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TAC-5, Contrails

Transmission formulation for random stacks of clouds

Klaus Gierens*

Deutsches Zentrum für Luft- und Raumfahrt (DLR) – Institut für Physik der Atmosphäre, Oberpaffenhofen, Germany

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Abstract

We present a formulation for calculations of photon transmission for random cloud stacks (maximum-random overlap) that can be applied in contrail avoidance experiments, for instance. Other applications are not excluded. In contrail avoidance experiments, the application of the method is to estimate in advance whether contrails are detectable with a satellite instrument and whether therefore the result of the experiment can be validated. For an established contrail avoidance procedure, the method can help to decide whether a contrail should be avoided if cirrus clouds in below the contrail would render the expected infrared radiation effect quite low. The method is derived in some detail, and a practical example is provided. For applications it is necessary to empirically determine or estimate a transmission threshold, that likely depends on the viewing geometry, the spatial resolution and the wavelength of the satellite instrument. An IDL-code is available to interested users.

Keywords: clouds and contrails, radiation transmission, satellite data, visibility

1 Introduction

Operational contrail avoidance is an important possibility to quickly reduce the climate effect of aviation. While theoretical studies of this concept have been performed in the past, first practical tests in the real air traffic have been undergone recently (Sausen et al., in press) and further experiments are planned in several national and international projects. The simple recipe for contrail avoidance is to avoid flying in ice supersaturated regions (Gierens et al., 2010), supposed that ice supersaturation can be forecasted reliably (Gierens et al., 2020). Ice supersaturation can occur in clear sky, but often cirrus clouds are present as well. For practical contrail avoidance, cirrus clouds should be taken into consideration for two reasons: first, the contrail climate effect is reduced if there are cirrus clouds nearby (Schumann et al., 2012). Second, in a practical trial it is necessary to check the success of avoidance maneuvers and for this purpose it is necessary that contrails can be seen, mainly in satellite imagery. Evidently, cirrus clouds hamper the detection of contrails in satellite images.

Thus, in the presence of cirrus clouds, their coverage and their optical thickness can be taken into account while contrail avoidance is planned, and one can decide in advance whether contrail avoidance would be beneficial for climate and whether a trial could be validated successfully. If clouds are expected to inhibit a successful validation, contrail avoidance trials should be postponed. This helps to minimise additional work load for air traffic controllers.

*Corresponding author: Klaus Gierens, Deutsches Zentrum für Luft- und Raumfahrt (DLR) – Institut für Physik der Atmosphäre, Münchener Str. 20, 82234 Weßling, Germany, e-mail: klaus.gierens@dlr.de

Air traffic control receives aviation weather data from weather services. If contrail avoidance was standard practice, the weather data would need to contain information on regions where persistent contrail formation was to be expected, based on temperature and relative humidity on flight levels. For the decision whether clouds may disturb the mitigation actions too much, the weather data should additionally contain information on cloudiness and expected detectability of contrails. Such information, cloud fraction and their optical thickness, is given in the weather forecast models. These fields can be combined to an effective transmission probability which informs the user whether a contrail in a certain flight level will, on average, be detectable or not. This information should be provided to all stakeholders who receive aviation weather forecasts.

In this paper I will show how a product "effective transmission" can be provided. This will be derived in the next section. Section 3 gives a short example, while Section 4 ends the paper with a short discussion and a summary.

2 Theory

Consider a numerical weather or climate model with L vertical levels, numbered from top to bottom with an index $\ell \in [1, L]$. In the following, we constrain ourselves to one model column (that is, we consider a certain longitude-latitude grid point). Let b_ℓ be the cloud fraction in level ℓ, with $0 \leq b_\ell \leq 1$. Let the optical thickness of the cloud be uniformly $\tau_\ell \geq 0$, such that the corresponding transmission probability for this cloud is $t_\ell = e^{-\tau_\ell}$.

