

FLIGHT DYNAMIC STABILITY PREDICTION FOR AN AIRCRAFT IN TRANSONIC SEPARATED FLOW CONDITIONS WITH A LINEAR FREQUENCY DOMAIN SOLVER

Markus Widhalm

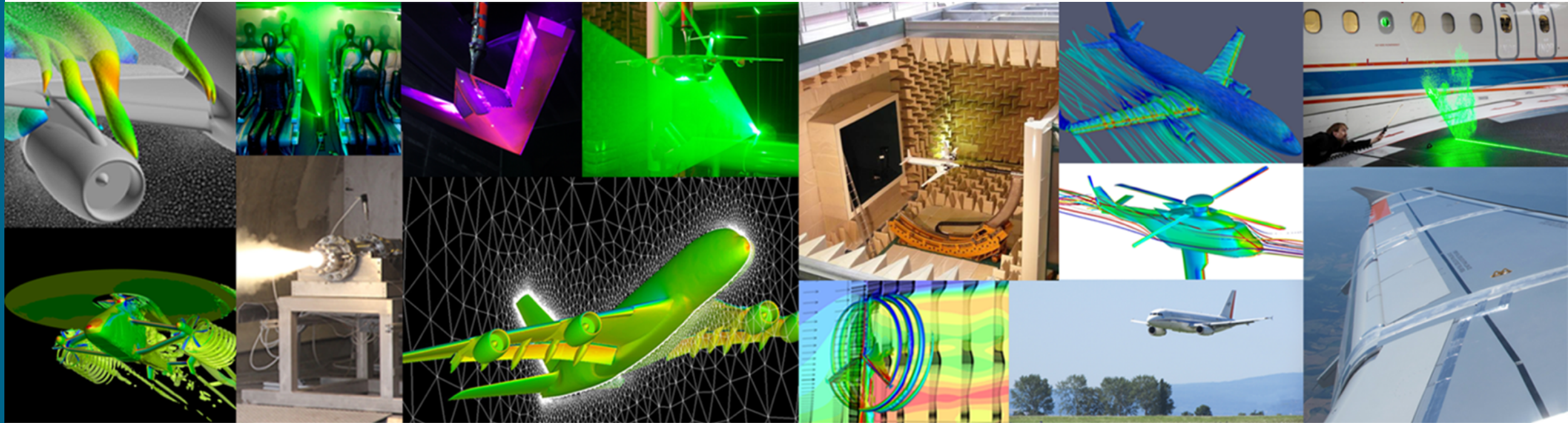
DLR, Institute of Aerodynamics and Flow Technology, C²A²S²E

FH Wiener Neustadt, 1. December 2023

Intro
Theory
Results
Conclusion



Research Areas at the Institute of Aerodynamics and Flow Technology



In the research domain of fluid physics the institute provides the link between basic research and industrial implementation. Our knowledge base is embedded into an international framework of research establishments and industry.

Organization | Sites of the Institute

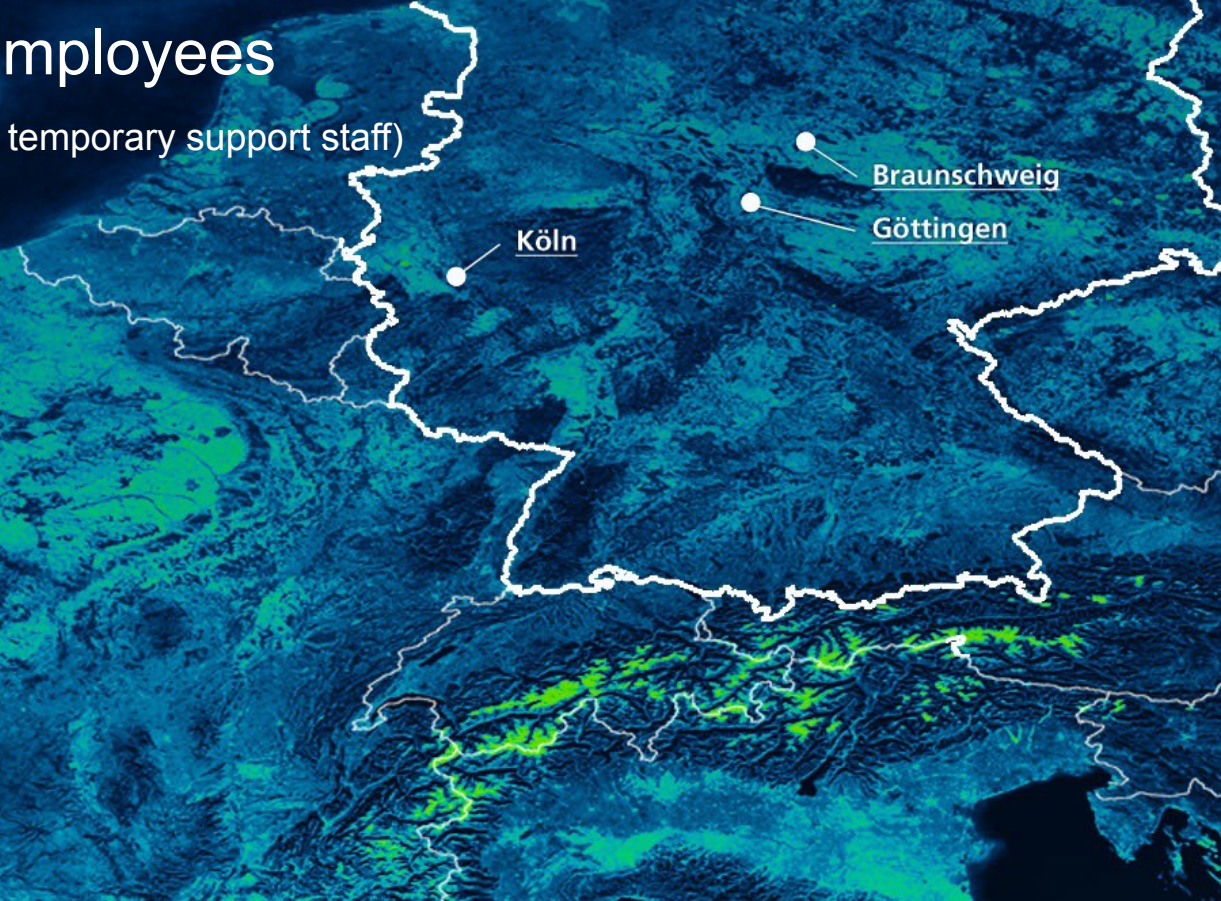
Braunschweig: ~ 150 Employees

Göttingen / Köln: ~ 200 Employees

Gesamt: ~ 350 Employees

(except internships, students, temporary support staff)

Status: May 2023



C²A²S²E Braunschweig | Where am I located ?



C²A²S²E
Braunschweig

SUM
Surrogate and
Uncertainties
Modeling

MDAO
Multidisciplinary
Design Analysis
and Optimisation

CODA
Next Generation
flow solver



**Numerical
Methods**
TAU

Ice Accretion

**Data Driven
Modeling**

**Adjoint Technique
Frequency
Domain solver**

**Interaction
aerodynamic/
aeroelasticity**

Status: May 2023

FLIGHT DYNAMIC STABILITY

Flight Dynamics

Aircraft motion - three axes (t/r)

Arbitrary flight-orientation

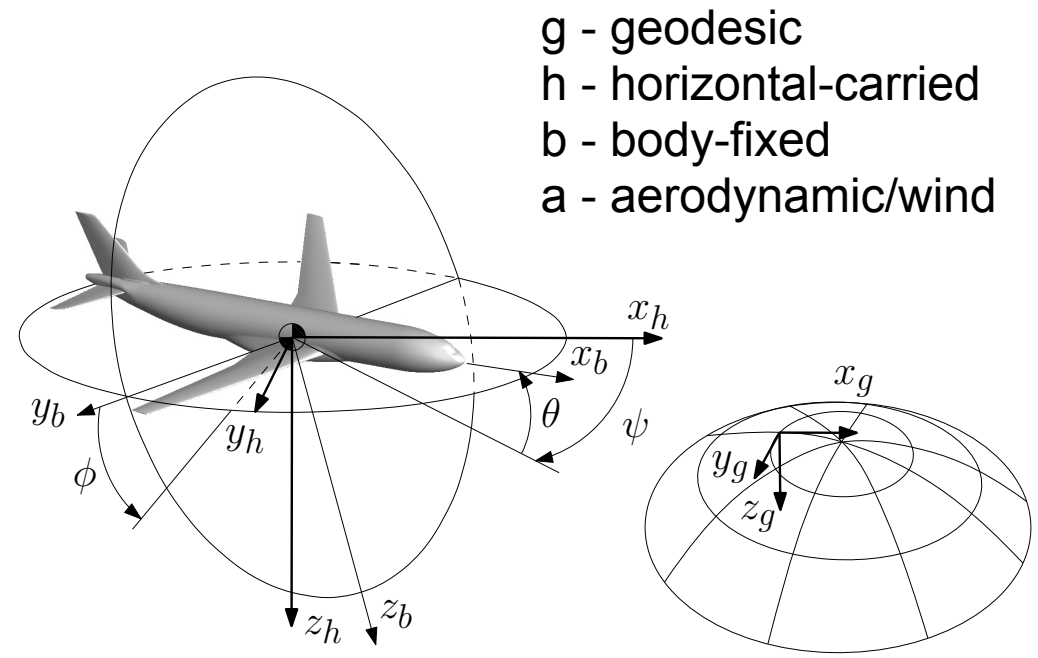
Considerations

Flight trajectory

Aircraft - Fluid interaction

Subsonic-Transonic flow region

Question: Is flight vehicle stable or unstable after perturbation from trimmed state?



Flight Dynamics

Aircraft motion - three axes (t/r)

Arbitrary flight-orientation

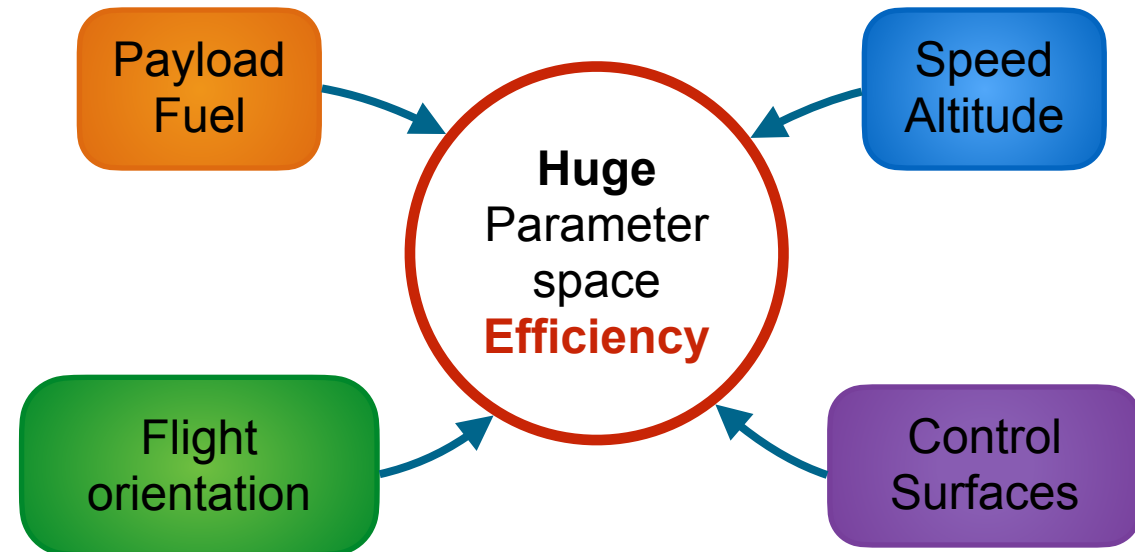
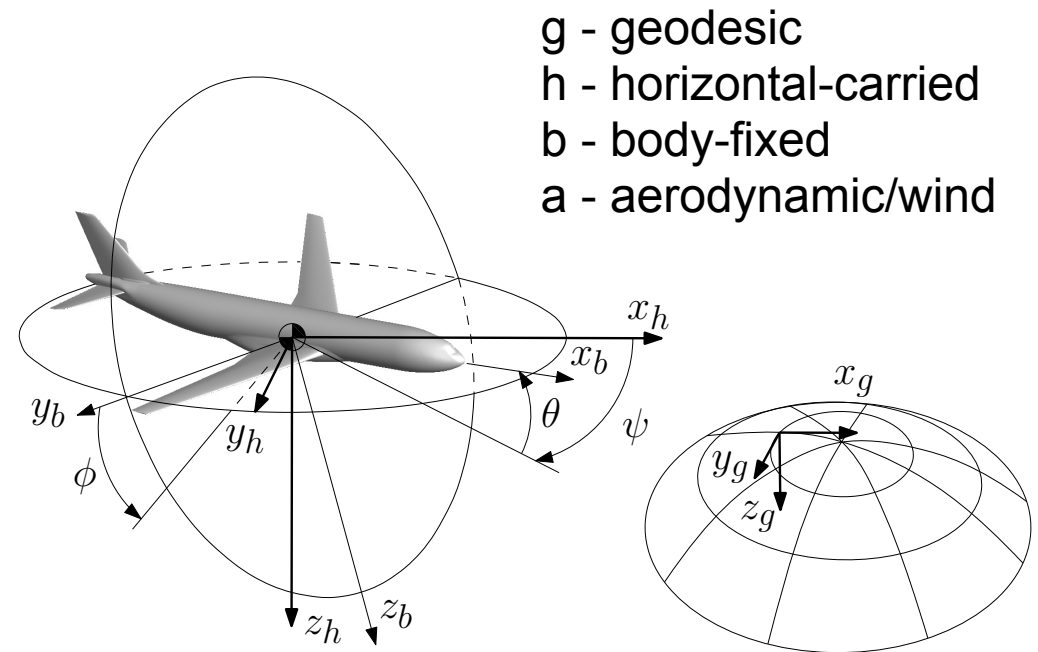
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Question: Is flight vehicle stable or unstable after perturbation from trimmed state?



Why so difficult? Transonics and its traps

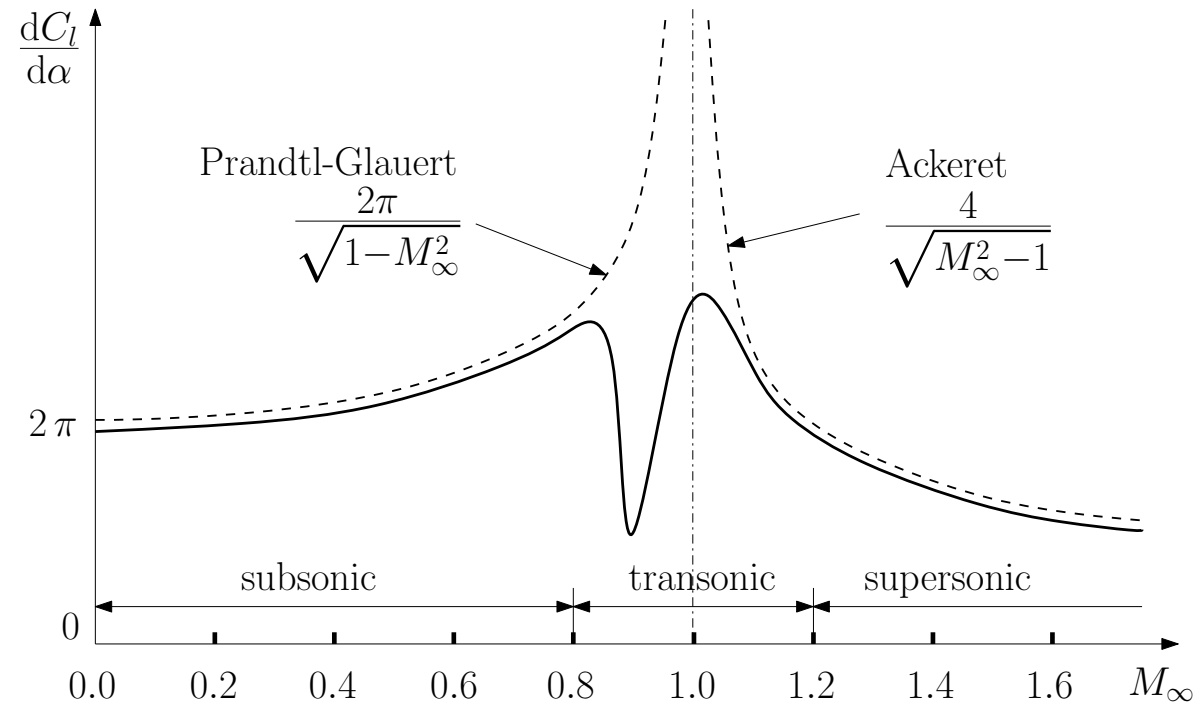
Transonic flow region
linear - nonlinear

Shocks

Influence of viscosity
(flow separation)

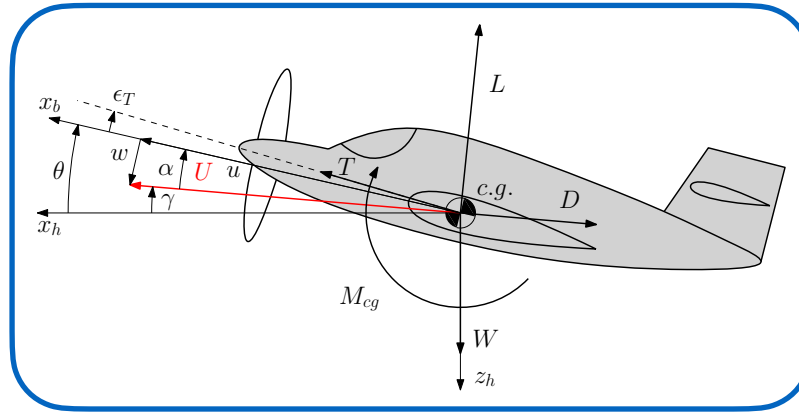
To reflect this flow phenomena,
demand of certain numerical schemes:
Navier-Stokes Eqn.

