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On stacking-sequence independent bending properties of Double-Double laminates—A short communication

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Abstract

Double-Double (DD) laminates are discussed as an alternative to conventional laminates used in aerospace practice, which are usually composed of 0°, 45°, 90°, and −45° plies. Double-Double laminates promise advantages mainly due to simplification of design, optimization, and manufacturing processes. Stacking-sequence independent bending properties of DD laminates allow for significant simplification in context of buckling analysis, as a recent publication shows. However, not all conceivable ply-orientation combinations in DD's building block lead to it. The present short communication outlines briefly how stacking-sequence independent bending properties arise from DD's laminate architecture. The analysis reveals two valid and also a single invalid angle selection, for DD's balanced 4-ply building block.

Keywords

composite design, Double-Double, buckling, optimization

Introduction

Double-Double laminates show advantageous aspects for composite design[1,2] in terms of optimization[3] and also for buckling problems. A recent study on buckling of simply supported rectangular DD laminates[4] outlines the effect of the stacking-independence of DD laminates on identifying optimum laminates for certain buckling-load states. As a consequence, stacking-sequence discussions, known from conventional laminates, with up to millions of conceivable combinations, are obsolete for DD, which represents a remarkable simplification for designers. It has neither been outlined in the literature why DD laminates show stacking-independent bending properties, nor it has been verified that independence is achieved for all conceivable ply orders in the building block.

The present short communication outlines why some DD laminates show stacking-sequence independent bending properties. The parameters φ and Ψ denote ply orientations in the following. The presented analyses refer to the conventional notation of the classical laminate theory (CLT), which is well described in Nettles,[5] for example. A brief summary of relevant entities used hereafter is provided in the Appendix.

Analysis

The building block (BB) of a DD laminate is defined by four plies. The general form is described by

$$BB : [angle_1, angle_2, angle_3, angle_4]. \tag{1}$$

the corresponding angle-specific ply-stiffness matrices are denoted as $angle_i \rightarrow [\overline{Q}_i]$ hereafter. The thickness-normalized bending-stiffness matrix $[D^*]$ is defined as

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$$[D^*] = \frac{12}{t_{lam}^3} \cdot [D] = \frac{12}{t_{lam}^3} \cdot \frac{1}{3} \cdot \sum_{k=1}^{n} [\overline{Q}]_k \left(h_k^3 - h_{k-1}^3\right) = \frac{1}{16 \cdot r^3 \cdot t_{ply}^3} \cdot \sum_{k=1}^{n} [\overline{Q}]_k \left(h_k^3 - h_{k-1}^3\right). \tag{2}$$

it is well known that $[D]$ depends on the stacking sequence for conventional laminates. However, the particular DD architecture leads to the fact that thickness-normalized bending properties are independent from the repeat parameter r, which is the key for remarkable simplification in context of laminate optimization when buckling problems are examined.

The following section outlines how stacking independence arises from DD's laminate architecture. Therefore, equation (2) is analyzed in more detail. Table 1 summarizes the relevant terms for the analysis.

The term $h_k^3 - h_{k-1}^3$ determines the relative contribution of a specific ply in the laminate stack to the total bending stiffness of the whole laminate. From Table 1, one can deduce the parametric expression depending on r and k, which covers each entry.

$$h_k^3 - h_{k-1}^3 = f(k, r) = t_{ply}^3 \cdot \left(12r^2 + 6r \cdot (1 - 2k) + 1 + 3k \cdot (k - 1)\right) \tag{3}$$

When $r = 1$ only four plies contribute. The dependence of the term from r changes when the total number of plies increases. The repetitive pattern of the DD laminate leads to the fact that the total number of ply contributions can be summarized in four groups, referring to the BB-plies' stiffness matrices: $[[\overline{Q}_1], [\overline{Q}_2], [\overline{Q}_3], [\overline{Q}_4]]$. Note that $[\overline{Q}_1]$ refers to the first ply in the DD building block, while in conventional CLT calculation $[\overline{Q}]_k$ it refers to the k-th ply in the whole laminate stack. Table 1 shows that the factor t_{ply}^3 is found in all $h_k^3 - h_{k-1}^3$ terms. In combination with equation (2), it blanks out. A simple case study is executed hereafter to outline how stacking independence arises. The study covers the repeat values r = 1, 2, 3.

