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Hierarchical sensitivity study on the aeroelastic stability of the IEA 15 MW reference wind turbine

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Abstract. Increased rotor size and blade flexibility are leading to new stability characteristics for current and future wind turbines. Numerical aeroelastic models are an essential tool to understand these new mechanisms and to design next-generation wind turbines suitably. A comprehensive understanding of how model input parameters influence the stability analysis will enhance the confidence in these simulations. This article presents a study on the sensitivity of uncertain parameters on aeroelastic stability predictions of the IEA 15 MW turbine model in HAWCStab2. It uses a hierarchical approach to handle the large number of investigated model parameters. Relevant uncertainties are identified through one-at-a-time and elementary effects analyses first. The remaining parameters are fed into a variance-based uncertainty quantification (UQ), that employs polynomial chaos expansion representations as surrogates for a full nonlinear description of the sensitivities. A robust post-processing of the stability analysis results is the main challenge of the presented methodology, but it is shown how the UQ process can still be used to explore the parameter space effectively. The presented study shows that the structural blade properties have the highest sensitivity and that a set of relatively small parameter variations can lead to unstable behavior of the reference model.

1. Introduction

The aeroelastic stability analysis of wind turbines is an active research topic. Increasing rotor sizes and corresponding increases in blade flexibility lead to new stability behavior and potentially new instability mechanisms. Volk et al. showed that edgewise whirling instabilities appear in an overspeed experiment of a 7 MW turbine [1]. The fundamental mechanism of this type of instability has been investigated by Kallesøe et al. and Stäblein et al. [2, 3]. The stability analysis of wind turbines is commonly done with linearized low-fidelity numerical models [4]. Thoroughly understanding the sensitivity of model input parameters on the stability prediction can provide more confidence in these predictions and highlight those parameters which should be handled with particular care. Sensitivity studies on the influence of aeroelastic model parameters on wind turbine loads have been done recently by multiple authors [5, 6, 7]. Fewer studies have been carried out with the wind turbine aeroelastic stability as sensitivity subject [8, 9].

The first objective of this work is to define a comprehensive uncertainty quantification (UQ) process and to identify the most influential parameters. A comprehensive sensitivity study on the influence of 46 input parameters of the HAWCStab2 aeroelastic model on the stability prediction of the IEA 15 MW reference wind turbine is performed. The methodology consists of three hierarchical steps. The most sensitive parameters are selected through a one-at-a-time (OAT) and elementary effects (EE) screening step, followed by an in-depth variance-based UQ



with a polynomial chaos expansion (PCE) surrogate model. A second objective in this study is to investigate the stability margin of the IEA 15 MW turbine, i.e. which minimal input parameter modifications would lead to an instability. This stability margin is explored by means of a minimization experiment with the PCE models.

2. Reference model

The onshore variant of the IEA 15 MW wind turbine serves as reference model in this work [10]. The aeroelastic stability analysis is done with HAWCStab2 [11, 12]. The model was established on the basis of the v1.1.6 HAWC2 model [13]. The turbine was modelled with a free-free drivetrain and a full description of the unsteady aerodynamics. An open-loop control system with fixed operating conditions is used. The influence of varying the operating conditions with the model variations in the sensitivity studies was verified to be negligible. All simulations in this work have therefore been done with the same set of steady-state operating points. Figure 1 shows the stability analysis result of the baseline model. Only a selection of the aeroelastic modes, highlighted in color in the diagram, will be analyzed in this study. The selection consists of all modes with a dominant participation of the 1st and 2nd edgewise blade bending modes. This includes the backward whirl, forward whirl and 1st and 2nd collective edgewise modes, but also includes an additional mode with a structural eigenfrequency of 1.95 Hz. This mode has a clear participation of the 2nd tower fore-aft mode, but also a significant participation of a complex blade shape including in-plane, out-of-plane and torsional motion. Despite its complex composition, this mode will be referred to as the 2nd tower fore-aft mode. A clearer identification and analysis of its modal participations is subject for future studies. The line markers indicate the dense discretization of the operating points, especially around rated wind speed, to improve the mode tracking robustness. In the UQ studies the frequency and damping values at all operating points will be analyzed as quantities of interest, i.e. all 350 markers in this figure will be tracked and evaluated.

