



# Collaborative High Precision GNSS

## ITSNT 2024

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# Overview on GNSS



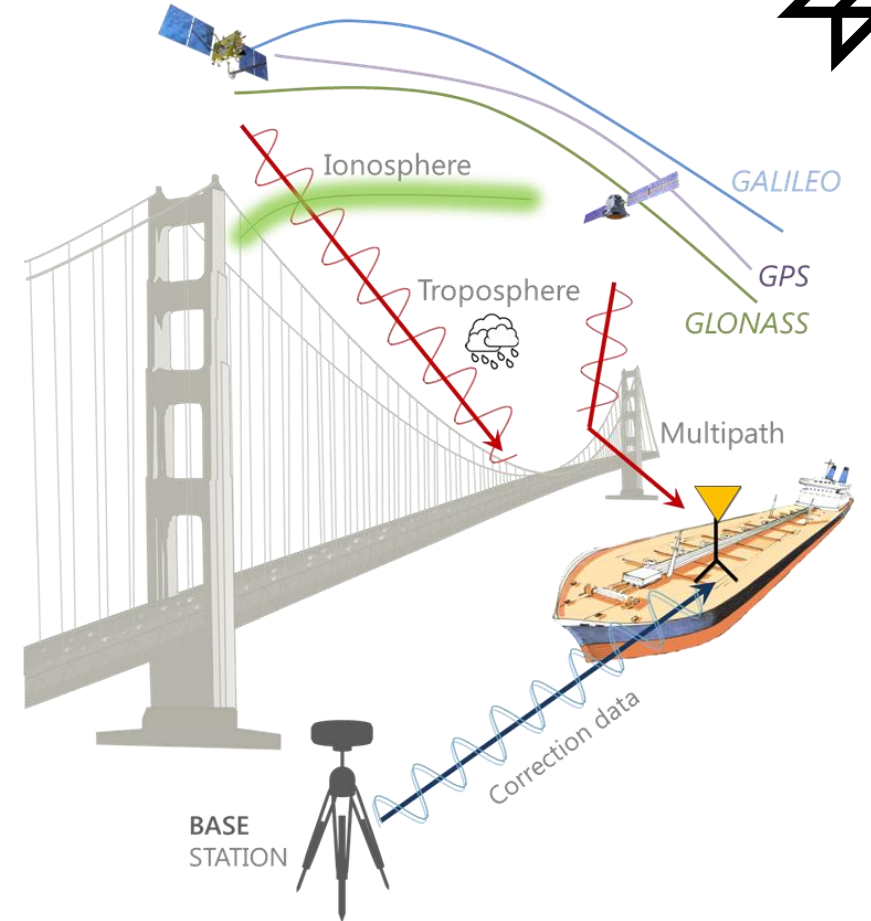
**GNSS** is the main information source for **Positioning, Navigation and Timing (PNT)**

## Challenge #1: Precision

The **accuracy** of standard **code-based** navigation is **limited** → <10 meters positioning & poor attitude

## Challenge #2: Robustness

**Multipath** and other local effects strongly degrade the performance → **large errors, low availability**

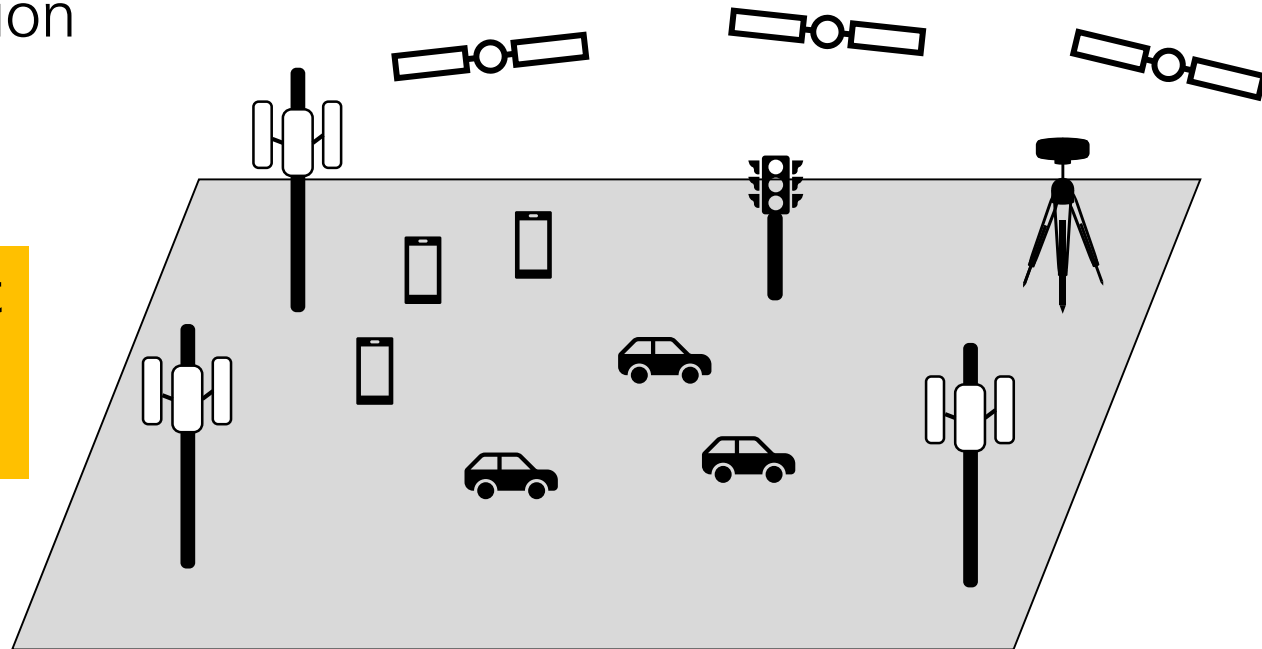


- The use of carrier phase observations is the key for high precision navigation
- Collaborative positioning may overcome the limitations on carrier phase obs.

# Why Collaborative?

- GNSS is **least performing when most needed**: poor visibility, multipath, etc.
- GNSS chipsets are now integrated with communication ones
- With the **advances on V2X technology**:
  - (1) Broad and heterogeneous network of users
  - (2) Fast and low-latency communication
  - (3) Ranging capabilities

Collaborative approaches may be the right solution for precise & reliable navigation (the quest for autonomy)

