Considering Reactive Power in Generation Expansion Planning for Energy System Design

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Abstract: Planning models for studying future energy system designs predominantly use the "DC" power flow approximation, a linearization of the AC power flow equations which ignores reactive power. This can lead to results that are physically infeasible ("AC-infeasible"). We investigate whether the use of an approximation which incorporates reactive power can improve the designed system with respect to AC-feasibility. As a first experiment, we solve a generation expansion problem using aggregated data representing the European energy system. The results show that the AC-feasibility improves significantly with the alternative approximation, leading to 97% AC-feasible snapshots, whereas the DC approximation only leads to ~73%. Since we observe an increase of one billion € in capacity investments and 239 million € in operating costs, this can likely be attributed to the improved visibility of grid bottlenecks. Hence, in our experiment the DC approximation leads to an underestimation of the total system costs by 1.239 billion €.

Keywords: Energy System Modelling, Power Flow Approximations, Generation Expansion Planning

1 Introduction

Energy system planning models are central to decisions of policy-makers and transmission system operators. The increased complexity of highly renewable systems due to the time-varying availability of generation demands a high spatial and temporal model scale, leading to large-scale optimization problems. Hence, researchers aim at reducing complexity in order to make the problems computationally tractable. To this end, the so-called DC approximation [1] is commonly used, which linearizes the physical laws of power flow. As this approximation ignores reactive power, its use has been criticized [2]. Hence, we investigate whether an alternative approximation [3], which incorporates reactive power, leads to a higher share of feasibility with respect to AC-feasibility, i.e. to the true power flow physics. We also compare both approximations in terms of system cost. As a means to keep the problem class manageable, the expansion problem is limited to generation expansion. Furthermore, as a first use case wind power was chosen as the only expandable generation technology with reactive power capability. Storage systems, solar power and HVDC lines could be modeled analogously in accordance with their technical capabilities.

2 Generation Expansion Problem

In the following, we adopt much of the notation of [2]. As we additionally consider reactive power, we separate the active and reactive power parts of each complex power variable $x = x^p + \mathrm{j}x^q$ via superscripts p and q, respectively. We define a generation expansion problem for a single year over snapshots $t \in \mathcal{T}$, of which each is weighted by its time span ω_t such that $\sum_t \omega_t = 8760\,\mathrm{h}$. The base of our problem formulation is formed by a representation of the power network as a graph $(\mathcal{N}, \mathcal{L})$ with $\mathcal{N}$ the set of buses, which are interconnected by alternating current (AC) and direct current (DC) lines $\mathcal{L}_{AC}$ and $\mathcal{L}_{DC}$, respectively. The set of branches is given by $\mathcal{L} = \mathcal{L}_{AC} \cup \mathcal{L}_{DC}$. For each snapshot $t \in \mathcal{T}$, an exogenous demand $d_{n,t} = d^p_{n,t} + \mathrm{j}d^q_{n,t}$, different generator types $r \in \mathcal{R}$ injecting power $g_{n,r,t} = g^p_{n,r,t} + jg^q_{n,r,t}$, and different storage system types $s \in \mathcal{S}$ supplying active power $h^-_{n,s,t}$ or consuming active power $h^+_{n,s,t}$ are attached to each bus $n \in \mathcal{N}$. Power flow between buses n and m via a line $(n,m) \in \mathcal{L}$ is given by the flow variable $f_{nm,t} = f^p_{nm,t} + \mathrm{j}f^q_{nm,t}$. The aim is to minimize the costs of fulfilling the exogenous demand $d_{n,t}$ for each snapshot t by expanding generator capacities $G_{n,r}$ at a marginal capital cost $c_{n,r}$ and dispatching generators and storage units at marginal operational costs $o_{n,r}, o_{n,s}$, respectively. Thus, we define the objective function

$$\min_{G,g,h} \sum\nolimits_{n,r} c_{n,r} G_{n,r} + \sum\nolimits_{n,r} \omega_t o_{n,r} g^p_{n,r,t} + \sum\nolimits_{n,s,t} \omega_t o_{n,s} h^-_{n,s,t}\,, \tag{1}$$

which minimizes both capital and operational costs. The extension of generator capacities is constrained by

$$\underline{G}_{n,r} \leq G_{n,r} \leq \bar{G}_{n,r}. \tag{2}$$

The active power dispatch of every generator is then constrained by

$$0 \leq g^p_{n,r,t} \leq \overline{g}^p_{n,r,t} G_{n,r}, \tag{3}$$

where $\overline{g}^p_{n,r,t} \in [0,1]$ is the per unit availability at time t. For conventional generators, the availability is constant, while for renewable generators it varies over time depending on weather conditions. Moreover, the complex generator dispatch must lie within a convex polytope which approximates the typical P-Q capability of the respective generator, i.e.

