Fatigue Simulation Using Phase-Field Modeling to Predict the Crack Propagation Direction for Tooth Root Failure of Helical Gears

FEniCS 2024 - 2024-06-13

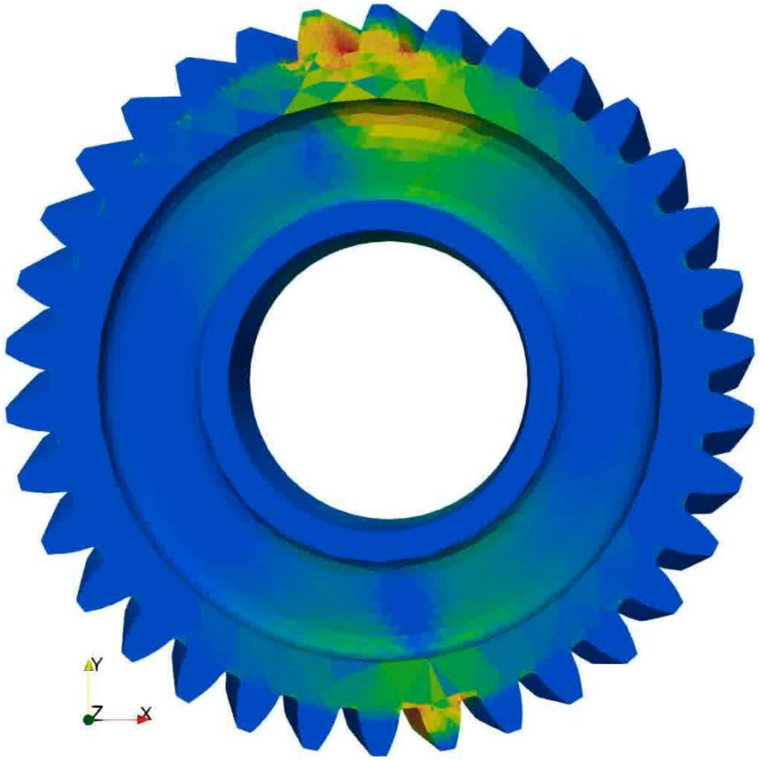
German Aerospace Center (DLR)
DLR-Institute of Test and Simulation for Gas Turbines

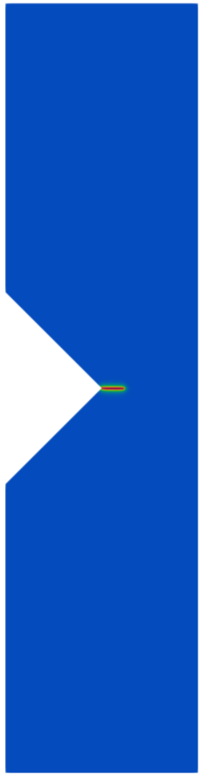
Daniel Martini, Lisa Reischmann

DLR

Agenda

- Propagation direction of gear tooth root cracks
- Governing equations
- Cycle Jump technique
- Submodeling technique
- Model setup
- Crack propagation study
- Outlook

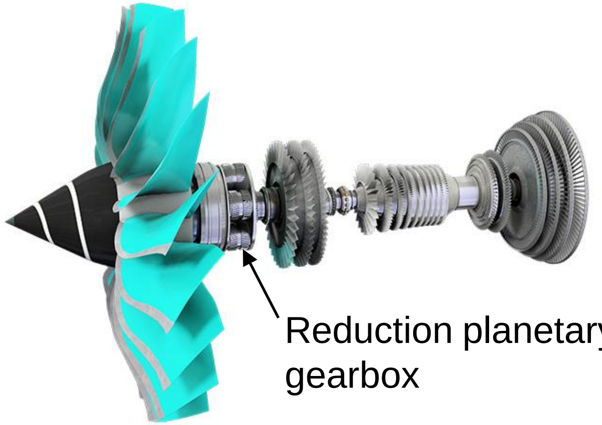




Propagation direction of gear tooth root cracks

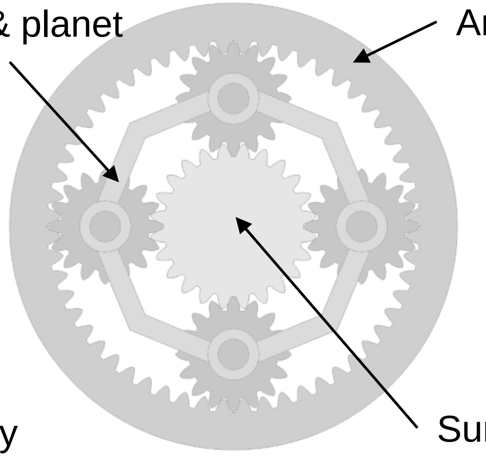
Failure of planet gears

Reduction gearbox between fan and low pressure turbine



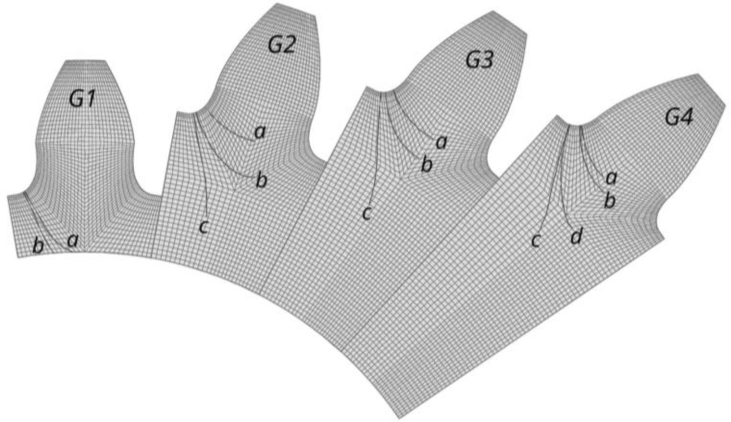
UltraFan Concept [Source: Rolls-Royce plc,]

Planetary gearbox with fixed annulus



Planetary gearbox [Source: wikimedia.org, CC0]

Crack directions dependent on rim thickness and shaft fits



Crack directions [Source: Güven 2022]

- Reduction gear box allows higher bypass ratio of the aeroengine.
- Containment to be ensured to show airworthiness of the gear box.
- Rim cracks in planet gears may cause uncontained bursting.