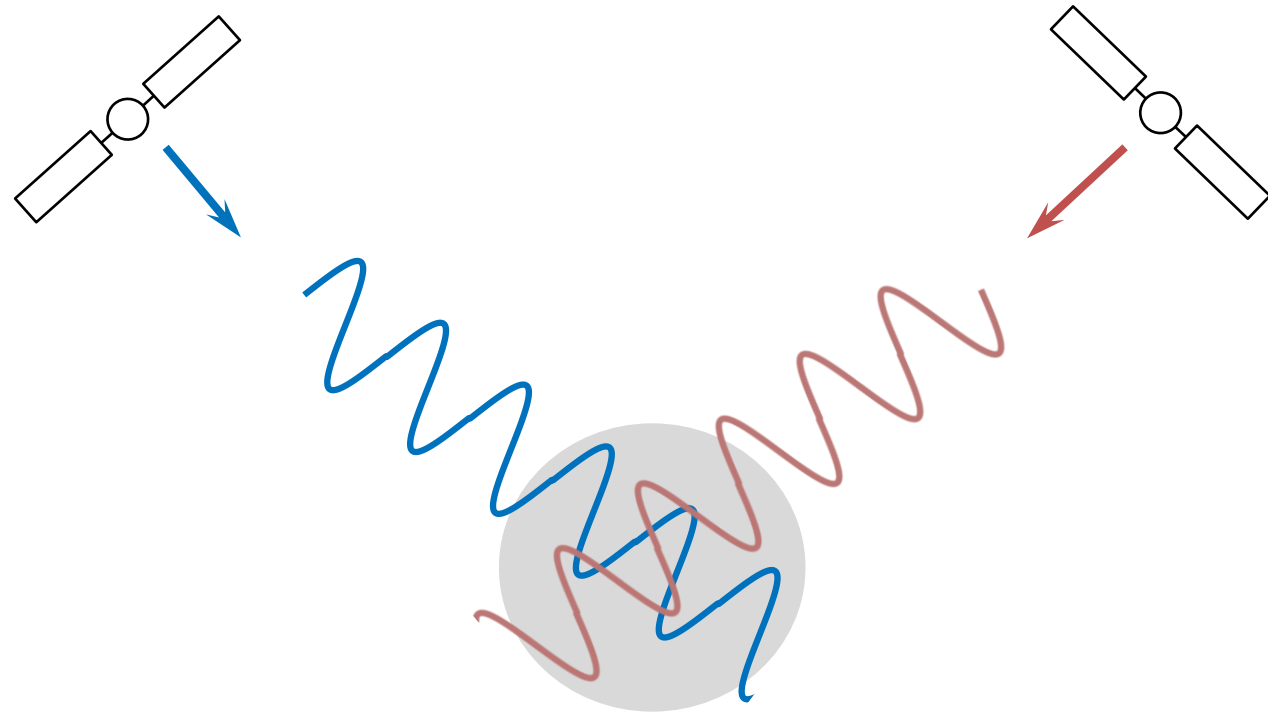


# In today's conversation...



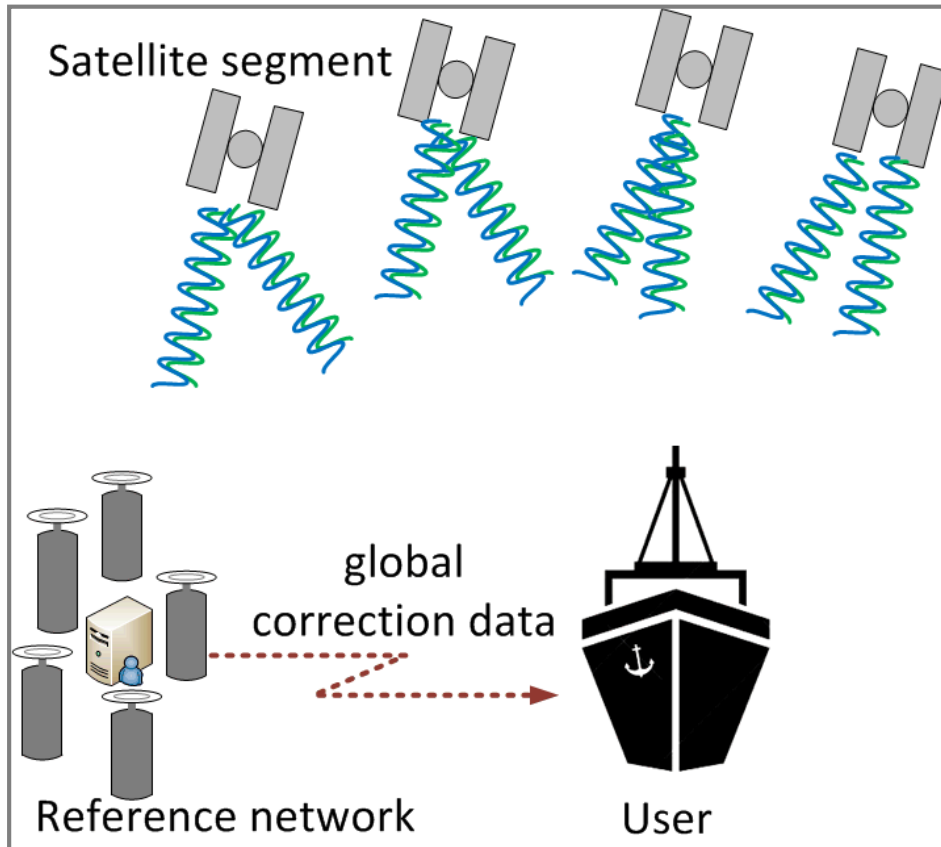
- **Basics of precise GNSS:** estimation problems for RTK, PPP-RTK & limitations
- **Understanding cooperation:** architectures, advantages, limitations
- **Cooperative RTK (C-RTK):** the idea behind, performance evaluation
- **Research & industry perspectives**

Precise  
Positioning  
Reaching cm-level  
positioning accuracy

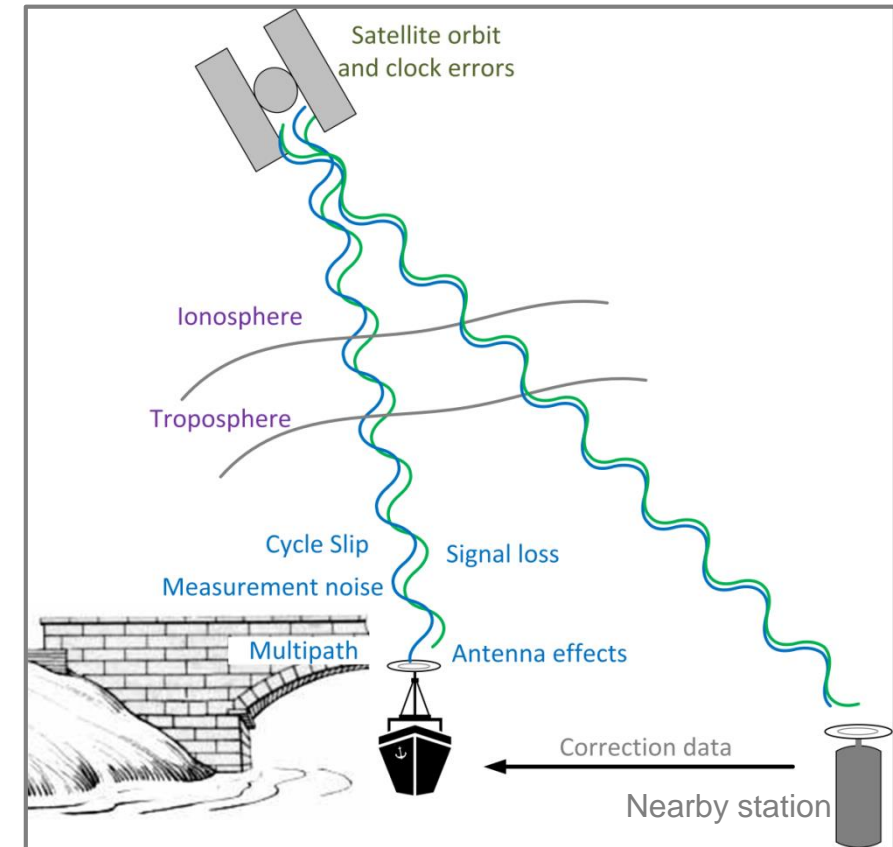


# High Precision GNSS Techniques

## Precise Point Positioning (PPP)



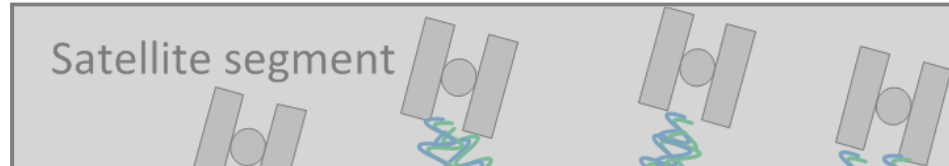
## Real Time Kinematic (RTK)



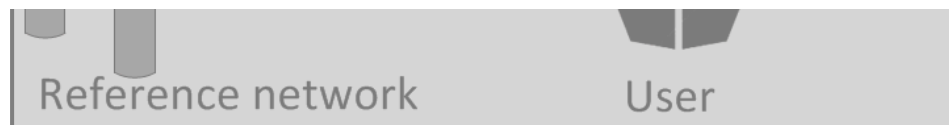
High precision GNSS is all about correction data streams & Integer Ambiguity Resolution  
Nearby users are exposed to the same atmospheric (and sometimes local) effects

# High Precision GNSS Techniques

## Precise Point Positioning (PPP)



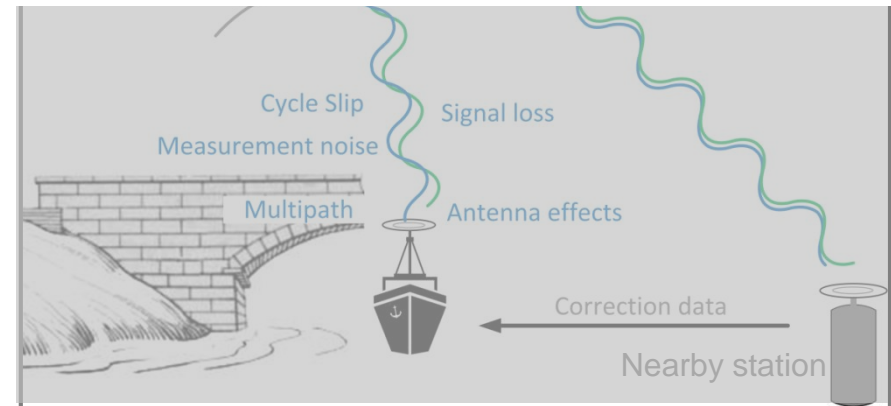
- Limited by convergence time
- “low observability” of nuisance parameters: iono, tropo, etc.
- Limited real-time usability due to delay on the corrections (\*HAS)