Evaluation for different *r*-cases

For the simple case r = 1, the normalized bending stiffness from equation (2) is determined to

$$[D_{r=1}^*] = \frac{1}{16}\left[[\overline{Q}_1], [\overline{Q}_2], [\overline{Q}_3], [\overline{Q}_4]\right] \cdot \begin{bmatrix} 7 \\ 1 \\ 1 \\ 7 \end{bmatrix}. \tag{4}$$

for the case r = 2 (eight-ply laminate), one finds

$$[D_{r=2}^*] = \frac{1}{16}\frac{1}{8}\left[[\overline{Q}_1], [\overline{Q}_2], [\overline{Q}_3], [\overline{Q}_4], [\overline{Q}_1], [\overline{Q}_2], [\overline{Q}_3], [\overline{Q}_4]\right] \cdot \begin{bmatrix} 37 \\ 19 \\ 7 \\ 1 \\ 1 \\ 7 \\ 19 \\ 37 \end{bmatrix}. \tag{5}$$

which can be summarized to

Table 1. Relevant terms for bending-stiffness matrix.

r	k	$[\overline{Q}]_i$	h_k	h_{k-1}	$h_k^3 - h_{k-1}^3$
1	1	$[\overline{Q}]_1$	$-r \cdot 4 \cdot t_{ply}/2 + 1 \cdot t_{ply}$	$-r \cdot 4 \cdot t_{ply}/2 + 0 \cdot t_{ply}$	$12r^2t_{ply}^3 - 6rt_{ply}^3 + t_{ply}^3$
	2	$[\overline{Q}]_2$	$-r \cdot 4 \cdot t_{ply}/2 + 2 \cdot t_{ply}$	$-r \cdot 4 \cdot t_{ply}/2 + 1 \cdot t_{ply}$	$12r^2t_{ply}^3 - 18rt_{ply}^3 + 7t_{ply}^3$
	3	$[\overline{Q}]_3$	$-r \cdot 4 \cdot t_{ply}/2 + 3 \cdot t_{ply}$	$-r \cdot 4 \cdot t_{ply}/2 + 2 \cdot t_{ply}$	$12r^2t_{ply}^3 - 30rt_{ply}^3 + 19t_{ply}^3$
	4	$[\overline{Q}]_4$	$-r \cdot 4 \cdot t_{ply}/2 + 4 \cdot t_{ply}$	$-r \cdot 4 \cdot t_{ply}/2 + 3 \cdot t_{ply}$	$12r^2t_{ply}^3 - 42rt_{ply}^3 + 37t_{ply}^3$
2	5	$[\overline{Q}]_5 = [\overline{Q}]_1$	$-r \cdot 4 \cdot t_{ply}/2 + 5 \cdot t_{ply}$	$-r \cdot 4 \cdot t_{ply}/2 + 4 \cdot t_{ply}$	$12r^2t_{ply}^3 - 54rt_{ply}^3 + 61t_{ply}^3$
	6	$[\overline{Q}]_6 = [\overline{Q}]_2$	$-r \cdot 4 \cdot t_{ply}/2 + 6 \cdot t_{ply}$	$-r \cdot 4 \cdot t_{ply}/2 + 5 \cdot t_{ply}$	$12r^2t_{ply}^3 - 66rt_{ply}^3 + 91t_{ply}^3$
	7	$[\overline{Q}]_7 = [\overline{Q}]_3$	$-r \cdot 4 \cdot t_{ply}/2 + 7 \cdot t_{ply}$	$-r \cdot 4 \cdot t_{ply}/2 + 6 \cdot t_{ply}$	$12r^2t_{ply}^3 - 78rt_{ply}^3 + 127t_{ply}^3$
	8	$[\overline{Q}]_8 = [\overline{Q}]_4$	$-r \cdot 4 \cdot t_{ply}/2 + 8 \cdot t_{ply}$	$-r \cdot 4 \cdot t_{ply}/2 + 7 \cdot t_{ply}$	$12r^2t_{ply}^3 - 90rt_{ply}^3 + 169t_{ply}^3$
	9				
	…				

