
$$\begin{pmatrix} 0 \\ \frac{\partial B_x}{\partial z} \\ 0 \end{pmatrix} = \frac{\Delta B_x}{d} = \mu_0 \, j_y$$

Reconnection



Ampere's Law:

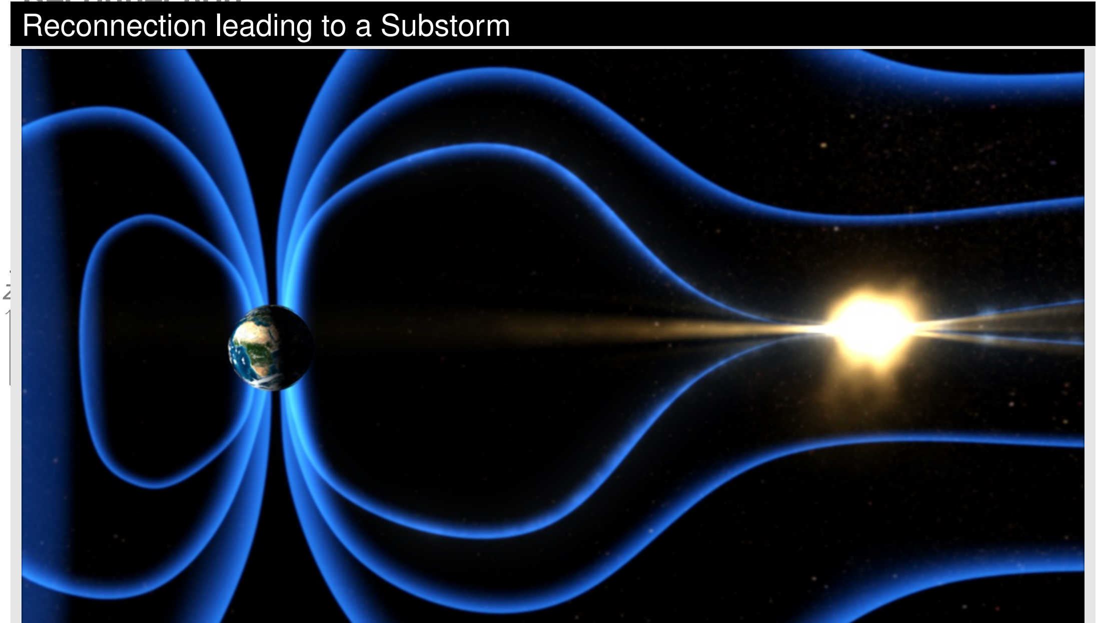
$$\nabla \times \vec{B} = \mu_0 \, \vec{j}$$

$\vec{j}$ current density

$$\begin{pmatrix} \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} \\ \frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x} \\ \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \end{pmatrix} = \mu_0 \, \vec{j}$$

$$\begin{pmatrix} 0 \\ \frac{\partial B_x}{\partial z} \\ 0 \end{pmatrix} = \frac{\Delta B_x}{d} = \mu_0 \, j_y$$

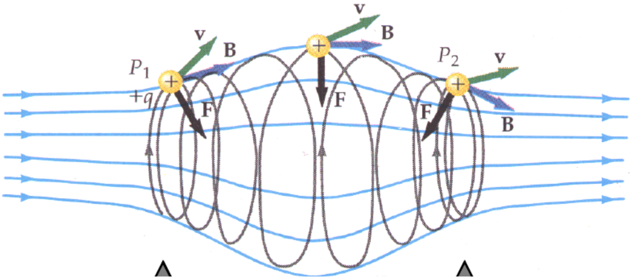
Reconnection


Reconnection leading to a Substorm

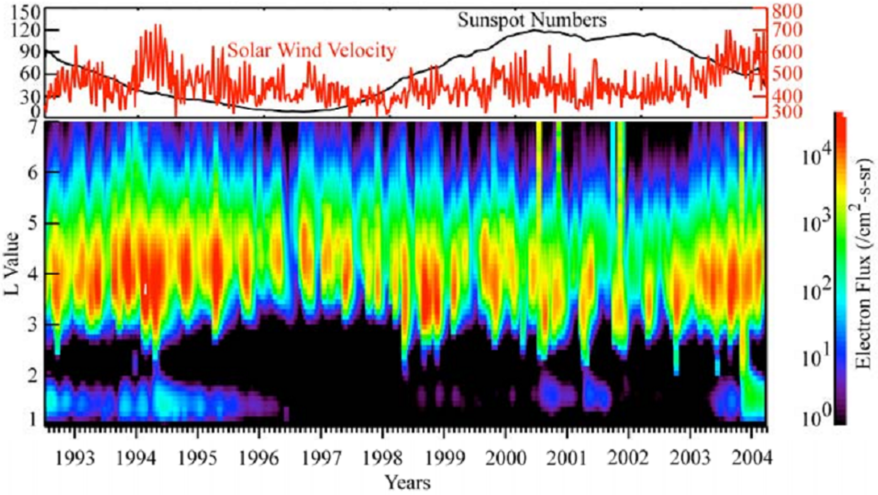
Acceleration

Longitudinal invariant (2nd adiabatic invariant):

$$J_2 = \int_{s_1}^{s_2} m v_{\|} \, \mathrm{d}s = const.$$

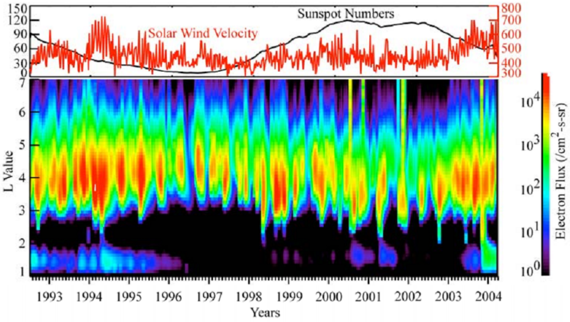


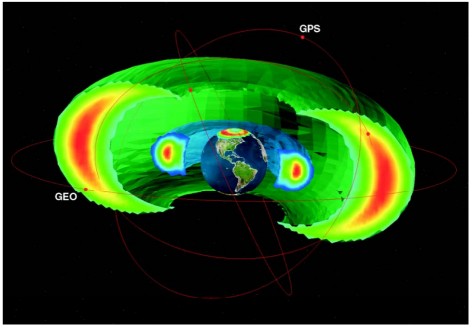
Radiation belts



Baker et al., 2004

Radiation belts





Baker et al., 2004

Air Force Research Laboratory

Inner Van Allen belt 10 000km – $\sim$ 2 r_E, peak at 1.5 r_E

protons 10-100 MeV, GAKs to multiple GeV

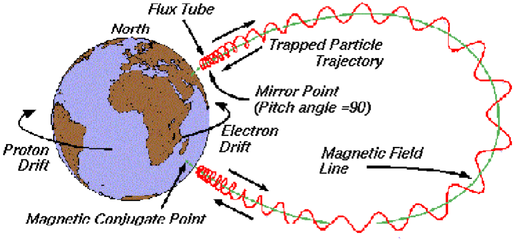
protons relatively stable, electrons fluctuating

Slot region -2–3 r_E

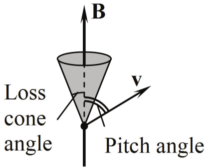
gap may be filled with electrons during geomagnetic storms

Outer Van Allen belt 3–6.5 r_E, mostly relativistic electrons (100 keV-10 MeV)

Where do they precipitate?



