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Master‘s Thesis

Remote Monitoring of Correlated Sources Over Random Access Channels in IoT Systems

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With my signature below, I assert that the work in this thesis has been composed by myself independently and no source materials or aids other than those mentioned in the thesis have been used.

München, 06.05.2024

Place, Date

Signature

Abstract

In the present-day communication technology involving fast-evolving Internet of Things (IoT) systems, accurate and timely information delivery is paramount. This thesis investigates an interesting concept of information freshness along with accuracy in a system where the transmitting sources are correlated. We focus on two key parameters to carry out this study: Average Age of Incorrect Information (AoII) and Average Estimation Error.

We design a Markov Model to depict the intricacies of real-world correlated sources within communication systems. This Markov Model aids us in investigating our key parameters (AoII and Average error) under a range of correlation coefficients. We further explore different transmission and estimation methods to understand the variations in the above-mentioned parameters to comment on the optimal method for a given system condition.

We further develop a strong theoretical framework for the transmission and estimation methods tailored to the unique challenges posed by correlated sources. This caters to our goal to understand the evolution of the AoII and Average Estimation Error throughout the range of the correlation coefficients.

A rigorous investigation using empirical methods is conducted through carefully designed simulations to depict the real-world applications as closely as possible. This reveals interesting insights into the behavior of AoII and Average Error in the presence of correlated sources accessing a wireless random access channel. The results not only advance our understanding of these metrics but also provide practical guidance for optimizing communication systems under real-world conditions.

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Chapter 1

Introduction

1.1 Motivation

The Internet of Things (IoT) in today's world has given rise to a new era of connectivity, enabling day-to-day objects to collect and exchange data in a reliable manner and process it in ways that were once unimaginable. This not only applies to everyday objects but also plays a major role in industrial setups that need high accuracy.

The IoT ecosystem, consisting of a vast network of interconnected devices and sensors, has revolutionized various industries in no time. Healthcare, entertainment, mass production, transportation, smart cities, and agriculture are only a few. The paradigm shift induced by IoT across all areas of life has transformed the way information is generated, viewed, and communicated.

However, with the expansion of interconnected devices and sensors, an important aspect of consideration in the IoT or Wireless Sensor Network (WSN) ecosystem is the intricate relationship between the transmitting sources.

In this dynamic environment, the age of incorrect information (AoII) within the correlated source is a critical challenge. Not only is it the question of identifying and correcting inaccurate data, but also of understanding how inaccurate or misleading information can remain on the network and potentially have cascading consequences. Considering the vast application area of such networks, studying the error in the estimation of the status of the transmitting sources (sensors) proves to be of utmost importance. It is interesting in these scenarios to understand the duration for which the estimator results in an incorrect estimation.

As we now know the benefits of focusing on parameters such as estimation error, AoII, and average error length in estimation, we can move on to briefly look at the major questions that are addressed in the thesis.

1.2 Problem Statement

In this thesis, we intend to investigate the problem of gauging the status of correlated sources. The system of linked sources working in a random-access channel environment is the setting in which this study is conducted.

This thesis investigates the following research questions:

1. How does the level of correlation in a system using a random-access channel affect the accuracy of source status estimation?

2. What effects does source correlation have in the Age of Incorrect Information, where data relevance and accuracy are crucial?

3. Does the correlation coefficient between the sources impact the duration of estimating a continuous error state?

4. Can we achieve meaningful improvements in state estimation when the estimator has access to a vast knowledge base about the correlation between sources?

The answers to these questions capture the essence of the research and point us in the direction of a deeper comprehension of the complex interactions between source correlation, and estimation accuracy in a digital ecosystem. With a foundation of correlated sources using random-access channel to communicate, we work on leveraging the knowledge of correlation for dependable and more accurate decisions.

1.3 Thesis Overview

A Markov Model is developed to simulate the transitions in the state of two sources correlated (positively and negatively) with each other. These two sources (eg: sensor nodes in an IoT setup) use a random access channel to send updates to a receiver. Various transmission policies are discussed to optimize the channel usage and hence achieve better throughput. On the receiver end, the estimator may or may not receive an update from either of the two sources. We design a MAP (Maximum a posteriori) estimator to efficiently use the knowledge of correlation to make better estimations. We focus on estimation error and compare it with a simple estimator that does not use the correlation data during statute estimation. Outcomes are compared at various levels of correlation to meaningfully explain the impact of correlation between the two sources. In addition to this Average Age of Incorrect Information and Average Error lengths are studied to show the best transmission and estimation policies.

1.4 Thesis contribution

As we move ahead in the thesis we witness significant improvement in the estimation while there is more knowledge about the sources and their relationship with each other. We study the peak estimation errors and notice how the correlation of the system is closely related to estimation accuracy. We investigate different transmission techniques and discuss the best transmission method for a given correlation coefficient. The age of Incorrect Information which is an important metric in decision-making for modern-day networks is investigated in detail. The average duration in the error state gives us some important insights into the estimator efficiency and system behavior.

1.5 Thesis Structure

The remainder of the document is organized as follows. In Chapter 2 recent work on correlated sources in an IoT or similar setups are reviewed. Relevant literature on random access, Error Estimation, and the Age of Incorrect Information in a wireless environment are studied in depth.

In Chapter 3 a detailed description of the followed methodology is introduced. The considered setup is discussed at length. In Chapter 5 the simulation results are discussed. In this section, we interpret our findings and discuss their broader implications, highlighting the critical importance of addressing AoII and Average Error in the design and operation of systems with correlated sources. We also acknowledge the limitations encountered during our research, opening avenues for future investigations.

In conclusion, with this thesis, we strive to contribute valuable insights into the behavior of Average Age of Incorrect Information and Average Error, in the context of systems containing correlated sources. By shedding light on the intricate connection between correlation coefficient, accuracy, and information timeliness, this research aims to enhance the performance of communication systems in our increasingly interconnected world.

Chapter 2

Background

2.1 IoT and WSNs Status updates

Through pervasive computing, multiple communication technologies, sensors, and actuators, the Internet of Things (IoT) links the physical and digital worlds [4]. Sensors play an important role in IoT systems and the primary function of the sensor nodes is to detect a parameter of interest (such as weather, air quality, etc.) and feed the system with information about the physical environment [26]. Many applications such as health monitoring, and environmental sensing are supported by sensing devices in the network (nodes) sending status updates to the receiver (sink). Similar to IoT, Wireless sensor networks (WSNs) have also been thoroughly researched for a number of industrial and environmental monitoring and surveillance applications [2]. A common ground in IoT and WSNs is the highly meshed network of sensor nodes whose status are likely to be highly correlated with each other due to vicinity in space or closeness in time of capturing the parameter of interest.

Random access channels become essential due to the high number of sensors, and several protocols are taken into consideration for various network setups. In our work, we study contention-based protocols over contention-free protocols to minimize significant signaling overhead for coordinated transmissions of the status updates from the sensor nodes. Slotted ALOHA, for instance, can be used with WSNs [17]

In an IoT system, the correlation between several observations or source nodes in time and space is highly likely with the considerable number of sensors placed close to each other. Network performance can be significantly enhanced by using correlated sensor data, or information sources [10].

In [10] two sources are considered, one being the source of interest and the other correlated source (secondary source). The system utilizes the information from the secondary source to reduce the frequency of updates from the source of interest by inferring valuable data from the correlated secondary source. The focus of this study is to schedule the optimum time for status updates so as to keep the information as fresh as possible. In this study, the scheduling is extensively studied to optimize the parameter of interest (freshness of information). Whereas our work majorly focuses on the estimation of the information(status) and decreasing the estimation error using the knowledge of correlation.

In [8] an ecosystem with correlated sensor signals that uses a random access channel (ALOHA) for communication is studied. An adaptive transmission approach is used to exploit the correlation between signals that results in an efficient WSN. In [32] an observation is conducted on a highly correlated spacial and temporal sensor domain. Several key elements are investigated to utilize the correlation in the sensor network in order to realize an efficient communication protocol. This paper clearly leverages the knowledge of the correlation between the sources and improves the existing system to increase reliability in medium access.

A specially correlated landscape is studied in [31] where the sensors use a common wireless communication channel to update the sink with the information. Although this paper studies the correlation, medium access, and the effect it has on the efficiency of the system, the collisions are minimized using a coordinated system which might add an overhead compared to the random access.

Regulating and optimizing the rate at which the sources transmit messages to update the information about the sensor observations in a network of correlated information sources allows for an increase in the throughput. These notifications also referred to as status updates, are sent to the receiver by accessing a common wireless random access channel. By Making use of the correlation information it is possible to anticipate the status of various sensor devices at different locations in an instant using the data collected at another location and a different time instant. Therefore, it is possible to reduce the rate at which information sources submit status updates to a gateway by utilizing the correlation between many observations of the same phenomena [9].

2.2 Estimation Methods

Estimation of the concerned parameters in the field of sensor networks has been extensively studied in recent times since the widespread applications in all the areas are evident. In [37] a specially correlated dynamic field is observed. A traditional

Kalman buck filter (KBF) that gives the minimal root mean squared (RMS) error is studied against a reduced order KBF to highlight the tradeoffs of communication cost. In this paper, estimation error is used as a metric to compare the cost of communication which gives an important insight into the trade-offs in this study. An interconnection matrix is used to represent the to represent connection and in turn the correlation between the nodes in the network.

In [36] a graph approach towards the estimation of the sensor data is considered. In a wireless sensor network setup, this study tries to exploit the spatial correlation phenomenon between the sensors to estimate the state at a particular physical location where the sensors are not present.

In the paper [11] a sequential estimation problem is studied with two decision-makers. The observer looks at a certain stochastic process and makes sequential observations for a certain duration. This observer decides to transmit a message to the estimator. The estimator continuously estimates the state of the stochastic process. The major goal of this study is to minimize a performance criterion with the constraint that the observer agent may only act a limited number of times.

In [18] a Kalman-like filter is used for remote estimation in a communication system. An entity called a pre-processor sends a message to the estimator irregularly with a positive cost. The estimator with limited information sequentially estimates the state of the system. In this work, the combination of pre-processor and estimator is said to achieve the optimal cost for communication.

The authors [22] in this paper take into account the limited energy in energy harvesting sensors to propose optimal strategies in communions and achieve optimal results while remote estimation.

The paper [5] presents the fundamental limits of remote estimation of autoregressive Markov processes under communication constraints. A sensor estimator pair is employed to study the remote estimation system. A senor observes a Markov process and at each time instant there may or may not be a transmission. The estimator estimates the Markov process based on the observations. In such systems, there is a chance of estimation error and hence a trade-off between communication cost and estimation accuracy. This work gives us some interesting details about the minimum achievable estimation error. Some of the transmission and estimation strategies that achieve the fundamental limits are also identified in this work.

Our system uses Markov's model to simulate the changes in the parameters of interest and the Hidden Markov Model is used as an estimation tool, In [24, 21] the theoretical aspects of the hidden Markov Model (HMM) are carefully and methodically reviewed. Although this tutorial applies the HMM in machine speech recognition, due to its rich mathematical structure HMM can be applied to wide areas of research such as ours.

2.3 Age of Incorrect Information

With the surge in the demand for real-time monitoring, the need for up-to-date information in an IoT setup has increased. Recent studies have primarily focused on the Age of Information (AoI) [35] metric that measures the freshness of the information by means of time-stamped status updates [16]. The paper [13] focuses on minimizing the AoI to maintain maximum freshness where status updates are generated by the sensor nodes and transmitted randomly. In [34, 12] although multiple transmitting sources are considered the sources are assumed to be independent which in a practical sensor network such as an IoT system is unlikely. In our work, the noticeable correlation between the various transmitting nodes is explored to improve the overall performance of the system.

Tn the paper [30] formulated a simple model to study the timely monitoring of correlated sources over a wireless network. Using this model, in the presence of correlation new scheduling policies that optimize weighted-sum average Age of Information (AoI) were introduced. The paper illustrates AoI improves in large networks in the presence of correlation.

In [29] the authors try to encounter the minimization problem of AoI within a correlated environment with stochastically identical and non-identical multichannel. Although our work focuses on the correlated environment, the use of random access channels for communication is the goal of the thesis. In [38] the authors investigate another parameter called Urgency of Information focusing on the context of the data. In [15] the problem of mismatch in achieving minimum AoI at the minimum estimation error is discussed. An alternate metric called effective age is proposed. A Markov chain source is observed, with various transmission strategies for transmission of these observations.

However, in real-time applications in the remote estimation of the status of the sources, a challenge arises that is closely tied to monitoring the status of the sensors of interest. Reducing AoI although ensures the freshness of the information, does not give any insight into the accuracy of the said status update. AoI on its own may not be a sufficient parameter to fully comprehend the behavior of the system in case of error estimation.

Age of Incorrect Information (AoII) theory was put forth that describes the price of a monitor being in an incorrect position. A new performance matrix: Age of

Incorrect Information is introduced (AoII) [20] which tries to tackle the shortcomings of AoI and conventional error penalty functions. The authors define the status update as "informative" considering the need for correct information on the monitoring side. This paper draws attention to the existing shortcomings in the current metric of AoI in the remote process estimation context. The work justifies the need for AoII by highlighting the detrimental effect of wrong information on the overall efficiency of a sensor network. An N-state Markovian information source sends the status updates using an unreliable channel. In [14] the authors investigate AoII a basic environment for binary information monitoring sources over a feedback-controlled delay mechanism.

The paper [7] studies the minimization problem of AoII on an unreliable channel. The paper [19] works on minimizing the average AoII in a transmitter-receiver pair scenario. The paper summarizes that the optimal transmission strategy is a randomized threshold policy, and proposes an algorithm to find the optimal parameters. Performance advantages of AoII are highlighted over other performance metrics such as AoI and the error-based measure approach.

Similar to [20, 19] this thesis focuses on studying the AoII. However, the thesis uses a random access channel open for collisions and assumes a correlation between the transmitting sources a common phenomenon in practical sensor network scenarios.

2.4 Literature Review Outline

Notable differences between the existing work and the thesis are :

1. The majority of the research assumes independence on the transmitting end which in practice is not the case. Hence exploring the correlation and exploiting this knowledge is the major focus of our work.

2. A centralized approach is considered in terms of scheduling or channel access to achieve optimal results. However, this might lead to an overhead in the design or implementation. Our work focuses on random transmission or a decentralized approach to the transmission of sensor data: thus reducing the complexity of the system.

3. A simple yet efficient metric of estimation accuracy, the estimation error is studied in detail to give a clear understanding of leveraging the knowledge of correlation.

4. Age of Incorrect Information which not only guarantees freshness but also gives valuable insights into the accuracy is studied, unlike the Age of Information in various literary works.

Chapter 3

Methodology

3.1 System Model

3.1.1 Introduction

In the following chapter, we introduce the system model and discuss the construction of various elements of the model in detail. In order to design an effective estimation model for a system with correlated transmitters, we aim to construct a simple yet practical transmitter section, considering various transmission techniques, and measure the effectiveness of the estimator by analyzing the errors in estimation. We design a simulation to mirror the real-time scenarios, capturing the dynamics of data transmission among correlated sources.

In the study, we focus on a system with two transmitting sources namely X and Y. We introduce a level of correlation between the two sources to study the impact of correlation (could be positive, negative, or uncorrelated) on the evaluation parameters such as Estimation error and Age of incorrect information. The two sources $\mathbf{X}$ and $\mathbf{Y}$ share a common wireless random access channel to transmit the status updates to the receiver.

Time is slotted and the value generated by the sources can be represented as follows:

$$\mathbf{X} = X_1, X_2, X_3....X_n$$

$$\mathbf{Y} = Y_1, Y_2, Y_3....Y_n$$

Where X_i and Y_i are the values at time slot $i \in N$

For this study, we look into one of the transmitting sources (Source $\mathbf{X}$) and address that source as the source of interest. The evaluation of parameters is based on the observation of that particular source. The other source is referred to as the correlated source (source **Y**). Note that the system model is symmetrical and we can monitor either of the two sources to study the evaluation parameters.

