**CSPBankability Project Report**

# Draft for an Appendix M – Uncertainty Analysis

# to the SolarPACES Guideline for Bankable STE Yield Assessment

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# Contents

# M. Uncertainty Analysis

Assigning meaningful values of uncertainty to the results of CSP yield assessments is essential for the general financial evaluation and economic risk analysis of commercial CSP projects. Therefore, those values of uncertainty characterizing the quality and reliability of predicted yields and economic performance benchmarks should be given.

A modeling approach is a simplified mathematical representation of a physical effect. It should be valid regardless of the size or the quality of the technical system. The technical parameters used in the modeling equations adapt the general approach to the characteristics of a specific technical system with a certain size, capacity and quality. High quality simulation results are only obtained if the chosen parameters closely represent the technical system under consideration.

The chosen parameter values themselves are associated with uncertainty originating from their experimental or theoretical determination. Further uncertainty arises as actual technical parameters are not constant but may vary during operation while typically only considered as average values in the simulation. E.g. mirror reflectivity due to soiling introduces some variability over time, which is usually not addressed in yield modeling, but only approximated by a constant average cleanliness index - this can lead to deviations of actual yields from results of the simplified yield calculation used for financing. Finally, all technical parameters show an uncertainty since the actual realization of components in the construction process always deviates from the specifications to some level.

Since the output of the power plant directly depends on technical parameters it is recommended to assign an uncertainty value at least to those model parameters that have significant impact on the generated electricity. At an early stage of project development, these uncertainty values will be quite high representing the spectrum of technical realization. If the yield analysis is based on a very detailed plant layout, the uncertainty values of the parameters more or less reflect the natural variability of technical parameters in general.

In addition to the uncertainty of technical parameters the approach to the determination of overall uncertainty described in the guideline (chapter 13) also includes the uncertainty contributions of the solar resource data and the STE plant performance model used (refinement, operation strategy). The latter are not covered in this appendix yet.

## M.1. Analysis of Parameter Uncertainty

### M.1.1. Introduction and Basics

In general, the uncertainty of a result determines its significance: A measurement reading or simulation result as such is essentially a random sample of the measurand or simulated quantity. The sole value itself does not contain any information on how likely it is to represent the actual measurand. Its uncertainty distribution and the resulting width of the associated uncertainty band need to be determined by means of uncertainty analysis.

Carrying out a comprehensive uncertainty analysis for simulated results requires a thorough knowledge of the relevant input data and the simulation method. It constitutes a valuable means of reviewing the input data and simulation process with respect to possible errors or inconsistencies. Thus, uncertainty analysis may reveal gaps in such analyses and help to find solutions for improvement.

Results of more complex tests and simulations are typically derived from various measurands or input parameters with a certain functional relationship linking them to the desired output quantity. Analyzing individual contributions to the overall uncertainty helps identifying the most relevant influences as well as prioritizing and solving the strongest sources to reduce the overall uncertainty most effectively.

**Definition of uncertainty**

The uncertainty of a measurement is defined as a parameter that characterizes the dispersion of the measured values around the true value or the interval about a measured value that is likely to encompass the actual value with a certain probability. The uncertainty of a simulation result should represent the vagueness of the predicted quantity for a certain coverage probability. As no measurement process nor equipment, nor set of input parameters, nor simulation approach is perfect, every measurement and simulation result must be fraught with uncertainty.

Formerly, uncertainty was often considered as “error”. The concept of error implies that there is a true value relative to which the magnitude of errors can be quantified. In practice however, such true values are never known. Uncertainty in contrast, refers to the measured or predicted value itself. In terms of the mathematics involved, the two concepts are very similar and commonly described in literature, but the concept of uncertainty is preferred today. The term error rather tends to be associated with actual errors that can occur due to deficient measurement set-ups or inadequate modelling. Such errors are to be reduced to a minimum with reasonable care and identified remaining risks are to be translated into uncertainty contributions.

The general rules for expressing and evaluating measurement uncertainty are set in the “Guide to the Expression of Uncertainty in Measurement” – the so called GUM [GUM 2008]. GUM is a standard by the International Standards Organization and the basis for the following uncertainty evaluation in general. It can also be applied for propagation of uncertainty into predicted/simulated quantities. In order to adequately consider and evaluate measurement and simulation uncertainty the contributing effects are classified according to their nature, value and characteristic probability density distribution and modeled accordingly.

#### M.1.1.1. Uncertainty characteristics

**Accuracy versus precision**

For the qualification of the uncertainty of a data set the terms 'accuracy' and 'precision' are to be distinguished. The accuracy of a measurement refers to the closeness of the measurement result to the true value. It is an expression of the distance a measurement might have from the reference. In statistical terms this translates as the offset of the average value obtained from a series of measurements or the mean of their distribution from the reference value as shown in Figure M-1.