For the following derivations it is advantageous to use the transmission probability (short "transmission")

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instead of the optical thickness, just because it is a probability, a property that it shares with the cloud fraction, which can be considered a probability as well. This character of cloud fraction allows to compute the total cloud cover over the L levels using common assumptions of maximum-random overlap (Geleyn and Hollingsworth, 1979; a good illustration can be found in Hogan and Illingworth, 2000). Concretely, clouds located in adjacent levels are treated as one large cloud that is vertically stacked as narrowly as possible, such that its cloud cover equals the maximum cloud fraction of the considered levels. Clouds that are separated by one or more clear levels are treated as different clouds which overlap randomly. Assume, there are N cloud blocks, each consisting of maximally overlapping cloud layers. Let them be indexed with $1 \le n \le N$ and let their block-individual maximum cloud cover be $B_n = \max(b_i)$ where the index i runs over all levels that belong to cloud block n. The total coverage over all separated cloud blocks then is

$$B = 1 - \prod_{n=1}^{N} (1 - B_n). \tag{2.1}$$

If clouds were all optically thick (transmission zero), then transmission calculations could be done with cloud fractions alone. But in particular cirrus clouds are often translucent, so that a contrail underneath thin cirrus can sometimes be seen from a satellite. Therefore, for such and similar questions it is necessary to take the transmission of the stacked clouds into account.

For a single level ℓ with cloud fraction b_ℓ and transmission t_ℓ the mean (or effective) transmission, T_ℓ, is the weighted mean of the transmissions in the clear and the cloudy parts with weights $1 - b_\ell$ and b_ℓ

$$T_\ell = (1 - b_\ell) \times 1 + b_\ell \times t_\ell = 1 - b_\ell(1 - t_\ell). \tag{2.2}$$

Now assume two adjacent partly cloudy layers. Let them be indexed for the moment with numbers 1 and 2. Assume $b_1 \ge b_2$. If this is not the case, we can simply change the indexing. In this situation the effective transmission through the two levels is a weighted mean over the clear part (fraction $1 - b_1$), over the part that is only covered by the larger cloud (fraction $b_1 - b_2$) and the part covered by both clouds (fraction b_2). The mean transmission through both levels is thus

$$\begin{aligned} T_{1,2} &= (1 - b_1) + (b_1 - b_2)t_1 + b_2 t_2 t_1 \\ &= 1 - b_1(1 - t_1) - b_2(1 - t_2)t_1. \end{aligned} \tag{2.3}$$

This pattern can be extended to three and more cloudy levels. Each time we need to first sort the levels according to their individual cloud fractions. For this let us introduce indices such that

$$b_{\ell_1} \ge b_{\ell_2} \ge b_{\ell_3} \ge \cdots \tag{2.4}$$

With this ordering, the effective transmission through k adjacent cloudy levels is

$$T_{1,k} = 1 - \sum_{i=1}^{k} b_{\ell_i}(1 - t_{\ell_i}) \prod_{j=1}^{i-1} t_{\ell_j}, \tag{2.5}$$

where the product is to be taken as 1 for $i = 1$.

One can easily check that this formula reduces to the maximum overlap formula in the case that all clouds are opaque, that is, $t_{\ell_i} = 0$ for all i. Then, $T_{1,k} = 1 - b_{\ell_1}$, and b_{ℓ_1} is the maximum cloud fraction according to the stipulated ordering. Note also that effective transmissions cannot be identified with actual optical thicknesses in the system, that is, there is no $\tau_{1,k}$ corresponding to $T_{1,k}$ because $T_{1,k} \ne T_{1,m}T_{m+1,k}$. For later use, the part of transmission from between levels k and 1, that is, through the whole cloud stack is calculated as if there was one big cloud with a cloud fraction equal to the maximum value b_{ℓ_1}:

$$T^c_{1,k} = \frac{T_{1,k} - 1 + b_{\ell_1}}{b_{\ell_1}}. \tag{2.6}$$

The above procedure has to be followed for all blocks of adjacent cloud layers individually, that is, for other than the uppermost cloud block the transmissions are computed in the same way ignoring for the moment that there are other clouds above and below. Through this procedure, preliminary transmissions are computed for each block. For the uppermost cloud block, the calculated transmissions are already the final ones, but for the lower blocks the effect of the clouds above needs to be incorporated.