This wireless channel resembles slotted ALOHA where a packet from the transmitting source takes exactly one slot to send an update (sources are slot-synchronized). The transmitting sources compete to access the channel randomly without any centralized system to allot the slots or time to any source in particular. Therefore the transmitting sources are slot-synchronized and we see no partial transmission of the packets. The channel is collision-prone and a collision results in a complete packet loss. A packet is successfully transmitted to the receiver if only one of the sources accesses the channel at any given instance.

At the receiver, the channel is observed continuously and the possible observations for a slot are:

1. Idle: No transmissions from either of the sources

***2. Collision*:** Both the sources try to access the channel at the same point.

***3. Successful status update*:** Only one of the two transmitting sources access the channel and this results in a successful transmission.

The sequence of observations up to slot n can be represented as:

$$\mathbf{O} = O_1, O_2, O_3 .... O_n$$

We make the following assumptions in order to continue this study.

1. The receiver is able to detect a collision as well as an idle slot in the channel.

2. In case of a packet reception, the receiver is able to identify the transmitting source of the packet.

At the receiver, we design an estimator which provides an estimate of the state of the source of interest based on the sequence of observations. An estimate is produced at each slot and we denote the sequence of estimates as follows:

$$\mathbf{Z} = Z_1, Z_2, Z_3 .... Z_n$$

We compare the estimated value Z_n with the real value X_n to study the accuracy of the designed estimator.

We focus on the following evaluation parameters: average state estimation error of one of the sources, AoII, and average duration of error estimate (of either source **X** of source **Y**) to study the discrepancies in the estimation at the receiver. An understanding of how far the estimated values are from the real status gives us a clear view of the effectiveness of the estimator design.

The source modeling and the correlation between the transmitting sources, transmission methods, the random access channel, and the estimator at the receiver serve as the foundation on which our entire investigation stands. Through an exploration of diverse scenarios and analysis, this framework aims to capture some key aspects of the use cases closely.

We use a system model that is expected to provide insights into our investigation. A comprehensive system model is constructed to capture the dynamics of two correlated sources. Within our architectural model, we encounter the following essential components:

3.1.2 Correlated Sources

To establish an interdependency between the considered data sources, we construct a Markov Model. It is employed as a framework to mirror the intricacies of dependencies inherent within sensor clusters observed in both environmental and industrial domains. We achieve this by integrating transition probabilities between the states of the Markov model. These connections between the states of this model aim to simulate the correlation between the transmitting sources. Often such relationships exist among sensor clusters, however, dependencies within the sensor data are concealed.

We first assume two sources, source **X** and source **Y** with alphabets:

$$\mathcal{X} = \{ 0, 1, 2, 3 \}$$

$$\mathcal{Y} = \{ 0, 1, 2, 3 \}$$

The above alphabet is considered to achieve a wide range of correlation coefficients between the sources while representing various states of transitions. It is limited to four states for ease of understanding the underlying concepts.

Figure 3.1 gives an overview of the considered Markov model. Here we consider the joint state probability (X_n, Y_n) of both sources at the slot n.

We introduce dependencies between the two sources using transition probabilities to different states of the said Markov Model.

We introduce a Markov model with 16 states:

Markov States: { 00, 01, 02, 03, 10, 11, 12, 13, 20, 21, 22, 23, 30, 31, 32, 33}

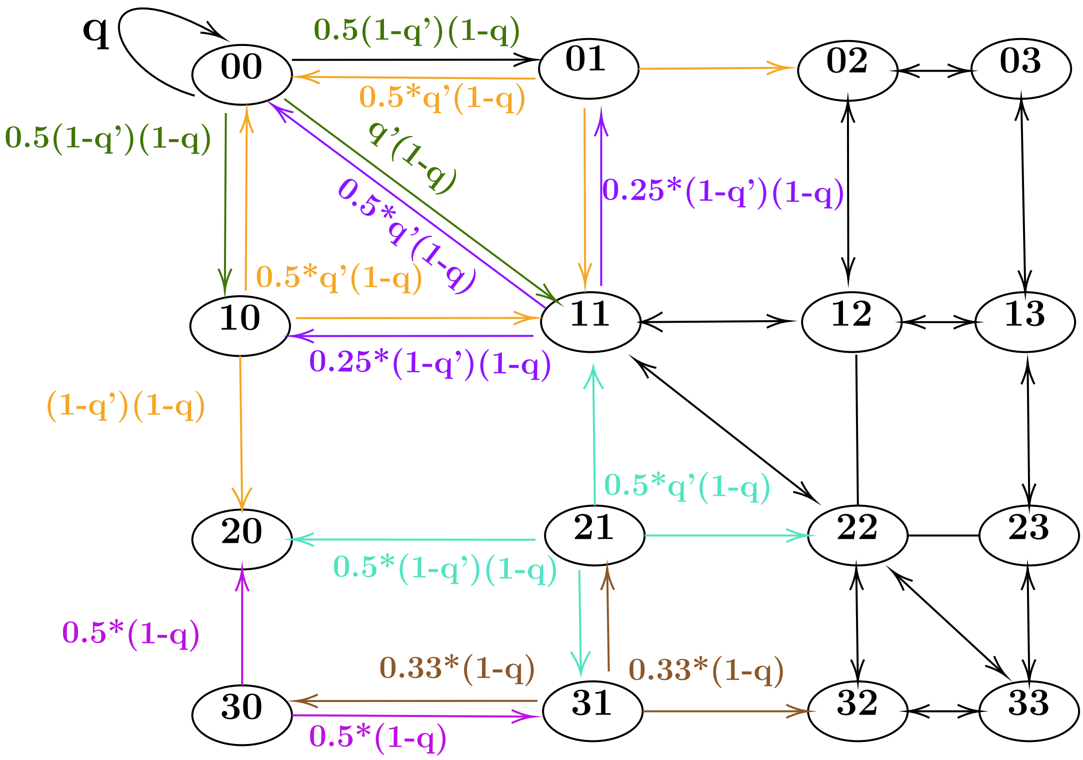

Figure 3.1: States and Transitions of the Markov Model

The first digit in the Markov state represents the state or value of the source **X** and the second represents the value of **Y**. We make an assumption that the state of the sources can either remain the same or only change by a magnitude of exactly one at any given slot, i.e., any n,

$| X_n - X_{n-1} |\leq 1$ and $| Y_n - Y_{n-1} |\leq 1$

We also design the model such that a positive change in one of the sources will result in either a positive change or no change in the other. The model is constructed such that the system moves along the main diagonal of the Markov model. This is to ensure cases that give a high correlation between the transmitting sources.

We consider the following parameters during the construction of the Markov model:
q: Same state probability, both the sources remain in the same state.
q': Parameter that influences the system to move towards a correlated state. Both the sources change their states simultaneously with equal magnitude.

The transition probabilities in terms of q and q' are shown in Figure 3.1.

The Markov model is designed in a way to provide a wide range of values for correlation coefficients to study the impact of the correlation on the considered evaluation parameters. We consider the elements of the main diagonal (00, 11, 22, 33) as highly correlated states. The parameter q' decides the probability of transitioning to these highly correlated states. We see from Figure 3.1 that if a state does not have a highly correlated state as an immediate neighbor (02, 13, 31, 20), the probability of moving is equally distributed among the neighboring states.

In the case of a state that has a highly correlated state as its neighbor, the factor q' decides the probability of moving to such a state. Therefore moving to a highly correlated state increases with an increase in the value of q'. The probability of moving to a highly correlated state is equally distributed in case a state has multiple highly correlated neighboring states.

The factor q indicates the probability of the system remaining in the same state (both sources retain the same value as the previous slot). As the value of q decreases the system becomes more dynamic and we observe the system changes the states more often. On the other hand, the parameter q' dictates how much the two interacting sources are interconnected. As the value of q' increases, we see the correlation between the sources also increases. Figure 3.2 illustrates the connection between the value of q' and the correlation coefficient ρ. The graph clearly represents the wide range of correlation coefficients considered in this study. By tuning the parameters q and q', we can achieve a wide range of correlations between the two sources X and Y.

To calculate the Correlation coefficient ρ for a given value of q', we have:

$$\rho = \frac{Cov(X,Y)}{\sigma_X \sigma_Y} \tag{3.1}$$

Where σ_X and σ_Y are the standard deviations of the distribution of the source **X** and **Y**. $Cov(X,Y)$ is the covariance of the joint distribution of the two sources.

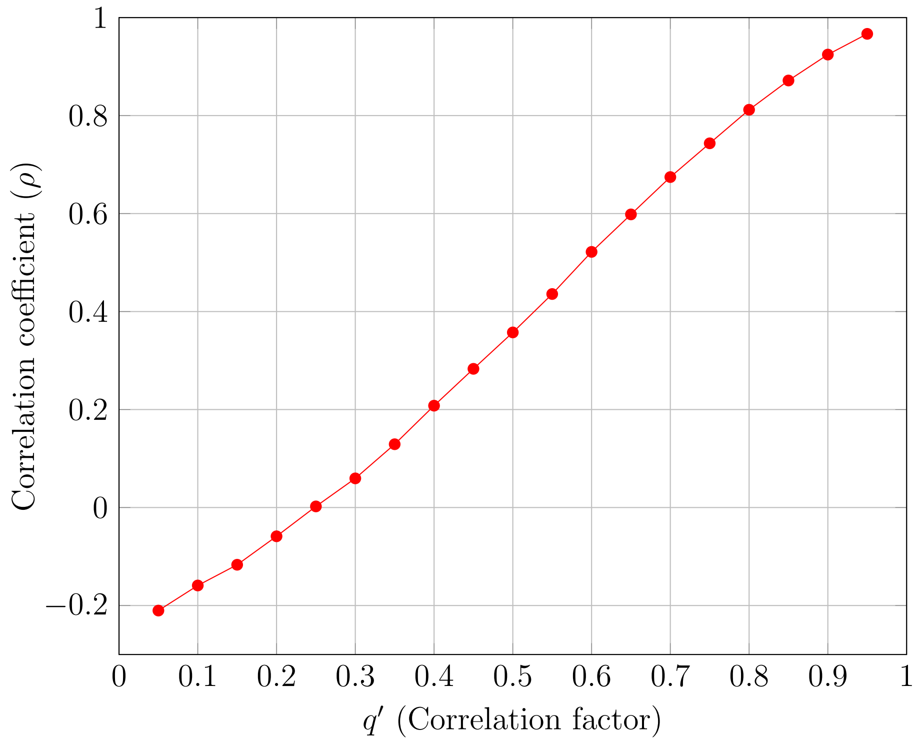

Figure 3.2: Correlation coefficient for the corresponding value of q'

To calculate the $Cov(X,Y)$ in (3.1), we have by definition,

$$\begin{aligned} Cov(X,Y) &= E[(X - m_x)(Y - m_y)] \\ &= E[XY] - E(X)m_y - E(Y)m_x + m_x m_y \\ &= E[XY] - m_x m_y - m_y m_x + m_x m_y \\ &= E[XY] - m_x m_y \end{aligned} \tag{3.2}$$

Where m_x and m_y are the mean of the distribution of the sources **X** and **Y**. $E[X]$, $E[Y]$ are the expectations of the distribution of the sources and

$$E[XY] = \sum_{x,y} xyp(x,y) \tag{3.3}$$

To calculate the standard deviations in (3.1), we have:

$$\sigma_x = \sqrt{\sum_x x^2 p(x) - m_x^2} \tag{3.4}$$

$$\sigma_y = \sqrt{\sum_y y^2 p(y) - m_y^2} \tag{3.5}$$

$p(x)$, $p(y)$ and the joint probability $p(x, y)$ can be obtained from the steady state vector calculated in (3.11). For example,

$p(0, 0) = \pi_{00}$ and $p(x = 0) = \pi_{00} + \pi_{01} + \pi_{02} + \pi_{03}$

The steady-state vector in (3.11) is calculated on the basis of the transition matrix **A** which is formulated using q and q' as will be discussed next. Therefore, we can calculate the correlation coefficient ρ for a given value q'. Hereafter we study the system on the basis of correlation coefficient as it is a known statistical parameter.

Transition Matrix

We have now designed a model with 16 states, each representing a value of the transmitting sources **X** and **Y** at any given slot. We next move towards calculating the transition matrix and steady-state vector for the designed Markov model.

To represent the transition between the different states in the Markov model, we formulate a transition matrix **A** as follows:

By considering all the possibilities of state transition, we denote:

u = Probability of moving to an uncorrelated state

v = Probability of moving to a highly correlated state

With this notation, the probability of moving to an uncorrelated state u is expressed as:

$$u = (1 - q')(1 - q) \tag{3.6}$$

And the probability of moving to a highly correlated state v is expressed as:

$$v = q'(1 - q) \tag{3.7}$$

Accordingly, we have

$$\begin{aligned} 1 = u + v + q &= (1 - q')(1 - q) + q'(1 - q) + q \\ &= (1 - q)[1 - q' + q'] + q \end{aligned} \tag{3.8}$$

The sum of every row of the state transition matrix **A** is 1.

The probability of changing the state $\bar{q}$ is expressed as:

$$\bar{q} = 1 - q \tag{3.9}$$

$$
\mathbf{A} = \begin{array}{c} \\ (00) \\ (01) \\ (02) \\ (03) \\ (10) \\ (11) \\ (12) \\ (1,3) \\ (20) \\ (21) \\ (2,2) \\ (23) \\ (30) \\ (31) \\ (32) \\ (33) \end{array}
\begin{array}{cccccccccccccccc}
(00) & (01) & (02) & (03) & (10) & (11) & (12) & (13) & (20) & (21) & (22) & (23) & (30) & (31) & (32) & (33) \\
\mathrm{q} & \frac{u}{2} & 0 & 0 & \frac{u}{2} & v & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{v}{2} & \mathrm{q} & \mathrm{u} & 0 & 0 & \frac{v}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\bar{q}}{3} & \mathrm{q} & \frac{\bar{q}}{3} & 0 & 0 & \frac{\bar{q}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\bar{q}}{2} & \mathrm{q} & 0 & 0 & 0 & \frac{\bar{q}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{v}{2} & 0 & 0 & 0 & \mathrm{q} & \frac{v}{2} & 0 & 0 & u & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{v}{2} & \frac{u}{4} & 0 & 0 & \frac{u}{4} & \mathrm{q} & \frac{u}{4} & 0 & 0 & \frac{u}{4} & \frac{v}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{u}{2} & 0 & 0 & \frac{v}{2} & \mathrm{q} & \frac{u}{2} & 0 & 0 & \frac{v}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\bar{q}}{3} & 0 & 0 & \frac{\bar{q}}{3} & \mathrm{q} & 0 & 0 & 0 & \frac{\bar{q}}{3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\bar{q}}{3} & 0 & 0 & 0 & \mathrm{q} & \frac{\bar{q}}{3} & 0 & 0 & \frac{\bar{q}}{3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{v}{2} & 0 & 0 & \frac{u}{2} & \mathrm{q} & \frac{v}{2} & 0 & 0 & \frac{u}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{v}{2} & \frac{u}{4} & 0 & 0 & \frac{u}{4} & \mathrm{q} & \frac{u}{4} & 0 & 0 & \frac{u}{4} & \frac{v}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & u & 0 & 0 & \frac{v}{2} & \mathrm{q} & 0 & 0 & 0 & \frac{v}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\bar{q}}{2} & 0 & 0 & 0 & \mathrm{q} & \frac{\bar{q}}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\bar{q}}{3} & 0 & 0 & \frac{\bar{q}}{3} & \mathrm{q} & \frac{\bar{q}}{3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{v}{2} & 0 & 0 & u & \mathrm{q} & \frac{v}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & v & \frac{u}{2} & 0 & 0 & \frac{u}{2} & \mathrm{q}
\end{array} \tag{3.10}
$$

The transition probability is shown in the matrix **A** in (3.10). The matrix represents the possibility of transitions of the sources from a state X_{n-1} and Y_{n-1} at the slot $n-1$ to any other state in the system X_n and Y_n at slot n.

