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NEW DEVELOPMENTS IN SODIX

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Abstract

The source localization method SODIX reconstructs a distribution of directive point sources from the data in the cross-spectral matrix of a microphone array in an optimization scheme. It has been first presented by Michel and Funke in 2008. The first Matlab version of SODIX has since been improved and rewritten in Python 3 by Oertwig. The improvements were mainly in the processing speed and the stability of the code. The mathematical formulation has been improved, e.g. by describing the sources using complex values, simplifying the formulation of partial derivatives of the cost function, and by using an improved starting solution for the iterations. SODIX has been extensively used and verified using synthetic and experimental data. The method is being used in industry by engine and airframe companies, other groups have started to develop their own flavours of SODIX, one of these is implemented in the open-source framework for acoustic beamforming Acoular. Oertwig has since extended the SODIX method from a formulation that was based on incoherent point sources to fully and partially coherent sound sources, enabling the application of SODIX to engine tones. This latest extension of SODIX has been verified using synthetic data from the DLR noise prediction tool PropNoise and has been successfully applied to engine noise data measured in outdoor and indoor testbeds.

1 INTRODUCTION

The source localization method SODIX is able to determine the positions, and the individual amplitudes of sound sources in the directions of every single microphone in an array. This capability to resolve the directivity of the sound sources makes it a useful tool for the analysis of the sound fields of aircraft engines that generate highly directive sound fields.

Since the first presentations of SODIX by Michel and Funke [1, 2], the method has been continually developed further [3] and tested against other advanced source localization methods [4, 5] and applied in different industrial and scientific projects [3, 6–14]. While the development has been driven by the demands of the acoustic analysis of turbofan engines in freefield tests or in enclosed test cells, the method is a general source localization method that can in principle be applied to any problem with stationary sound sources, especially when strong directivity effects can be expected.

Taking over the development from Funke, Oertwig reviewed the method and refactored the code, moving from Matlab to Python 3. He improved the mathematical formulation of SODIX and its gradient based solving method. He has also improved the general understanding of the SODIX method and how it compares with other source localization methods. Finally, Oertwig extended the analysis from strictly broadband sources to tonal sound sources that can be partially or fully coherent.

The aim of this paper is to present a brief overview on the development of SODIX and point to the relevant publications, because the original doctoral theses of Funke [15] and Oertwig [16] are written in German and may not be accessible to international readers. The development of SODIX will be explained together with the most important extensions of the method and its applications.

2 The original SODIX method by Michel and Funke

2.1 The source model of directive sound sources

The basic idea behind SODIX, driven by the need to resolve directivity effects from sources of turbofan aircraft engines, is to evaluate the positions and the directional amplitudes of the sound sources from the data measured with a microphone array. SODIX can be seen as generalization of the Spectral Estimation Method (SEM) by Blacodon und Élias [17, 18]. Figure 1 visualizes the difference between the SEM and the SODIX approach: while SEM uses monopole sources that cannot resolve sound sources with non-uniform directivities, the SODIX model allows different source amplitudes in the direction of every microphone and is therefore able to resolve the directive sound field.

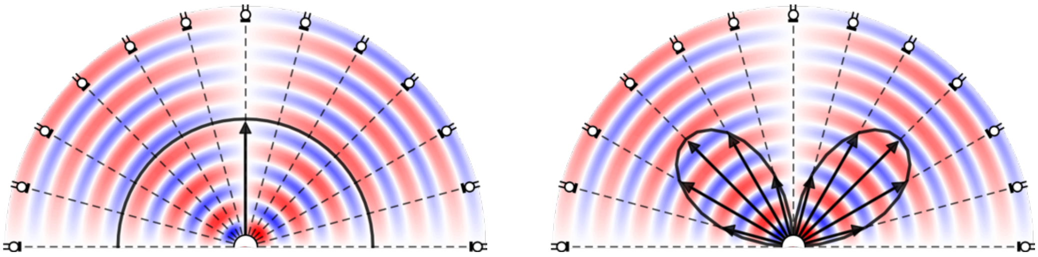


Figure 1: Comparison of a monopole source model with uniform directivity in the direction of all microphones (left) with the SODIX model of a directive sound source (right) that allows for different amplitudes in the direction of every microphone. The source is at the bottom centre, the microphones of the array are arranged on a semicircle around it, the colour code indicates the sound wave amplitude and phase of a dipole source.

