QUANTUM OPTIMIZATION FOR PHASE UNWRAPPING IN SAR INTERFEROMETRY

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ABSTRACT

Phase unwrapping is the reconstruction of a phase given its values mod 2π. It is an important image processing technique used in synthetic aperture radar interferometry, e.g., in the context of topography and ground deformation. In light of recent quantum algorithm developments for mathematical optimization problems, we explore the usage of gate-based quantum computers and hybrid quantum algorithms to create novel phase unwrapping approaches.

Index Terms— Phase Unwrapping, InSAR, QAOA, Quantum Computing

1. INTRODUCTION

Interferometric synthetic aperture radar (InSAR) is a well-known and powerful remote sensing technique used to measure geophysical features such as ground deformation and topography. By combining two radar images of the same scenery, acquired with a spatial or temporal baseline, used for topography or ground deformation models, respectively, phase interferograms are formed. However, due to the physical limitations of practical SAR system parameters, the acquisition system can only measure the absolute phase $\phi \in [-\pi, \infty)^{n\times m}$ modulo 2π. Here $n \times m$ refers to the image dimensions in azimuth and range. This limitation also arises in interferometric synthetic aperture sonar, magnetic resonance imaging and optical interference. Formally, we have

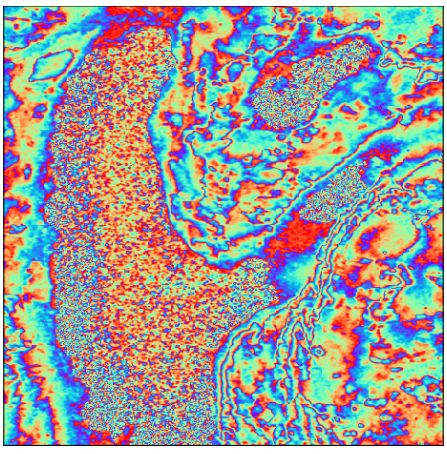

Fig. 1. Simulated Wrapped Phase, Ammersee, Germany.

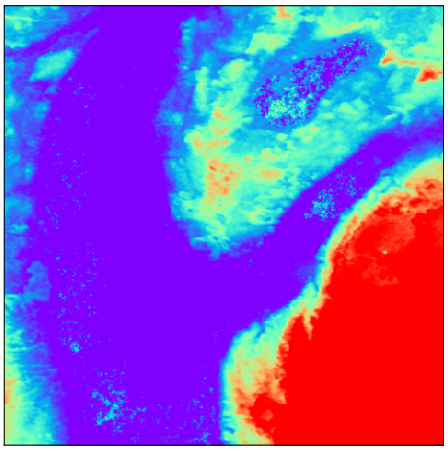

Fig. 2. Unwrapped Phase of Fig.1, Graph Cut Approach.

$$\phi = \psi + 2\pi k,$$

where $\psi \in [-\pi, \pi)^{n\times m}$ is the measured *wrapped* phase and $k \in \mathrm{N}_0^{n\times m}$ is the ambiguity correction. Phase unwrapping is the process of recovering the absolute phase ϕ from the wrapped phase ψ. Mathematically, this is a highly ambiguous problem. Therefore, virtually all phase unwrapping algorithms are based on the hypothesis that the phase surface is smooth enough to produce phase differences less than π for neighbouring pixels, as this allows the absolute phase to be easily determined. However, if the true phase surface is highly noisy or the ground resolution is too low, the above hypothesis will be violated, giving rise to the need for sophisticated algorithms to compensate for these errors. L^p-norm phase unwrapping algorithms, compare Ghiglia et al. [1], commonly used in InSAR applications, minimize differences of neighbouring pixels in ϕ through k

$$\min_{k\in \mathrm{N}_0^{n\times m}} \sum_{(s,t)\in\Omega} \left\| k_s - k_t - \hat{\psi}_{s,t} \right\|_p, \qquad (1)$$

https://doi.org/10.1109/IGARSS52108.2023.10282041

where Ω is a list containing all combinations of two different neighbouring image pixels and $\hat{\psi}$ depends on the wrapped phase.
Least-squares methods ($p = 2$) deal with large images efficiently, but tend to flatten discontinuities, spreading errors and thereby creating low-accuracy images. Graph-Cut methods ($p = 1$), such as outlined in Constantani [2], improve the unwrapping quality significantly, however, scale worse, and represent certain topologies wrongly. Lowering p below 1 further improves the image quality and increases computation complexity. Particularly the L^0-norm is accepted to produce the highest quality images, compare Bioucas-Dias et al. [3], however, its calculation poses an NP-hard problem and is therefore currently infeasible for practical grid sizes.

2. QUANTUM APPROXIMATE OPTIMIZATION ALGORITHM

In principle, quantum computers have been shown to speed up certain algorithms, scale well with large data sets and even calculate some NP problems in polynomial time. In this work we analysed the challenges, potential advantages and implementation of L^0-minimum norm methods utilizing the hybrid quantum approximate optimization algorithm (QAOA) introduced by Farhi et al.[4][5].