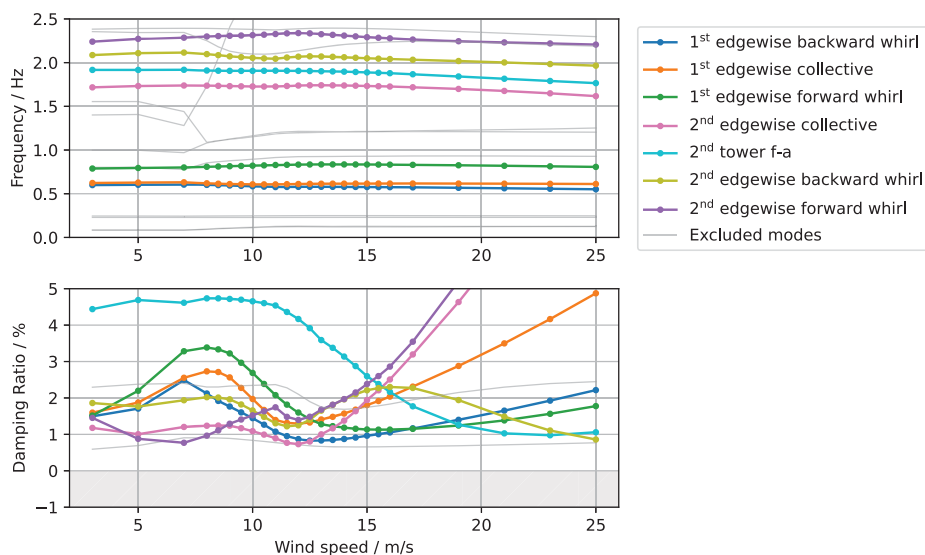


Figure 1. Reference Campbell diagram and selected modes for the UQ analysis

3. Uncertainty Quantification Methodology

This section describes the large set of input parameters and the hierarchical UQ procedure to investigate parameter sensitivities in an accurate and computationally efficient manner.

3.1. Input parameter description

In the optimal case, the uncertain input parameters and their respective statistical distributions are known through prior knowledge, e.g. from experimental tests or experience. Robertson et al. made a significant effort to define uncertain aeroelastic wind turbine model input parameters and their distributions through an extensive literature study, nevertheless expert opinions had to be used for the definition of multiple parameters [5]. Alternatively, in this work, it was decided to define the input parameter distributions as simple as possible, yet within realistic and relevant bounds. Therefore, almost all input parameters are described by uniform distributions with a range of $\pm 10\%$ around the nominal value. The aim of this choice was to make the definition of the input parameters as transparent as possible, which should help the understanding and generalization of the results and conclusions of the sensitivity studies. The selection of input parameters has been based on Robertson et al., but extended with parameters particularly interesting for the topic of stability analysis or unique to the HAWCStab2 model. Due to the large number of parameters, each one can only be briefly discussed, as summarized in table 1. Some noteworthy decisions should be highlighted:

- Model parameters which have a distribution along the radius of the blade or along the height of the tower are uniformly modified along the radius or height. The exception to this rule is the blade chord length, which is determined by the blade chord tip and blade chord root parameters, similar to the implementation in Robertson et al. [5]
- Modifications of the c.o.g. and shear center positions are normalized with the local chord length, e.g. a value of $+10\%$ implies: new position = old position + 10% local chord length. The positions perpendicular to the chord are given a uniform distribution of only $\pm 1\%$, because a $\pm 10\%$ distribution would lead to unrealistic properties.
- Figure 2 shows the parameterization of the steady state aerodynamic polars.

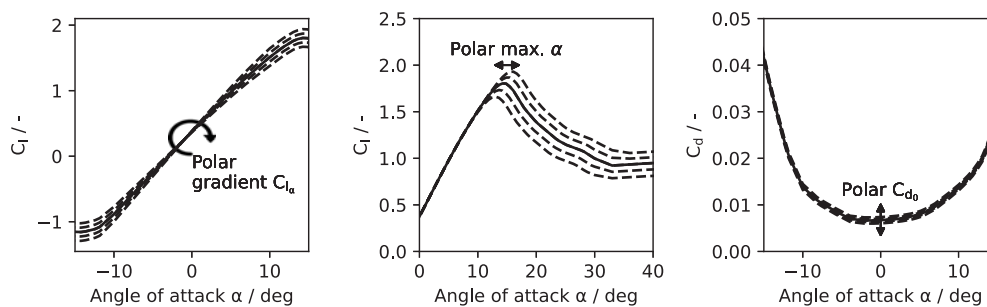


Figure 2. Modification of the steady polars by uncertain parameters polar gradient C_{l_α} , polar max. α and polar C_{d_0}

3.2. Hierarchical uncertainty quantification procedure

The large number of uncertain input parameters requires a specialized UQ methodology. A hierarchical procedure is applied, similar to the methodology used by Hübler et al. [14]. This stepwise approach is visualized in figure 3. The 46 defined input parameters largely exceed the range in which a global variance-based sensitivity analysis with a surrogate model is possible. The first two steps of the procedure, the OAT and EE, are therefore used as screening methods to identify the sensitive parameters and to narrow down the number of parameters which are analyzed with the variance-based approach, that gives a full in-depth insight into the sensitivities.