Accuracy



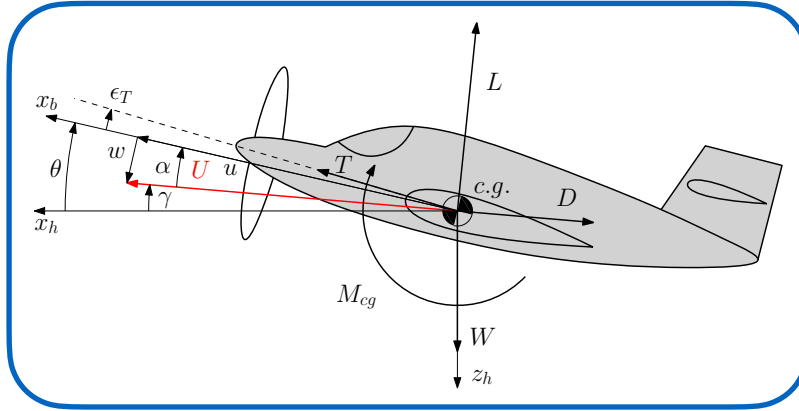
Problem reduction

Flight vehicle - Flight state

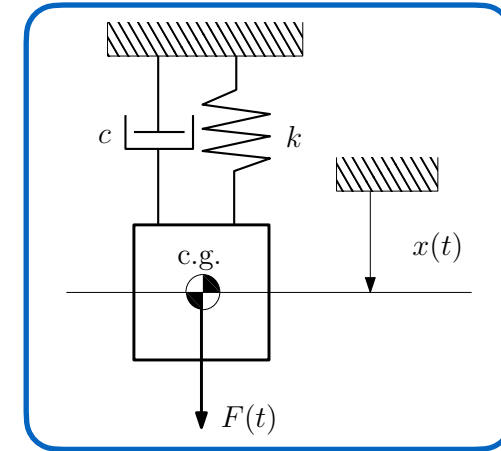


Problem reduction

Flight vehicle - Flight state



Spring-mass
damper oscillator



Nonlinear equations of motion

$$\begin{aligned}\mathbf{x} &= [x, y, z]^T, & \boldsymbol{\varphi} &= [\phi, \theta, \psi]^T, \\ \dot{\mathbf{x}} &= \mathbf{T}_{hb} \mathbf{U}, & \dot{\boldsymbol{\varphi}} &= \mathbf{R}_{bh}^{-1} \boldsymbol{\omega}_b, \\ \ddot{\mathbf{x}} &= \frac{1}{m} \mathbf{X} + \mathbf{T}_{hb} \mathbf{W} + \boldsymbol{\omega}_b \times \mathbf{U}, & \ddot{\boldsymbol{\varphi}} &= \mathbf{I}^{-1} (\mathbf{M} - \boldsymbol{\omega}_b \times \mathbf{I} \boldsymbol{\omega}_b)\end{aligned}$$

Initial conditions
Small perturbations
Linearisation - Taylor series

$$\begin{aligned}\mathbf{f}[\mathbf{x}(t), \dot{\mathbf{x}}(t)] &= 0, \quad \mathbf{f}[\bar{\mathbf{x}}, \dot{\bar{\mathbf{x}}}] = 0, \\ \mathbf{x}(t) &\approx \bar{\mathbf{x}} + \tilde{\mathbf{x}}(t), \\ \dot{\mathbf{x}}(t) &\approx \dot{\bar{\mathbf{x}}} + \dot{\tilde{\mathbf{x}}}(t), \\ \mathbf{f}(t) &\approx \mathbf{f}[\bar{\mathbf{x}}, \dot{\bar{\mathbf{x}}}] + \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \tilde{\mathbf{x}} + \frac{\partial \mathbf{f}}{\partial \dot{\mathbf{x}}} \dot{\tilde{\mathbf{x}}} = 0\end{aligned}$$

$\mathbf{x}(t) = \mathbf{x}_0 \sin(\omega t)$
Harmonic response
Stability $\mathbf{F}(t) = 0$

$$\dot{\tilde{\mathbf{x}}} = \mathbf{A} \tilde{\mathbf{x}}$$

Jacobian, state matrix **A**, exact symmetry x-z, **longitudinal** + lateral/directional

$$\mu = \frac{2m}{\rho_{\infty} S l_{\mu}}$$



body-fixed (u,v,w)

$$\mathbf{A}'_{\text{long},b} = \begin{bmatrix} \frac{1}{2\mu} C_{X_u} & \frac{1}{2\mu} C_{X_w} & \frac{1}{2\mu} C_{X_q} - \frac{\bar{w}}{U_{\infty}} & -\frac{1}{2\mu} C_G \cos \bar{\theta} \\ \frac{1}{2\mu} C_{Z_u} & \frac{1}{2\mu} C_{Z_w} & \frac{1}{2\mu} C_{Z_q} + \frac{\bar{u}}{U_{\infty}} & -\frac{1}{2\mu} C_G \cos \bar{\phi} \sin \bar{\theta} \\ \frac{1}{i_{yy}} C_{m_u} & \frac{1}{i_{yy}} C_{m_w} & \frac{1}{i_{yy}} C_{m_q} & 0 \\ 0 & 0 & \cos \bar{\phi} & 0 \end{bmatrix}$$

trimmed state
arbitrary flight orientation
no simplification velocity components

aerodynamic (U, α , β)

$$\mathbf{A}'_{\text{long},w} \approx \begin{bmatrix} -\frac{1}{2\mu} C_{D_u} & \frac{1}{2\mu} (\bar{C}_L - C_{D_{\alpha}}) & -\frac{1}{2\mu} C_{D_q} & -\frac{1}{2\mu} C_G \cos \bar{\theta} \\ -\frac{1}{\phi} C_{L_u} & -\frac{1}{\phi} (\bar{C}_D + C_{L_{\alpha}}) & \frac{1}{\phi} (2\mu - C_{L_q}) & -\frac{1}{\phi} C_G \sin \bar{\theta} \\ a_{31} & \frac{1}{i_{yy}} (C_{m_{\alpha}} - C_{m_{\dot{\alpha}}} \frac{(\bar{C}_D + C_{L_{\alpha}})}{\phi}) & \frac{1}{i_{yy}} (C_{m_q} + C_{m_{\dot{\alpha}}} \frac{(2\mu - C_{L_q})}{\phi}) & -\frac{C_{m_{\dot{\alpha}}} C_G \sin \bar{\theta}}{i_{yy} \phi} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

with : $\frac{1}{\phi} = 2\mu + C_{L_{\dot{\alpha}}}$, $a_{31} = \frac{1}{i_{yy}} \left(C_{m_u} - C_{m_{\dot{\alpha}}} \frac{C_{L_u}}{\phi} \right)$

trimmed state, second column: $\sin \alpha = w/U$
 $\alpha \approx \beta \approx 0^\circ$, $\sin \alpha \approx \alpha$, $\cos \alpha \approx 1$
 $v \approx w \approx 0$, $\dot{v} \approx \dot{w} \approx 0$

$$\mathbf{A} - I\lambda = 0 \rightarrow \lambda = \sigma \pm i\omega \rightarrow x(t) = x_0 e^{\sigma t} \cos(\omega t + \varphi)$$

state matrix **A**: **dynamic** / **steady-state static**, **quasi-steady** and **dynamic** derivatives

Jacobian, state matrix **A**, exact symmetry x-z, **longitudinal** + lateral/directional

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trimmed state

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$A'_{\text{long},w} \approx$

$$\begin{bmatrix} -\frac{1}{2\mu} C_{Du} & \frac{1}{2\mu} (\bar{C}_L - C_{D\alpha}) & -\frac{1}{2\mu} C_{Dq} & -\frac{1}{2\mu} C_G \cos \bar{\theta} \\ -\frac{1}{\phi} C_{Lu} & -\frac{1}{\phi} (\bar{C}_D + C_{L\alpha}) & \frac{1}{\phi} (2\mu - C_{Lq}) & -\frac{1}{\phi} C_G \sin \bar{\theta} \\ a_{31} & \frac{1}{i_{yy}} (C_{m\alpha} - C_{m\dot{\alpha}} \frac{(\bar{C}_D + C_{L\alpha})}{\phi}) & \frac{1}{i_{yy}} (C_{mq} + C_{m\dot{\alpha}} \frac{(2\mu - C_{Lq})}{\phi}) & -\frac{C_{m\dot{\alpha}} C_G \sin \bar{\theta}}{i_{yy} \phi} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

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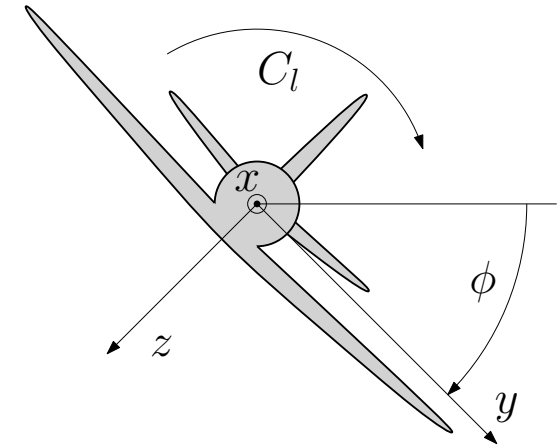
$$\mu = \frac{2m}{\rho_{\infty} S l_{\mu}}$$



Coupled motion

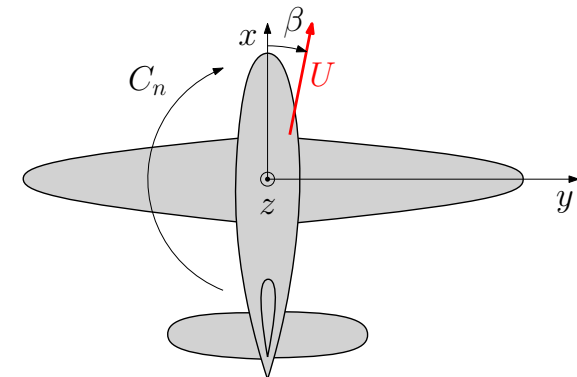
$$\mathbf{A}'_{\text{lat/dir},b} =$$

$$\begin{bmatrix} \frac{1}{2\mu} C_{Y_v} & \frac{1}{2\mu} C_{Y_p} + \frac{\bar{w}}{U_{\infty}} & \frac{1}{2\mu} C_{Y_r} - \frac{\bar{u}}{U_{\infty}} & \frac{1}{2\mu} C_G \cos \bar{\theta} \\ \frac{1}{i_D} (i_{zz} C_{l_v} + i_{xz} C_{n_v}) & \frac{1}{i_D} (i_{zz} C_{l_p} + i_{xz} C_{n_p}) & \frac{1}{i_D} (i_{zz} C_{l_r} + i_{xz} C_{n_r}) & 0 \\ \frac{1}{i_D} (i_{xz} C_{l_v} + i_{xx} C_{n_v}) & \frac{1}{i_D} (i_{xz} C_{l_p} + i_{xx} C_{n_p}) & \frac{1}{i_D} (i_{xz} C_{l_r} + i_{xx} C_{n_r}) & 0 \\ 0 & 1 & \tan \bar{\theta} & 0 \end{bmatrix}$$



$$\mathbf{A}'_{\text{lat/dir},w} \approx$$

$$\begin{bmatrix} \frac{1}{2\mu} C_{Y_{\beta}} & \frac{1}{2\mu} C_{Y_p} & \frac{1}{2\mu} C_{Y_r} - 1 & \frac{1}{2\mu} C_G \cos \bar{\theta} \\ \frac{1}{i_D} (i_{zz} C_{l_{\beta}} + i_{xz} C_{n_{\beta}}) & \frac{1}{i_D} (i_{zz} C_{l_p} + i_{xz} C_{n_p}) & \frac{1}{i_D} (i_{zz} C_{l_r} + i_{xz} C_{n_r}) & 0 \\ \frac{1}{i_D} (i_{xz} C_{l_{\beta}} + i_{xx} C_{n_{\beta}}) & \frac{1}{i_D} (i_{xz} C_{l_p} + i_{xx} C_{n_p}) & \frac{1}{i_D} (i_{xz} C_{l_r} + i_{xx} C_{n_r}) & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$



Stability Derivatives

VLM - Vortex-Lattice
DLM - Doublet-Lattice
URANS - Unsteady R-A Navier-Stokes
LFD - Linear Frequency Domain



Unsteady motion

Stability Derivatives

VLM - Vortex-Lattice
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Unsteady motion

VLM/DLM

Lifting surface theory

Inviscid treatment - no
viscosity

No shock treatment

Extremely fast

Resolution of aircraft ?

Stability Derivatives

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URANS

Complete resolution of motion in the time domain

Excitation dependent

Full frequency spectrum

Intensive in time and computational resources

High resolution of aircraft

Stability Derivatives

VLM - Vortex-Lattice
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Unsteady motion

VLM/DLM

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Resolution of aircraft ?

URANS

Complete resolution of motion in the time domain

Excitation dependent

Full frequency spectrum

Intensive in time and computational resources

High resolution of aircraft

LFD

URANS-based - RANS properties remain

Small perturbations
Excitation independent

Solved in the frequency domain for one frequency

High resolution of aircraft remains

Linear Frequency Domain Method - LFD



Small perturbation approach, periodic motion, harmonic response

$$\begin{aligned} W(t) - \bar{W} = \tilde{W}(t) &\approx \text{Real}(\hat{W}e^{i\omega t}) & W \dots \text{conservative flow state vector} \\ x(t) - \bar{x} = \tilde{x}(t) &\approx \text{Real}(\hat{x}e^{i\omega t}) & x \dots \text{grid-node vector} \end{aligned}$$

Semi-discrete URANS (**Spalart-Allmaras one equation turbulence model**)

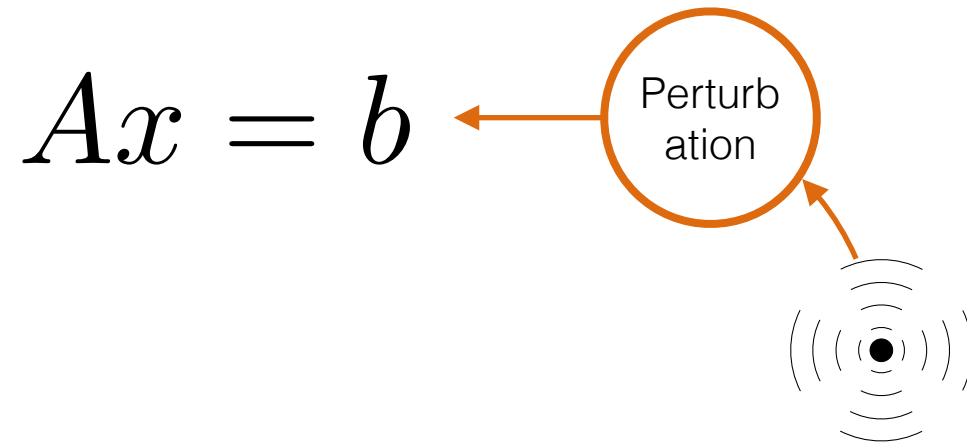
$$\frac{d(MW)}{dt} + R(W, x, \dot{x}) = 0$$

Consistent linearisation - complex-valued linear system of equations

$$\left[i\omega\bar{M} + \frac{\partial R}{\partial W} \right] \hat{W} = - \left[\frac{\partial R}{\partial x} + i\omega \left(\frac{\partial R}{\partial \dot{x}} + \bar{W} \frac{\partial M}{\partial x} \right) \right] \hat{x} \quad \rightarrow \quad \mathbf{A} \hat{W} = b$$