The cross-spectral matrix (CSM) of the measured microphone array data is approximated by a model CSM

$$C_{mn}^{\mathrm{mod}} = \sum_{j=1}^{J} g_{jm} D_{jm} D_{jn} g_{jn}^{*} \tag{1}$$

with the amplitudes of the directive point sources D_{jm} between all sources $j = 1,...,J$ and all microphones $m = 1,...,M$ and the steering vectors g_{jm}, which describe the propagation of sound between the sources and the microphones. Figure 2 presents a schematic for measurements of a turbofan engine with a linear microphone array and the source directivities D_{jm} that can have different values for one source towards the different microphones. SODIX solves for the directive source amplitudes D_{jm} with a least-squares approach by minimizing the difference between the cross-spectral matrices C_{mn} of the measured data and the modelled distribution of sources with the condition that the amplitudes must remain positive $D_{jm} \geq 0$:

$$F(D) = \sum_{m,n=1}^{M} ||C_{mn} - C_{mn}^{\mathrm{mod}}||^2 \tag{2}$$

2.2 The limitation of the analysis to broadband sources

The classical SODIX by Michel and Funke is based on the assumptions of incoherent sources and real valued source amplitudes. The limitation to fully incoherent sources ignores the contributions from off-diagonal elements in the source directivity and limits the application to sound fields that are made up of strictly broadband sources. For applications to turbofan engines this limitation leads to non plausible source distributions for fan tones that radiate from both the inlet and the bypass of the engine. For the classical SODIX variant, tonal components in the signals have to be ignored and should be removed from the signals in a preprocessing step.

2.3 Assumption of real valued amplitudes

The condition of real valued amplitudes is already implicitly included in the definition of the cross-spectral matrix in Equation 1 where the cross-spectral amplitude of a sound source has real number values in the direction of all microphones. The phase information rests only in the sound propagation that is described in the steering vectors g_{jm}. This source model cannot resolve phase jumps that would occur when multiple radiation lobes are superposed and is only valid for spherical sound propagation within the main lobe of a sound source.

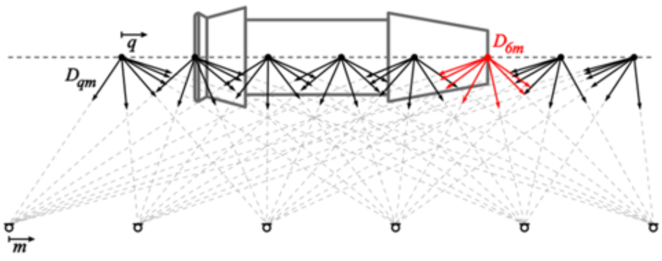


Figure 2: A distribution of directive sources with different source amplitudes D_{jm} in the directions of the microphones in the linear array.

2.4 Regularization strategies

The number of unknowns in the SODIX formulation, i.e. the amplitudes of the sources in the direction of all array microphones, is much greater than the number of unknown values in other source localization schemes that only solve for the position and the averaged amplitude of a source and assume uniform directivity. Therefore, the cost function (Equation 2) usually leads to an ill-posed problem that requires some kind of regularization. The original SODIX formulation uses a regularization strategy that restrains the directivity of the sources to relatively smooth shapes, which is a plausible assumption for broadband sources. The directivity of a source is not allowed to have discontinuities or large local changes in amplitude between neighbouring microphones. The regularization scheme implemented by Funke [15] takes the differences between source amplitudes in the direction of neighbouring microphones into account. This scheme has been implemented in a generalized way that allows the application to any three-dimensional microphone arrangement and is not restricted to linear microphone arrays.

2.5 The solver in SODIX

The unknown source directivities D_{jm} are calculated by minimizing the cost function in Equation 2 (with an added regularization term) using a numerical optimization scheme. Michel and Funke used a conjugate gradient scheme by Rasmussen [19]. The partial derivatives of the cost function $\partial F / \partial D_{jm}$ have been calculated analytically and written into the code in order to improve the convergence and the computational efficiency. The alternative solution of a numerical approximation of the partial derivatives is not as accurate as the analytical solution.

2.6 Limiting the values of the source amplitudes to positive values

The acoustic energy must be preserved because sources with negative amplitudes are not possible, therefore a limitation of D_{jm} to positive amplitudes is required. The Rassmussen solver, however, does not allow for additional constraints on its parameters. The way around this limitation was to substitute the directivities D_{jm} by d_{jm}^2, solve the modified cost function for the source amplitudes d_{jm} and then re-substitute the derived solution for the desired source amplitudes D_{jm} [3].

2.7 Starting conditions for the SODIX solver

The gradient based optimization scheme of SODIX requires an initial guess for the source amplitudes $D_{jm}^{t_0}$ to start the calculation. The choice of the initial guess influences the stability and the convergence of the solver. Funke tested several different schemes [15], among these the simple assumption of a distribution with constant values $D_{jm}^{t_0} = 1$, the application of the solution from the SEM method [17, 18] with uniform source directivity towards all microphones, before he finally settled for an energy-equivalent initial solution that fulfills the constraint for positive source amplitudes and which exactly reproduces the spectral powers of the microphone signals. This initial distribution assumes that all sources j have equal amplitudes in the direction of a particular microphone m and the incoherent superposition of the sound waves from all sources on the microphone position results in the measured auto-power spectrum of the microphone signal. This energy-equivalent initial guess assumes equal values of the source amplitudes from all source points towards one microphone, which is usually quite different from the real distribution of the sources and their directivities.