Loss Cone



Loss cone pitch angle: $\alpha_{LC} = \arcsin\sqrt{\frac{B_{eq}}{B_m}}$

The loss cone angle at the equator is about:

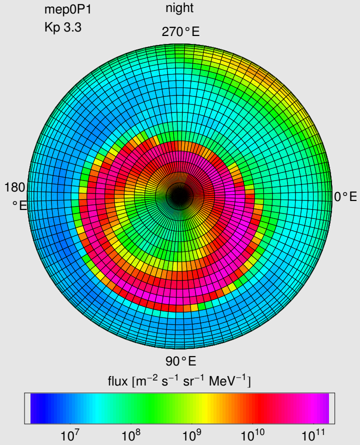
e.g. 2° at L=6.6 (geostationary orbit), 16° at at L=2

$\alpha > \alpha_{LC}$ trapped, $\alpha <= \alpha_{LC}$ precipitate

PA-scattering pushes trapped particles into loss cone

Particle Precipitation Pattern

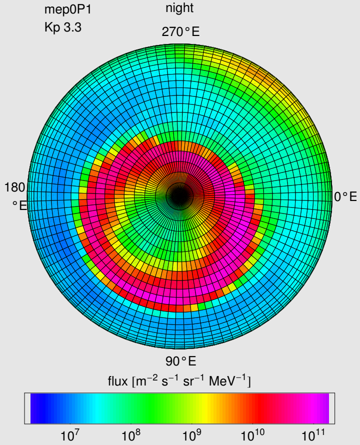
30–80 keV low energy but high spatial variability



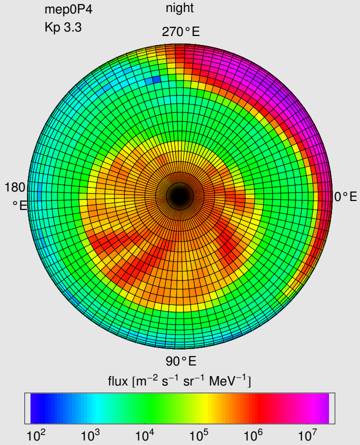
Wissing et al. 2008

Particle Precipitation Pattern

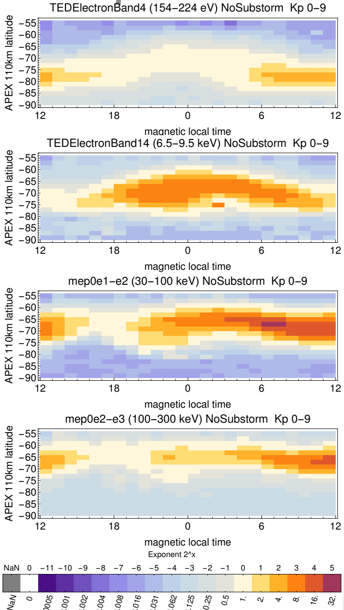
30–80 keV low energy but high spatial variability



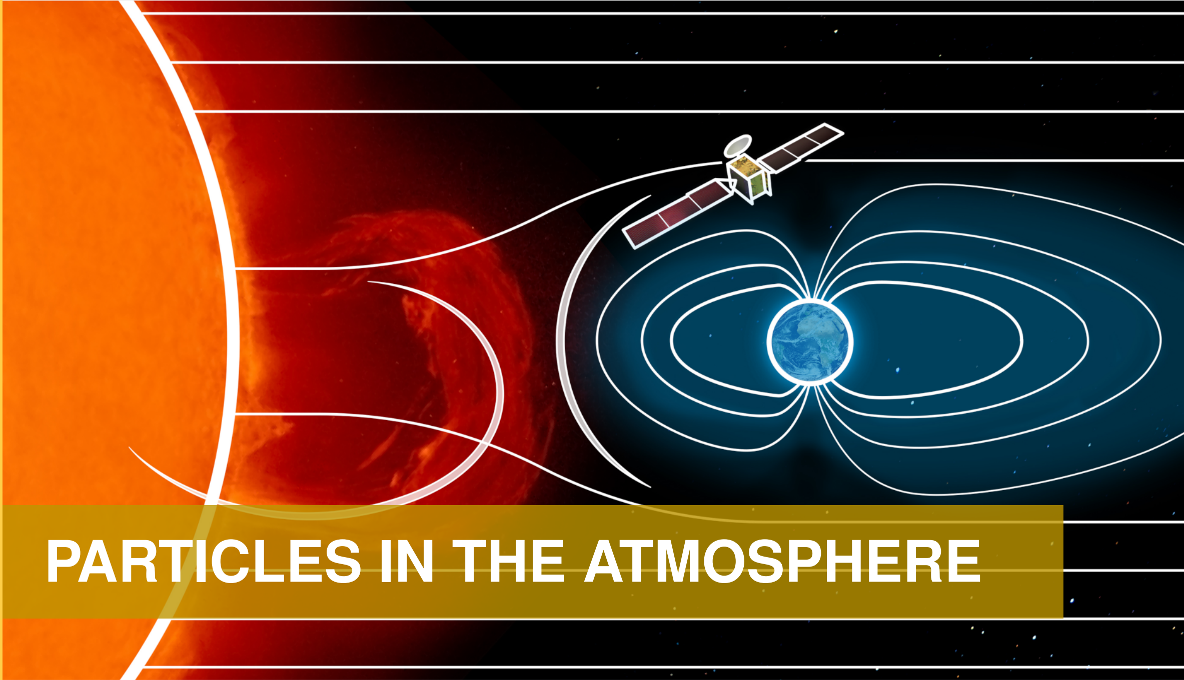
0.8–2.5 MeV high energy but low spatial variability



Precipitation Pattern for different Energies and MLTs



Electrons, Yakovchuk und Wissing (2019)



PARTICLES IN THE ATMOSPHERE

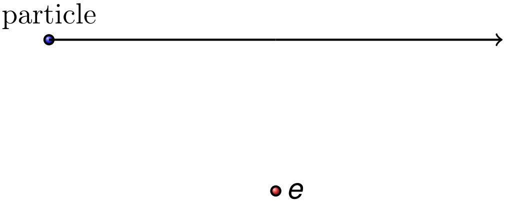
Particles in the atmosphere: Why do we care?

Particles in the atmosphere: Why do we care?

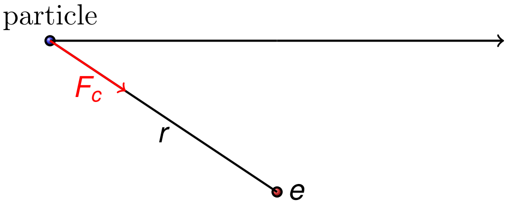
- Impacts on Chemistry:
    - NOx, HOx
    - Ozone
- Direct Impacts on Life:
    - Ground level events
    - during flights (talk by Meier, tomorrow)
- Impacts on GPS:
    - TEC

Interaction with the Atmosphere

Interaction with the Atmosphere

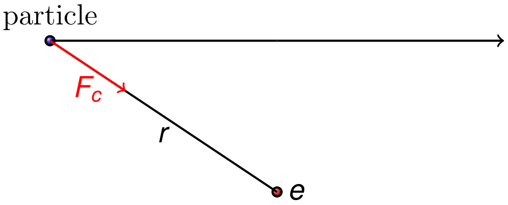


Interaction with the Atmosphere



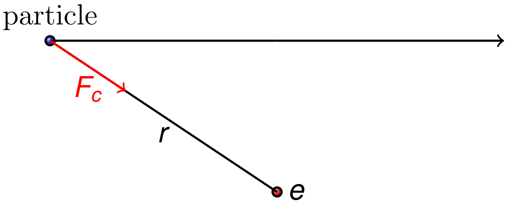
$$F_c = \frac{1}{4\pi\epsilon_0}\frac{q_1 \cdot q_2}{r^2}$$

Interaction with the Atmosphere



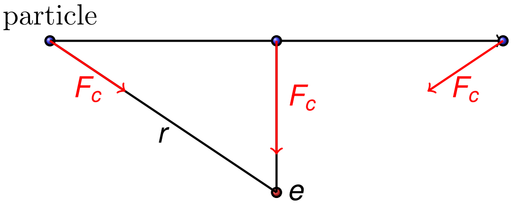
$$F_c = \frac{1}{4\pi\epsilon_0}\frac{z\ e \cdot e}{r^2}$$