Governing Equations (following Seleš 2021)

Quasistatic balance of momentum

$$div\,\boldsymbol{\sigma} + \boldsymbol{b} = \boldsymbol{0}$$

$$\boldsymbol{u} - \bar{\boldsymbol{u}} = \boldsymbol{0}$$

$$\boldsymbol{t} - \bar{\boldsymbol{t}} = \boldsymbol{0}$$

Phase-field model of fatigue

$$(1-\varphi)\mathcal{H} - (\varphi - l^2\Delta\varphi) = 0 \qquad \forall\, \boldsymbol{x} \in \mathcal{B}$$

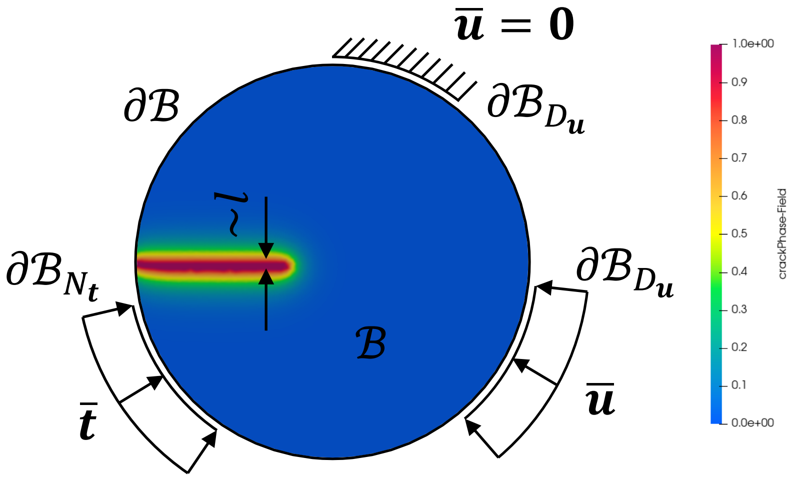
$$\mathbf{grad}\varphi \cdot \boldsymbol{n} = 0 \qquad \forall\, \boldsymbol{x} \in \partial\mathcal{B}$$

$$\mathcal{H} = \max_n \left\langle \frac{\psi^+}{F(\bar{\psi})\psi_C} - 1 \right\rangle_+$$

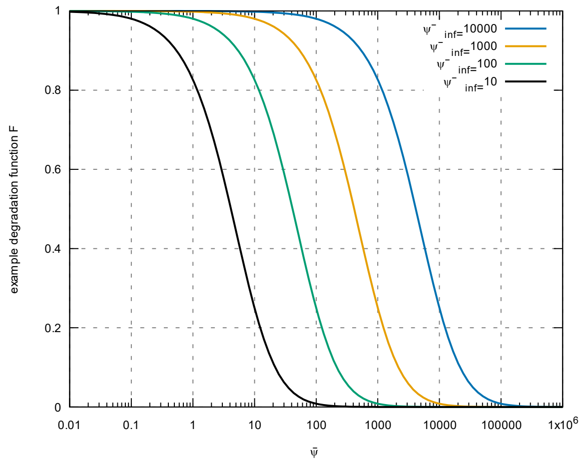
Crack driving force
(with threshold)

$$F(\bar{\psi}) = \left(1 - \frac{\bar{\psi}}{\bar{\psi} + \bar{\psi}_\infty}\right)^2$$