## Real Time Kinematic (RTK)

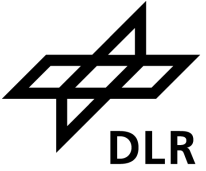


- Unfeasible to deploy a sufficiently dense network of base stations



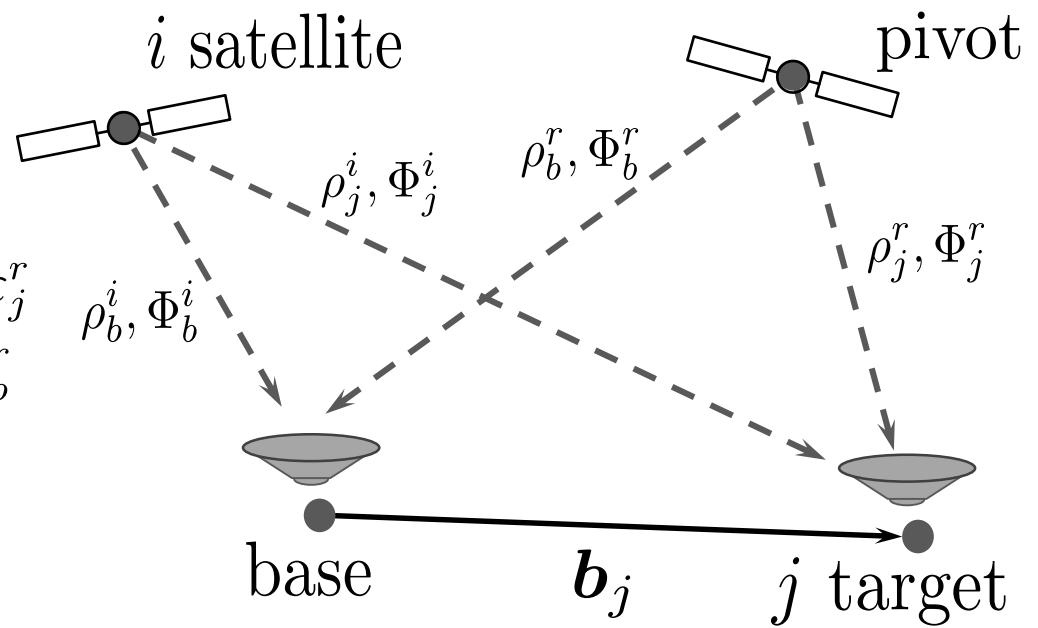
High precision GNSS is all about correction data streams & Integer Ambiguity Resolution  
Nearby users are exposed to the same atmospheric (and sometimes local) effects

# RTK Positioning Model



RTK is a differential positioning method → code & carrier phase observations from the  $j$ th vehicle and the base station are used

$$\begin{aligned} \Phi_j^i &= \|\mathbf{p}^i - \mathbf{p}_j\| - I^i + T^i + c(-dt^i + dt_j) + \lambda N_j^i + \varepsilon_j^r \\ (-) \Phi_b^i &= \|\mathbf{p}^i - \mathbf{p}_b\| - I^i + T^i + c(-dt^i + dt_m) + \lambda N_b^i + \varepsilon_b^r \\ \Phi_j^r &= \|\mathbf{p}^r - \mathbf{p}_j\| - I^r + T^r + c(-dt^r + dt_j) + \lambda N_j^r + \varepsilon_j^r \\ (-) \Phi_b^r &= \|\mathbf{p}^r - \mathbf{p}_b\| - I^r + T^r + c(-dt^r + dt_m) + \lambda N_b^r + \varepsilon_b^r \end{aligned}$$



Set of observations

$$\mathbf{y}_j = \left[ \mathbf{DD}\Phi_j^\top, \mathbf{DD}\rho_j^\top \right]^\top$$

The *Mixed Real-Integer Model*

$$\mathbf{y}_j \sim \mathcal{N}(\mathbf{A}\mathbf{a}_j + \mathbf{B}\mathbf{b}_j, \Sigma_j), \quad \mathbf{a}_j \in \mathbb{Z}^n, \mathbf{b}_j \in \mathbb{R}^3$$

And... how to solve the mixed model?



# RTK Positioning Model

## Estimation Process and Bounds

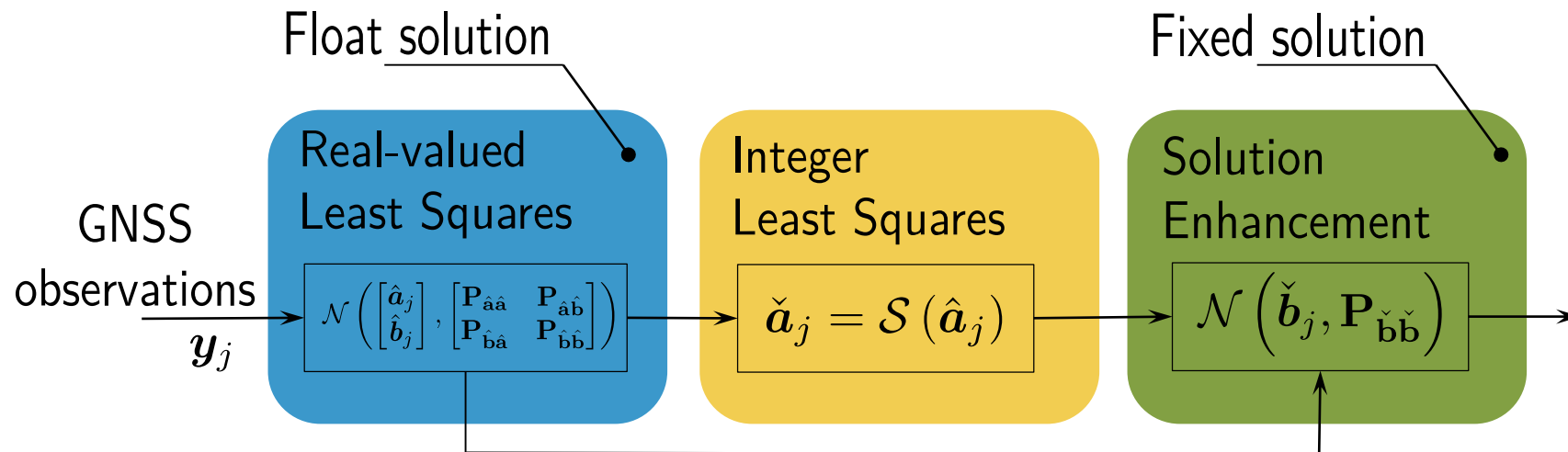


$$\left( \|\hat{\mathbf{e}}\|_{\Sigma_j}^2 + \min_{\mathbf{a}_j \in \mathbb{Z}^n} \left( \|\hat{\mathbf{a}}_j - \mathbf{a}_j\|_{\mathbf{P}_{\hat{\mathbf{a}}\hat{\mathbf{a}}}}^2 + \min_{\mathbf{b}_j \in \mathbb{R}^3} \left\| \hat{\mathbf{b}}_j(\mathbf{a}) - \mathbf{b}_j \right\|_{\mathbf{P}_{\hat{\mathbf{b}}(\mathbf{a})}}^2 \right) \right)$$

Float solution

Integer Ambiguity Resolution (IAR)

Fixed solution



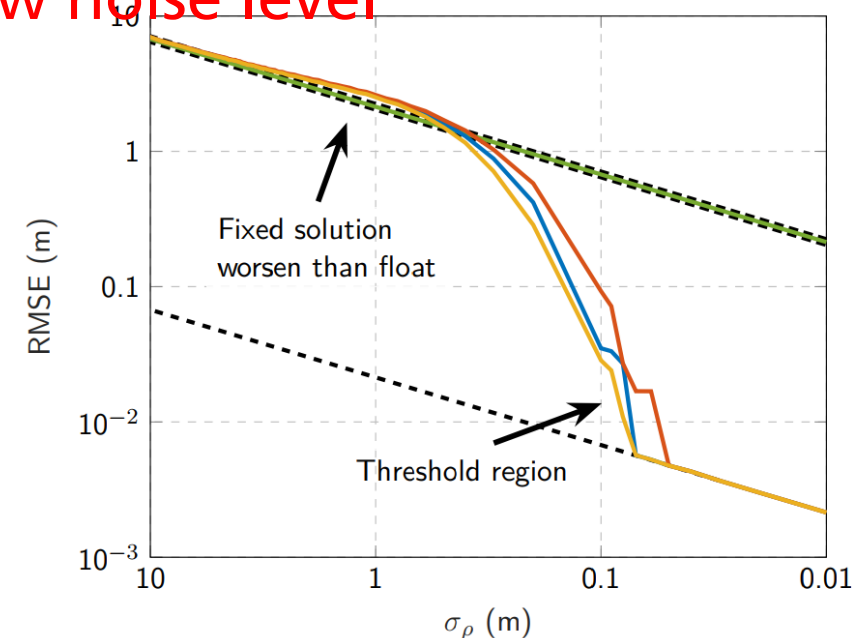
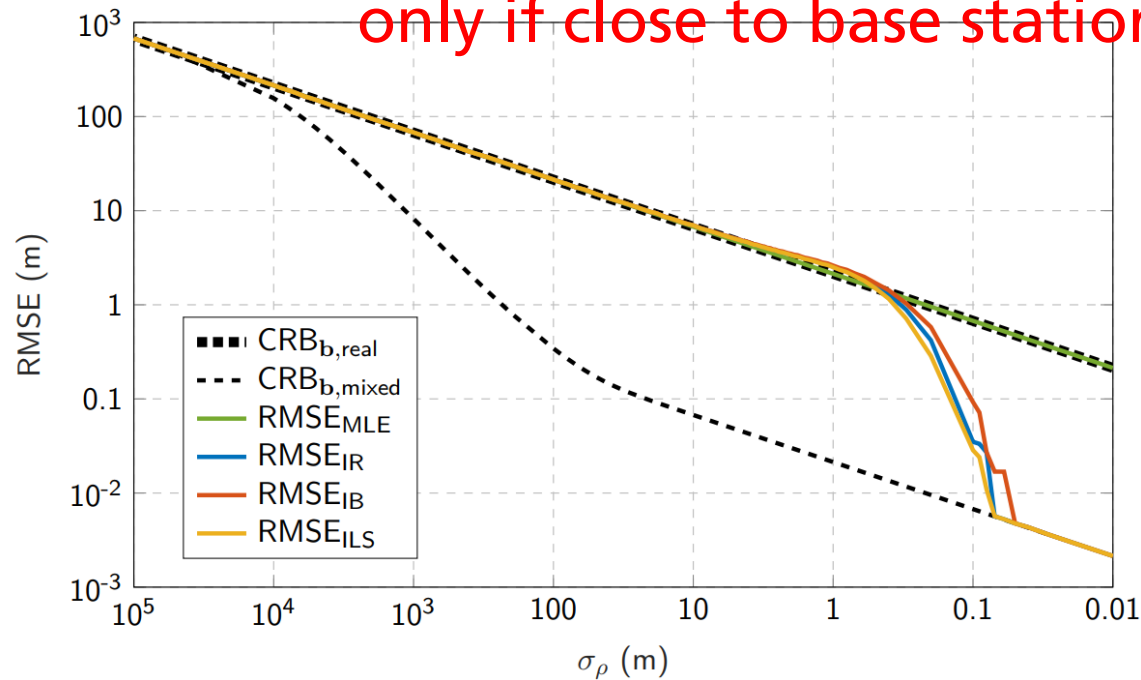
# RTK Positioning Model

