Quantum Computation for SAR Antenna Optimization Problems

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Abstract

This paper addresses antenna optimization problems in the context of spaceborne SAR systems for Earth observation. Theses kind of antennas require careful and precise design of their radiation characteristics. Special focus is laid on the optimization of transmit patterns, suitable for wide-swath SAR scenarios. Based on a selection of classical problem formulations, quantum computational concepts and approaches are investigated, which have the potential to outperform classical optimization routines, both in terms of speed and solution quality.

1 Introduction

Antenna pattern optimization or antenna synthesis, as it is called synonymously, is a subject which dates back to the first ground based radar systems developed and put into operation during World War II. Today, for modern radar remote sensing, the optimization of the radar antenna still plays an important role. Most notably, the spaceborne Earth observation missions NISAR [32], ROSE-L [14], Sentinel-1 Next Generation [34] and ALOS-4, to be launched in the near future, employ large antennas with electronic beam control and sophisticated beamforming techniques [23, 19].
Where in the past SAR sensors covered swath widths in the order a few ten kilometers at medium resolution, e.g., in stripmap modes, these new generation of SAR satellites aim for several hundred kilometer swath widths at even finer resolution. This requires specially designed SAR antennas with complex feed networks and receiver hardware. As illustrated in Fig. 1, illuminating a wide swath (indicated by the yellow patten and the blue curve) means distributing the available power in such a way, that the imaging performance in far range isn't degraded too much. One way to deal with this is to synthesize a transmit antenna pattern which compensates the power loss due to the larger distance in far range by an increased gain pattern in this region. An example of such patterns versus elevation angle is shown in Fig. 2, where the swath has been marked by vertical dashed lines. Other optimization goals are motivated by the fact that interference from outside (or inside) the imaging swath might corrupt the signal of interest. The interferences are typically range ambiguities with the most prominent the so called nadir return, which would lie somewhere between $-40°$ and $-20°$ in Fig. 2. For this, constraints can be imposed (slanted dashed lines), which help to keep the sidelobes at or below a certain level while maintaining the shape of the mainlobe.

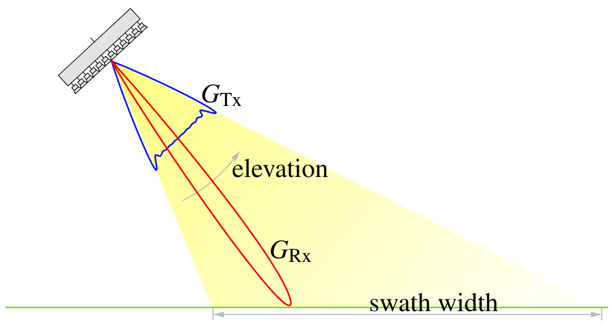

Figure 1 Sidelooking SAR. The radar satellite, symbolized by the array antenna, illuminates a swath on the earth's surface with a wide pattern (blue curve, yellow pattern) and records the radar echoes using a narrow receive beam (red curve).

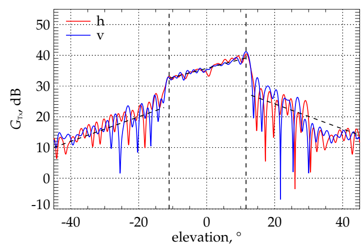

Figure 2 Transmit antenna diagrams computed for two polarizations as a function of the elevation angle (ϑ). The dashed lines mark the target pattern. The vertical dashed lines separate the swath to be illuminated from the region outside the swath.

Antenna pattern optimization problems come in a wide variety, depending on the type of antenna or depending on the mode an antenna is operated, e.g., transmit or receive mode. Many publications, such as [22] and, in a broader sense books, as for example [35], look at antenna optimization from a signal processing (or in that case array processing) perspective, with the goal of improving the signal-to-noise performance or the interference rejection capabilities. Other antenna optimization concepts are focused more towards the robustness of beamforming system concepts [21, 25].

