# A Simplified Method to Approximate the AOI Coverage Duration in Single Acquisition Direction for SAR Satellites with Repeat GroundTrack Orbits 

Allan Bojarski, Markus Bachmann<br>German Aerospace Center (DLR), Microwaves and Radar Institute, Weßling, Germany


#### Abstract

In early stages of SAR mission design the coverage capabilities of a system are either tailored to fulfil the mission objectives or drive their development. Traditionally, this involves using complex software that evaluates the intersections of an Area of Interest (AOI) with the footprint of the SAR instrument throughout the orbit using a grid-point approach. Although this methodology yields precise results, it requires cumbersome software setups and scenario configurations in order to achieve a complete representation of the system's coverage performance. To address this challenge, this paper proposes a simplified methodology based on the geometric relations between the instrument's swath width, the ascending orbital ground-tracks, and the potential positions of an AOI. By adopting this approach, streamlined analysis of the minimum and maximum access possibilities for various AOIs, or different system and orbit configurations becomes feasible. Consequently, an approximate representation of the system's coverage capabilities can be obtained straightforwardly.


## 1 Introduction

At the beginning of every new SAR mission, preliminary mission objectives are identified that further determine the performance requirements of the SAR system or vice versa. One key aspect of the performance in this case is denoted by the coverage capability of the system. In general, it can be stated that the faster mission relevant areas of interest (AOIs) can be acquired, the more data can be generated and therefore mission objectives can be accomplished earlier.
To evaluate the coverage performance of a system, conventional approaches often employ the grid-point method [1], which is applied by generating graphical propagation scenarios using a uniformly sampled, equidistant grid. The achievable coverage and corresponding duration for any AOI are then determined through orbit simulations, where intersections between the system's footprint, i.e. the swath, and AOI grid points are evaluated over the simulation time. Prominent software utilizing the grid-point technique are for example the Systems Toolkit (STK) [2] or the FreeFlyer Astrodynamics Software [3]. While this methodology yields highly accurate and precise outcomes, it suffers from cumbersome scenario configurations and analysis, making simulations for a wide range of AOIs or different SAR systems time-consuming.
Therefore, this paper introduces a simplified approach dedicated to determine the coverage duration based solely on a set of basic parameters. The methodology in this case is presented only for single direction acquisitions from either ascending or descending orbits. A description of all acquisition direction possibilities is out of scope of this paper. The basic parameters encompass the orbit characteristics, such as the number of days per repeat cycle (RC), the number of completed orbits per repeat cycle $n_{\text {orbits,RC }}$, and the orbit's inclination along with the effective swath width of
the SAR system. The selection of the orbit parameters relies on fundamental considerations of orbit geometry for repeat ground-track orbits, while the swath width, as a design parameter, is derived from early SAR instrument assumptions.
To support the simplification of the approach, the following assumptions are made for parameters which only contribute to minor improvements or for which an explanation is out of scope of this paper: The Earth's rotation is neglected, the orbit inclination equals the swath inclination, applicable orbit inclinations regard near-polar inclinations as used for Earth observation satellites and the change in inclination with respect to the meridian is neglected.
The calculation of the required acquisitions for each Area of Interest (AOI) is then conducted by analyzing the geometric relationships between the swath width and the AOI's geographical dimensions in latitude and longitude direction. Additionally, this approach takes into account the impact of overlapping access ranges (ARs) towards the poles and the time required to access the orbits from which a coverage of the AOI is achieved. By considering these factors, the final coverage duration for a given AOI is estimated by means of minimum and maximum possible time required for a complete coverage.
In this way, we present an efficient solution for analyzing diverse orbit and system parameters, along with varying Area of Interest (AOI) ranges and locations. Although the results may not achieve the same level of precision as conventional methods, they rely on only a few fundamental parameters providing an effortless and suitable assessment of the coverage capabilities for the analyzed configuration.