Solution scheme: Direct solver, ILU-preconditioner, Krylov-GMRes

$$Ax = b$$



Change of vector **b**

Self-induced forces

Rigid body motion

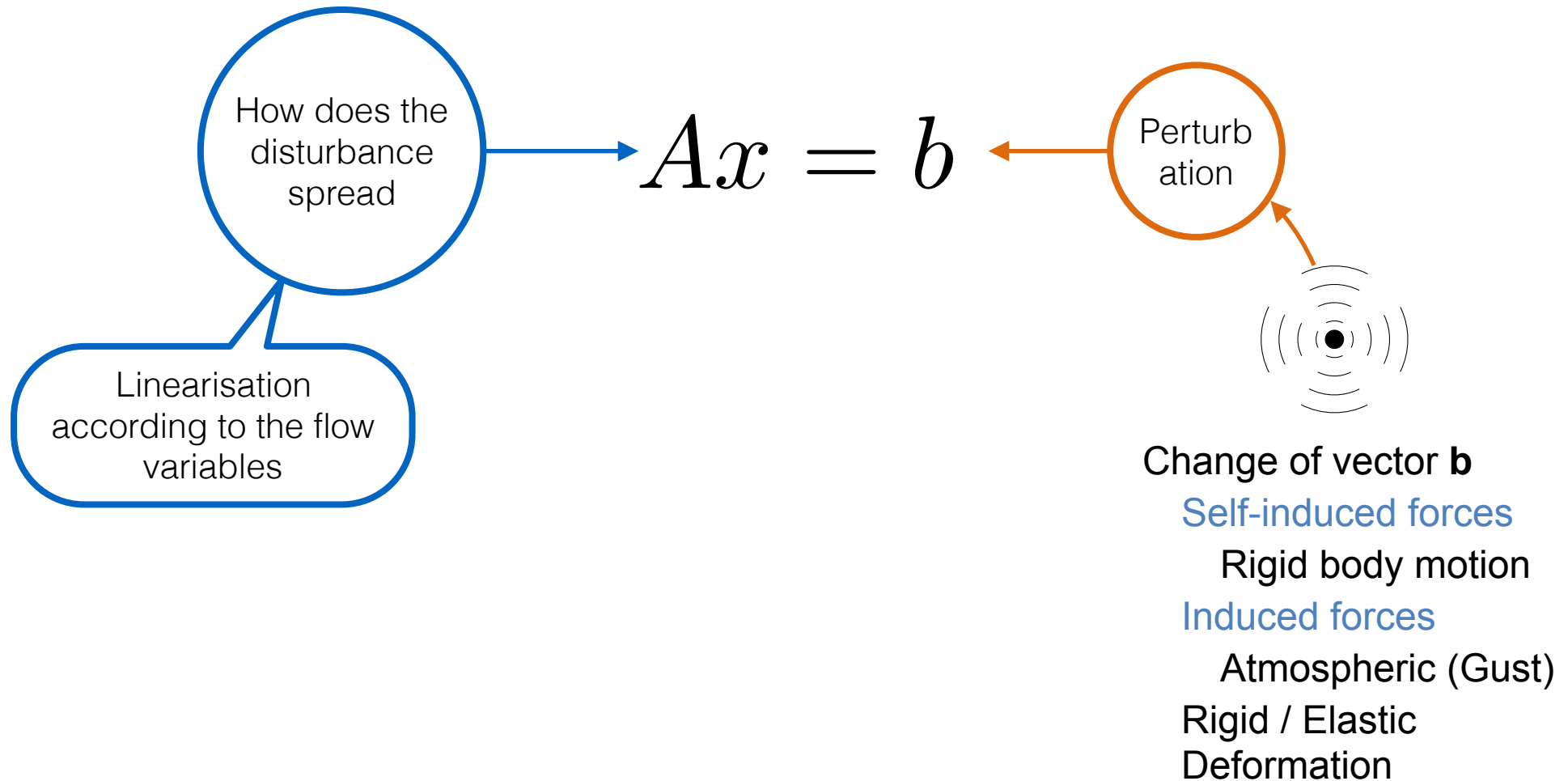
Induced forces

Atmospheric (Gust)

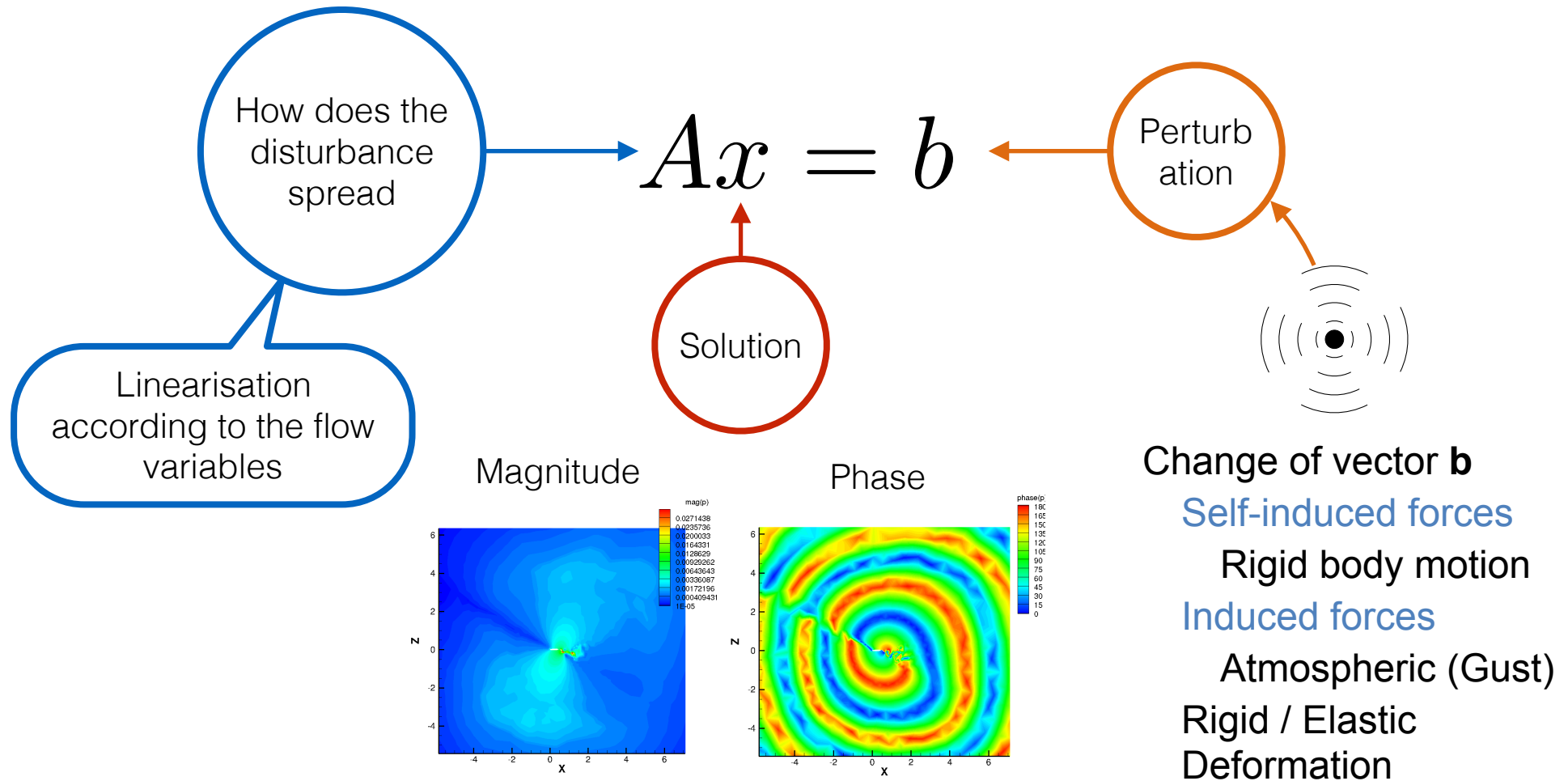
Rigid / Elastic

Deformation

Problem reduction | Complex-valued linear system



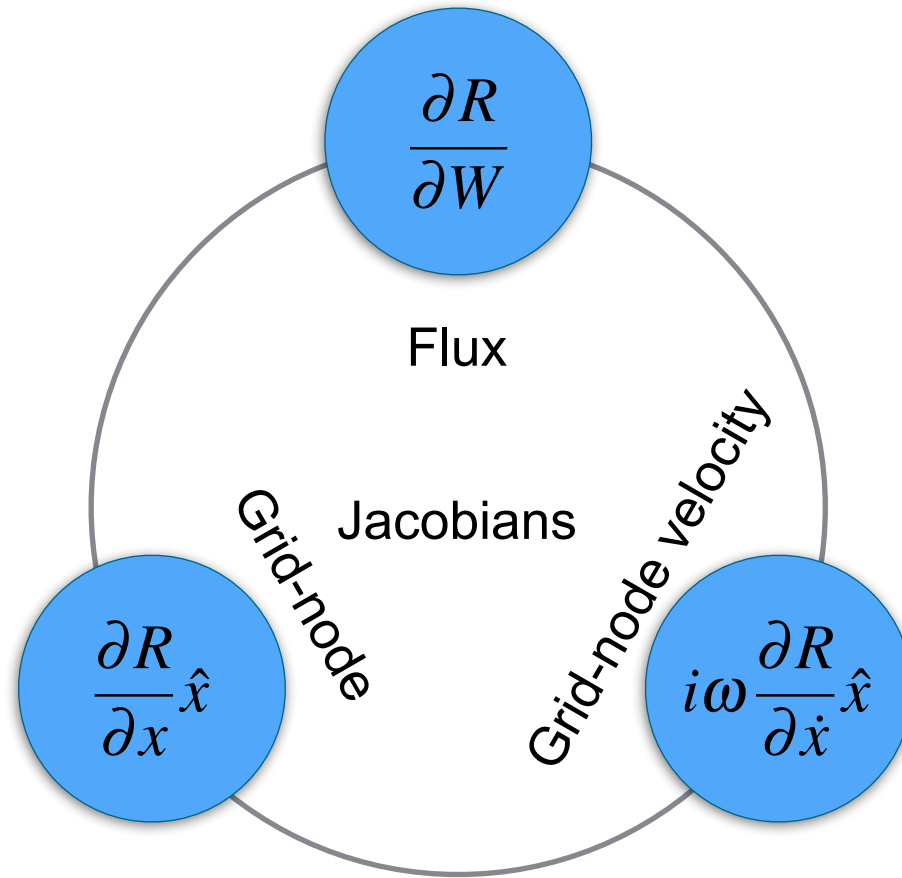
Problem reduction | Complex-valued linear system



Jacobians and Perturbation



Jacobians and Perturbation



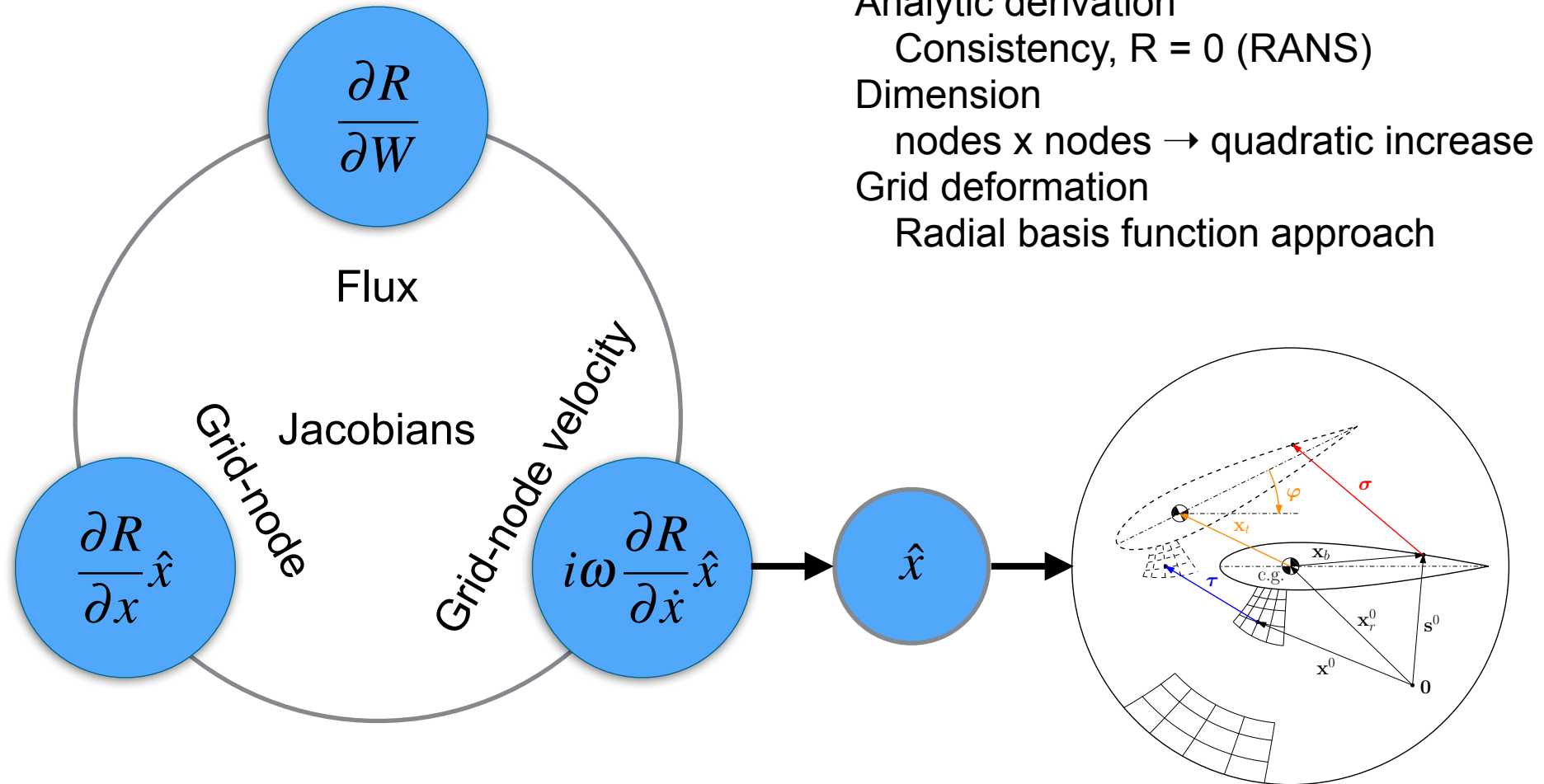
Analytic derivation

Consistency, $R = 0$ (RANS)

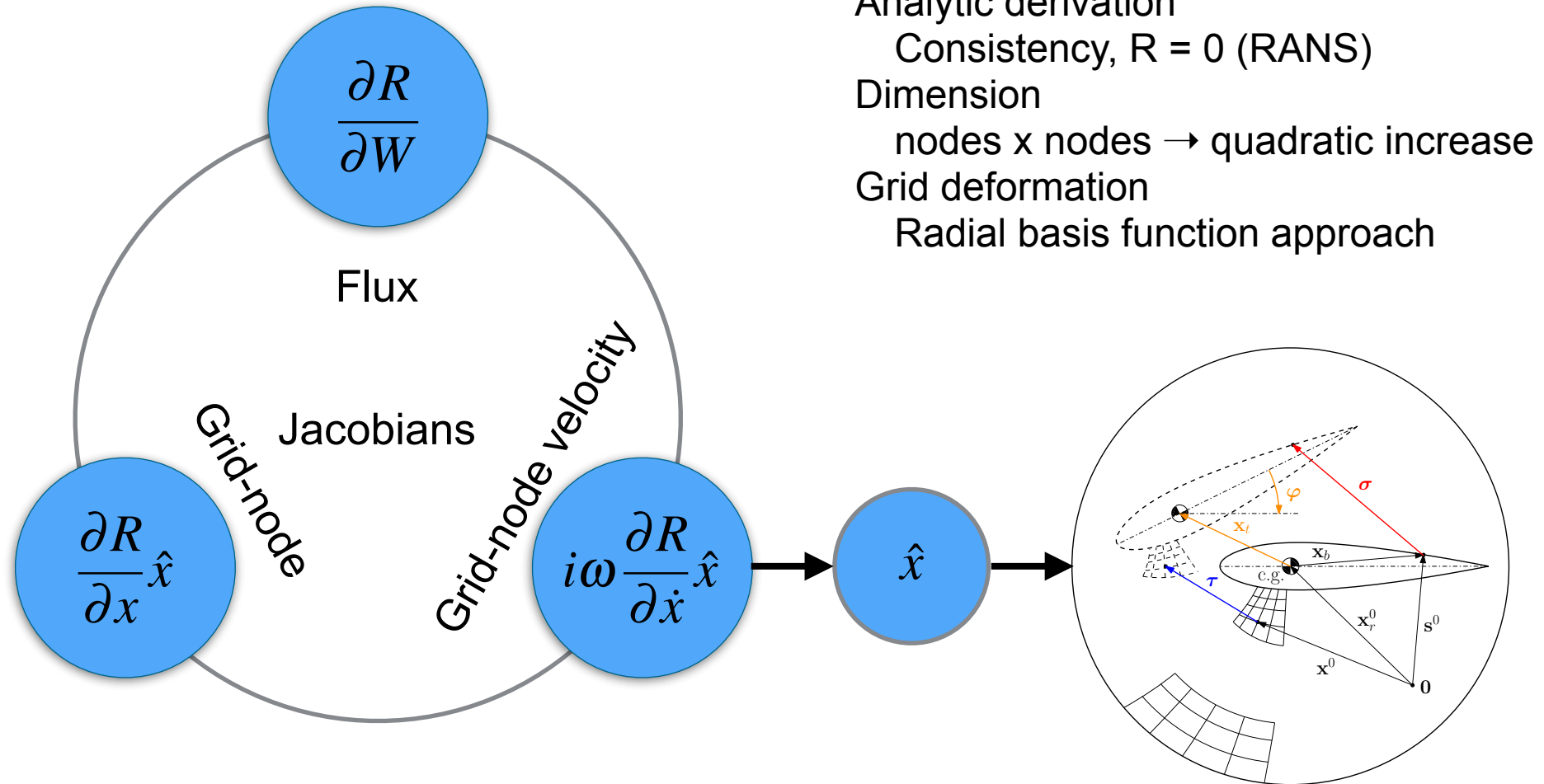
Dimension

nodes x nodes $\rightarrow$ quadratic increase

Jacobians and Perturbation



Jacobians and Perturbation



Turbulence model - Spalart-Allmaras

Importance of Linearisation of Turbulence Model

NACA 64A010, $M=0.8$, $Re=12.5$ mill., $\alpha=0/2$ deg



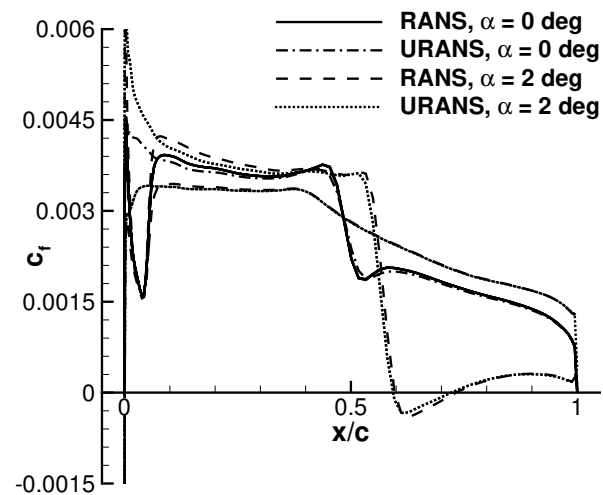
Why so important?