In this work we focus on the optimization of the transmit characteristic of active phased array antennas as introduced above. The literature on antenna synthesis covers of course a wide spectrum, for instance with global techniques [9], formulating concepts for convex problems including inequality constraints [24] or weighted least squares [10] approaches. Of special interest for transmit pattern optimization is so called phase-only pattern synthesis [16, 7], which has been investigated since more than three decades. These techniques allow transmitting with unit power per transmit channel and by this ensure maximum radiated power towards the scene of interest. Those kind of optimization problems are non-linear and hard to solve on classical computers.

Quantum computation as a new paradigm offers a radically different way to look at such optimization problems. Certain mathematical operations and algorithms are specifically suitable to be executed on a quantum computer, since the quantum pendant to a classical algorithm may run quadratically or even up to exponentially faster. An example for this would be the quantum Fourier transform (QFT), which offers an exponential speedup compared to the classical discrete Fourier transform [13]. However, for optimization problems not only the execution time plays an important role but even more so the quality of a solution. Many iterative gradient-based optimization procedures suffer from the well know problem of tending to get 'stuck' in local minima. Here, quantum computers can potentially deliver significantly better results. One of the first papers proposing a quantum version for solving semidefinite programs [5] offers a square-root speed-up over classical methods. A more application oriented paper [27] presents a quantum version of particle swarm optimization for linear array antenna synthesis and other problems in electromagnetics. Another application of quantum computing to antenna design is presented in [12], where a quantum genetic algorithm is considered. Both papers claim better convergence behaviour of the quantum algorithms compared to the classical counterparts. A further paper [30] exploits the quantum Fourier transform in array thinning. Finally, concepts of adiabatic quantum computation applied to array processing have been investigated in [20, 33]. In this paper different novel concepts and ideas are presented, which aim at an efficient solution of difficult pattern synthesis problems, both in terms of solution quality as well as run time. These new quantum optimization approaches will help improving the imaging quality and performance of future SAR sensors.

2 Classical Problem Formulations

Pattern synthesis aims at shaping the radiation characteristic of an antenna. Usually, this radiation characteristic is described by its power or gain pattern [2]. A good approximation for the gain pattern $G(\vartheta)$ of an array antenna in transmit mode can be expressed in the form

$$G(\vartheta) \sim \frac{1}{P_{\text{rad}}} \left| \sum_i w_i a_i(\vartheta) \right|^2 , \quad w_i, a_i \in \mathbb{C} , \tag{1}$$

where w_i are complex coefficients which represent the gain and phase setting of a transmit channel and a_i denote the copolar electric far fields of the individual antenna elements. In this model, the gain is expressed as a function of the spherical angle ϑ, which corresponds to the elevation angles in Figs. 1 and 2. Typically, SAR antennas in transmit mode are operated such that the high power amplifiers are in saturation. This implies maximized radiated power

$$P_{\text{rad}} \sim \sum_i |w_i|^2 . \tag{2}$$

For the excitation coefficients

$$w_i = \mathrm{e}^{\mathrm{i}\phi_i} \tag{3}$$

this means that they all have unit-magnitude, with ϕ_i the individual phases of the antenna transmit channels. Then, the radiated power of an active phased-array antenna is simply proportional to the number of antenna elements n_c

$$P_{\text{rad}} \sim n_\text{c} . \tag{4}$$

Based on this antenna model, different problem formulations shall be considered, all aiming at pattern optimization results similar to the ones presented in Fig. 2. Described as so called feasibility problem [4]

$$G(\vartheta_l) \geq \tilde{G}(\vartheta_l) \quad \forall\, l , \tag{5}$$

$$G(\vartheta_m) \leq \tilde{G}(\vartheta_m) \quad \forall\, m , \tag{6}$$

this formulation consists of two sets of inequality constraints, where the desired pattern G shall lie above a given target pattern $\tilde{G}$ for angles ϑ_l inside the mainlobe domain and below a certain level in the sidelobe regions ϑ_m. This formulation offers the advantage of a larger solution space without a single global optimum. On the other hand this optimization problem, together with requirement (3), is highly non-linear and therefore hard to solve on classical computers.