$$= \frac{1}{16}\frac{1}{8}\left[[\overline{Q}_1],[\overline{Q}_2],[\overline{Q}_3],[\overline{Q}_4]\right]\cdot\begin{bmatrix}38\\26\\26\\38\end{bmatrix}. \quad (6)$$

adapted to the format in equation (4), this leads to

$$= \frac{1}{16}\left[[\overline{Q}_1],[\overline{Q}_2],[\overline{Q}_3],[\overline{Q}_4]\right]\cdot\begin{bmatrix}4.75\\3.25\\3.25\\4.75\end{bmatrix}. \quad (7)$$

for $r = 3$, one obtains similarly after summarizing

$$\begin{aligned}[D^*_{r=3}] &= \frac{1}{16}\frac{1}{27}\left[[\overline{Q}_1],[\overline{Q}_2],[\overline{Q}_3],[\overline{Q}_4]\right]\cdot\begin{bmatrix}117\\99\\99\\117\end{bmatrix}\\ &= \frac{1}{16}\left[[\overline{Q}_1],[\overline{Q}_2],[\overline{Q}_3],[\overline{Q}_4]\right]\cdot\begin{bmatrix}4.33\\3.67\\3.67\\4.33\end{bmatrix}.\end{aligned} \quad (8)$$

The previous cases show commonalities, as the following list shows.

$$[D^*_{r=1}] = \frac{1}{16}\left[[\overline{Q}_1],[\overline{Q}_2],[\overline{Q}_3],[\overline{Q}_4]\right]\cdot[7 \quad 1 \quad 1 \quad 7]^T \quad (9)$$

$$[D^*_{r=2}] = \frac{1}{16}\left[[\overline{Q}_1],[\overline{Q}_2],[\overline{Q}_3],[\overline{Q}_4]\right]\cdot[4.75 \quad 3.25 \quad 3.25 \quad 4.75]^T \quad (10)$$

$$[D^*_{r=3}] = \frac{1}{16}\left[[\overline{Q}_1],[\overline{Q}_2],[\overline{Q}_3],[\overline{Q}_4]\right]\cdot[4.33 \quad 3.67 \quad 3.67 \quad 4.33]^T \quad (11)$$

the vector $1/16 \cdot [\ldots]^T$ is found in all three previous equations. It can be denoted as a normalized-contribution vector

$$\overrightarrow{nc} = \frac{1}{16}\cdot[nc_1 \quad nc_2 \quad nc_3 \quad nc_4]^T. \quad (12)$$

the examples in the equations (9)–(11) all show

$$\frac{1}{16}\sum_{k=1}^{4} nc_k = 1. \quad (13)$$

the vector determines the relative contributions of the individual plies to the total normalized bending stiffness. In fact, one observes a kind of homogenization for increasing r, as the normalized relative contributions nc_i of the plies approach a value of 1/4. When the relative ply contributions are compared, one finds differences for the provided r-cases. However, one also finds important parallels, which are important in context of the examined stacking-sequence independence. Careful observation shows that the sums $nc_1 + nc_2 = 8$ and $nc_1 + nc_3 = 8$ are independent from r. This is not true for $nc_1 + nc_4$ and $nc_2 + nc_3$. Those observations are the key to the stacking sequence independence of DD laminates. It is essential to recall that a single BB in a DD laminate is always balanced. The global ply-stiffness matrices for a $+\varphi$ ply and a $-\varphi$ ply are

$$\begin{aligned}[\overline{Q}_{+\varphi}] &= \begin{bmatrix}\overline{Q}_{\varphi,11} & \overline{Q}_{\varphi,12} & \overline{Q}_{\varphi,16}\\ \overline{Q}_{\varphi,12} & \overline{Q}_{\varphi,22} & \overline{Q}_{\varphi,26}\\ \overline{Q}_{\varphi,16} & \overline{Q}_{\varphi,26} & \overline{Q}_{\varphi,66}\end{bmatrix},\\ [\overline{Q}_{-\varphi}] &= \begin{bmatrix}\overline{Q}_{\varphi,11} & \overline{Q}_{\varphi,12} & -\overline{Q}_{\varphi,16}\\ \overline{Q}_{\varphi,12} & \overline{Q}_{\varphi,22} & -\overline{Q}_{\varphi,26}\\ -\overline{Q}_{\varphi,16} & -\overline{Q}_{\varphi,26} & \overline{Q}_{\varphi,66}\end{bmatrix}\end{aligned} \quad (14)$$