$$g_{n,r,t} \in PQ_r. \tag{4}$$

The dispatch and charging of a storage unit is limited by its capacity

$$\begin{aligned} 0 &\leq h^-_{n,s,t} \leq \overline{h^-_{n,s,t}}, \\ 0 &\leq h^+_{n,s,t} \leq \overline{h^+_{n,s,t}} \end{aligned} \tag{5}$$

and depends on the state of charge

$$e_{n,s,t} = e_{n,s,t-1} + \omega_t \left(\eta^+_{n,s} h^+_{n,s,t} - \left(\eta^-_{n,s}\right)^{-1} h^-_{n,s,t} + h^{\mathrm{in}}_{n,s,t} - h^{\mathrm{sp}}_{n,s,t} \right), \tag{6}$$

which considers the charging and dispatch efficiencies $\eta^+_{n,s}$, $\eta^-_{n,s}$ as well as the inflow and spillage $h^{\mathrm{in}}_{n,s,t}, h^{\mathrm{sp}}_{n,s,t}$. The state of charge is constrained by

$$0 \leq e_{n,s,t} \leq E_{n,s}. \tag{7}$$

For each bus, Kirchhoff's current law is enforced via the nodal balance constraints

$$\sum_r g^p_{n,r,t} + \sum_s h^-_{n,s,t} - \sum_s h^+_{n,s,t} - d^p_{n,t} = \sum_{(n,m)\in\mathcal{L}} f^p_{nm} + \sum_{(m,n)\in\mathcal{L}} f^p_{mn}, \tag{8}$$

$$\sum_r g^q_{n,r,t} - d^q_{n,t} = \sum_{(n,m)\in\mathcal{L}_{AC}} f^q_{nm} + \sum_{(m,n)\in\mathcal{L}_{AC}} f^q_{mn}, \tag{9}$$

where the left side corresponds to the nodal power imbalance and the right side represents the sum of inflow and outflow. The flow over a DC line $(n,m) \in \mathcal{L}_{DC}$ can be actively controlled. It is bidirectional and only limited by the line capacity, i.e.

$$-F_{nm} \le f^p_{nm,t} \le F_{nm}. \tag{10}$$

Since it is a DC line, the reactive power flow is equal to zero, and we furthermore assume that DC converter stations do not exchange reactive power with the network. The AC line flows are passively determined and depend on the power flow model, which uses some representation of the flow physics. Hence, we consider the different definitions separately in the following subsections.

2.1 AC Power Flow

Each AC line $(n,m) \in \mathcal{L}_{AC}$ has an admittance $Y_{nm} = g_{nm} + jb_{nm}$, where g_{nm} is the conductance and b_{nm} the susceptance. We denote the per unit voltage magnitude of a bus $n \in \mathcal{N}$ at time t as $v_{n,t} \in [0.9, 1.1]$ and the corresponding voltage angle as $\theta_{n,t} \in [-\pi/2, \pi/2]$. Then, the active and reactive power flows over the line in direction from bus n to bus m are given as

$$\begin{aligned} f^p_{nm,t} &= g_{nm} v^2_{n,t} - v_{n,t} v_{m,t} \big(g_{nm}\cos(\theta_{n,t} - \theta_{m,t}) - b_{nm}\sin(\theta_{n,t} - \theta_{m,t})\big), \\ f^q_{nm,t} &= -b_{nm} v^2_{n,t} - v_{n,t} v_{m,t} \big(b_{nm}\cos(\theta_{n,t} - \theta_{m,t}) - g_{nm}\sin(\theta_{n,t} - \theta_{m,t})\big). \end{aligned} \tag{11}$$