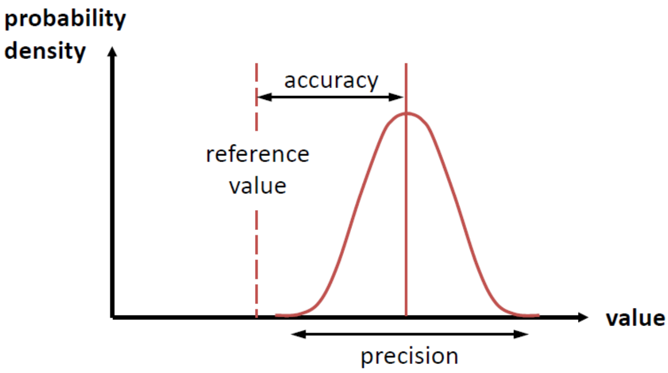


Figure M-1: Illustration of precision and accuracy in measurement (source: DLR).

The precision of a measurement system in contrast, describes the degree to which repeated measurements under unchanged conditions show the same results. This is also referred to as reproducibility (between-run precision, variability on different occasions) or repeatability (with-in run precision, variability on an occasion). The random process of repeated measurements or simulation results with slightly variable output values is characterized by the variance of its probability density characteristic (see bell-shaped distribution in Figure M-1). The lower this variance, the higher is the precision of the measurement.

The fact that precision and accuracy are independent characteristics of a measurement is illustrated in Figure M-2: using the examples of targets with the reference value at the center. This also shows that the terms precision and random uncertainty as well as accuracy and systematic uncertainty are closely linked.

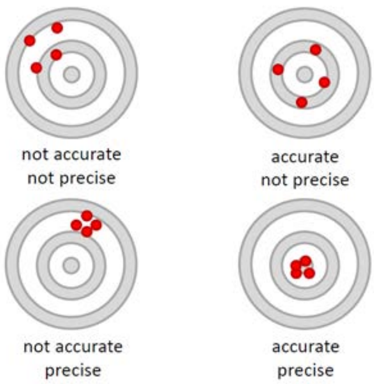


Figure M-2: Illustration of independence of accuracy and precision for an exemplary measurement. (source: DLR)

At the same time as clarifying the meaning of accuracy and precision, Figure M-2 indicates the challenge in distinguishing them from measurement data: while precision can be directly deduced from the measurements and their reproducibility, an assessment of accuracy requires the knowledge of the true value of the target quantity which is hardly available. In practice the accuracy of measurements can only be assessed by comparison to reference instruments of higher accuracy.

**Random versus systematic uncertainty**

Random and systematic uncertainty is a more abstract, mathematical approach classifying observed uncertainty effects that are technically described as precision and accuracy.

Random uncertainty effects result in independent and differing random value for every occasion a measurement is taken although the measurand is unchanged. They are generally assumed to follow a normal distribution and are typically described in terms of the dispersion of the measured values, hence standard deviation or variance. Pure random behavior is typical for independent sampling from a large population, but less relevant for predicting performance or annual yields of CSP systems.

Systematic uncertainty effects in contrast, lead to results all differing from the actual “true” value in the same way and are thus more difficult to detect without any indication of this “true” value.

**Dependent versus independent uncertainties**

Depending on their origin, the contributing uncertainties of a measurement quantity or simulation inputs can be dependent on one another or not. Independent uncertainty contributions are characteristic for different instruments while dependent uncertainties often result from when the same instrument is used or several (linear dependent) parameters are determined from one data set. As the dependence of uncertainty affects influences the width of the resulting budget, this relation is to be considered carefully.

**Parameter uncertainty**

Most importantly in terms of yield and performance prediction, measurement/test results are typically used to identify characteristic parameters of a tested component or subsystem. In combination with an adequate model, these then serve for computational simulation of the behavior of the system under various boundary conditions.

Typical parameter identification methods are least square fitting and optimization. At the same time as yielding the values of best fit parameters, these standard fitting methods can be used to derive their uncertainties. In doing so, particular care is required however, as the evaluation of parameter uncertainty included is typically based on the assumption of independence of measurements points. This assumption is generally violated when a single test set-up and/or set of instruments/sensors is used for all measurement points. In case of predominant uncertainty contributions due to accuracy effects (compared to precision), the measurements are potentially subject to an offset random in value but identical (systematic) for all points.

By assuming independent data points, standard identification methods however, only account for the effect of random data variation (precision) and thus tend to underestimate parameter uncertainty in particular for large series of low accuracy. The sensitivity of identification results to data shifts (due to accuracy effects) is to be tested in a separate step of the analysis and the two results to be combined. Due to calibration procedures, characteristics of individual instruments and high repeatability uncertainty budgets are typically dominated by accuracy rather than precision effects. Thus, the above considerations are vital for correctly concluding on the uncertainty of the identified parameters.