Interaction with the Atmosphere



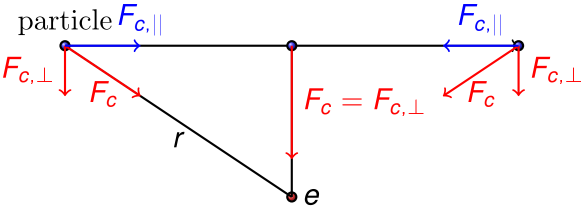
$$F_c = \frac{1}{4\pi\epsilon_0}\frac{z\ e^2}{r^2}$$

Interaction with the Atmosphere



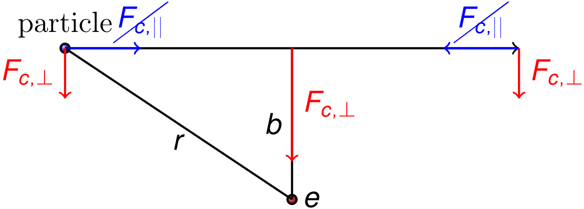
$$F_c = \frac{1}{4\pi\epsilon_0}\frac{z\ e^2}{r^2}$$

Interaction with the Atmosphere



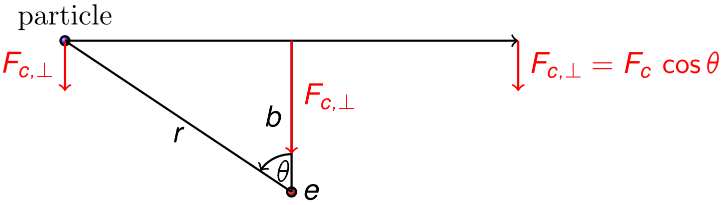
$$F_c = \frac{1}{4\pi\epsilon_0}\frac{z\ e^2}{r^2}$$

Interaction with the Atmosphere



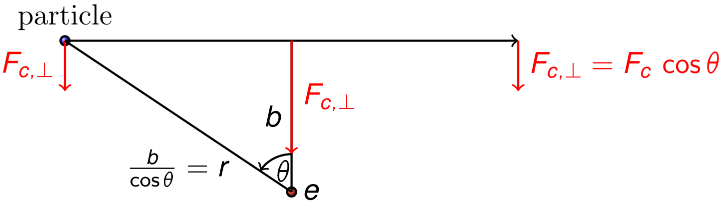
$$F_c = \frac{1}{4\pi\epsilon_0}\frac{z\ e^2}{r^2}$$

Interaction with the Atmosphere



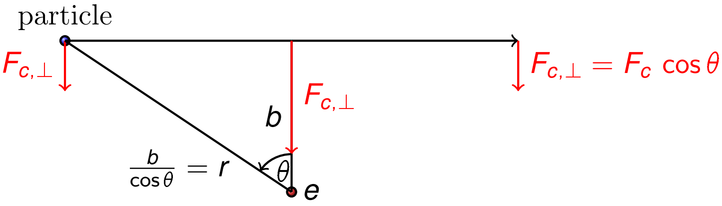
$$F_{c,\perp} = \frac{1}{4\pi\epsilon_0}\frac{z\ e^2}{r^2}\ \cos\theta$$

Interaction with the Atmosphere



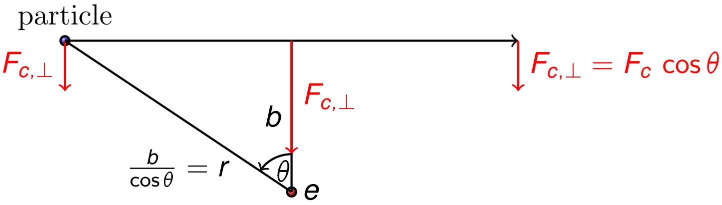
$$F_{c,\perp} = \frac{1}{4\pi\epsilon_0} \frac{z\ e^2}{r^2}\ \cos\theta$$

Interaction with the Atmosphere



$$F_{c,\perp} = \frac{1}{4\pi\epsilon_0} \frac{z\ e^2}{b^2} \cos^3\theta$$

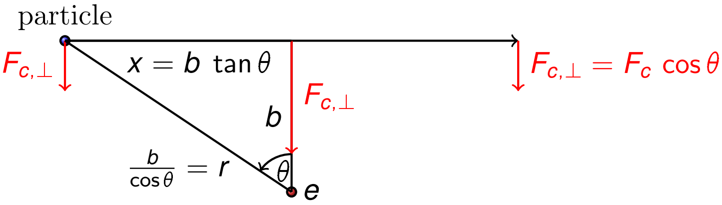
Interaction with the Atmosphere



$$F_{c,\perp} = \frac{1}{4\pi\epsilon_0}\frac{z\ e^2}{b^2}\ \cos^3\theta$$

$$|\Delta p_\perp| = \int_{-\infty}^{\infty} F_{c,\perp}\,\mathrm{d}t$$

Interaction with the Atmosphere

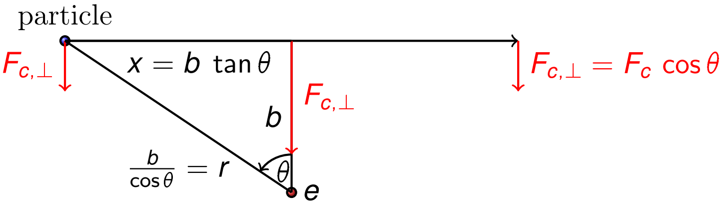


$$F_{c,\perp} = \frac{1}{4\pi\epsilon_0}\frac{z\ e^2}{b^2}\ \cos^3\theta$$

$$|\Delta p_\perp| = \int_{-\infty}^{\infty} F_{c,\perp}\ \mathrm{d}t$$

$$\mathrm{d}t = \frac{\mathrm{d}x}{\frac{\mathrm{d}x}{\mathrm{d}t}} = \frac{\mathrm{d}x}{v}$$

Interaction with the Atmosphere

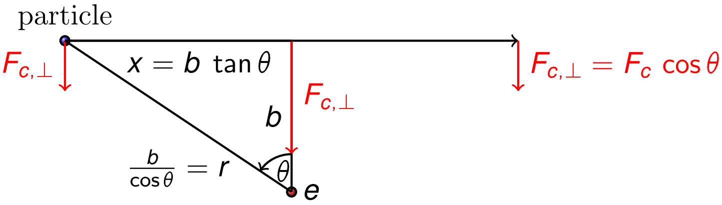


$$F_{c,\perp} = \frac{1}{4\pi\epsilon_0}\frac{z\ e^2}{b^2}\ \cos^3\theta$$

$$|\Delta p_\perp| = \int_{-\infty}^{\infty} F_{c,\perp}\,\mathrm{d}t$$

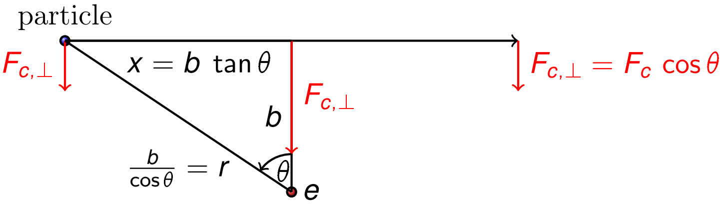
$$\mathrm{d}t = \frac{\mathrm{d}x}{\frac{\mathrm{d}x}{\mathrm{d}t}} = \frac{\mathrm{d}x}{v} = \frac{1}{v}\frac{1}{\frac{\mathrm{d}\theta}{\mathrm{d}x}}\mathrm{d}\theta = \frac{1}{v}\frac{b}{\cos^2\theta}\ \mathrm{d}\theta$$

