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Inverse Level-set Problems for Capturing Calving Fronts

Introduction

Ice sheet models are an important part of the scientific effort to understand Earth system dynamics. How to best represent calving (the process of iceberg detachment at the front of glaciers) is an active area of research. Physical models so far do not convincingly reproduce the real movement and position of the front. Novel data processing methods provide observations (see Fig. 1) for data driven approaches. In the TerraByte-DNN2Sim project [7], we aim to compute calving rates (and later parameters of calving laws) from known front positions by solving inverse problems of the Level-set Method.

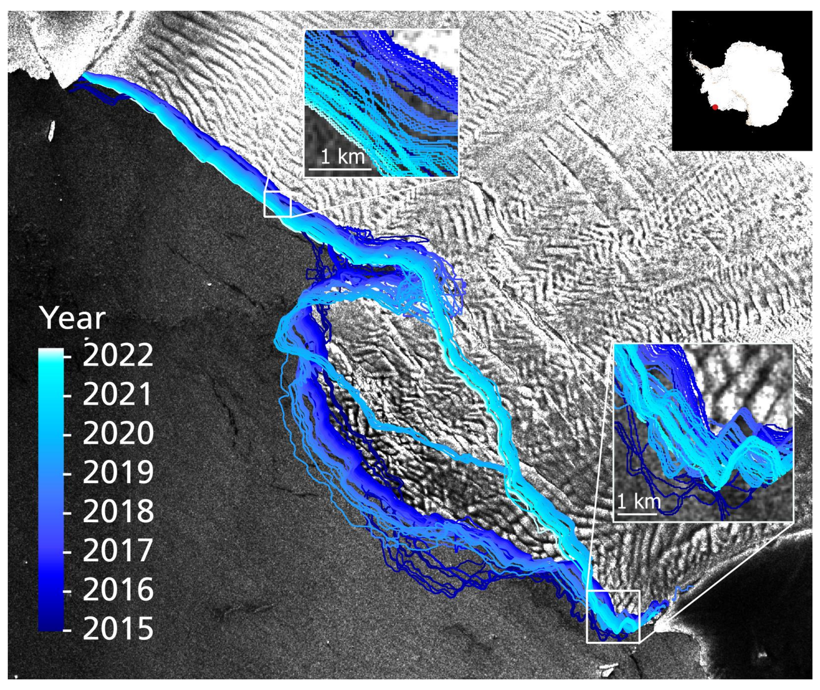

Fig. 1: The IceLines [1] data set (here: DeVicq Glacier, Getz Ice Shelf, Antarctica) provides monthly calving front positions.

Level-set Method

The front is represented by an implicit function φ with

$\varphi \leq 0$ inside the ice
$\varphi > 0$ outside the ice
$\varphi = 0$ at the interface

A common choice for φ is a signed distance function (SDF, Fig. 2).

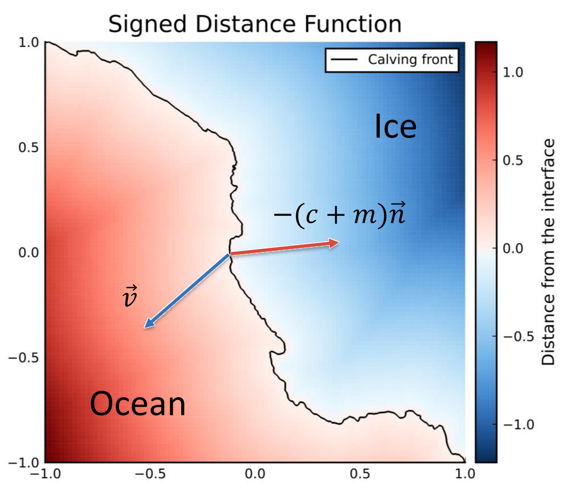

Fig. 2: Signed distance function and advection coefficients.