3 Variants of the classical SODIX formulation

Since the publication of SODIX, several other authors have picked up the idea and developed methods that are built on the idea. A variant of SODIX has been implemented in Acoular, the source localization package that has been developed at TU Berlin [20, 21]. The Acoular version of SODIX is an implementation of the classical SODIX by Michel and Funke with some additions and improvements like the constraint on positive source amplitudes and a simplified regularization strategy. It has been used by Jekosch and Sarradj for the analysis of rotating sound sources [22, 23].

Lian et al. [24] presented SODIX-Bes (SODIX based on beamforming source), an extension of SODIX that combines results calculated with CLEAN-SC into the SODIX model of the cross-spectral matrix and uses an adapted formulation of the cost function. SODIX-Bes first uses the cross-power spectra of the microphone signals to determine the CLEAN-SC results and then models the auto-power spectra of the microphone signals in order to calculate the source directivities. Lian et al. [24] report the analysis of the directive sources on the leading edge of a wing in wind tunnel experiments with good agreement of their results with an analytical model.

4 The further development and improvement of SODIX

The classic SODIX formulation was a huge step forward especially for the analysis of the directive sound sources of turbofan engines. However, some features were still missing that were desirable for the analysis of sound sources of turbofan engines, especially the analysis of tonal sources with mutual coherence. This motivated further work towards a second generation of SODIX.

During the development of the new SODIX method, a number of issues have been addressed:

- the representation of the source amplitudes by complex numbers in order to solve problems with stationary multipole sources and coherent sources,
- the choice of a gradient based solver that directly enforces the condition of positive source amplitudes,
- an improved criterium for the convergence of the solution, and
- an optimized initial guess.

4.1 An improved mathematical formulation of SODIX

Oertwig [16] presented an improved mathematical formulation of SODIX that generalizes the formulation of the problem and makes it more compatible with the formulations of other source localization methods. The new formulation allows the treatment of broadband and tonal sound sources, both problems can be treated as special cases of the general SODIX formulation. He introduced a matrix of weighting factors that allows to pose additional constraints on the expected directivities of the coherence of the sound sources. This leads to two variants of SODIX, which both are implemented in the second generation code, that treat either

- incoherent sources of sound, where the contributions from cross-spectral components of sound sources are not included in the model of the cross-spectral matrix in Equation 1, or
- coherent sources of sound, where all cross-spectral components of the sound sources are modelled. This leads to an ill-posed problem that can only be solved by applying assumptions and additional constraints to the directivities and the mutual coherence of the sound sources. This variant has been implemented for both fully and partially coherent sources of sound.

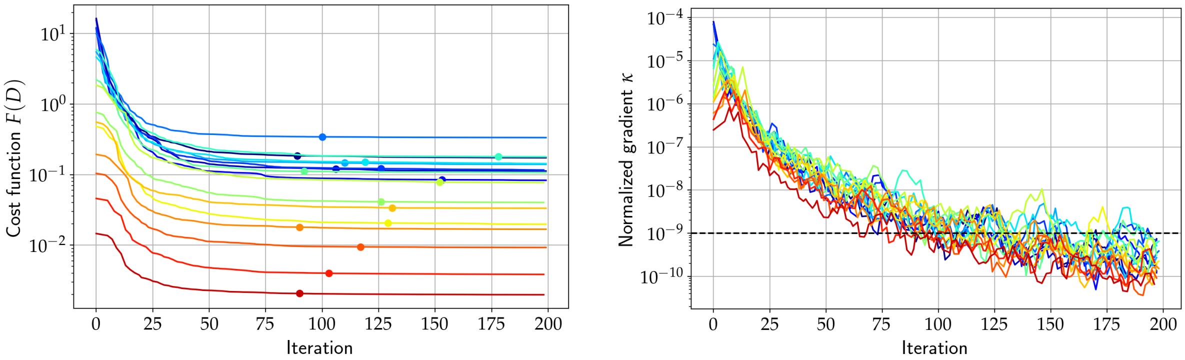


Figure 3: Cost function and normalized gradient of the cost function during the iterative minimization process for an application of SODIX to engine noise measurements on an outdoor test stand. The coloured lines show the corresponding values for the centre frequencies of the one-third octave bands from 50 Hz to 2 kHz (blue to red). The round markers in the cost function indicate the iteration number when the new SODIX version exits the minimization process (from [26]).

4.2 Complex source amplitudes

The classic SODIX formulation represents the source amplitudes by real numbers only and models only the amplitudes, but not the phase of the sound sources. This limits the solution for the source directivity. The representation of the source amplitudes with complex numbers in the second generation code helps to obtain more realistic models of the sound sources.

4.3 Partial derivatives of the SODIX cost function

The classical SODIX already featured pre-calculated analytical solutions of the partial derivatives of the cost function because they lead to more stable solutions and speed up the calculation compared to schemes where the partial derivatives need to be calculated numerically at runtime.