Performance predictions and yield analysis using models for bankability purposes are typically based on three determining elements:

- Solar field and power block model,
- Performance parameters and
- Weather data.

Thus, they involve modelling uncertainty as well as uncertainty of pre-evaluated parameters as well as pure data uncertainty.

#### M.1.1.2. Types of statistical distributions

Any parameter or measurement value merely constitutes one particular occurrence of the basic population of the described quantity. To complete this information different assumption on the general statistical distribution of the quantity are necessary and may be obtained on the basis of additional information on the particular quantity or the way the value is obtained:

**Normal or Gaussian distributions** are used for characterizing distributions in nature and other random quantities as well as process with numerous contributing effects (according to the central limits theorem). Normal distributions are described in terms of mean μ (expected value) and standard deviation σ (or variance $\sigma^2$). Their characteristic bell-shaped probability density function shown in Figure M-3 (left) illustrates the different degree of likelihood for individual occurrences.

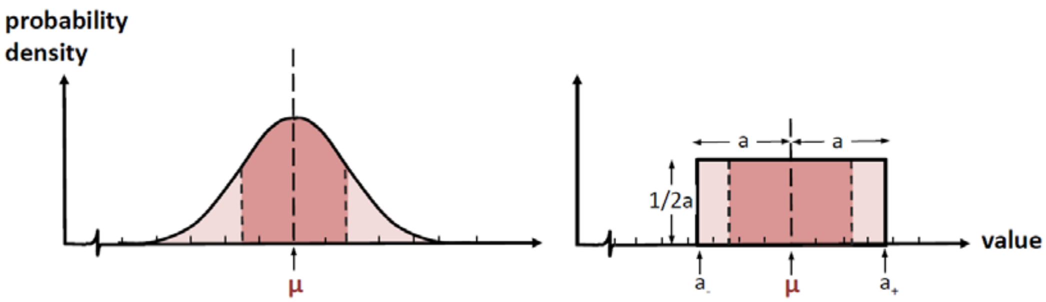


Figure M-3: Probability density functions of normal and uniform uncertainty distributions (source: DLR)

**Uniform or rectangular distributions** as shown in Figure M-3 (right) characterize cases with no preference for any particular values (or range thereof) within certain bounds (± a). They often constitute a conservative estimate if little information on an uncertainty distribution is available and only bounds are named.

Beyond these basic and useful distributions there are numerous alternative modeling approaches to match the particular shapes of uncertainty characteristics such as triangular, M-shaped and loop-sided distributions. However, these are only rarely needed in typical applications and generally require a more detailed knowledge of the individual uncertainty distribution.

### M.1.2. Determining Parameter Uncertainty According to GUM

Any uncertainty assessment of (simulation) results requires concise (pre-evaluated) information on the uncertainty of the contributing/input quantities. Should this not be available to a sufficient degree of detail, it has to be evaluated from background information prior to the actual assessment. The standard “Guide to the Expression of Uncertainty in Measurement” [GUM 2008] provides the basis for such an evaluation of combined standard uncertainty including uncertainty effects of various types and sources. Regardless of their classification according to accuracy or precision GUM distinguishes two types of uncertainties.

#### M.1.2.1. Standard types of uncertainty

**Type A**

Type A encompasses all uncertainty effects that become manifest as variation of the measured values such as signal noise or general reproducibility. These are characterized and evaluated by repeated measurements of the same quantity. Based on $n$ repeated measurements $x_k$, (which are completely random and independent) the standard uncertainty contribution of a Type A quantity is calculated as the experimental standard deviation of the mean according to

$$u(x) = \sqrt{\frac{s(x)^2}{n}} = \sqrt{\frac{1}{n(n-1)} \cdot \sum_{k=1}^{n} (x_k - \bar{x})^2} \qquad \text{(M.1)}$$

**Type B**

Type B uncertainty effects encompass all uncertainty effects that are not purely statistical, e.g. all additional knowledge on the measurement process. This includes various sources of information such as the instruments themselves (characterized by their calibration certificates, manufacturer’s specifications), any available experience from previous measurement series as well as any additional experience or knowledge like results of instrument characterization

Such information is included in the calculation of the resulting measurement uncertainty by modeling the underlying individual uncertainty effects. Depending on the kind of information different modeling approaches can be applied:

- For uncertainty specified as multiples of standard deviations normal distributions are suitable. This mainly applies to previously evaluated uncertainties for example resulting from instrument calibration.

- In other cases limiting (maximum) values of uncertainty might be specified for which rectangular distribution are most suitable.

Selecting the correct modeling approach and assumptions requires experience and is decisive for type B results.