Interaction with the Atmosphere



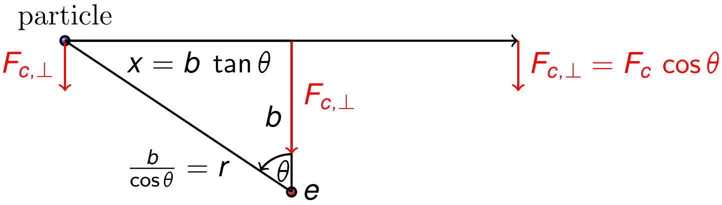
$$|\Delta p_\perp| = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{4\pi\epsilon_0} \frac{z\ e^2}{b^2}\ \cos^3\theta\ \frac{1}{v} \frac{b}{\cos^2\theta}\ \mathrm{d}\theta$$

Interaction with the Atmosphere



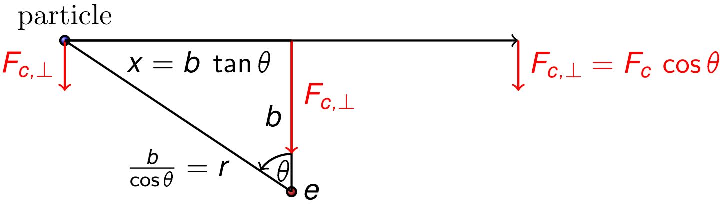
$$|\Delta p_\perp| = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{4\pi\epsilon_0} \frac{z\ e^2}{b^{\cancel{2}}}\ \cos^{\cancel{3}}\theta\ \frac{1}{v} \frac{\cancel{b}}{\cancel{\cos^2\theta}}\, \mathrm{d}\theta$$

Interaction with the Atmosphere



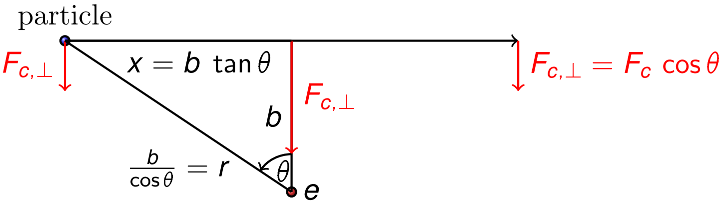
$$|\Delta p_\perp| = \frac{1}{4\pi\epsilon_0}\frac{z\ e^2}{v\ b}\underbrace{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\theta\ \mathrm{d}\theta}_{2}$$

Interaction with the Atmosphere



$$|\Delta p_\perp| = \frac{1}{2\pi\epsilon_0}\frac{z\ e^2}{\underbrace{v}_{=\beta\ c}\ \ b}$$

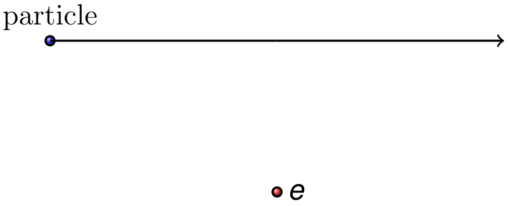
Interaction with the Atmosphere



Momentum transfer between particle and a single air-electron at distance *b*:

$$|\Delta p_\perp| = \frac{1}{2\pi\ \epsilon_0} \frac{z\ e^2}{\beta\ c\ b}$$

Interaction with the Atmosphere



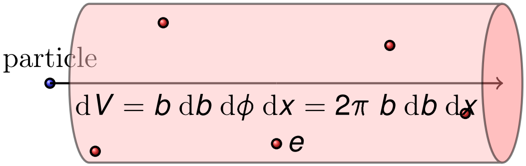
Momentum transfer between particle and a single air-electron at distance b:

$$|\Delta p_\perp| = \frac{1}{2\pi\ \epsilon_0} \frac{z\ e^2}{\beta\ c\ b}$$

Energy transmitted from the particle to **one** single air-electron at distance b:

$$\Delta E = \frac{(\Delta p_\perp)^2}{2\ m_e} = \left(\frac{1}{2\pi\epsilon_0}\right)^2 \frac{z^2\ e^4}{2\ m_e\ \beta^2\ c^2\ b^2}$$

Interaction with the Atmosphere



$$\mathrm{d}V = b\,\mathrm{d}b\,\mathrm{d}\phi\,\mathrm{d}x = 2\pi\ b\,\mathrm{d}b\,\mathrm{d}x$$

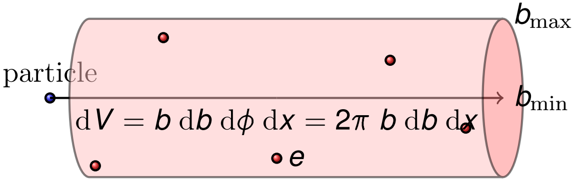
Energy transmitted from the particle to **one** single air-electron at distance b:

$$\Delta E = \frac{(\Delta p_\perp)^2}{2\,m_e} = \left(\frac{1}{2\pi\epsilon_0}\right)^2 \frac{z^2\ e^4}{2\ m_e\ \beta^2\ c^2\ b^2}$$

Energy loss of the energetic particle to **all** air-electrons of the volume:

$$-\mathrm{d}E = \Delta E\ \underbrace{n_e}_{=\frac{\rho\,Z}{u\,A}}\ \mathrm{d}V$$

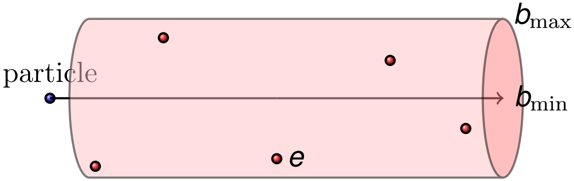
Interaction with the Atmosphere



Energy loss of the energetic particle to all air-electrons of the volume:

$$-\mathrm{d}E = \frac{e^4}{4\pi\;\epsilon_0^2\;m_e\;c^2\;u}\frac{z^2}{\beta^2}\;\frac{\rho\;Z}{A}\;\frac{1}{b}\mathrm{d}b\,\mathrm{d}x$$

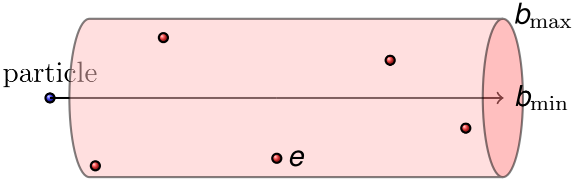
Interaction with the Atmosphere



Energy loss of the energetic particle to all air-electrons of the volume:

$$-\mathrm{d}E = \frac{e^4}{4\pi\ \epsilon_0^2\ m_e\ c^2\ u} \frac{z^2}{\beta^2}\ \frac{\rho\ Z}{A} \qquad \underbrace{\frac{1}{b}\mathrm{d}b}_{\int_{b_{\mathrm{min}}}^{b_{\mathrm{max}}} \frac{1}{b}\mathrm{d}b = \ln \frac{b_{\mathrm{max}}}{b_{\mathrm{min}}}} \qquad \mathrm{d}x$$

Interaction with the Atmosphere



Energy loss of the energetic particle to all air-electrons of the volume:

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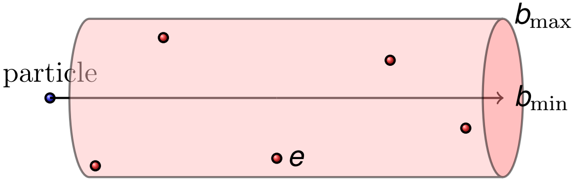
Interaction with the Atmosphere