As we have a 16-state Markov Model, with the help of the transition matrix **A** we calculate the steady state vector Π for the Markov model in the figure 3.1.

The steady-state vector is of the dimension a 1 **x** 16. The elements of this vector give us the probability of finding the system at that particular state for $n \to \infty$.

$$\Pi = [\pi_{00}, \pi_{01}, \pi_{02}, ...\pi_{33}] \tag{3.11}$$

E.g. π_{01} represents the probability of $X_n = 0$ and $Y_n = 1$.

We obtain the steady state vector by solving the system of equations:

$$\Pi = \Pi\mathbf{A} \tag{3.12}$$

and

$$\sum \Pi = 1 \tag{3.13}$$

Examples of the system equations:

$$\pi_{00} = q\pi_{00} + v\pi_{11} + \frac{u}{2}\pi_{01} + \frac{u}{2}\pi_{10} \tag{3.14}$$

$$\pi_{32} = q\pi_{32} + u\pi_{31} + \frac{v}{2}\pi_{22} + \frac{v}{2}\pi_{33} \tag{3.15}$$

16 such equations along with (3.13) are solved to obtain all the elements of the steady state vector Π

We further denote the transition matrix for the source **X** individually. The matrix can be calculated based on the previously obtained combined transition matrix **A** of the sources.

$$Q = \begin{bmatrix} P[X_n=0|X_{n-1}=0] & P[X_n=1|X_{n-1}=0] & P[X_n=2|X_{n-1}=0] & P[X_n=3|X_{n-1}=0] \\ P[X_n=0|X_{n-1}=1] & P[X_n=1|X_{n-1}=1] & P[X_n=2|X_{n-1}=1] & P[X_n=3|X_{n-1}=1] \\ P[X_n=0|X_{n-1}=2] & P[X_n=1|X_{n-1}=2] & P[X_n=2|X_{n-1}=2] & P[X_n=3|X_{n-1}=2] \\ P[X_n=0|X_{n-1}=3] & P[X_n=1|X_{n-1}=3] & P[X_n=2|X_{n-1}=3] & P[X_n=3|X_{n-1}=3] \end{bmatrix}$$

An example of calculation of the elements of the matrix Q is as follows:

From the definition of conditional probability,

$$P[X_n = 0|X_{n-1} = 1] = \frac{P[X_n = 0, X_{n-1} = 1]}{P[X_{n-1} = 1]} \tag{3.17}$$

In turn, the probability of the source **X** assuming the value 1 at any n-1 is given by marginalizing the joint stationary distribution:

$$\text{P}[X_{n-1} = 1] = \pi_{10} + \pi_{11} + \pi_{12} + \pi_{13}$$

Plugging this in (3.17) and considering all possible values of Y_{n-1}, we have

$$\begin{aligned} P[X_n = 0|X_{n-1} = 1] = \frac{1}{\pi_{10} + \pi_{11} + \pi_{12} + \pi_{13}} \Big[& P[X_n = 0, X_{n-1} = 1, Y_{n-1} = 0] + \\ & P[X_n = 0, X_{n-1} = 1, Y_{n-1} = 1] + \\ & P[X_n = 0, X_{n-1} = 1, Y_{n-1} = 2] + \\ & P[X_n = 0, X_{n-1} = 1, Y_{n-1} = 3] \Big] \end{aligned} \tag{3.18}$$

We have,

$$P[X_n = 0, X_{n-1} = 1, Y_{n-1} = 0] = P[X_n = 0|X_{n-1} = 1, Y_{n-1} = 0] P[X_{n-1} = 1, Y_{n-1} = 0] \tag{3.19}$$

By substituting 3.19 in 3.18 we get:

$$\begin{aligned} P[X_n = 0|X_{n-1} = 1] = \frac{1}{\pi_{10} + \pi_{11} + \pi_{12} + \pi_{13}} \Big[& P[X_n = 0|X_{n-1} = 1, Y_{n-1} = 0]\pi_{10} + \\ & P[X_n = 0|X_{n-1} = 1, Y_{n-1} = 1]\pi_{11} + \\ & P[X_n = 0|X_{n-1} = 1, Y_{n-1} = 2]\pi_{12} + \\ & P[X_n = 0|X_{n-1} = 1, Y_{n-1} = 3]\pi_{13} \Big] \end{aligned} \tag{3.20}$$

$P[X_n = 0|X_{n-1} = 1, Y_{n-1} = 0]$ is the probability of $X_n = 0$ when $X_{n-1}Y_{n-1} = 10$. This is the probability of going from state '10' to any of the following states: (00, 01, 02, 03) where the value of the source **X** is zero. These values are obtained by the transition matrix **A** and the π values are obtained by the steady-state vector.

3.1.3 Transmission Techniques

A random access channel is a fundamental communication channel used extensively in various wireless communication systems. Its primary function is to allow multiple users or devices to access the shared channel with low complexity and without centralized coordination. This uncoordinated access is crucial in scenarios where multiple devices need to transmit data without any external scheduling[8]. This capability is particularly advantageous in scenarios such as IoT networks where there is a need for low-power and low-complexity solutions [25].

Here, multiple users contend for access to the channel, often resulting in potential collisions when two or more users attempt to transmit simultaneously. In our case where the sources are limited to two devices, a collision may occur in case both the sources transmit. The collisions lead to data loss and we do not consider any re-transmissions or feedback of the lost packets. In our system, we use a Slotted ALOHA-like protocol in which we divide time into slots for transmission.

Understanding and optimizing random access channels involves addressing various challenges such as managing the trade-off between system throughput and collision probability, designing efficient protocols to mitigate collisions, and optimizing channel access for diverse traffic patterns. The relevance of the random access channel within the scope of IoT networks is highlighted in literature [25]. Its capability to enable decentralized access to the transmitting sources aligns with the operational necessities of IoT devices [3].

We investigate two methods for transmission strategies that can be used by the transmitting sources to access the channel.

1. A random transmission method.
2. An event-driven transmission method.

Random Transmission Method

The random transmission method is predicated on uncoordinated access, allowing devices to opportunistically access the channel without prior scheduling or synchronization. The probability of transmission at any given slot is purely based on a predetermined parameter.

In our system, the transmitting sources X and Y decide to randomly access the channel at each time slot with a preassigned value of probability p_t and stay silent with a probability of $1 - p_t$. This decision is independent of the evolution of the system till this particular point and of the behavior of the other source.

The probability for source **X** to successfully deliver the packet (P_S) during a slot can be formulated based on binomial distribution and is given by,

$$P_s = p_t(1 - p_t) \tag{3.21}$$

where p_t denotes the probability of transmission by the source.

The probability of success in a two-source system is if only one of the two transmitters access the channel.

In our system, the only way a packet fails to reach the receiver is when there is a collision with a packet from another source. This is the probability of both the sources X and Y transmitting during the same slot. Hence, at any given slot, the probability of collision is:

$$P_c = p_t^2 \tag{3.22}$$

The fundamental premise of a random access channel rests upon the absence of prior coordination between transmitting devices. However, this lack of coordination presents a significant drawback: the potential for channel collisions.

This has a detrimental impact on the channel throughput. Channel throughput refers to the fraction of slots that experience successful transmission(S). In our case, this can be written as:

$$S = 2p_t(1 - p_t) \tag{3.23}$$

Collisions effectively disrupt this flow, hindering the amount of data that can be transmitted. The Random access method also proves to be a disadvantage in case of high correlation between the contending sources as the correlation information is not leveraged to improve the efficiency of the system.

Despite these inherent limitations, random access channels possess some advantages due to the ease of implementation. The absence of a complex scheduling mechanism translates to reduced overhead and lower latency for devices attempting to access the channel. One such example is LoRa, [23] which uses this simple method of transmission. We can also consider Zigbee [6] which uses the IEEE 802.15.4 [1] standard under the random transmission method. However, this benefit comes at the cost of sacrificing predictability and efficiency in channel access.

Hence, the adoption of a random access channel for communication presents a trade-off between simplicity and performance. While it offers a readily deployable solution, the stochastic nature of transmissions introduces significant challenges in the form of collision-induced data loss and in turn reduced channel throughput.

Reactive method: An Event-Driven Transmission Strategy

In contrast to the random transmission method, the event-driven transmission method relies on specific triggers or events dictating channel access. Devices access the channel selectively, prompted by predetermined events such as the availability of data for transmission, or environmental changes within the network.

In this study, the change in the state of the Markov model is an Event for transmissions. This becomes important as it avoids transmitting any redundant information to the receiver while there is no new information generated. This approach introduces a level of control over channel access, enabling devices to transmit when an event arises.

By harnessing this event-driven strategy, devices can potentially mitigate contention by transmitting data only when necessary, thereby reducing collision occurrences and optimizing overall channel efficiency.

Although this is true in many cases, as we discuss the correlation between the two sources, we see that the increase in the dependencies between them results in simultaneous changes in their states resulting in the simultaneous need to access the channel and therefore increased collision and packet drops.

Comparative Analysis

It is also noteworthy to mention that although the event-driven method shows an advantage in cases of low correlation coefficients, as the correlation between the interacting sources increases, the efficiency of the event-driven system significantly decreases because of the increased collisions in the channel.

The examination of the Markov model within the context of collision occurrences is shown in Figure 3.3. This provides valuable insights into the dynamics of channel access and transmission behaviors. The parameter denoted as q drives the frequency with which sources persist in their current state. For low values of q, the Markov model exhibits a dynamic behavior. This results in a higher rate of state changes among the sources in the case of the event-driven transmission method, leading to increased transmission of these transitions. However, these heightened state changes also correspond to a lower probability of sources remaining in their current state, subsequently contributing to a surge in collision occurrences where the reactive transmission method is employed. Conversely, as q escalates to higher values, the model tends to experience fewer state changes, resulting in a diminished number of transmission events and relatively lower collision instances.

The parameter q' governs the degree of correlation between the sources within the Markov model. A higher value of q' signifies a heightened correlation level between

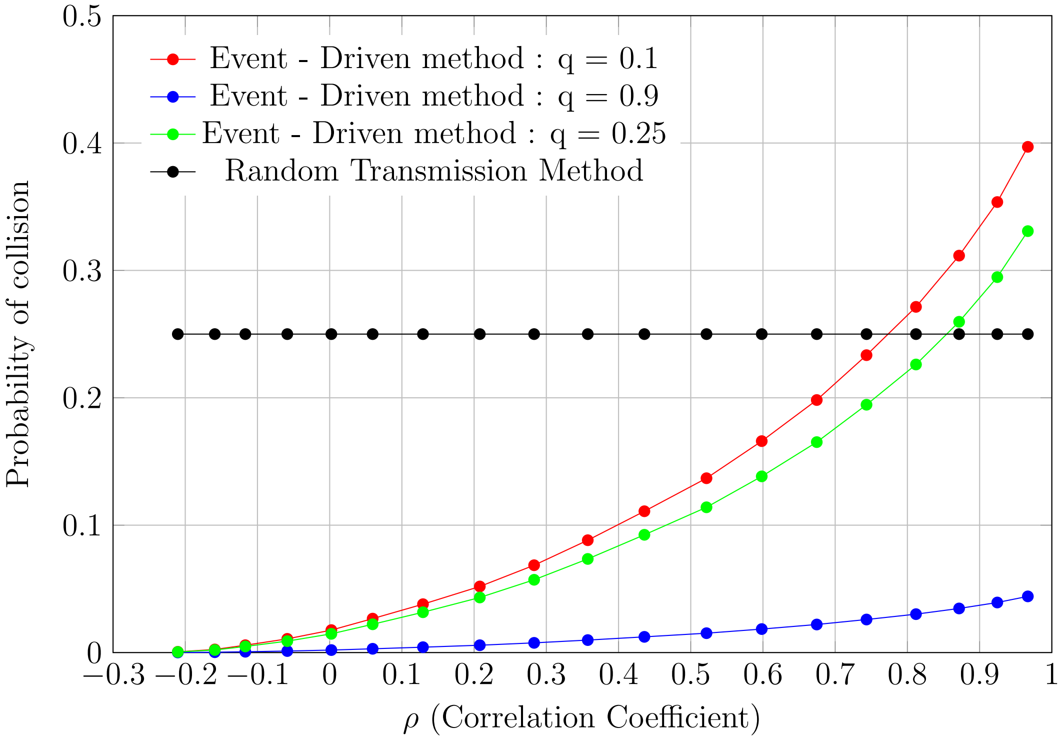

Figure 3.3: Probability of collision for different levels of correlation

the sources, indicating a synchronized state change between the sources. As both sources intend to access the channel concurrently in the reactive method, collision instances increase, diminishing the chances of successful transmissions.

3.2 Estimator Design

In this section, we introduce the receiver-side estimator.

We make a realistic assumption that the receiver can sense the channel. At any given slot the receiver either observes the updates sent by one of the sources or observes an idle slot or a collision in the channel.

Hence we can define the alphabet of the observations as

$$O_a = \{0_x,\ 1_x,\ 2_x,\ 3_x,\ \mathrm{I},\ \mathrm{C},\ 0_y,\ 1_y,\ 2_y,\ 3_y\}$$

Where I = Idle state and C = collision in the channel.

0_x, 1_x, 2_x, 3_x are the denotes reception of a packet with values $\{0,\ 1,\ 2,\ 3\}$from the source **X**, and 0_y, 1_y, 2_y are the values from the source **Y**.

The estimated value by the estimator at a given slot is represented as Z_n. For a given slot n, based on the observations, the estimator intends to accurately estimate the value of the state at the source. The alphabet for the estimation can be represented as:

$$Z_a = \{0,\ 1,\ 2,\ 3\}$$

In this study, the system on the receiving end although estimates values of both X and Y, analysis can be done considering either of the sources. This leads us to consider one source as 'the source of interest' (e.g. X) and the other source as a correlated source. In one case, source Y contributes as a factor of collision while trying to access the random access channel during the same time as the source of interest. In the other case, it contributes as the source of information when the correlation with the source of interest is known.

The ultimate goal of the system is to estimate the values of the source to optimize the parameters such as the error, age of incorrect information, or the duration of the error depending on the requirement of the system.

We try to explore two methods of estimation in the following section.

3.2.1 Update and Hold Estimator

The receiver using the "update and hold" estimation method follows a straightforward approach, retaining the last received sensor value when there are no new updates available from the transmitting source. This approach is beneficial in scenarios where data transmission might be sporadic, or subject to delays, commonly encountered in various real-world IoT setups [33] [28].

The advantages of this approach lie in its simplicity and ability to ensure continuity in estimation. It helps mitigate the impact of missing updates, allowing the system to maintain a consistent output based on the latest available information.

However, the downside of the "update and hold" method is its inherent limitation in adapting to changes in the system. If the system undergoes significant transitions while no updates are received, the estimator might continue to output the last known value, potentially leading to inaccuracies in estimation. Moreover, prolonged gaps between updates could cause the estimator to lag behind the current state of the system. This lag can impact the responsiveness of the estimation process, particularly in scenarios where real-time adjustments to changing conditions are critical [27].

In practical terms, when the estimator receives sensor data, it estimates the sensor value based on the available information. In our experiment, the estimation is simply updating the most recent value received from the corresponding source. If subsequent updates from the transmitter fail to arrive the estimator maintains the previously received value as the current state. This "holding" or "updating" mechanism works well under the assumption that the system remains relatively constant or changes gradually over time. This is not true for all the cases in our experiment.

For example, the sources **X** and **Y** are less likely to change when the value of the system parameter q is high. In this case, we will observe that the simple "holding" or "updating" mechanism indeed proves to be of advantage because of its simplicity and ease of design. In cases where the value of system parameter q is low, the probability of the sources retaining the previous state is low. Thus the results are worse when compared to randomly guessing the value, as the estimator tends to estimate wrongly and keep the older value while the probability of change is high thus making the simple hold and update estimator an infeasible option.