**Combined standard uncertainty**

Similarly to the formerly used propagation of error, there is a propagation of uncertainty that is more commonly called determination of combined uncertainty. As indicated by the name, the combined uncertainty encompasses all uncertainty contributions so that their resulting effect on the target value of a measurement can be quantified. The combination of uncertainty effects can be calculated on several levels of an uncertainty analysis:

- One particular measurand (like temperature or flow rate) can be influenced by several uncertainty effects (for example one Type A and one Type B) so that the calculation of combined uncertainty makes sense on this level.
- Or the target value of a measurement (like useful heat or thermal efficiency as in the case of performance testing) is calculated from several measurands which also necessitates an evaluation of combined standard uncertainty.

And in case of later parameter identification, the separation of precision and accuracy effects yielding two combined uncertainties can be very relevant.

In any of the mentioned cases a functional relationship linking the individual uncertainty effects and the target value is required.

$$y = f(x_1, x_2, x_3, \dots, x_N) \qquad \text{(M.2)}$$

With the help of this functional relationship, and the standard uncertainties of the contributing measurands, the combined standard uncertainty of the target value is calculated according to:

$$u_c(y) = \sqrt{\sum_{i=1}^{N} \left[\frac{\partial y}{\partial x_i}\right]^2 \cdot u(x_i)^2} + 2\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\partial f}{\partial x_i} \cdot \frac{\partial f}{\partial x_j} \cdot u(x_i, x_j) \qquad \text{(M.3)}$$

Combined uncertainty thus is a function of sensitivities and individual standard uncertainties as well as correlation of measurands or uncertainty effects. This formula is valid for correlated and uncorrelated input quantities as the contribution of the second summand is negligible for the latter.

In the context of uncertainty evaluation “standard” always refers to a coverage of 1σ or k = 1. A normal distribution of the resulting uncertainty is assumed so that this value corresponds to covering 68.2% of all possible outcomes. If larger fractions are to be covered, a coverage factor of k>2 must be chosen and stated. The typical values are k = 2, covering about 95 % of all possible outcomes, and k = 3 for 99 %.

$$U(y) = k \cdot u(y) \tag{M.4}$$

### M.1.2.2. Determining parameter uncertainty from measurement samples

Some model inputs, such as the reflectance of reflector materials for example, can be determined by direct measurement of samples. This results in a simplified application of the assessment of their uncertainty discussed above reducing the contributing effects to two, namely the dispersion of the measurements (Type A) and the typical instrument/ process uncertainty of the reflectance measurement (Type B). The former is evaluated according to equation (M.1), while the latter constitutes pre-evaluated information (Type B, potentially available as combined standard uncertainty). The simplest functional relationship linking the two to the final quantity of interest is a mere sum (superimposing the actual measurement values with a normally distributed measurement uncertainty about the same mean and with a standard deviation corresponding to the combined standard uncertainty characteristic for this type of measurement). The resulting standard uncertainty of the reflectance is then determined according to equation (M.3).

### M.1.2.3. Determining parameter uncertainty using manufacturer's specification

Uncertainty information is typically not given by manufacturers in a standard form or coverage but presented as they see fit for the characterization of an instrument or component. This generally leaves room for interpretation and complementary assumptions.

General specifications as stated in a manual or specification sheet are typically valid / guaranteed for an entire series whereas a particular component or instrument may well fulfil tighter individual uncertainty bands. However, these can only be determined by individual characterization or calibration.
Typical uncertainty specifications include:

- Overall uncertainties (according to different operation ranges)
- Specification of tendencies to drift and temperature sensitivities
- Calibration information (typical or instrument specific)

Depending on the type of information and details stated these can be modeled by different types of probability density functions:
Information obviously derived from considerable pre-examinations with numerous contributing effects is best modeled using normal distributions. This typically applies to calibration uncertainties and overall uncertainties if stated with coverage factor. Effects stated in terms of limiting values are often modeled using uniform distributions for lack of more detailed distribution information.

### M.1.2.4. Determining parameter uncertainty from complex measurement data

Other input parameters of the yield simulation like system performance parameters such as optical efficiency or thermal losses cannot be measured directly but have to be derived in more sophisticated ways [Janotte 2012]. These parameters are typically identified from series of test data by means of least square optimization. Consequently, a different approach for determining their combined standard uncertainties is required distinguishing the following uncertainty effects:

- Random effects result from signal noise and uncompensated cross sensitivities and are expressed in terms of repeatability of a measurand.
- Systematic effects characterize every member of a set of measurements. Their actual values are unknown and may be random but the same for all data points of a measurement. These effects are typically caused by uncertainties of individual calibrations or set-ups.

