

Investigation of Multiple-Satellite Formation Configurations for Single-Pass Synthetic Aperture Radar Interferometry

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Abstract

This paper examines multiple distributed synthetic aperture radar (SAR) configurations for obtaining accurate digital elevation models (DEMs) and suppressing range ambiguities in SAR missions, involving multiple platforms. Building upon the findings of the TanDEM-X mission, this study explores different formation configurations, including fixed baselines and multi-satellite systems, to enable range ambiguity suppression across multiple platforms. It investigates the control effort, formation geometries, and interferometric requirements to understand the potentiality of across-track interferometric SAR distributed systems. The analysis considers parameters such as spacecraft separation, orbit altitude, external perturbations, and interferometric performance to identify optimal design strategies for future SAR missions. This analysis is essential for identifying the best combination of orbit altitude, spacecraft separation, and antenna parameters for various operational scenarios in future SAR missions.

Keywords: distributed space system, formation flying, across-track interferometry, synthetic aperture radar

1. Introduction

In the context of future SAR missions, accurate digital elevation models (DEMs) and range ambiguity suppression techniques are essential to enhance SAR interferometry performance. The TanDEM-X mission, which began in 2009, has provided accurate DEMs using a system of two spacecraft flying in a helix formation [1]. Such configuration consists of one daughter spacecraft orbiting the mother satellite, creating slowly varying across-track baselines for SAR interferometry purposes. The TanDEM-X mission has achieved an unprecedented 12 m horizontal resolution and 2 m high accuracy of the DEM [1][2].

Building on these achievements, this study explores multiple formation configurations to enable multi-platform range-ambiguity suppression, particularly focusing on small across-track separations (< 100 m). We investigate configurations with more than two spacecraft to enable multiple baseline acquisition in a single pass (see Fig. 1). Specifically, we study and compare two different approaches for across-track SAR interferometry, with multiple satellites (> 2):

- nested helix relative trajectories, with time-varying baselines [3][4];
- parallel relative tracks, with fixed (time-invariant) baselines [5][6].

Based on these premises, we have identified two main research objectives. First, we investigate the feasibility of formation maintenance from a delta-velocity and

propellant consumption perspective [6]. Second, we combine the control effort with the geometry requirements for across-track SAR interferometry, as detailed in [4]. This aspect is crucial for properly setting up the relative baselines of a distributed system.

As a result, we conduct a parametric analysis to obtain delta-velocity maps, for different across-track baselines and orbit altitudes, coupled with the height accuracy of the across-track interferometry. The delta-velocity maps can be used to optimise the mission design parameters, such as orbit altitude and baseline, to minimise the control effort. The results of these analyses, combined with the delta-velocity budget and power demand, are essential for identifying the optimal combination of orbit altitude, spacecraft separation, and antenna parameters for different operational scenarios, given future interferometric SAR missions.

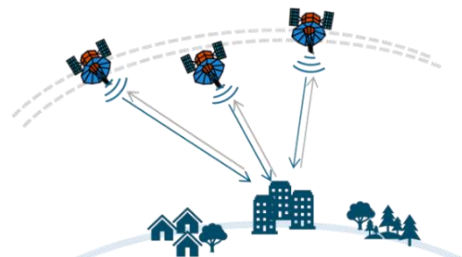


Fig. 1: Graphical representation of a distributed SAR system, with three satellites, for single-pass SAR interferometry.

2. Single-Pass Across-Track SAR Interferometry

As mentioned in Section 1, we have analysed two different configurations to perform across-track SAR interferometry: time-varying baselines, and fixed time-invariant baselines. In the followings, both approaches are described and analysed.

2.1 Time-varying baselines

The first concept has been derived based on the helix relative trajectory of the TanDEM-X mission, resulting in a time-varying baseline. Starting from the analyses in [3][4], where more than two spacecraft are considered in the distributed system, we can identify at least three baselines during a single-pass: a large, a medium and a small baseline, as shown in Fig. 2.

To keep the advantages of the passive safe configuration of TanDEM-X, we envision the possibility of distributing three or more satellites in a nested helix relative trajectory. The main advantages of this concept are the natural passive safety of the relative motion [7] and the small control effort to maintain the formation against external orbital perturbations. However, this configuration does not guarantee a constant ratio among the baselines during the SAR acquisition.