Example fatigue degradation function



Crack phase-field φ of a half-cracked unit square



Example fatigue degradation function $F(\bar{\psi})$

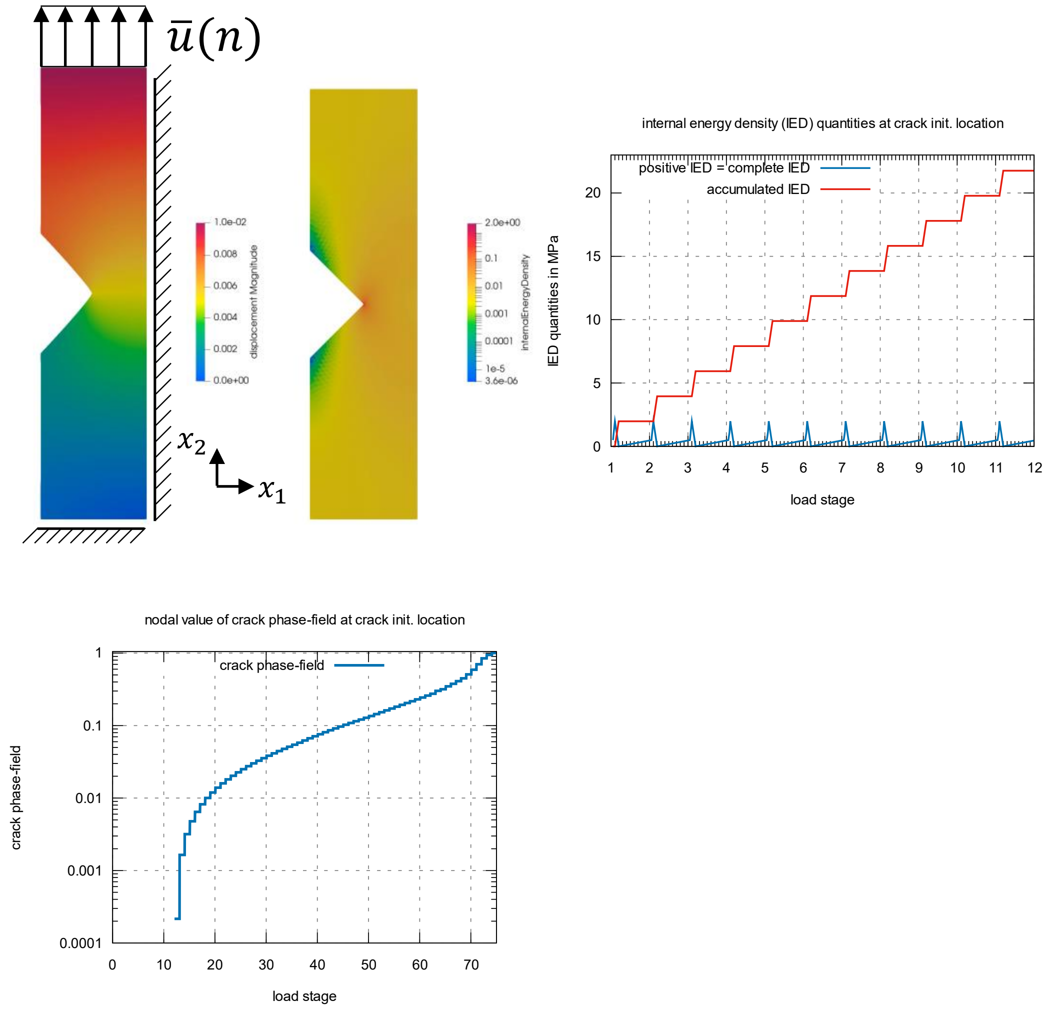
Cycle jump up to the onset of $\mathcal{H}$ (following Seleš 2021)

Preliminary remarks

- Same k cycle steps j for all cycles i
- Load stage $n = i.j$

$$\bar{\psi}_n = \bar{\psi}_{n-1} - \langle \psi_n - \psi_{n-1} \rangle_-$$ accumulation of internal energy density after stage n

- Reversible / elastic behavior assumed.
- Crack driving state function must exceed threshold value (SCT-model).



Example fatigue calculation with cycle jump to onset of $\mathcal{H}$

Cycle jump up to the onset of $\mathcal{H}$ (following Seleš 2021)

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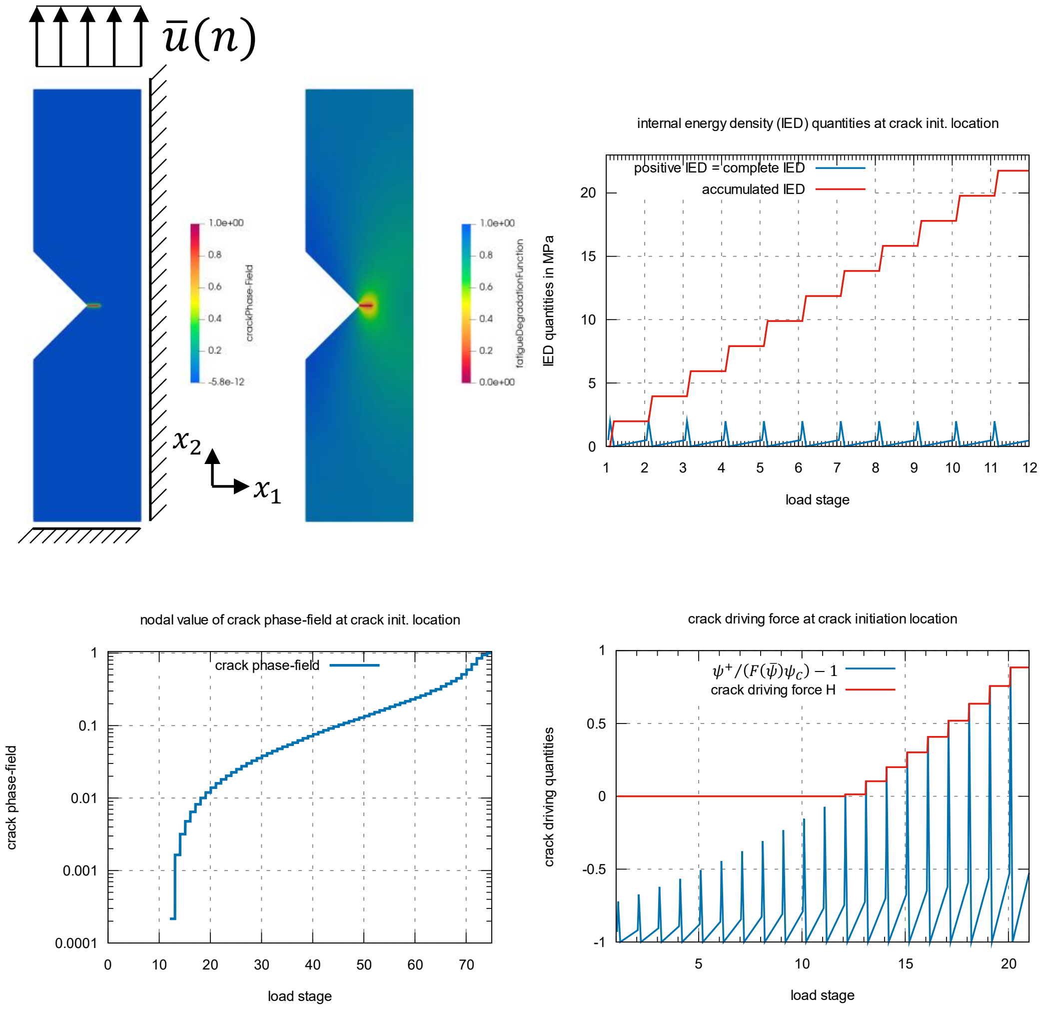
In which cycle N is $\mathcal{H} > 0$?