Table 1. Overview of the uncertain input parameters

Variable	Variation
Tower: bending stiffness, torsion stiffness, shear stiffness, mass	Tower properties are modified at all tower stations by $\pm 10\%$ of the local original value.
Damping of 1 st tower mode, Damping of 2 nd tower mode	Structural critical damping of the 1 st and 2 nd tower mode. Original scalar values are modified by $\pm 10\%$.
Nacelle: mass, yaw inertia, nodding inertia, rolling inertia, c.o.g. position vertical, c.o.g. position horizontal	Inertial properties of the nacelle and center of gravity position with respect to the tower top. These are all scalar values which are modified by $\pm 10\%$ of the original value.
Drivetrain: stiffness and damping	Torsional shaft stiffness and damping around the rotor axis. Both original scalar values are modified by $\pm 10\%$.
Cone angle	Original value $\pm 10\%$
Blade: stiffness (flap, edge, torsion), mass, principal axis orientation, twist angle	Blade properties are modified at all blade stations by $\pm 10\%$ of the local original value.
Blade: c.o.g. position \parallel to chord, shear center position \parallel to chord	C.o.g. and shear center position parallel to the chord with respect to the half chord point. The positions at all blade stations are modified by $\pm 10\%$ of the local chord length.
Blade: c.o.g. position \perp to chord, shear center position \perp to chord	C.o.g. and shear center position perpendicular to the chord with respect to the half chord point. The positions at all blade stations are modified by $\pm 1\%$ of the local chord length.
Blade chord length (root), Blade chord length (tip)	Blade chord length separately modified at the root and tip of the blade by $\pm 10\%$ of the original value. The chord length modifications over the blade are interpolated between the root and tip modification values.
Blade prebend, Blade sweep	Structural out-of-plane (prebend) and in-plane (sweep) pre-deformation of the blade at all blade stations modified by $\pm 10\%$ of the local original value.
Damping of 1 st to 4 th blade mode	Structural critical damping of the 1 st - 4 th blade mode. Original scalar values are modified by $\pm 10\%$.
Polar gradient $C_{l\alpha}$, Polar max. α , Polar C_{d0}	Static airfoil polar properties. Respectively, gradient in the linear domain, maximum angle of attack, and drag coefficient at zero angle of attack, see figure 2. All airfoils along the blade are modified by $\pm 10\%$ of the local original value.
Dynamic stall parameters: a1, a2, b1, b2	Coefficients in the Jones approximation of the exponential potential flow step response in the dynamic stall model. All scalar original values are modified by $\pm 10\%$.
Dynamic stall parameters τ_{pre} , τ_{bly}	Non-dimensional parameters for pressure time lag and boundary layer separation time lag in the dynamic stall model. These are both scalar values which are modified by $\pm 10\%$ of the original value.
BEM: far wake mixing ratio, far wake polynomial coefficients, near wake mixing ratio, near wake polynomial coefficients	Parameters in the description of the near wake and far wake parts of the dynamic induction. The polynomial coefficients cover four coefficients which are modified simultaneously. These are all scalar values which are modified by $\pm 10\%$ of the original values.

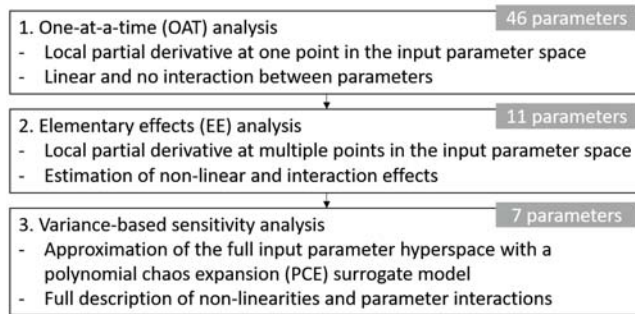


Figure 3. Hierarchical uncertainty quantification procedure, adapted from [14]