To guarantee a proper interferometry acquisition, a certain ratio should be kept among the large and small orthogonal baselines to enable multi-platform range-ambiguity suppression. As identified in [4], the ratio between the large and small orthogonal baselines shall be kept in the range of 2.4 to 4, for across-track SAR interferometry. Fig. 3 shows an example of a formation of three satellites, respecting such requirements. The chief, represented by the grey line, is placed at the centre of the formation. The two deputies follow helix trajectories around the chief, with different relative distances. The radius of the two nested helix trajectories was selected to guarantee a baseline ratio in the range of 2.4 to 4, for interferometric purposes.

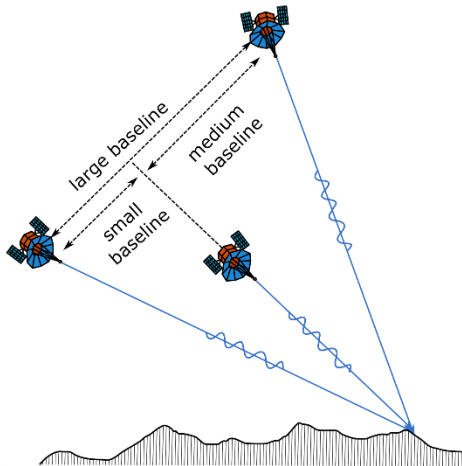


Fig. 2: Multi-platform SAR system with large, small, and medium baselines, under a nested helix configuration.

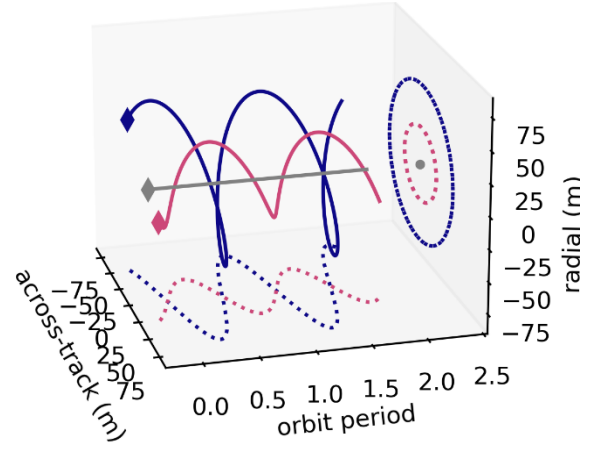


Fig. 3: Evolution over 2 orbital periods of a 3-satellites formation in a nested helix orbit configuration.

2.2 Fixed baselines

The second concept is based on the possibility to maintain fixed, time-invariant, baselines. This can be realized with multiple spacecraft flying on parallel relative tracks, as shown in Fig. 4. Following the approach in Section 2.1, the ratio between the large (b_{34}) and the small (b_{13}) baselines shall be selected to respect the interferometric requirement from [4]. This configuration can be designed for three or more satellites with a fixed-baseline in the across track direction. Keeping a constant baseline goes in the direction of generate highly accurate DEM, thanks to a constant baseline ratio during the SAR acquisition.

However, it is important to study the feasibility of this concept. First, keeping parallel tracks among the satellites requires a continuous control against the natural relative motion, which induces oscillations in the across-track direction [6]. Such behaviour could pose safety problem in case of failure of the on-board control. Consequently, safety strategies have been investigated in a previous work [5], to guarantee the safety of the concept from an operational point of view. Another challenge

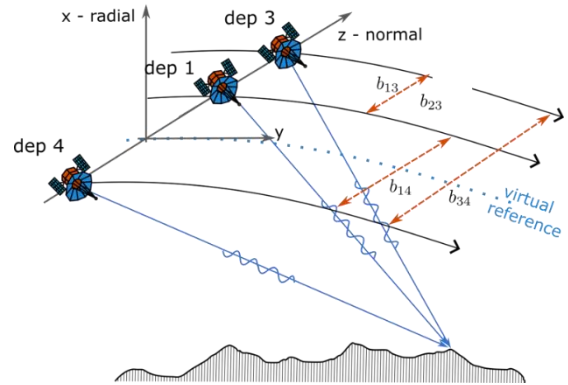


Fig. 4: Multi-platform SAR system with large, small and medium across-track baselines, following parallel relative tracks.