The actual values of random effects for every measurement are independent and normally distributed as indicated by the bell-shaped distributions about the measurements in Figure M-4. Their independence implies that the data/parameter uncertainty of a set of measurements decreases as the number of measurements increases - in other words, the probability of all data points being subject to the same deviation decreases with an increasing number of data points [Press 1992]. The resulting uncertainty is a measure of the scatter or fluctuation of data points about the model function and is determined from the covariance matrix that is calculated based on the Jacobian matrix at the solution. In contrast, systematic effects have a lasting impact on identified parameters requiring a different kind of analysis. Identifying the parameters for a performance characteristic with potential systematic uncertainty corresponds to doing so for the same characteristic but shifted by the value of the systematic contribution, as represented by dotted lines in Figure M-4. Thus, the effect of systematic uncertainty contributions on identified performance parameters can be derived from an evaluation of the sensitivity of parameter identification with respect to data inputs.

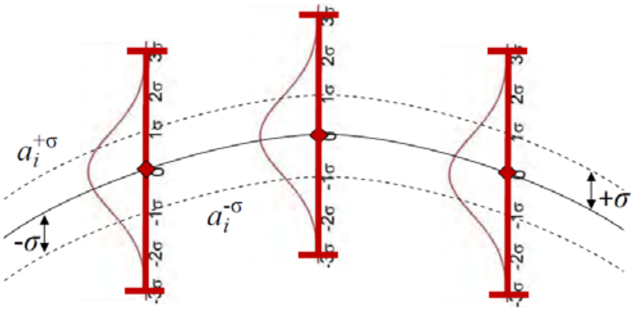


Figure M-4: Systematic and random uncertainties in measurement data and their influence on identified parameters [Janotte 2012]

On the assumption of the independence of the individual parameter uncertainties resulting from random and *l* systematic influences in the data, the combined uncertainty of the parameter $a_i$ is calculated as

$$u_{\mathrm{c}}(a_i) = \sqrt{u_{\mathrm{rnd}}^2(a_i) + \sum_{k=1}^{l} u_{\mathrm{sys},k}^2(a_i)} \qquad \text{(M.5)}$$

## M.2. Overview of methods for the evaluation of parameter uncertainty with respect to yield of STE plants

There are a number of approaches sharing certain aspects in determining the impact of variation of input parameters on simulation results that are (theoretically) applicable for the assessment of the uncertainty of annual CSP plant performance:

- **Extreme Scenarios:** (Realistic) Worst case scenarios for model inputs are established and the outcome in terms of variations in target quantities investigated.
- **Model sensitivity study:** The general sensitivity of annual performance models to variation in inputs is investigated and uncertainties derived.
- **Application of GUM:** The principles of GUM are applied to generate combined standard uncertainty information for annual performance results for uncertain input parameters.
- **Probabilistic Uncertainty Assessment:** Based on simulation results for a large number of random parameter sets, the uncertainty of the target quantity is evaluated in terms of the distribution of the results.

All four methods are based on the principle of monitoring the simulation result as a function of the variation in input parameters using simpler or more sophisticated, empirical or analytical approaches resulting in more or less comprehensive and systematic results.

**Extreme scenarios** generating delimiting worst/optimum case predictions can give but a very rough estimate of the uncertainty of performance predictions. They typically neither include occurrence probabilities, nor uncertainty distribution, nor effects of correlations (if not worst/optimum case) and are thus not constructive for generating qualified uncertainty and thus risk assessments. They may be instructive to show maximum deviations of results.

**Model sensitivity studies** investigate model behaviour for a broad and systematic variation in input parameters resulting in a clear picture of the model behaviour as a function of individual inputs. Knowledge on the model sensitivities is clearly advantageous for any uncertainty evaluation. In a first step, the effort for establishing such relationships by parameter variation is manageable and can be completed without insight into the model. Should there be correlations between model inputs and these impact the target quantity however, the sensitivity analysis becomes multi-dimensional and more complex. As the number of input parameters increases, combining inverse trends and different magnitude impacts on the target quantity becomes increasingly complex. Consequently, an overall picture of the probable distribution of the target quantity is difficult to obtain in spite of the vast information and risk analysis hindered.

The **application of GUM** not only to generate input parameter uncertainties but also to propagate them into the simulation result offers a systematic and analytical approach to the problem on the