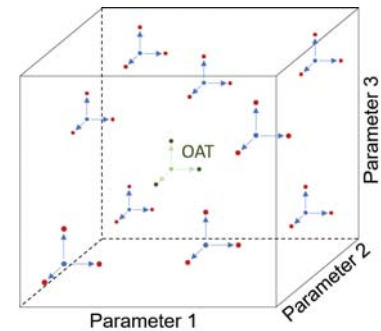


Figure 4. Visualization of OAT and EE approach, adapted from [5]

4. Parameter screening: OAT and EE

OAT analysis is a local, linear sensitivity approach. As the name suggests, each parameter is varied individually, while keeping all other parameters fixed. This can be understood as the partial derivative of the quantities of interest with respect to each input parameter. This is done once, so this method will only need $N + 1$ samples, with N the number of parameters. The EE analysis extends this by doing the same analysis at multiple points in the input parameter space, as visualized in figure 4. As a consequence, the required number of model evaluations will grow to $M \cdot (N + 1)$, with M the number of locations for the sampling. Through a convergence study, the required number of locations for the EE analysis was defined at 50 for this analysis. The OAT and EE sensitivity indices can be computed as

$$S_{ij} = \frac{f(x_{1j}, \dots, x_{ij} + \Delta, \dots, +x_{Nj}) - f(x_{1j}, \dots, x_{ij}, \dots, +x_{Nj})}{\Delta}, \quad \text{for } i \in [1, N], \quad j \in [1, M] \quad (1)$$

with S_{ij} the sensitivity index corresponding to the i th input factor at the j th location and $f(\dots)$ representing the model. Note that M is 1 for OAT, which will therefore result in a single sensitivity index for each input parameter. EE will result in multiple values for each parameter from which a mean and variance can be obtained. The variance of the sensitivity index at different points in the input parameter space is therefore a measure of the non-linear sensitivity effects and the interaction between parameters. As both methods are based on a locally linear assumption, the disturbance Δ should be chosen sufficiently small. A disturbance of $5E-3$ times the input parameter distribution width was chosen. This results for the chosen input parameters with a uniform $\pm 10\%$ distribution in a disturbance of 0.1% of the original values.

The results of the OAT and EE analysis are shown in figure 5. The bar plots in this diagram represent the $S_{\text{OAT/EE}}$ index averaged over all operating points, for both the frequency and damping. The initial OAT analysis shows that the sensitivity for a large number of input parameters is negligible across all modes. Therefore, only the eleven input parameters above the lower black line were retained for the EE study. Overall, the mean EE values are close to the OAT results. The sensitivities of the 2nd edgewise modes are significantly higher compared to the 1st edgewise modes. The sensitivity of the input parameters differs significantly across the modes. The spread in the EE results, indicated by the standard deviation of the sensitivity index at different locations in the parameter space, is significant. This indicates that the sensitivities have a non-linear character over the full uncertainty domain and/or that interactions between parameters are important to consider. Based on the EE results, the number of parameters is further reduced to seven, to reduce the computational cost of the variance-based UQ. The parameters with the overall lowest sensitivity, the blade flapwise stiffness, tower mass, polar gradient $C_{l\alpha}$ and blade prebend parameters, are excluded.

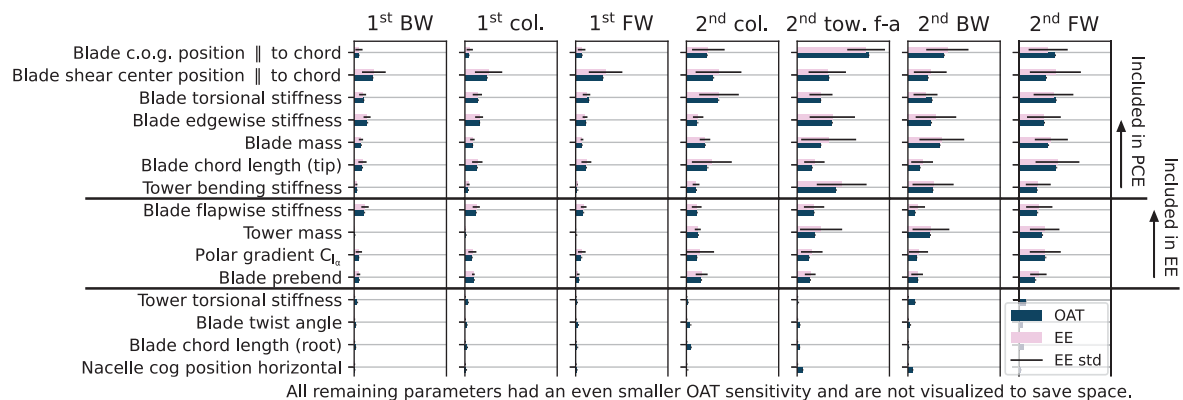


Figure 5. Combined plot of the OAT and EE results. Each bar represents the averaged sensitivity index over all operating points for both frequency and damping.