## 2 Number of Required Acquisitions

For the computation of the coverage duration, first the number of necessary acquisitions $n_{\text {acq }}$ to entirely cover the

AOI has to be determined. The AOI in this case is assumed to be of reactangular shape on ground with a height $h_{\text {AOI }}$ in latitude direction and a length $l_{\mathrm{AOI}}$ in longitude direction. Based on these measures as well as the inclination delta $\Delta i=i-90^{\circ}$ of the orbit and the effective swath width $r_{\text {swath,eff }}$ of the intrument, both displayed in Figure 1, we can derive the number of required acquisitions. For the following derivations, in a first step we assume a best-case orientation between the AOI and the swath in which the near-range side of the swath intersects with the westward lower AOI edge. Such a configuration is shown in the top part of Figure 1 which displays the coverage of an AOI performed by a minimum number of three ascending Stripmap acquisitions i.e. three ascending Stripmap beams. Due to the inclined orbit the beams are angled against the AOI thus the effective coverage differs from the effective swath width. Instead we can define various parameters $x_{1}$, $x_{2}$ and $x_{3}$ which represent the intersections of one beam with the AOI and which are displayed in Figure 1.


Figure 1: AOI coverage with four ascending beams and effective coverage parameters $x_{1}-x_{3}$ for best-case configuration (top) and worst-case configuration (bottom).
According to the geometric relations in the top part of Figure 1, $x_{1}-x_{3}$ can be expressed as:

$$
\begin{align*}
& x_{1}=\frac{r_{\text {swath }, \text { eff }}}{\cos \Delta i}  \tag{1}\\
& x_{3}=h_{\text {AOI }} \cdot \tan \Delta i \text { for } h_{\text {AOI }}<\frac{r_{\text {swath,eff }}}{\sin \Delta i}  \tag{2}\\
& x_{2}=x_{1}-x_{3} . \tag{3}
\end{align*}
$$

From Figure 1 we can derive that the first part of the AOI (orange) is effectively covered by $x_{2}$ while the remaining part is effectively covered by multiples of $x_{1}$. This means we can obtain the number of required acquisitions $n_{\text {acq }}$ from:

$$
\begin{equation*}
n_{\mathrm{acq}}=\left\lceil\frac{l_{\mathrm{AOI}}-x_{2}}{x_{1}}+1\right\rceil . \tag{4}
\end{equation*}
$$

In order to determine the number of necessary acquisitions for a worst-case orientation of the swath and the AOI, we consider the far-range side of the first beam intersecting with the eastward upper edge of the AOI as shown in the bottom part of Figure 1. In this case, the first beam covers a minimum part $x_{3}$ of the longitude dimension of the AOI while still covering the entire extent in latitude direction thus maximizing $n_{\text {acq. }}$. However, the first part does not cover an entire rectangle of the AOI in latitude and longitude. Therefore, derived from (4) we can express the worstcase number of beams to:

$$
\begin{equation*}
n_{\mathrm{acq}, \max }=\left\lceil\frac{l_{\mathrm{AOI}}}{x_{1}}+1\right\rceil . \tag{5}
\end{equation*}
$$

Generally, the worst-case only exceeds the best-case scenario by one beam. In rare cases both worst- and best-case result in an equal number of beams.

## 3 Determination of AOI Access Possibilities

### 3.1 Overlapping Access Ranges

Sun-synchronous repeat ground-track orbits are the most commonly used ones for Earth observation with SAR. These orbits offer several advantages, including constant exposure of solar panels to sunlight, stable thermal environmental conditions and global coverage capability [4]. Another significant benefit of these repeating groundtracks is the consistent viewing geometry in each repeat cycle, making them ideal for applications like repeat-pass interferometry [5].
Additionally, the distance between the orbital groundtracks is largest at the equator and gradually decreases towards the poles [6]. As a result, higher latitudes gain increased accessibility to an AOI. In order to quantify the accessibility at different latitudes we analyse the ranges that are accessible from each orbit. For a gapless coverage these access ranges (ARs) need to cover at least the distance between the satellite ground-tracks on the equator, also referred to as the minimum interval. The extent of the ARs depends on the incidence angle range of the SAR system and remains constant over the latitude $\Lambda$. Hence, the ARs between adjacent orbits begin to overlap with absolute increasing latitude as shown in Figure 2. This overlap can be expressed as overlap factor $f_{\text {ovl }}$ :

$$
\begin{equation*}
f_{\mathrm{ovl}}=1-\cos \Lambda \tag{6}
\end{equation*}
$$

By using the overlap factor, we can determine the size of the overlaps of the adjacent orbits. In Figure 2, we select a central orbit $(j=0)$, represented by the orange central access range $\left(\mathrm{AR}_{0}\right)$ and analyse the overlap range in both positive $(+j)$ and negative $(-j)$ longitude directions. With $r_{\mathrm{AR}}$ as the extent of the AR, which is identical for each orbit, we can then write for the overlap range $r_{\text {ov }, j}$ :