QAOA consists of two principle components: Deriving optimal classical parameters $\alpha \in [-\pi, \pi)^q$ and $\beta \in [0, \pi)^q$ and executing a circuit on a quantum computer based on these parameters. The value q determines the depth of the quantum circuit.
We chose the operator C to be the amount of fulfilled clauses, so the 0-contributions, in (1), where $p = 0$. Inspired by [4] we treat the phase differences between neighbouring pixels as regular 4-graphs while q determines the distance up to which qubits of neighbouring pixels interact. We construct a corresponding subgraph of interaction for every two neighbouring pixels. Due to symmetry, these subgraphs can be categorized into different subgraph isomorphy classes g. Subgraphs of the same isomorphy class have the same influence on the following calculation, therefore we only need to count the respective amount, denoted ω_g, and multiply with the contribution associated with this isomorphy type. By classically calculating the maximum expected value of

$$\langle \alpha, \beta | \, C \, | \alpha, \beta \rangle = \sum_g \omega_g \sum_{s=0}^{2^{\mathcal{O}(q)}} \left| \eta_g^s(\alpha, \beta) \right|^2,$$

for measuring the operator C of the quantum state

$$|\alpha, \beta\rangle = \prod_{i=q}^{1} U(\beta_i, X) U(\alpha_i, C) \, |+\rangle \, ,$$

we find optimal values of α and β. Here $|+\rangle$ denotes the equal superposition state and U the exponential operation.

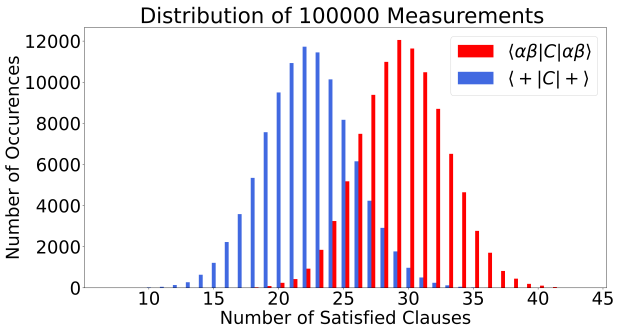

Fig. 3. $5 \times 6 \times 1$: Effect of the QAOA operation on the number of fulfilled clauses C.

Afterwards, we use X- and rotation-gates to build the corresponding quantum state $|\alpha, \beta\rangle$ on either a quantum computer or simulator. Subsequent measurements yield maximum estimates of the operator C. QAOA is an approximate algorithm, i.e. the results improve with the circuit depth q as long as the parameters can be optimized, however for specific problems, smaller values of q can in principle yield the optimal unwrapped phase.

3. EXPERIMENTS AND DISCUSSION

We conducted two sets of experiments for validation purposes. For both real and simulated quantum computers, the number of accessible qubits is limited. At the time of writing, around 32 logical qubits can be accessed with reasonable effort on simulated machines. Therefore we designed our validation experiments with these restrictions in mind, limiting the grid sizes, bitrate b_k of the ambiguity correction k (with higher bitrates resulting in higher accuracy) and setting $q = 1$. For experiment 1 the following wrapped phase grid of size 5×6 and bitrate of $b_k = 1$ was chosen:

$$\psi_1 = \pi \cdot \begin{bmatrix} -0.8 & -1 & -0.2 & 0.6 & -0.6 \\ -0.8 & -1 & -0.2 & 0.6 & -0.6 \\ -0.3 & 0.1 & 0.9 & -0.3 & 0.1 \\ -0.3 & 0.1 & 0.9 & -0.3 & 0.1 \\ 0.6 & -0.6 & 0.2 & 0.8 & 0.4 \\ 0.6 & -0.6 & 0.2 & 0.8 & 0.6 \end{bmatrix}$$

The parameter optimization yielded $\alpha \approx 0.1747, \beta \approx 0.6290$. $5 \times 6 \times 1 = 30$ qubits were simulated. Measurement of a quantum mechanical state is a statistical process, mathematically equivalent to sampling from a probability distribution. Therefore repeated sampling is necessary to achieve statistical significance. For both experiments, 100000 samples were generated.

Fig. 3 shows the corresponding statistics, where results of equal satisfied clauses are accumulated once for the QAOA (red bars) and with sampling the equal superposition state (blue bars) as a reference, to visualize improvements. The