For each cloud block except the uppermost one the calculated transmissions are now adapted for the effect of the clouds above. This is done in a random-overlap sense, because we consider the different cloud blocks as independent. Above, cloud transmissions and maximum cloud fractions at the bottom of each cloud block have been defined. Let there be N cloud blocks stacked above each other with $N - 1$ clear layers between them. Let furthermore T^c_n and B_n be the cloud transmission and maximum cloud fraction for each block $n \in [1, N]$. We define the following function:

$$W(n) = \prod_{i=1}^{n-1} [1 - B_i(1 - T^c_i)], \quad W(1) = 1 \tag{2.7}$$

Note that $W(n)$ is simply the product of the effective (block) transmissions, T_i, but in the form that uses cloud transmissions the similarity between this equation and the formulation for cloud cover in the random-overlap situation becomes clearer. If one was not interested in the transmission profile within the individual clouds, $W(n), 1 < n \le N$ would be the expected transmission below each of the N clouds. For the cloud layers that belong to, say, cloud n, the effective transmissions calculated above ignoring the other clouds are now multiplied with $W(n)$, and the transmission in the cloud free layers is set to the effective transmission ($T_\cdot$, not $T_\cdot^c$) at the bottom of the cloud above.

The result of this is a profile of effective transmission probabilities from top ($\ell = 1$) to bottom ($\ell = L$). $T_{1,\ell}$ is a monotonically decreasing function of ℓ. In practice, one can stop the calculation once a critical level is reached for which one expects no longer good visibility. The critical level must be determined experimentally.

3 Example

As an example let us consider 16 levels, partly cloudy, partly clear, see Fig. 1. Let the cloud fractions from top to bottom be

$$b = [0.25, 0.45, 0.6, 0.5, 0.3, 0.4, 0., 0., 0.3, 0.6, 0.4, 0.5, 0.25, 0., 0.01, 0.] \quad (3.1)$$

and the corresponding transmissions through the individual cloud layers

$$t = [0.02, 0.4, 0.9, 0.25, 0.3, 0.7, 1., 1., 0.8, 0.96, 0.3, 0.4, 0.85, 1., 0.4, 1.] \quad (3.2)$$

Of course, the transmissions are unity for clear levels. In this example there are three cloud blocks that need to be treated according to the maximum overlap assumption. These extend from level 1 to 6, from 9 to 13, and there is a single cloudy level 15. The remaining levels are clear.

Now, Eq. (1) is used to compute preliminary transmissions within each of the three blocks. These calculations lead to values from 0.755 to 0.513 for the uppermost block, values from 0.94 to 0.57 for the second, and the preliminary transmission for the third block is 0.994.

The cloud transmissions at the bottom of the upper two clouds (the third one is not needed since there are no more clouds below) are 0.188 and 0.284. Both maximum cloud fractions are 0.6 such that $W(1) = 0.513$ and $W(2) = 0.293$. Thus, the preliminary transmissions of the second cloud are multiplied with $W(1)$, giving values from 0.48 to 0.293, and the transmission of the lowermost cloud is 0.291 after correction with $W(2)$. The figure shows the resulting transmission profile.

A couple of things can be noted. The uppermost two cloud layers reduce the transmission considerably. The next thin one has a small effect, but the following one is thick again and accordingly the transmission is reduced substantially. But then the next thick cloud layer in level 5 has hardly any effect, which is surprising on first sight. The reason for this is that the cloud fraction in level 5 is smaller than in level 4, that is, cloud level 5 is completely covered by another thick cloud in level 4. Therefore its effect on transmission is surprisingly small. A similar situation is given in levels 11 and 12, but the other way round: the lower level has the larger cloud fraction. Thus, this cloud is not completely in the shadow of another thick cloud and therefore it causes a notable reduction of transmission. This shows, that not only the cloud fractions and their individual transmissions are important for the profile calculation but their stacking order as well.

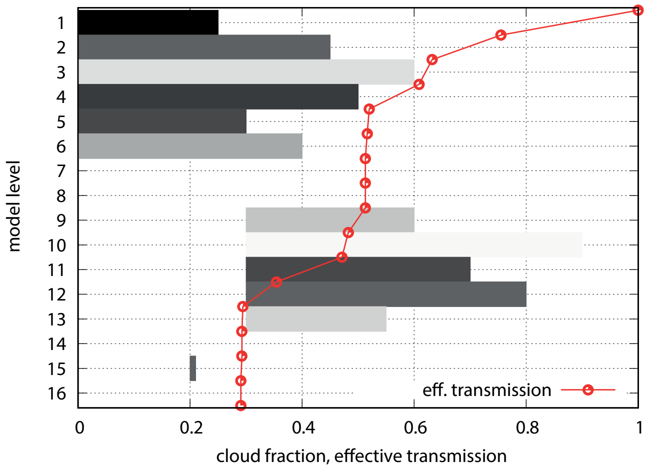

Figure 1: Example of a cloudy scene with three cloud blocks extending over levels 1 to 6, 9 to 13, and the single level 15. The clouds are stacked with maximum overlap within each block, but the three blocks overlap randomly. Clouds within the levels are indicated by the grey bars whose lengths represent the cloud fractions and whose darkness the individual transmissions from translucent (light grey) to opaque (dark grey). The red line shows the effective transmission from top to the bottom of the respective layer. Each cloud causes a decrease of this function. Clear layers leave the transmission function unchanged.