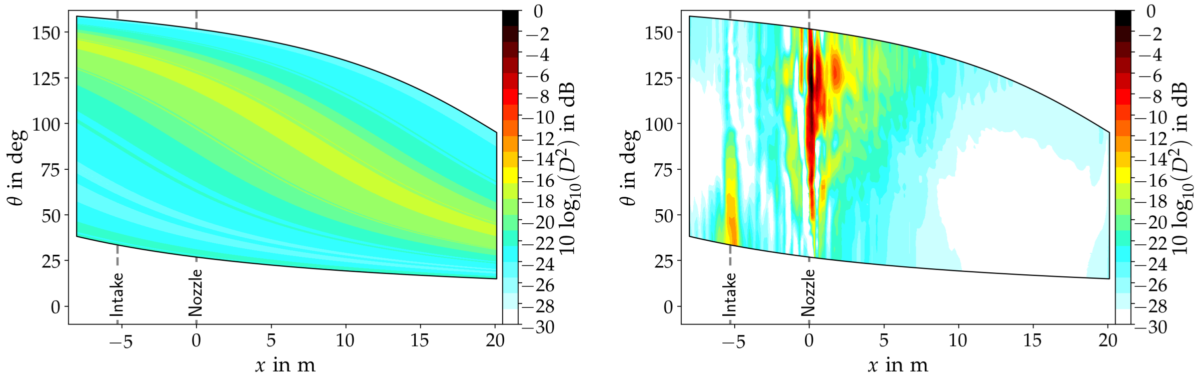


Figure 4: SODIX source plots with the source amplitudes as a function of position and emission angle. Left: for the energy-equivalent initial guess, right: result after the optimization has been calculated in one frequency band that is used as the initial guess for the next frequency band (from [26]).

In the second generation of SODIX, the calculation has been improved by taking complex values into account and by introducing a new formalism for calculating the partial derivatives based on the total derivate of the cost function [25]. The new implementation also performs faster numerically.

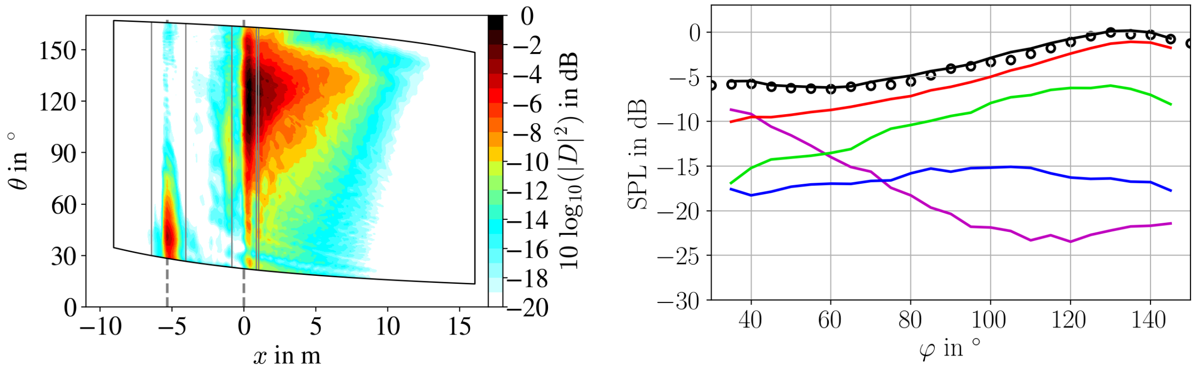
4.4 Regularization

The regularization in the classical SODIX formulation works by restricting the source directivities to relatively smooth shapes. This is achieved by adding a regularization term to the cost function of Equation 2 with a weight factor σ_D. The value for the factor σ_D has to be chosen based on experience. While this approach has some advantages for experimental data that are distorted by noise, it unnecessarily restricts the source directivities. In order to improve this situation, an L_2 regularization has been implemented in the second generation SODIX together with a scheme that adapts the regularization parameter in every step of the iteration.