Since flows are not symmetric in general, we suppose that $(n,m) \in L_{AC} \Leftrightarrow (m,n) \in L_{AC}$ such that we have a flow variable for each direction of each line. Furthermore, each AC line has a thermal limit F_{nm} restricting the apparent power flow. By squaring both the apparent power flow and the thermal limit, this can be formulated as the convex quadratic constraint

$$\left(f^p_{nm,t}\right)^2 + \left(f^q_{nm,t}\right)^2 \le F^2_{nm}. \tag{12}$$

The AC power flow equations are nonlinear and nonconvex, and in fact finding a feasible solution is an NP-hard problem [4]. In practice, locally feasible solutions for small problem instances can be obtained using a nonlinear solver such as IPOPT [5]. However, as energy system planning models require a large spatial and temporal scale, they usually contain an approximation of the AC power flow equations.

2.2 DC Power Flow

A common approximation of the AC power flow equations is the “DC” approximation [1]. By making the assumptions that

i. the differences of voltage angles are small, such that $\cos(\theta_{n,t} - \theta_{m,t}) \approx 0$ and $\sin(\theta_{n,t} - \theta_{m,t}) \approx \theta_{n,t} - \theta_{m,t}$,
ii. the per unit bus voltage magnitudes $v_{n,t}$ are close to one,
iii. line conductances g_{nm} are much smaller than susceptances b_{nm},
iv. reactive power flows $f^q_{nm,t}$ can be neglected,

the AC power flow equations are simplified to

$$f_{nm}^{p} = -b_{nm}(\theta_n - \theta_m). \tag{13}$$

There is no reactive power flow, and thus the nodal balance constraint for reactive power is ignored. Furthermore, the thermal limit constraint simplifies to

$$\left|f_{nm,t}^{p}\right| \leq F_{nm}. \tag{14}$$

Losses are ignored, as the flows are symmetric (i.e. $f_{nm,t}^{p} = -f_{mn,t}^{p}$). Since all equations are linearized, a globally optimal solution can be reliably determined by a linear solver in polynomial time [6]. For this reason, it is a widely used approximation. However, its use in energy system planning models has been criticized, as e.g. voltage drops or overloaded lines due to reactive power flows cannot be considered [2].

2.3 Approximation incorporating reactive power

In [3], a linearization of the AC power flow equations is introduced which, in contrast to the DC approximation, includes reactive power flows and voltage magnitudes. In the derivation, the authors assume that

i. the differences of phase angles are small,
ii. the per unit voltage magnitudes are close to one,

which allows obtaining the approximation

$$\begin{aligned} f_{nm}^{p} &= g_{nm}(v_n^2 - v_m^2)\left(\frac{1}{2}\right) - b_{nm}(\theta_n - \theta_m) + f_{nm}^{p,l}, \\ f_{nm}^{q} &= -b_{nm}(v_n^2 - v_m^2)\left(\frac{1}{2}\right) - g_{nm}(\theta_n - \theta_m) + f_{nm}^{q,l}, \end{aligned} \tag{15}$$

where $f_{nm}^{p,l}, f_{nm}^{q,l}$ are the active and reactive power losses. As this approximation only contains the squared voltages, it can be regarded as linear if the squared voltage itself is taken as the decision variable. Using a base solution of the AC power flow equations, the losses are linearized around the resulting operating point. However, this is impractical for planning models, as the network topology changes during the optimization, making the base solution obsolete. Hence, we must neglect the loss terms. Moreover, the authors use a linearization of the thermal line limit constraint by approximating the circle formed by the constraint in the complex plane with a set of linear inequalities. This again can be impractical in planning models, whose large temporal and spatial detail would lead to a large number of additional constraints. Instead, we use the true thermal limit constraint, which leads to a convex quadratic program but reduces the number of constraints. Moreover, in the course of our experiments, we observed that this improves the accuracy of the approximation.

To assess the quality of the approximation, we apply it on a selection of optimal power flow (OPF) test cases from the PGLIB library [7] and compare the results to an AC-OPF solution. We focus on the flows, as they are passively determined by the generator injections, phase angles, bus voltages, and the line admittances. Thus, a close match to the AC solution would indicate a good quality of the approximation. Figures Figure 1 and Figure 2 show the flows of the approximation OPF solution in relation to the flows of the AC-OPF solution: Flows match perfectly if they lie on the diagonal. Flows in the upper left triangle are overestimated by the approximation, flows in the lower right triangle are underestimated. For all test cases, the active power flows of the approximation coincide well with the AC solution. The reactive power flows match well in some smaller test cases, but in general deviate more strongly from the diagonal. There is no clear tendency for over- or underestimation of reactive flows.

