# Vibration correlation technique: Methodology applied to buckling of large-scale sandwich cylindrical shell

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**ABSTRACT** This paper assesses the Vibration Correlation Technique's (VCT) applicability within the context of sandwich cylindrical shells through detailed numerical investigation. The study utilizes a sandwich shell from NASA's Shell Buckling Knockdown Factor project. Finite element models are implemented, incorporating initial mid-surface and thickness imperfections to replicate NASA's buckling test campaign. The numerical results are compared to the experimental data, validating the nonlinear solution with an approximate deviation of 1 %. Subsequently, the VCT experimental campaign is numerically conducted through free vibration analyses at different load levels, enabling a comprehensive evaluation of VCT's applicability. The results verified the effectiveness of the VCT, demonstrating a deviation of less than 10 % when estimating the buckling load for load levels below 65 % of the nonlinear buckling load. Overall, the findings confirm the non-destructive nature of the VCT when employed on such structures, supporting future practical applications.

**KEYWORDS** Non-destructive technique; Buckling; Imperfection-sensitive structures; Nonlinear FE analysis

## 1. Introduction

Cylindrical or conical shells are often found in the launch vehicle primary structures due to their inherently optimized strength-to-weight ratio. In such applications, the design, driven by the operational load envelope, predominantly focuses on mitigating buckling failure (Degenhardt et al., 2010). Due to the high imperfection-sensitive of these structures, slight deviations from the ideal parameters related to geometry and boundary conditions, e.g., geometric shape and minor irregularities, result in a measurable difference between the theoretical buckling load, obtained by classical theory, and the corresponding experimental one (Amabili, 2008; Degenhardt et al., 2010; Hoff and Soong, 1965).

With this gap between the results and the loads, the NASA SP-8007 guideline was established, proposing a design factor, commonly known as the knockdown factor (KDF), multiplied by the critical buckling load. This approach, even though it is nowadays considered overly conservative, provided successful shell design for several generations of space projects (Arbocz and Starnes Jr, 2002). Recently, a second review of NASA SP-8007 was published, offering alternative methods for improving KDF. Advanced design criteria were proposed to consider each particular structure imperfection characteristics. This approach ensures that the structural design is less conservative; however, the buckling test is still required (Hilburger, 2020). Due to its destructive nature, the buckling test cannot be repeated under the same conditions without affecting the integrity of the structure. Non-destructive tests ensure that the same structure can be used in other tests, which is particularly relevant from a financial and time-consuming point of view.

The vibration correlation technique (VCT) estimates in situ buckling load based on a series of vibration tests conducted at different load levels. The frequency modes and their corresponding natural frequencies are determined for each vibration test. The buckling load is estimated by establishing the relationship between the natural frequency magnitudes and the corresponding applied load. In the early 20th century, Sommerfeld correlated natural frequency and applied load, when analyzing a cantilever beam with a variable mass at its free end. As the variable mass approached the buckling load, the first natural frequency of the structure tended toward zero (Singer et al., 2002). Subsequently, Massonnet (1940) expanded the analysis to the analytical field by correlating the square of the natural frequency ($\omega^2$) with the applied load ($P$) for beams, plates, and rods. The author identified that the relation between the variables was precisely linear if the vibration mode was identical to the buckling mode, establishing the relation

$$f^2 + p = 1 \tag{1}$$

where the parameter f is the ratio between the natural frequency of the loaded structure ($\bar{\omega}_{mn}$) and the unloaded structure ($\omega_{mn}$). The subscripts $n$ and $m$ represent the number of circumferential waves and axial half-waves for analysis in cylindrical structures, respectively. The variable p represents the ratio between the applied load ($P$) and the critical buckling load ($P_{CR}$).

For structures sensitive to imperfections, the relationship between the square of the natural frequency and the applied load does not exhibit linear behavior as it approaches the critical buckling load (Lurie, 1952). Therefore, the VCT applied to columns cannot be used for


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cylindrical shells and it is currently under development, as indicated by Kalnins et al. (2015), Skukis et al. (2017), Franzoni (2020), and Baciu et al. (2023).