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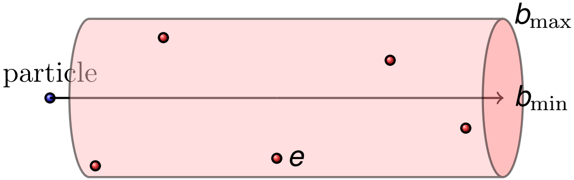
$$-\mathrm{d}E = \frac{e^4}{4\pi\ \epsilon_0^2\ m_e\ c^2\ u}\frac{z^2}{\beta^2}\ \frac{\rho\ Z}{A}\ \ln\frac{b_{\mathrm{max}}}{b_{\mathrm{min}}}\ \mathrm{d}x$$

b_{max}: close enough that mean excitation energy I is transferred to the electron

b_{min}: maximum momentum transfer in a central elastic relativistic collision is $\Delta p = 2\ m\ v\ \gamma$

Interaction with the Atmosphere



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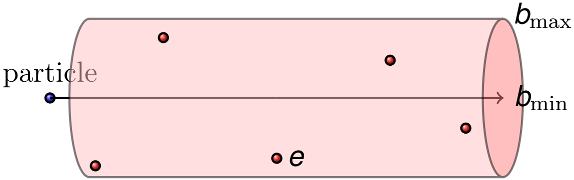
$$-\mathrm{d}E = \frac{e^4}{4\pi\ \epsilon_0^2\ m_e\ c^2\ u}\frac{z^2}{\beta^2}\ \frac{\rho\ Z}{A}\ \ln\frac{b_{\mathrm{max}}}{b_{\mathrm{min}}}\ \mathrm{d}x$$

b_{max}: close enough that mean excitation energy I is transferred to the electron

$$\Delta E = I = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{2z^2e^4}{m_e\beta^2c^2b_{max}^2} \quad \rightarrow \quad b_{max} = \frac{1}{4\pi\epsilon_0}\frac{ze^2}{\beta c}\sqrt{\frac{2}{m_e\,I}}$$

Interaction with the Atmosphere



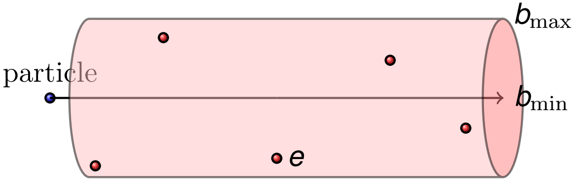
Energy loss of the energetic particle to all air-electrons of the volume:

$$-\mathrm{d}E = \frac{e^4}{4\pi\ \epsilon_0^2\ m_e\ c^2\ u}\frac{z^2}{\beta^2}\ \frac{\rho\ Z}{A}\ \ln\frac{b_{\mathrm{max}}}{b_{\mathrm{min}}}\ \mathrm{d}x$$

b_{min}: maximum momentum transfer in a central elastic relativistic collision is:

$$\Delta p = 2 m_e v \gamma = \frac{1}{4\pi\epsilon_0}\frac{2ze^2}{b_{min}\,v} \qquad \to \qquad b_{min} = \frac{1}{4\pi\epsilon_0}\frac{ze^2}{m_e\beta^2 c^2\gamma}$$

Interaction with the Atmosphere



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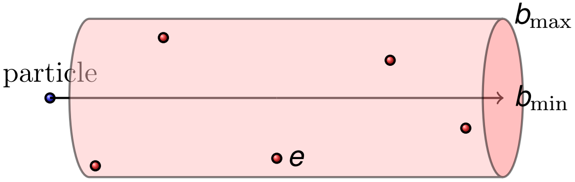
Energy loss of the energetic particle along path (by Niels Bohr):

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{e^4}{8\pi\ \epsilon_0^2\ m_e\ c^2}\,\frac{z^2}{\beta^2}\ \frac{\rho\ Z}{A\ u}\ \ln\left(\frac{2\ m_e\ \beta^2\ c^2\ \gamma^2}{I}\right)$$

Interaction with the Atmosphere



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$$-\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{e^4}{8\pi\, \epsilon_0^2\, m_e\, c^2} \frac{z^2}{\beta^2} \frac{\rho\, Z}{A\, u} \ln\left(\frac{2\, m_e\, \beta^2\, c^2\, \gamma^2}{I}\right)$$

Behte formula (based on quantum mechanics):

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{2\, e^4}{8\pi\, \epsilon_0^2\, m_e\, c^2} \frac{z^2}{\beta^2} \frac{\rho\, Z}{A\, u} \left[\ln\left(\frac{2\, m_e\, \beta^2\, c^2\, \gamma^2}{I}\right) - \beta^2\right]$$

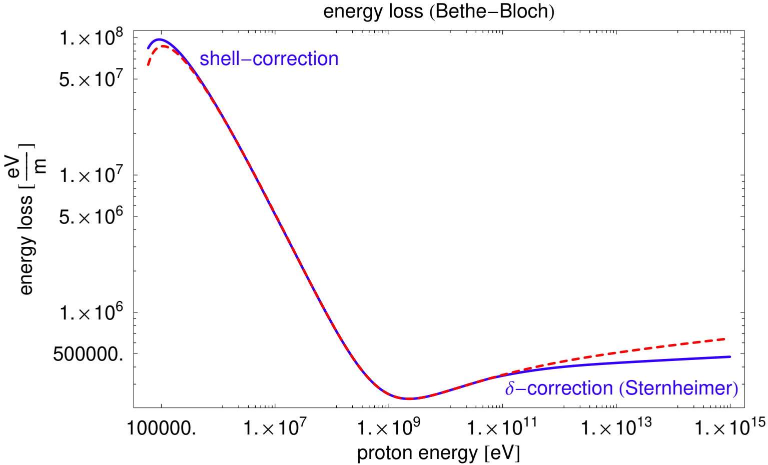
Bethe formula

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{2\ e^4}{8\pi\ \epsilon_0^2\ m_e\ c^2} \frac{z^2}{\beta^2}\ \frac{\rho\ Z}{A\ u}\ \left[\ln\left(\frac{2\ m_e\ \beta^2\ c^2\ \gamma^2}{I}\right) - \beta^2\right]$$

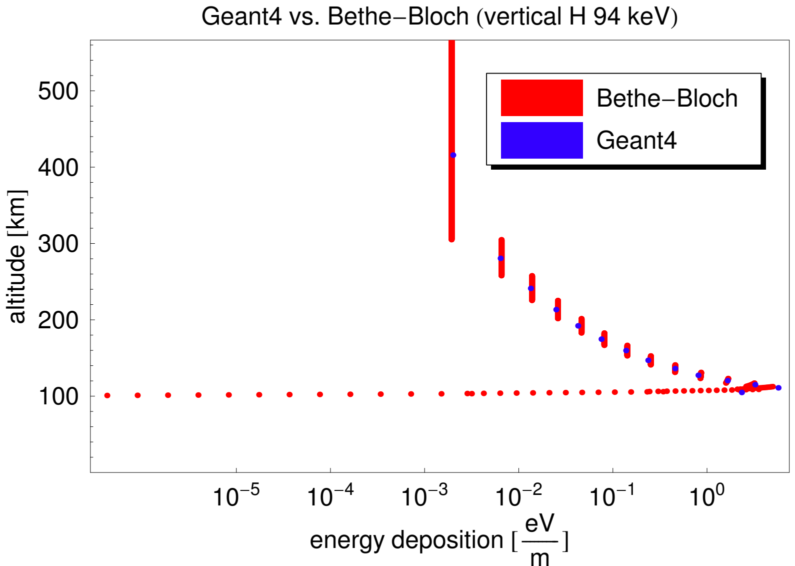
e	elemental charge
m_e	electron mass
ρ	density (matter)
$\epsilon_0 = \frac{1}{\mu_0 c^2}$	permittivity of free space
u	atomic mass constant
$\beta = \frac{v}{c}$	
$\gamma = \frac{1}{\sqrt{1-\beta^2}}$	Lorentz factor
z	# of charges (particle)
Z	atomic number (matter)
A	relative atomic mass
I	mean excitation energy