5. Variance-based uncertainty quantification

The detailed uncertainty quantification of the seven most relevant parameters is made with a variance-based methodology which describes the full uncertainty domain, including nonlinearities and parameter interactions. This is achieved by fitting a PCE model to the uncertainty domain for each of the quantities of interest. Each of these models takes the uncertain parameters as input and gives one of the quantities of interest, in this case one single frequency or damping value for one mode at one single operating point, as output. The generation of the PCE models is done by sampling the model at sufficient points in the domain to determine the coefficients of the PCE polynomial basis through a least-squares fit. For detailed information on variance based UQ with a PCE model, readers are referred to Sudret [15]. The order of the polynomial and the number of sampling points was determined through a convergence study. A fourth order polynomial sampled at 1000 points was used. The sampling points were determined with a quasi-random Sobol scheme.

5.1. The main challenge

The large set stability analysis samples have to be automatically post-processed to select the correct modes in the full stability result. The main obstacle in this work is to make this procedure robust. The input parameter modifications lead to differences in the frequency trends over the wind speed and in some cases, this can lead to mode tracking errors, i.e. the mode tracking of a sampled Campbell diagram in the UQ study can differ from the reference mode tracking. If not handled correctly, this will lead to a comparison of different modes between different samples, which will ultimately lead to a false uncertainty quantification. The implemented automatic post-processing procedure is made up out of two steps to guarantee 1) a correct selection of the modes, and 2) a verification that the mode tracking was similar to the reference run. The selection of the modes is made based on a comparison of the Modal Assurance Criterion (MAC) between the aeroelastic modes at the first operating point with the selected modes from the reference stability analysis. The modes with the closest approximation to the reference mode were selected, if the MAC value was at least above a mode-specific threshold (1st BW, 1st coll., 1st FW, 2nd coll. = 0.8, 2nd tower f-a = 0.85, 2nd BW = 0.82, and 2nd FW = 0.9). The mode tracking check is done by a similar MAC comparison between the modes of the sampled Campbell diagram and the selected reference run modes. If this MAC comparison was less than 0.5, it was assumed that the mode tracking was different from the reference run and the sample was rejected. In the presented study, the mode picking and tracking was robust for the first four modes (1st BW, 1st coll., 1st FW, 2nd coll.). None of the samples had to be rejected. However,

for the 2nd tower f-a, 2nd BW and 2nd FW, respectively 16.2%, 14.4% and 32.1% of the samples had to be rejected. The consequence of this issue is, that the PCE model for these latter modes is not a representation of the full uncertainty domain, but rather a representation of the subset of the domain in which the post-processing was successful. This makes that these models can not be used for the sensitivity quantification. Some closing remarks with respect to this topic:

- A *false* mode tracking does not imply that the stability analysis result is wrong. It means that the mode tracking was different compared to the reference run. In theory, it should be possible to correct the mode tracking differences, instead of rejecting the samples. This was not attempted in this work.
- The setup of the stability analysis was attempted to be optimal for a robust mode tracking. The operating points started at a low wind speed, and a dense wind speed discretization was used around rated wind speed, where most mode tracking differences occurred.
- It was not possible to conclusively define the input parameter ranges in which the post-processing failed. Had this been possible, the UQ study could have been done on the reduced input parameter ranges. Alternatively, proving that post-processing failures were entirely random would enable a solution through additional sampling.

5.2. Results

Verification of the PCE model is necessary to confirm that the surrogate is a correct representation of the true model. This is done with a leave-one-out test. A new leave-one-out surrogate model is established for each of the training data samples, yet without this sample included in its training data. The evaluation of the leave-one-out model at the sample point is then compared with the evaluation of the true model. The verification results are shown in figure 6. A highly accurate match between the leave-one-out verification model and the true training data can be seen for all modes, with only a slightly larger spread for the 2nd edgewise modes. This verification proves that the PCE models are a highly accurate representation of the true model. However, as discussed in the previous section, the PCE models of the last three modes are only a representation in the subset of the domain where the post-processing is successful. The leave-one-out test requires post-processed training data for the verification. It is therefore only possible to verify the accuracy of the PCE model in that subset of the full domain.