Another way to optimize transmit patterns is by means of a least squares formulation:

$$\text{minimize} \quad \sum_{lm} \left(G - \tilde{G}\right)^2 (\vartheta_{lm}) , \tag{7}$$

$$\text{subject to} \quad |w_i| = 1 \quad \forall\, i . \tag{8}$$

Here, the quadratic deviation from the target pattern is minimized, both in the mainlobe and the sidelobe region. In this formulation the unit-magnitude requirement for the coefficients w_i is enforced by additional non-linear constraints $|w_i| = 1$. Such an optimization problem is therefore interesting for quantum computational approaches.

A third formulation, which may be promising for circuit based quantum computation is the following:

$$E(\vartheta_l) = \sum_i w_i a_i(\vartheta_l) \quad \forall\, l , \tag{9}$$

$$E(\vartheta_m) = \sum_i w_i a_i(\vartheta_m) \quad \forall\, m . \tag{10}$$

This ansatz could be phrased field inversion approach, where one deals with a set of linear equations with the total transmitted field $E(\vartheta)$ on the left side. It can be shown that this formulation has a strong resemblance to linear constrained minimum variance beamforming [35], where E would be proportional to $\sqrt{\tilde{G}}$. This means of course that the coefficients w_i won't have unit magnitude in general. Nevertheless, close to unit coefficients may be achieved by a proper selection of the target field E and therefore render this approach attractive for transmit pattern optimization, too.

3 Quantum Optimization

On the basis of the above mathematical problem formulation, different quantum computational concepts shall be investigated in the following.

3.1 Phase-Only Pattern Synthesis by Grover-Search

Classically, problems according to equations (5) and (6) would be solved by iterative methods or simply by searching the entire problem space. This is of course not possible for most practical applications, however, Grover's quantum search algorithm [17, 28] allows searching a problem space quadratically faster than any classical search algorithm. This algorithm adopts a so called oracle, who's purpose is to evaluate equations (5) and (6) in terms of being part of the solutions space or belonging not to the solution space. In oder to derive a quantum circuit it is convenient to reformulate these inequalities in terms of clauses

$$f(\vartheta_j) := G(\vartheta_j) \circ \tilde{G}(\vartheta_j)\,, \quad \circ \in \{\geq, \leq\}\,. \tag{11}$$

For a Grover search the oracle has to perform the following action

$$|q\rangle \mapsto |q \oplus [f(\vartheta_1) \wedge f(\vartheta_2) \wedge \cdots \wedge f(\vartheta_{n_\mathrm{d}})]\rangle\,. \tag{12}$$

Figure 3 presents a quantum circuit, where each clause $f(\vartheta_j)$ corresponds to a different angle ϑ_j and is evaluated

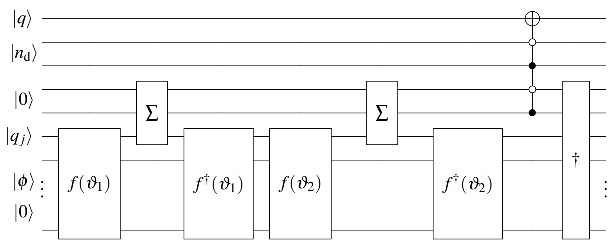

Figure 3 Oracle circuit evaluating the clauses $f(\vartheta_j)$ in a sequential manner. In this example, if both clauses are fulfilled the controlled XOR-gate will perform a swap on the query-qubit $|q\rangle$. The dagger operator at the end of the circuit symbolizes the reverse operation of all sub-circuits before the controlled XOR-gate.

in a serial connection. A first register counts the fulfilled clauses and the result is compared to a second register containing the number of angles n_d.