$$
\begin{equation*}
r_{\mathrm{ovl}, j}=r_{\mathrm{AR}} \cdot\left(|j| \cdot f_{\mathrm{ovl}}-(|j|-1)\right) \tag{7}
\end{equation*}
$$

$$
\text { for } j= \pm 0,1,2, \ldots, n_{\text {orbits,RC. }}
$$

According to this equation the overlap range $r_{\text {ovl } 1, j}$ can result to a negative value. This is intended as an AOI is potentially accessible from adjacent ARs that are not yet overlapping with $\mathrm{AR}_{0}$. In order to cover this case we accept negative values for $r_{\mathrm{ovl}, j}$. How these negative values are used will be further explained in section 3.3.


Figure 2: Overlapping ARs and decreasing distance between ground-tracks towards the north pole, shown for three orbits.

### 3.2 AOI Positioning

The first step is now to determine the accessibility of an AOI in combination with the overlapping ARs determined in the preceding chapter. Due to the overlapping ARs and the extent of the AOI in longitude direction, an AOI might be located in such a way that it becomes accessible from a maximum number of orbits. This means that these positions provide the highest number of access possibilities thus the fastest coverage duration for the given latitude. We therefore analyse the potential positions of the AOI along the AR of a central orbit with respect to the ARs of the adjacent orbits. For this purpose, Figure 3 shows a scenario with a central orbit AR ( $\mathrm{AR}_{0}$ in orange) and two adjacent overlapping $A R s$ ( $\mathrm{AR}_{-1}$ in blue and $\mathrm{AR}_{+1}$ in green).


Figure 3: Scenario showing a central AR (orange) and the two adjacent ARs (blue and green) that overlap with the central AR.

In this scenario it can be seen that an AOI can potentially be accessed from two ARs (blue and orange or orange and green) at the same time. By defining a coordinate system, we determine the exact positions for which this becomes true for a given AOI. This is visualized in Figure 4 with the locations $r_{21, \min }$ as the western limit for a potential AOI location which grants access from $\mathrm{AR}_{-1}$ and $\mathrm{AR}_{0}$ and $r_{21, \max }$ as the according eastern limit. The same is shown for $r_{22, \min }$ and $r_{22, \max }$ as the respective limits for a simultaneous AOI access from $A R_{0}$ and $A R_{+1}$. All four are indicated by the dashed lines. The first index of these parameters specifies the maximum number of orbits with an access possibility
and the second index specifies the position from left to right i.e. from negative to positive longitude direction.


Figure 4: Position limits that determine the range for an AOI access from two orbits in a scenario with a central AR (orange) and two adjacent overlapping ARs (blue and green).

Now if the AOI is located between the min and max positions of each index pair, e.g. between $r_{21, \min }$ and $r_{21, \max }$, enables an access from two orbits. Independent from the longitude extent $l_{\mathrm{AOI}}$ this means, that within this range the AOI can be accessed at least with one beam from one AR and with the remaining beams from its adjacent AR. Ideally the number of acquisitions is shared equally between both ARs. As an additional note, if the number of acquisitions is odd, it is assumed that the greater fraction of the AOI is covered by the central AR as this way the fastest coverage is achieved.
From Figure 4 it becomes furthermore clear that for the quantification of the parameters $r_{21, \text { min }}$ to $r_{22, \max }$ obviously the extent of the AR and the swath width but also the overlap $r_{\text {ovl }, j}$ as well as the AOI length $l_{\mathrm{AOI}}$ have to be considered. If the overlap becomes bigger $r_{21, \max }$ is shifted further in the positive direction i.e. eastward and $r_{22, \min }$ is shifted further westward. For an increasing $l_{\mathrm{AOI}} r_{21, \max }$ but also $r_{22, \max }$ are both shifted eastwards. From this point of view it could be concluded that $r_{21, \min }$ remains in a fixed position. However, the position of $r_{21, \min }$ depends on the relation between the swath width and the length of the AOI. In the above example in Figure 4 it specifies the position where maximum one acquisition can be performed from $A R_{0}$ while the remaining ones have to be performed from $A R_{-1}$. Moving the AOI further west would mean even though the AOI is still accessible from $\mathrm{AR}_{0}$, an acquisition from the according orbit does not contribute to reduce the number of beams necessary to entirely cover the AOI. So, in other words it is the minimum part of an AOI that needs to be accessible by the AR of one orbit so that it contributes to the reduction of $n_{\text {acq }}$ that otherwise need to be performed only from the adjacent orbit. As the identical distance as $r_{21, \text { min }}$ can be found between all $r_{j p, \min }$ and the western limit of the according orbital ARs, the parameter is generalized and referred to as $r_{\text {min }}$ in the following. With $l_{\mathrm{AOI}}$ and the swath width $r_{\text {swath,eff }}, r_{\text {min }}$ can be expressed as:

$$
\begin{equation*}
r_{\min }=l_{\mathrm{AOI}}-\left(\left[\frac{l_{\mathrm{AOI}}}{r_{\mathrm{swath}, \mathrm{eff}}}\right\rceil-1\right) \cdot r_{\mathrm{swath}, \mathrm{eff}} \tag{8}
\end{equation*}
$$

The effective swath width in this case is approximated by using the value as assumed in the design process since an accurate description is out of scope of this paper. Based on $r_{\text {min }}$ and the overlap obtained from (7) we can now determine the remaining $\mathrm{min} / \mathrm{max}$ parameters for any number of overlapping ARs and numbers of orbits with AOI access. They can be expressed as:

$$
\begin{equation*}
r_{j p, \min }=r_{\mathrm{AR}}+r_{\min }-r_{\mathrm{ovl}, p-1} \tag{9}
\end{equation*}
$$

$$
\begin{align*}
& r_{j p, \max }=l_{\mathrm{AOI}}-r_{\min }+r_{\mathrm{ovl}, j-p}  \tag{10}\\
& \text { for } p=1,2, \ldots, n_{\mathrm{orbits}, \text { max }}
\end{align*}
$$

The parameter $n_{\text {orbits,max }}$ represents the maximum number of orbits with a potential AOI access. It is obtained by iterating the overlap range $r_{\text {ovl }, j}$ over the possible number of orbits and evaluating it against $r_{\text {min }}$, the effective swath width and the length of the AOI. The parameter is explained in the following section in more detail as it is a pivotal element in the approximation of the coverage duration.

### 3.3 Determining the Number of Orbits with AOI Access

For the final determination of the coverage duration, the key outcome of the preceding considerations is the maximum number of orbits with a potential AOI access $n_{\text {orbits,max. }}$. Its precise determination can be achieved by analysing the access possibilities at latitudes at which the second adjacent $A R$ begins to overlap with $\mathrm{AR}_{0}$. In Figure 5 this is shown for $\mathrm{AR}_{-2}$ (yellow) and $\mathrm{AR}_{0}$ (orange) at a latitude of $\Lambda=54^{\circ}$.


Figure 5: AOI between $\mathrm{AR}_{-2}$ and $\mathrm{AR}_{0}$ at $\Lambda=54^{\circ}$
As can be seen, both ARs are getting close to overlapping with each other. Depending on $l_{\mathrm{AOI}}$, if the distance between $A R_{-2}$ and $A R_{0}$ is small enough, i.e. the overlap is large enough, the AOI then also becomes accessible from AR-2 and $n_{\text {orbits,max }}$ increases by one. This is of course also valid for an AOI position between $\mathrm{AR}_{-1}$ and $\mathrm{AR}_{+1}$ or $\mathrm{AR}_{0}$ and $A R_{+2}$ due to the symmetry of the overlap on both sides of $A R_{0}$. The overlap on the right side of $\mathrm{AR}_{0}$ is not displayed in Figure 5. Considering the length of the AOI $l_{\mathrm{AOI}}$ and employing the geometric considerations in Figure 5, we establish a criterion for the overlap obtained from (7) at which an increase of $n_{\text {orbits, max }}$ occurs:


Figure 6: Iteration of (11) to obtain $n_{\text {orbits,max }}$

$$
\begin{equation*}
r_{\mathrm{ovl}, j} \geq-\left(l_{\mathrm{AOI}}-r_{\mathrm{swath}, \mathrm{eff}}-r_{\min }\right) \tag{11}
\end{equation*}
$$