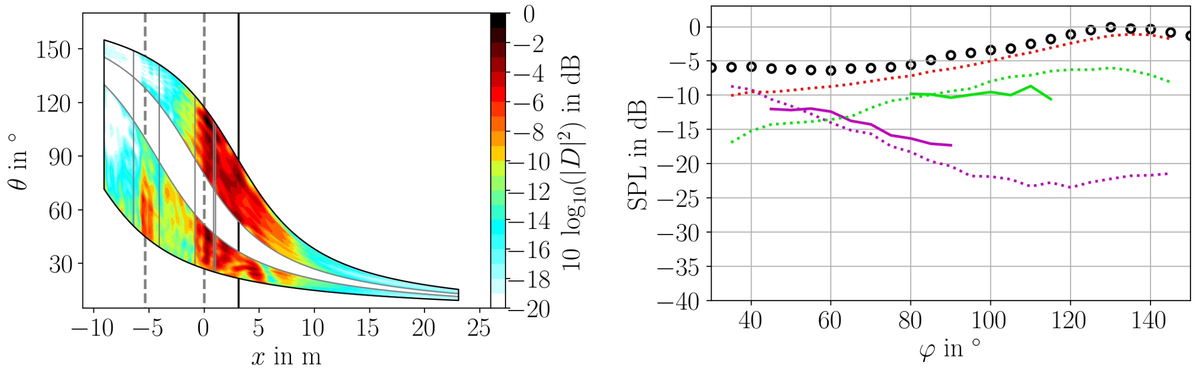
4.5 A new solver

A major improvement of SODIX has been achieved by applying a modern optimization method that includes the constraint for positive source amplitudes directly. The second generation SODIX uses the L-BFGS-B gradient method by Byrd and Zhu et al. [27, 28], which is a variant of the better known BFGS-algorithm by Broyden, Fletcher, Goldfarb und Shanno.

The classical SODIX formulation usually converges to stable solution well within 200 iteration steps and yields sensible solutions already after a few iterations. In order to speed up the calculations, a convergence criterium has been added later that can stop the iterations well before the predefined maximum number of iterations has been reached (see [26]), which cuts the CPU time roughly in half in most cases. Figure 3 presents a typical example for the evolution of the cost function and the normalized gradient of the cost function with the increasing number of iteration steps.



(a) Freefield measurements



(b) Indoor measurements

Figure 5: SODIX results for broadband sources of a turbofan engine in the 400 Hz one-third-octave band . Left: SODIX source directivity charts, right: extrapolated farfield directivity of the total sound field (solid black line) and the source regions at the inlet (violet), nozzle (green), engine casing (blue), and the jet region (red). The dotted lines refer to the freefield results from Figure 5a. Black circles: directivity measured directly with the sparse farfield microphones in the freefield test. (figures from [16]).

4.6 An optimized initial guess to start the optimization

The energy-equivalent initial guess in the original SODIX version is an unrealistic solution for most practical cases (shown on the left-hand side of Figure 4). SODIX calculations are usually performed for many adjacent frequency bands and the assumption that the source distributions do not change much from one narrow band frequency to the next seems reasonable for stationary, broadband sound sources. The second generation SODIX code uses a modified approach: while the calculation for the first frequency band starts with the energy-equivalent initial guess, for the following frequency bands, the solution of the previously calculated source distribution is used as the new initial guess (a practical example from an outdoor static engine test is shown in Figure 4).

Table 1: SODIX for broadband sources - comparison between the original SODIX and the second generation SODIX

	SODIX I	SODIX II
source model	incoherent sources $D_{jm}D_{kn}^* = 0$ for $j \neq k$	
source amplitudes	real values $\boldsymbol{D} \in \mathbb{R}^{J\times M}$	complex values $\boldsymbol{D} \in \mathbb{C}^{J\times M}$
regularization	regularization of source directivities	L2 regularization
initial guess	energy equivalent directivity	result from previous frequency
constraint for positive amplitudes	indirectly: substitution $D_{jm} = d_{jm}^2$	directly: $D_{jm} \geq 0$
solver	conjugate gradient method by Rasmussen [19]	L-BFGS-B gradient method by Byrd and Zhu et al. [27, 28]

4.7 Verification and testing of SODIX for broadband sources

The second generation of the SODIX method has been tested by applying it to microphone array measurements with the same turbofan engine on a freefield testbed and in an indoor testcell. Oertwig [16] reports the analysis of these experiments. For the SODIX analysis, the freefield measurements have been taken with a nearfield microphone array running in parallel with the engine axis. Reference data have been measured simultaneously using a sparsely populated farfield microphone arc with a radius of 150 foot (45 m) around the engine. For the indoor test, data were measured using a linear microphone array running along one edge of the test-cell that had to be split into an forward and a rear segment due to other installations in the testcell.

The SODIX results for the broadband sources in the 400 Hz one-third-octave band from the freefield and the indoor test are presented in Figure 5. The source maps show a high dynamic range of over 20 dB for both test environments. The individual source regions (inlet, casing, nozzle, and jet) of the engine are separated by vertical gray lines in the SODIX source charts on the left-hand side of Figure 5. The directivies of the casing, the nozzle and the jet sources show the expected behaviour: the inlet sources radiate mainly into the forward arc while sources in the nozzle and the jet dominate the rear arc of the engine. The farfield extrapolated SODIX results from the freefield test closely match the directivity measured with the farfield microphones. The SODIX results of the indoor test are in good agreement with the freefield test results in the regions that can be resolved by the smaller indoor arrays. The directivities of the sources near the engine inlet and the nozzle agree within ±2 dB with the freefield results, which is remarkable considering the different geometries of the test setup and the difficult acoustic environment of an indoor testcell.

Table 1 presents a schematic overview to illustrate the main features and processing steps of the improved SODIX method for broadband noise sources compared to the original method by Michel and Funke.