The quantum circuit for the individual clauses requires the efficient implementation of complex arithmetic. Here, we follow a design strategy which aims at a minimal qubit usage. This typically comes at the cost of deeper circuits as compared to parallel implementations. Note, for the purpose of a clearer notation the index for the angles j shall be dropped in the following. Rewriting the clauses (11) substituting equations (1), (3) and (4) and using $a_i = \bar{a}_i \mathrm{e}^{\mathrm{i}\bar{\phi}_i}$, $\bar{a}_i, \bar{\phi}_i \in \mathbb{R}$ yields

$$f = \left|\sum_i \bar{a}_i \mathrm{e}^{\mathrm{i}(\phi_i + \bar{\phi}_i)}\right|^2 \circ n_\mathrm{c} \tilde{G}\,. \tag{13}$$

This formulation suggests an algorithm design where, first, the real-valued phases for each antenna channel are added. One way to do this is by means of a quantum Fourier transform based modulo-2 adder [31], as shown in Fig. 5, which takes a superposition of a binary representation of the phases ϕ_i

$$\phi_i = \frac{2\pi}{2^{n_\mathrm{b}}} \sum_{k=0}^{n_\mathrm{b}-1} 2^k x_{ik} \tag{14}$$

as input. The fixed pattern phases $\bar{\phi}_i$ are hard-coded into the circuit as illustrated in Fig. 5.

Another important building block is the computation of the weighted complex exponential which performs the following action on the state

$$|\phi_i + \bar{\phi}_i\rangle \mapsto |\bar{a}_i \exp[\mathrm{i}(\phi_i + \bar{\phi}_i)]\rangle \tag{15}$$

For this, a unitary

$$U_i = \sum_{\{\phi_i + \bar{\phi}_i\}} |\bar{a}_i \exp[\mathrm{i}(\phi_i + \bar{\phi}_i)]\rangle \langle \phi_i + \bar{\phi}_i| \tag{16}$$

realizing a lookup table (or dictionary) has been constructed (for an example see Fig. 4), where the summation goes over all possible values $\phi_i + \bar{\phi}_i$. Finally, the

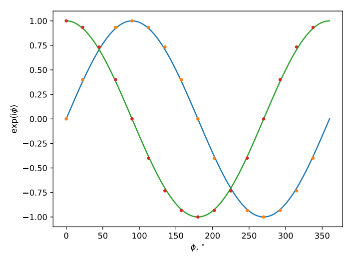

Figure 4 Weighted complex exponential function (here $\bar{a}_i = 1$) realized as unitary operator. In this example the output state containing the function values of $\exp(\mathrm{i}\phi)$, indicated by the red and orange dots, uses ten qubits, five for the real- and imaginary part each.

sub-circuit contains adder networks for the real- and imaginary part and a parameterized comparator. To give a proof of concept, a small artificial problem with a single clause, two channels and two bits per channel phase-quantization has been optimized:

$$|\mathrm{e}^{\mathrm{i}(\phi_1 + \bar{\phi}_1)} + \mathrm{e}^{\mathrm{i}(\phi_2 + \bar{\phi}_2)}|^2 \geq n_\mathrm{c} \tilde{G}\,, \tag{17}$$