- $\mathcal{H} = \max_n \left\langle \frac{\psi^+}{F(\bar{\psi})\psi_C} - 1 \right\rangle_+$
- From $\frac{\psi^+}{F(\bar{\psi})\psi_C} - 1 > 0$ follows an estimate for the number of cycles N, when $\mathcal{H} > 0$



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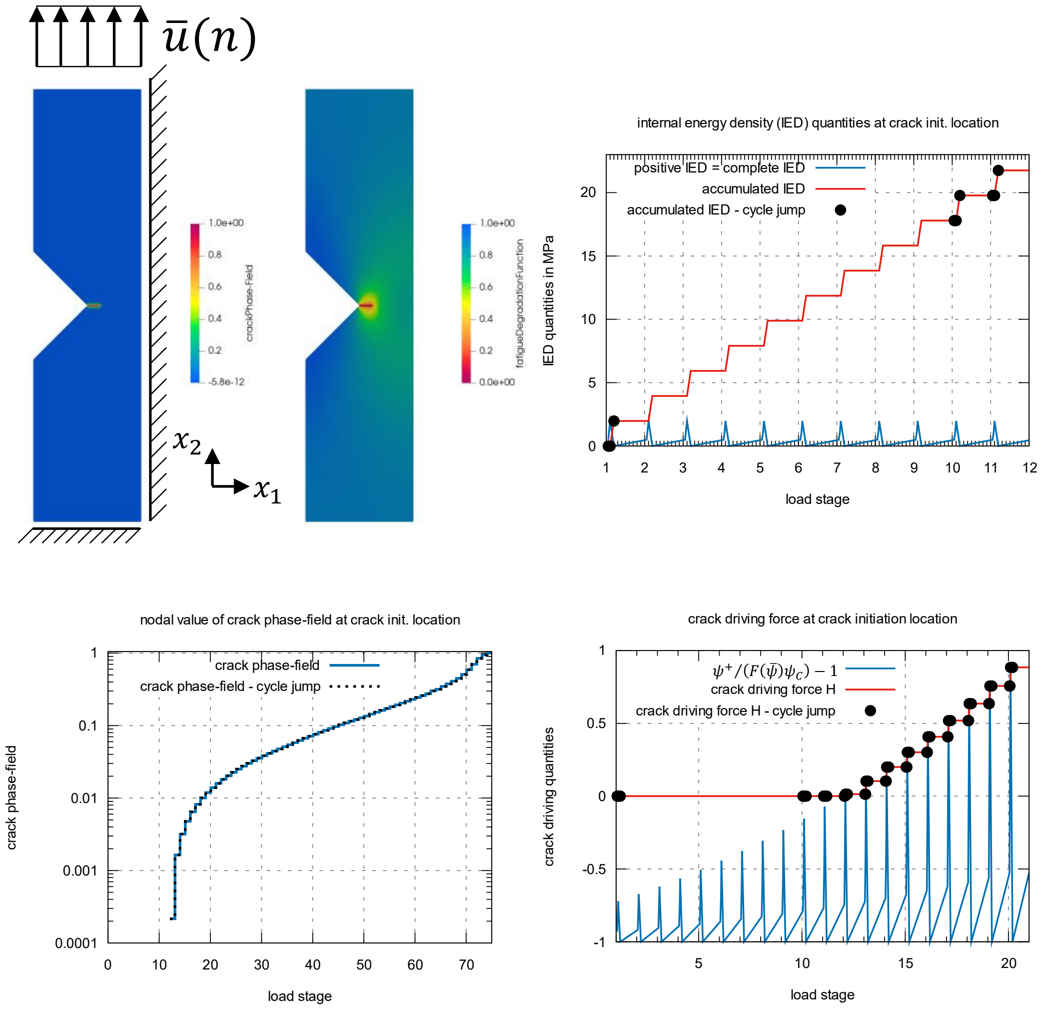