One-equation Spalart-Allmaras

Attached flow, $\alpha = 0$ deg

Partly separated flow, $\alpha = 2$ deg

FEVA - Frozen Eddy Viscosity Approach



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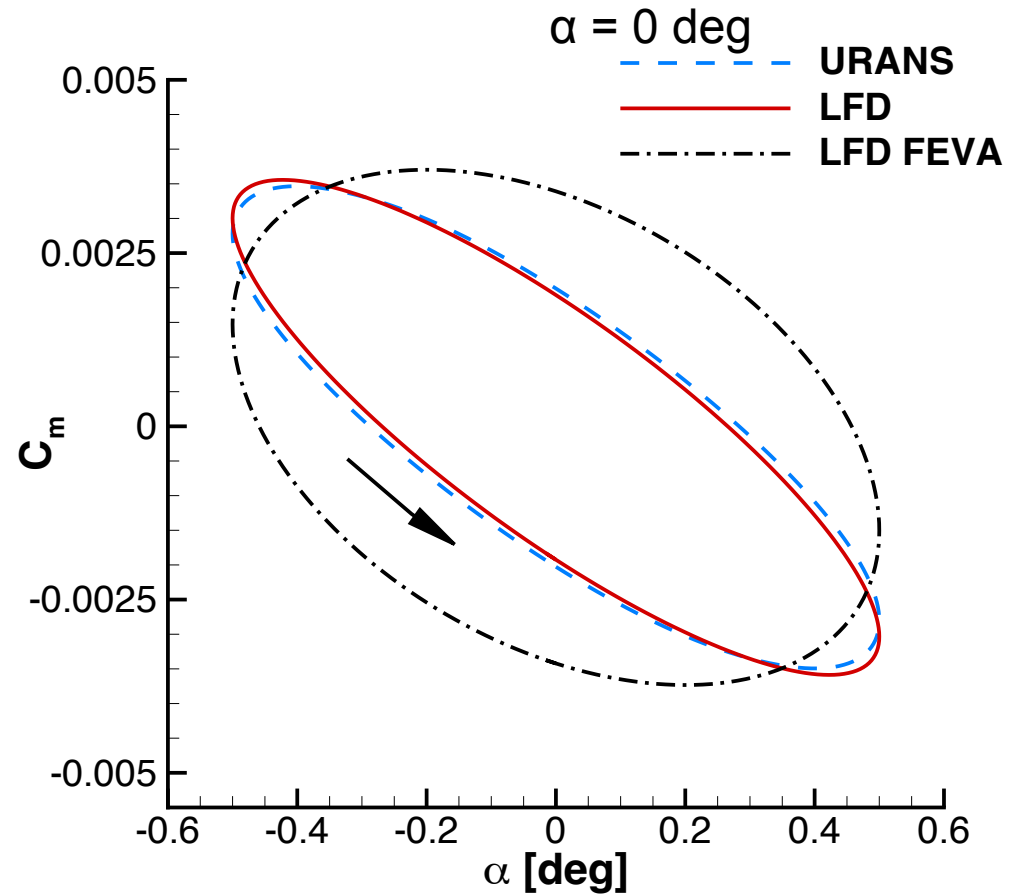
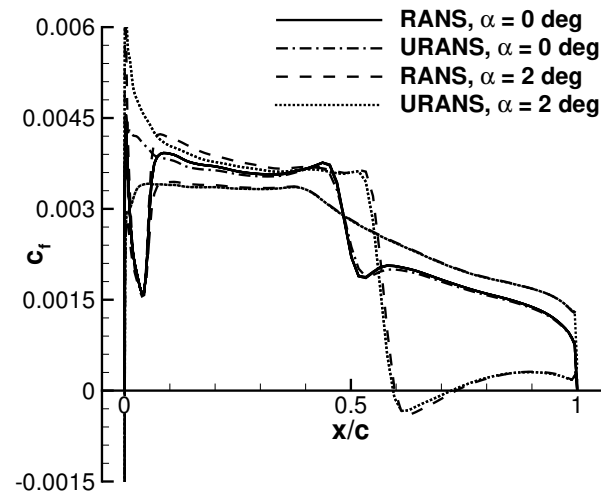
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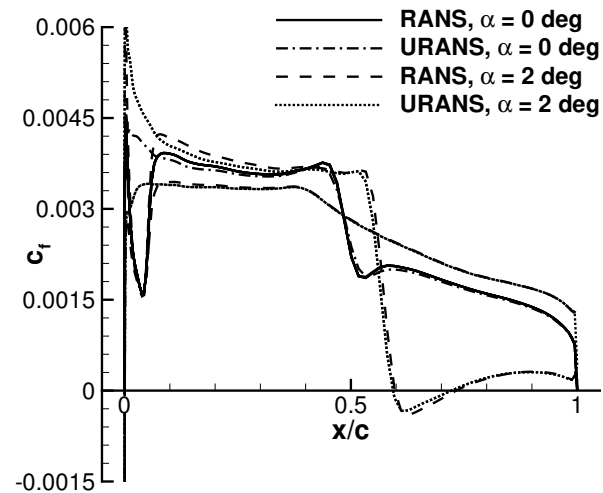
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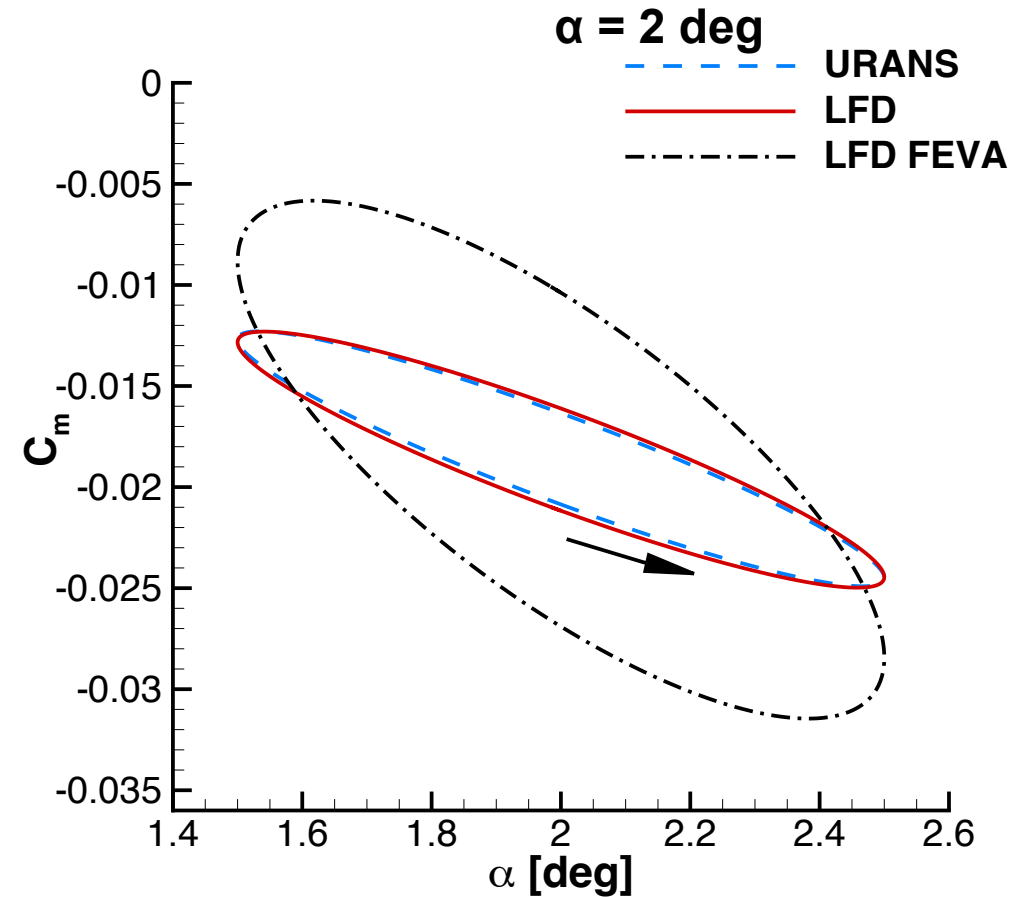
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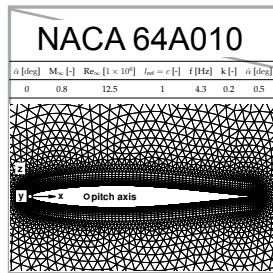
FEVA - Frozen Eddy Viscosity Approach



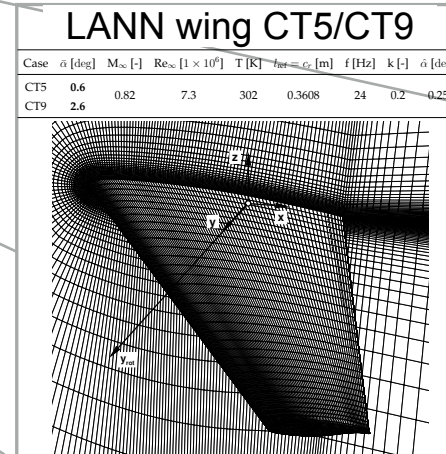
FEVA - unacceptable



LFD - Validation

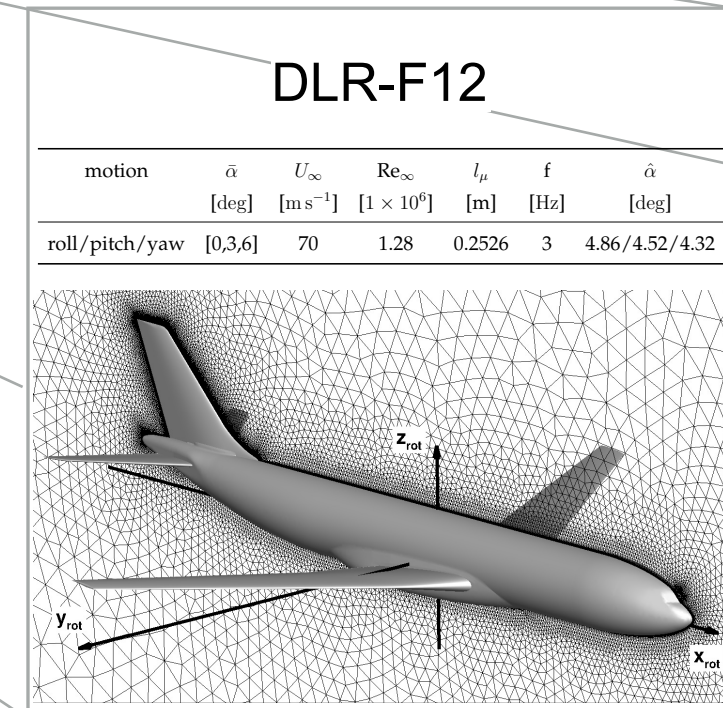


Initial investigation
Turbulence Model



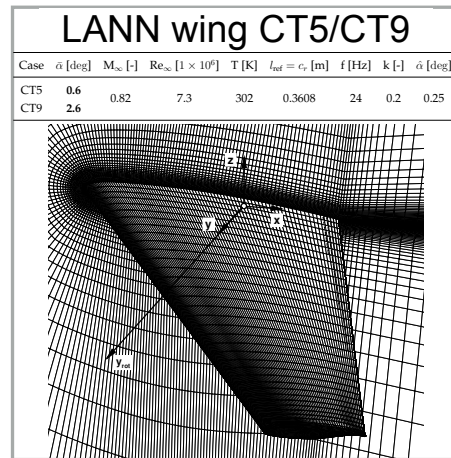
Solution techniques
breakthrough
speed up

years of extensive testing

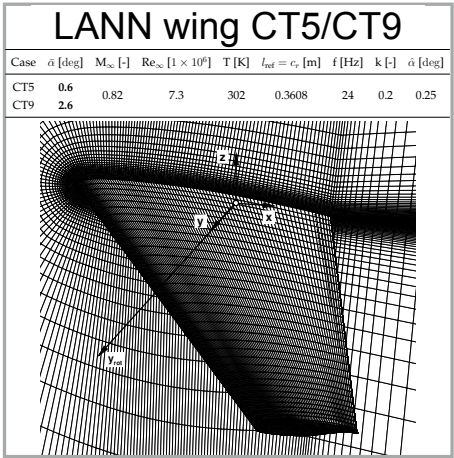


Experimental data
roll/pitch/yaw

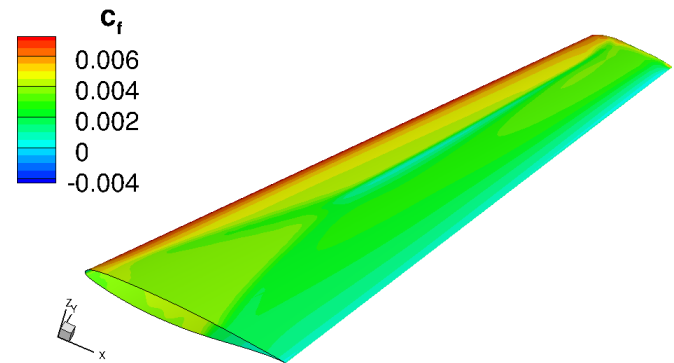
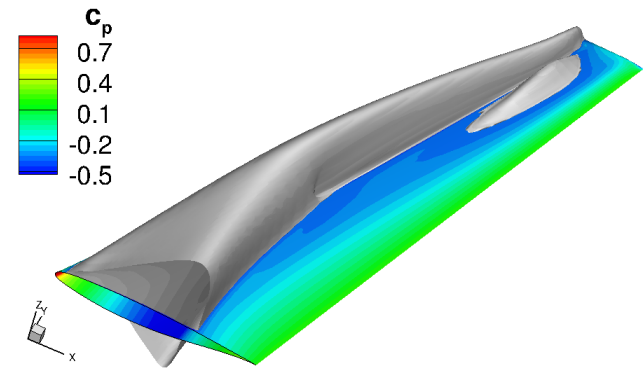
LFD - Validation



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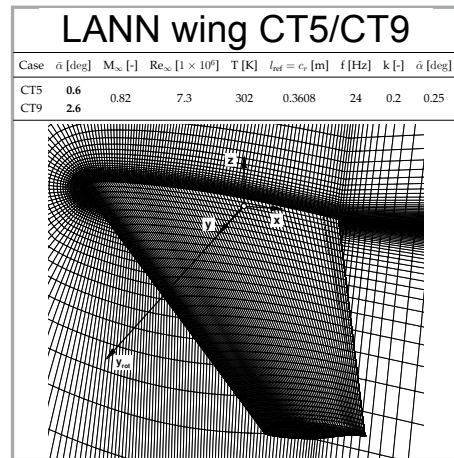


Case	$\bar{\alpha}$ [deg]	M_∞ [-]	Re_∞ [1×10^6]	T [K]	$l_{ref} = c_r$ [m]	f [Hz]	k [-]	$\hat{\alpha}$ [deg]
CT5	0.6	0.82	7.3	302	0.3608	24	0.2	0.25
CT9	2.6							

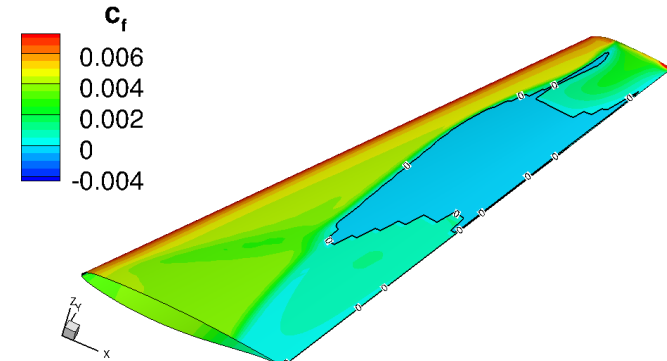
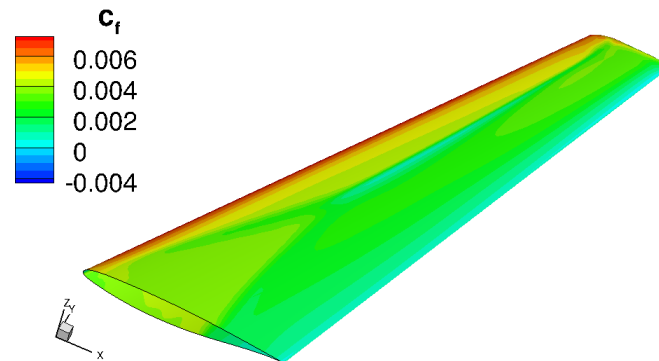
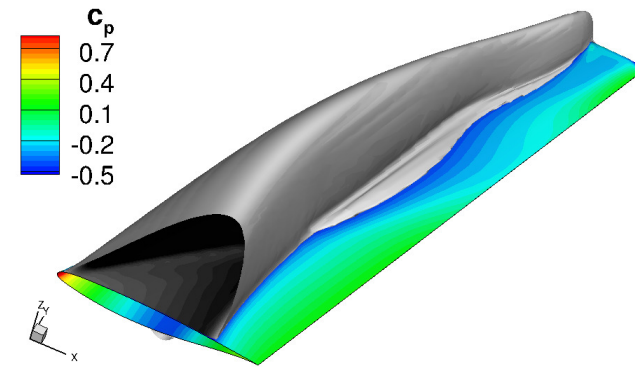
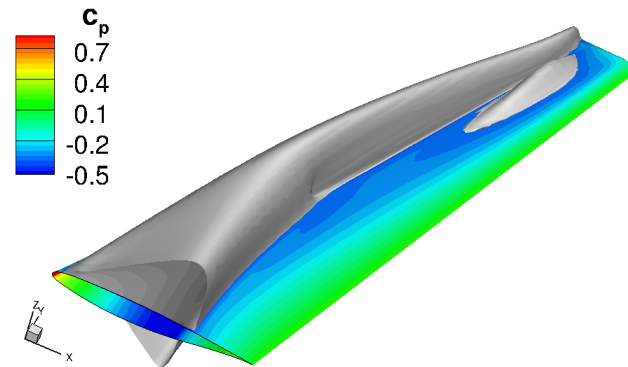