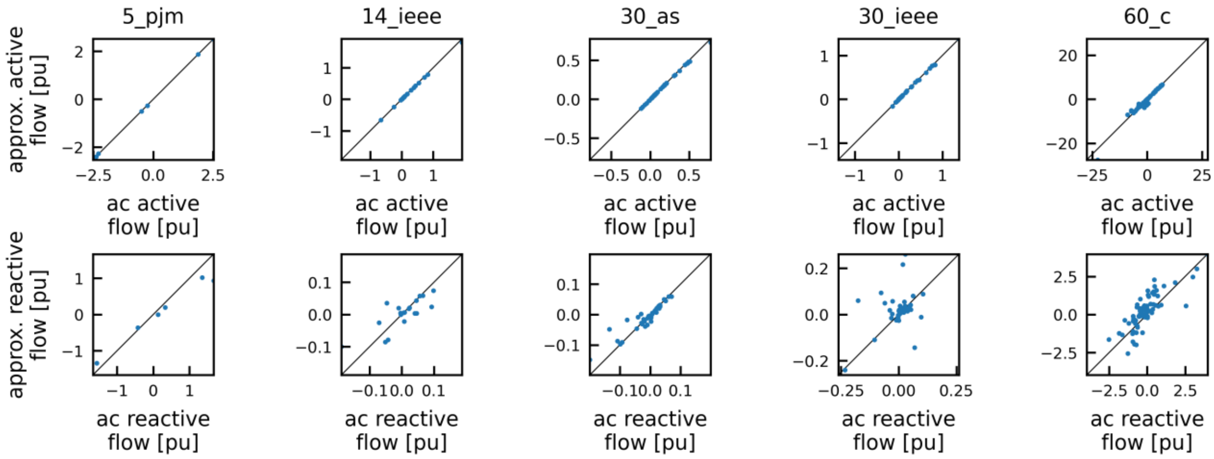

Figure 1: Active and reactive power flows of approximation in relation to AC power flows for five OPF test cases.

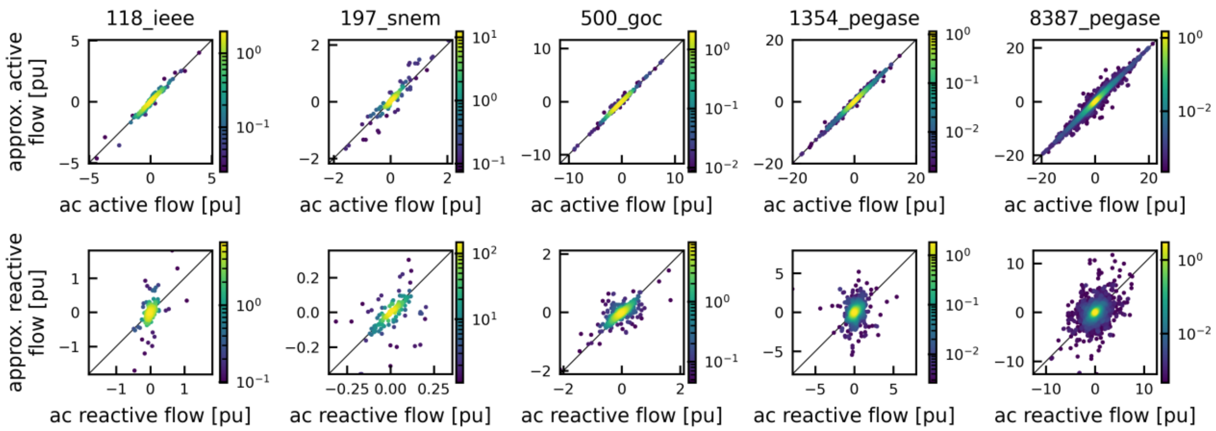

Figure 2: Active and reactive power flows of approximation in relation to AC power flows for five larger OPF test cases. The colorbar shows the density of points on a logarithmic scale.