related to this concept is the control effort required to keep the relative tracks parallel and, thus, a constant baseline. In a first approximation, we can derive the acceleration to counteract the natural oscillations in the across-track direction from the Hill-Clohessy-Wiltshire (HCW) equations:

$$|a_z| = n^2 z_0, \quad (1)$$

where $|a_z|$ is the absolute value of the acceleration in the across-track direction from the solution of the forced HCW equations, n is the mean motion of the reference orbit and z_0 is the across-track separation of each satellite with respect to the virtual reference. Consequently, larger the baseline, higher will be the control effort for maintenance.

3. Parametric Analysis

This section presents the parametric analysis to combine the performance of the across-track SAR interferometry and the cost of the formation maintenance, in terms of delta-velocity and propellant consumption.

3.1 Interferometric Performances

As introduced in Sections 1 and 2, this work investigates the possibility of implementing single-pass across-track interferometry with both time-varying and fixed orthogonal baseline.

For a preliminary assessment of the interferometric performance for accurate DEM, we computed the 90th percentile of the height accuracy, namely $\sigma_{h,90p}$. This value depends on different parameters. First, it depends on the acquisition wavelength λ , the look angle θ , the number of looks n_L , and the coherence level γ . Then, we can correlate the range R and the orthogonal baseline $b_\perp$ via the look angle θ and the orbit altitude H . We obtain the following relation for the interferometric phase difference σ_ψ and the height accuracy σ_h [2]:

$$\sigma_\psi = \sigma_\psi(n_L, \gamma), \quad (2)$$

$$\sigma_h = \frac{\lambda R \sin \theta}{2\pi b_\perp}, \quad (3)$$

As a result, the ratio $b_\perp/R$ and, specifically, the ratio $b_\perp/H$ play an important rule in the definition of the height accuracy. Fixing the wavelength during the acquisition, the look angle and the number of looks, we performed a parametric analysis with the numerical values reported in Table 1. Considering the parameters in Table 1, we compute the SAR height accuracy $\sigma_{h,p90}$ (90th percentile) for various orthogonal baseline to altitude ratios, and different coherence levels. Figure 5 presents these results, where coherence levels vary within the range [0.5, 0.99]. The figure shows how the height accuracy improves with larger $b_\perp/H$ ratios and higher coherence levels.

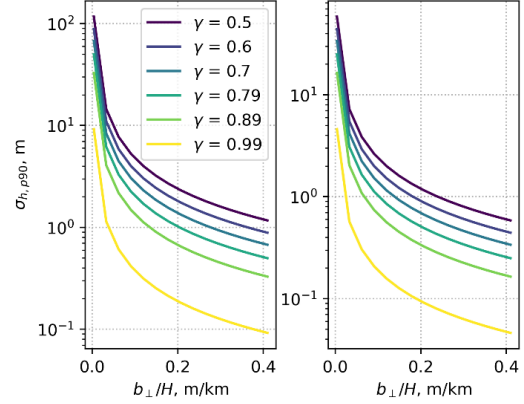


Fig. 5: Height accuracy $\sigma_{h,p90}$ (90th percentile) for the bistatic (left) and the alternating bistatic modes (right), with varying $b_\perp/H$ ratios and coherence levels.

Table 1: Parameters for the parametric analyses.

Parameter	Symbol	Value
Altitude range	H	[200; 1000] km
Baseline range	$b_\perp$	[5; 100] m
Coherence level	γ	[0.5; 0.99]
SAR frequency	f	35.5 GHz
Bandwidth	B	100 MHz
Look Angle	θ	35 deg
No. of looks	n_L	24

3.2 Formation Maintenance

Starting from the derivations in [2], we performed a parametric analysis for a preliminary assessment of the delta-velocity budget for the two configurations under analysis in Section 2. The cost for formation maintenance is based on two main contributions:

- Cost to maintain the absolute orbit
- Cost to maintain the relative trajectory

The analysis was performed including in the dynamical description the external orbital perturbations present in low Earth orbit, such as the mean earth oblateness and the atmospheric drag. The analysis is performed considering altitude and baseline variation as in Table 1. We implemented a closed-loop control strategy, based on a linear quadratic regulator (LQR) [6][8] to feedback the control accelerations on the relative dynamics.