## Estimation Process and Bounds



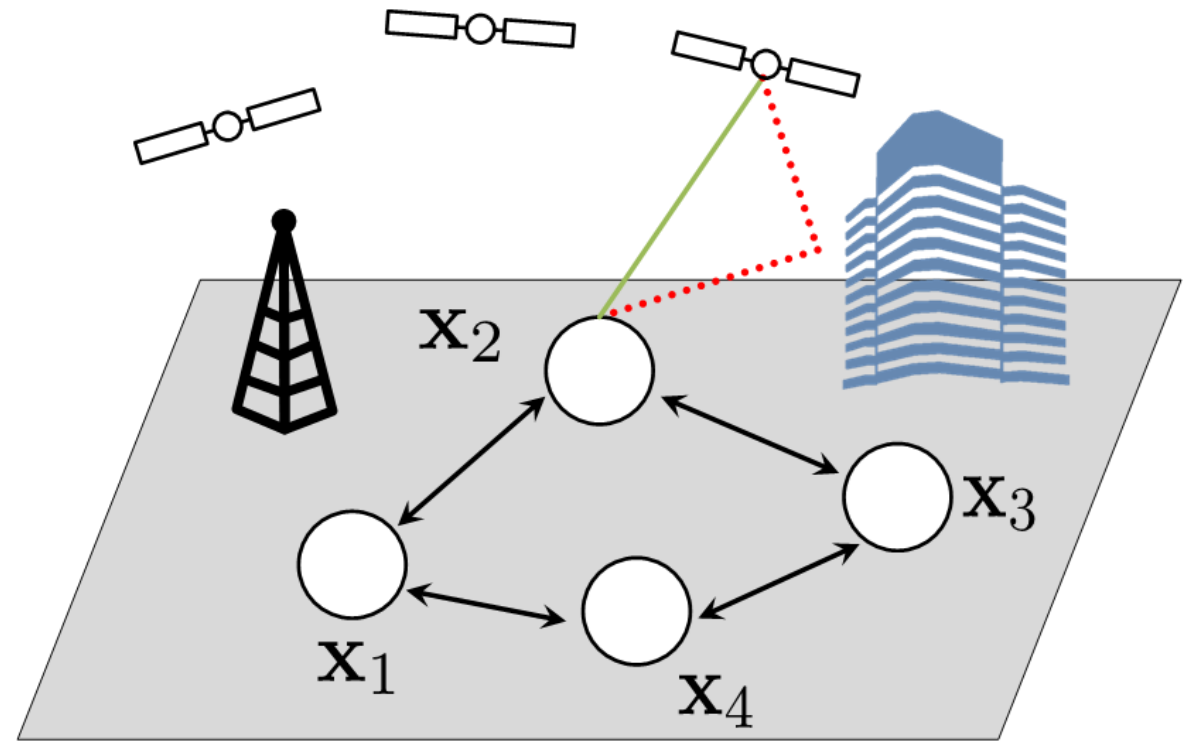
$$\left( \|\hat{\mathbf{e}}\|_{\Sigma_j}^2 + \min_{\mathbf{a}_j \in \mathbb{Z}^n} \left( \|\hat{\mathbf{a}}_j - \mathbf{a}_j\|_{\mathbf{P}_{\hat{\mathbf{a}}\hat{\mathbf{a}}}}^2 + \min_{\mathbf{b}_j \in \mathbb{R}^3} \left\| \hat{\mathbf{b}}_j(\mathbf{a}) - \mathbf{b}_j \right\|_{\mathbf{P}_{\hat{\mathbf{b}}(\mathbf{a})}}^2 \right) \right)$$

**! We have efficient estimators, but...  
only if close to base station, low noise level**



# Cooperative Positioning

Network of users  
helping each other



# Collaborative Localization (with or without GNSS)



## ▪ **Type of collaboration**

- Active – inter-user ranging, exchange location information
- Passive – broadcast of observations, non-location info

## ▪ **Architectures**

- Centralized – the solutions for all users are jointly estimated
- Distributed – each user estimates their solution

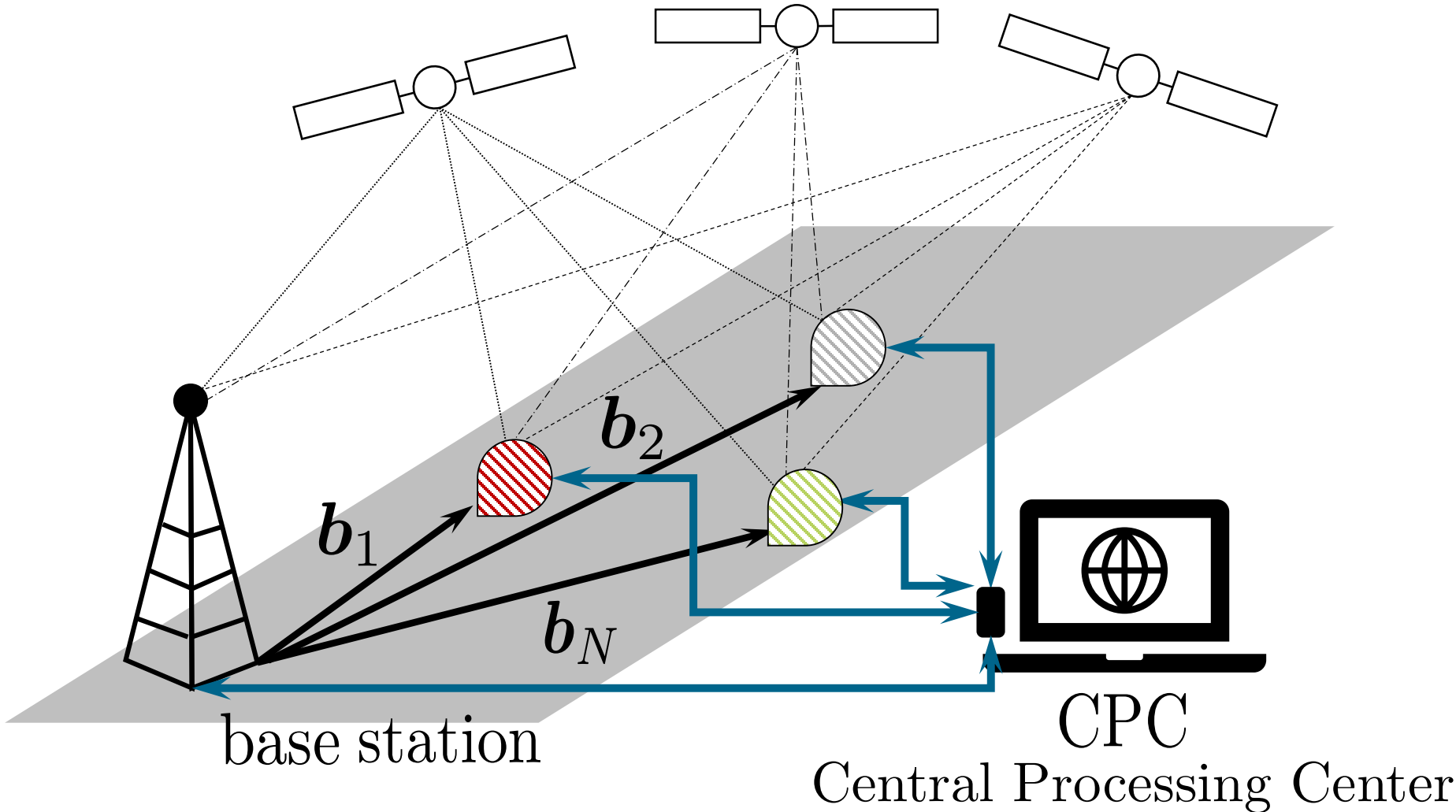
## ▪ **Points of concern**

- Intentional / unintentional interference: MP
- Respect of privacy

## ▪ **Existing literature**

- Estimation for cooperative localization: works of Wymeersch, Win, Buehrer
- For GNSS: works of Caceres (GNSS+ranging), Minetto (measurement correlation), Calatrava (Massive Differentiation)