Example fatigue calculation with cycle jump to onset of $\mathcal{H}$

Submodeling technique

Implementation in FEniCSx

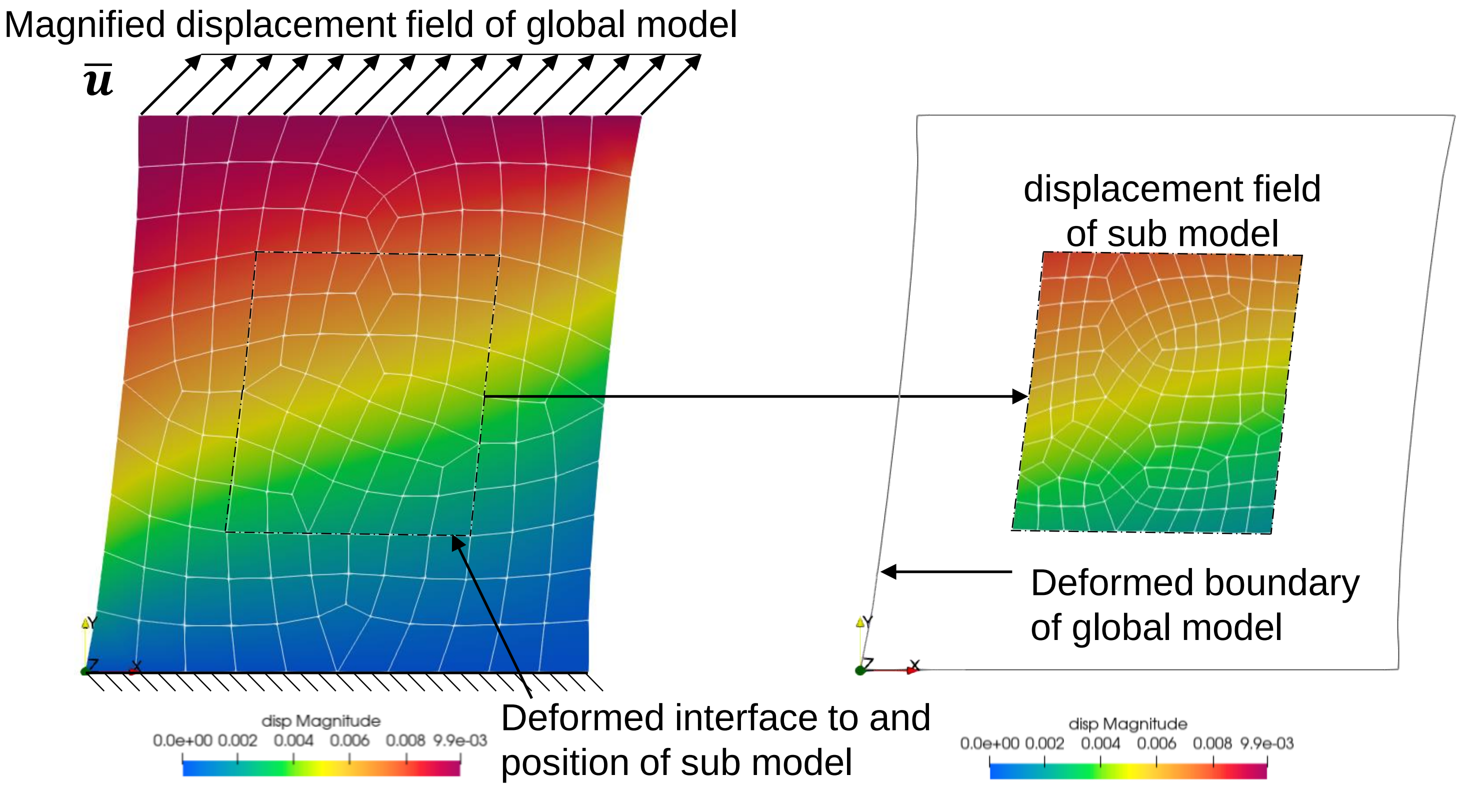
FEniCSx-code

```
# meshes for globalM and sub model
globalModel = ...
subModel = ...

# function spaces
globalU = dolfinx.fem.VectorFunctionSpace(globalModel, ("CG", 1))
subU = dolfinx.fem.VectorFunctionSpace(subModel, ("CG", 1))

# degrees of freedom of sub model on interface
interfaceOfSubModel = ...
dofsOfSubModel = dolfinx.fem.locate_dofs_topological(subU, localDomain.topology.dim-1, interfaceOfSubModel)
dofCoordinatesOfSubModel = subU.tabulate_dof_coordinates()[dofsOfSubModel]

# bounding box tree and colliding cells of global model
boundBoxTreeOfGlobalModel = dolfinx.geometry.BoundingBoxTree(globalModel, globalModel.topology.dim)

cellsOfGlobalModel = []
pointsOnProcOfSubModel = []

# 1st filtering: find cells whose bounding-box collide with the points
cellCandidatesOfGlobalModel = dolfinx.geometry.compute_collisions(boundBoxTreeOfGlobalModel,
                              dofCoordinatesOfSubModel)

collCellsOfGlobalModel = dolfinx.geometry.compute_colliding_cells(globalModel,
                         cellCandidatesOfGlobalModel, dofCoordinatesOfSubModel)
for i, point in enumerate(dofCoordinatesOfSubModel):
    if len(collCellsOfGlobalModel.links(i))>0:
        pointsOnProcOfSubModel.append(point)
        cellsOfGlobalModel.append(cellsOfGlobalModel.links(i)[0])

pointsOnProcOfSubModel = np.array(pointsOnProcOfSubModel, dtype=np.float64)

# Interpolation from
uGlobal = dolfinx.fem.Function(globalU)

to interface points:
uInterpolatedToSubModelPoints = []
uInterpolatedToSubModelPoints = uGlobal.eval(pointsOnProcOfSubModel, cellsOfGlobalModel)

# write interpolated DOF values to uSub = dolfinx.fem.Function(subU) via uSub.x.array[j]
```

Adapted code based on [Dokken 2022]

Global model

Sub model

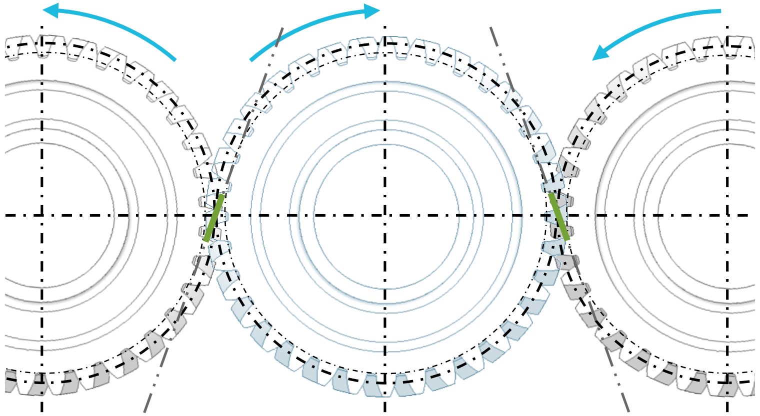


Model setup and overview

FE-model configuration

Test rig configuration

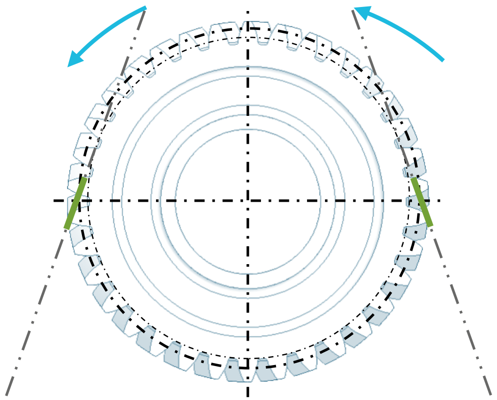
Fixed planes of action und
zones of action