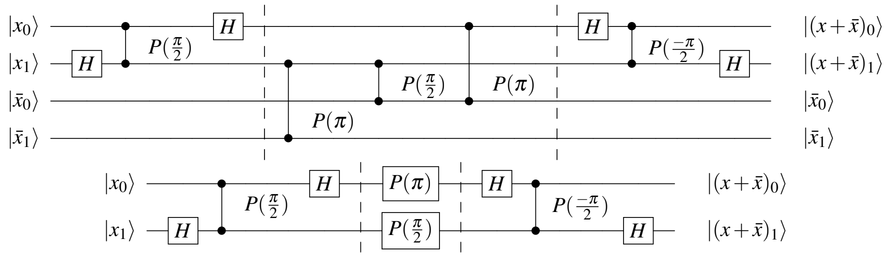

Figure 5 Upper circuit: 2-bit QFT modulo-two adder. The first circuit part performs a QFT. Addition is done in the Fourier basis, realized by conditional rotations. The last step is an inverse QFT giving the result in the first two qubits. Lower circuit: In a practical implementation this addition would be implemented via a parametrized circuit, effectively removing the second register for $\bar{\phi}$. In this example the angle in binary representation $|\bar{x}_1\bar{x}_0\rangle = |01\rangle$ is encoded in the circuit. Note, for notational simplicity the counting index for the antenna channels i has been omitted.

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where $\bar{\phi}_1 = 90°$, $\bar{\phi}_2 = 180°$ and $n_c\tilde{G} = 3$. Given the total number of states $N = 2^{2\cdot 2}$ with $M = 4$ solutions, the number of Grover-iterations

$$r = \mathrm{NINT}\left(\frac{\pi}{4\arcsin\sqrt{M/N}} - \frac{1}{2}\right) \tag{18}$$

is exactly one. NINT is the nearest integer function. Figure 6 shows the probability distribution of the output state and a summary of the results can be found in Table 1. This

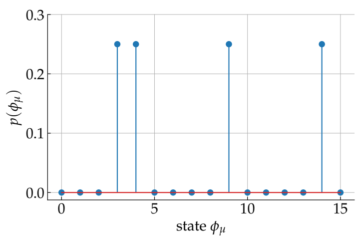

Figure 6 Grover output state probabilities for a simple example with a single clause, two antenna elements and 2-bit quantization for each transmit channel phase.

state binary	state dec.	ϕ_1	ϕ_2	G
0 0 1 1	3	0°	270°	2
0 1 0 0	4	90°	0°	2
1 0 0 1	9	180°	90°	2
1 1 1 0	14	270°	180°	2

Table 1 Output state configurations in binary and decimal encoding, which have been identified as solutions by the Grover-search algorithm. The last column shows the corresponding gain, e.g. the numerical value of left hand side of equation (17).

particular example required a total of thirteen qubits, four qubits for the two phases with two-bit quantization each and nine workspace qubits.

3.2 Optimization via Quantum Annealing

Optimizing problem (7) and (8) is again a hard problem, especially due to the non-linear constraints. Here, a quantum annealing approach [15, 26] shall be investigated. This requires to translate the constrained optimization problem in an unconstrained one, which can be achieved by including penalty terms for the constraints, e.g.

$$e_\mu \sim \sum_{l,m}\left(G-\tilde{G}\right)^2(\vartheta_{lm}) + \sum_i \gamma_i\left(|w_i|^2-1\right)^2 . \tag{19}$$

In this context the γ_i are real-valued scaling factors and the w_i are the complex weights to be optimized. By decomposing the coefficients $w_i = w_i^{\mathrm{R}} + \mathrm{i}w_i^{\mathrm{I}}$ into their real and imaginary parts a fixed precision approximation using K binary digits $x_{i,k}^{\mathrm{R/I}}$

$$w_i^{\mathrm{R/I}} = \frac{2}{2^{n_\mathrm{b}}-1}\sum_{k=0}^{n_\mathrm{b}-1} 2^k x_{ik}^{\mathrm{R/I}} - 1 \tag{20}$$

can be used. In this way the constraints $|w_i|^2 = 1$, and similarly the objective function in the first term in equation (19), become polynomials of degree four of the binary variables. In order to solve this optimization on a quantum annealer, slack variables (for products of two variables) are required in order to cast the polynomial in quadratic form [8]. This reduction of the degree and the subsequent optimization and evaluation is handled by our python package quark [1]. It also provides functionality to automatically introduce the penalty terms and therefore handle the constraints more conveniently. The convergence with K for a small demonstration problem with a single coefficient is illustrated in Fig. 7.