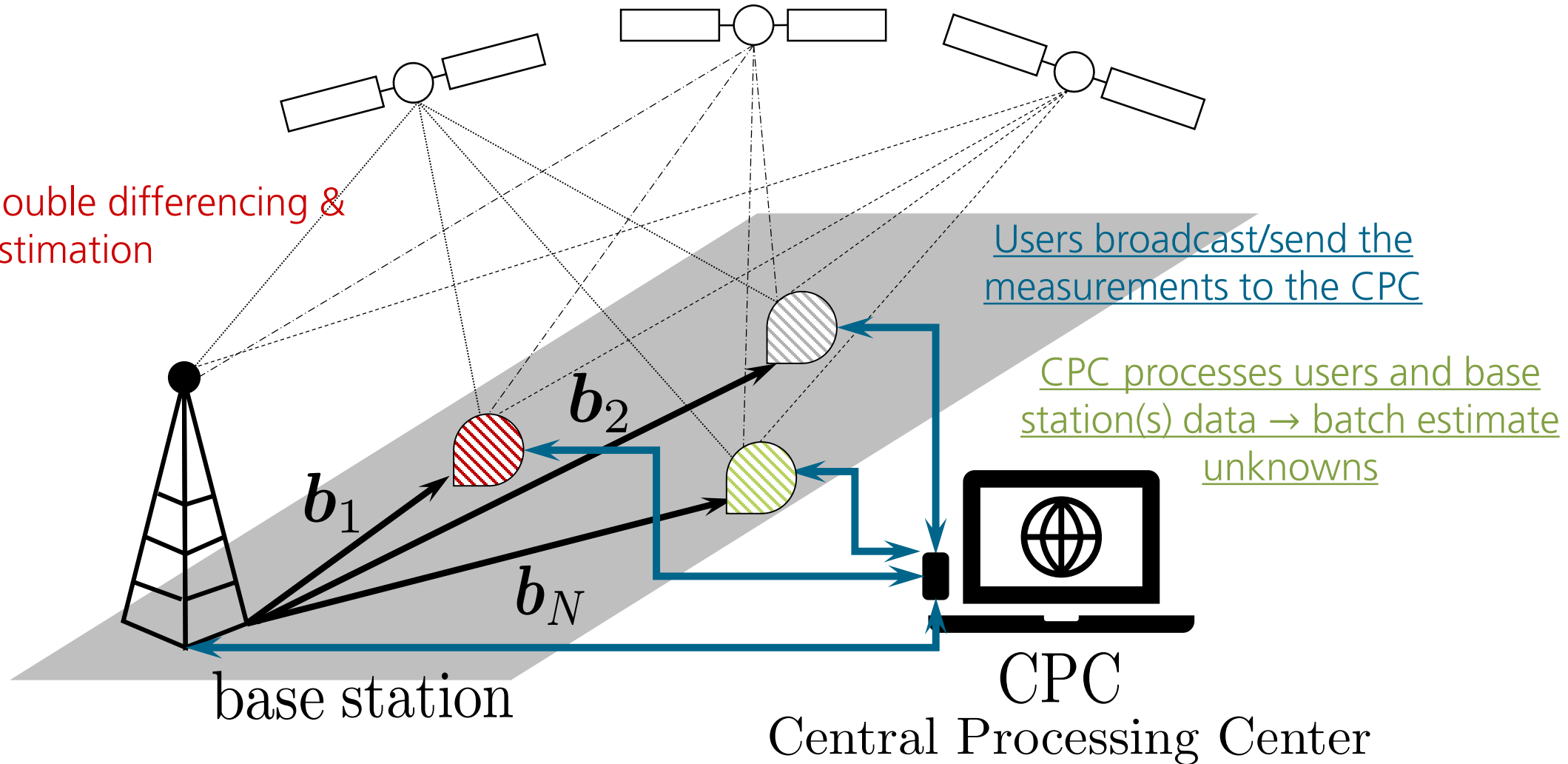
# Collaborative-RTK (C-RTK) – Conceptual & Technical Idea



# Collaborative-RTK (C-RTK) – Conceptual & Technical Idea



Processing → double differencing & mixed model estimation



# C-RTK – Positioning Problem



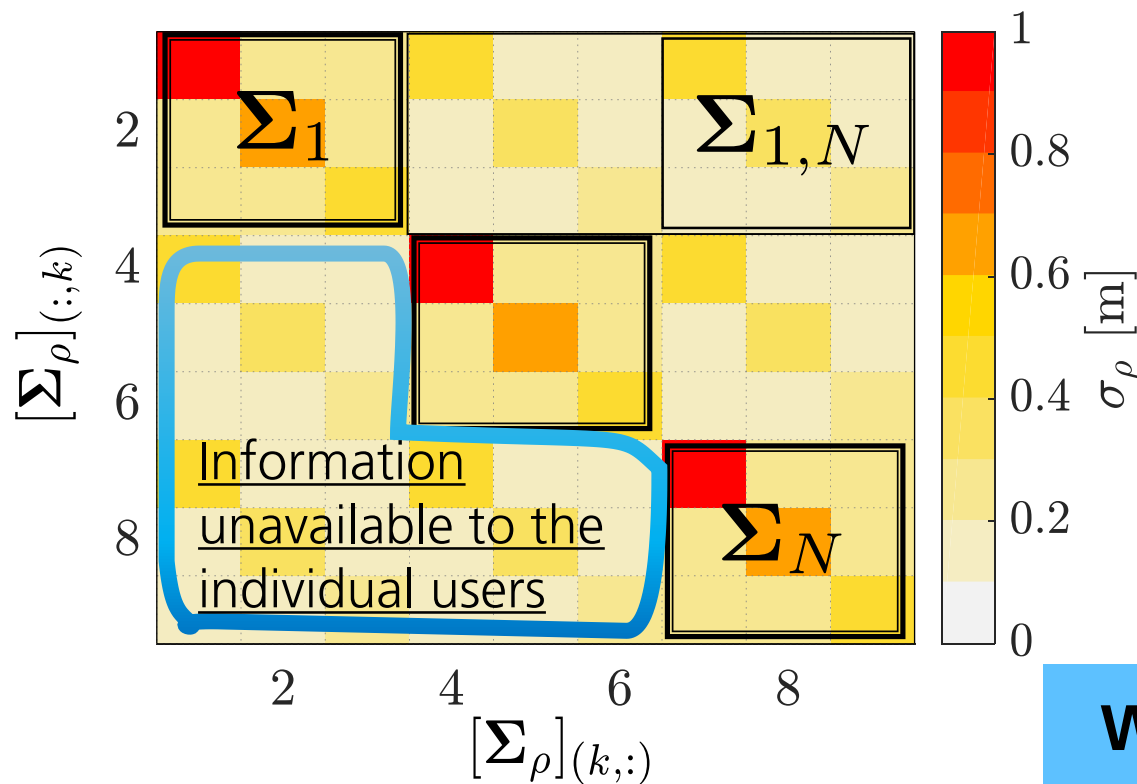
## Conventional RTK

$$y_j \sim \mathcal{N}(Aa_j + Bb_j, \Sigma_j)$$

for  $N$  users  
 $\longrightarrow$

## Collaborative RTK

$$\tilde{y} \sim \mathcal{N}(\tilde{A}\tilde{a} + \tilde{B}\tilde{b}, \tilde{\Sigma}), \quad \tilde{a} \in \mathbb{Z}^{n \cdot N}, \tilde{b} \in \mathbb{R}^{3 \cdot N}$$



“Extended” version of obs., unknowns, matrices

$$\tilde{y} = [DD\Phi_1^\top, \dots, DD\Phi_N^\top, DD\rho_1^\top, \dots, DD\rho_N^\top]^\top$$

$$\tilde{a} = [a_1^\top, \dots, a_N^\top]^\top, \quad \tilde{b} = [b_1^\top, \dots, b_N^\top]^\top$$

## The importance of stochastic modeling

- The cross-correlations due to combining observations wrt. base station → **fundamental information!**

**We can leverage on the existing CRBs and estimators for the mixed model problem**

# Monte Carlo Performance Analysis



## Study Cases:

### 1) Open Sky

- $N=6$  vehicles, "fully connected network"