This approach not only offers an initial estimation of the delta velocity budget, but also assesses the control accuracy of the relative distance between the spacecraft. Additionally, we have considered a continuous thrust to maintain the formation under external perturbations, assuming electric thrusters on board. In the following sections, we show the results of the analysis for the two formation configurations described in Section 2. For both cases we consider an area-to-mass ratio of $0.005 \text{ m}^2/\text{kg}$, and we assume that all the spacecrafts have the same mass and physical properties.

3.2.1 Parametric analysis for time-varying baselines

When considering the case with time-varying baseline, the main contribution to the control effort is given by the absolute orbit maintenance against the atmospheric drag, for orbit altitude of around 500 km or below. In fact, the cost of keeping a helix trajectory is relatively small, as the controller only need to compensate for differential perturbing effects. Fig. 6 shows the control effort in terms of daily velocity variation. As expected the daily control effort is in the cm/s range, especially for altitude above 400 km and baseline to altitude ratio below 0.8. For configuration with altitude below 400 km, the contribution of the atmospheric drag on the absolute orbit maintenance increases exponentially.

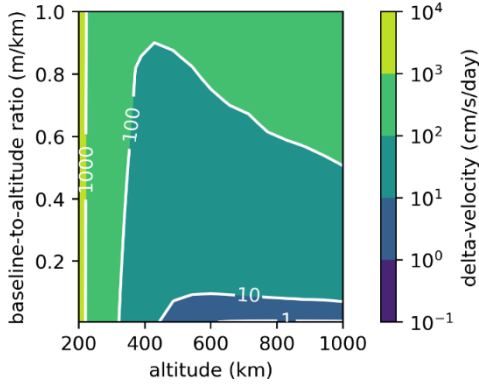


Fig. 6: Daily delta-v budget for time-varying baseline under nested helix orbit configuration.

3.2.2 Parametric analysis for fixed baselines

Differently from the previous case, when we consider fixed baselines and, consequently, parallel relative orbit tracks, the cost of keeping the relative trajectory becomes more relevant compared to the absolute orbit maintenance, especially above 400 km. Fig. 7 shows the control effort in terms of daily velocity variation. We can observe how the daily delta-velocity is significantly higher than the case of nested helix trajectory, being in the order of few m/s/day. As shown in [6], this is the result of a forced motion against the natural relative oscillations in the across-track direction.

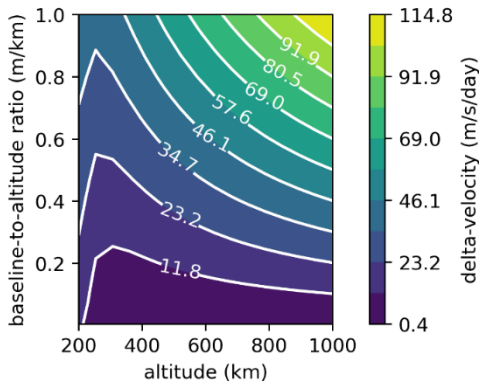


Fig. 7: Daily delta-v budget for fixed baseline case.

This result has to be carefully considered when designing a new mission configuration, as the total delta-v budget influences the mission lifetime and the propellant required on board.

3.3 Combination of interferometric performance and formation maintenance

Considering a SAR mission with a requirement on the height accuracy $\sigma_{h,p90}$ of 1 m and the coherence level γ of 0.79, we can compute the cost of the formation maintenance for the corresponding baseline-to-altitude ratio of 0.1 m/km for an alternating bistatic mode. The results for these parameters are shown in Table 2.

Table 2: Example for the combination of the parametric analyses for interferometric performance and formation maintenance.