Bethe formula

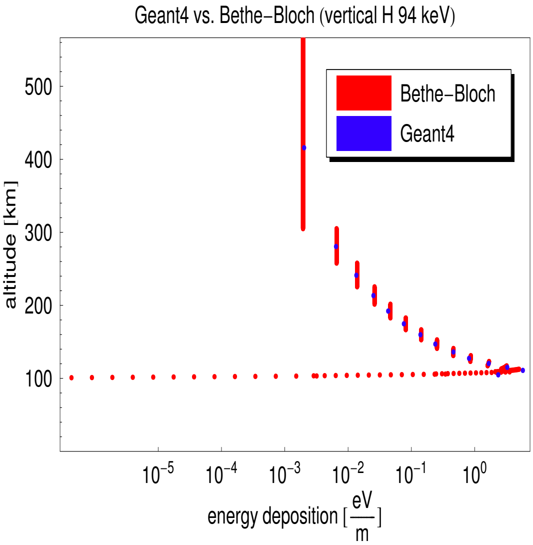
$$-\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{2\ e^4}{8\pi\ \epsilon_0^2\ m_e\ c^2}\frac{z^2}{\beta^2}\ \frac{\rho\ Z}{A\ u}\ \left[\ln\left(\frac{2\ m_e\ \beta^2\ c^2\ \gamma^2}{I}\right) - \beta^2\right]$$



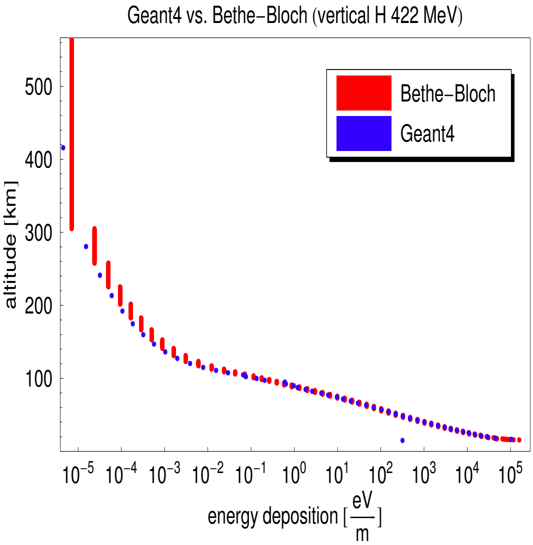
Bethe applied to different energies



Bethe applied to different energies



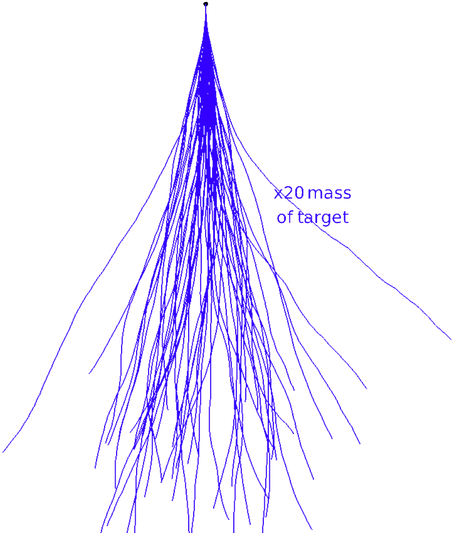
Geant4 vs. Bethe−Bloch (vertical H 94 keV)

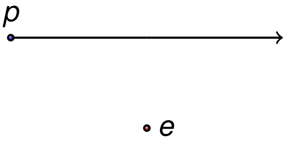


Geant4 vs. Bethe−Bloch (vertical H 422 MeV)

What about lighter particles?

Classic elastic collision:





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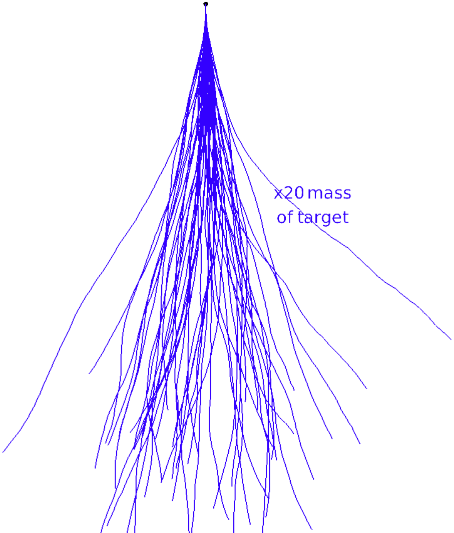
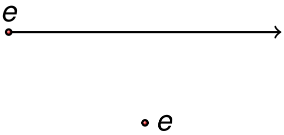
Classic elastic collision:





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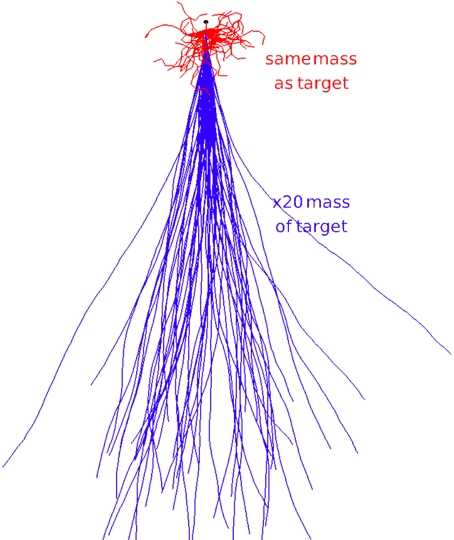
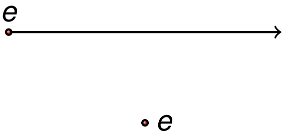
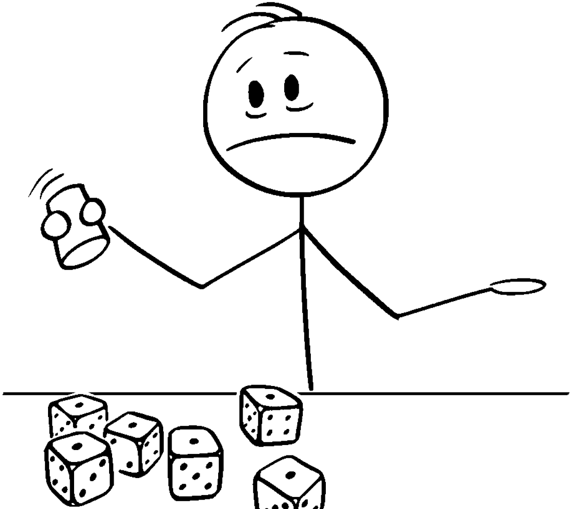




Gambling!

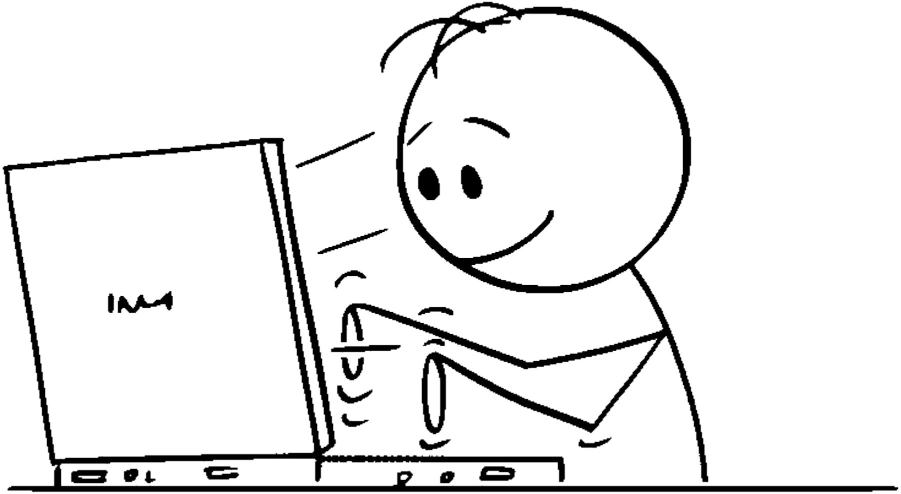
Gambling!