### 2) Urban Scenario

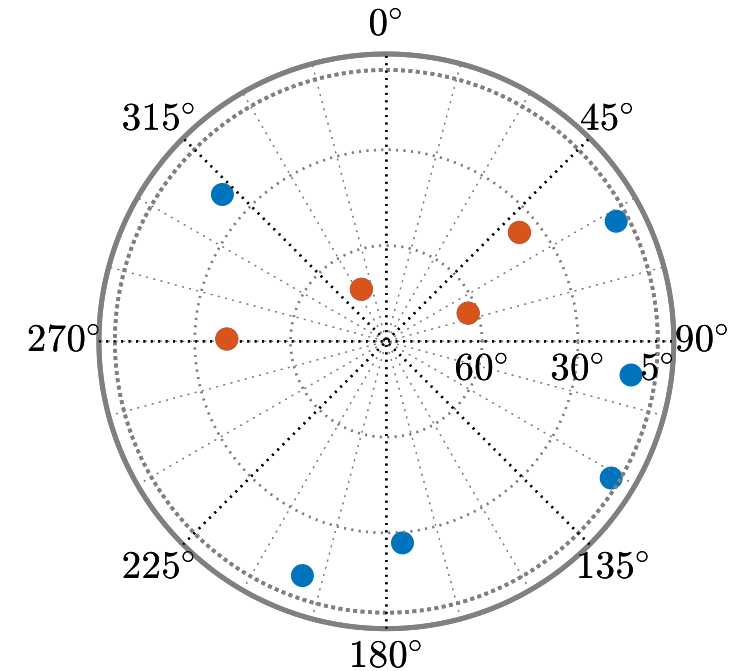
- 2 vehicles limited view + 4 open sky

## Stochastic modelling

- Unweighted model, equal noise across users
- 1 to 100 ratio for phase/code std
- A range of precision levels is evaluated

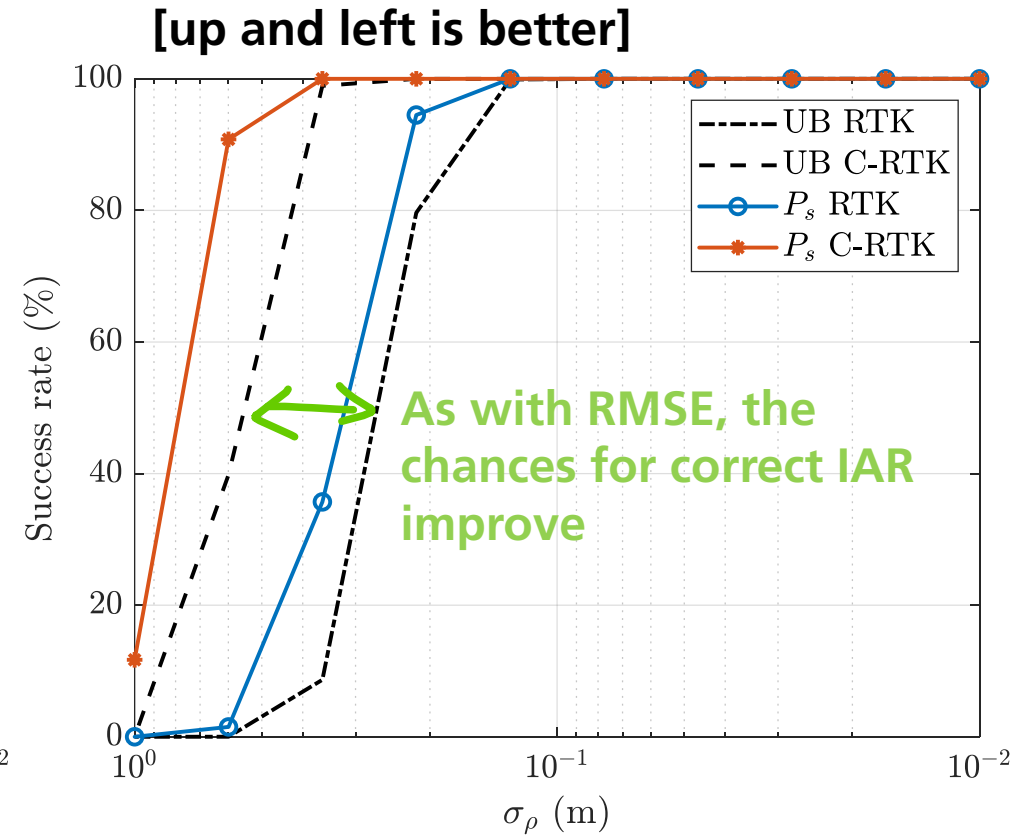
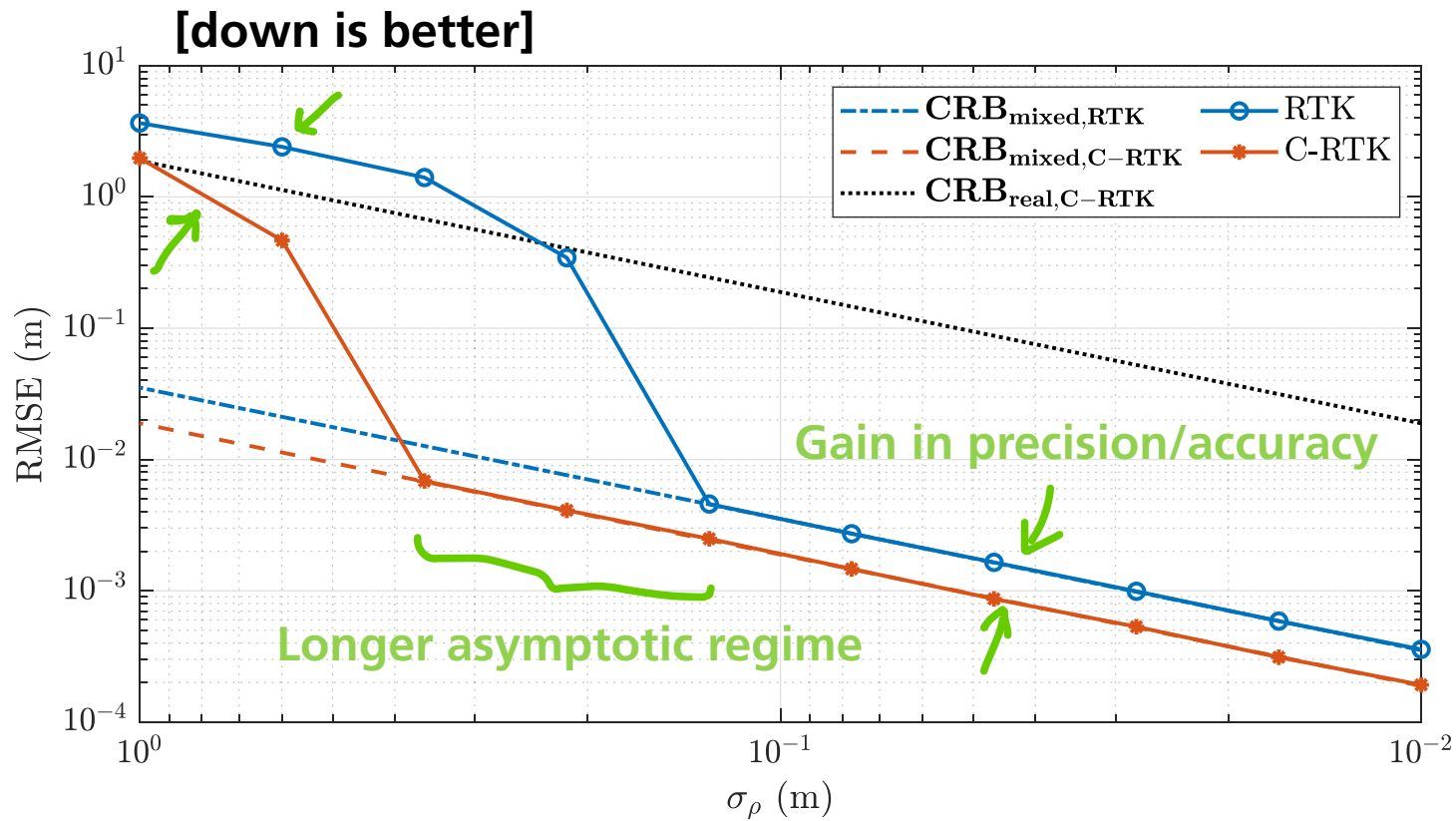
## Metrics:

- Comparison of the positioning RMSE & IAR performance





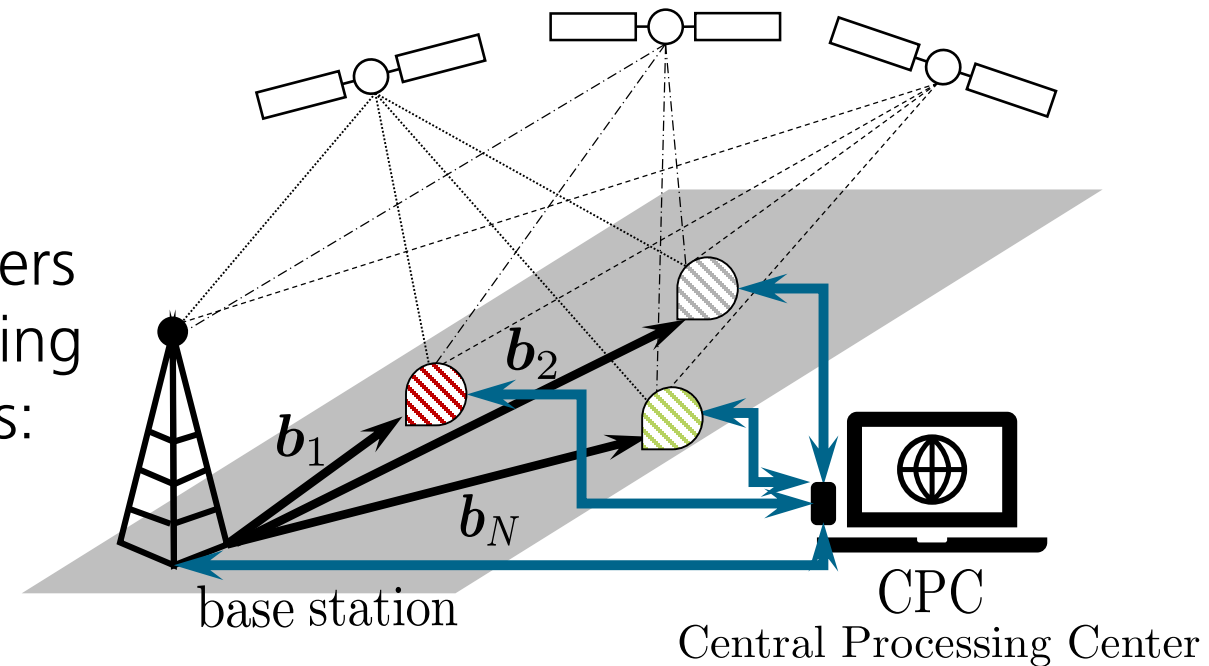
# Monte Carlo based Performance Analysis



# C-RTK – Overview, Benefits, Limitations

- Regular RTK: involves base station to users communication
- C-RTK is a centralized, passive collaboration architecture
- ✓ **Privacy preserving:** users do not compromise their localization information
- ✓ **Performance gain:** greater availability and precision
- ? **Can we live without base stations?**

- ✗ A low-latency, 2-way communication
- ✗ Growing complexity with number of users
- ✗ Careful with: *i)* undetected faults affecting the overall estimation; *ii)* “byzantine” users: those purposely attacking the network



# Industry and Research Perspectives



## From industry & standards:

- Advancement on communication & ranging technologies
- Protocols for the exchange of GNSS-related messages
- **Attractive solution for the largest GNSS market share**

## From research:

- Feasibility of **base station-less RTK** and **regional PPP-RTK**
- The role of Machine Learning: network-wide stochastic modeling, better IAR
- **Robust estimation**: detecting/dealing with non-Gaussian effects
- Relationship to robotics community: active SLAM, distributed perception, ...

- Cooperative localization is an attractive paradigm with lots to do (regarding GNSS) + solution to main issues for urban

**Say hi at:  
daniel.ariasmedina@dlr.de**

**Thanks for your attention!**



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# Impressum



**Title:** Collaborative High Precision GNSS  
ITSNT 2024

**Date:** 2024-06-27

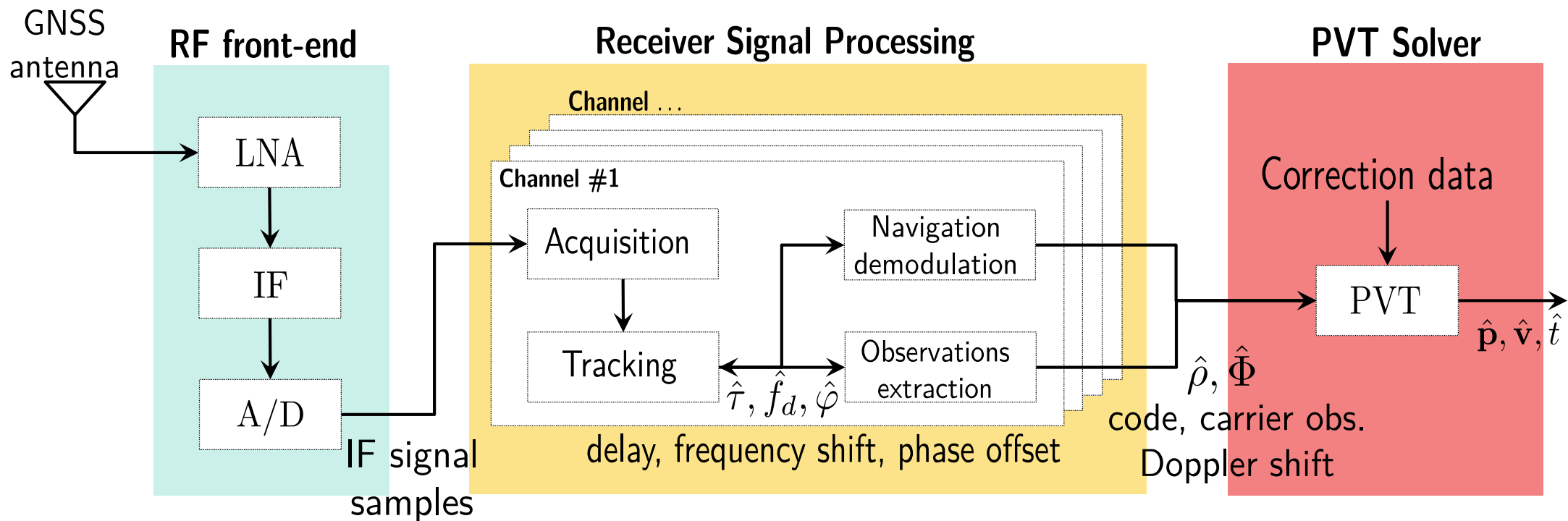
**Author:** Daniel Medina ([daniel.ariasmedina@dlr.de](mailto:daniel.ariasmedina@dlr.de))

**Institut:** Communications and Navigation

**Credits:** All pictures are „DLR (CC BY-NC-ND 3.0)“, unless otherwise stated

**BACK UP SLIDES**

# General GNSS Receiver Architecture





# RTK Processing



State estimate

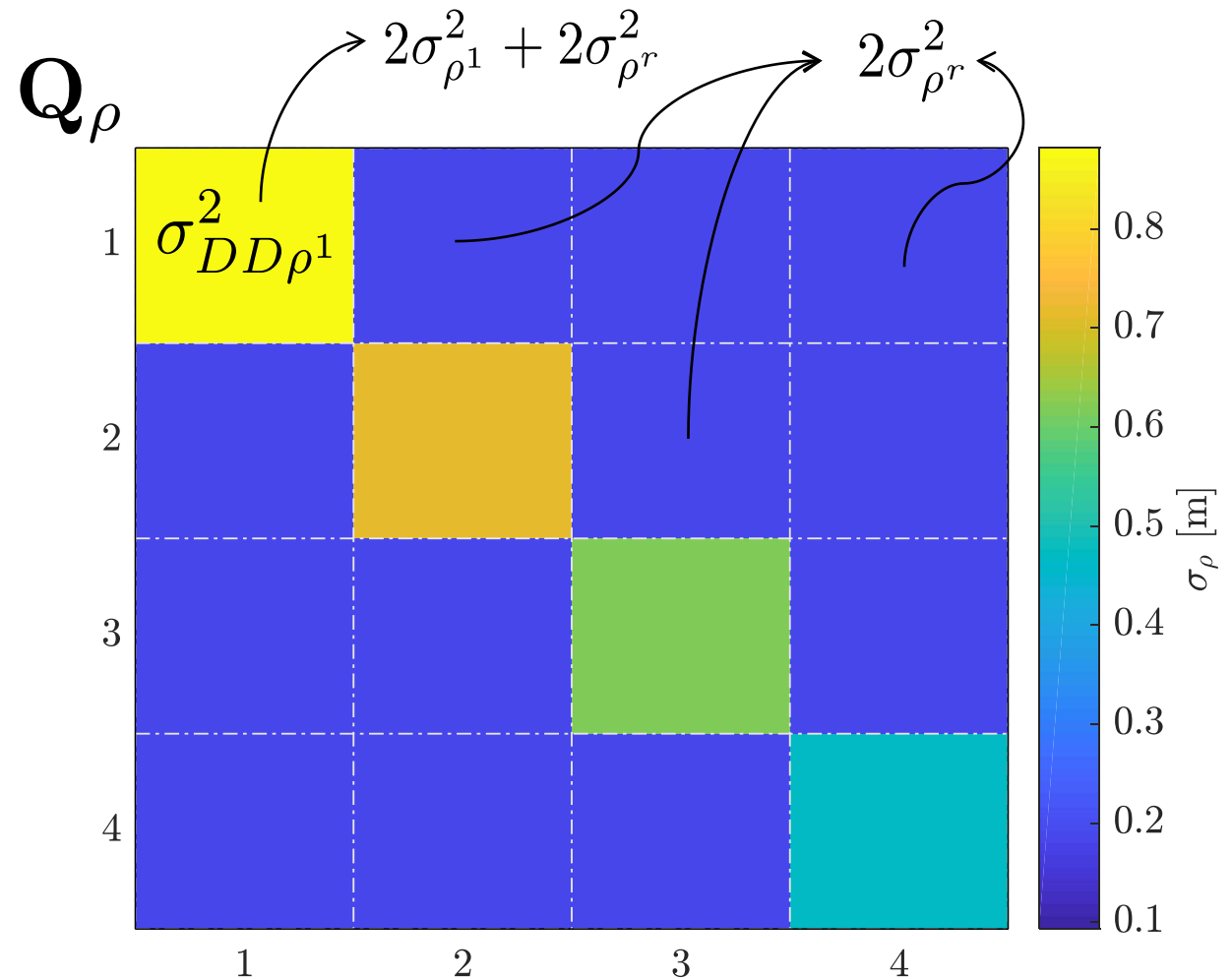
$$\mathbf{x} = \begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix}, \mathbf{a} \in \mathbb{Z}^n, \mathbf{b} \in \mathbb{R}^3$$