Parameter	Symbol	Value
Height Accuracy	$\sigma_{h,p90}$	1 m
Coherence level	γ	0.79
Interferometric Outputs		
Baseline to altitude ratio	b/H	0.1 m/km
Fixed baseline		
Optimal orbit altitude	H_{opt}	360 km
Daily delta-velocity	Δv_{day}	~ 5 m/s/day
Time-varying baseline		
Optimal orbit altitude	H_{opt}	600 km
Daily delta-velocity	Δv_{day}	~ 0.1 m/s/day

4. Final considerations

Starting from the results in Section 3, we have identified some practical application that can benefit from the two configurations under analysis.

As shown in Fig. 7 and in Table 2, the control effort to keep a constant baseline is proportional to the baseline to altitude ratio. To keep the control cost low, a small ratio is preferred, and this can be achieved as a combination of orbit altitude around 400 to 500 km and small orthogonal baselines, in the order of few tens of meters. A practical SAR interferometry application in combination with short baselines (< 100 m) is envisioned with interferometric acquisition in the Ka-band of the electromagnetic spectrum. Acquisitions in Ka-band reduce the penetration in the terrain and, at the same time, achieve compliant performance with a small baseline. The former aspect is important to obtain observations less affected by the penetration and accurate DEMs. The use of short baseline results in a lower control cost to keep fixed - time invariant - baselines, and can be further exploited for mission design.

An additional possible application of fixed-baseline acquisition to keep the control cost low, is the possibility of designing a mission with alternating acquisition of fixed and time-varying baseline. We are currently investigating the possibility of designing a mission based

on the nested helix configuration to minimise the control cost, with few months of operations in the parallel track configuration. This approach could be beneficial to demonstrate the advantages of a fixed baseline for across-track interferometry and, at the same time, keeping the delta-v and the mass of propellant as low as possible.

Finally, as part of future across-track SAR interferometry, the beneficial effects of multi-static configuration flying under parallel tracks can benefit from a demonstration using a swarm of drones. Multi-static concepts necessitate the engagement of multiple transmitters and/or receivers, which can be effectively evaluated by employing swarms of drones equipped with radars and localization systems, presenting a cost-effective solution [9][10]. An example of an across-track InSAR acquisition with two drones is shown in Fig. 8 (see ref. [11][13] for full derivation of this concept). Such approach can provide a showcase of spaceborne Synthetic Aperture Radar (SAR) techniques, and at the same time, it can be used to characterize the bistatic scattering process of surfaces and monitor local areas with sub-hourly temporal resolution [11][12]. This research is currently being developed at the Microwaves and Radar Institute of the German Aerospace Centre, in the framework of a European Research Council (ERC)-funded initiative, named "Distributed Radar Interferometry and Tomography Using Clusters of Smallsats (DRITUCS)".

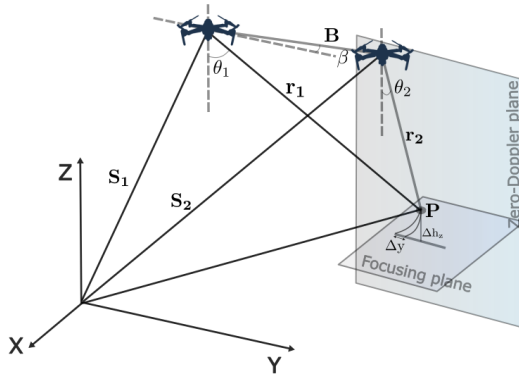


Fig. 8: Geometry of an across-track InSAR acquisition, where the SAR images are focused to a common plane [13].

5. Conclusions

The proposed parametric analysis for the identification of the delta-v budget demonstrates the synergy of the parametric maps with DEM performance indicators for preliminary mission design. We demonstrate that feasible levels of delta-v for fixed-baseline configuration require further investigation, while the nested helix configuration proves its advantage in terms of control cost. Additionally, fixed-baseline configurations with large delta-v requirements may be employed for short operational phases (e.g., 1-2 months), to mitigate the impact on the overall budget and acquire an

up-to-date global DEM, under similar environmental conditions between neighbouring swaths. The method is promising for the design of future multi-satellite mission for across-track SAR interferometry.

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