basis of partial derivative (sensitivities) and parameter covariance matrices. To this end however, the model must be transformable into a coherent set of model equations and partial derivatives with respect to input quantities be calculable. Should this not be the case - due to the complexity or confidentiality of the model - this type of analysis must be discarded. Furthermore, due to the analytic approach the effort required for setting up an individual GUM evaluation increases as the model complexity and number of equations increases.
The attractiveness of the **probabilistic uncertainty evaluation** (described in detail in section M.3) is largely based on the fact that no other than the annual performance model itself and standard statistical evaluation methods are required to obtain established uncertainty information on the predicted annual yields. The approach can be implemented in various environments (Matlab, Excel, etc.) or integrated in the performance model itself according to individual needs and preferences.
Furthermore, the advantages are that any kind of uncertainty information available in form of distributions can be integrated even without characterizing or classifying the distributions themselves. Like the two first simple methods presented and in contrast to GUM no explicit functional relationship between model input and target quantity is required. As long as the model can be fed with various inputs, generating corresponding results no further information is needed and the model can be treated as a black box. This enables uncertainty evaluation even for confidential models as well as facilitating incorporating complex models with non-continuous elements such as operating strategies etc.). Setting up a GUM evaluation for a complex performance model accounting for all its effects is relatively costly and time-consuming and requires individual solutions in every application, whereas the only parameters undergoing change in probabilistic approaches are the sample size and numbers of parameters.

While alternative approaches offer insight into particular aspects of uncertainty, **probabilistic uncertainty evaluation is judged most effective, versatile and comprehensive for CSP plant performance uncertainty analysis.**

## M.3. Methodology for probabilistic uncertainty modeling of CSP power plant yields

### M.3.1. Approach and General Conditions for CSP Applications

Probabilistic modeling approaches are a common means of accounting for the fact that the inputs to a model are typically clouded with uncertainty: Instead of solely considering their most probable value (as in the case of deterministic models), they incorporate input parameters and their respective probabilities by statistical distributions (probability density functions) and repeated evaluations of the target function. To this end, random samples are created from the probability density functions of the individual input parameters and grouped into sets of parameter inputs according to prevailing correlations between the input quantities. The model is evaluated for this large number of different sets of input parameters. Consecutively, the uncertainty distribution and best estimate of the desired output quantity are derived from all model results as a whole.

In view of potentially complex annual CSP performance/yield models requiring considerable amounts of time for each simulation run, reducing the number of parameter samples sets to a minimum without compromising the significance of the result is particular relevant. Correlations among input quantities resulting from previous system analyses are common and need to be allowed for. Typical target values characterizing the overall annual plant performance and the impact of model and parameter uncertainty are LEC or annual solar or net electric yields.

[Janotte 2012]

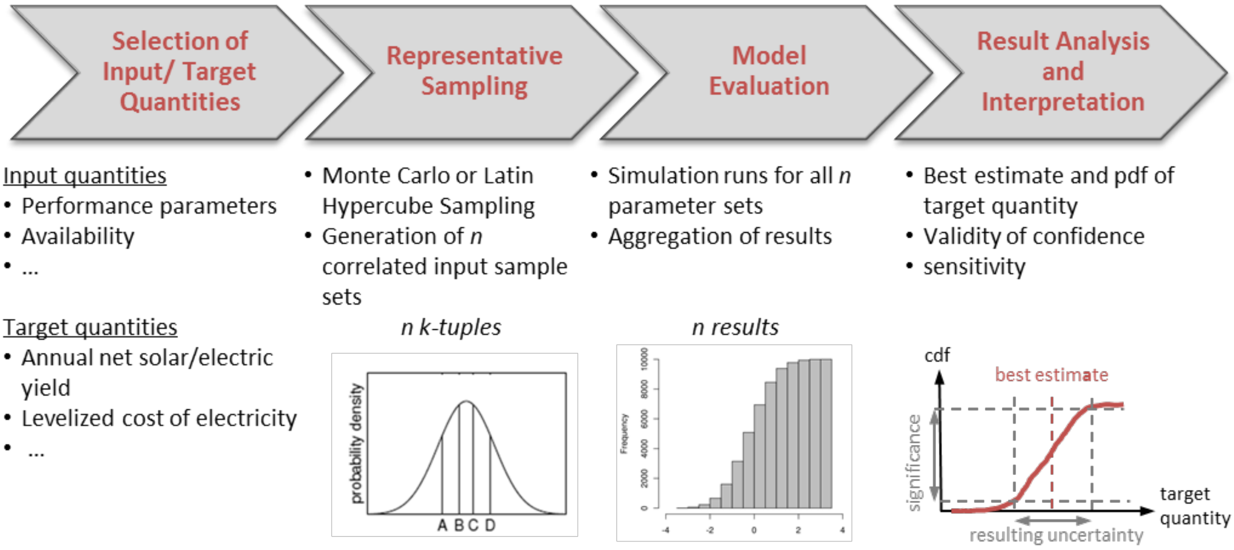


Figure M-5: Estimating parameter uncertainty by probabilistic assessment (source DLR)

### M.3.2. Representative Sampling

In the case of probabilistic uncertainty analysis sampling aims at the selection of a limited number of subsets of input parameters from within their statistical population that ideally describe the characteristics of the whole population. Monte Carlo and Latin Hypercube Sampling are the most commonly used methods for random sampling respecting the distributions of a random quantity.