In summary, while the "hold-last" method ensures stability and continuity in estimation in the absence of new data, its limitations in adapting to real-time changes and potential delays in reflecting current system states might prompt the consideration of alternative estimation strategies for enhanced accuracy and responsiveness in IoT setups.

3.2.2 Estimator using Hidden Markov's Model

Constructing an estimator based on HMM [24] for our system model that is built on the basis of Markov's progression clearly proves to be an advantage in capturing the states and the observations they generate. In this study, the HMM process corresponds to the source of interest, and the observations are given by the succession of slots This involves determining the potential hidden states that characterize the system's behavior. The information about the ideal state or collided state of the channel along with the correlation between the two transmitting sources **X** and **Y** is leveraged to derive the state of the transmitting sources.

To design a HMM we have to specify how the system transitions between hidden states over time, represented by transition probabilities (parameters q and q'). These probabilities dictate the likelihood of moving from one state to another, mirroring the system's dynamics.

In this system, the observations can be:
1. An idle slot 2. A collision slot
3. Successful reception of the transmitted packets from either of the sources.
The hidden states are the state transitions or the information generated at the sources that are either not transmitted or lost due to collision.

At any given slot n the goal of the HMM estimator is to provide an estimate of Z_n using the observations **O** up to the slot n.

We start with the goal of finding the value of Z_n that has the maximum likelihood to occur at the slot n. Hence, we have:

$$Z_n = \arg\max_x P[X_n = x|\mathbf{O}] \tag{3.24}$$

Where x ∈ 0, 1, 2, 3 and **O** capture the entire sequence of observation by the estimator till the slot n.

By the definition of conditional probability, we have:

$$P[X_n = x|\mathbf{O}] = \frac{P[X_n = x, \mathbf{O}]}{P[\mathbf{O}]} \tag{3.25}$$

Following the forward recursion of HMM [24], we can write:

$$\alpha_n(x) = P[X_n = x, \mathbf{O}] \tag{3.26}$$

We understand that the vector α_n reads the best state sequence up to slot n and all the observations till slot n

where α is a vector whose elements contain value for each $x \in \{0, 1, 2, 3\}$

$$\alpha_n = \begin{bmatrix} \alpha(0) & \alpha(1) & \alpha(2) & \alpha(3) \end{bmatrix}$$

Hence the equation 3.24 can be written as:

$$Z_n = \arg\max_x \alpha_n \tag{3.27}$$

Z_n assumes the value $x \in \{0,\ 1,\ 2,\ 3\}$ that is most likely to be the state of X_n: the value of the source.

We now focus on calculating the vector α

$$\begin{aligned}\alpha_n &= P[X_n = x, \mathbf{O}] \\ &= P[X_n = x, X_1, X_2 ... X_{n-1}, O_n, O_{n-1} ... O_1] \\ &= P[X_n = x, O_n | X_{n-1} ... O_{n-1} ... O_1] P[Z_{n-1} ... O_{n-1} ... O_1]\end{aligned} \tag{3.28}$$

Note that Markov's assumption is that the current output depends only on the current observation. Therefore we have,

$$P[X_n = x, O_n | X_{n-1} ... O_{n-1} ... O_1] = P[X_n = x, O_n] \tag{3.29}$$

From the equation 3.26 we have the value of α_{n-1} as:

$$\alpha_{n-1} = P[X_{n-1} = x, ... O_{n-1} ... O_1] \tag{3.30}$$

Hence α_n after the substitution is:

$$\begin{aligned}\alpha_n &= P[X_n = x, O_n | O_{N-1} ... O_1] \alpha_{n-1} \\ &= P[X_n = x, O_n] \alpha_{n-1} \\ &= P[O_n | X_n = x] P[X_n = x] \alpha_{n-1}\end{aligned} \tag{3.31}$$

Hence we can write the equation as follows:

$$\alpha_n = P[O_n | X = x] P[X = x] \alpha_{n-1} \tag{3.32}$$

Hence, we see that the vector α and in turn, the estimation depends on three elements,
1. The previous alpha vector α_{n-1}.
2. $P[X_n = x]$ which can be calculated using the Matrix Q 3.1.2
3. $P[O_n | X_n = x]$ which depends on the transmission strategy. We formulate an observation matrix to calculate this term.

This method closely follows the forward propagation in the Hidden Markov Model [24] where the previous α is weighted by their transition probabilities (used in the calculation of $P[O_n|X = x]$) and multiplied by the state observation likelihood (P[X = x]).

To calculate the elements of α we have:

$$\alpha_n(x) = \sum_{i=0}^{i=3} Q_{ix} P[O_n|X_n = x]\alpha_{n-1}(i) \tag{3.33}$$

For example: let n = 1, x = 0 and O_n = I. We consider an idle channel observation (I) by the receiver at the first slot. We calculate the likelihood of the source value X_1 being 0.

$$\alpha_1(0) = \sum_{i=0}^{i=3} Q_{i0} P[Z_n = I|X_n = 0]\alpha_0(i) \tag{3.34}$$

$$\alpha_1(0) = P[Z = I|X = 0]\{Q_{00}\alpha_0(0) + Q_{10}\alpha_0(1) + Q_{20}\alpha_0(2) + Q_{30}\alpha_0(3)\} \tag{3.35}$$

Q_{jk} indicates the probability of transitioning from j^{th} state to k^{th} state. In the above example, we calculate the probability of the source **X** coming to state 0 from any of the other states using the matrix Q.

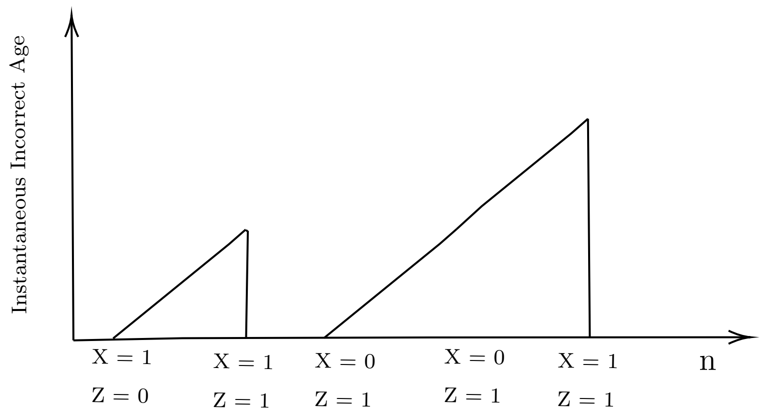

Figure 3.4: Propagation of instantaneous incorrect age

3.3 Evaluation Parameters

The following three parameters are used as metrics to analyze the system
1. Age of Incorrect information (AoII)
2. Average Estimation Error
3. Average duration of Error Estimation

3.3.1 Age of Incorrect information (AoII)

We analyze the performance of the presented schemes in terms of the Age of Incorrect Information (AoII) [14]. Age of Information (AoI) [35] is a well-documented metric used in communication systems to measure the information freshness of a given source. Although AoI successfully captures how recent the knowledge about the state of a source is at the destination, it does not account for the accuracy of that information.

To account for the above discrepancy, we move towards the more recently introduced metric Age of Incorrect Information [14] which can capture the amount of time spent by the estimator at the receiver estimating the wrong status value and penalizing for wrong estimation. We assume each status update from the source of interest to contain a time stamp, denoting the instant at which the message was generated.

Figure 3.4 gives us an overview of AoII. The age is reset in case the receiver correctly estimates the source state. If $X_n = Z_n$, the instantaneous age of AoII is zero. In other words, when the receiver has successfully estimated the perfect knowledge about the status of the source of interest, we set or reset the value of AoII to zero. However, as the state of the sources changes the estimator is ideally expected to keep up with the correct estimation which in reality is highly unlikely. We record this error in estimation until the estimator again gives the true value of the source of interest. The age of incorrect information grows linearly with time as long as the estimator is in an erroneous state. Therefore the metric Age of Incorrect Information (AoII), can be written as follows:

$$\delta_n = f(n) \times g(X(n), Z(n))$$

where $f(n)$ is an increasing time function, that can be considered a penalty paid for being unaware of the correct status of the source X (source of interest) for a certain amount of time. It reflects the cost or detriment associated with the receiver's lack of awareness regarding the accurate state of the source over time. $f(n)$ profoundly influences how the AoII grows with time.

On the other hand, $g(X(n), Z(n))$ is a function that reflects the difference between the current estimate at the receiver Z_n and the actual state of the source of interest X_n. It quantifies the error in estimation and plays a crucial role in determining the magnitude of AoII.

There exists a wide variety of choices for f and g that we can pick from.

In this study, we use the linear time-dissatisfaction function:

$$f(n) = n - V(n)$$

where V(n) is the last time instant where the estimator had accurate information of the source of interest and n is the current time slot.

We consider the indicator error function:

$$g(n) = \begin{cases} 0 & \text{if } X_n = Z_n \\ 1 & \text{otherwise} \end{cases} \tag{3.36}$$

When there is any mismatch between the status of the sources $\mathbf{X}$ at a slot X_n and the estimated status value at the receiver Z_n a penalty is imposed by the system in terms of $g(n)$. We consider this approach with the error function $g(n)$ for studying the results further.

Another known way of penalizing the incorrect estimation is to consider the absolute value of inaccuracy. The penalty is higher for a larger gap in the estimation. In other words, the farther the estimated value from the real value of the system, the higher the impact on the Age of Incorrect information.

We further consider the indicator error function $g'(n)$ to give a better picture of the accuracy of the estimator.

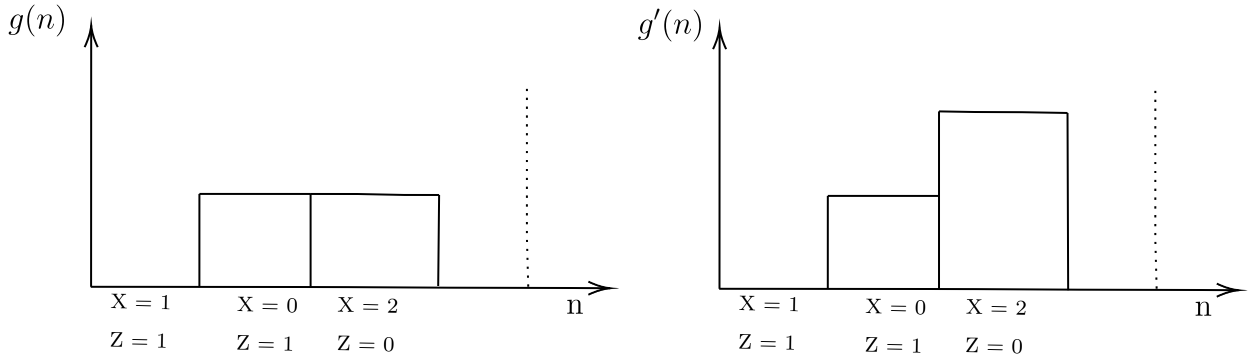

Figure 3.5: Error functions for AoII

$$g'(n) = \begin{cases} 0 & \text{if } X_n = Z_n \\ |X(n) - Z(n)|; & \text{otherwise} \end{cases} \tag{3.37}$$

In summary, both $f(n)$ and $g(X(n), Z(n))$ are important components of the AoII metric, each contributing to the dynamics of information freshness. By carefully selecting and calibrating these functions, the impact of incorrect information can be studied for diverse applications.

3.3.2 Average Estimation Error

We consider the simplest form of error evaluation to measure the estimation error. The error in estimating the value of the source **X** at the time slot n is as follows:

$$E_n = |X(n) - Z(n)| \tag{3.38}$$

We span through N slots in the system to gain a stable understanding of the error in the estimation of transmitter status.

The mean absolute estimation error is defined as:

$$E_e = \frac{1}{N} \sum_{n=1}^{N} |X_n - Z_n| \tag{3.39}$$

3.3.3 Average Error length

In some systems, not only the absolute error but also the duration the receiver spends estimating the wrong status plays an important role.

Let us consider an example,

X: 0, 0, 1, 2, 2, 3, 2, 2, 2, 1, 1, 2, 1, 0, 0

Z: 0, 0, 0, 1, 2, 3, 3, 2, 2, 3, 1, 2, 1, 1, 0

E: 0, 0, 1, 1, 0, 0, 1, 0, 0, 2, 0, 0, 0, 1, 0

In the above example, we see there are 4 error streaks. One of the error length of two slots and three of the error length of one slot. Hence we consider 4 error instances and the total number of errors is 5.

To capture this, we define E_d:

$$E_d = \frac{N_e}{N_i} \tag{3.40}$$

Where E_d is the average error duration, N_i is the number of estimation error instances, and N_e is the total estimation error.

This parameter gives us valuable insight into the duration for which the system assumes a wrong value of a transmitter without much focus on how far the estimation is from the true value.

3.3.4 Contextual Understanding E_e and E_d Metrics

Although both E_e and E_d evaluate the error in the system, the context under which each one of these errors becomes significant varies. In case it is important that the average error of the system is low and the accuracy is of higher importance, it is better to evaluate the system based on E_e, which considers error magnitude. This could play a role in safety-critical applications. In case the goal is to learn the estimator based on the duration (or the number of error instances) the receiver spends in error, regardless of magnitude, E_d proves to give us a better understanding. This could be important in time-critical applications where timely status updates are important.

Error Magnitude Evaluation E_e

E_e focuses on evaluating the error in the system based on the magnitude of the discrepancies between the actual and estimated states.

Duration-Based Error Evaluation E_d

E_d focuses on quantifying the duration or frequency of time spent in error states by the receiver. It disregards the magnitude of individual errors and instead emphasizes the persistence of inaccuracies over time. By tracking the duration of error instances, E_d provides insights into the system's robustness. Prolonged periods of error states may indicate inefficiencies in the estimation process. It prioritizes the exploration of error patterns over time. By quantifying the duration of erroneous states,

To summarize, While E_e emphasizes accuracy and error magnitude for precision-centric tasks, E_d prioritizes duration-based analysis for systems where error dynamics and system stability take precedence.

3.4 E_e and E_d in the context of AoII

In the context of AoII, large error magnitudes indicate significant deviations from the true state of the source, which will contribute to increased AoII. An appropriate error function is to be selected to account for the error magnitude. The larger the error magnitude, the greater the penalty imposed on the system's freshness of information, reflecting the impact of inaccurate estimates on the currency of information.

E_d evaluates the duration or frequency of error instances, irrespective of their magnitude. In the context of AoII, longer duration of error instances contributes to increased AoII, as they indicate prolonged periods during which the receiver is unaware of the correct state of the source. Even if individual errors have small magnitudes, persistent errors over time will lead to a higher AoII, reflecting the accumulation of incorrect information over prolonged durations. In this case the error function ($g(n)$ 3.36 is selected as it accounts for the accuracy of the system. The impact of using different transmission methods and estimation strategies can be studied individually without the influence of an additional parameter of the error magnitude.

Higher values of E_d correspond to higher AoII, as they indicate longer periods of erroneous states during which the receiver lacks accurate information about the source. The results in the upcoming section are based on this error function.

3.5 Random Transmission Strategy

3.5.1 Update and Hold Estimator

The Random transmission strategy with an Update and hold estimation model is a simple approach by the receiver to realize the status of the transmitter with minimum effort on the receive receiver end.

The possible channel observations by the receiver will be

$$\mathcal{O} = \{0_x,\, 1_x,\, 2_x,\, 3_x,\, \text{I},\, \text{C},\, 0_y,\, 1_y,\, 2_y,\, 3_y\}$$

Thus, using the channel observation, the update and hold estimator takes the following steps,

$$Z_n = \begin{cases} Z_{n-1}, & \text{if } O_n = \{Y_n,\, \text{I},\, \text{C}\ \} \\ \text{X}_n, & \text{if } O_n = X_n \end{cases} \tag{3.41}$$

Figure 3.5.1 gives an example of a possible evolution of sources **X** and **Y** over time slots, and their decision to access the channel. We see the collision and the idle states in the channel and how the Update and Hold leverages this information. The knowledge of the state of the other sources do not provide information about the state of the source of interest in this simple estimator.