CT5

LFD - Validation



Case	$\bar{\alpha}$ [deg]	M_∞ [-]	Re_∞ [1×10^6]	T [K]	$l_{ref} = c_r$ [m]	f [Hz]	k [-]	$\hat{\alpha}$ [deg]
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CT5

CT9

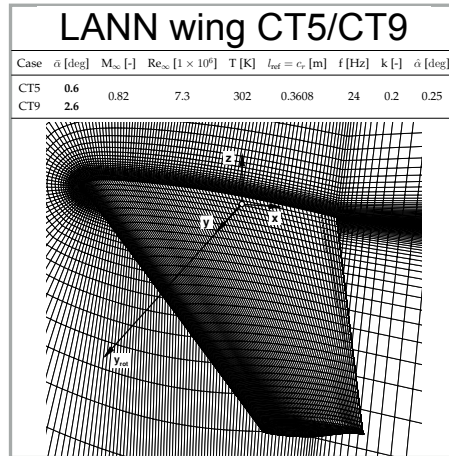
Frequency/Amplitude Response Function (FRF/ARF)

Rigid motion in pitch - Lift

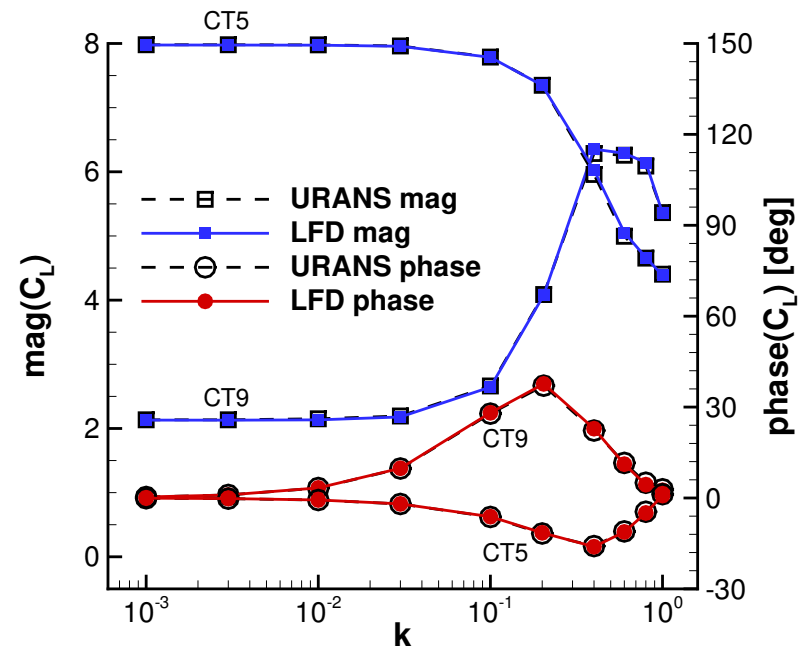


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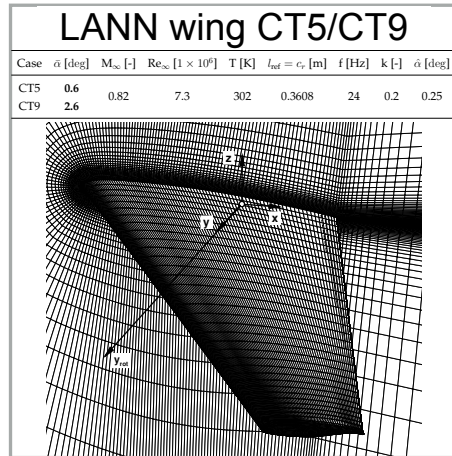
Phase lag / lead - Dynamically linear / non-linear



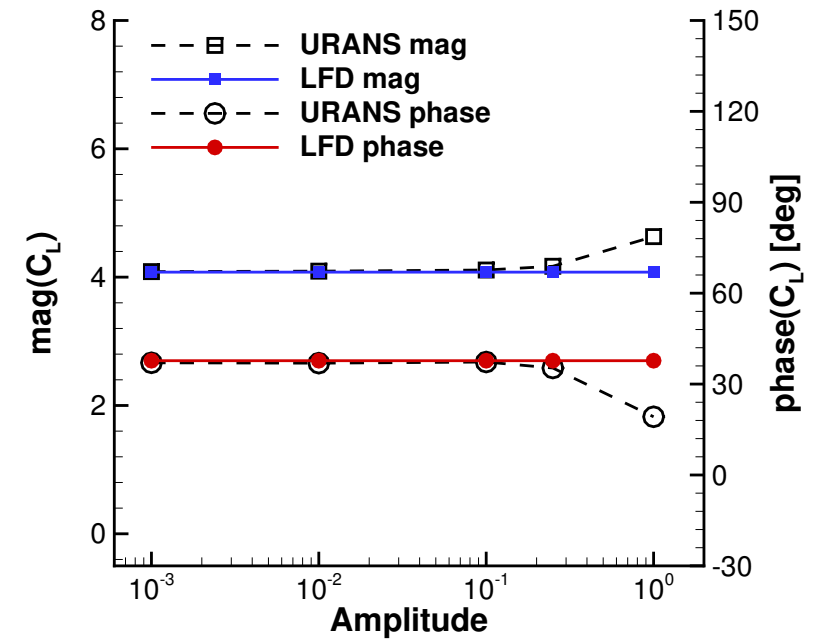
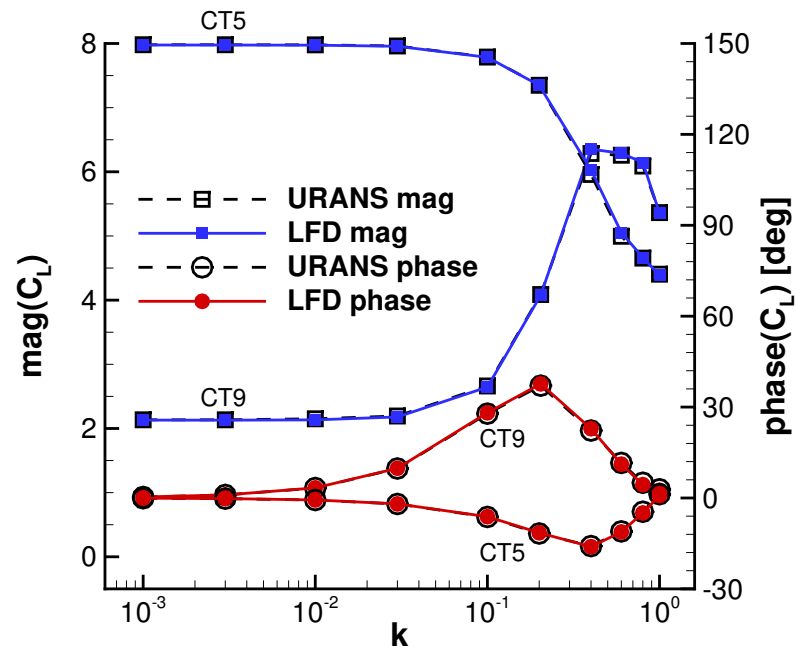
Pitch FRF CT5/CT9

Frequency/Amplitude Response Function (FRF/ARF)

Rigid motion in pitch - Lift

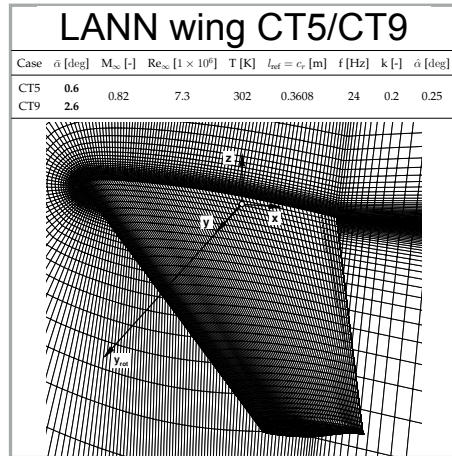


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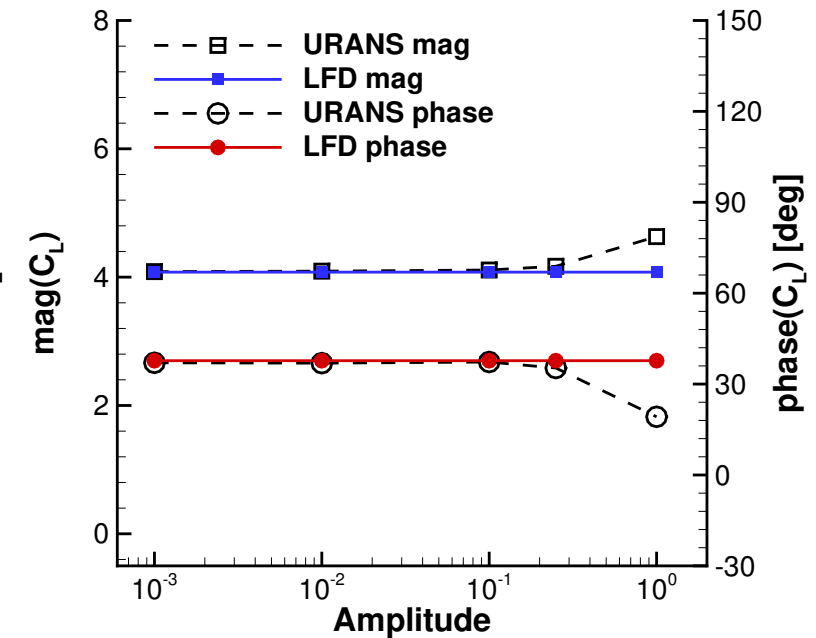
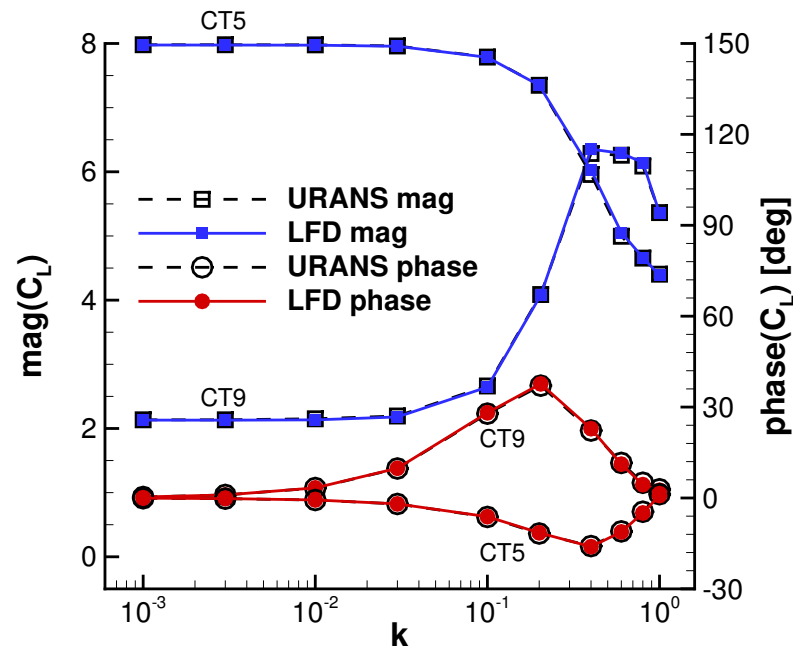
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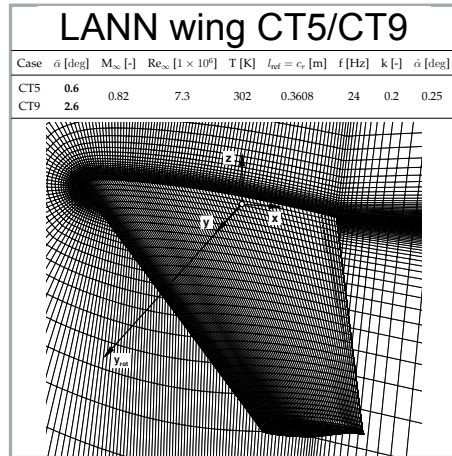
Phase lag / lead - Dynamically linear / non-linear

CASE	CPU-time (h)		URANS/LFD
	URANS	LFD	
CT5	994	6.7	148
CT9	5512	32	172



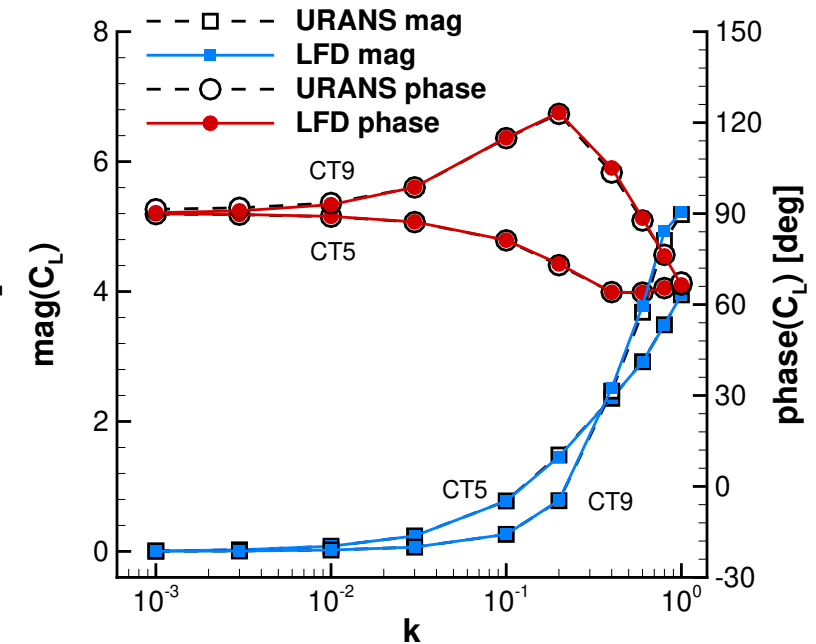
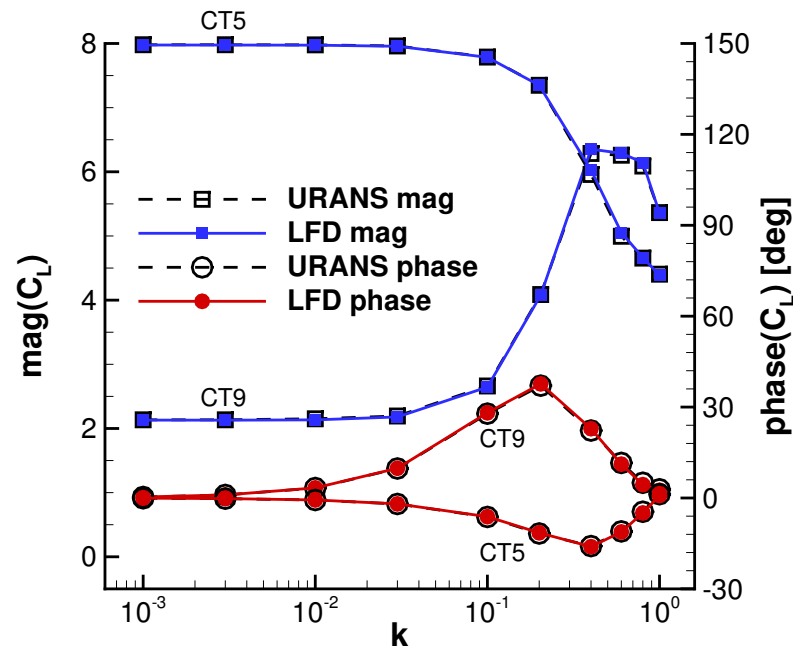
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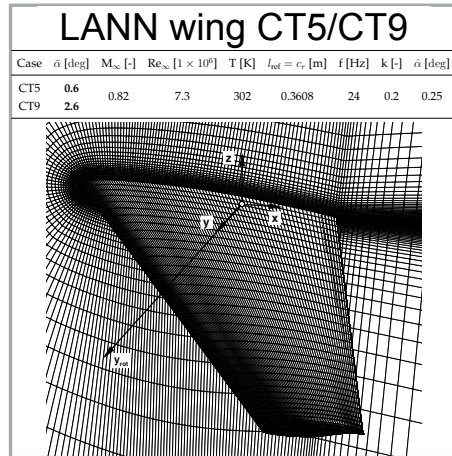