3 Numerical Experiment

We conduct a numerical experiment based on data from PyPSA-Eur [8], an energy system model covering the European energy system at the transmission level. It includes existing conventional and run-of-river generators, hydroelectric storage units, and potentials for the installation of renewable generators based on weather data. Moreover, it includes a year of hourly time series for the active power demand, renewable generation potentials, as well as inflow and spillage for hydroelectric storage units. For computational reasons, we aggregate the model in time and space: We use a two-hourly resolution and represent Germany with 20 buses, while each other country is represented by a single bus. Due to the limited model size we can further assume that the linearization presented in Equation (15) approximates the reactive power flow sufficiently well. To reflect the recent decommissioning of German nuclear power plants, we remove them from the existing conventional generator park.

As PyPSA-Eur does not include reactive power demands or reactive power capabilities for generators, we have to make additional assumptions. To set the reactive power demand, we assume a lagging power factor of $cos(\varphi_d) = 0.95$ for buses in Germany and $cos(\varphi_d) = 0.98$ for the remaining (country) buses. The reactive power demand is then defined as $d^q_{n,t} = \tan(\varphi_d)\, d^p_{n,t}$. The P-Q capabilities of synchronous and wind generators are determined by approximating typical P-Q diagrams, shown in Figure 3. We neglect the ability of HVDC converter stations as well as solar and storage units like pumped-hydro storage to provide reactive power compensation. Furthermore, we only allow extending the capacities of solar and wind generators and do not consider battery storage.

We run a generation expansion with two different models: The "DC" model represents a generation expansion model with the established DC power flow approximation. It is defined by Equations (1) – (8), (10), (13), (14). The generation expansion model using the alternative approximation, which incorporates reactive power, is called the "QP" model, as it is a convex quadratic program. It is given by Equations (1) – (10), (12), (15).

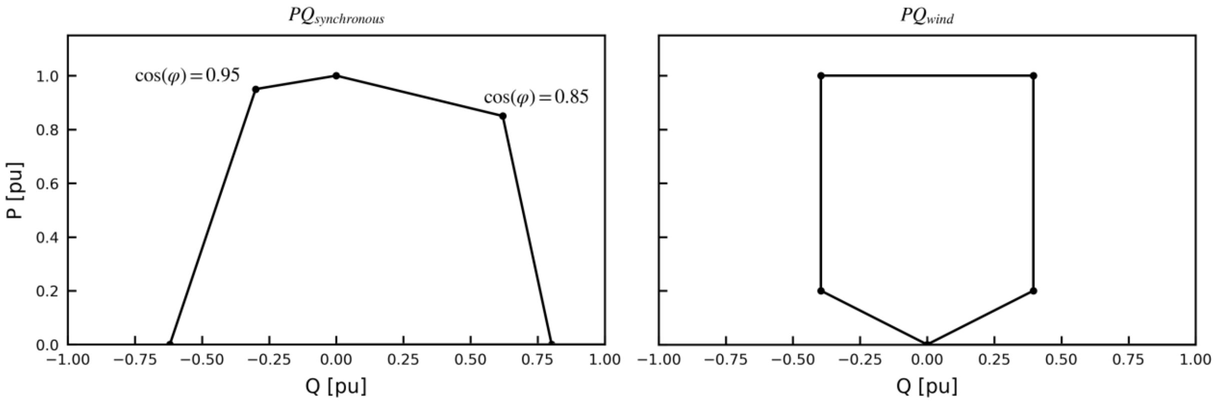

Figure 3: Convex polytopes approximating typical P-Q diagrams of synchronous machines and converters of wind turbines [9], [10].

3.1 Evaluation of AC-feasibility

We evaluate the AC-feasibility of the resulting systems by solving an AC dispatch problem separately for each snapshot, ignoring costs. That is, we try to find a feasible point for the problem defined by Equations (2) – (12) while fixing both t and the generator capacities, which correspond to the solution of the DC or QP model. The states of charge for storage units are initialized to the same values for both models. We use IPOPT [5] with an upper bound of 10000 maximum iterations. Note that IPOPT converging to a locally infeasible solution or not converging within the iteration limit is not a proof of infeasibility. However, as AC-feasibility is an NP-hard problem, a polynomial time algorithm for this is not known.