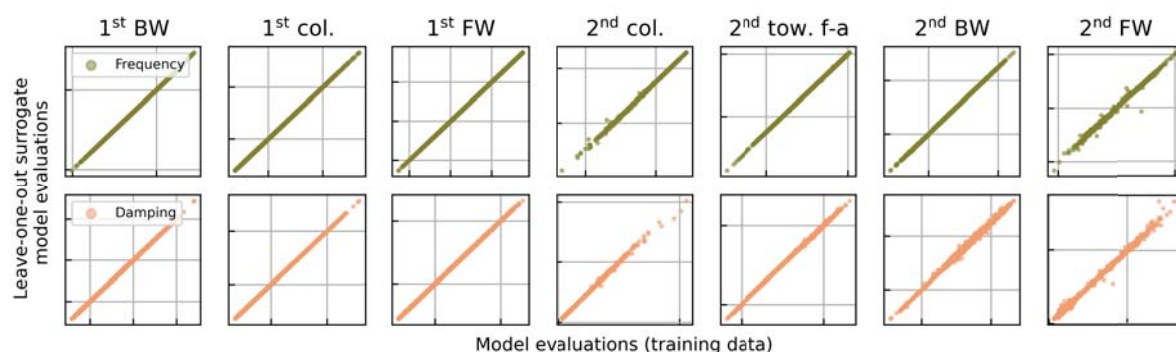


Figure 6. Leave-one-out cross-validation of the PCE surrogate models

The total variance of the stability analysis results can be decomposed in its separate contributors by an analysis of the PCE models. This is done for the PCE models of the 1st edgewise modes and the 2nd edgewise collective mode, as shown in figure 7. This visualizes the standard deviation of the total output distributions in the top left plot, the isolated contribution