We can intuitively notice that the error significantly decreases as the probability of remaining in the same state q increases. With a system that tends to remain in the same state for a longer duration, an Update and Hold estimator shows an increased advantage. This estimator with limited computational complexity can be used as a benchmark against the more complex estimators to measure improvements in terms of estimation error and various other parameters.

For a two-source system such as ours, the best usage of the channel is observed when the probability of transmission is 0.5. Anything above this value results in increased collision, and anything below 0.5 results in more idle slots (3.23).

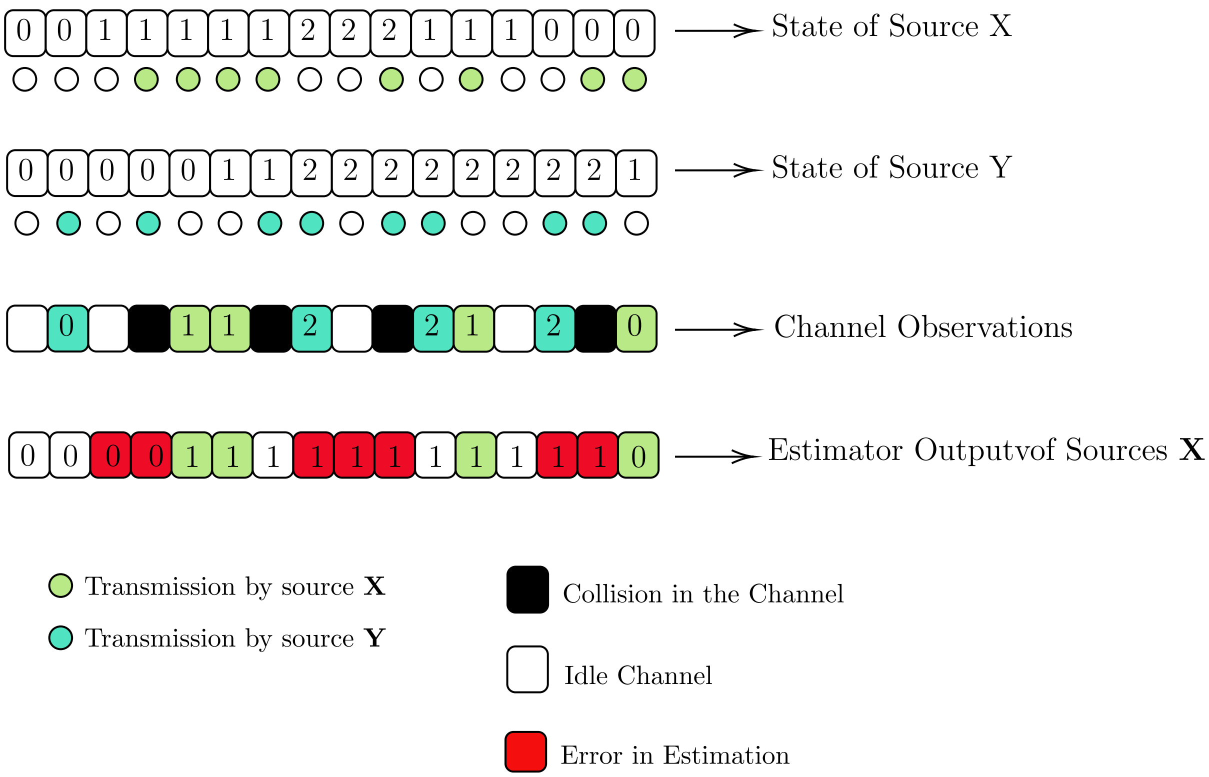

Figure 3.6: Representation of a system using random transmission method and update and hold estimator at the receiver

3.5.2 MAP Estimator based on Hidden Markov's Model

By combining a Random Transmission method at the source with a Hidden Markov Model (HMM) at the estimator, we explore the possibility of optimization, leveraging probabilistic inferences (correlation between the two sources).

The Random Transmission method introduces a stochastic element into the data transmission process, whereby data is sent sporadically at the source, with a certain predetermined probability, regardless of the system state.

Meanwhile, at the estimator, the integration of a Hidden Markov Model approach to parameter estimation enables the inference of underlying system dynamics (correlation between the sources) that exert influence on the received data.

As we have discussed earlier, HMM estimation is a recursive method and uses the observation till that point (in terms of α) to make an estimate at the current slot.

We now formulate an observation matrix O_{ran} for the random transmission method to calculate $P[O|X]$: The probability of making an observation conditioned on the current observation of the source. This value is further used in status estimation in section 3.2.2.

Observation matrix: O_{ran}

$$
\begin{array}{c|cccccccccc}
 & 0_x & 1_x & 2_x & 3_x & I & C & 0_y & 1_y & 2_y & 3_y \\
0 & k & 0 & 0 & 0 & (1-p)^2 & p^2 & kP[O=0_y|0] & kP[O=1_y|0] & kP[O=2_y|0] & kP[O=3_y|0] \\
1 & 0 & k & 0 & 0 & (1-p)^2 & p^2 & kP[O=0_y|1] & kP[O=1_y|1] & kP[O=2_y|1] & kP[O=3_y|1] \\
2 & 0 & 0 & k & 0 & (1-p)^2 & p^2 & kP[O=0_y|2] & kP[O=1_y|2] & kP[O=2_y|2] & kP[O=3_y|2] \\
3 & 0 & 0 & 0 & k & (1-p)^2 & p^2 & kP[O=0_y|3] & kP[O=1_y|3] & kP[O=2_y|3] & kP[O=3_y|3]
\end{array}
$$

The last four columns are in the form of $P[O_n = y|X_n = x]$. The matrix calculates the probability of observing a condition in the channel $\in \mathcal{O}$ given the value of the source $\mathbf{X}$.

Also p is the value of the probability of transmission assigned to both sources, in the first column of O_{ran}, the probability of observing 0_x when 0 is transmitted by the source $\mathbf{X}$ is when $\mathbf{X}$ transmits (with the probability of p) and $\mathbf{Y}$ does not (with the probability of $1 - p$).

$$k = p(1-p) \tag{3.42}$$

Thus $P[O = 0_x|X = 0] = k$. There is no way the receiver observes 0 when any other value is transmitted. Hence all the other values in the first column are zero.

Further, the probability of observing the idle state is $(1-p)^2$, and the probability of observing collision p^2.

An example for calculating the last four columns of the matrix O_{ran}:

$$\begin{aligned} P[O = 0_y | X = 0] &= \frac{P[O = 0_y, X = 0]}{P[X = 0]} \\ &= \frac{\pi_{00}}{\pi_{00} + \pi_{01} + \pi_{02} + \pi_{03}} \end{aligned} \tag{3.43}$$

$P[X = 0]$ = Probability of finding the value of the source **X** as 0. From the steady state 3.11 we can calculate the value of $P[X = 0] = \pi_{00} + \pi_{01} + \pi_{02} + \pi_{03}$.

3.6 Reactive Transmission Strategy

3.6.1 Update and Hold Estimator

As we have studied earlier, the Update and Hold estimator is a simple estimation method and in this experiment used to measure the performance of the advanced estimation strategy (HMM) adapted to improve the performance parameter.

The reactive transmission strategy dictates that the transmitter only initiates status update transmission when there is a change in the system state. By transmitting data reactively, the system aims to avoid collisions by limiting unnecessary data updates. This additionally conserves energy and bandwidth, as it avoids transmissions during periods of stability or when there is no new data to transmit.

When a change in the system state triggers a reactive transmission event, the transmitter sends data to the receiver. Upon receiving this data, the receiver utilizes its update and hold estimator to update its estimates of the state of the transmitting source based on the newly received information.

We can define the estimation decision by the transmitters as

$$X_t^n = \begin{cases} 0, & \text{if } X_n = X_{n-1} \\ 1, & \text{otherwise} \end{cases} \tag{3.44}$$

$$Y_t^n = \begin{cases} 0, & \text{if } Y_n = Yn - 1 \\ 1, & \text{otherwise} \end{cases} \tag{3.45}$$

Where X_t^n and Y_t^n indicate the transmission decisions taken by the sources at the time slot n to transmit a status update depending on the information available. The sources X and Y transmit if X_t^n or Y_t^n are 1. The possibility of collision in this system is only when both sources have changed and have new information to transmit.

The alphabet of channel observations by the receiver is $\mathcal{O}$

The estimator makes the decision based on the channel observations:

$$Z_n = \begin{cases} Z_{n-1} & \text{if } O_n = \{Y_n, \text{I}, \text{C}\} \\ \text{X}_n & \text{if } O_n = X_n \end{cases} \tag{3.46}$$

Figure 3.7 gives an example of the progression of state changes in sources X and Y. We see the sources trying to transmit an update using the random access channel while there has been a state change.

Although the only possible collision scenario is the state change of both sources, this knowledge of the state change is not utilized by the simple hold and update estimator. This deterministically results in an error in the estimation.

Similar to the Random transmission method, we can intuitively derive that the error significantly decreases as the probability of remaining in the same state q increases. With a system that tends to remain in the same state for a longer duration, we observe a large reduction in the channel access by the transmitting sources resulting in reduced collision. The update and hold estimator retains the latest update from the source of interest, unless a new packet from that particular source arrives, hence showcasing increased advantage.

The parameter q' plays a major role in the performance of the reactive transmission. The probability that both X and Y change states is indicated by q'. In other words, q' is the parameter that decides the correlation between the two sources. As the correlation between the sources increases, both sources likely change their states simultaneously resulting in accessing the channel together and in turn colliding. This failed transmission due to collision results in an error in estimation as the Update and Hold estimator still holds on to the previous value whereas there has been a change of state at the sources. The simple estimator although observing the collision is not able to capture this information for estimation.

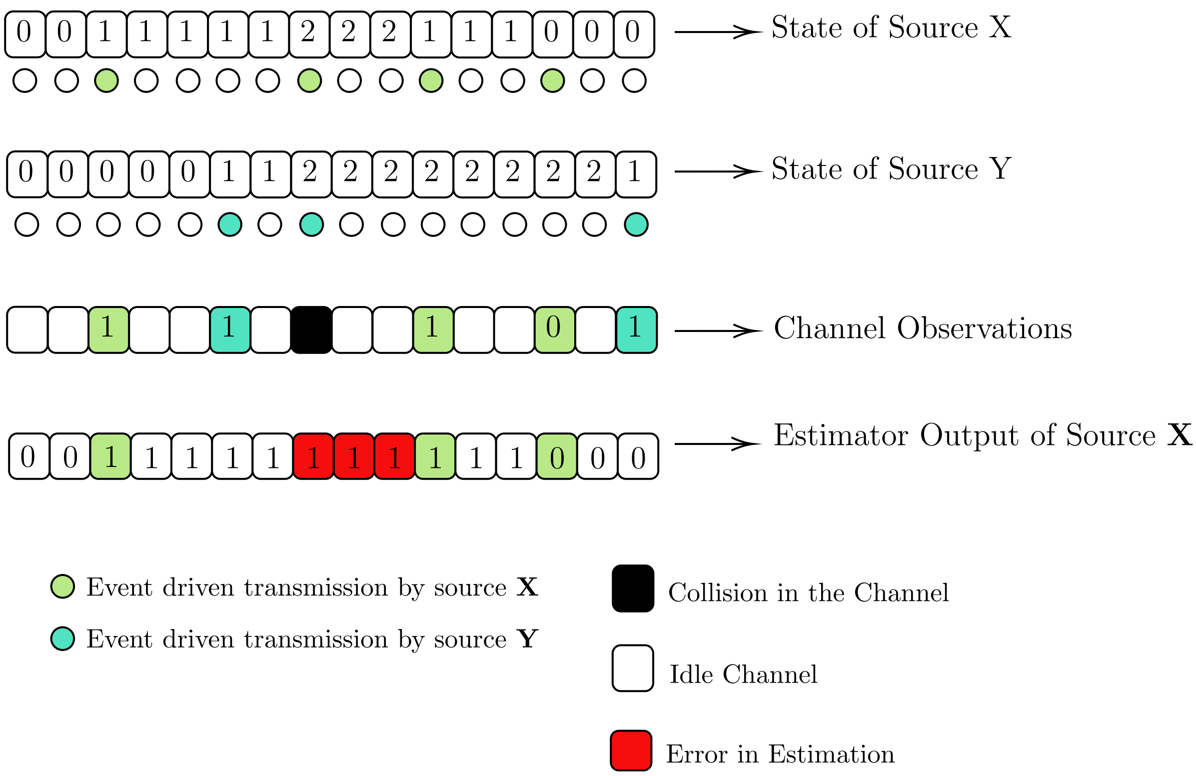

Figure 3.7: Representation of a system using reactive (event-driven) transmission method and Update and Hold estimator at the receiver

3.6.2 MAP estimator based on Hidden Markov's Model

When an event triggers the source node to transmit data, the receiver begins to observe a status update in case there is no collision. The MAP estimator on the receiver end is designed on the basis of HMM and hence will effectively use the correlation information to optimize the estimator output.

We design the HMM estimator considering the transmission methodology. We construct the O_{react} on the basis of the transition matrix $\mathbf{A}$ in (3.10) which is based on the parameters q and q'.

We see that when the value of q' is lower, the correlation between the transmitting sources is low which results in a high number of successful transmissions as the sources do not often try to access the shared channel simultaneously.

Conversely, when the value of q' is higher, although the system results in many more collisions compared to the earlier example, the knowledge at the estimator that a collision is a result of a change in both the transmitting sources helps in improved estimator performance. When the receiver witnesses a channel collision, although it does not know the current status of the source, it knows there has been a change. Using this knowledge the receiver intends to estimate the new state. This method shows a significant improvement over the Update and Hold estimation strategy.

We formulate the observation matrix O_{react} for the reactive transmission method as follows:

$$\begin{array}{c} \\ 0_x \\ 1_x \\ 2_x \\ 3_x \end{array} \begin{array}{c} \begin{array}{cccccccccc} 0_x & 1_x & 2_x & 3_x & I & C & 0_y & 1_y & 2_y & 3_y \end{array} \\ \begin{pmatrix} P[0_x|0] & 0 & 0 & 0 & P[I|0] & P[C|0] & P[0_y|0] & P[1_y|0] & P[2_y|0] & P[3_y|0] \\ 0 & P[1_x|1] & 0 & 0 & P[I|1] & P[C|1] & P[0_y|1] & P[1_y|1] & P[2_y|1] & P[3_y|1] \\ 0 & 0 & P[2_x|2] & 0 & P[I|2] & P[C|2] & P[0_y|2] & P[1_y|2] & P[2_y|2] & P[3_y|2] \\ 0 & 0 & 0 & P[3_x|3] & P[I|3] & P[C|3] & P[0_y|3] & P[1_y|3] & P[2_y|3] & P[3_y|3] \end{pmatrix} \end{array}$$

The elements of O_{react} represent $P[O = o|X = x]$: If the probability of observing a said observation given the value of the transmitting source $\mathbf{X}$.

In the following let us consider a few examples to compute the elements of O_{react} :

Example 1:

$$P[O = 0_X|X = 0] = \frac{P[\text{Change in X, No change in Y, X} = 0]}{P[X = 0]} \tag{3.47}$$

Reason for the above expression:

a. A Successful transmission from the source **X** indicates no collision and hence source **Y** has retained its earlier value and source **X** has transitioned to the state 0 from an unknown state.

b. As we know the states can only move up and down to their immediate neighbor, from the Markov model, we know that if $X_n = 0$ after a change, then $X_{n-1} = 1$.