Iterating over the adjacent orbits $j$ and evaluating the according overlaps $r_{\text {ovlj,j }}$ then yields the correct $n_{\text {orbits,max }}$ at the given latitude. If the criterion fails, the iteration is stopped and $n_{\text {orbits,max }}$ is eventually obtained. This process is illustrated in Figure 6.
For the remaining positions along $\mathrm{AR}_{0}$ for which $n_{\text {orbits,max }}$ cannot be achieved, we define a minimum number of orbits with AOI access $n_{\text {orbits,min }}$ according to:

$$
\begin{equation*}
n_{\text {orbits, } \min }=n_{\text {orbits, } \max }-1 \tag{12}
\end{equation*}
$$

If for example $n_{\text {orbits, } \text { max }}=2$ then $n_{\text {orbits,min }}$ which has access to the entire AOI results to 1 .
By performing this iteration for all overlaps over the entire latitude range we can visualize the access possibilities for a certain AOI and orbit configuration on a global scale. Figure 7 shows the result for $n_{\text {orbits,max }}$ as a global heatmap for an AOI with $l_{\text {AOI }}=75 \mathrm{~km}$ and the TerraSAR-X orbit as well as the TerraSAR-X swath width for a Stripmap beam $r_{\text {swath,eff }}=24 \mathrm{~km}$ [7]. As expected the results are symmetrical to the equator, due to the cosine function in (6), and increase rapidly starting from $\Lambda>53^{\circ}$. Due to the inclination, the satellite's ground-track is limited to a certain latitude range depending on the inclination difference towards $90^{\circ}$. Therefore, the computation does not adapt the full global latitude spectrum which explains the white fringes that appear at the top and bottom of the global map.


Figure 7: Global distribution of $n_{\text {orbits, } \text {, }}$ ax for $l_{\mathrm{AOI}}=75 \mathrm{~km}$, seen from the TerraSAR-X orbit.

## 4 Approximating the Coverage Duration

In the preceding sections the number of required acquisitions $n_{\text {acq }}$ as well as the maximum and minimum number of ARs with AOI access $n_{\text {orbits,max }} / n_{\text {orbits, min }}$ have been quantified. Now the next step to estimate the coverage duration is to determine the duration $d$ to reach the orbits within $n_{\text {orbits,max }} / n_{\text {orbits, }}$ min.
This is achieved by computing the repeating ground-track pattern of the ascending orbits within two consecutive equatorial crossings of the ground track, also called the fundamental interval $S_{\mathrm{Q}}$ which can be seen in Figure 8. As described in [8] this pattern, also referred to as sub-cycles, is obtained from the orbit parameters themselves. This includes the number of days $D$ within a full repeat cycle and the integer number $K$ that corresponds to the fractional part of revolutions within one day. In mathematical terms this
is expressed as $Q=I+K / D$ with $Q$ being the revolutions per day and $I$ being the integer part of $Q$. According to [8] the number of days $d$ to reach the adjacent orbits to a central orbit $(d=0)$ are obtained from:

$$
\begin{equation*}
d=\frac{k+m \cdot D}{K} . \tag{13}
\end{equation*}
$$

The number $k$ represents the number of the adjacent orbits to be analysed, which is also displayed in Figure 8, and $m$ being an additional parameter to provide a single unique solution to the above equation for each $k$. In addition, $d, m$ and $k$ are integer numbers with $k \neq 0$.
In order to illustrate the sub-cycles, we again utilize the TerraSAR-X orbit which was already taken as an example in the preceding sections. With the orbital parameters $D=11 \mathrm{~d}, K=2$ and $I=15$ the resulting sub-cycle pattern is depicted in Figure 8. The pattern is symmetrical to the centre orbit which appears at 0 and $D$ days. The figure also shows the fundamental interval $\mathrm{S}_{\mathrm{Q}}$ which for TerraSAR-X results to 2640 km .


Figure 8: Sub-cycle pattern of ascending orbital ground-tracks for TerraSAR-X.

Since we cannot specify if an AOI is placed left, middle or right to the center orbit, we analyze $d$ for all possible orbit combinations that can be generated with $n_{\text {orbits,max }}$. Given for example the parameters $n_{\text {orbits, } \max }=3$ and $n_{\text {acq }}=2$ to cover an AOI, the resulting combinations of $d$ are $[10,5,0] \mathrm{d},[5,0,6] \mathrm{d}$ and $[0,6,1] \mathrm{d}$ which is illustrated in Figure 9 in blue, orange and green.


Figure 9: orbit combinations for $n_{\text {orbits, max }}=3$ shown for the subcycle pattern of ascending ground-tracks for TerraSAR-X.