Rotating double helical gears

Change of observer for global FE-model

Rotating planes of action and
zones of action



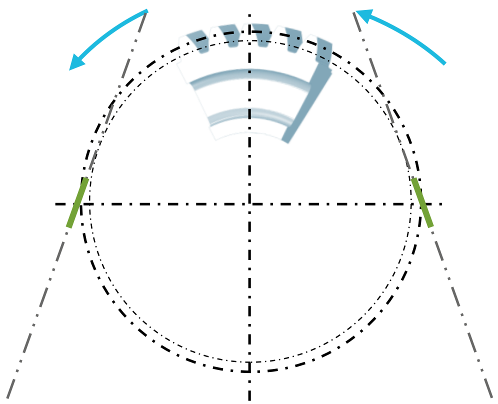
Fixed planet gear

Solution via FEM in FEniCSx:

- Balance of momentum (quasi-static)

Sector model

Rotating planes of action and
zones of action



Fixed planet gear sector

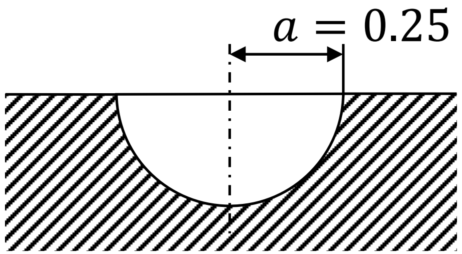
Solution via FEM in FEniCSx:

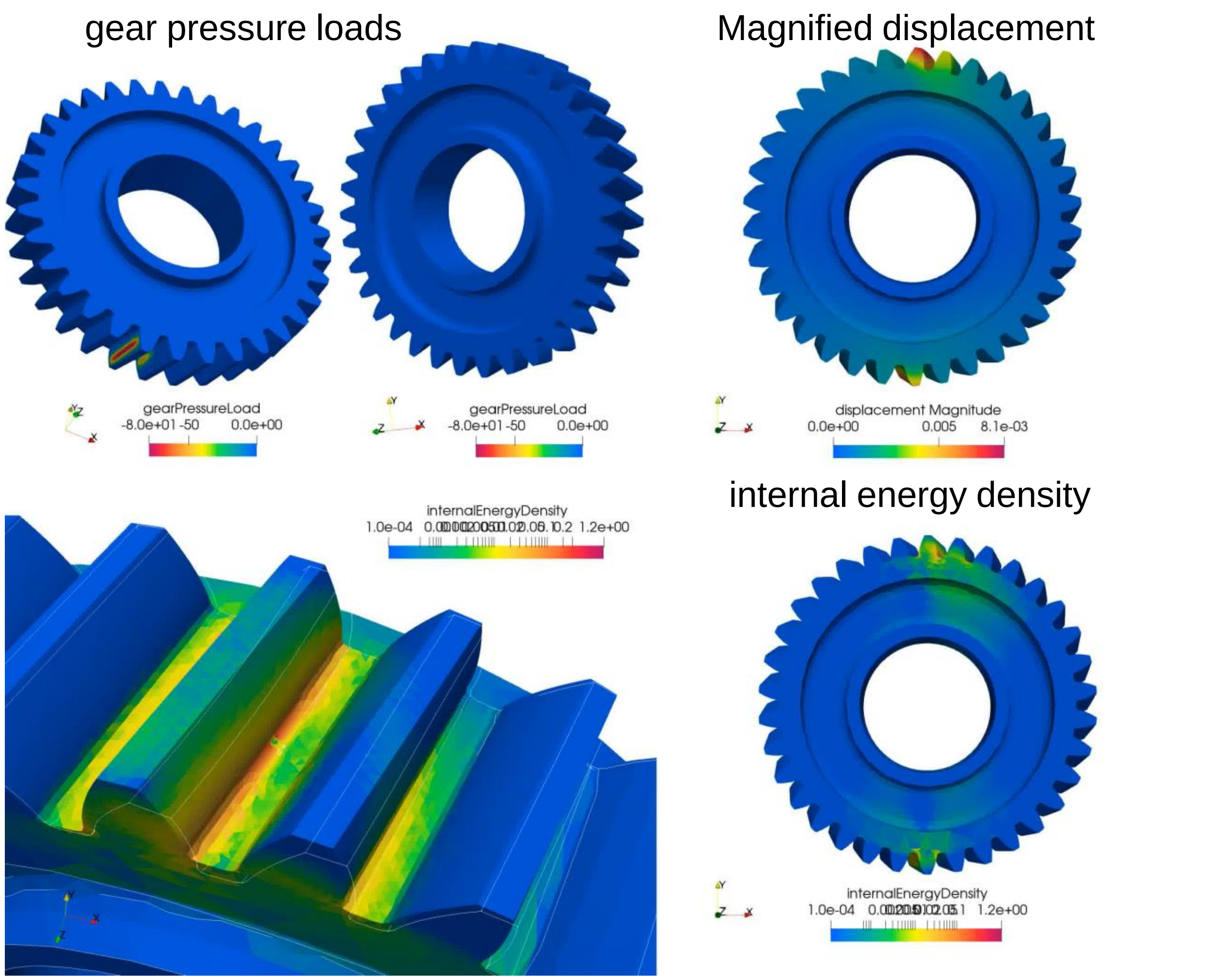
- Balance of momentum (quasi-static)
- Phase-field model of fatigue