3.2 Results

The system determined by the QP model achieves ~97% AC-feasible snapshots, a significantly higher share than that of the DC model at ~73%. This is visible in Figure 4, which also relates the feasibility of the snapshots to the total system demand. Especially for the DC approximation, infeasibility coincides with high demand. Figure 5 visualizes the allocation of the capacities to the buses. While many buses in the South show similar investments, there is a significant difference in the North: Sweden and Finland invest into a much larger share of solar generators. Norway visibly increases its total investments, invests into a larger share of onshore wind and replaces DC-connected offshore wind by AC-connected offshore wind. The differences in the total capacity investments are displayed in Figure 6, which shows that the QP model invests into an additional 3.1 GW of wind and 10 GW of solar generators, leading to an increase of one billion € in capacity costs. The operating costs in the QP model are higher by 239 million €. Apparently, the need for additional reactive power capacities is not as strong as the need for active power capacities: Despite our assumption that solar generators do not supply reactive power, the QP model invests significantly more into solar.

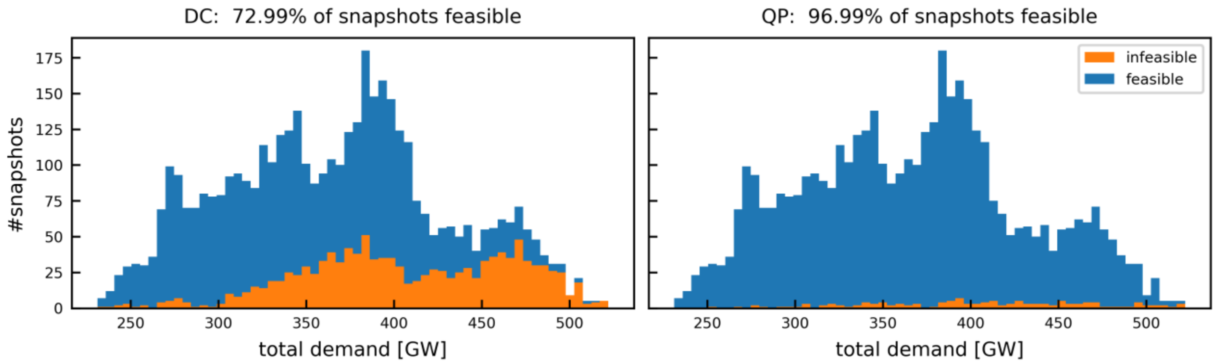

Figure 4: Relation of AC-infeasible snapshots to the total demand.

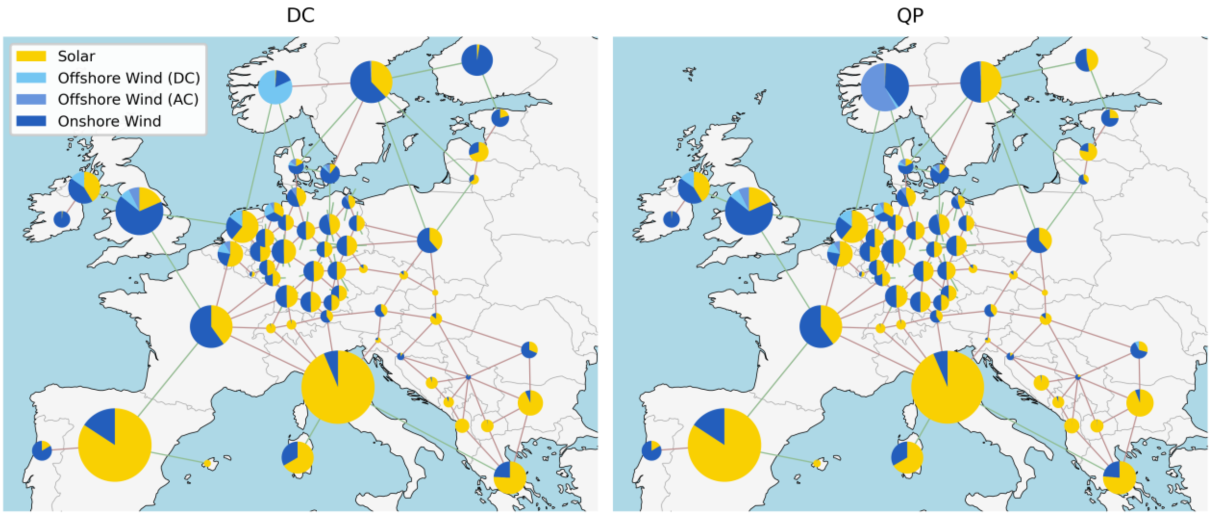

Figure 5: Results of the generation expansion problems. Pie charts display the share of investments at each bus. Their size indicates the size of the investments and has been normalized between the two model results such that they are visually comparable.