5 SODIX for tonal sources

The classical SODIX has been derived under the assumption of fully incoherent sources and is therefore only applicable to problems with broadband sources. With the first SODIX version, tonal components in the microphone signals have to be removed in a pre-processing stage. However, especially for turbofan engines, there is a great interest in the analysis of tonal contributions that radiate with strong directivities. These source signals radiate at least partially coherent from the inlet and the nozzle or from the bypass exit of the engine and combine at the microphone positions depending on their frequency and phase. Therefore, SODIX has been extended to include methods to evaluate both fully and partially coherent tonal sound sources.

5.1 Fully coherent sources

In order to consider the coherence of sound sources at different positions j and k, the formulation of the cross-spectral matrix of the classical SODIX in Equation 1 is modified to

$$C_{mn}^{\mathrm{mod}} = \sum_{j=1}^{J}\sum_{k=1}^{J} g_{jm} D_{jm} D_{kn}^{*} g_{kn}^{*} \tag{3}$$

with an additional summation over the source positions k. Usually, the number of microphones M is much lower than the number of unknown directivities of the sources, which is M times the number of sources J.

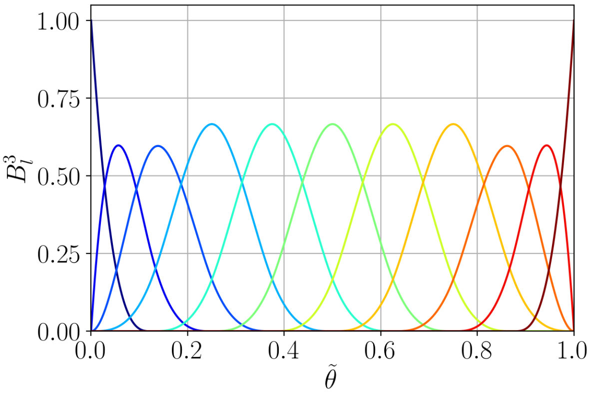


Figure 6: Cubic B-splines B_l^3 as base functions with $l = 1, \ldots, 11$ elements over a unit interval.

5.2 Parametrization of the source directivities

In order to reduce the number of unknowns, the source directivities are described in parametric form, as the weighted sum of the elements of a base function B. The base elements are evaluated at specific coordinate positions $\tilde{\theta}$ for every combination of a sound source j and a microphone m. For problems with a linear microphone array and a parallel source grid, these positions are located at specific emission angles θ between the source grid axis direction and the line connecting a particular source position with a microphone. The base functions that are used in SODIX are cubic B-splines, i.e. polynomials that are locally defined between individual sections of a computational domain [29]. After initial tests with piecewise linear, harmonic, and Bernstein polynomial base functions, cubic B-splines have been chosen, because they offer a good compromise between the overall and the local accuracy of the reconstruction of the source directivity [30, 31]. Figure 6 shows an example of the B-Spline parametrization with eleven base functions over a unit interval.

Figure 7 shows the SODIX results of data for a simulated source directivity generated with the DLR fan noise prediction tool PropNoise [32]. In this example, the parametrization of the directivity of the sound field with cubic B-splines is able to reproduce the amplitude and the phase of the simulated source directivity over a large angular range and a large dynamic range. The simulated source directivity has strong phase jumps between the radiation lobes at normalized angles of $\theta \approx 0.6$ and $\theta \approx 0.75$ that are caused by different radial modes that are overlaid. The correct determination of the phase is crucial for the analysis of cases with coherent sound sources because the phase does affect the sound pressure levels in the farfield, causing constructive as well as destructive interference of sources in contrast to incoherent sound sources that simply add up.

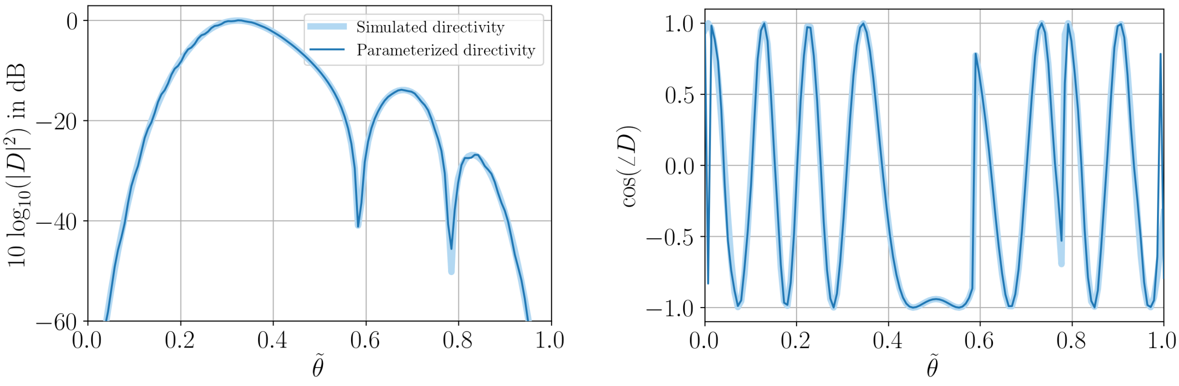


Figure 7: Directivity of the amplitude (left) and phase (right) obtained from a simulated source distribution (wide light blue line) with a parametrization with cubic B-splines (thin dark blue line) from [33].