Set of observations

$$\mathbf{y} = \begin{bmatrix} DD\Phi \\ DD\rho \end{bmatrix}, \mathbf{y} \in \mathbb{R}^{2n}$$

Careful with noise statistics

$$\eta \sim \mathcal{N}\left(\mathbf{0}_{2n,1}, \underbrace{\begin{bmatrix} \mathbf{Q}_\Phi & \\ & \mathbf{Q}_\rho \end{bmatrix}}_{\mathbf{Q}_y}\right)$$

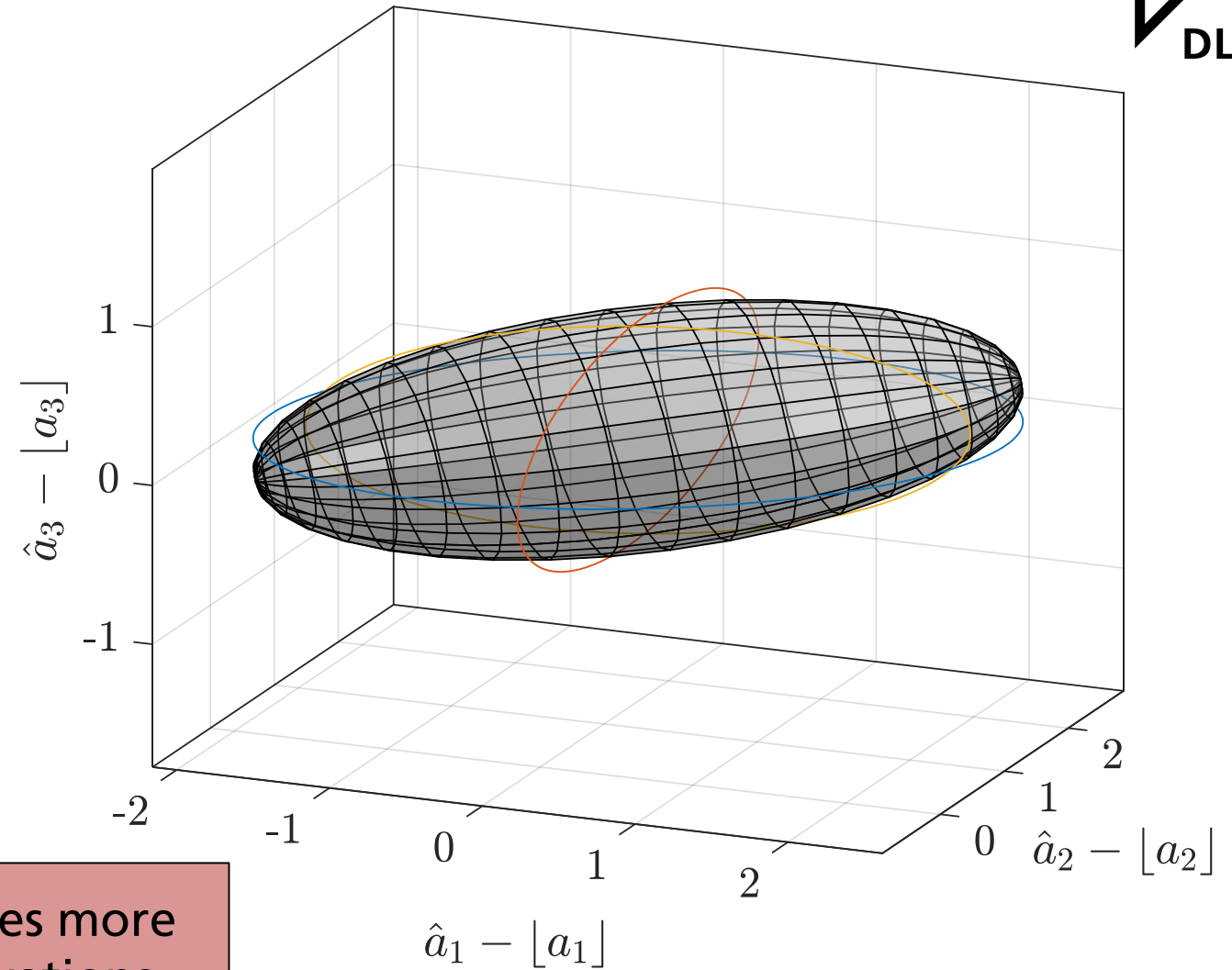


# Integer Ambiguity Resolution

Some basic integer solving

$$\hat{a} = \begin{bmatrix} 15.23 \\ -36.55 \\ 44.11 \end{bmatrix}$$

$$Q_{\hat{a}} = \begin{bmatrix} 2 & 0.5 & 0.2 \\ 0.5 & 1.5 & 0.05 \\ 0.2 & 0.05 & 0.2 \end{bmatrix}$$



Integer ambiguity resolution becomes more complex with the number of observations

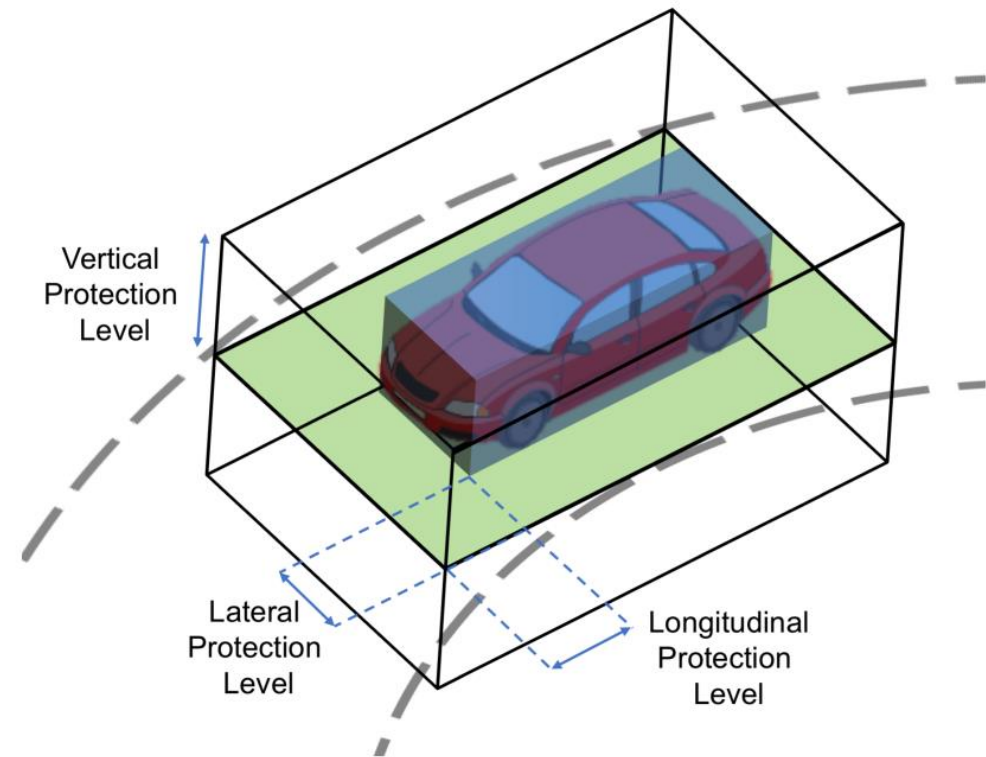
Integrity monitoring measures the trust on the navigation estimates & provides timely warnings when an unacceptable fault occurs / system is unreliable

## Navigational requirements

- Accuracy
- Continuity
- Availability

## Integrity components

- Alert Limit
- Integrity Risk
- Time to Alert
- Protection Level



Reid, Tyler GR, et al. "Localization requirements for autonomous vehicles." *arXiv preprint arXiv:1906.01061* (2019).

# State of the Art on Integrity Monitoring: the limitations



- Standard solutions are derived specifically for aviation purposes:
  - open sky assumption
  - **very low number of faults** (only due to satellite faults)
  - not applicable to landing / take-off maneuvers
- Typically, **only code observations** are used (or code-carrier smoothing)
- **Only snapshot solutions** are considered (no recursive estimation)
- Multi sensor integration and related challenges are not contemplated
- Availability of Integrity Support Message (ISM), meaning “perfect” stochastic modeling



**There is a need for new methods on Integrity Monitoring!**