Pitch FRF CT5/CT9

Heave - FRF CT5/CT9

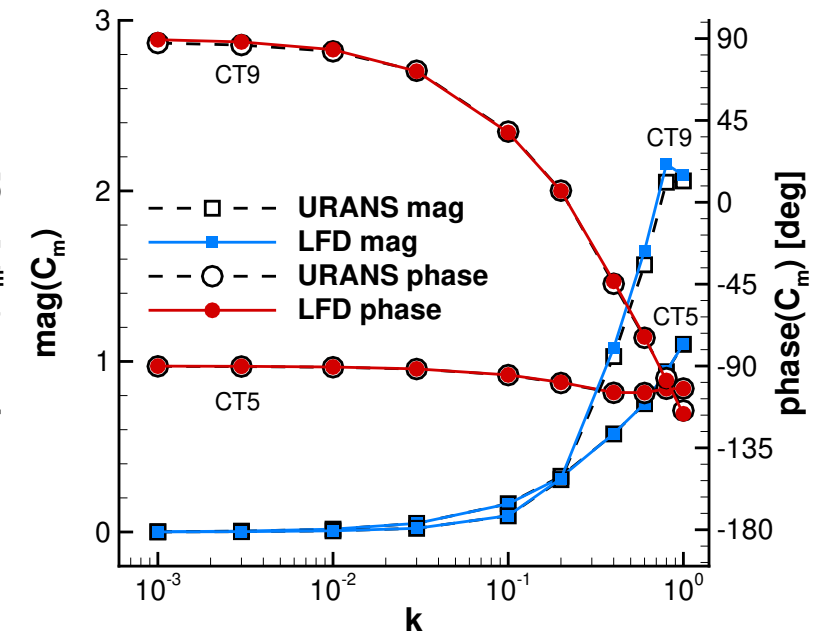
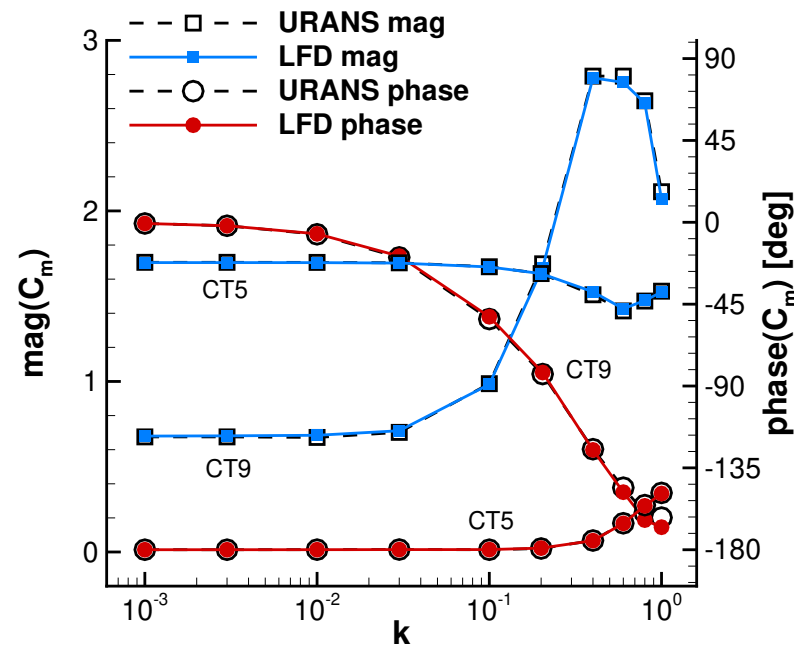
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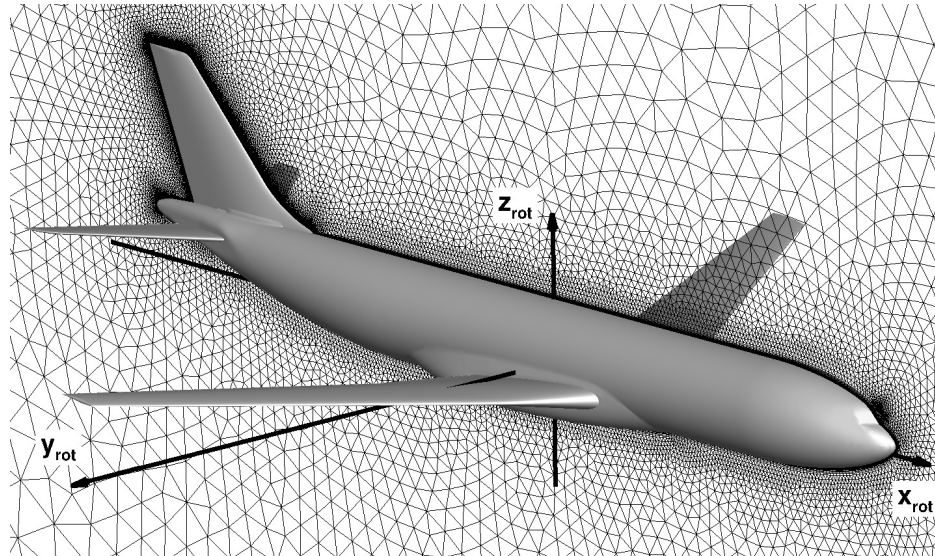


Pitch FRF CT5/CT9

Heave - FRF CT5/CT9

Flight stability prediction

DLR-F12 subsonic case



Full scale DLR-F12

Parameter	value
l_μ (MAC)	7.2705 m (0.252 625 m)
scale	28.78
S	367.87 m ² (0.444 14 m ²)
m	193 t
I_{xx}	1.50×10^7 kg m ²
I_{yy}	2.02×10^7 kg m ²
I_{zz}	3.46×10^7 kg m ²
I_{xz}	-7.70×10^5 kg m ²
$\mathbf{x}_{c.g.}$	[30.18, 0.0, -0.87] m ([1.049, 0.0, -0.030] m)
x_{LE-MAC}	27.0 m (0.9381 m)

Parameter space DLR-F12

Parameter	value/range
f [Hz]	[(0.0), 0.15, 0.3, 0.8, 1.5, 3.0, 8.0, 15.0, 33.0]
$\bar{\alpha}$ [deg]	[0, 3, 6]
U_∞	70 m s ⁻¹
ρ_∞	1.225 kg m ⁻³
p_∞	101 325 Pa
T_∞	288.15 K
$\mu = \frac{2m}{\rho_\infty S l_\mu}$	118.2
I_{ref}	21 651 kg m ²

Full scale DLR-F12

Subsonic case - Flight Stability



LFD (filled) - VLM (lucent, M. Hepperle)
 $M = 0.206$, $Re = 1.28 \text{ m.}$, $\alpha = \beta = 0^\circ$
Trimmed case
Five typical aircraft modes

Longitudinal

Short period mode

Phugoid (viscosity)

2nd order time response

Lateral/Directional

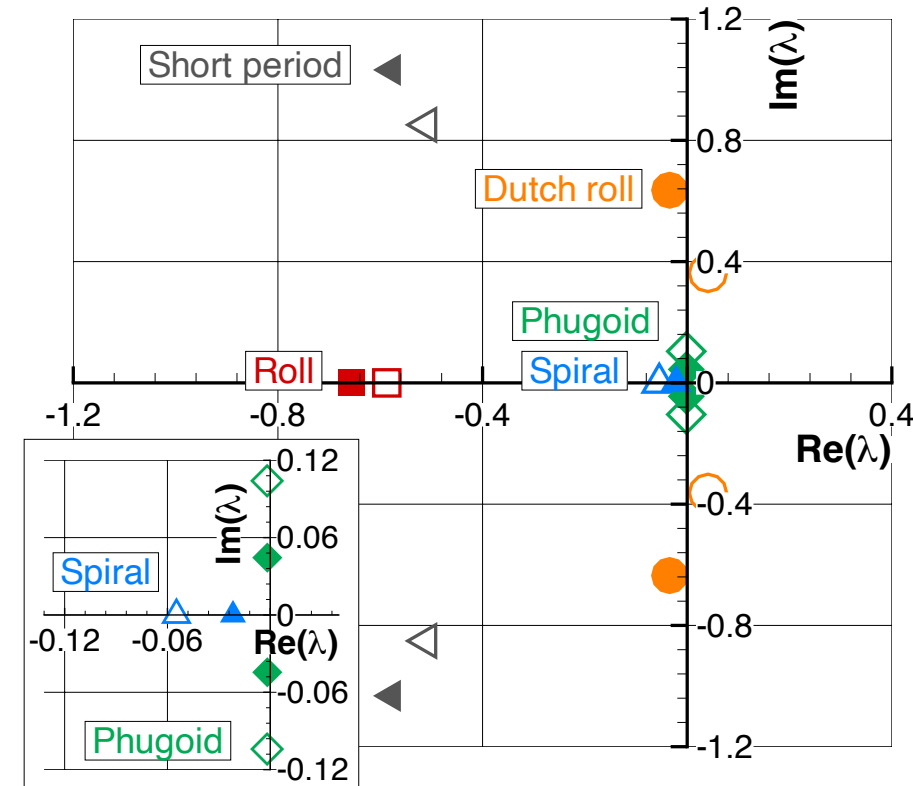
Dutch roll mode

2nd order time response

Roll mode

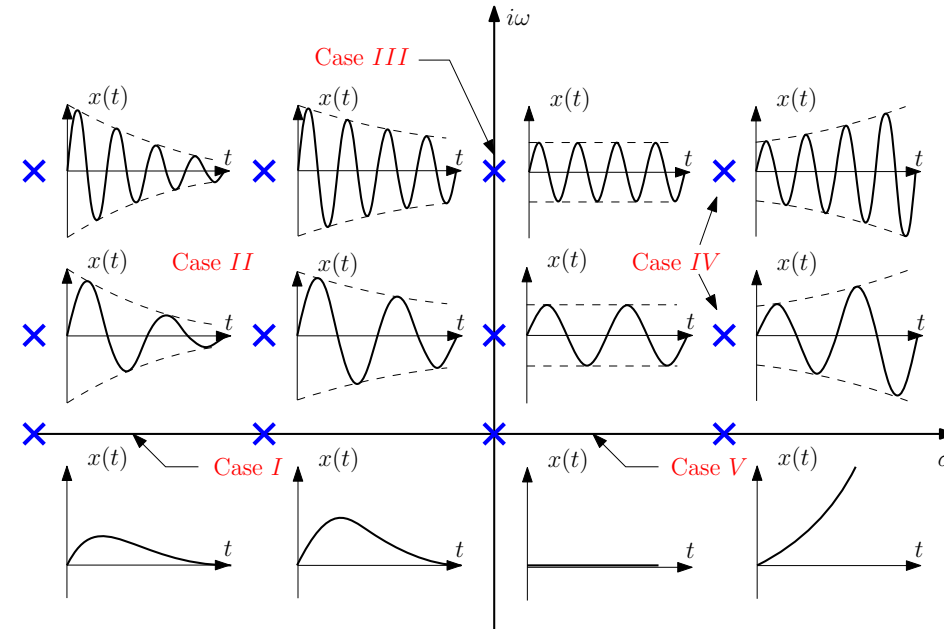
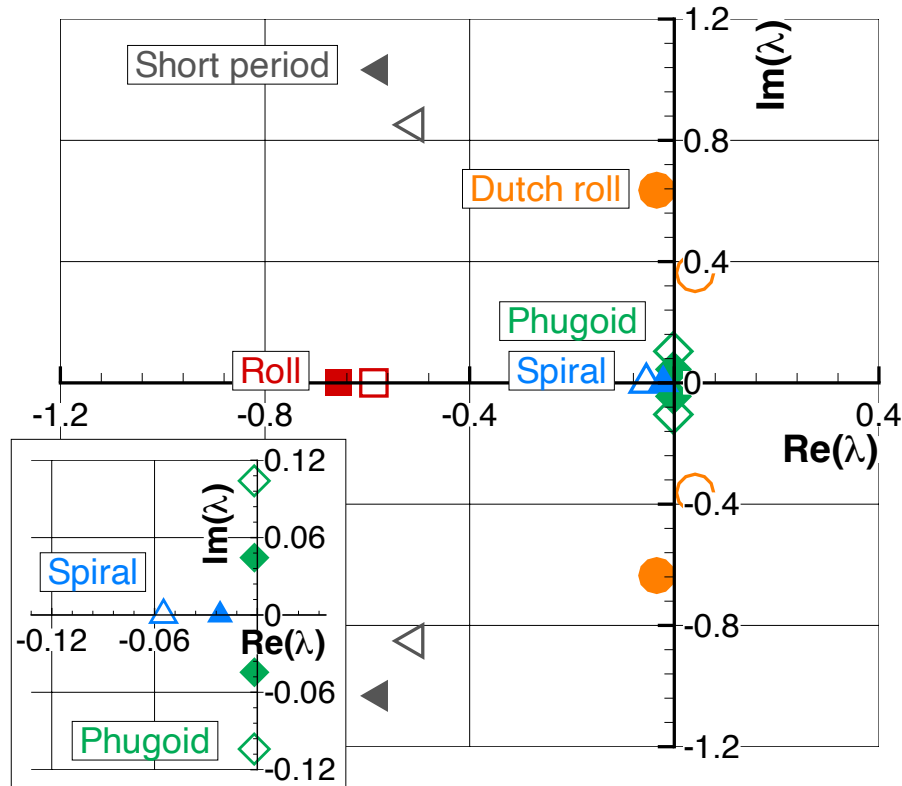
Spiral mode

1st order time response



Full scale DLR-F12

Subsonic case - Flight Stability



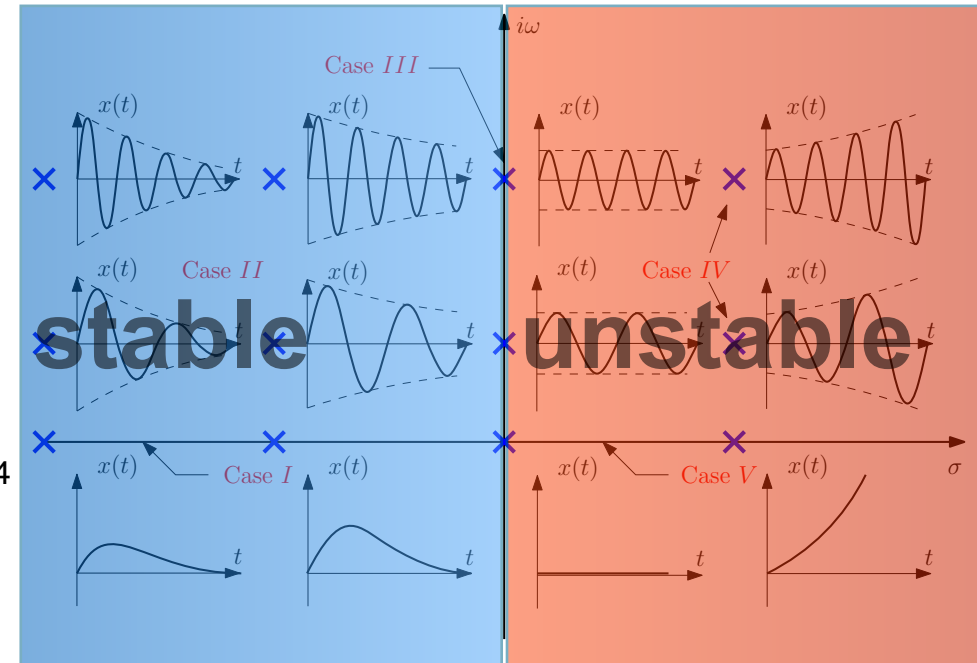
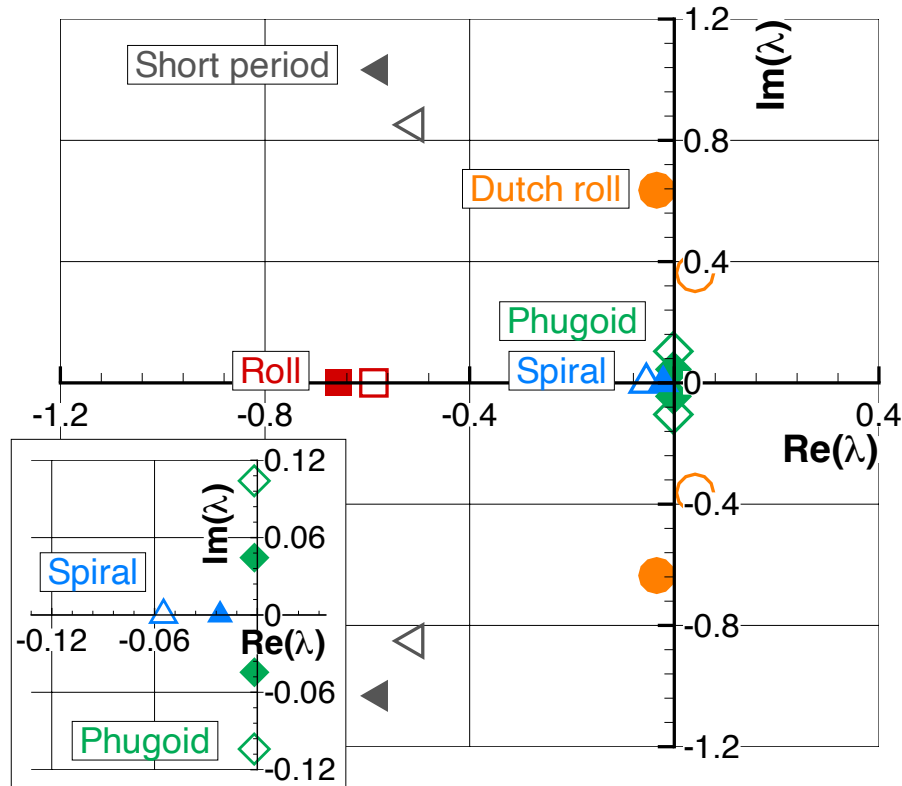
CASE IV
1st order response
transient