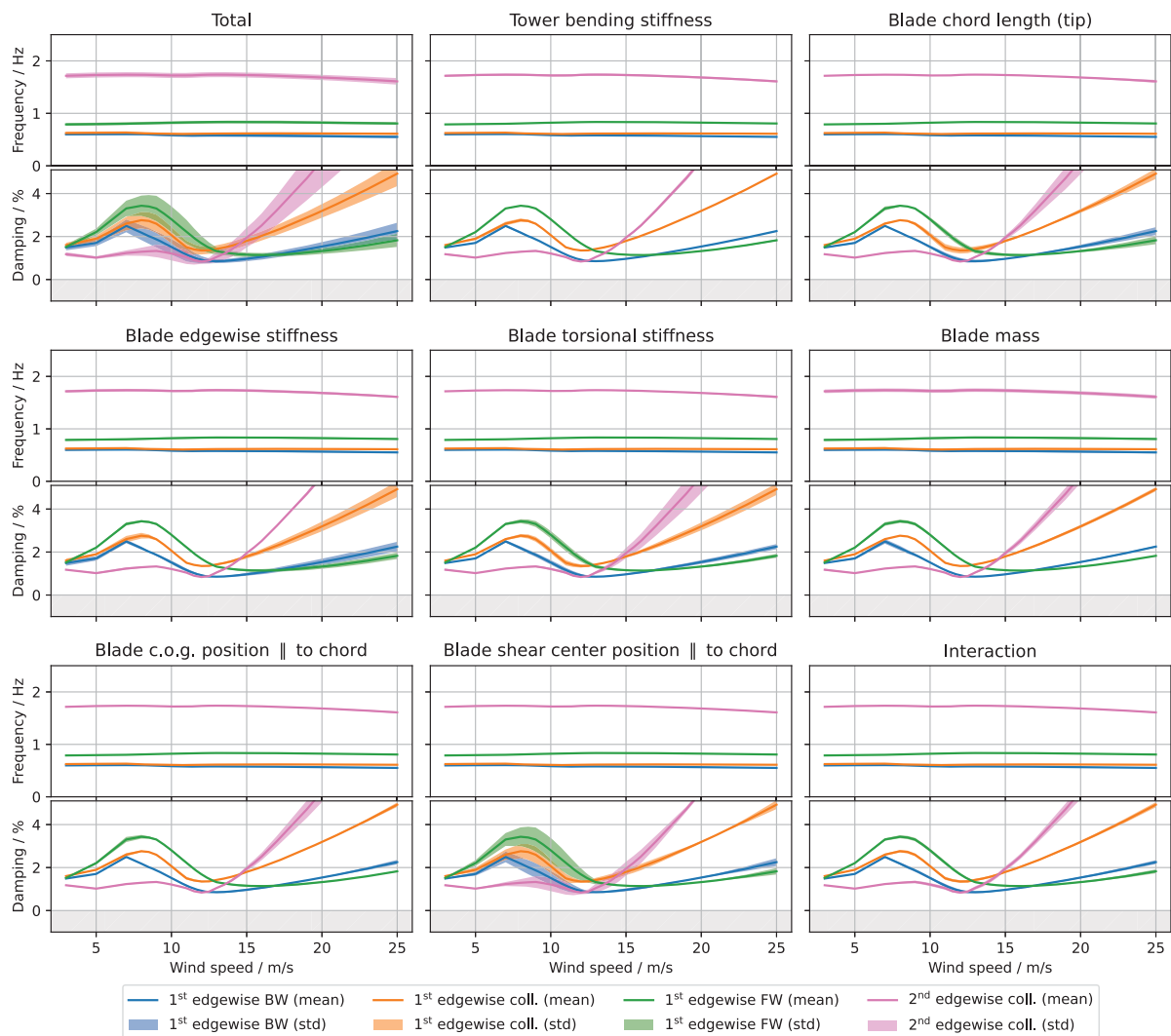


Figure 7. Variance decomposition of the Campbell diagram results.

of each of the seven input parameters and the contribution due to interaction between the parameters in the bottom right plot. As expected, the sensitivity of the damping values is significantly higher compared to the frequency values. The shear center position parallel to the chord has the highest sensitivity for these modes, which agrees with the results of the EE and OAT study, as seen in figure 5. The sensitivities of the parameters depend strongly on the operating points, e.g. the sensitivity of the shear center position on the damping of all modes is significant before rated, but limited after rated wind speed. The variance contribution due to the interaction between the input parameters is small, which can not be correlated with the large spread observed in the EE analysis.

The obtained PCE models can also be used in an efficient way to explore the stability margin. Multiple experiments were performed to investigate how far the design of the IEA 15 MW turbine is from an instability. This is done by using the standard Python minimization routines to find the minimum required input parameter modifications (= sum of all relative modifications) for an instability to occur (= damping < 0). The PCE models of the 2nd tower f-a, 2nd BW and 2nd FW modes are again considered in this experiment, as they are valid in the main part of

the domain. Instabilities within the chosen parameter ranges are observed for the 2nd tower f-a and 2nd edgewise BW modes. The most interesting results are visualized in figure 8. The minimum required parameter modification for a 2nd tower f-a instability is a combined -6.3% c.o.g. displacement and 1.5% shear center displacement. The minimum required parameter modifications for a 2nd edgewise BW instability is a combined -4.5% c.o.g. displacement and 0.2% shear center displacement. Furthermore, even when the c.o.g. and shear center positions are left unaltered, an instability of the 2nd edgewise BW mode can occur within the given input parameter ranges, by reducing the torsional stiffness and increasing the tower bending stiffness, blade chord length and blade mass.