For an analysis under the assumption of fully coherent sound sources, the cross-spectral matrix can be reduced to a vector of sound pressures and the mathematical problem is reduced to a linear system of equations.

5.3 Compressed sensing based solver

The parametrized equation system is still underdetermined because in practical applications, the number of microphones is lower than the number of the unknown parametrized source amplitudes: $M \ll J \cdot L$. A compressed sensing based algorithm is used to determine the unknown source directivities. Compressed sensing algorithms are used to reconstruct sparse signals [34] and a sparse distribution is a good assumption for tones from sources inside turbofan engine ducts that propagate through the intake and the bypass. One algorithm that is very suitable for the analysis of engine tones with SODIX is the Block Orthogonal Matching Pursuit (BOMP) [35], which exploits the special block-structure inherent to the SODIX formulation.

5.4 Partially coherent sources

In technical applications, sound sources are neither completely incoherent nor fully coherent. Therefore, SODIX has been extended to resolve partially coherent sound sources and their directivities. The analysis technique is a variant of the SODIX method for fully coherent sources and is based is based on an eigenvalue decomposition of the measured cross-spectral matrix and a compressed-sensing based algorithm proposed by Behn et al. [36] for the analysis of turbomachinery induct modes. The method for partially coherent sources is generalization of the method for fully coherent sources [25] and has been presented by Oertwig et al. [30, 31].

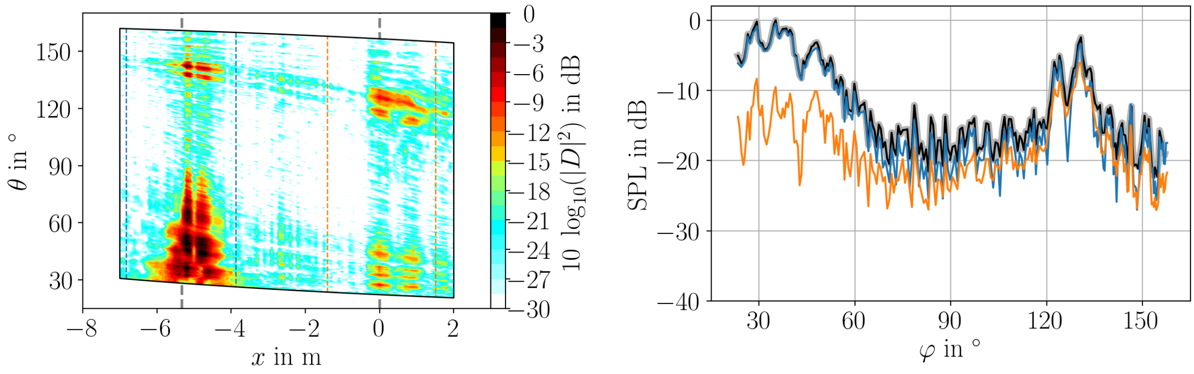
5.5 Extrapolation of the source amplitudes to positions in the acoustic farfield

In most practical cases, the scientific or technical interest in the results of the source localization is in the farfield directivity of the sound field and the individual sources (or source regions for distributed sources). In static acoustic tests with turbofan engines, the farfield directivity of the total sound field is measured with a polar array, which is usually 150 ft. (45,72 m) away from the engine. The farfield array is relatively sparse and used to calculate the directivity of the sound field at these farfield positions. In order to compare the direct farfield measurements with the SODIX solution for the sound fields of the total and the individual sources, the directivities of the SODIX sources are extrapolated to the reference positions of the farfield microphones. The amplitudes at the farfield positions have to be calculated from a superposition of the amplitudes of all SODIX sources in the direction of the farfield points (which have to be interpolated from the microphone positions in the array used to measure the SODIX input data). For tonal sources with mutual coherence, the extrapolation of the SODIX sources has to include the phase information in the direction of the extrapolation.