The 2D field (and the interface) is evolved by the Level-set (LS) equation

$$\dot{\varphi} + \big(\vec{v} - \vec{n}(m + c)\big)\nabla\varphi = 0$$

with $\vec{v} \coloneqq \vec{v}(x,t)$ the horizontal velocity of the ice and $m \coloneqq m(x,t)$ and $c \coloneqq c(x,t)$ the melt- and calving rates in direction of the normal of the front $\vec{n}$.

Optimization

For given $\vec{v}$ and m find

$$\min_c \arg \int_T \|\varphi - \tilde{\varphi}\|^2 + \lambda \cdot r(\varphi, c)\, dt$$

with observations $\tilde{\varphi}$ and Tikhonov regularization term r. The shape of the calving rate function is not known with certainty. We want to assess different continuous and discontinuous shapes using different regularization methods.

E.g., limiting the gradient of the calving rate gives a continuous calving rate as in [4]:

$$r(\varphi, c) = \|\nabla c\|^2$$

Discontinuous calving rates, e.g., piecewise constant, require special methods like [6].

We only have data for $\tilde{\varphi}$ at the front, so φ and $\tilde{\varphi}$ are set to signed distance functions when evaluating the objective function.

Because of this, information from the LS solver is lost and there are different calving rate functions that produce the same front.

Implementation

The LS equation is solved by a Discontinuous Galerkin method implemented using the Trixi.jl framework [2] with a homogeneous rectangular mesh and element basis functions of degree 1. Time discretization is a 4th order explicit Runge-Kutta method with adaptive step size.

The optimization problem is solved using the LBGFS of Optim.jl [3]. The gradient of the objective function is evaluated by solving the adjoint of the LS equation with the same method as the LS equation itself.

In the gradient, we currently ignore the discrepancy between the signed distance function used in the objective and the actual result of the solver, so the gradient is only approximately correct.

Numerical Experiment

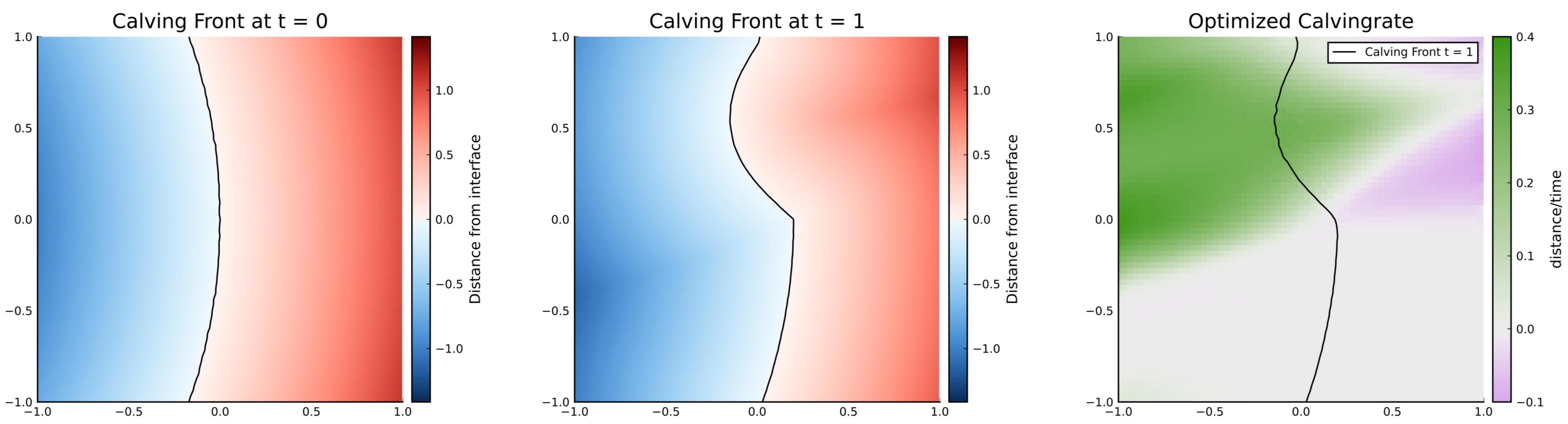

Fig. 3: Experimental Setup and result: Initial and final calving front and calving rate result from the optimization with regularization parameter $\lambda = 0.2$. The final front is only approximately reproduced by the optimal calving rate.