Okubo and Whittier (1967) analyzed six spherical shells to assess the relation described by Eq. 1 for the first free natural frequency under pressure. The authors identified that each structure would have a specific curve for the VCT and associated this behavior with geometric imperfections and residual stresses.

Subsequently, research conducted at the Technion's Laboratory of Space Structures, indicated that in reinforced cylindrical shells, the support conditions have a greater influence, while imperfections have less impact on buckling behavior compared to unreinforced cylindrical shells. Additionally, the corresponding buckling and vibration modes are similar for lower natural frequencies. A curve fitting for the test data was suggested through the following relationship

$$f^q = A + Bp \tag{2}$$

where $A$ and $B$ are adjustment constants and $q$ is the exponent that represents the optimal value leading to the extrapolation point exactly matching the experimental buckling load.

Another parameterization relation was proposed by Souza et al. (1983), where the authors reviewed some of the VCT methodologies to determine the buckling load of structures characterized by unstable equilibrium paths, empirically proposing the parameterization of Eq. 1 for lightly reinforced cylindrical shells in the form of

$$1 - f^4 = (1-p)^2 \tag{3}$$

the instability is evaluated when the natural frequency of the loaded structure approaches zero, where the relationship is given by

$$(1-p)^2 + \left(1+\xi^2\right)\left(1-f^4\right) = 1 \tag{4}$$

the parameter $\xi = 1 - p_0$ represents the reduction in the buckling load due to the initial imperfection, where $p_0$ is the load associated when the natural frequency approaches zero. The parametric form in Eq. 3 ensures a linear relation for the data evaluated by the authors, deviations from this behavior can be interpreted as changes in boundary conditions during the test.

Arbelo et al. (2014) analyzed the VCT using Eq. 3 for composite unstiffened cylindrical shells and obtained negative values for $\xi^2$, which do not have a physical meaning. Therefore, the authors empirically proposed replacing the parameterization $1 - f^4$ with $1 - f^2$. With this modification, the expected behavior resembles a second-order equation, and the value of $\xi^2$ represents the square of the reduction in load-carrying capacity due to the initial imperfection, as in Souza et al. (1983).

The new parameterization was validated for benchmark structures available in the literature and various experimental campaigns, including Arbelo et al. (2015), Kalnins et al. (2015), Skukis et al. (2016), Skukis et al. (2017), Shahgholian-Ghahfarokhi and Rahimi (2018), Labans et al. (2019) and Franzoni (2020). According to Tian et al. (2022), numerical VCT can be implemented by following the same steps as the experimental procedure. In this study, the authors proposed an optimization design to enhance the buckling load based on the implemented approach. They conducted analyses on three cylindrical shell structures: one with variable stiffness, one isotropic, and one with a combination of various types of imperfections.

In Franzoni et al. (2018), the empirical parameterization proposed by Arbelo et al. (2014) is analytically demonstrated, confirming that $(1-p)^2$ is related to $\left(1-f^2\right)$ by a second-order equation in which

$$(1-p)^2 = \left[1-\left(1-f^2\right)\right]^2 \tag{5}$$

and the concept of the KDF factor is related to the minimum magnitude of $(1-p)^2$ without the need to reach the natural frequency equal to zero, i.e., the minimum magnitude is achieved at the exact point of the buckling load. The equivalent KDF for the VCT is proposed by Eq. 6.

$$\gamma_{VCT} = 1 - \xi \tag{6}$$

Gliszczyński et al. (2022) analyzed the capability of numerical VCT in conical shells with variations in the imperfection parameter, geometry, and thickness using the parameterization proposed by Arbelo et al. (2014). The results obtained for the buckling load prediction were conservative in 99.93 % of the 2826 predictions.

Recently, some research has been addressing cylindrical and conical sandwich shells within the context of VCT, evaluating the technique through numerical models that represent imperfections of these structures, such as mid-surface and thickness imperfections. Additionally, they have extended the analysis to experimental tests to assess the convergence of results between the approaches, as exemplified in Shahgholian-Ghahfarokhi et al. (2020) and Zarei and Rahimi (2022).