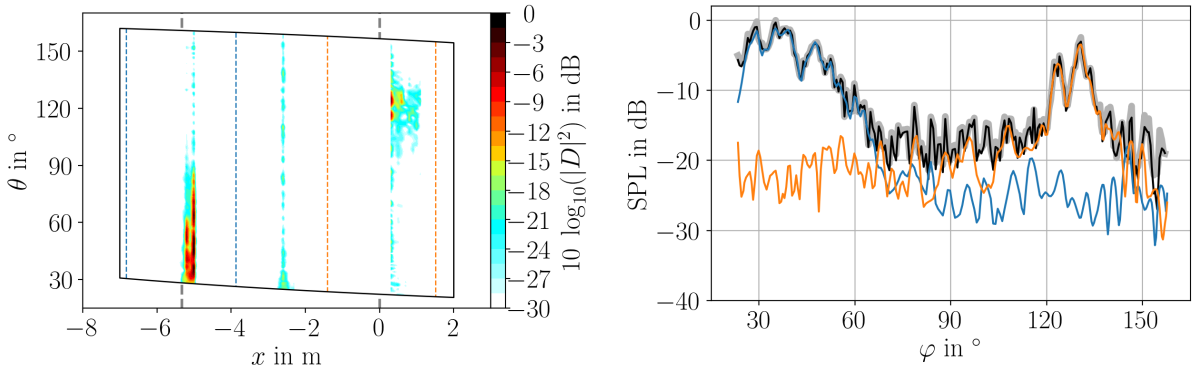
Table 2 presents a schematic overview on the key points of the new additions to SODIX for the analysis of tonal sources, which feature two different sub-models for tonal sources for fully coherent and partially coherent sources. The other steps are the same, i.e. the definition of the source amplitudes, the regularization strategy, the constraints imposed to the directivity by the cubic B-Spline base functions, and the compressed sensing solver.

5.6 Verification and testing

The extension of SODIX for tonal sound sources has been developed to determine the directive source amplitudes of partially coherent tonal sources from a turbofan engine intake and nozzle. The new SODIX method for coherent sound sources has been validated using data that have been simulated using the DLR noise prediction tool PropNoise [32]. These tests confirm that SODIX is able to reproduce the directivity of the simulated coherent sound sources very accurately [31]. The new SODIX method for tonal sound sources has also been used to quantify the differences of the sound pressure levels of engine tones measured with microphone arrays in an indoor testcell and on a freefield test rig (which have been presented in Section 4.7).



(a) Incoherent, classic SODIX source model.



(b) New partially coherent SODIX source model.

Figure 8: SODIX results from a freefield test with a turbofan engine for a 3.3 kHz tone. Left: SODIX source directivity charts, right: extrapolated farfield directivity of the total (black), the source regions at the inlet (blue) and the nozzle (orange), the wide gray line shows the sound pressure levels at the microphones (figure from [16]).

Figure 8 presents the results of the SODIX analysis of the freefield test data for an engine tone with a frequency of 3.3 kHz. The analysis has been performed with both the incoherent source model from the original SODIX version and the new partially coherent source model. The original SODIX method with the incoherent source model can localize the sources at the correct positions at the inlet and the nozzle, but the directivities do not match the expected radiation patterns of the tone from the inlet into the forward arc and from the nozzle to the rear. The partially coherent source model can resolve the directivities of the tones from the inlet and from the nozzle much better than the original SODIX with the incoherent source model. The source map has a high dynamic range of over 30 dB and does not show any spurious sources away from the expected positions. The farfield extrapolated overall directivity of the tones from the SODIX solution matches the directivity determined from a direct evaluation of the microphone signals very well for both models. When the directivities of the inlet and nozzle tones are extrapolated to farfield positions, only the partially coherent model delivers the expected results that the source at the engine inlet radiates mainly into the forward arc and the source at the nozzle to the rear. The new SODIX version also reduces the influence of

Table 2: The extended SODIX method for tonal sources

source model	coherent sources $D_{jm}D^*_{kn} \neq 0$ for $j \neq k$	
source sub-model	fully coherent $\frac{\lvert D_{jm}D^*_{kn}\rvert^2}{\lvert D_{jm}\rvert^2 \cdot \lvert D^*_{kn}\rvert^2} = 1$	partially coherent $\frac{\lvert D_{jm}D^*_{kn}\rvert^2}{\lvert D_{jm}\rvert^2 \cdot \lvert D^*_{kn}\rvert^2} \leq 1$
source amplitudes	complex values $\boldsymbol{D} \in \mathbb{C}^{J \times M}$	
regularization	L2 regularization	
constraints for the directivity	parametrization with cubic B-splines	
solver	compressed sensing BOMP method	

reflections from the rear of the testcell, allowing for a more accurate assessment of the sound pressure levels of engine tones in indoor measurements [16].

6 Conclusions

This paper presents the historical development of the source localization method SODIX. First, with a general description of the original SODIX by Funke and Michel, including its limitations to broadband sound sources and real-valued source amplitudes, followed by the improvements that have been included in the current second generation version of SODIX.

A number of improvements, mainly the representation of the source amplitudes by complex numbers and the implementation of a new regularization strategy, lead to a SODIX method that is more robust and computationally more efficient. The minimization of the cost function converges much faster and the reconstruction of the measured sound pressure levels has been improved.

With the new extended model for coherent sound sources, the second generation SODIX can be used to analyze broadband as well as tonal sources. Depending on the experimental setup, SODIX now can be applied to the analysis of incoherent, fully and partially coherent sources solving different cost functions that are adapted to the particular problem with different mathematical methods. With these improvements and extensions, the source localization method SODIX is a robust method for the experimental determination of the broadband and tonal sound fields of aircraft engines.

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