Monte-Carlo

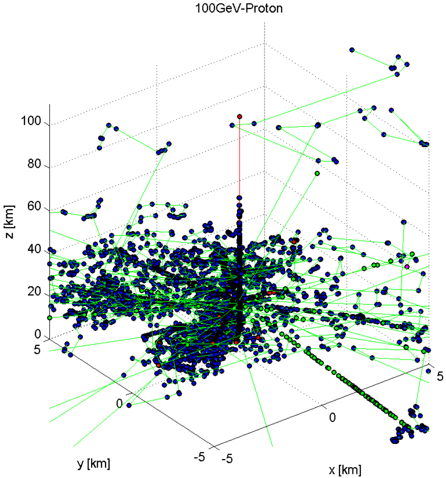


Gambling!

Monte-Carlo



Modeling energy deposition for single particle incidents

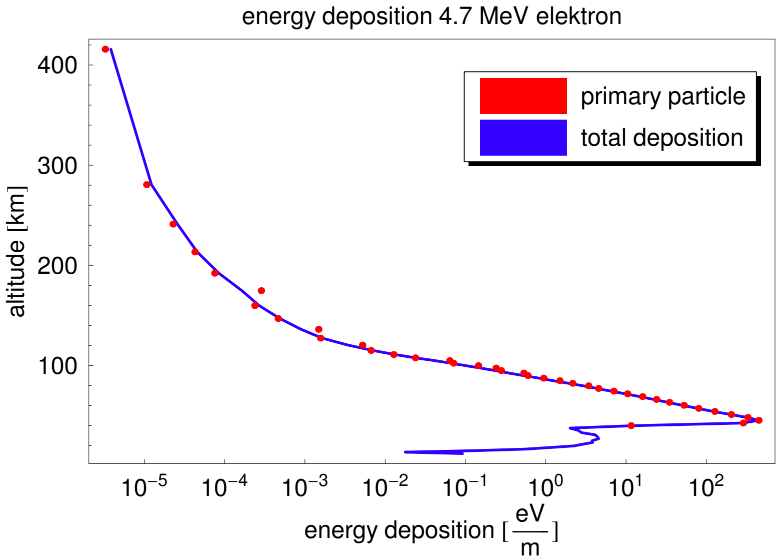


Monte-Carlo (e.g. Geant4)

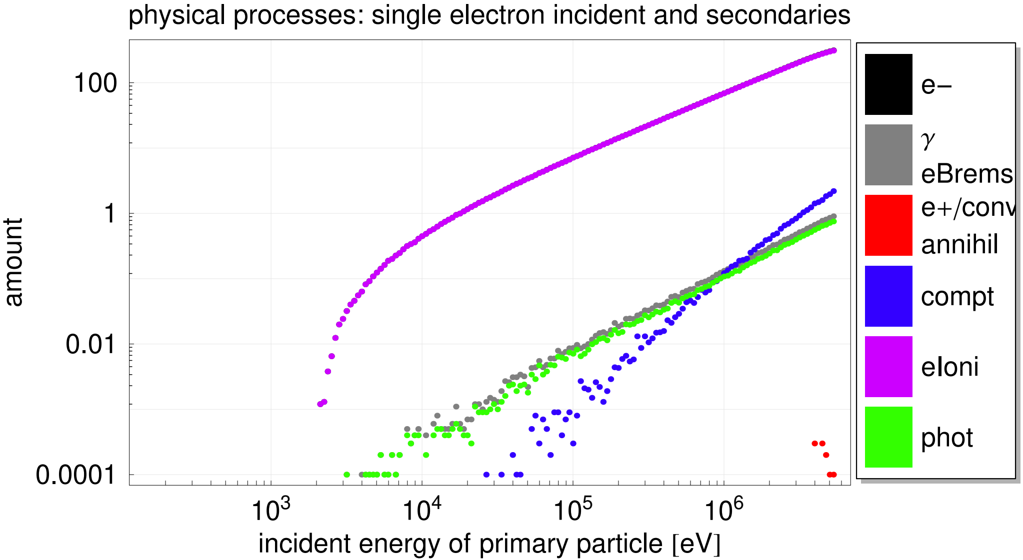
* provides energy deposition per particle

* is transformed into ionization rates (35 eV/ion pair, proved exp+theory)

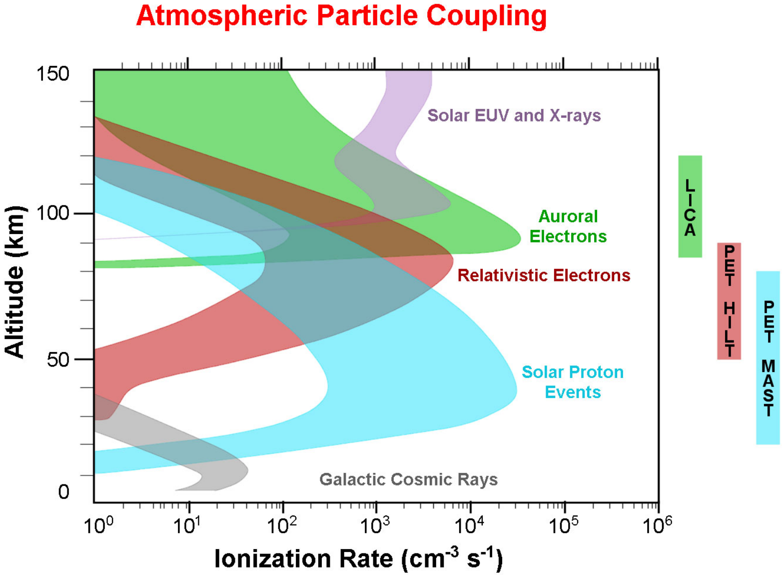
Geant4 Monte-Carlo Simulation: Electron incident