Crack propagation study: balance of momentum of global model

Example geometry based on Wu 2022

- Number of teeth $z = 34$
- Modulus $m = 2$
- Pressure angle $\alpha = 20°$
- Helix angle $\beta = 15°$
- Pitch circle radius $r = 35{,}2\ mm$
- Assumed gear line loads
- Semi-circular surface crack as initial crack at tooth root





Balance of momentum of global model with initial crack

Crack propagation study: balance of momentum of global model

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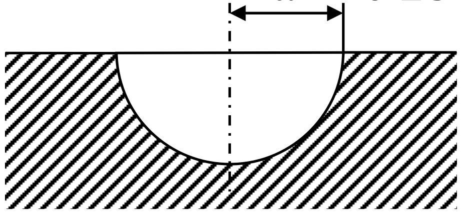
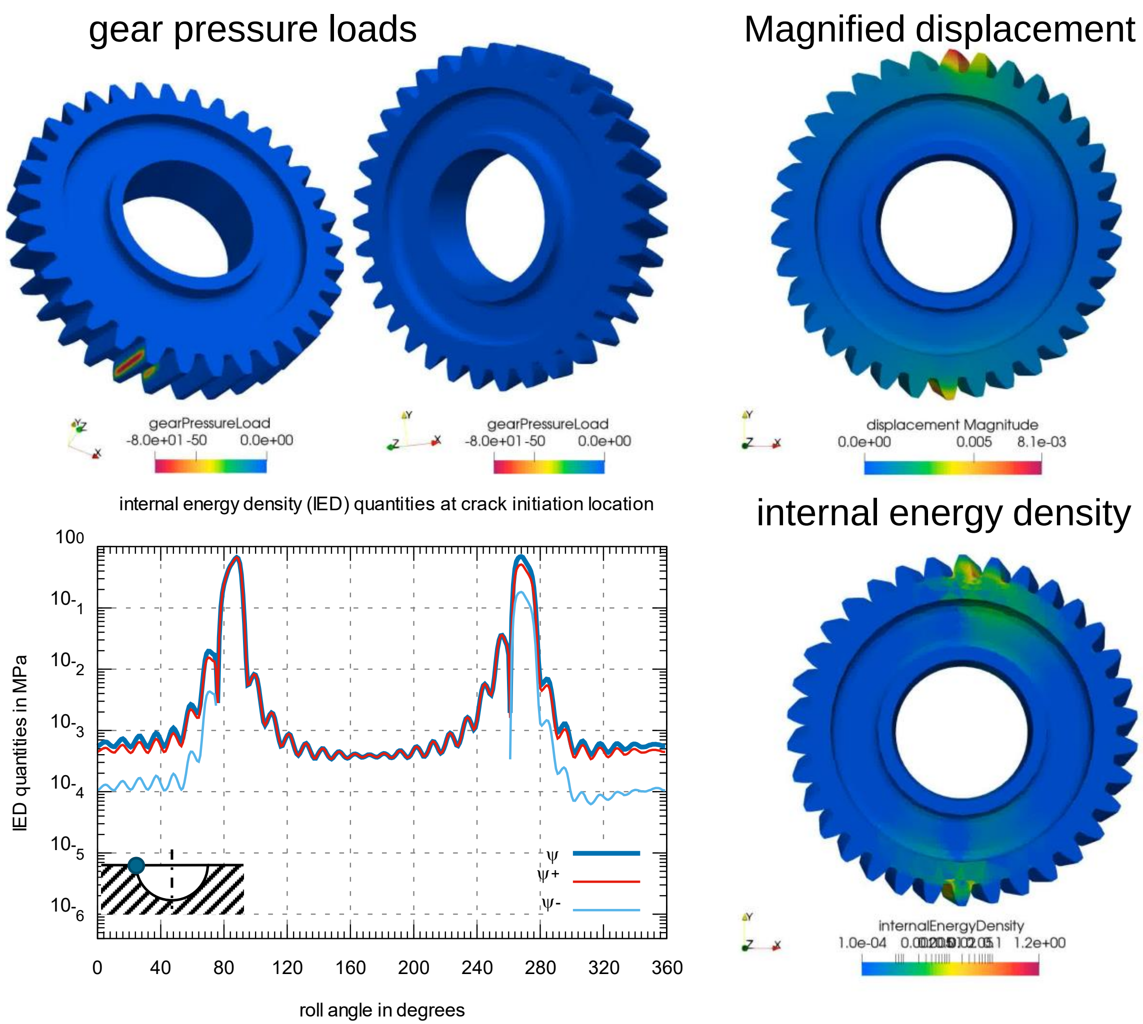
$a = 0.25\ mm$

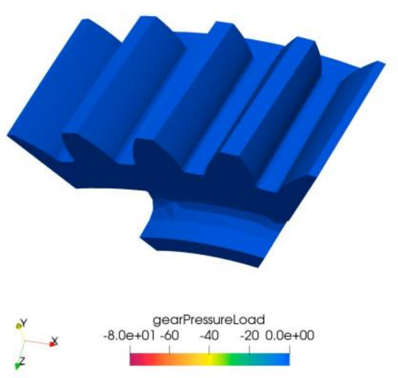




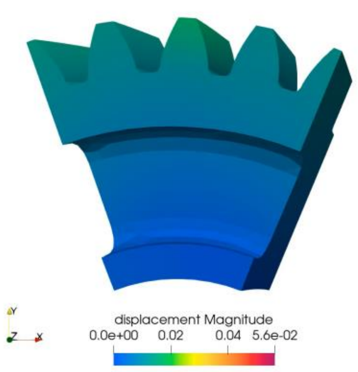
Balance of momentum of global model with initial crack