Monte Carlo Sampling consists of mere repeated random sampling from parameter populations to obtain the desired subsets. Latin Hypercube sampling (LHS) in contrast is a constrained or stratified Monte Carlo sampling scheme:

For *n* samples, the range of every parameter (*k*) is divided into *n* non-overlapping, equally probable intervals as illustrated in Figure M-6. Respecting the local probability distribution of the parameter, one value is randomly generated for every interval. These sampled values of the parameters are then combined at random to form *n k*-tuples, the so-called LHS sample. Instead of random pairing, known correlations among sampled parameters can be incorporated at this step by means of rearrangement of *n* samples of the individual parameters. The minimum number of required samples in order to honor such parameter correlations is 4*k*/3.

[Ho 2010; Wyss 1998]

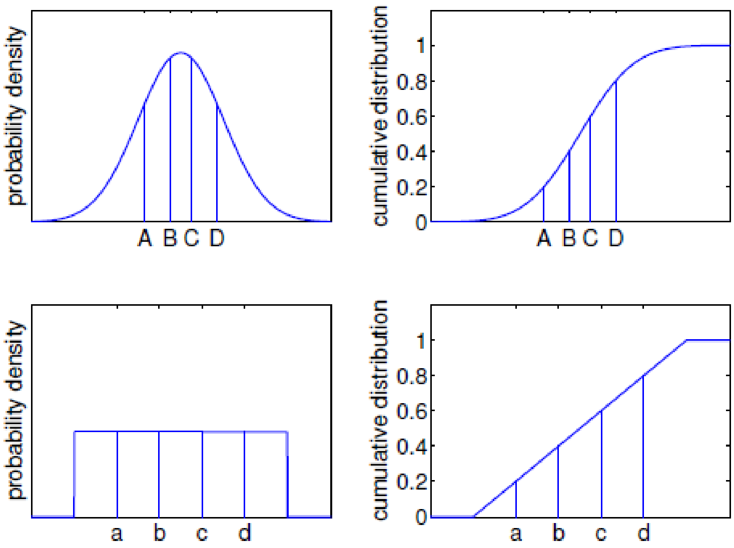


Figure M-6: Application of the concept of equally probable intervals to normal and uniform uncertainty distributions

The stratification of the sampling by division of the total range of values into intervals ensures an adequate representation of all parts of a parameter's uncertainty distribution (including the tails) for relatively small numbers of samples. This way, the computational effort of repetitive simulations in a probabilistic approach is minimized without compromising the representativeness of the result.

### M.3.3. Evaluation of Sampling Quality

In random sampling the quality of a sample is evaluated in terms of its capacity to adequately represent the population. Should this not be sufficient, the size of the sample needs to be increased. The number of sets of samples actually required can only be estimated once the probabilistic modeling results are available. A first (iterative) indication for the sufficiency of the sample size is the change of the mean of the results as the sample size increases or decreases. Using the posterior evaluation of the validity of confidence the capacity of a particular sample and distribution of results in representing the population can also be evaluated directly. The validity of confidence *c* is calculated by the following equation and expresses the probability with which the population mean $\bar{x}$ can be claimed to fall within certain bounds *b* (i.e.0.5% corresponding to a confidence level of 99%) of the simulated mean based on the sample [Ho 2011]

$$c = P\left(-b\bar{x}\frac{\sqrt{n}}{s} < t_{n-1} < b\bar{x}\frac{\sqrt{n}}{s}\right) \quad \text{(M.6)}$$

This evaluation is based on the fact that mean values of random samples from a population of normal distribution can be shown to be normally distributed themselves. As the unknown standard deviation *s* of the mean values is estimated from the samples in the present case, the Student's t-distribution

with a degree of freedom of *n-1* is applied instead of a normal distribution. This is particularly relevant for small numbers of samples (*n* < 100), for larger numbers the t-distribution tends towards a normal distribution and deviations are negligible. [Mohr 2008]
In accordance with the above equation, the determining factors for the required sample size *n* are the standard deviation of the mean $\bar{x}$, the chosen width of bounds and the desired level of confidence. As the standard deviation of the mean and the mean itself result from the analysis of the sample itself, the required number of samples which is indispensable for the generation of the sample can only be estimated roughly prior to the actual analysis. As the distribution of parameters generally broadens with increasing uncertainty resulting in larger variability of predictions, increasing standard deviation of means are expected. These imply the need for larger numbers of samples to ensure a particular level of confidence.

In practice, the required sample size for the probabilistic analysis differs according to the sensitivity of the target quantity to variation in input quantities and the standard uncertainty of model inputs. In general, large standard uncertainties of parameters lead to large standard deviations of the sample mean and thus imply large required numbers of samples. This can be investigated by increasing the number of samples until a satisfactory value of the validity of confidence is reached. Furthermore, the larger the desired coverage probability for the evaluation of the target quality, the more detailed information of its distribution towards the tails is generally required. This typically implies the need for increased sizes of sets of parameter samples. Optically, the smoothness of the resulting cumulative probability density function can serve for the illustration and evaluation of the impact of small numbers of samples especially towards the edges as illustrated in Figure M-7:.