4 Discussion and summary

In the present paper I propose a method to decide in advance whether aircraft should be tactically rerouted for contrail avoidance (that is in most cases, whether they should change flight level) given the situation of clouds in the close environment. If there are too many and too thick cirrus clouds close to the aircraft flight track, the contrail may not get climatically effective, and it might be difficult to check the result of rerouting if clouds hamper the contrail detectability from space.

In order to avoid unnecessary additional work for air traffic controllers and pilots, one can use cloud fractions and their optical thicknesses from the output of a weather forecast model and compute the transmission profile for each longitude and latitude and each time where persistent contrails are expected and where one thinks of avoiding them. A transmission profile gives for each flight level the probability that a contrail on this level can be seen through the overlying cirrus clouds. The transmission at flight level ℓ is the probability that a photon emitted in upward direction reaches space and can thus be detected by a satellite instrument.

The proposed method is based on given cloud fractions and optical thicknesses. The latter are equivalent to transmission probabilities for individual clouds. All this information is combined in the sense of maximum and random overlap, in a way similar to classical treatment of cloud stacks for the radiation calculation in models. The calculation is based on data, the weather model can offer. The method works in principle for each wavelength for which the optical thickness and transmission can be derived (or estimated) from model quantities.

The method contains a free parameter that needs to be determined empirically. This is a threshold value for the effective transmission which marks the flight level below which no contrail avoidance trials should be performed because of a low chance for validation possibility. This free parameter is also required because the method does not take into account a number of boundary conditions, for instance the viewing geometry. The calculation proceeds in the vertical (nadir direction), while the observing satellite is not in the zenith. The spatial resolution of the instrument is not considered, but evidently an instrument can better see between overlapping clouds with higher spatial resolution than one with lower spatial resolution. The contrail detectability between clouds can also vary with the solar zenith angle (if the detection method uses channels in the visible wavelength range) and it might therefore even be beneficial to use a variable threshold that accounts for the illumination geometry. Finally, cloud inhomogeneity cannot be considered, because clouds are homogeneous in the model world.

The same method can as well be used for a related purpose. The warming effect of contrails is generally low if cirrus clouds exist not far below the contrail such that the temperature difference between contrail and cirrus is small. Also in such a case one can look at the transmission function (for infrared radiation) to determine or estimate how much radiation from the ground or the lower troposphere can reach and be absorbed by the contrail. For this purpose another threshold value would need to be determined, in order to decide in practical application whether the a contrail needs to be avoided or not.

Evidently, it needs a lot of contrail and cirrus scenes to statistically determine useful thresholds. This is future work. For ongoing experiments, the thresholds may be estimated and later adapted if the initial estimate turns out unsatisfying.

An IDL-code for the calculation of transmission profiles is available and can be obtained from the author on request.

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References

Geleyn, J.F., A. Hollingsworth, 1979: An economical analytical method for the computation of the interaction between scattering and line absorption of radiation. – Contrib. Atmos. Phys. **52**, 1–16.

Gierens, K., P. Spichtinger, U. Schumann, 2010: Ice Supersaturation. – In: Atmospheric Physics. Background – Methods – Trends. Schumann U. (Ed.), Springer, Heidelberg, Germany, 135–150.

Gierens, K., S. Matthes, S. Rohs, 2020: How Well Can Persistent Contrails Be Predicted? – Aerospace **7**, 169, DOI: 10.3390/aerospace7120169.

Hogan, R.J., A.J. Illingworth, 2000: Deriving cloud overlap statistics from radar. – Quart .J.Roy. Meteor. Soc. **126**, 2903–2909.

Sausen, R., many coauthors, 2023: Can we successfully avoid persistent contrails in the real world? – Meteorol. Z.

Schumann, U., B. Mayer, K. Graf, and H. Mannstein, 2012: A parametric radiative forcing model for contrail cirrus. – J. Appl. Meteor. Climatol. **51**, 1391–1406.