the 11, 12, 22, and 66 coefficients are independent from the ply-angle sign. Thus, when, for example, we set $[\overline{Q}_1] = [\overline{Q}_{+\varphi}]$, $[\overline{Q}_2] = [\overline{Q}_{-\varphi}]$, $[\overline{Q}_3] = [\overline{Q}_{+\Psi}]$ and $[\overline{Q}_4] = [\overline{Q}_{-\Psi}]$, in the equations (9)–(11), the 11, 12, 22, and 66 coefficients of $[D^*]$ will be identical for all the provided r-cases. The $[D^*]$ matrix will be of the form

$$[D^*] = \begin{bmatrix} const & const & \frac{1}{r^2}\cdot(\ldots)\\ const & const & \frac{1}{r^2}\cdot(\ldots)\\ \frac{1}{r^2}\cdot(\ldots) & \frac{1}{r^2}\cdot(\ldots) & const\end{bmatrix}. \quad (15)$$

as the sign of the 16 and 26 coefficients in $[\overline{Q}_{+\varphi}]$ and $[\overline{Q}_{-\varphi}]$ changes with the ply-angle sign, the term $12r^2t^3_{ply}$ in $(h^3_k - h^3_{k-1})$ (see Table 1) blanks out when ± terms are summed. This leads to the fact that D^*_{16}, D^*_{26} are found proportional to $1/r^2$.

Generalization

The preceding numerical examples led to the observation that the sum of the normalized contribution vector entries is always

$$\frac{1}{16}\sum_{k=1}^{4} nc_k = 1.$$

the following analyses aim to the verify this observation, based on equation (3). Utilizing equation (13) allows for writing the normalized bending-stiffness matrix as

Table 2. Contributions of the i-th ply in the building block for multiple repeat values *r*.

nc_i contr.	$r = 1$	$r = 2$	$r = 3$
nc_1	$12r^2 - 6r + 1$	$12r^2 - 6r + 1$ $12r^2 - 54r + 61$	$12r^2 - 6r + 1$ $12r^2 - 54r + 61$ $12r^2 - 102r + 217$
nc_2	$12r^2 - 18r + 7$	$12r^2 - 18r + 7$ $12r^2 - 66r + 91$	$12r^2 - 18r + 7$ $12r^2 - 66r + 91$ $12r^2 - 114r + 271$
nc_3	$12r^2 - 30r + 19$	$12r^2 - 30r + 19$ $12r^2 - 78r + 127$	$12r^2 - 30r + 19$ $12r^2 - 78r + 127$ $12r^2 - 126r + 331$
nc_4	$12r^2 - 42r + 37$	$12r^2 - 42r + 37$ $12r^2 - 90r + 169$	$12r^2 - 42r + 37$ $12r^2 - 90r + 169$ $12r^2 - 138r + 397$

Table 3. Summed contributions.

Contr. to $[D^*]$	$r = 1$	$r = 2$	$r = 3$
$'[\overline{Q}_1] + [\overline{Q}_2]'$	$1/r^3(24r^2 - 24r + 8)$ $= 8$	$1/r^3(48r^2 - 144r + 160)$ $= 8$	$1/r^3(72r^2 - 360r + 648)$ $= 8$
$'[\overline{Q}_1] + [\overline{Q}_3]'$	$1/r^3(24r^2 - 36r + 20)$ $= 8$	$1/r^3(48r^2 - 168r + 208)$ $= 8$	$1/r^3(72r^2 - 396r + 756)$ $= 8$
$'[\overline{Q}_1] + [\overline{Q}_4]'$	$1/r^3(24r^2 - 48r + 38)$ $= 14$	$1/r^3(48r^2 - 192r + 268)$ $= 9.5$	$1/r^3(72r^2 - 432r + 882)$ $= 26/3 \approx 8.667$
$'[\overline{Q}_2] + [\overline{Q}_3]'$	$1/r^3(24r^2 - 48r + 26)$ $= 2$	$1/r^3(48r^2 - 192r + 244)$ $= 6.5$	$1/r^3(72r^2 - 432r + 846)$ $= 22/3 \approx 7.333$