The possible cases are as follows:

X_{n-1}	Y_{n-1}	X_n	Y_n
1	0	0	0
1	1	0	1
1	2	0	2
1	3	0	3

Hence from the transition matrix **A** in (3.10) we can formulate

$$P[O = 0_X/X = 0] = \frac{A(10,00)\pi_{10} + A(11,01)\pi_{11} + A(12,02)\pi_{12} + A(13,03)\pi_{13}}{P[X = 0]} \tag{3.48}$$

Example 2:

$$\begin{aligned} P[O_n = 0_y|X_n = 0] &= \frac{P[}{P[X_n = 0]} \\ &= \frac{A(01,00)\pi_{01}}{P[X_n = 0]} \end{aligned} \tag{3.49}$$

Example 3:

Estimation on witnessing an idle state:

$$P[O_n = I|X_n = 0] = \frac{P[}{P[X_n = 0]} \tag{3.50}$$

Reasoning:

An idle slot is observed with $X_n = 0$ only if

a. $X_n = 0$ and the channel is in an idle state shows that $X_{n-1} = 0$

b. Y_n and Y_{n-1} can assume any value but $Y_n = Y_{n-1}$ as the channel is in an idle state.

The possible cases are as follows:

X_{n-1}	Y_{n-1}	X_n	Y_n
0	0	0	0
0	1	0	1
0	2	0	2
0	3	0	3

$$P[O = I|X = 0] = \frac{A(00,00)\pi_{00} + A(01,01)\pi_{01} + A(02,02)\pi_{02} + A(03,03)\pi_{03}}{P[X = 0]} \tag{3.51}$$

3.7 A Hybrid Transmission Strategy

In this method, we focus on combining the random and reactive strategies. The reactive strategy works very effectively for the cases of low correlation (low q'). However, due to higher chances of collision in the highly correlated systems, the estimator that uses the reactive transmission at the sources will not see an update for a long time. Due to this, the receiver continuously estimates incorrect values for a long duration.

To avoid this we try to induce some randomness into the reactive method. The sources transmit when there is a change in the state with a probability of 1. Additionally, when there is no state change, the sources randomly decide to transmit with a non-zero probability of p_h. This helps the system with high q' escape the collision and update the receiver, thus helping in the status estimation.

It is important to note that although this strategy improves the performance in some cases, the knowledge of collision in the fully reactive method can not be completely used. There are chances of collision when one of the sources has changed the state and the other has decided to randomly send an update to the receiver. Now, when the receiver notices a collision, it can not know with certainty if there was a state change.

The grey slots in the demonstration in Figure 3.8 showcase the possibility of an error. Since there is a collision, although the HMM estimator knows there is a chance of change in the state of the sources, it can not be certain. The collision could also be because of the induced random transmission.

3.8 An Adaptive Estimator

The next phase involves training the Hidden Markov Model to learn the system parameters q and q'. The Baum-Welch algorithm [24] iteratively updates the transition probabilities of the HMM to best fit the data available at the receiver.

In this case, we make assumptions that

1. The receiver knows the transmission method the sources are using to access the channel and send the status updates.

2. The Markov model at the transmitter end is known to the estimator. The number of sources, the alphabet of the two sources, and the condition that the state of the sources either remains the same or increments or decrements by exactly one.

We initially start with an arbitrary value of q and q'. As the receiver gets the

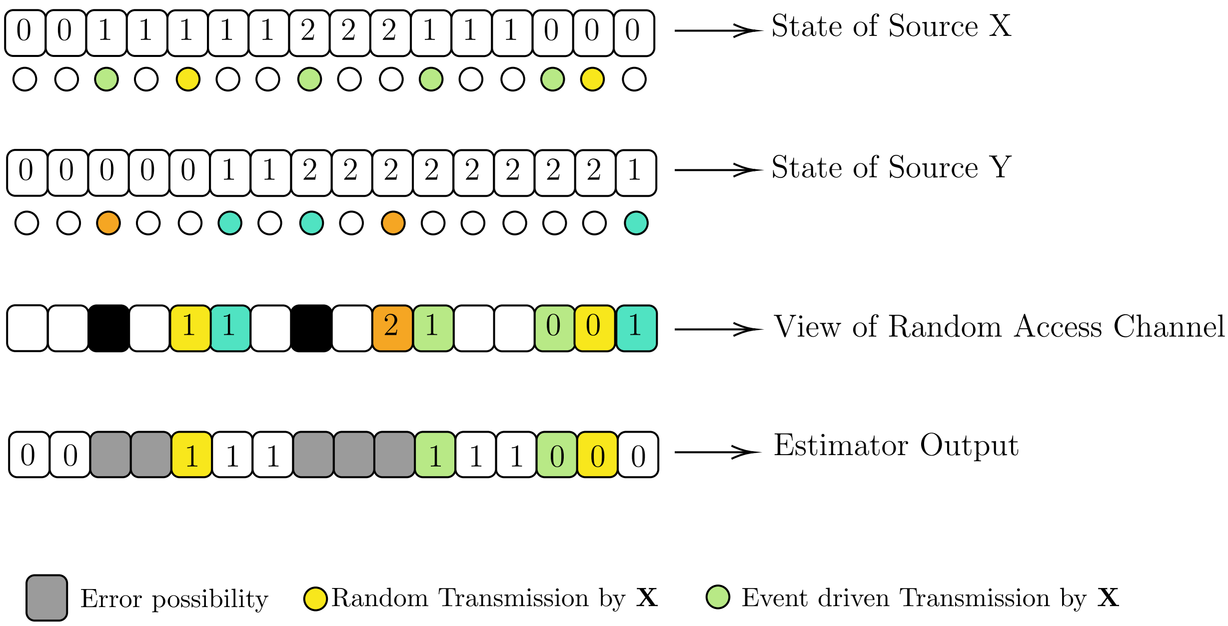

Figure 3.8: Representation of a system using a hybrid transmission method with HMM estimator at the receiver

updates, the HMM reads the values and tunes the system parameters accordingly forming the transition matrix.

We use the below formula to form the transition matrix:

$$a_{ij} = \frac{\text{expected number of transitions from state the state } i \text{ to state } j}{\text{expected number of transitions from state } i} \tag{3.52}$$

This step is iteratively followed to update the transition matrix, the steady state vector, and the observation matrix (dependent on the transmission method).

Chapter 4

Results and Discussions

In this chapter, the proposed research questions at the beginning of the document are answered by evaluating the strategies that were developed in the methodology section.

We analyze the impact the estimator using the Hidden Markov Model has on the status estimation in comparison with the simple update and hold estimation method. We take into account the effectiveness of the status estimation of the correlated sources when they adapt both random and reactive transmission strategies.

All the simulations are carried out in MATLAB. The insights on the behavior of the different transmission strategies are discussed in detail. Note that as our system is designed symmetrically monitoring either of the sources (source **X** or source **Y**) gives the same results.

The experimental setup has two sources transitioning according to the correlation between the two sources. We evaluate the system under various conditions by picking up different q (probability of remaining in the same state), q' (probability factor of moving to a highly correlated state), and p_t (probability of transmission in applicable cases). This allows us to evaluate the system under various correlation conditions which is the base of our study.

During the analysis different evaluation parameters such as Age of Incorrect Information (AOII), average error, and duration of the error are studied. Unless otherwise specified the simulations are conducted for N = 100000 slots.

4.1 Impact of q

4.1.1 Estimation error

We first analyze the impact of q (same state probability) on the evaluation parameters. As we observe in Figures 4.3 and 4.4, we study the estimation error for different values of q through the range of correlation coefficient ρ for a system using a reactive transmission strategy (event-driven). Figures 4.1 and 4.2 show the estimation error when a random transmission strategy is used. We see a significant dip in the estimation error as the value of q increases when the random transmission is used, as opposed to the reactive method where the system parameter q does not show any impact in the estimation error.

As the system is more dynamic in the lower value of q, there is a high chance of state transitions. In the case of the random transmission method, the sources will transition more often but might fail to update the status to the receiver or the update is lost during a collision. While the state at the sources has changed, the estimator is still holding to the previous value thus resulting in an error more often. In case of a higher value of q, the system does not transition frequently. Although the sources might delay or fail to transmit or a possibility of collision is similar to the cases of low q, we see an advantage in higher q in terms of estimation. The sources are less likely to have transitioned and hence the lost update could be the same as the previous value.

When the sources use the reactive method (an event-driven strategy) to transmit in Figures 4.3, 4.4, irrespective of how often the system transitions to a different state, every transition is updated and is a success unless a collision occurs. In case of the lower value of q, if the receiver incorrectly estimates the state of the source, there is a chance the sources are changing frequently and a new update is received shortly. On the contrary, while q is high, due to fewer transitions, the receiver will not frequently receive an update. Hence, in the case of high q, if the receiver estimates an incorrect value (although less frequent), the error persists for longer until an update is received. Therefore, the frequency of transitioning in the low value of q balances the persistence of an error in the estimation in the high value of q nullifying the impact of q during the use of reactive transmission strategy. This trend is clearly shown in Figures 4.3 using the update and hold method and in Figure 4.4 using the HMM estimation method.

Hence it is important to note that q shows a significant impact on the estimation error in the random transmission strategy while this is not observed in the reactive method of transmission.

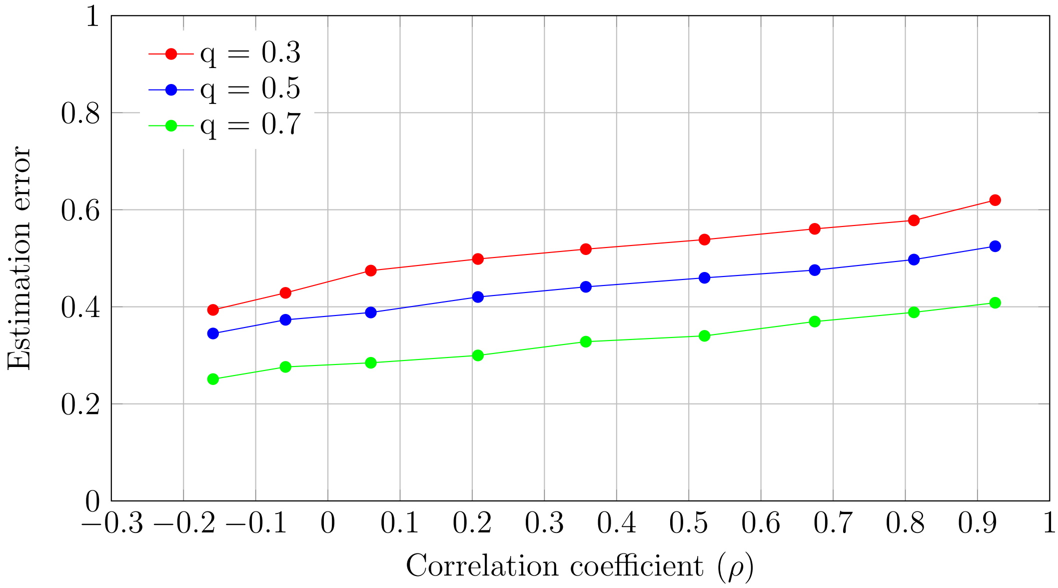

Figure 4.1: Estimation Error of a system using random transmission method for the status updates and a simple estimator at the receiver. q = probability of the system to remain in the same state

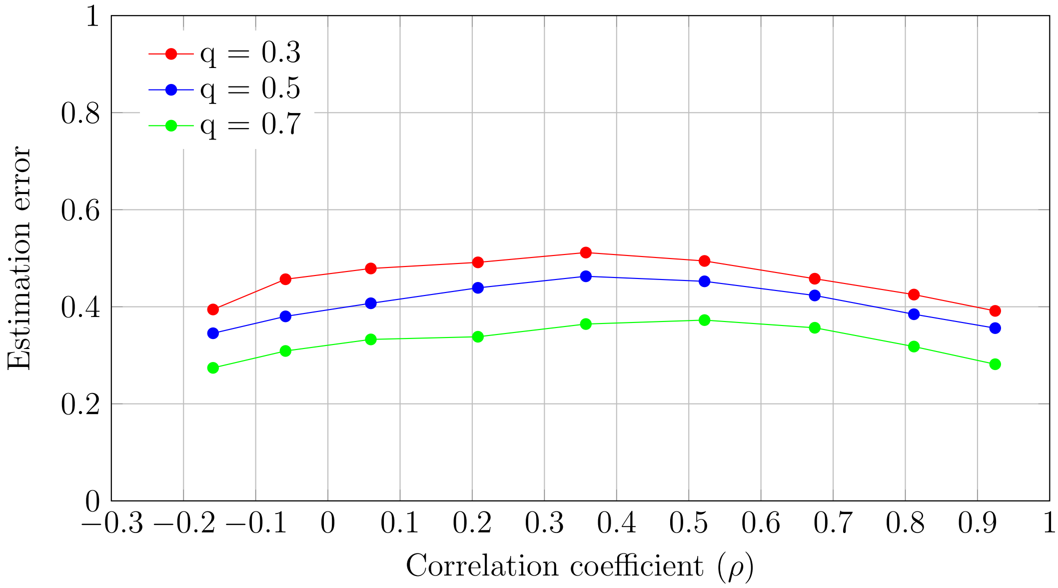

Figure 4.2: Estimation Error of a system using random transmission method for the status updates and a Hidden Markov Model (HMM) estimator at the receiver. q = probability of the system to remain in the same state

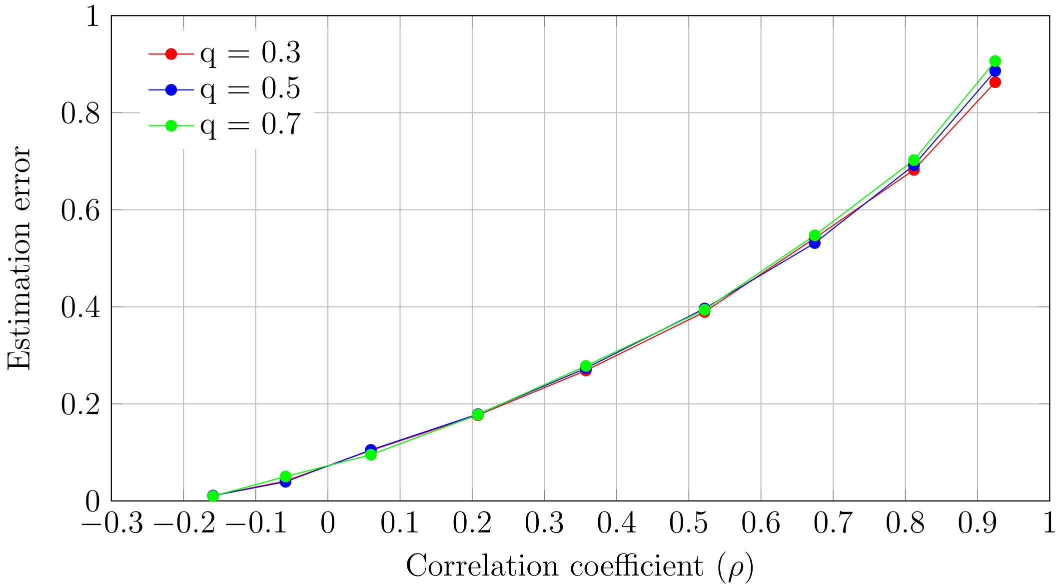

Figure 4.3: Estimation Error of a system using reactive transmission for the status updates and a simple estimator at the receiver. q = probability of the system to remain in the same state

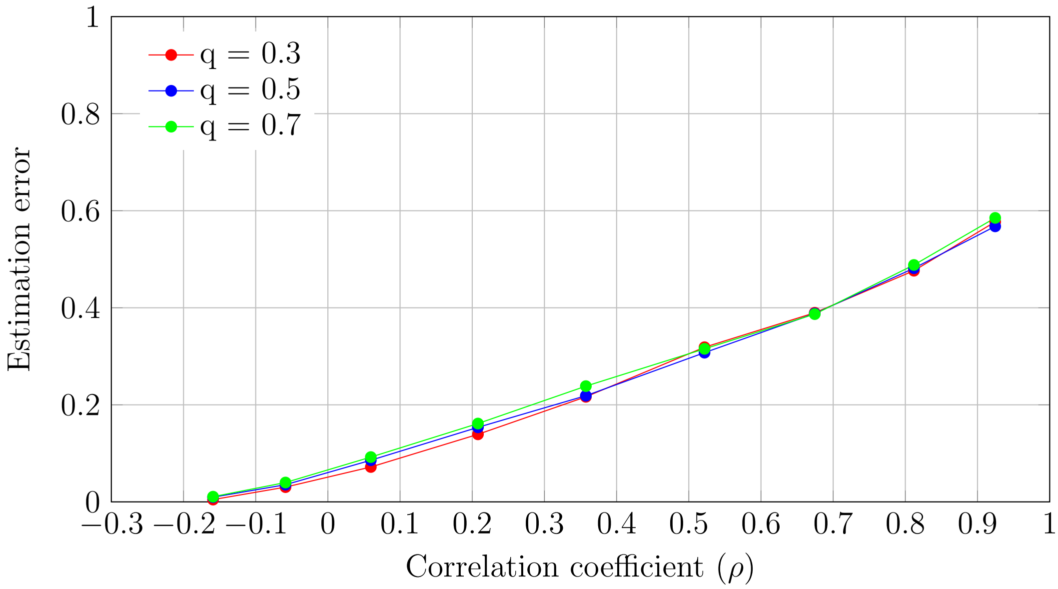

Figure 4.4: Estimation Error of a system using random transmission method for the status updates and a Hidden Markov Model (HMM) estimator at the receiver. q = probability of the system to remain in the same state

4.1.2 AoII

We now analyze the Age of Incorrect Information in the context of q. Figures 4.5 and 4.6 show the AoII for the random method for transmission. Similar to the estimator error, we see the value of AoII is less when the sources do not transition to other states frequently. This trend can be noticed clearly in Figure 4.5.