Assuming the acquisitions $n_{\text {acq }}$ are equally distributed between $n_{\text {orbits, max }}$ we can obtain the coverage duration by choosing the $n_{\text {acq }}$-smallest value of $d$ for each of the different combinations and then averaging the results. Given for example an AOI that requires $n_{\text {acq }}=2$ acquisitions to be completely covered, this means we take the second smallest value from each combination. This results to $d\left(n_{\mathrm{acq}}=2\right)=[5,5,1]$ for left, middle and right positioning and thus an average coverage duration of $t_{\mathrm{cov}}=3.67 \mathrm{~d}$. The according acquisition combinations in terms of $d$ then result to $[0,5] \mathrm{d},[0,5] \mathrm{d}$ and $[0,1] \mathrm{d}$.
In case $n_{\text {acq }}$ is higher than $n_{\text {orbits, max }}$, the total number of required repeat cycles $n_{\text {RC }}$ to complete the AOI coverage needs to be added to the previous calculation. In summary
the coverage duration estimation $t_{\text {cov }}$ can then be generalized to:

$$
\begin{align*}
& t_{\mathrm{cov}}=\mu\left(D \cdot\left(n_{\mathrm{RC}}-1\right)+d\left(n_{\text {acq }}\right)\right)  \tag{14}\\
& \text { with } n_{\mathrm{RC}}=\left\lceil\frac{n_{\mathrm{acq}}}{n_{\text {orbits }}}\right\rceil . \tag{15}
\end{align*}
$$

Given for example $n_{\text {acq }}=5$ means we again take the second smallest value in the above combinations (due to $n_{\text {acq }} \% n_{\text {obits,max }}=2$ ). But this time we have to add $n_{\mathrm{RC}}=2$ to (14) which results in an average coverage duration of $t_{\text {cov }}=14 \mathrm{~d}$. As an additional note, in (14) and (15) $n_{\text {acq }}$ and $n_{\text {orbits }}$ represent a placeholder for either the maximum or the minimum number of orbits $n_{\text {orbits,max }} / n_{\text {orbits,min }}$ and the worstor best-case number of required acquisitions $n_{\text {acq }} / n_{\text {acq.max. }}$. In this way the minimum as well as the maximum coverage duration can be obtained from both equations.
By applying the above equations to AOIs with various lengths and computing the result for $n_{\text {orbits, max }}$ and for each $l_{\mathrm{AOI}}$ over the entire latitude range, we can visualize the effects of both parameters on the coverage duration. Figure 10 therefore shows the estimation results for $l_{\mathrm{AOI}}=[40,200] \mathrm{km}$.


Figure 10: Coverage duration $t_{\text {cov }}$ estimated for $n_{\text {orbits,max }}$ and $l_{\mathrm{AOI}}=[40,200] \mathrm{km}$ for all latitudes.

In order to validate the approximation, we compare the simulation outcome for a Stripmap acquisition for a certain AOI to the according planning results from the Ter-raSAR-X mission (TSM). We therefore choose an AOI with $l_{\mathrm{AOI}}=100 \mathrm{~km}$ and $h_{\mathrm{AOI}}=45 \mathrm{~km}$. For the swath width with $r_{\text {swatheeff }}=24 \mathrm{~km}$ as well as for the orbit characteristics, the parameters from the TSM are used. The scenario with the TSM data is depicted in Figure 11 for a best-case (top) and a worst-case (bottom) at a latitude of $\Lambda=48.3^{\circ}$ in the region of Bavaria.
We can see that the actual mission uses four Stripmap beams in both cases (best-case: beams $7-10$, worst-case: beams $5-8$ ). The detailed results of the comparison are listed in Table 1. For the approximation these are given in a range according to the $\mathrm{min} / \mathrm{max}$ values of the number of required acquisitions, described in (4) and (5), and the difference between $n_{\text {orbits, max }}$ and $n_{\text {orbits, min }}$ generating different outcomes in (14) and (15). It shows that the approximation corresponds very well to the acquisition planning outcome. The difference in the best-case of 0.5 d for the fastest
achievable coverage results from the averaging of the coverage duration for different AOI positions in (14). The average in this case is computed from 16 d for orbits $j=0$ and $j=-1$ and 17 d for orbits $j=0$ and $j=+1$. As the acquisition planning ground-track is also labeled +1 , the methodology proves to correctly estimate the coverage duration. The parameters in the best-case appear to be optimal for a fast coverage in this latitude which is accurately represented by the approximation. Additionally, the slowest coverage estimation of 33 d in the worst-case is obtained from only one orbit $(j=0)$ with four required acquisitions. Since for one orbit no averaging is necessary and the number of required beams also results to four in the approximation according to (5), the outcome for the approximation and the acquisition planning data is identical.