CASE III
Undamped Oscillation

CASE II/IV
2nd order response
oscillation

Full scale DLR-F12

Subsonic case - Flight Stability



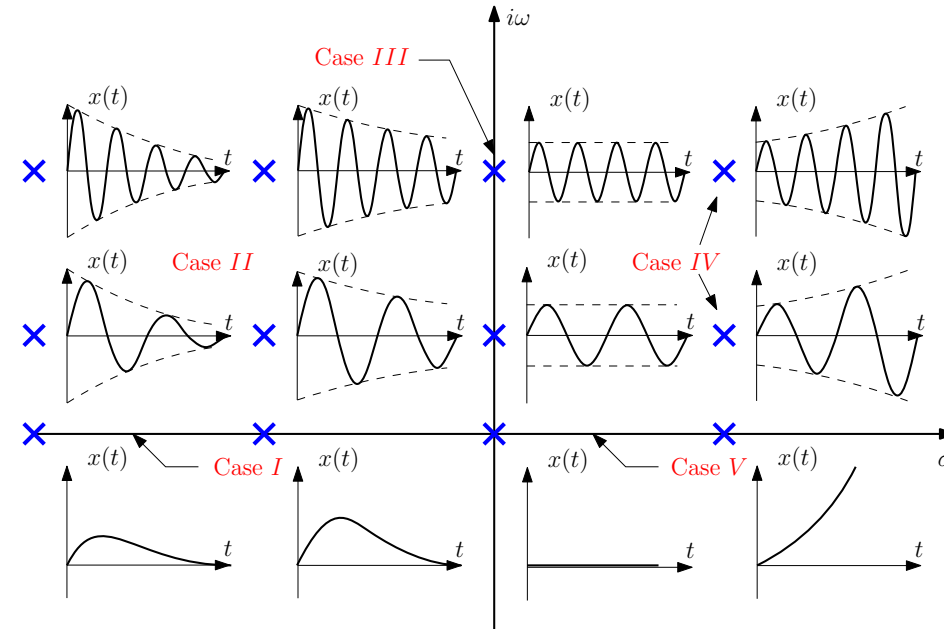
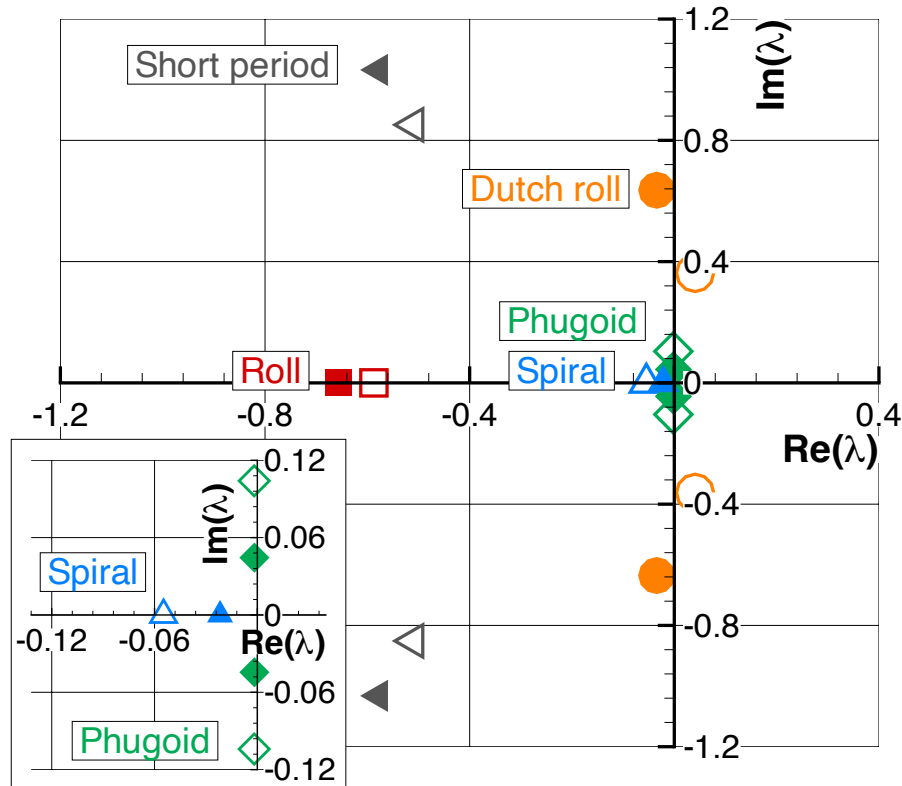
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Full scale DLR-F12

Subsonic case - Flight Stability



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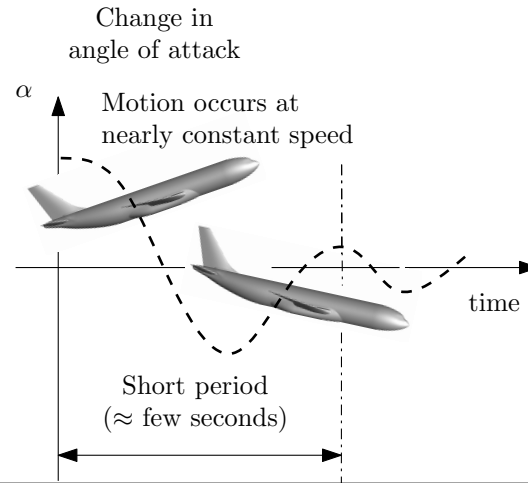
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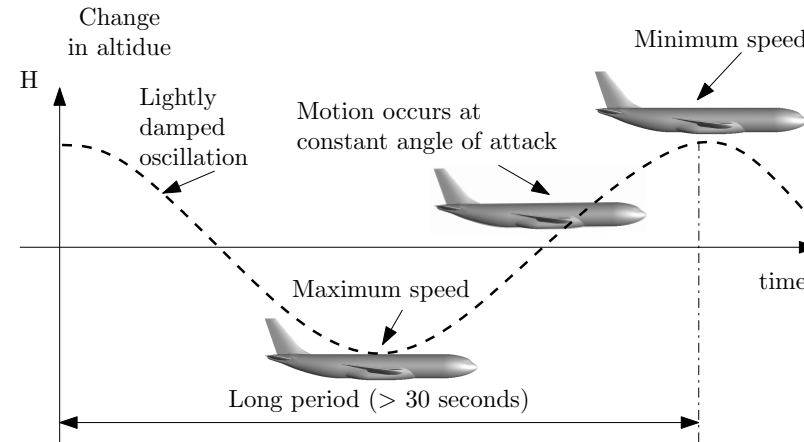
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Longitudinal

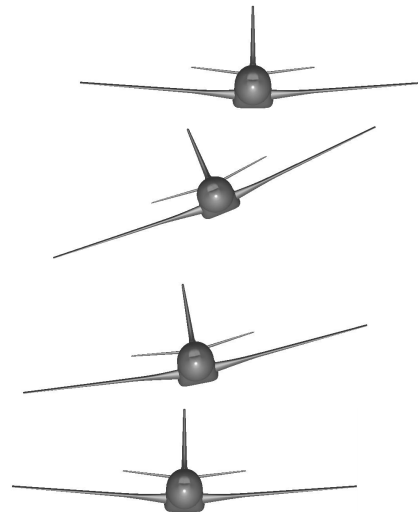
Short period mode



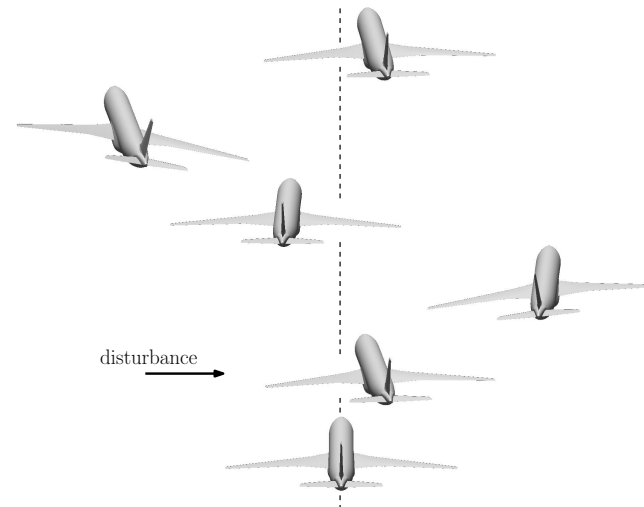
Long period mode (Phugoid)



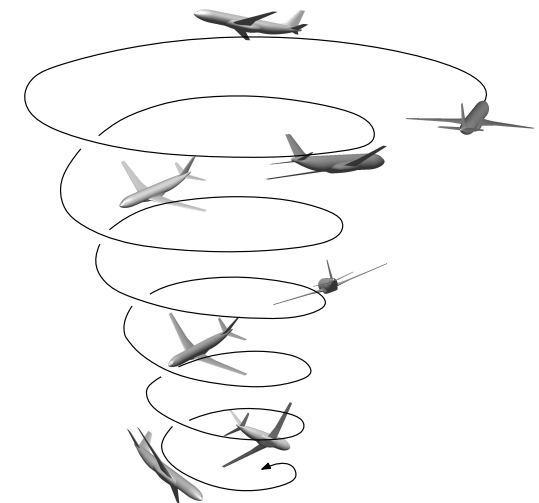
Roll mode



Dutch roll mode



Spiral



Lateral/ Directional

Full scale DLR-F12

Subsonic case - Flight Stability

$M = 0.206$, $Re = 1.28 \text{ m.}$, $\alpha = 0^\circ$, $\beta = 0^\circ$

LFD - 2nd order time response

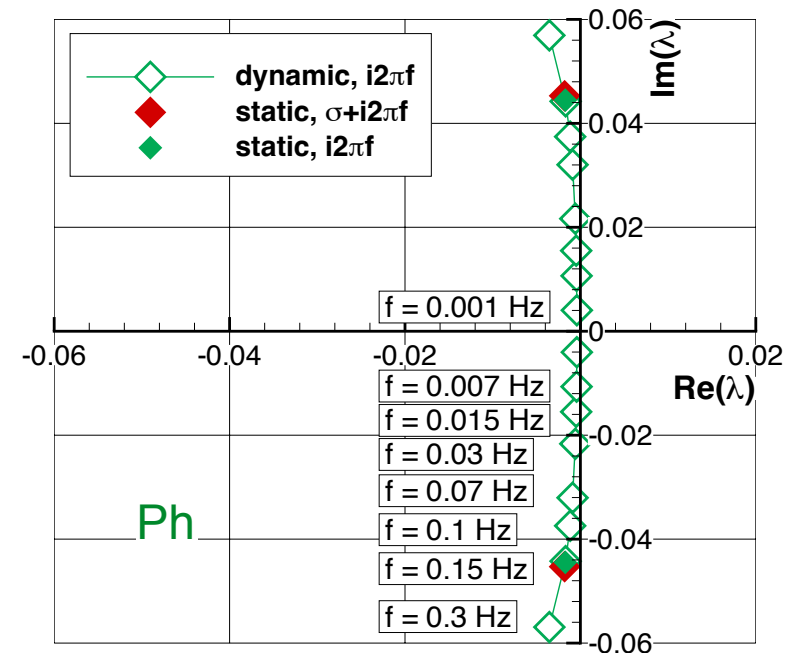
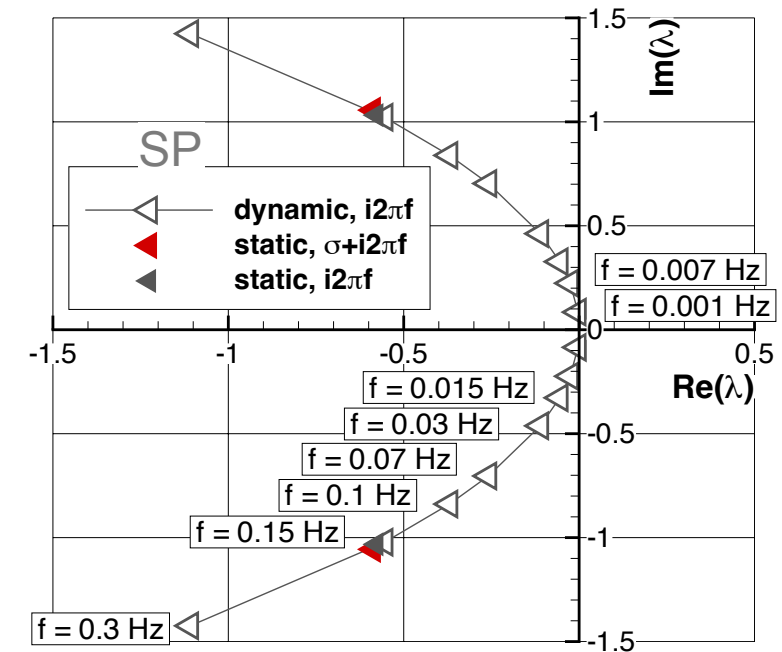
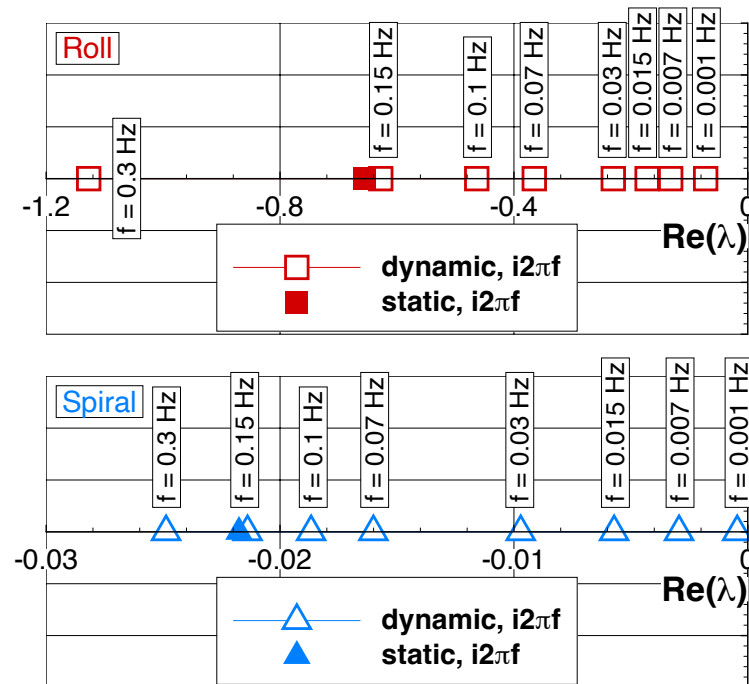
Short period mode