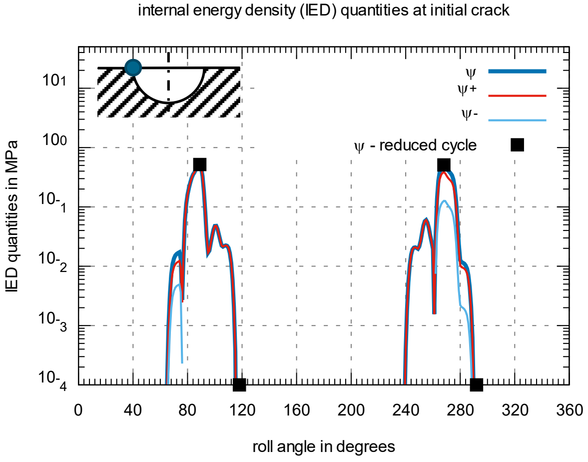
Crack propagation study: phase-field model of fatigue of sector model

gear pressure loads

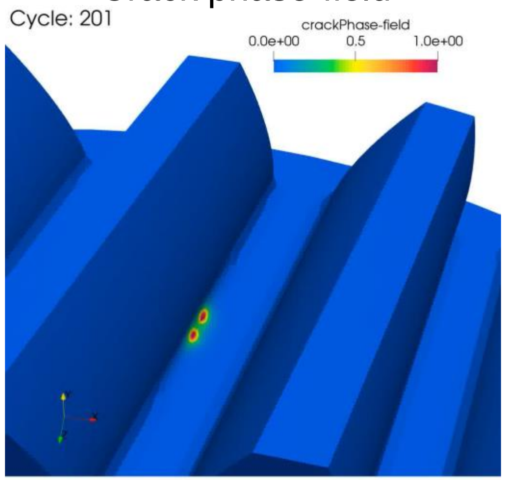


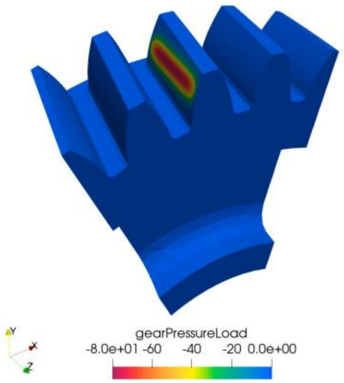
Magnified displacement



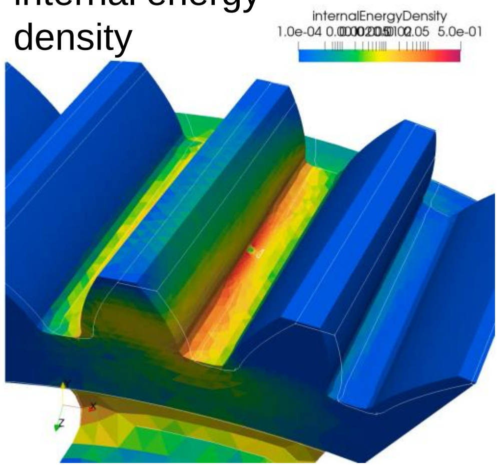


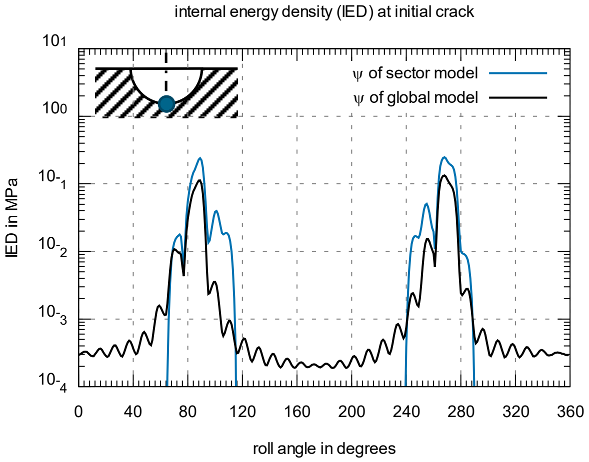
Crack phase-field

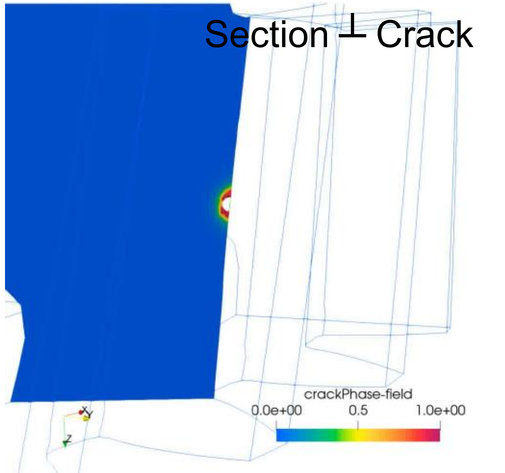




internal energy density







Fatigue simulation for the gear sector model based on gear characteristics from Wu 2022

Outlook

- Sector model as sub model
- Robustness of load-driven calculations
- Extension of the cycle jump to the crack propagation regime
  (in accordance e.g. to Seleš 2021)
- Investigation of observed stress ratios
- Residual stresses from manufacturing steps
- Validation of crack path with the help of tests

Acknowledgements

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THANK YOU FOR YOUR ATTENTION!

Impressum

Topic: Fatigue Simulation Using Phase-Field Modeling to Predict the Crack Propagation Direction for Tooth Root Failure of Helical Gears
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Authors: Daniel Martini, Lisa Reischmann

Institute: DLR-Institute of Test and Simulation for Gas Turbines

BACK-UP

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