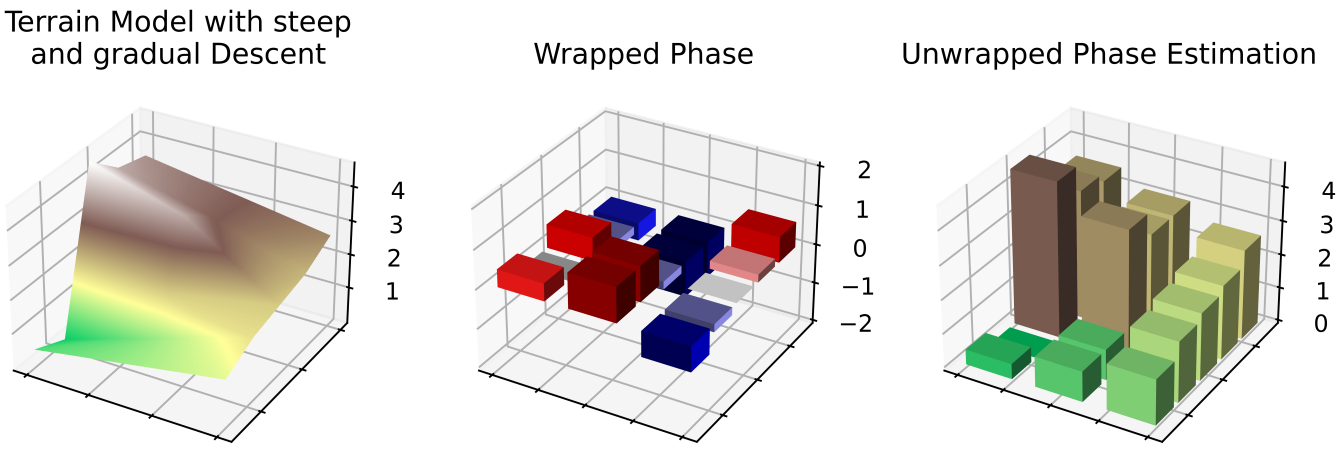

Fig. 4. $5 \times 3 \times 2$: The Phase Unwrapping Procces: Observing the terrain on the left yields the wrapped phase values in the centre. Unwrapping the phase with the best ambiguity correction in Fig. 5 creates the reconstruction on the right.

figure shows an upwards shift of the probability distribution with the mean improving by roughly 6.3, similar variance and a maximum returned value of 43 fulfilled clauses. This is equivalent to the maximum amount possible. QAOA with the above parameters and 100000 samples was able to find an optimal ambiguity correction k.

For experiment 2 the terrain model on the left, in Fig. 4, was chosen due to L^p methods with $p \neq 0$ having trouble recreating this shape. This resulted in the following wrapped phase grid of size 5×3:

$$\psi_2 = \pi \cdot \begin{bmatrix} -0.45 & -0.9 & 0.65 \\ -0.2 & -0.9 & 0.2 \\ 0.6 & -0.2 & 0 \\ 0 & 0.9 & -0.2 \\ 0.45 & 0.9 & -0.65 \end{bmatrix}$$

Choosing a bitrate $b_k = 2$, the parameter optimization yielded $\alpha \approx 0.2029, \beta \approx 0.7098$. $5 \times 3 \times 2 = 30$ qubits were simulated. Fig. 5 shows the corresponding statistics where results of equal satisfied clauses are accumulated once for the QAOA (red bars) and with sampling the equal superposition state (blue bars) as a reference. The figure shows only a slight upwards shift in the probability distribution. However, the maximum returned number of fulfilled clauses increased significantly to 19. The corresponding ambiguity correction k allows for a perfect reconstruction of the original topography, as seen in Fig. 4.

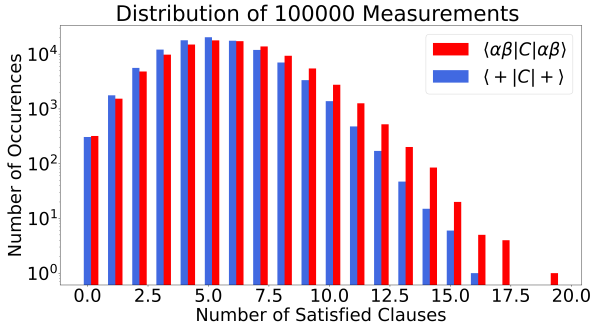

Fig. 5. $5 \times 3 \times 2$: Effect of the QAOA Operation on the Number of Fulfilled Clauses C. Logarithmic Scale.

4. CONCLUSION AND OUTLOOK

The potential of using quantum algorithms in the context of phase unwrapping applications was recognized previously: Kelany et al. [6][7] and Otgonbaatar et al.[8] have explored possibilities of implementing least-squares and network approaches on quantum annealers respectively. In this work, we studied the L^0-norm phase unwrapping approach by utilizing the QAOA algorithm. We were successful in reconstructing validation samples of small grid sizes in simulation.

Two major limitations became apparent: The amount of qubits currently available is not sufficient to unwrap relevant image sizes. However, since the demand of qubits needed to hold the phase values grows efficiently with size and bitrate, L^0-Phase unwrapping should be feasible for larger fields if quantum computers improve accordingly.

The second limitation comes in the form of the parameter q. The classical optimizations scale badly with increasing q and experiment 2 suggests that for higher bitrates larger q produce better results. To implement L^0 phase unwrapping with QAOA for more complex fields, a reasonable upper bound for q needs to be set. The significant positive shift of the distributions in experiment 1 and the local nature of the phase unwrapping problem suggest that such a bound may still result in optimal solutions.

Due to the increasing complexity induced by implementing higher q the classical optimization becomes difficult. Us-

ing machine learning frameworks such as Jax [9] allows for automatic differentiation with linear algebra acceleration on GPUs. This enables gradient descent methods facultatively with the integration of arbitrary neural networks. Future work, focussing on implementing larger q through the use of neural networks' ability to approximate the effects of the different subgraph isomorphy classes is planned.

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