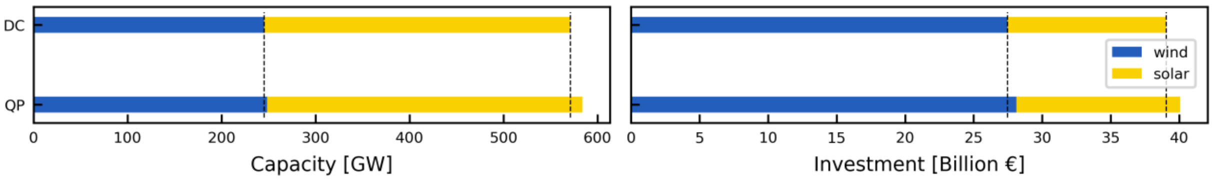

Figure 6: Differences of generation expansion results of DC and QP models.

3.3 Discussion

A likely explanation for the improved AC-feasibility is that the QP model is able to allocate capacity for avoiding grid bottlenecks: Lines congested due to additional reactive power flows are not visible to the DC model. Naturally, line congestion occurs more often during times of high demand, which again coincide with a large share of the DC model's infeasible snapshots. The QP model avoids bottlenecks by increasing the total capacity and diversifying the generator mix e.g. at buses in the North, despite high per-MWh costs. Grid bottlenecks translate to higher operating costs, as the cheapest available generators cannot be fully utilized. Both the higher capacity and the higher operating costs are visible in the QP model results, whose solution is 1.239 billion € more expensive.

As solar is in general cheaper than wind, it is used to cover the mismatch created by bottlenecks. Moreover, Northern countries diversify their capacities by investing more into solar. The lower need for reactive power capacities can also be attributed to the large number of conventional generators in our model.

Possibly, allowing battery storage would lead to a higher AC-feasibility of the DC system, as the decentralization could prevent grid bottlenecks. For example, the authors' DC model in [2], which allows investments into battery storage, shows at least 92% AC-feasibility. However, due to many differences in the assumptions to our model, the results are not directly comparable. The authors e.g. assume that there is always sufficient supply of reactive power, i.e. they do not put limits on the reactive power injections. Moreover, they investigate a different problem class by allowing transmission expansion, which is not considered in this paper.

4 Conclusion

We compare the results of two generation expansion models using different approximations of the power flow equations: The DC model uses the established DC approximation, which ignores reactive power. In contrast, the approximation of the QP model includes reactive power. First results show that the AC-feasibility improves significantly with the QP model, leading to ~97% feasible snapshots, whereas the DC model only shows ~73%. Since the QP model makes one billion € of additional capacity investments and diversifies some bus capacities at high per-MWh costs, this can likely be attributed to the improved visibility of grid bottlenecks in the model. Thus, the DC model potentially underestimates the total system costs by 1.239 billion €. The need to invest in additional measures, such as reactive power compensation systems, is shown by the frequent inability to achieve feasible dispatch situations using generator capacities from the DC-based expansion. As a result, the cost of expansion cannot be directly compared to that of the QP approach. In addition, many of the system snapshots are not operated at the optimal operating point, resulting in additional operating costs. Future work is planned to compare and analyze the causes of the additional costs based on the dual variables of line flow and voltage constraints.

It remains to be seen if this holds for more complex models. On the one hand, extension of transmission capacities and storage should improve the AC-feasibility of the established DC model. On the other hand, a higher spatial resolution should reveal additional grid bottlenecks, which again could lead to more infeasibilities. Moreover, future work could model the P-Q capabilities of the remaining energy carriers, including e.g. the possibility of solar parks. Additionally, different converter types with different P-Q capabilities and reactive power compensation devices could be modelled. This would be particularly interesting in a 100% renewable scenario, where the reactive power capabilities of conventional generators cannot be used. A possible pathway to close further close the gap between power flow studies and planning models could be using a relaxation of the power flow equations such as [11].

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