# Yannick Kuhna,b (yannick.kuhn@dir.de), Birger Horstmanna,b,c, Arnulf Latza,b,c <br> a Institute of Engineering Thermodynamics, German Aerospace Center (DLR), Pfaffenwaldring 38-40, 70569 Stuttgart, Germany b Helmholtz Institute Ulm for Electrochemical Energy Storage (HIU), Helmholtzstraße 11, 89081 Ulm, Germany <br> c Institute of Electrochemistry, University of Ulm, Albert-Einstein-Allee 47, 89081 UIm, Germany 

## MOTIVATION AND AIM

- Many models are available for each process in a battery cell:
- intercalation,
- SEI growth,
- cracking, ..
- Selection from many combinations leads to a vast zoo of possible models to describe a particular battery cell.
- Bayesian methods are best suited to perform honest parameterization and selection in the face of uncertainty.
- Aim: a model selection algorithm that is both flexible and stable enough to handle the variety in battery models.


## BAYES' THEOREM

- P(parameter | data) $\propto \mathrm{P}($ data | parameter) $\cdot \mathrm{P}$ (parameter)
- Read: „The Likelihood of the model parameters matching the data updates the Prior knowledge to Posterior knowledge."


## bayesian machine learning

Learn a function describing uncertainty

- The target to learn from will be ||model(parameter) - data||.
- Active Learning: leverage the included uncertainty to decide on the most informative next parameter sample.

- Choice of fit function: Gaussian Process [2].


## BAYESIAN OPTIMIZATION (LII)

Substitute Likelihood with ML function

- Likelihood is often intractable.
- Approximation: integral of ML fit function below a certain threshold.
- Optimal threshold can be calculated from ML fit function automatically [2].


## BAYESIAN QUADRATURE

## Evidence calculation for model selection

- Bayes' Theorem hides a normalizing factor, the so-called Evidence: $\int \mathrm{P}$ (data | parameter) $\cdot \mathrm{P}($ parameter $) \mathrm{d}$ (parameter).
- The Evidence is a reliable measure for the question "Could this data have originated from this model?" [1].
- BQ can efficiently calculate the Evidence.


## EXAMPLE APPLICATION: ECM

The simplest ECM to model arbitrarily many time constants is a chain of RC pairs (to infinity: DRT).

- Question: how many time constants / RC pairs are visible in any given impedance spectrum?
- High amount of noise makes classical optimization inaccurate [1].
- Overlapping time constants further complicate the task.

impedance spectrum of a R-RC-RC-RC circuit with one RC element "hidden" in the noise at high frequency


## BQ ISSUE: SENSITIVITY TO RANDOM <br> INITIALIZATION

- The randomly chosen initial samples greatly affect BQ convergence.
There is no "best" design of experiment to counteract this.
- While parameterization consistency is acceptable, model selection consistency is not.


Predictive posterior visualization after failed parameterization

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## BO PRECONDITIONING OF BQ

Preemptively ended Bayesian Optimization gives a Posterior that is not much narrower, but greatly increases $B Q$ success rate if used as a
"preconditioned" Prior.

- Tests with a simply narrower Prior did not improve results.


Correlation matrix computed by EP-BOLFI [2]; variances barely changed

## MODEL SELECTION

- With data from R-RC-RC, Evidence is computed once for R-RC-RC-RC.
Without BO preconditioning, only 1 out of 6 times the Evidence for the correct $\mathrm{R}-\mathrm{RC}-\mathrm{RC}$ is higher.
- With BO preconditioning, 6 out of 6 times R-RC-RC scores higher.


Predictive posterior visualization after successful parameterization

## SUMMARY

We suggest that Bayesian Quadrature as a model selection algorithm synergizes perfectly with Bayesian Optimization to reliably deliver automated model selection in complex scenarios.
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