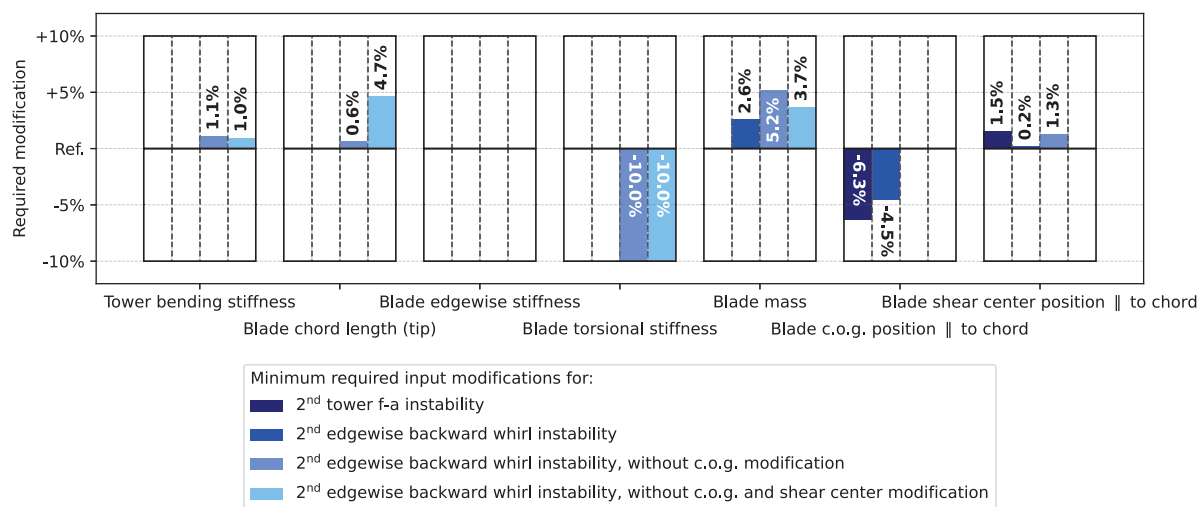


Figure 8. Minimum required input parameter modification to enforce an instability of the 2nd tower f-a (+ complex blade shape) or 2nd edgewise BW mode

The results of these minimization experiments were verified by a comparison of the PCE models with a new HAWCStab2 simulation for the obtained input parameters. The comparison for the minimum required input parameter modification for an instability of the complex 2nd tower f-a mode can be seen in figure 9. The prediction of the PCE model matches almost exactly with the true HAWCStab2 result. Similar accurate results were found for all four optimization results presented above. It has to be noted that some of the other minimization tests lead to false results, which is due to the mode tracking challenge and the boundaries within which the PCE models are valid.

6. Conclusion

A comprehensive hierarchical sensitivity study of 46 input parameters of the HAWCStab2 aeroelastic model on the stability prediction of the onshore variant of the IEA 15 MW reference wind turbine was performed. An initial screening step using a OAT and EE analysis was used to reduce the number of parameters down to seven, which were analyzed in depth with a variance-based UQ based on a PCE surrogate model.

This study showed that the hierarchical UQ procedure works efficiently and that the variance-based methodology can be used for an in-depth analysis if the mode tracking and post-processing of the stability analysis is robust. If the mode tracking fails, the PCE models can still be used for further analysis, but can not be trusted blindly and can not be used to make claims on the sensitivity estimation in the entire domain. The UQ of the IEA 15 MW turbine showed that structural blade properties and tower bending stiffness were the most sensitive

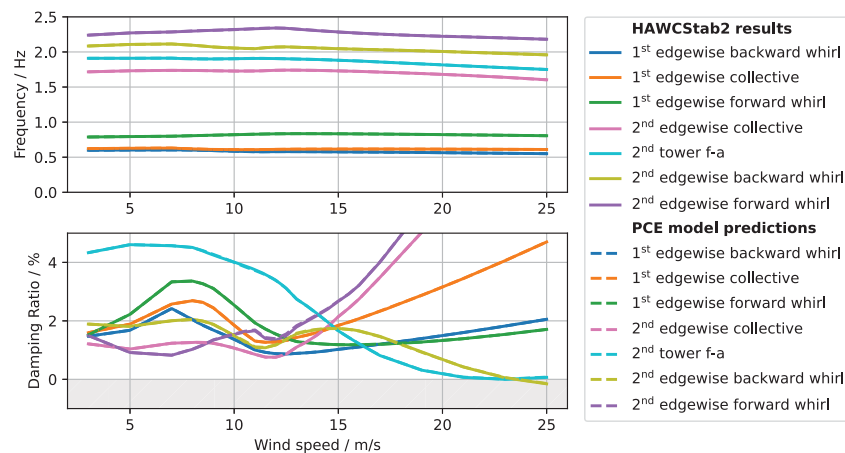


Figure 9. Campbell diagram with minimum required parameter modifications for an unstable complex 2nd tower f-a mode: comparison between PCE prediction and exact HAWCStab2 result

parameters, for the given input parameter definitions. The modal properties of the 2nd edgewise modes are more sensitive compared to 1st edgewise modes. The EE analysis showed a large spread, indicating significant non-linearity or interaction between parameters. The variance-based analysis, however, indicated only a limited variance contribution due to the parameter interaction for the 1st edgewise modes and the 2nd edgewise collective mode. Minimization experiments with the PCE models showed that the complex 2nd tower f-a mode and especially the 2nd edgewise backward whirling mode can become unstable with relatively small input parameter modifications. The instability mechanism and modal description of the complex 2nd tower f-a mode should be investigated further in future studies.

Acknowledgments

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