Particularly, Shahgholian-Ghahfarokhi et al. (2020) conducted a numerical and experimental approach to VCT, where the results demonstrated a good correlation when considering the nonlinear effects of mid-surface and thickness imperfections. The correlation between the results showed deviations below 5 %.

In the study by Zarei and Rahimi (2022), VCT provided reliable estimates for conical sandwich shells when at least 65 % of the experimental buckling load is used while indicating the inadequacy of using the linear parameterization proposed by Eq. 1 in this type of structure.

The VCT is a well-established technique in conventional structures such as beams and plates. However, for structures in cylindrical shells, particularly in the sandwich configuration, the technique is still under development (Shahgholian-Ghahfarokhi et al., 2020). Whithin this context, this paper proposes the evaluation of VCT in a cylindrical sandwich shell through a numerical approach, considering a finite element model that addresses the specificities of the real structure used as a reference.

Table 1: The mechanical properties.

| Properties | IM7/MTM45 | AS4/8552 | Core |
|---|---|---|---|
| Elastic modulus (Msi) | $20.662+613\varepsilon_1$, 1.250, n.d | $9.4\cdot10^6$, $9.4\cdot10^6$, n.d | $50\cdot10^{-6}$, $38\cdot10^{-6}$, $60\cdot10^{-3}$ |
| Shear modulus (ksi) | 770, 770, 770 | 720, 720, 720 | $17.5\cdot10^{-3}$, 29.5, 12 |
| Poisson ratio's | 0.36, n.d, n.d | 0.04, n.d, n.d | 0.45, 0.0001, 0.0001 |
| Density $\rho$ (lbs$^2$/in$^4$) | $110.327\cdot10^{-6}$ | $147.6\cdot10^{-6}$ | $6.729\cdot10^{-6}$ |
| Thickness (in) | 0.005679 | 0.014 | 0.25 |

## 2. Methods

### 2.1 Structure

The analyzed model represents a sandwich cylindrical shell with a honeycomb core of average radius $R=48$ in, length $L=100$ in, average thickness 0.33 in, and slenderness ratio of 145. The inner and outer layers are laminated composite material with a stacking sequence of $[45/-45/0/90/-45/45]$ using IM7/MTM45 material. At the upper and lower ends of the cylinder, represented by detail B in Fig. 1, there are two double pad-ups with three thickness variations ($t_1$, $t_2$, $t_3$) and a total length of de 20 in. The pad-ups have a stacking sequence of $[45/-45/45]_{t_1}$, $[-45/45]_{t_2}$, $[45]_{t_3}$ at the upper end and $[45/-45/45]_{t_1}$, $[45/45]_{t_2}$, $[45]_{t_3}$ at the lower end, both made of AS4/8552 material. The reference structure was tested within the SBKF (Shell Buckling Knockdown Factor) described by Schultz et al. (2018). The mechanical properties are shown in Table 1.

### 2.2 Numerical assessment

Two numerical models were developed for this study: a linear model and a nonlinear model. The linear model considers the perfect structure, while the nonlinear model incorporates middle surface imperfections (MSI) and thickness irregularities (TI). The MSI and TI imperfections are added to the numerical model using an inverse-weighted algorithm devised by Castro et al. (2014). The script calculates a disturbed nodal position (MSI) or a new thickness magnitude (TI), based on the initial nodal positions or element center of gravity positions of the perfect shell. Moreover, the TI imperfections are assumed to be irregularities generated by the core.

The numerical models utilize S4R finite elements, with simple support boundary conditions (SS4) and multiple point restrictions of the pin type applied to the central control node. A mesh size convergence study is conducted to ensure a deviation of less than 1 % for the chosen mesh compared to the finest mesh. Since the most critical conditions for buckling load occur for the first modes, the first buckling mode is used as a reference to establish the appropriate number of finite elements. The analyzed range of finite elements around the diameter is between 50 and 1600 elements, considering multiple numbers. The established criterion is met at 800 elements, as indicated in Fig. 2.