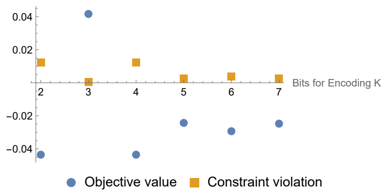

Figure 7 Convergence behaviour for a small demonstration example $1 - 0.894427w^{\mathrm{R}} - 0.447214w^{\mathrm{I}}$ under the constraint $|w|^2 = 1$ with the number of qubits K used for the binary encoding of real and imaginary parts of the variable w. The exact minimum is 0.

3.3 A Quantum Field Inversion Approach

Transmit pattern optimization by simulating equations (9) and (10) on a quantum computer presents an interesting problem for circuit based quantum computation. One can use the HHL algorithm [18] to solve the system of equations with a quantum speedup. However, in our case a more specialized approach could potentially be more beneficial. The interference of electromagnetic fields is completely analogous to the interference of quantum states. Therefore, calculating the transmit pattern and optimizing it appears to be a natural tasks for a quantum computer. To benefit from the 'natural interference', amplitude encoding has to be used, so the total field E is represented by the state

$$|E\rangle = \sum_{ij} w_i a_i(\vartheta_j) \, |j\rangle \ , \tag{21}$$

where j counts all angles from the mainlobe domain indexed by l and the sidelobe regions (m). We explored several different approaches on how to exploit this description, two of which we will sketch in the following.

The first one is to perform the inversion of equations (9) and (10) by inverting a circuit that prepares the state in equation (21). We start from unitaries U_i that prepare the state $|E_i\rangle = \sum_j a_i(\vartheta_j)\,|j\rangle$, i.e. that fulfill $U_i\,|0\rangle = |E_i\rangle$. They can be constructed with state preparation methods like [29], while again more specialized approaches might be possible. From the U_i we construct a controlled operation $U = \sum_i |i\rangle\langle i| \otimes U_i$. Applying this operation to the state $|w\rangle\,|0\rangle$ with $|w\rangle = \sum_i w_i\,|i\rangle$ and post-selection on $|+\rangle$ on the control register prepares $|E\rangle$ as desired. Inverting this operation is equivalent to inverting the order of the basic operations and complex conjugation. Note that post-selection and state preparation change their role. Because the post-selection affects the runtime of this approach, it can be beneficial to employ amplitude amplification [6] or rephrase it as a deterministic calculation.

The second approach is to use the quantum computer only for simulating the electric field and comparing it to the target pattern for weights given as input. Following the scheme of variational quantum algorithms [11], an outer optimization loop is then carried out on a conventional computer to obtain optimal weights. In order to compare the power and not the field to the target pattern we employ an additional trick. Note that the state $|E\rangle\,|E\rangle^*$ contains $|E(\theta_j)|^2$ in the jj-component, i.e. $\langle jj|\,|E\rangle\,|E\rangle^* = |E(\theta_j)|^2$. The selection to only the jj-components can be combined with the SWAP-test [3] that is used to calculate the overlap. We verified both approaches but additional investigations are required to prove or disprove an advantage compared over classical methods.

4 Conclusion

Three promising quantum optimization strategies tailored to specific problem formulations have been presented. Grover-search-based optimization has been successfully demonstrated at the example of a small problem. The applicability of such an optimization technique for relevant antenna problems on real quantum computers will of course depend on the availability of numerous error corrected qubits. Quantum annealing based approaches for beamforming problems on receive as well as sparse antenna configurations have been demonstrated for instance in [20]. Here, a demonstration example shows that nonlinear optimization problems can in principle be solved on quantum annealers. The performance of such annealing concepts in terms of qubits usage depends on the variable quantization and the number of slack variables introduced. Finally, an antenna pattern optimization Ansatz by field inversion has been investigated and two optimization strategies for gate-based quantum computers have been presented. Today, these quantum algorithms are only applicable to toy problems, but with the expected advancement of future quantum computer generations, these quantum optimization concepts will have the potential to find antenna optimization solutions better and faster.

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