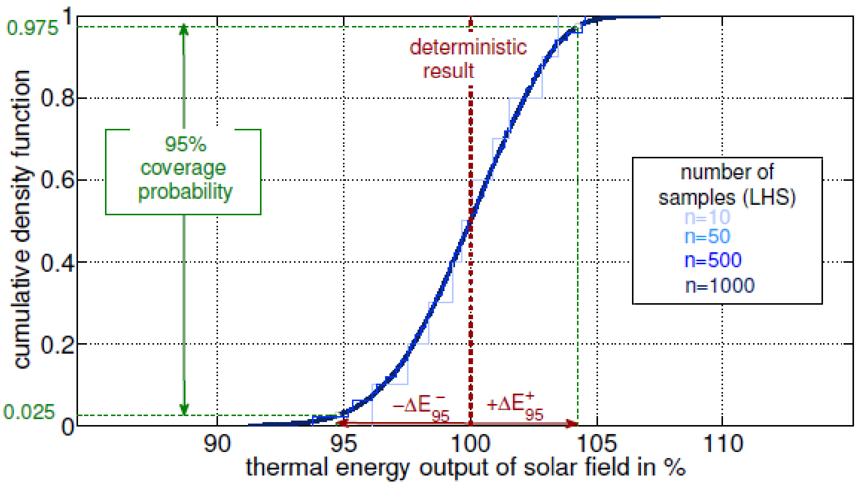


Figure M-7: Exemplary result of a probabilistic uncertainty evaluation for different sample sizes

### M.3.4. Evaluation and Interpretation of Results

Provided a satisfactory validity of confidence, the distribution of the target quantity serves as the basis for further assessment of the uncertainty of the prediction. The level of confidence can be set according to the requirements of the investigation. For scientific purposes 95% coverage (corresponding to 2σ) is typical. As illustrated in Figure M-7, the annual yields for which the cumulative density function amounts to 0.025 and 0.0975 confine the range of possible yields to be expected with a 95% coverage probability for the given parameter set. The smaller this range of the mean/deterministic result delimited by $\Delta E_{95}^{-}$ and $\Delta E_{95}^{+}$ is, the higher the quality of the prediction is. Likewise, alternative measures of uncertainty such as the standard deviation $\sigma$ can be derived from probabilistic uncertainty results in form of cumulative density functions. For conservative performance estimates the yield that will be exceeded or the cost that will be undershot with a probability of 50% (P50) is typically referred to. The predicted value corresponding to this P50 exceedance probability (or median) is also derived P(x) = 0.5 from the previously discussed cumulative density function of the target quantity. Only in the case of large samples and symmetrical distributions of the target quantity this result converges towards the mean value and even more rarely towards the deterministic result.

For an identification of the most influential factors and a better comprehension of the underlying principles a sensitivity analysis of the results can be instructive. The impact of individual input parameters on the annual yield is determined by the degree of correlation between the yield and the respective parameter and the sensitivity of the yield to changes in the related quantity. These are expressed in terms of Spearman's rank or Pearson's linear correlation coefficient and the slope of a linear regression. The coefficient of determination calculated as

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y}_i)^2} \qquad \text{(M.7)}$$

is a measure of the fraction of the total variance of the output that can be explained by the variability in input quantities or predictors ($\hat{y}_i$). Thus, it is a means of evaluating the goodness of fit. Starting with the most influential factor, a stepwise regression analysis investigates the effect of the successive addition of predictors on the model quality. The change in the (adjusted) coefficient of determination $R^2$ serves as criterion for the significance of model improvement. If it tends towards zero or results in negative values, the analysis is stopped as no further improvement is expected.

## M.4. Symbols used in this appendix

| Symbol | Description | Unit |
|---|---|---|
| $b$ | Bounds for interval into which mean is likely to fall with the calculated probability corresponding to a certain confidence level | - |
| $c$ | Validity of confidence | - |
| $k$ | Number of parameters constituting one parameter set<br>Coverage factor | - |
| $n$ | Samples size, number of samples | - |
| $P()$ | Probability | - |
| $R^2$ | Coefficient of determination | - |
| $s, \sigma$ | Standard deviation | * |
| $u(x)$ | Standard uncertainty | * |
| $u_c(y)$ | Combined standard uncertainty | * |
| $\bar{x}, \bar{y}$ | Mean value | * |
| $\hat{y}$ | Predictor value | * |

*individual units according to the quantity evaluated

# List of Figures



# List of literature

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