Considering the stability of the results, computational cost, and the convergence criterion, 850 finite elements around the diameter are chosen for this study. In total, there are 240,264 S4R elements and 241,116 nodes, with an approximate average size of 0.35 in, corresponding to a deviation of 0.5 %.

With the defined mesh, the analysis is performed to determine the buckling load through direct loading modeling. A uniform and unitary static load is applied to the center of the structure for the linear numerical model. In practice, buckling occurs according to the first modes, so the solution for the first 5 eigenvalues is obtained. The Lanczos method is used for solving the eigenvalue problem. Similarly, in the free vibration condition, the perturbation is added in a linear form. The results obtained for the first mode of free vibration and buckling are shown in Fig. 3(a)

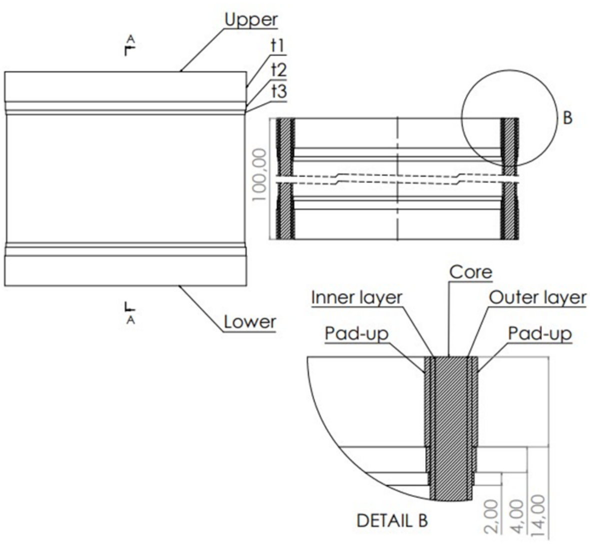



Figure 1: Sandwich composite cylinder.

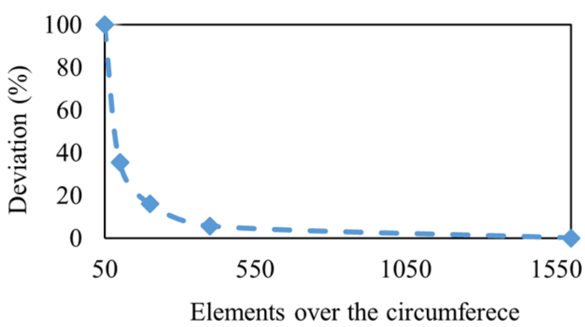
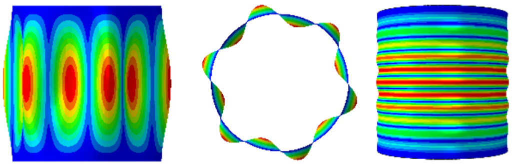
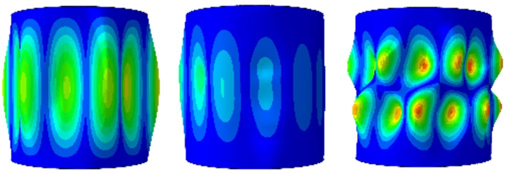



Figure 2: Mesh convergence for linear numerical model.



(a) First vibration mode (129.28 Hz). (b) First buckling mode (940,845 lb).

Figure 3: First free vibration and buckling modes.



(a) 129.28 Hz (b) 88.86 Hz (c) 16.63 Hz

Figure 4: First free vibration modes for the imperfect structure.

and (b), respectively.

Evaluating the mentioned figures, there is no similarity between the first buckling and vibration modes. The first vibration mode is governed by $m = 6$ circumferential waves and $n = 1$ axial half wave, while for the buckling mode, the number of circumferential waves is $n = 1$. The buckling load for the linear case is $P_{lin} = 940,845$ lb, which represents a deviation of 11.3 % compared to the nonlinear buckling load of the reference model $P_{ref} = 845,000$ lb (Schultz et al., 2018).