Phugoid

Dutch roll mode

1st step: $\lambda = i\omega$, harmonic

iterative: $\lambda = \sigma \pm i\omega$, damped harmonic



Full scale DLR-F12

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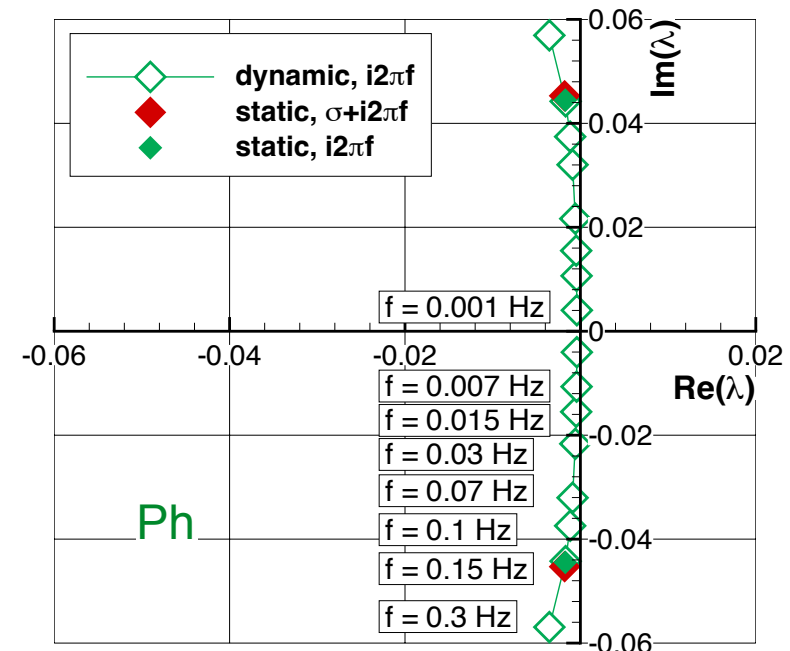
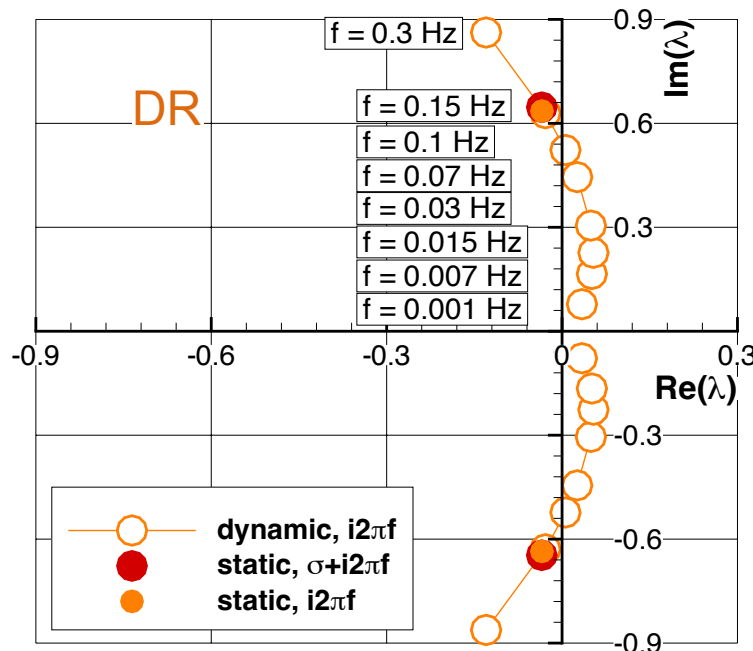
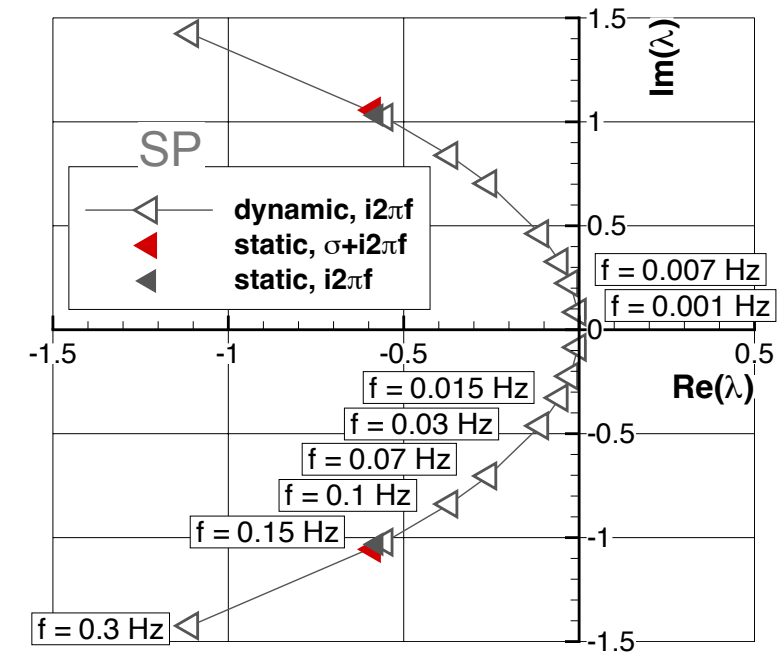
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Full scale DLR-F12

Subsonic case



LFD investigation - Angle of attack variation

$M = 0.206$, $Re = 1.28 \text{ m.}$, $\beta = 0^\circ$

Phugoid: C_{Xu} , C_{Zu} , and C_g

Dutch roll mode: C_{Yv} , C_{Yr} , C_{lv} , C_{lr} , C_{nv} and C_{nr} ($v - \beta$)

Comparison between wind and body-fixed Deviation from

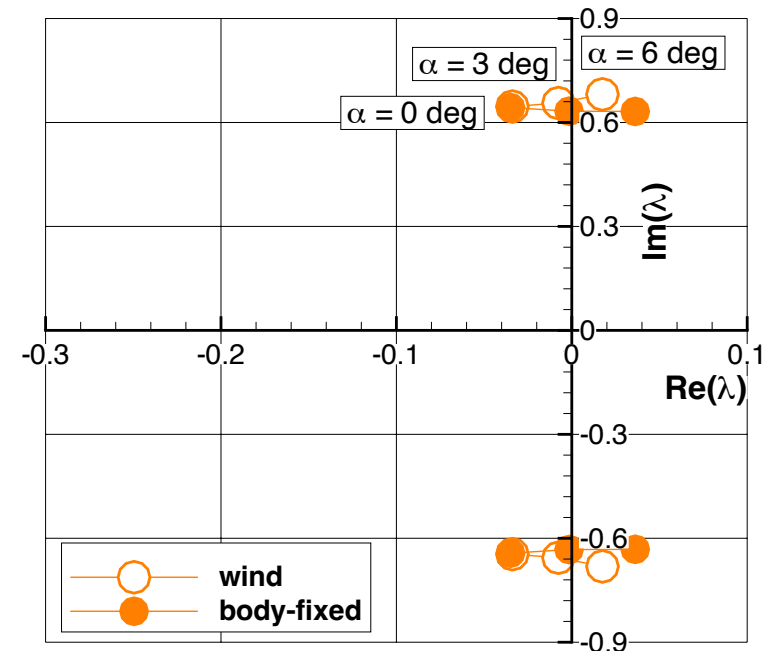
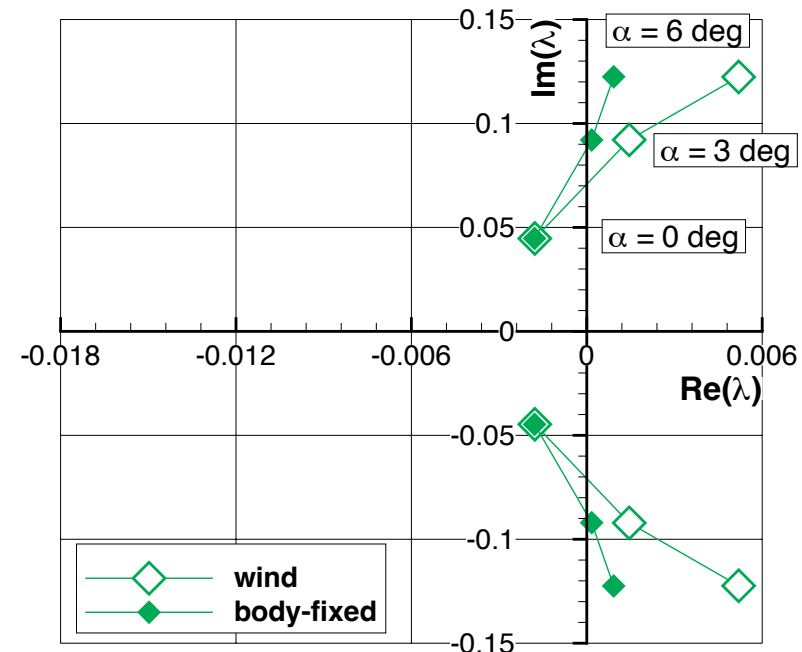
Oscillation frequency almost inherent
Wind

PH, **DR**: Damping deviates

Body-fixed

PH: Accurate C_{Xu} and C_{Zu}

DR: Coupling of roll-yaw



Full scale DLR-F12

Subsonic case



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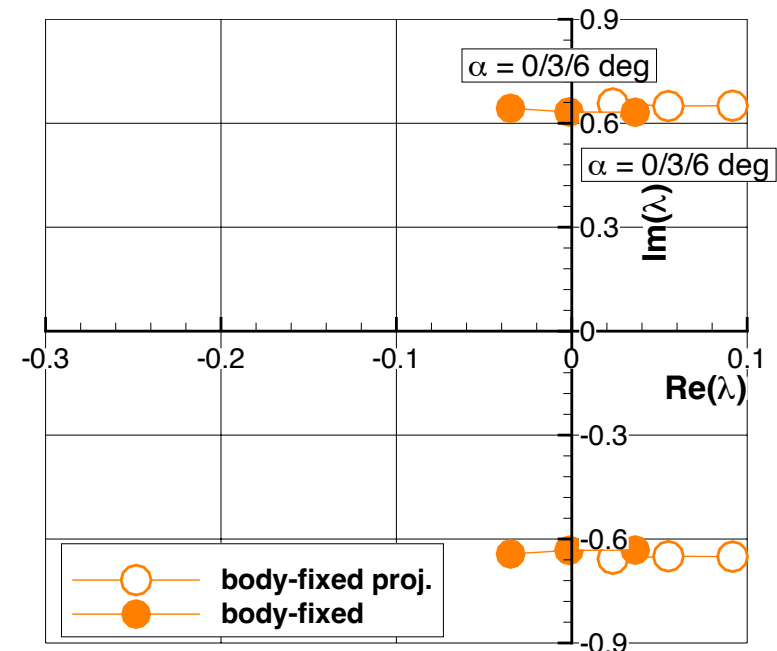
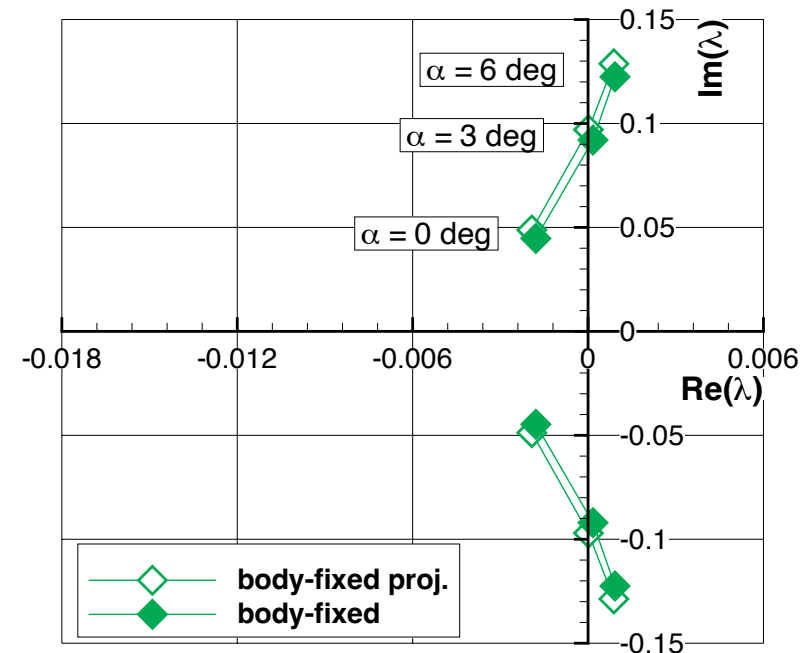
Comparison between body-fixed projection and body-fixed

Scalar treatment - Pendulum Approach

PH: Similar results

DR: Projection - C_{lv} and C_{nr} are similar to values of VLM and thus smaller in magnitude

Projection shifts EV entirely into unstable region



Time reduction - full scale DLR-F12 - transonic



Time reduction - full scale DLR-F12 - transonic

Transonic parameter space

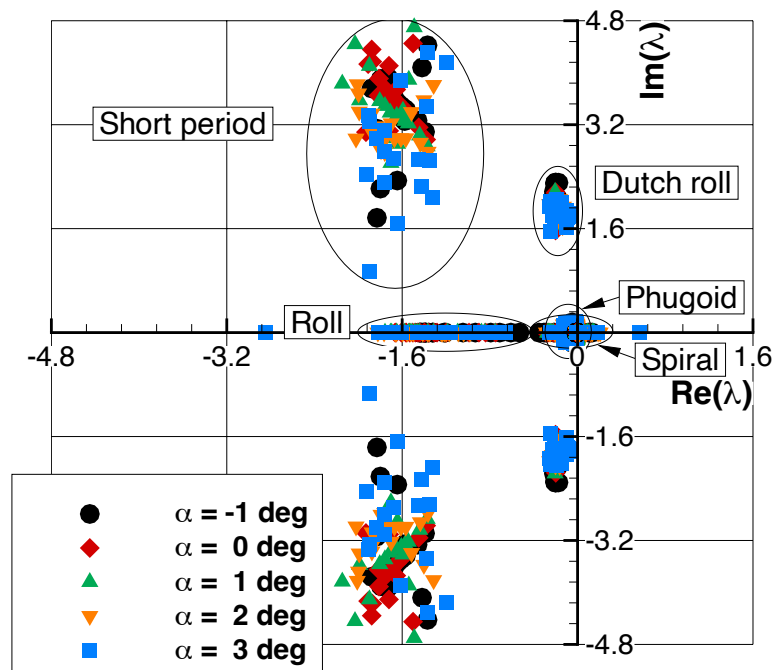
19 Mach numbers: 0.7-0.96

5 angles of attack: -1,0,1,2,3 deg

8 frequencies

6 motions (trans+rot)

4845 LFD simulations



Time reduction - full scale DLR-F12 - transonic

Transonic parameter space

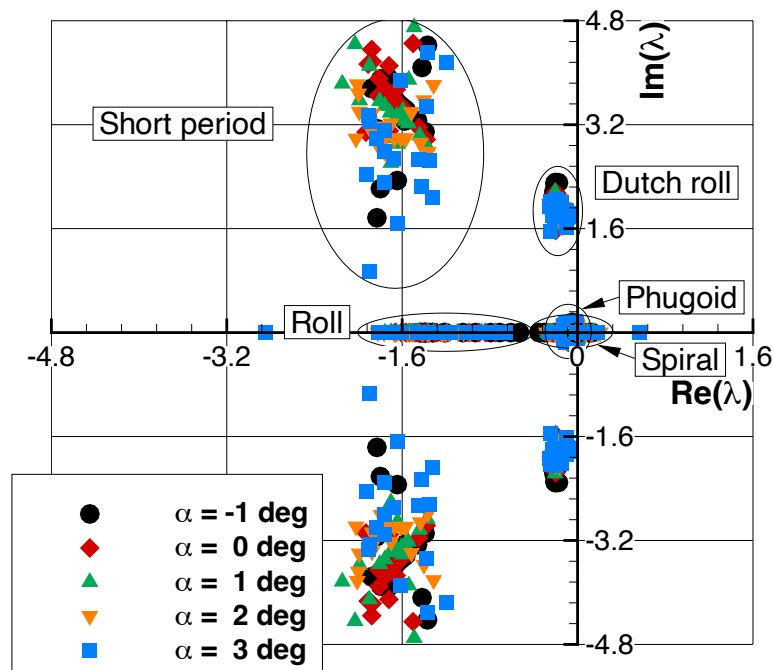
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4845 LFD simulations



Method	CPU	# sim.	t/sim. [hours]	t_{tot} [days]
RANS	72	19 x 5	12.1	48
LFD	72 (120)	19 x 5 x 48 ¹	1.67	≈ 318 (191)
URANS (estimated)				
$M_\infty = 0.70 - 0.85$	72	8 x 5 x 48	$\approx 77^2$	6160
$M_\infty = 0.86 - 0.90$	72	5 x 5 x 48	$\approx 96^3$	4813
$M_\infty = 0.91 - 0.96$	72	6 x 5 x 48	$\approx 116^4$	6960
$\sum$ URANS				17933
ζ_{CPU}				≈ 56

¹ 19 Mach numbers, 5 angles of attack, 48: 8 frequencies x 6 motions

² 300 pseudo time iter., 3W, 80 steps/period, 4 periods

³ 1000 pseudo time iter., sg, 80 steps/period, 5 periods

⁴ 300 pseudo time iter., 3W, 80 steps/period, 6 periods

RANS simulations are neglected URANS/LFD

All LFD simulations converged

URANS simulations are estimated

Time reduction URANS / LFD: **about 60**

Effective time:

RANS/LFD about 1 year

RANS/URANS about 60 years

Conclusion



Linear Frequency Domain method (LFD) - Small perturbation approach

LFD provides a **speed-up** of about **2 orders of magnitude** with the accuracy of URANS for small perturbations of motions

RANS (viscosity) properties remain

Good agreement of LFD results in comparison with URANS for harmonic oscillations

Consistent linearisation of the Jacobians

Linearization of **turbulence model** is a **key feature**

Robust method, almost inherent of excitation frequency

Preconditioner and **Krylov-GMRes** dramatically **increase** the **robustness** compared with multigriding

Direct solver: **Trade off between robustness and memory usage**

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Viscosity/Turbulence

Game changer whenever **separation** appears

Conclusion



Conclusion



Body-fixed versus aerodynamic frame of reference

Body-fixed is necessary for **arbitrary oriented aircraft**

Critical aircraft modes - Phugoid, Spiral and Dutch roll mode

Scalar versus vector-valued treatment

Critical aircraft modes - Dutch roll mode and Spiral

Scalar - similar to VLM results

Plain input of LFD results into state matrix - one motion/one column

Straightforward extension to three dimensional stability state matrix

Drawback: Identification of typical aircraft modes challenging

Retaining with vector based notation

Note: All six degree of freedom need to be evaluated

Conclusion



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Description in principal axes

Most accurate - Lagrangian equation of motion

Outlook



High Resolution of flight vehicle (LFD vs VLM/DLM)

No more correction factors for body/any mountings/high-lift devices aso

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Damped harmonic oscillator - LFD

Problem reduction of complex aerodynamic system important for many unsteady applications

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Problem reduction of complex aerodynamic system important for many unsteady applications

Further applications of small disturbance RANS-based LFD method

Control circuits - Frequency response functions/modulation

Control surfaces - ailerons, rudder, elevator,

Air load alleviation - Active flow control - Actuation jet velocity

Optimisation including Stability & Control

MDO - air loads (LFD - ROM) - Uncertainty quantification



The end result of a well executed spiral dive

Topic: Flight Dynamic Stability Prediction for an Aircraft in Transonic Separated Flow Conditions with a Linear Frequency Domain Solver

Date: 1. December 2023

Author: Markus Widhalm

Institute: Institute of Aerodynamics and Flow Technology

Images: DLR

[DLR.de/as/en](https://www.dlr.de/as/en)