The trend although noticeable is not as prominent when the HMM estimator is used in Figure 4.6 in comparison with Figure 4.5 showing the ability of the HMM bases MAP estimator to leverage the channel observations. By resetting the AoII frequently (as a result of correct status estimation) the HMM base MAP estimator reduces the long error sequences. By avoiding the long error sequences, the HMM estimator stops the AoII from growing substantially over time and in turn results in lower AoII. Hence, the impact of q is lesser in the case of AoII for random transmission.

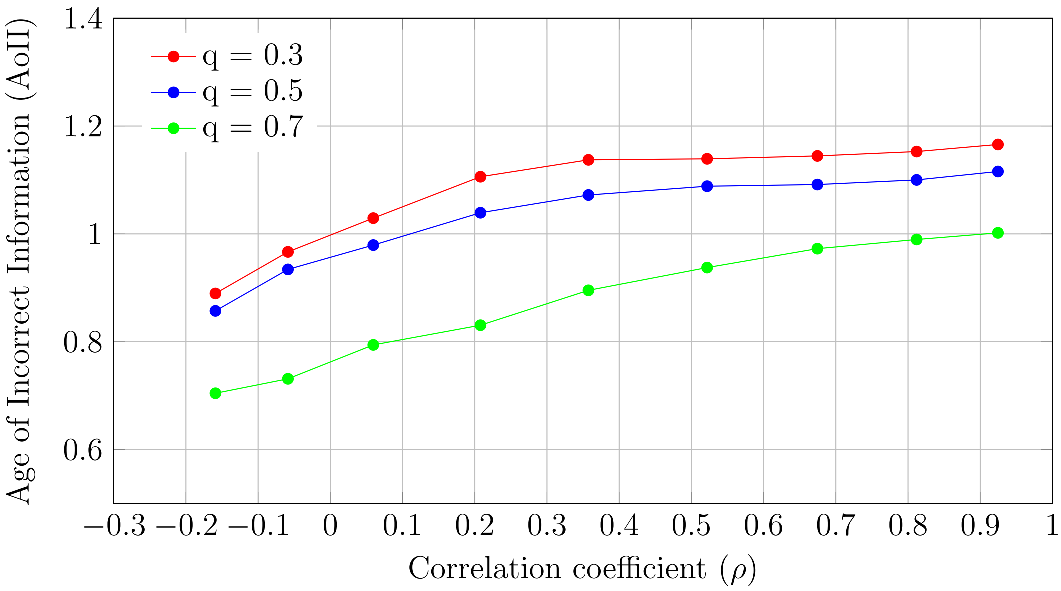

Figure 4.5: Analysis of Age of Incorrect Information (AoII) of a system using random transmission method for the status updates and a simple hold and update estimator at the receiver. q = probability of the system to remain in the same state

Now we study the influence of q on the AoII while the sources use the reactive transmission method. An important observation can be made from Figures 4.7 and 4.8: AoII is higher for a higher value of q. This observation proposes the opposite trend compared to the random transmission method. While the estimation error

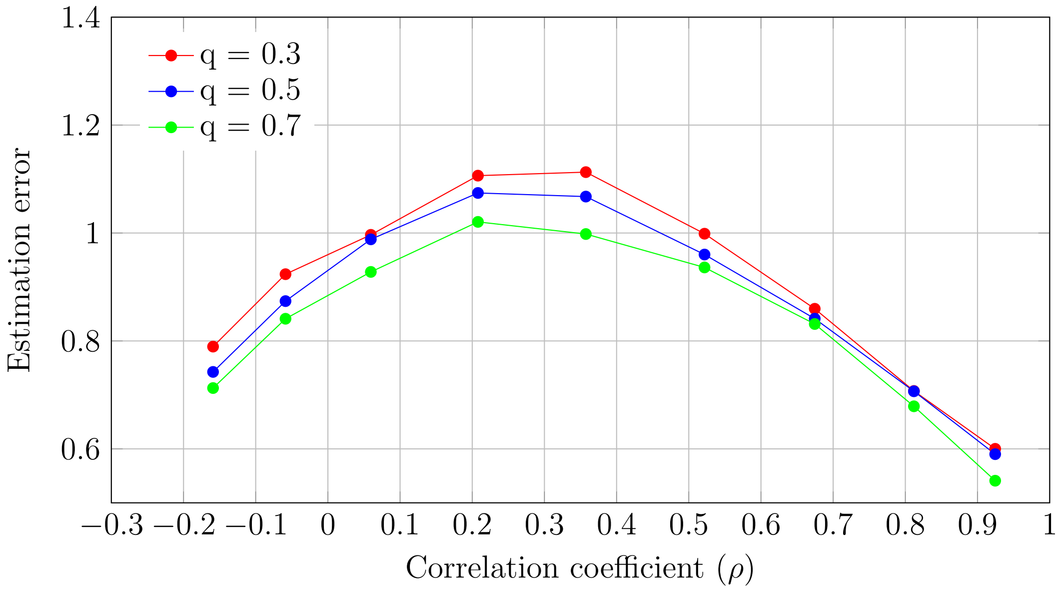

Figure 4.6: Analysis of Age of Incorrect Information (AoII) of a system using random transmission method for the status updates and an HMM estimator at the receiver. q = probability of the system to remain in the same state

is unaffected by the parameter q, the AoII in the reactive method increases as the probability of remaining in the same state increases.

We can explain this behavior by observing that the longer the estimation error stretches without a reset (without a correct estimation), the higher the AoII. The linear increase in age causes the average AoII to grow. As explained earlier, when the value of q is high, the probability of transitioning from a state is less. If there is an estimation error in the system while using an update and hold estimator, the error propagates until a new update is received. This is not frequent in the case of a high value of q. Hence we notice a substantial growth of AoII in higher q (close to 8 slots when $q = 0.7$).

This effect although significantly reduces when the HMM-based MAP estimator is used in Figure 4.8 (close to 1 slot for $q = 0.7$) showcasing the improvement achieved, the trend persists. Infrequent changes of states in the source result in fewer status updates to the receiver, resulting in errors at the estimator continuing for a longer duration and thus higher AoII. With lower values of q, the state changes result in updates which is used to reset the growing AoII, thus giving an advantage in terms of information freshness.

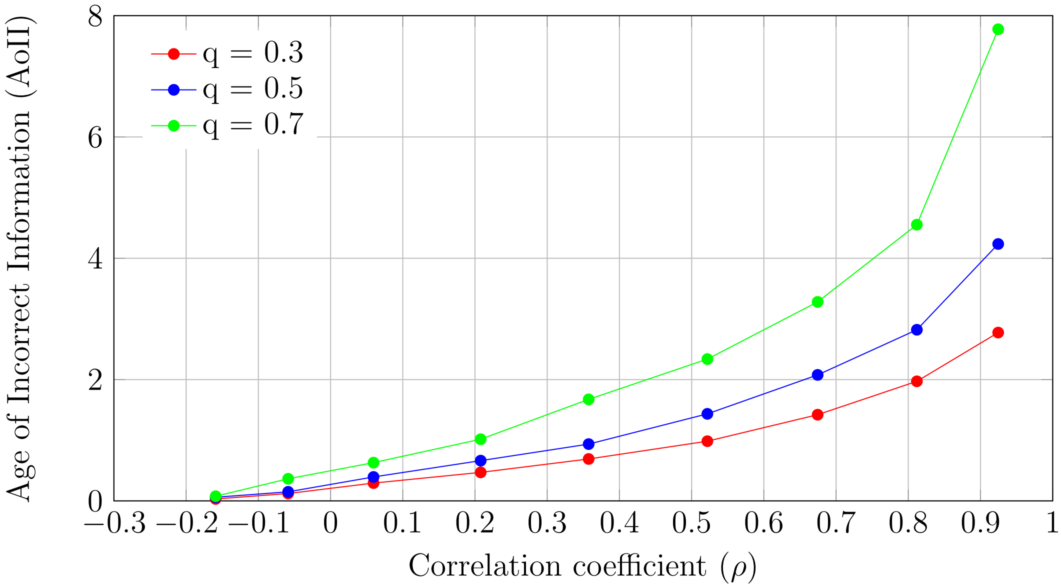

Figure 4.7: Analysis of Age of Incorrect Information (AoII) of a system using reactive transmission method for the status updates and a simple hold and update estimator at the receiver. q = probability of the system to remain in the same state

4.2 Impact of q': Correlation factor

While studying the transmission strategies and estimation methods, blue lines correspond to the reactive transmission approach, and the red lines represent the random transmission method. Solid lines refer to the HMM based MAP estimator and the dashed line shows the results for the hold and update method of estimation. The evaluation parameters (estimation error, AoII, error duration) are plotted on the y-axis against the correlation coefficient against the x-axis. The unit of evaluation parameters are considered in terms of slots.

For the below analysis, we consider a fixed value of $q = 0.5$ and study the behavior of the system for the range of correlation coefficient. The sources are negatively correlated at $q' = 0.05$ with correlation coefficient ρ = -0.2103 to positively correlated at $q' = 0.95$ with correlation coefficient $\rho = 0.998$. Figure 3.2 shows the value of q' corresponding to the correlation coefficient.

The parameter q' is the factor that decides the transition to a highly correlated state in the system. Figure 3.1 shows the transition probabilities.

From Figure 4.9 for a low value of ρ the reactive transmission works the best (blue

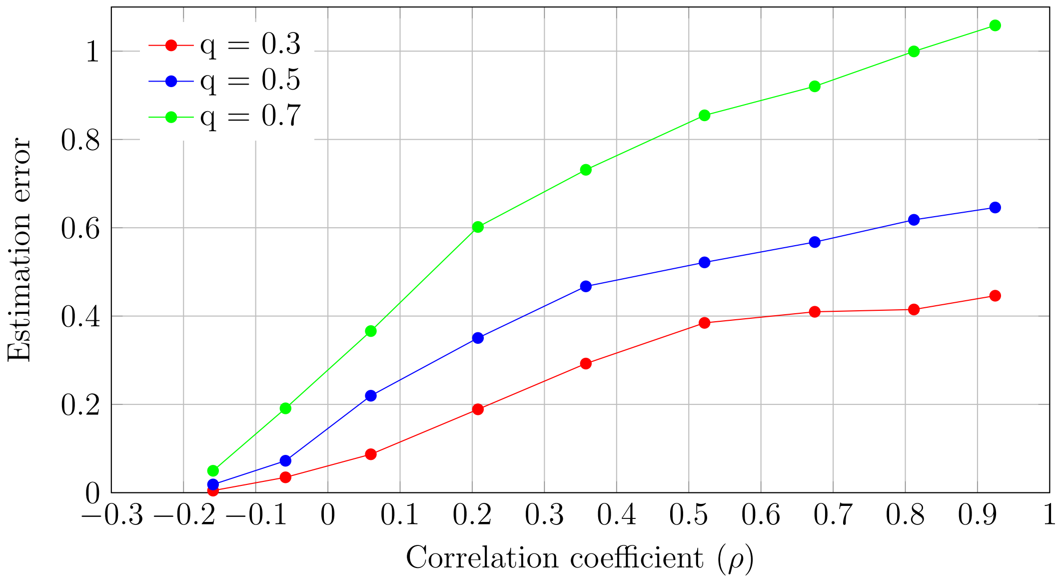

Figure 4.8: Analysis of Age of Incorrect Information (AoII) of a system using reactive transmission method for the status updates and an HMM estimator at the receiver. q = probability of the system to remain in the same state

curves). The probability of transitioning to a highly correlated state is less for a lower ρ value (low q'). Hence the two sources $\mathbf{X}$ and $\mathbf{Y}$ do not change their states simultaneously, avoiding collisions. Therefore most of the status updates are successfully transmitted resulting in low estimation error.

As the correlation coefficient increases, the system is more likely to transition to a highly correlated state, resulting in simultaneous state change by the involved sources. This results in a higher number of collisions and a reduced number of meaningful status updates at the receiver resulting in high estimation error. In such cases, the receiver does not get any update for an extended period of time due to collisions. This is applicable for a system using the reactive method (event-driven) irrespective of the estimation method.

When an HMM-based MAP estimator is used, as it has the ability to observe the channel behavior, it uses the collision information correctly and estimates a state change. This results in the improvement of reduced estimation error at the higher value of ρ, since a large number of collisions occur at higher ρ. The blue curves in Figure 4.9 towards a high ρ show a drastic reduction of around 0.4 slots in the estimation error.

Although the random transmission does not perform as well as the reactive for a low values of ρ, the transmission method performs significantly better for a highly correlated system.

In Figure, 4.9 for a system using the update and hold method for estimation along with the random transmission method (dashed red), there is an increase in the estimation error over the range of ρ. This is due to the transition of the states to a highly correlated state with an increase in ρ, but the inability of the estimator to utilize the correlation information. This is rectified when an HMM-based MAP estimator is used for the status estimation (solid red). Effective use of knowledge of the correlation between the sources can be noted with a reduction in error by 0.23 slots at high ρ

The estimator has the least information about the transmitting sources $\mathbf{X}$ and $\mathbf{Y}$ in the region between the values $\rho = 0$ and $\rho = 0.4$. Hence we do not see a prominent advantage in estimation error while using the HMM estimator over the simple estimator in this region.

In the Figure 4.9 we study a hybrid technique for transmission (black curve). This shows a slight advantage over the random and reactive transmission for a short region of correlation coefficient between $\rho = 0.2$ to $\rho = 0.6$. This gain as we see is not too significant and as we introduce a level of randomness into the reactive transmission, we also bring in some uncertainty which leads to the estimator losing some knowledge about the sources.

Study of AoII

In Figure 4.10 we report the Age of Incorrect Information for the combinations of transmission and estimation strategies. The random transmission follows the same trend as the estimation error. From the tables 4.1 and 4.3 even at $q' = 0.5$ (least correlated region), we can see that there is more success in the case of using the HMM estimator (higher proportion of zero errors). This results in a higher number of resets in the AoII thus reducing the AoII. The advantage of using an HMM is more prominent in AoII for the values between $\rho = 0$ and $\rho = 0.4$ as compared to the estimation error.

The AoII of reactive transmission also follows a trend similar to the random transmission method. In the case of reactive transmission, from the table 4.2 and 4.4 we observe that there is a noticeable portion of errors of magnitude two in highly correlated case ($q' = 0.9$). In the AoII, as the magnitude of error is not considered (which plays a significant role in estimation error), we see a greater advantage of using an HMM estimator over an update and hold estimator. Due to this, we can also observe the cross-over of reactive transmission performing worse than the random transmission occurring at $\rho = 0.82$. Whereas, this was seen at $\rho = 0.68$ in estimation error.