The nonlinear solution to buckling is obtained through the Newton-Rapshon method with artificial damping for controlling forced displacement. The artificial damping is added based on the specified dissipation energy, which represents a fraction of the strain energy. The damping coefficient is automatically calculated by the solver. The properties of the numerical model are listed in Table 2.

The nonlinear buckling load is $P_{NL} = 840,643$ lb, which represents a deviation of 0.5 % and 1.0 % from the nonlinear buckling load of the numerical model and the buckling load obtained in the experimental test available by Schultz et al. (2018), respectively. The quantified deviations may be associated with imperfection readings and the approximations made in the implemented numerical model, such as the simple support boundary conditions (SS4).

The vibration modes of the loaded structure are obtained by including an additional vibration step in the nonlinear solution for desired load levels. Once more, the first five vibration modes are requested. In Fig. 4(a), (b), and (c), the first free vibration modes for the imperfect structure are shown in the unloaded condition, nonlinear buckling increment, and first stable post-buckling increment, respectively.

The buckling and vibration modes under the analyzed conditions do not show similarity. Additionally, the natural frequency of the structure does not reach zero magnitudes, which reinforces the need for the application of a

Table 2: The properties of the numerical model.

| Parameters | Values |
|---|---|
| Nonlinear algorithm | Newton-Rapshon |
| Element type | S4R |
| Elements around circumference | 850 |
| Specify dissipated energy fraction | $1 \cdot 10^{-6}$ |
| Min. increment | $1 \cdot 10^{-7}$ |
| Max. increment | $1 \cdot 10^{-2}$ |
| Max. number of increments | 600 |

method that does not require this condition to be met. Extensive research conducted analyzing the various impacts of design parameters of cylindrical shells, as demonstrated in Baciu et al. (2023) and Jeon et al. (2023), indicates, based on the experimental and numerical results obtained, the lack of necessity for a frequency magnitude equal to zero.

## 3. Results

To evaluate the VCT, it is necessary to define the maximum load level used for estimating the buckling load. In experimental tests, it has been suggested that 60 % of $P_{CR}$ (Skukis et al., 2017; Shahgholian-Ghahfarokhi and Rahimi, 2018) serves as a safe percentage for the test to occur within the elastic range. Nevertheless, as this is a numerical study, a broader range of the load-displacement curve can be exploited. Thus, the displacement steps were divided into 10 sub-intervals ranging from 9.8 % to 87.6 % of $P_{CR}$; moreover, both the deviation of the VCT prediction $\delta_{VCT}$ and the maximum load level considered for such prediction $P_{MAX}$ in Fig. 5 refer to the nonlinear buckling load $P_{NL}$. All deviations fall within the negative range, indicating that all VCT estimations are conservative, thereby demonstrating its capability as a non-destructive technique.

In addition to determining $P_{MAX}$, according to the methodology proposed by Arbelo et al. (2014), a minimum of three points for the relationship between $(1-p)^2$ and $\left(1-f^2\right)$ is necessary for appropriate VCT predictions. However, the use of more points indicates better convergence between numerical and experimental results (Gliszczyński et al., 2022). Therefore, in this evaluation, 10 sub-intervals are again adopted for the prediction of the buckling load. The prediction curve is described by a second-order nonlinear interpolation, and the evaluated conditions for $P_{MAX}$ = 65 %, 75 %, 85 %, 98 %, as illustrated in Fig. 6. Maintaining the conservative behavior for the 10 analyzed sub-intervals, the VCT demonstrates its capacity to be a truly non-destructive technique. The values of applied and predicted loads, free fundamental frequencies, associated deviations, and equivalent KDFs by VCT are presented in Table 3.