So far, we can solve a simplified test problem (see Fig 3)

- Constant velocity field $\vec{v} = (0.2, 0.0)^\top$ and melting $m = 0.0$
- Known front positions at $t = 0.0$ and $t = 1.0$
- Grid of $2^6 \times 2^6 = 4096$ finite elements, 4225 vertices
- Target continuous constant calving rate with one parameter per grid vertex

The optimization is not sufficiently exact yet, but the regularization ensures a continuous result.

Challenges and Outlook

- Improve the optimization by using automatic differentiation
- Limit the number of "useless" parameters that have no influence on the front position
- Different regularization methods for discontinuous calving rate functions (Fig. 4)
- Coupling the Level-set problem to an ice sheet model for feedback with the momentum balance
- Improve performance by using adaptive meshes

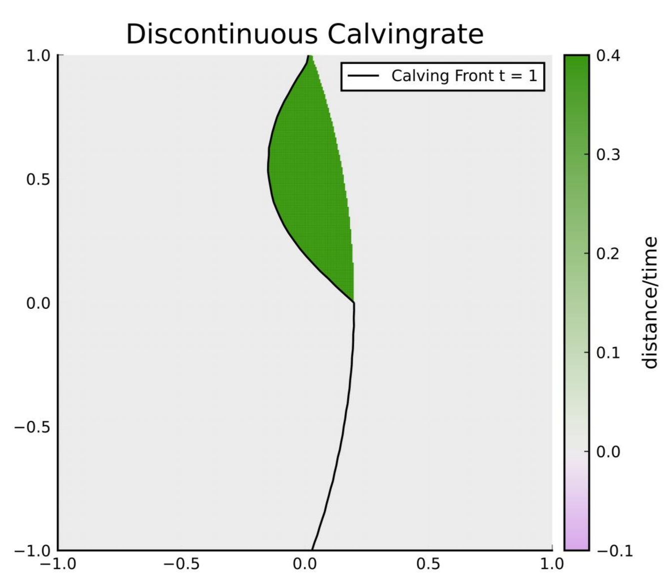

Fig. 4: One possible discontinuous calving rate distribution for the same setup as in Fig. 2

References:
1. Baumhoer, C.A., Dietz, A.J., Heidler, K. and Kuenzer, C. (2023) IceLines – A new data set of Antarctic ice shelf front positions. *Scientific Data*, **10**, 138, 10.1038/s41597-023-02045-x.
2. Schlottke-Lakemper, M., Gassner, G.J., Ranocha, H., Winters, A.R., Chan, J. (2021): Trixi.jl: Adaptive high-order numerical simulations of hyperbolic PDEs in Julia. 10.5281/zenodo.3996439
3. Mogensen, P.K., Riseth, A.N. (2018): Optim: A mathematical optimization package for Julia. JOSS, **3**, 10.21105/joss.00615
4. Morlighem, M., Bondzio, J., Seroussi, H., Rignot, E., Larour, E., Humbert, A., and Rebuffi, S.-A. (2016) Modeling of Store Gletscher's calving dynamics, West Greenland, in response to ocean thermal forcing, *Geophys. Res. Lett.*, **43**, 2659–2666, 10.1002/2016GL067695.
5. Chourdakis, G., Davis, K., Rodenberg, B., et al. (2022) preCICE v2: A sustainable and user-friendly coupling library [version 2; peer review: 2 approved]. *Open Res Europe 2022*, **2**:51, 10.12688/openreseurope.14445.2.
6. Egger, H. and Leitao, A. (2009) Nonlinear regularization methods for ill-posed problems with piecewise constant or strongly varying solutions. *Inverse Problems,* **25**, 10.1088/0266-5611/25/11/115014.
7. Project TerraByte-DNN2Sim, https://dnn2sim.de/

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