$$[D^*] = \frac{1}{16} \cdot \frac{1}{r^3} \left(nc_1 \cdot [\overline{Q}_1] + nc_2 \cdot [\overline{Q}_2] + nc_3 \cdot [\overline{Q}_3] + nc_4 \cdot [\overline{Q}_4] \right). \tag{16}$$

each contribution nc_i refers to a single ply of the building block. The total contribution of the first ply in the building block is composed of multiple fractions, when r increases. For $r = 3$, for example, the total contribution is composed of three summands, one from each of the three building blocks in the full laminate (Table 2). The following table plots the individual contributions, for the repeat range $r = 1, 2, 3$.

The numerical examples in Section 3 indicate that the summed contributions of two plies in the building block to $[D^*]$ are found constant for different building-block repeats. This represents the basis for r-independence of the $[D^*]$ matrix. Therefore, Table 3 provides different combinations of summed ply contributions. The utilized nomenclature is as follows. The ID $'[\overline{Q}_1] + [\overline{Q}_2]'$, for example, refers to the case when the first two plies in the building block refer to the ply angle φ ($[\overline{Q}_1] = [\overline{Q}_\varphi]$ and $[\overline{Q}_2] = [\overline{Q}_{-\varphi}]$).

The results in Table 3 substantiate the numerical examples from the previous section. For the cases $'[\overline{Q}_1] + [\overline{Q}_2]'$ and $'[\overline{Q}_1] + [\overline{Q}_3]'$, the summed contributions are found independent from the repeat parameter r.

Thus, $[\varphi, -\varphi, \Psi, -\Psi]$ and $[\varphi, -\Psi, -\varphi, \Psi]$ are valid BBs for DD. Both lead to the desired effect, that the normalized bending-stiffness matrix $[D^*]$ entries 11, 12, 22, and 66 are independent from the repeat parameter r. The other cases ($'[\overline{Q}_1] + [\overline{Q}_4]'$ and $'[\overline{Q}_2] + [\overline{Q}_3]'$) show that the BB angle selection $[\varphi, -\Psi, \Psi, -\varphi]$ is invalid, as it does not lead to the desired independence from r.

Conclusion

Building-block-repeat independent normalized bending properties are a unique aspect of DD laminates, which simplify laminate optimization tasks in context of buckling analysis. The present short communication outlines how independence arises from DD's particular laminate architecture, which is characterized by simply stacking multiple, balanced four-ply building blocks r-times $([angle_1, angle_2, angle_3, angle_4]_{rT})$.

Whenever the building block angles fulfill: $|angle_1| = |angle_2|$ or $|angle_1| = |angle_3|$ stacking independence is present for the 11, 12, 22, and 66 coefficients of the $[D^*]$ matrix. The analysis reveals that building blocks with $|angle_1| = |angle_4|$ are invalid selections, when looking for independence from r. Thus, $[\varphi, -\varphi, \Psi, -\Psi]$ and $[\varphi, -\Psi, -\varphi, \Psi]$ are valid building blocks for DD laminates, while $[\varphi, \Psi, -\Psi, -\varphi]$ is found an invalid building block stacking sequence.

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CRediT author contribution statement

As the sole author, Erik Kappel is responsible for the whole content.

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Appendix

Nomenclature CLT

The following definitions are in line with the CLT document provided by Nettles.[5] The compliance matrix of a ply in local coordinates in defined by

$$[S] = \begin{bmatrix} 1/E_1 & -\nu_{12}/E_1 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & 0 \\ 0 & 0 & 1/G_{12} \end{bmatrix} \tag{17}$$

with E_1, E_2, ν_{12}, and G_{12} being the ply's Young's modulus in fiber direction, in transverse direction, the Poisson's ratio, and the shear modulus, respectively. The ply's stiffness matrix in the local coordinate system $[Q]$ is defined as

$$[Q] = [S]^{-1}. \tag{18}$$

the ply's stiffness matrix in the global laminate coordinate system is defined as

$$[\overline{Q}] = [T]^{-1}[Q][T]^{-T}. \tag{19}$$

with

$$[T] = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \tag{20}$$

being a rotation matrix with $m = cos(\alpha)$, $n = sin(\alpha)$. α denotes the ply's alignment with respect to the global laminate x-direction.