The KDF obtained by Schultz et al. (2018), considering the NASA SP-8007 methodology for a similar structure to the one evaluated in this study, is $\gamma = 0.61$. While the VCT in the most conservative condition, i.e., with 65 $\%P_{NL}$, yields an equivalent KDF $\gamma_{VCT}$ equal to 0.80 for

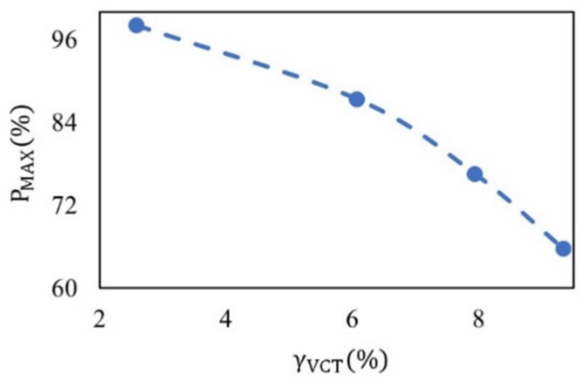



Figure 5: VCT deviation analysis according to the applied load.

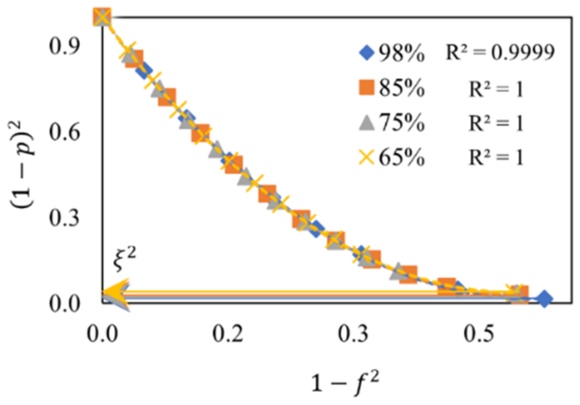



Figure 6: VCT application to 65 %, 75 %, 85 % and 98 % of $P_{NL}$.

a highly reliable numerical model. The deviation between these values is 31 %, which can be justified by the different tools implemented in their respective analyses. The value of $\gamma$ was empirically established based on the results obtained from experimental tests conducted between the 1930s and 1960s. The second published revision of NASA SP-8007 (Hilburger, 2020) already considers design approaches that lead to less conservative KDF results based on high-fidelity numerical modeling.

## 4. Conclusions

The VCT based on the parametrization proposed in Arbelo et al. (2014) demonstrated satisfactory and conservative results, with deviations below 10 % when applying 65 % of the nonlinear buckling load for an equivalent KDF of $\gamma_{VCT} = 0.80$ under these conditions. This demonstrates its capability as a non-destructive technique in sandwich cylindrical shells, showcasing its potential for future practical applications. In comparison to the KDF obtained from the NASA guideline of 0.61, there is a deviation of 31 %, which is attributed to the tools used. The VCT considers specific characteristics of the structure and inherently considers the effects of the imperfections and deviations of a given numerical model (test stand in case experimental results are evaluated), whereas the NASA

Table 3: VCT results for the models analyzed in the first frequency natural vibration.

| $P_{MAX}$ (lb) | $P_{VCT}$ (lb) | Frequency (Hz) | $\delta_{VCT}$ (%) | $\gamma_{VCT}$ |
|---|---|---|---|---|
| 65% $P_{NL}$ | 762,063 | 107.47 | 9.35 | 0.80 |
| 75% $P_{NL}$ | 773,874 | 103.97 | 7.94 | 0.81 |
| 85% $P_{NL}$ | 789,560 | 99.15 | 6.08 | 0.83 |
| 98% $P_{NL}$ | 818,951 | 88.56 | 2.58 | 0.87 |

SP-8007 guideline relies on extensive experimental results from the 1930s to 1960s to determine the KDF. The impact of imperfections is verified by observing the differences in behavior between the modes and frequencies of the perfect and imperfect structure. The minimum value of $(1-p)^2$ obtained is not associated with a zero magnitude of the natural frequency of the structure. Furthermore, the vibration and buckling modes for the structure were not similar under the evaluated conditions, emphasizing the need for a methodology not relying on these characteristics.

It is suggested that the deviations observed in the obtained values are primarily associated with the implementation of imperfections, boundary conditions, and loading conditions. Specifically, the imperfections were indirectly implemented based on data obtained from IE and MSI images. The simple support condition (SS4) is a conservative approximation to simplify the fixing apparatus of the testing machine.

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