Random transmission with update and hold				
Error magnitude	0	1	2	3
$q' = 0.1$	0.6966	0.2756	0.026	0.0018
$q' = 0.35$	0.6496	0.298	0.0478	0.0045
$q' = 0.5$	0.6167	0.3178	0.058	0.0075
$q' = 0.9$	0.5662	0.3479	0.0755	0.0104

Table 4.1: Proportion of error magnitudes for a system using random transmission method with update and hold estimator

Reactive transmission with update and hold				
Error magnitude	0	1	2	3
$q' = 0.1$	0.9869	0.0127	0.0003	0
$q' = 0.35$	0.8824	0.1036	0.0125	0.0015
$q' = 0.5$	0.6825	0.2464	0.0612	0.0098
$q' = 0.9$	0.3733	0.4062	0.1792	0.0414

Table 4.2: Proportion of error magnitudes for a system using reactive transmission method with an update and hold estimator

Random transmission with HMM				
Error magnitude	0	1	2	3
$q' = 0.1$	0.7174	0.2567	0.0221	0.0038
$q' = 0.35$	0.6633	0.2961	0.0375	0.003
$q' = 0.5$	0.6346	0.3171	0.0481	0.0002
$q' = 0.9$	0.7128	0.2693	0.0179	0

Table 4.3: Proportion of error magnitudes for a system using random transmission method with an HMM estimator

Reactive transmission with HMM				
Error magnitude	0	1	2	3
$q' = 0.1$	0.9967	0	0.0033	0
$q' = 0.35$	0.9474	0	0.0526	0
$q' = 0.5$	0.8426	0	0.1574	0
$q' = 0.9$	0.7147	0	0.2853	0

Table 4.4: Proportion of error magnitudes for a system using reactive transmission method with an HMM estimator

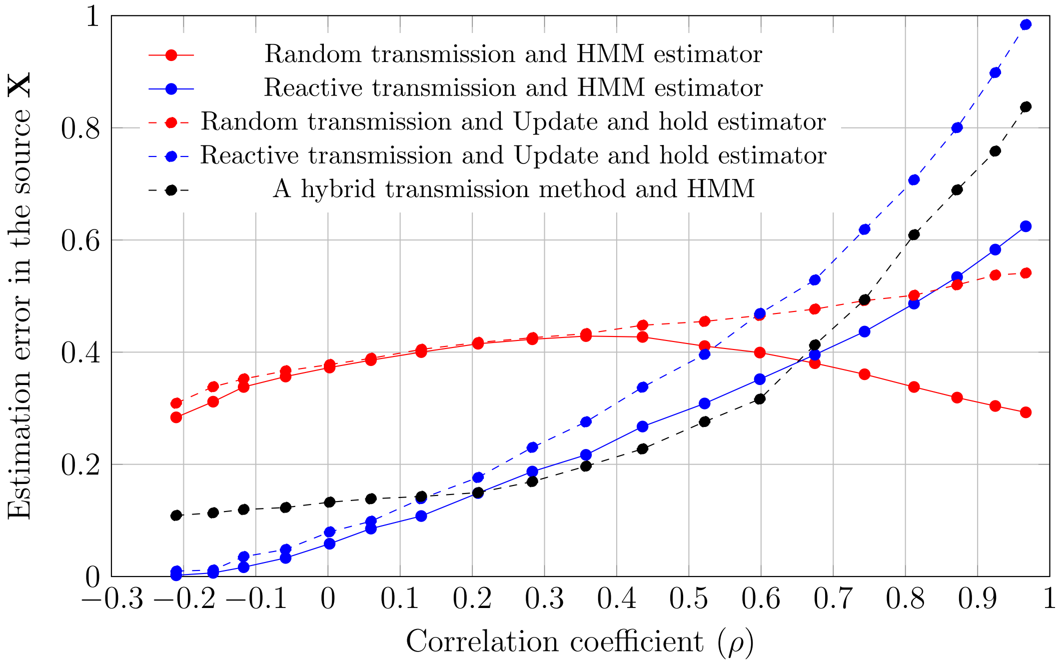

Figure 4.9: Estimation error analysis against correlation coefficient for combinations of reactive and random transmission with HMM and update and hold estimation

Study of Error Duration

The average duration of error plays a role in the evaluation of the estimator. Figure 4.11 outlines the trend for the four cases considered.

We first look into the random transmission method (red curves). Intuitively the highest error duration is noticed at the least correlation coefficient and the error duration is significantly less at the higher correlation between the sources.

In the reactive system, the receiver using the HMM-based MAP estimator shows a significant drop in the error duration at the high correlation region. Although this system consists of a considerably higher proportion of errors with higher magnitude in Figure 4.4, the error duration does not consider magnitude. The system also has a good proportion of zero errors, thus helping to shorten the error duration.

The counter-intuitive result is noted for the system using reactive transmission and Update and Hold estimator. For higher values of the correlation coefficient, the updates are lost in collision hence the error persists for longer. In the case of a low correlation coefficient, although there are fewer error instances, a small number of longer errors cause a significant rise in error duration. Having fewer

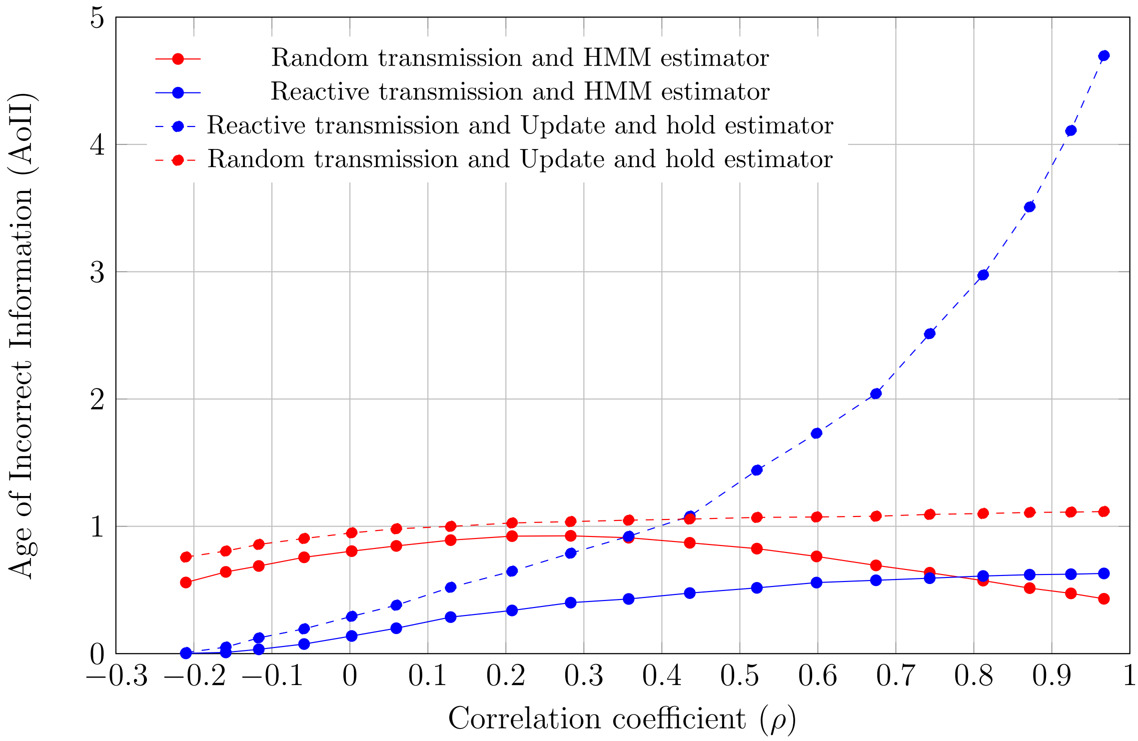

Figure 4.10: Age of incorrect information against correlation coefficient for combinations of reactive and random transmission with HMM and update and hold estimation

error instances does not balance the effect caused by some longer errors. Therefore we see a trend for the reactive updtae and hold method where the error duration is least at the least correlation coefficient.

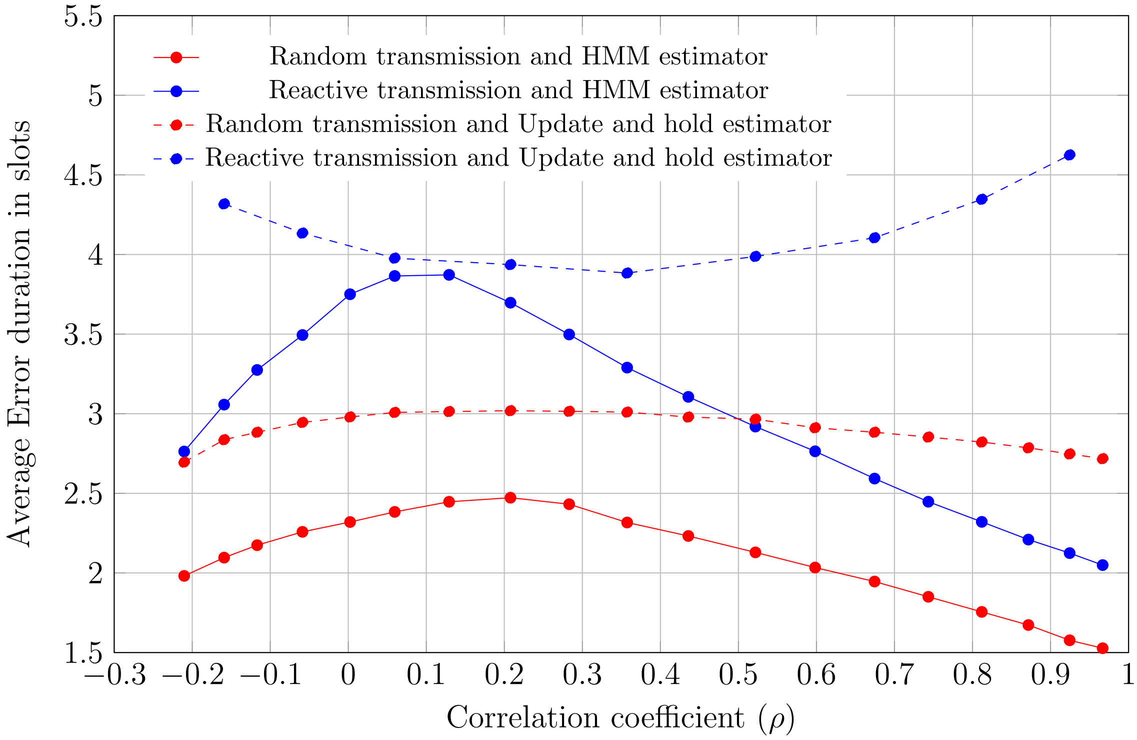

Figure 4.11: Average error duration against correlation coefficient for combinations of reactive and random transmission with HMM and update and hold estimation

Figure 4.12 shows the learning of the parameter q' by the estimator using the data available at the receiver. An iterative method is followed using the Baum-Welch algorithm (3.52) that considers all the observations till the current slot to learn the parameter q'. This method is particularly useful when the estimator does not know the behavior of the sources. We assume an arbitrary initial value for q' at the estimator (0.5) and this value is later learned as the estimator receives updates from the sources. The reference points in Figure 4.12 are the real parameter values at the source. We see the HMM-based MAP estimator observes the evolution of the sources through the channel and learns the true value of q'.

We notice that the reactive transmission is quicker in reaching the desired value as it generally has lesser uncertainty. The learning is slower as the number of slots progresses and it takes longer to reach the exact value as that on the transmission end since the HMM is only reading the channel to gain the information.

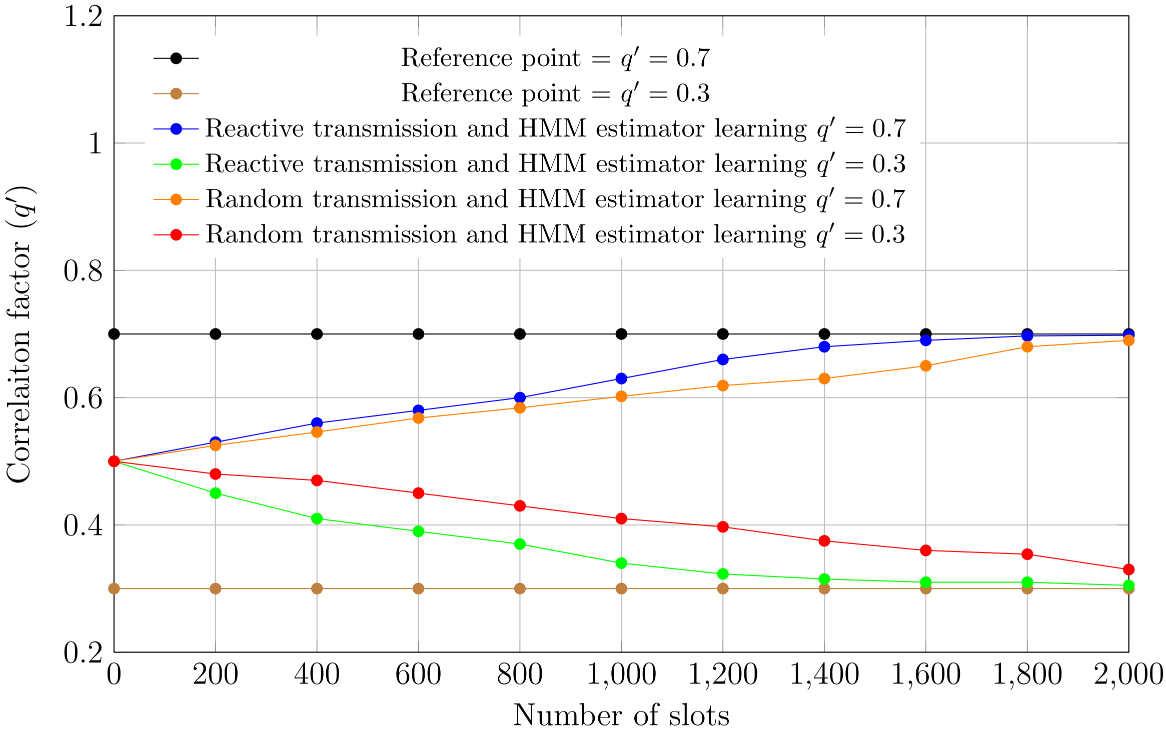

Figure 4.12: System learning the correlation factor q' over a period of time

Chapter 5

Conclusions and Outlook

In this thesis, we have studied the Markov Model of two sources under different correlation coefficients whose goal is to access the random access channel to transmit the status update to the receiver. We consider a two-source Markov model and through transitions to different states, achieve a wide range of correlation coefficients upon which the study is carried out. We report the impact of Random and Reactive transmission (event-driven) methods on the evaluation parameters.

We have extensively investigated an HMM-based MAP estimator against a simple Update and Hold estimator to understand the influence of the correlation coefficient on the status estimation. Through the results, we conclude the MAP estimator performs better in almost all cases of correlation across various evaluation parameters. This however comes with design complexity and increased computational time due to the iterative nature of the estimator.

We have then considered different evaluation parameters: Estimation error, Age of Incorrect Information (AoII), and average error duration. We study the system on the basis of different system parameters (q and q') and conclusively show the improvement achieved by the HMM-based MAP estimator. We also attempt to understand the difference in the status estimation and the system behavior when different transmission techniques are used. We study a hybrid transmission technique to achieve the best of both Random and Reactive methods. We further make an attempt to learn the system parameters (q and q') so as to make the estimator more dynamic.

In conclusion, we can derive that the correlation coefficient has a positive effect on the status estimation while using the Random transmission method. The Hidden Markov Model positively impacts the AoII, estimation error, and error duration in all the considered cases.

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