Figure 11: scenario with $l_{\mathrm{AOI}}=100 \mathrm{~km}$ and $h_{\mathrm{AOI}}=45 \mathrm{~km}$ at $\Lambda=48.3^{\circ}$ for best-case (top) and worst-case (bottom).

Table 1: Comparison between approximation and TSM planning results

|  | Approximation | TSM |
| :--- | :--- | :--- |
| $n_{\text {acq }}$ | 4 | 4 |
| $n_{\text {orbits, } \text { max }}$ | 2 | 2 |
| $n_{\text {orbits,min }}$ | 1 | 1 |
| $t_{\text {cov }}$ | $16.5-33 \mathrm{~d}$ | $17 \mathrm{~d}-33 \mathrm{~d}$ |
| Sub-cycle pattern for <br> $n_{\text {orbits, } \text { max }}$ | $[5,0,6] \mathrm{d}$ | $[5,0,6] \mathrm{d}$ |

## 5 Conclusion

Commonly the coverage performance of a SAR system is evaluated using cumbersome simulations to find the intersections between the system's footprint and the AOI gridpoints. By introducing a simplified approach that approximates the coverage duration based on a geometric analysis, the effort is significantly reduced. This enables a
streamlined comparison of the coverage performance between various AOI sizes as well as between different system and orbit configurations.
For this purpose, first the number of acquisitions to cover a given AOI is determined by evaluating the geometry of the footprint and the AOI. Since different alignments between the swath and the AOI are possible, this number is derived for a best- and worst-case alignment.
Then, the number of orbits that grant an access to the AOI is estimated. Due to overlapping ground-tracks towards the poles this number varies with the latitude at which the AOI is located as well as with the extent of the AOI particularly in the longitude direction. By analysing the potential AOI positions along the access range of a centre orbit and considering the overlap, the resulting number of orbits with AOI access is derived. Also in this case, the outcome is presented for both an optimal and a non-optimal case in terms of a fast coverage.
Based on these results, in the final step the coverage duration is computed. By using the sub-cycle pattern of the orbital ground-tracks, the duration in days to reach the according orbits with AOI access is derived and then applied to the number of acquisitions. In this way, the coverage duration results to a range representing the minimum and maximum achievable duration depending on the positioning of the AOI. A comparison with actual planning results of the TSM has shown that this methodology delivers an accurate approximation of the coverage duration.

## 6 Literature

[1] Z. Song, G. Dai, M. Wang, X. Chen: A Novel Grid Point Approach for Efficiently Solving the Constella-tion-to-Ground Regional Coverage Problem, IEEE Access, vol. 6, Aug. 2018.
[2] Satellite Tool Kit: https://www.ansys.com/prod-ucts/missions/ansys-stk, last visited 25. Sept., 2023
[3] FreeFlyer Astrodynamics Software: https://ai-solu-tions.com/freeflyer-astrodynamic-softwarel, last visited 25. Sept., 2023
[4] R. Boain: A-B-Cs of sun-synchronous orbit mission design, AAS/AIAA Space Flight Mechanics Meeting, vol. 14, Feb. 2004
[5] O. Alvarez-Salazar, S. Hatch, J. Rocca, P. Rosen, S. Schaffer, Y. Shen: Mission design for NISAR repeatpass interferometric SAR, Proc. SPIE 9241, Sensors, Systems, and Next-Generation Satellites, vol. 8, Nov. 2014.
[6] E. Ortore, M. Cinelli, C. Circi: A ground track-based approach to design satellite constellations, Aerospace Science and Technology, vol. 69, pp. 458 464, Oct. 2017.
[7] S. Buckreuss, R. Werninghaus, W. Pitz: The German Satellite Mission TerraSAR-X, IEEE Radar Conference, May 2008.
[8] X. Luo, M. Wang, G. Dia, X. Chen: A Novel Technique to Compute the Revisit Time of Satellites and Its Application in Remote Sensing Satellite Optimization Design, International Journal of Aerospace Engineering, vol. 2017, No. 6, Jan. 2017.