Processes: Electron incident



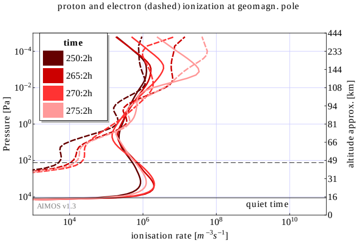
Overview: Precipitating particles



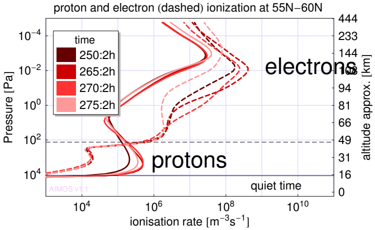
SAMPEX data center

Results

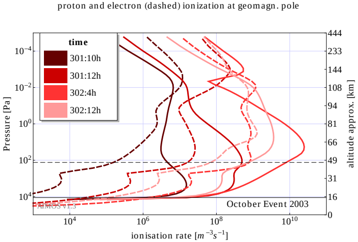
Geomagnetic pole - quiet



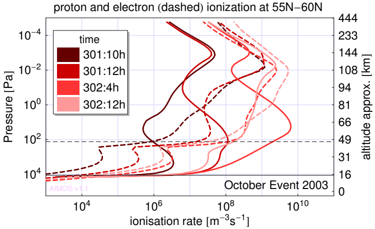
Auroral latitudes - quiet



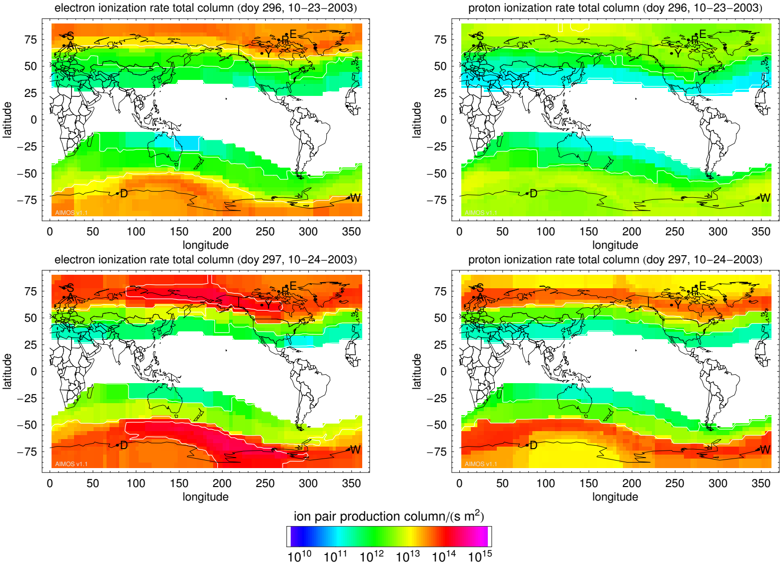
Geomagnetic pole - event



Auroral latitudes - event

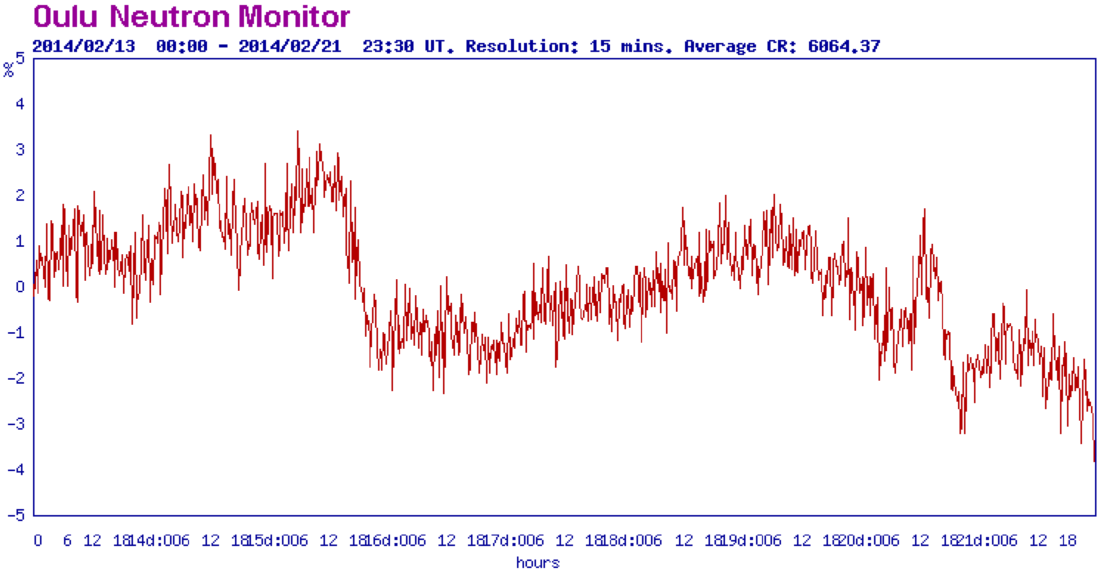


Global ionization rate



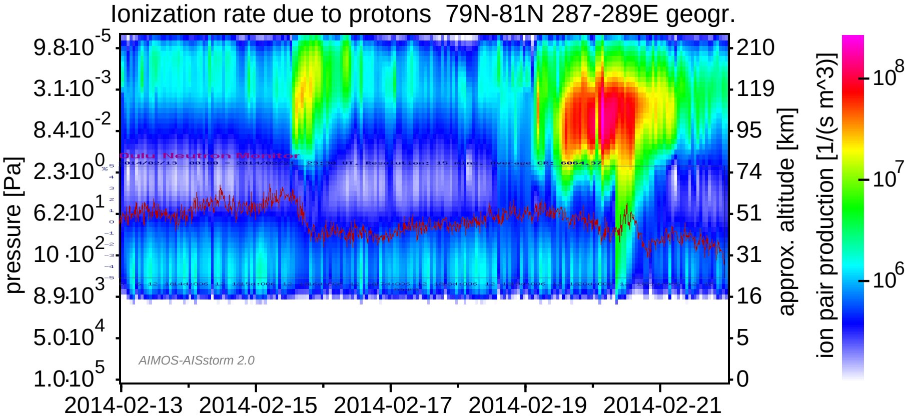
Wissing et al. (2010)

Forbush decrease

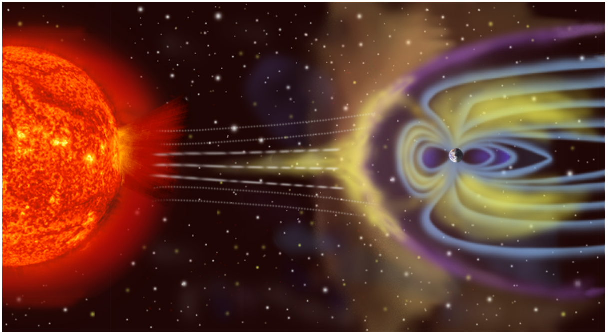


Cosmic Ray Station website, Ilya Usoskin, University of Oulu

Forbush decrease



Summary



NASA

Particles in the Magnetosphere
    Basic Charged Particle Movements in a B-Field
    Origin, Occurence and Loss of Particles
    Pattern of Particle Precipitation

Particles in the atmosphere
    Interaction with the Atmosphere
    Ionization Pattern

Thank you for listening!

[Wissing und Kallenrode 2009] WISSING, J. M. ; KALLENRODE, M.-B.: Atmospheric Ionization Module Osnabrück (AIMOS): A 3-D model to determine atmospheric ionization by energetic charged particles from different populations. In: Journal of Geophysical Research: Space Physics 114 (2009), Nr. A6. `http://dx.doi.org/10.1029/2008JA013884`. – DOI 10.1029/2008JA013884.

[Wissing et al. 2011] WISSING, J. M. ; KALLENRODE, M.-B. ; KIESER, J. ; SCHMIDT, H. ; RIETVELD, M. T. ; STRØMME, A. ; ERICKSON, P. J.: Atmospheric Ionization Module Osnabrück (AIMOS): 3. Comparison of electron density simulations by AIMOS-HAMMONIA and incoherent scatter radar measurements. In: Journal of Geophysical Research: Space Physics 116 (2011), Nr. A8. `http://dx.doi.org/10.1029/2010JA016300`. – DOI 10.1029/2010JA016300.

[Wissing et al. 2010] WISSING, J. M. ; KALLENRODE, M.-B. ; WIETERS, N. ; WINKLER, H. ; SINNHUBER, M.: Atmospheric Ionization Module Osnabrück (AIMOS): 2. Total particle inventory in the October–November 2003 event and ozone. In: Journal of Geophysical Research: Space Physics 115 (2010), Nr. A2. `http://dx.doi.org/https://doi.org/10.1029/2009JA014419`. – DOI https://doi.org/10.1029/2009JA014419.

[Yakovchuk und Wissing 2019] YAKOVCHUK, O. ; WISSING, J.M.: Magnetic local time asymmetries in precipitating electron and proton populations with and without substorm activity. In: Annales Geophysicae 37 (2019), Nr. 6, 1063–1077. `http://dx.doi.org/10.5194/angeo-37-1063-2019`. – DOI 10.5194/angeo–37–1063–2019.